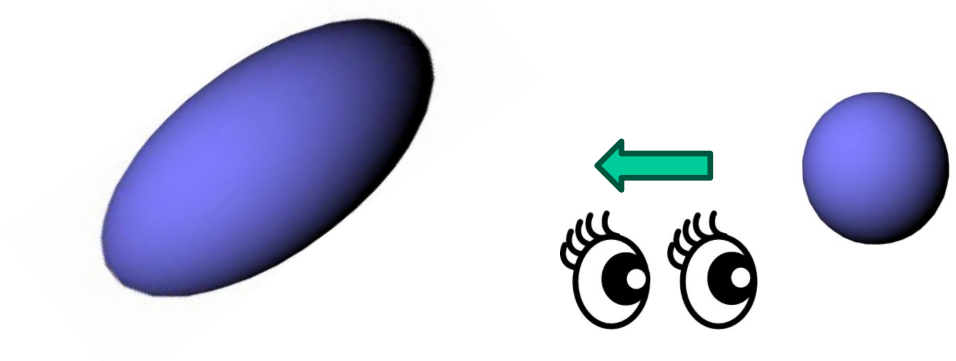
Taking a snapshot of a nucleus

Kouichi Hagino
Kyoto University, Kyoto, Japan



1. Introduction
2. Low-energy Nuclear Reactions: overview
3. Role of deformation in sub-barrier fusion reactions
4. A short comment on relativistic heavy-ion collisions
5. Summary

84th OMEG-SSANP workshop, Soongsil University, Seoul, 2025.4.18-20

Taking a snapshot of a nucleus

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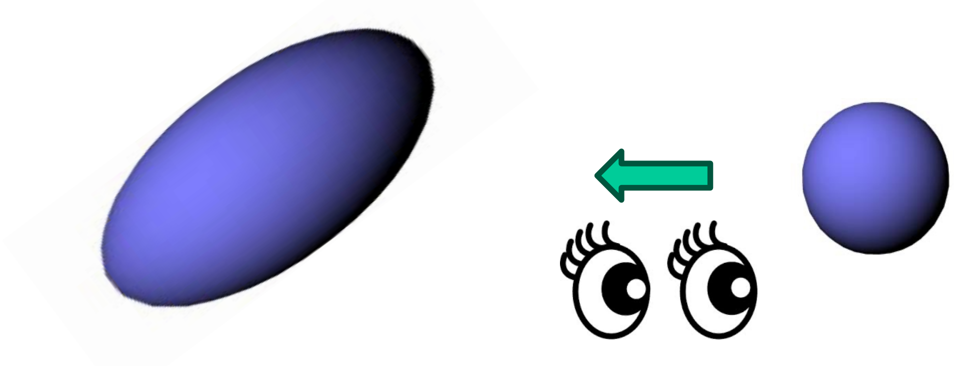
a *slow* mode

a *fast* mode



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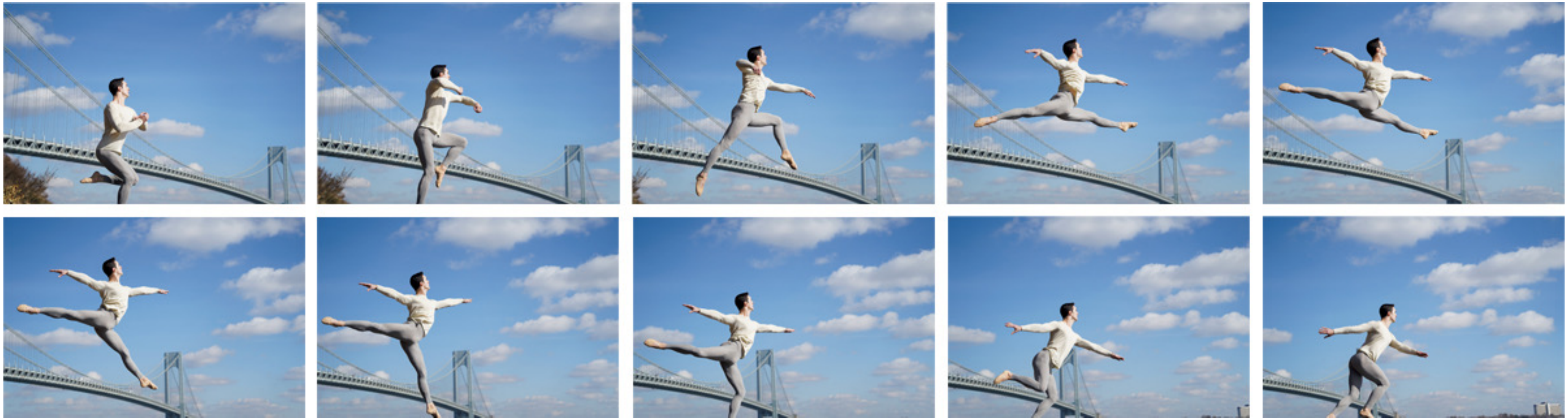
Snapshots

taking snapshots of a "slow" motion with a high-speed camera

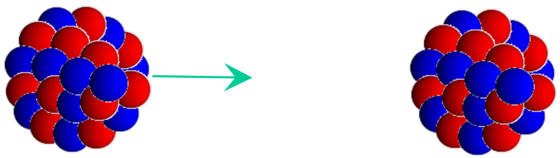
$$\tau_{\rm camera} \ll \tau_{\rm motion}$$



https://www.sony.jp/ichigan/products/ILCE-7M3/feature_3.html

(photos with a Sony camera α7III)

→ taking snapshots of a nucleus with a "fast" nuclear reaction

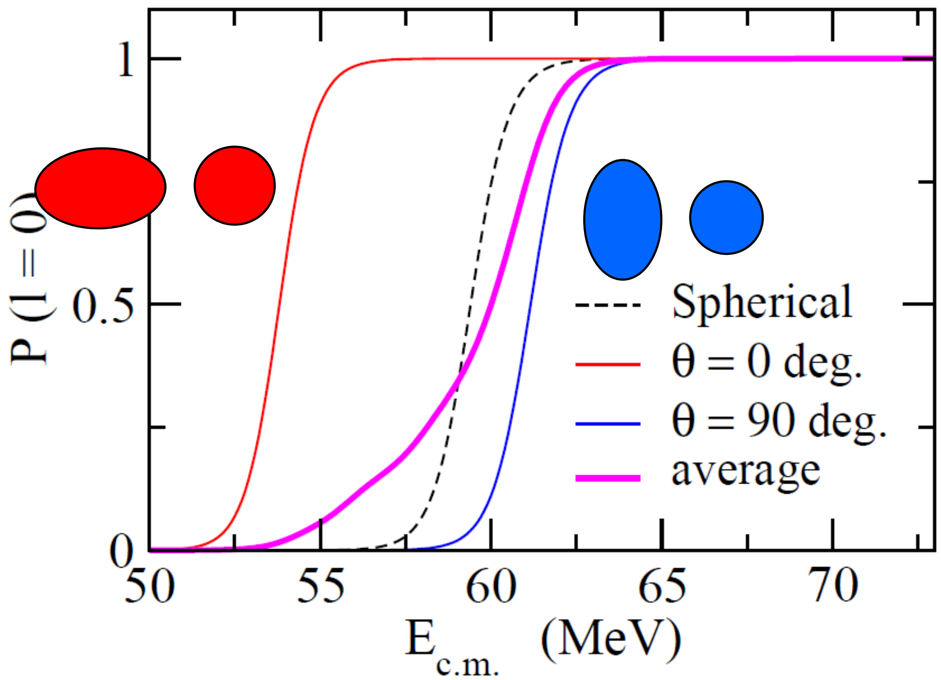


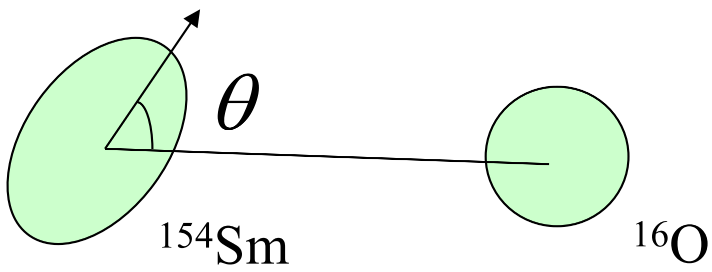
$$\tau_{\rm reaction} \ll \tau_{\rm nucleus}$$

Snapshots

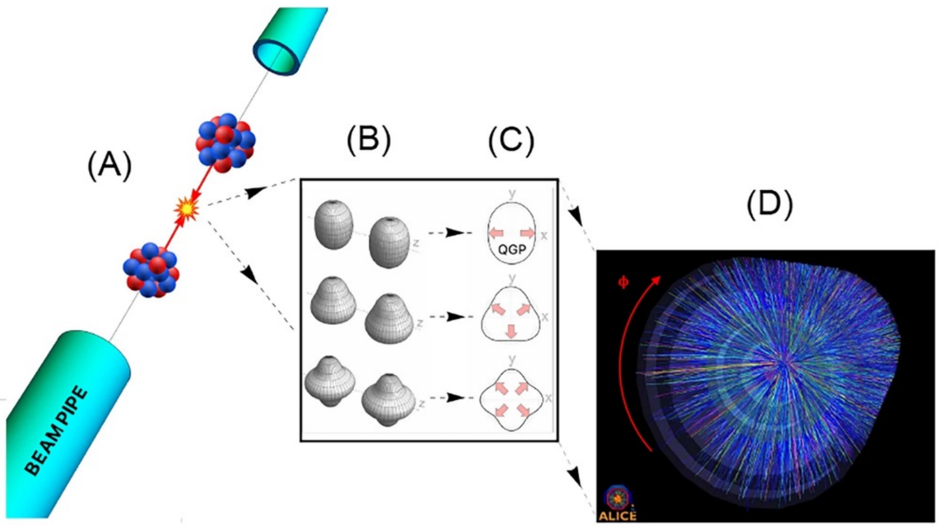
taking a snapshot of a nucleus with a "fast" nuclear reaction

low-energy H.I. fusion reactions of a deformed nucleus





relativistic H.I. collisions with a deformed nucleus



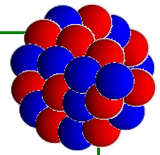
J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

increasing interests
in recent years

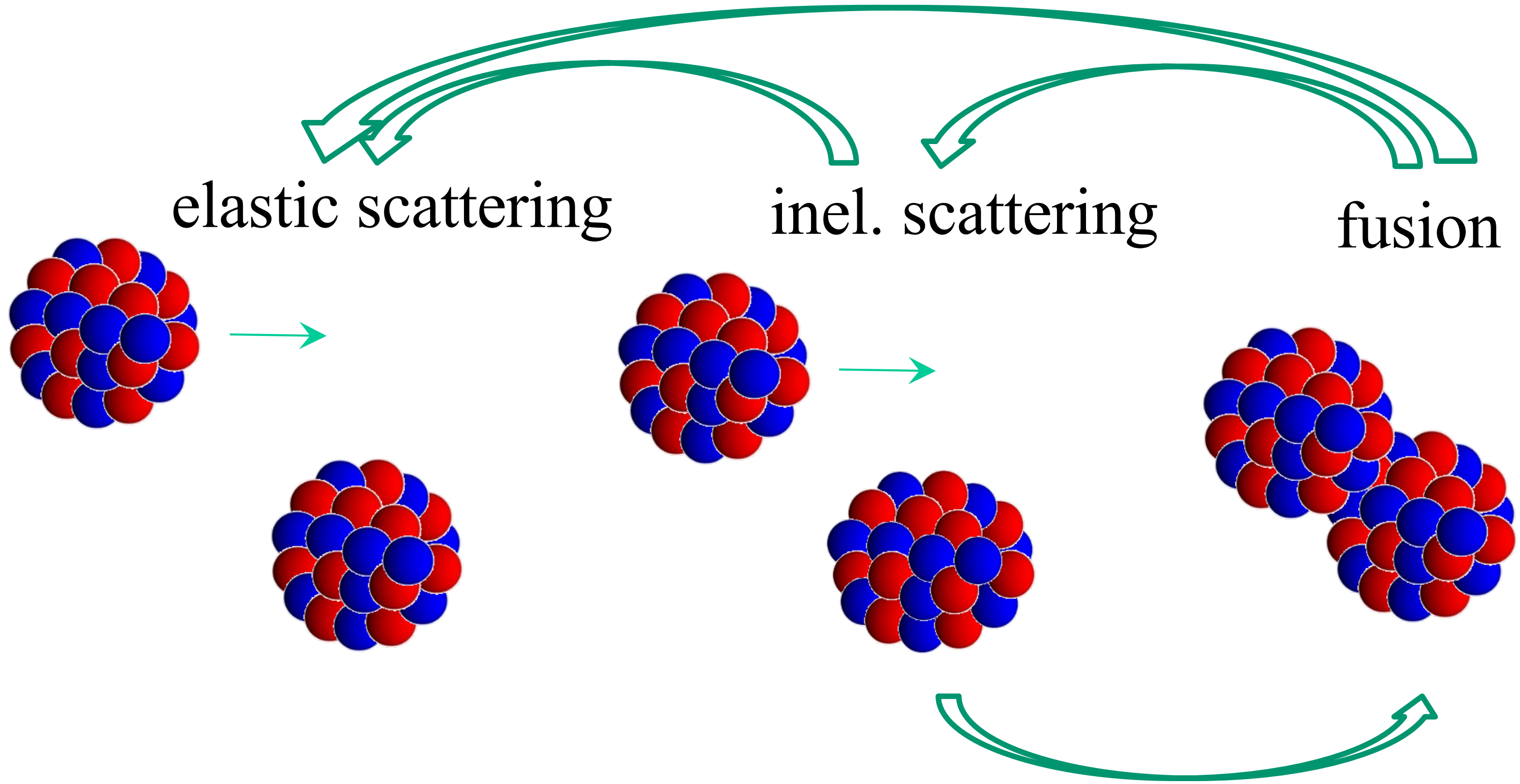
Introduction: low-energy nuclear reactions

nucleus: a composite system

✓ various sort of reactions

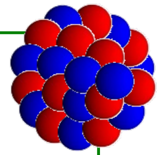


- elastic scattering
- inelastic scattering
- transfer reactions
- breakup reactions
- fusion reactions



Introduction: low-energy nuclear reactions

nucleus: a composite system

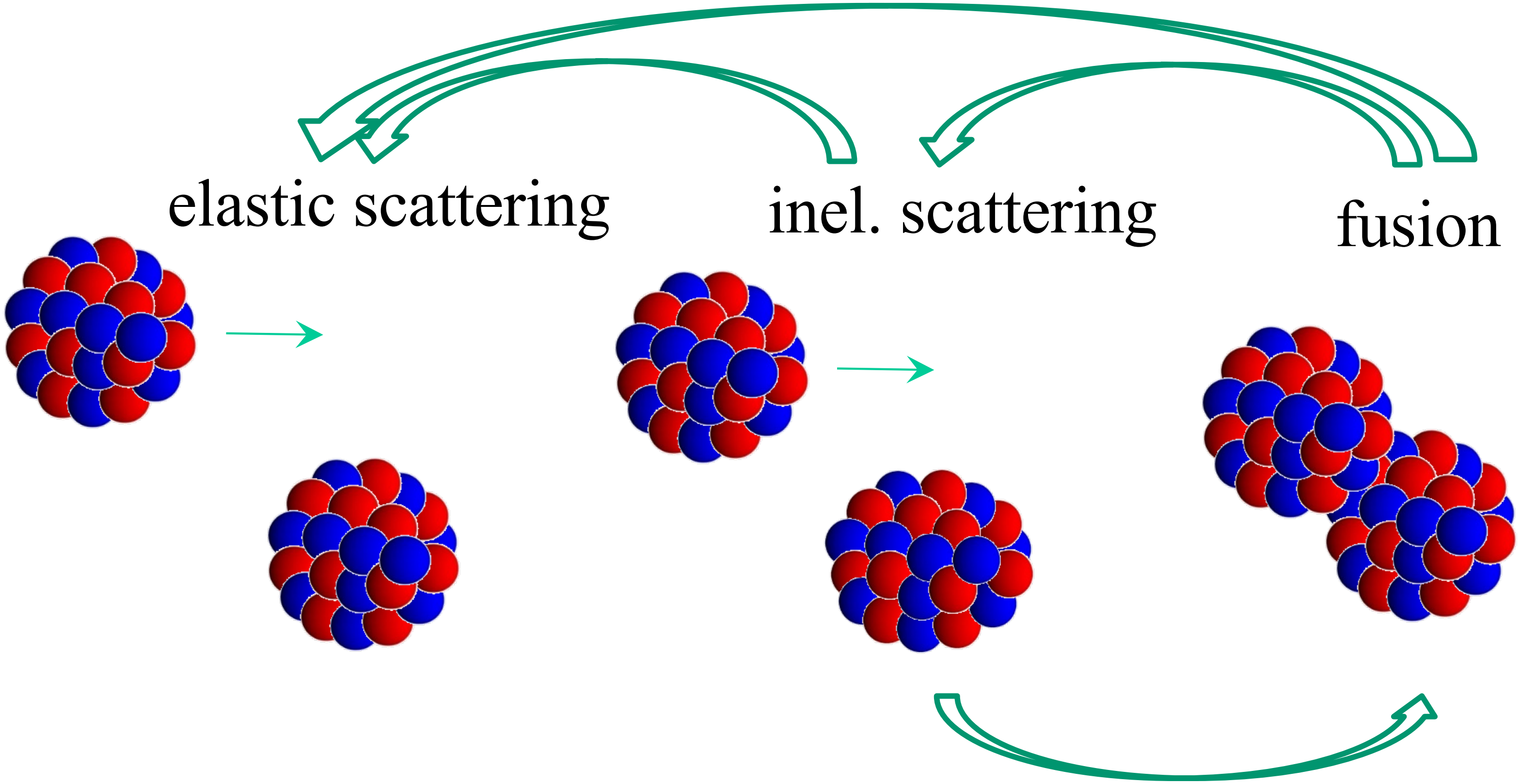


✓ various sort of reactions

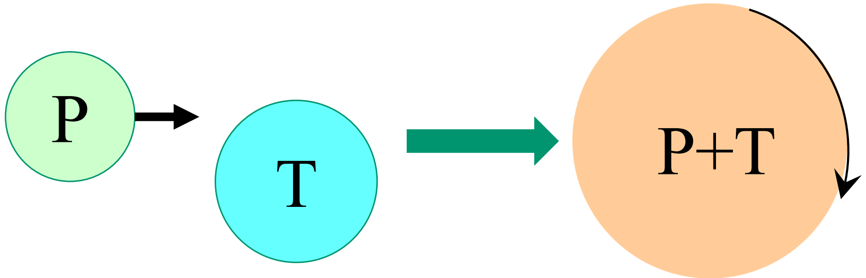
✓ an interplay between nuclear structure and reaction

shapes, excitations, …..

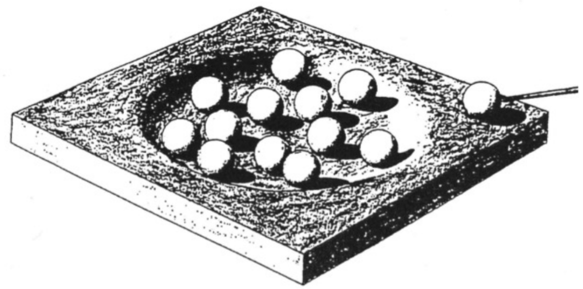
- elastic scattering
- inelastic scattering
- transfer reactions
- breakup reactions
- fusion reactions



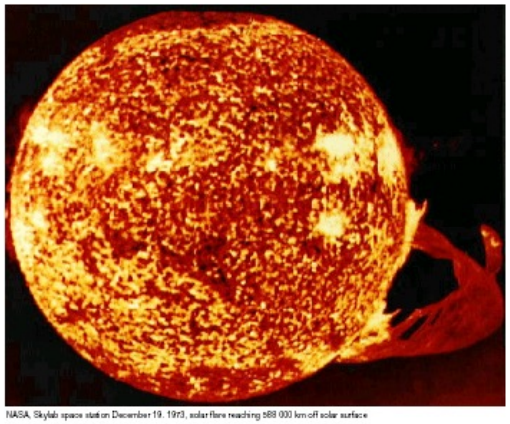
Fusion reactions: compound nucleus formation



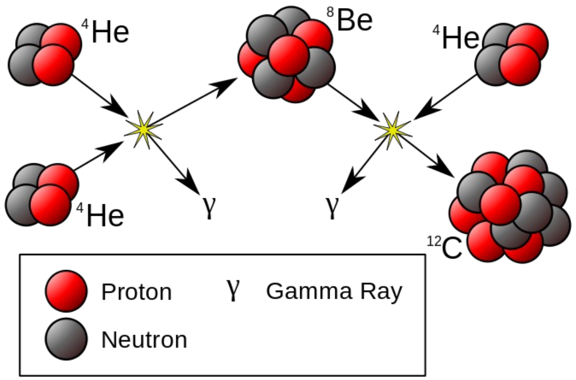
compound
nucleus



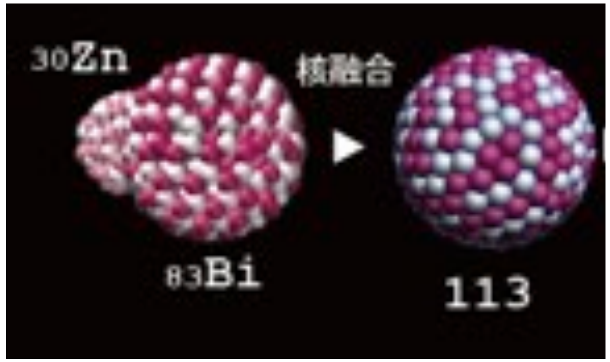
cf. Bohr '36



energy production
in stars (Bethe '39)



nucleosynthesis

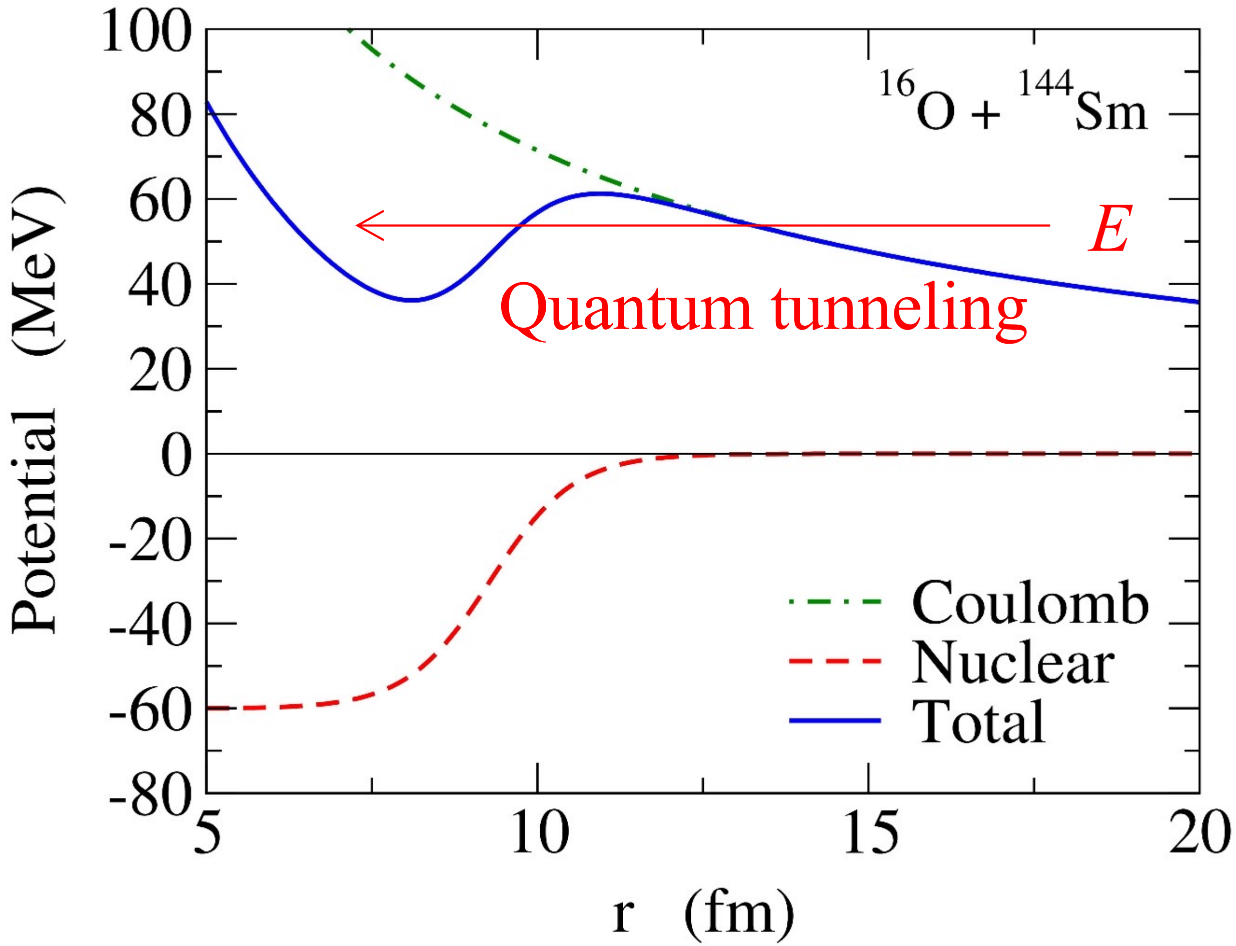


superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems with strong interaction

⟵ microscopic understanding: an ultimate goal of nuclear physics

Coulomb barrier



1. Coulomb interaction
   long range, repulsion
2. Nuclear interaction
   short range, attraction

↓

Potential barrier
(Coulomb barrier)

Fusion: takes place by overcoming the barrier

the barrier height → defines the energy scale of a system

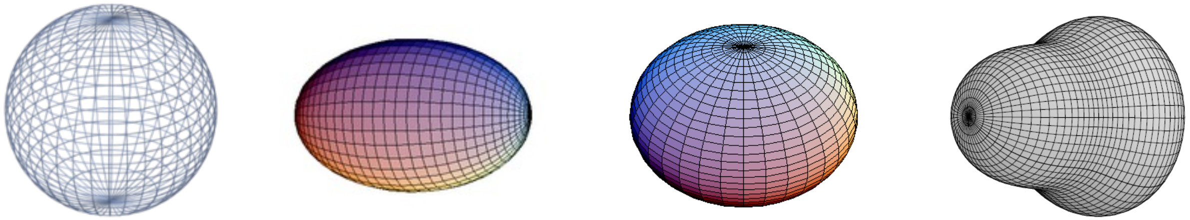
Fusion reactions at energies around the Coulomb barrier

Low-energy heavy-ion fusion reactions and quantum tunneling

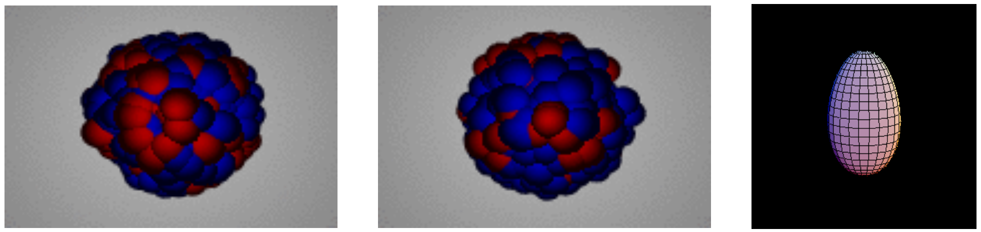
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes



- several surface vibrations



several modes and adiabaticities

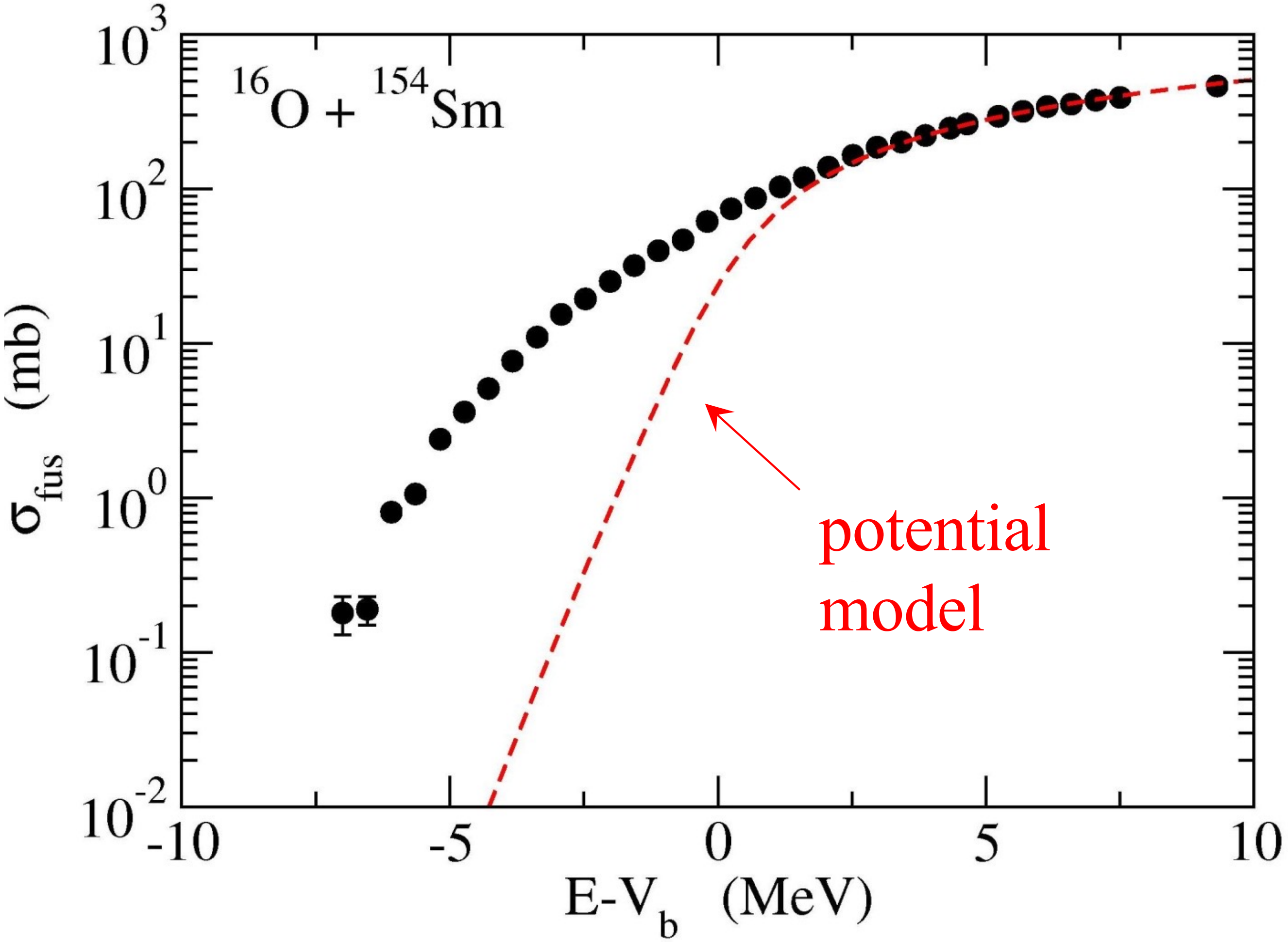
- several types of nucleon transfers

Tunneling probabilities: the exponential E dependence
→ nuclear structure effects are amplified

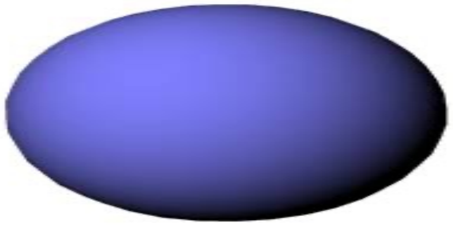
Discovery of large sub-barrier enhancement of σ_{fus} (~80's)

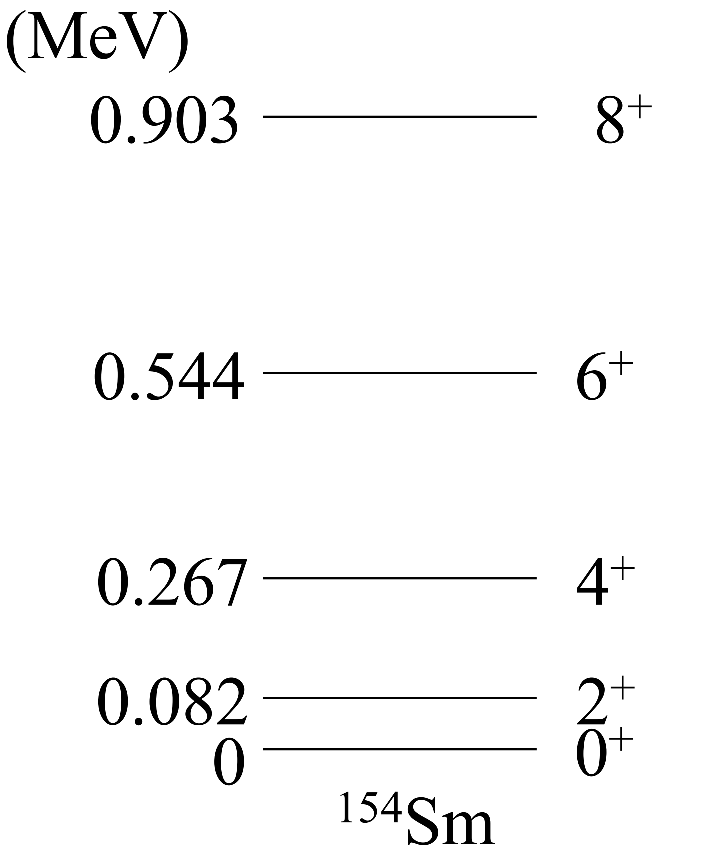
the potential model: inert nuclei (no structure)

$$\sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$

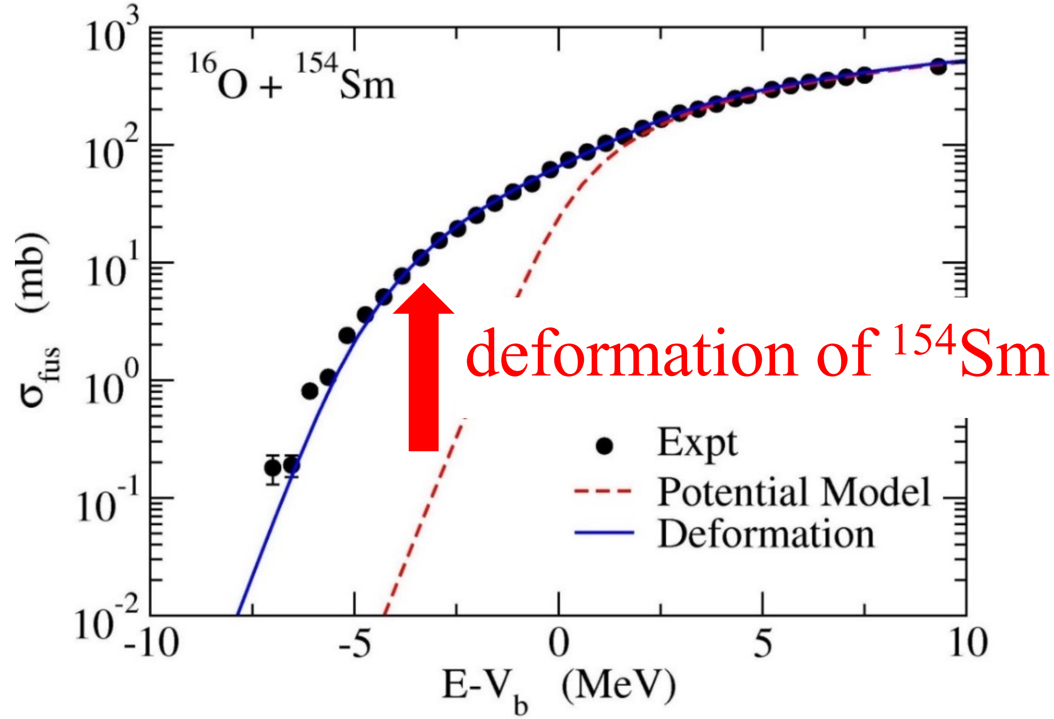


^{154}Sm : a typical deformed nucleus



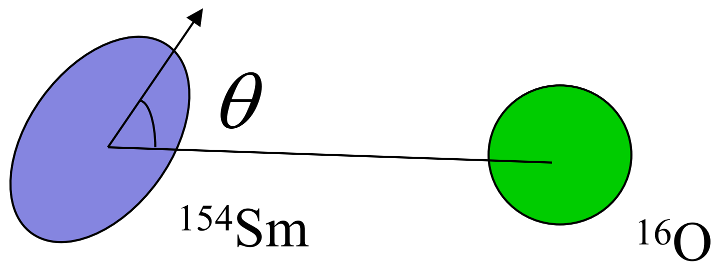


rotational spectrum

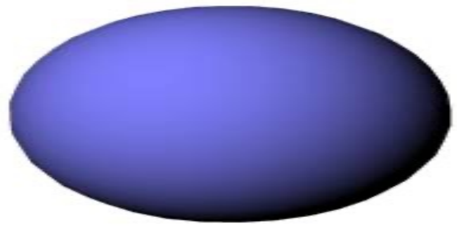


K. H. and N. Takigawa,
Prog. Theo. Phys.128 ('12)1061.

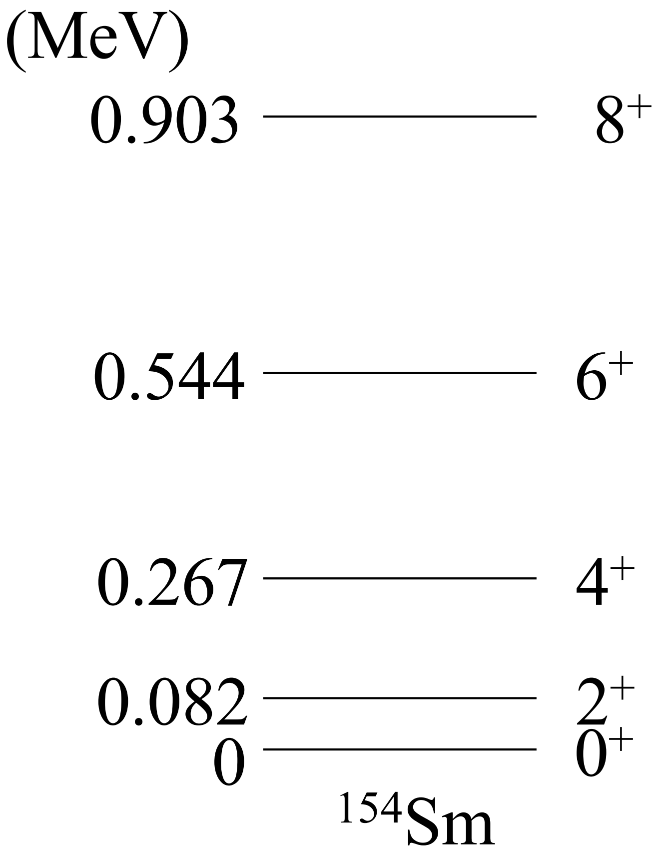
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$



Effects of nuclear deformation



^{154}Sm



rotational spectrum

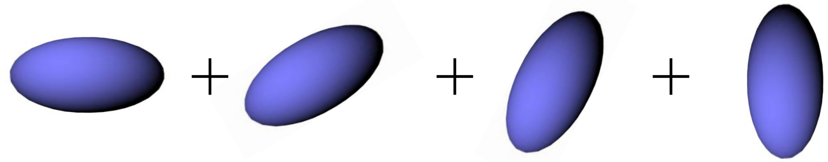
a small rotational energy

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

→ a large moment of inertia *J*

→ rotation: a slow deg. of freedom

$$E_{\rm rot} \sim E_{2^+} = 82 \text{ keV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

$\Psi_{0^+} =$ 

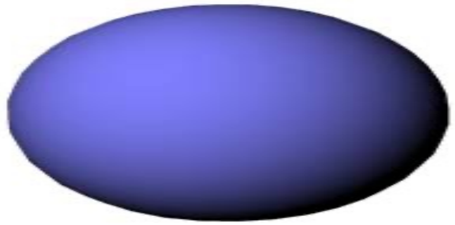
→ a spherical state in the lab. system

fix the orientation angle to calculate the fusion probability

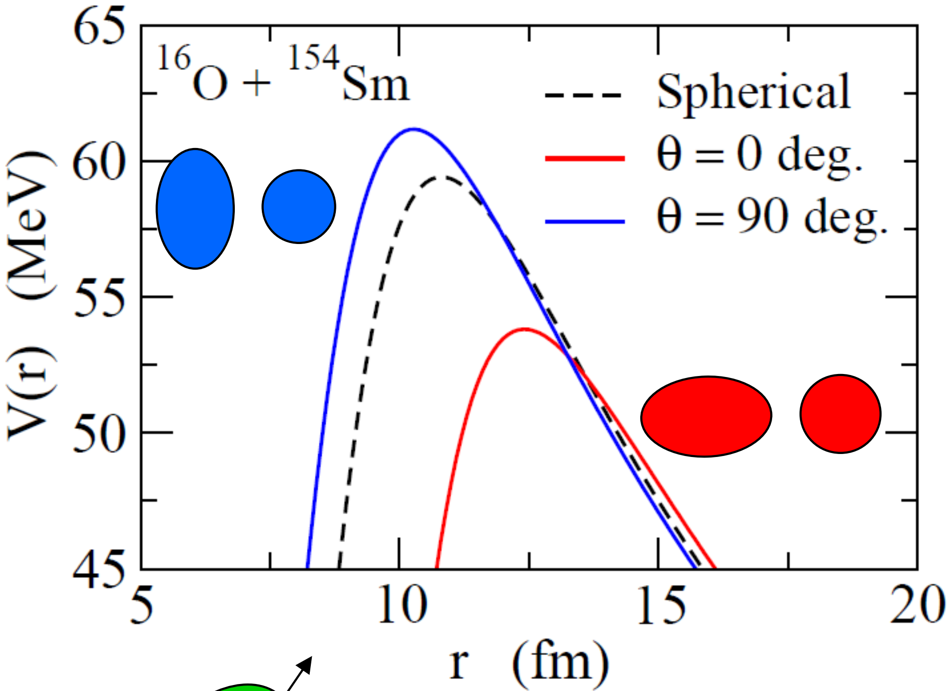
"a snapshot of a rotating nucleus"

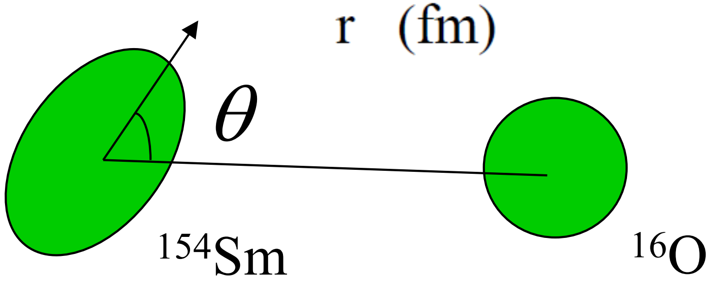
Effects of nuclear deformation

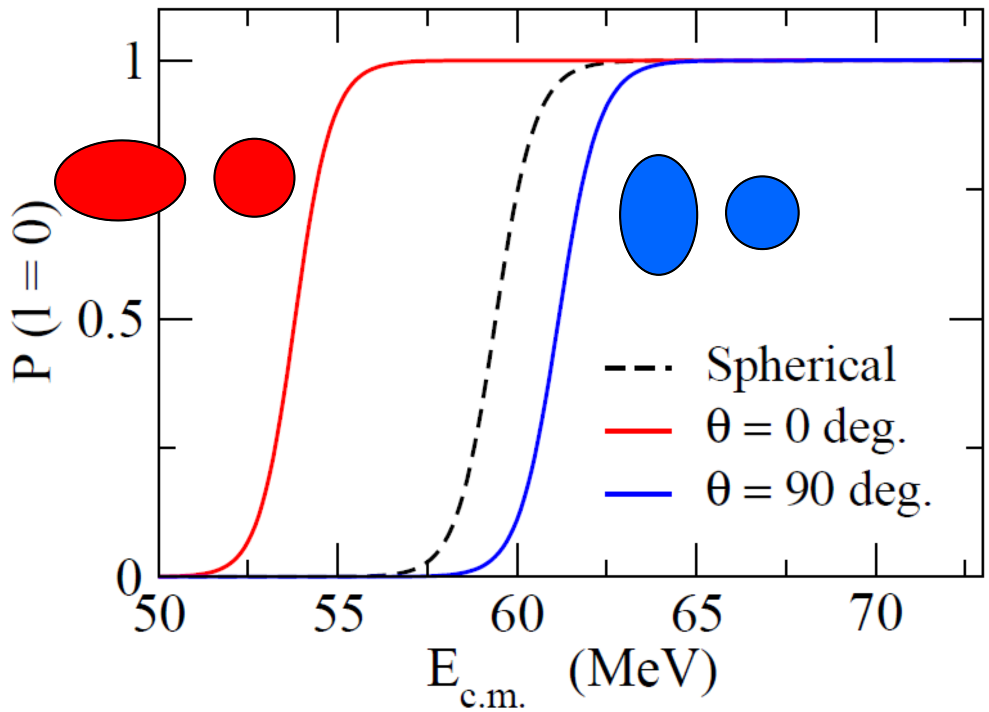
^{154}Sm : a typical deformed nucleus



^{154}Sm

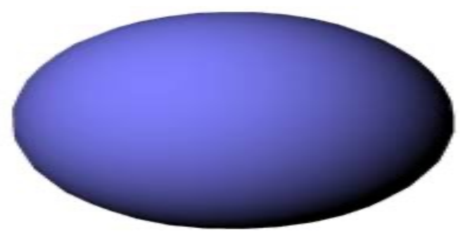




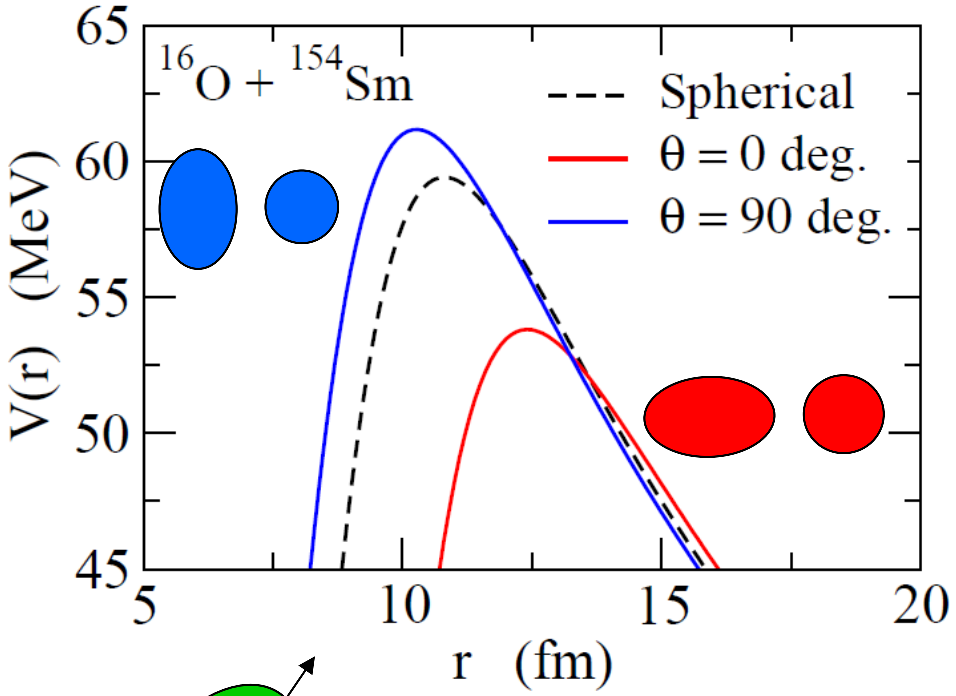


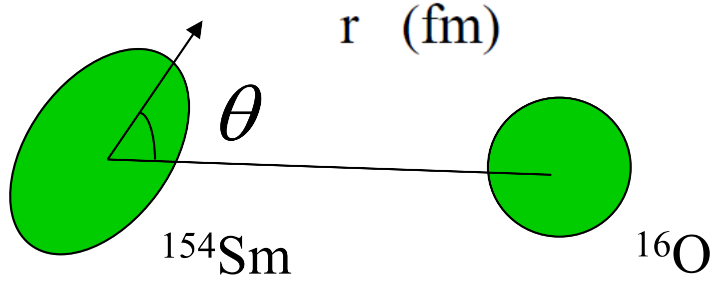
Effects of nuclear deformation

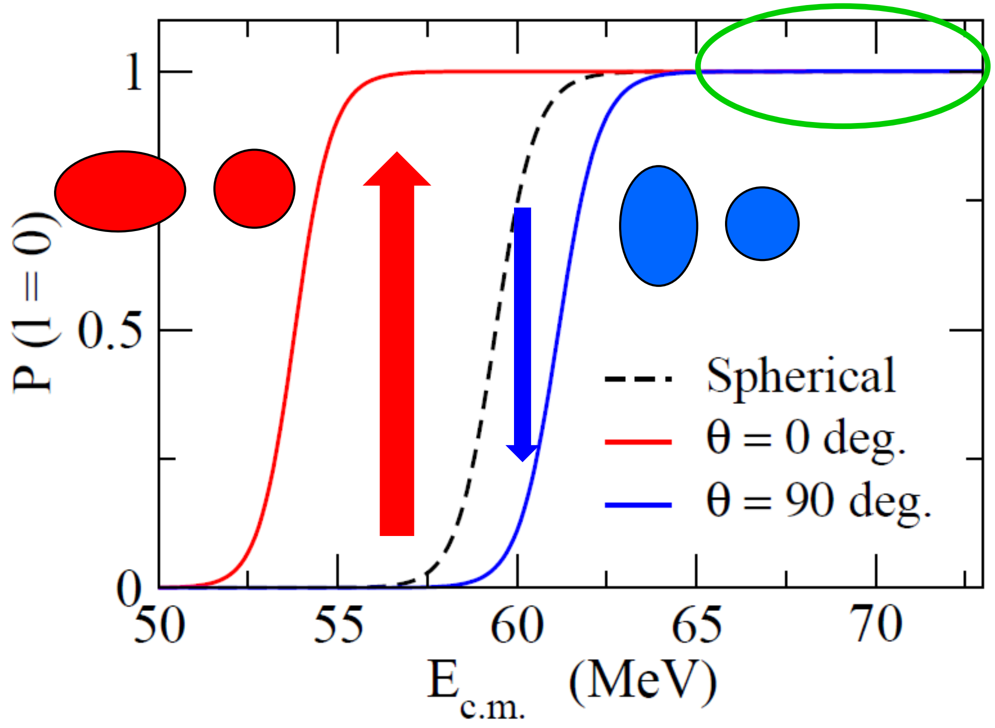
^{154}Sm : a typical deformed nucleus



^{154}Sm

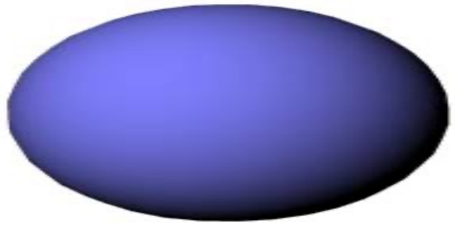


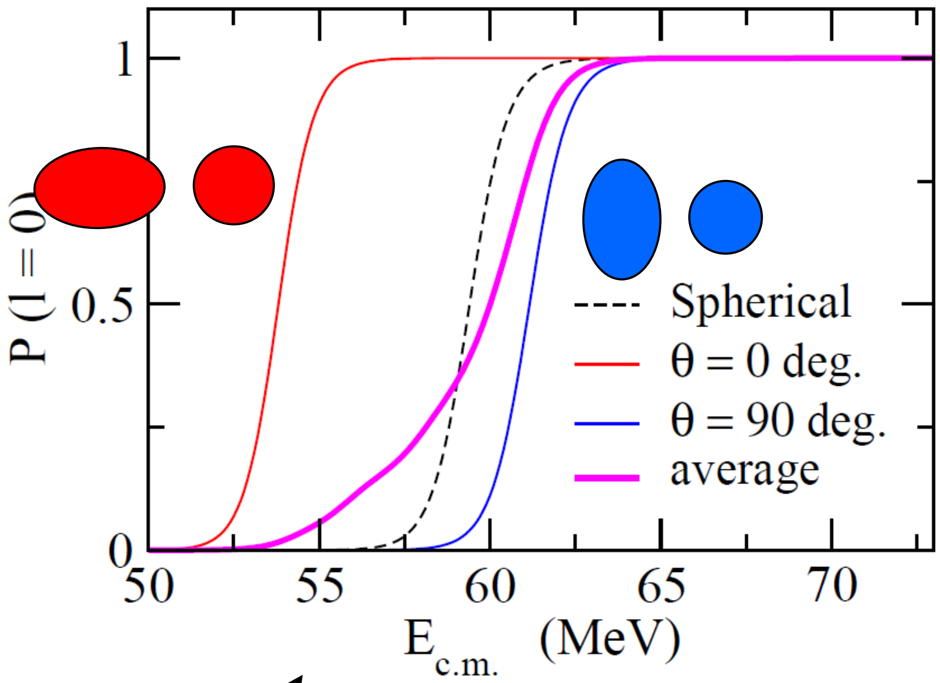


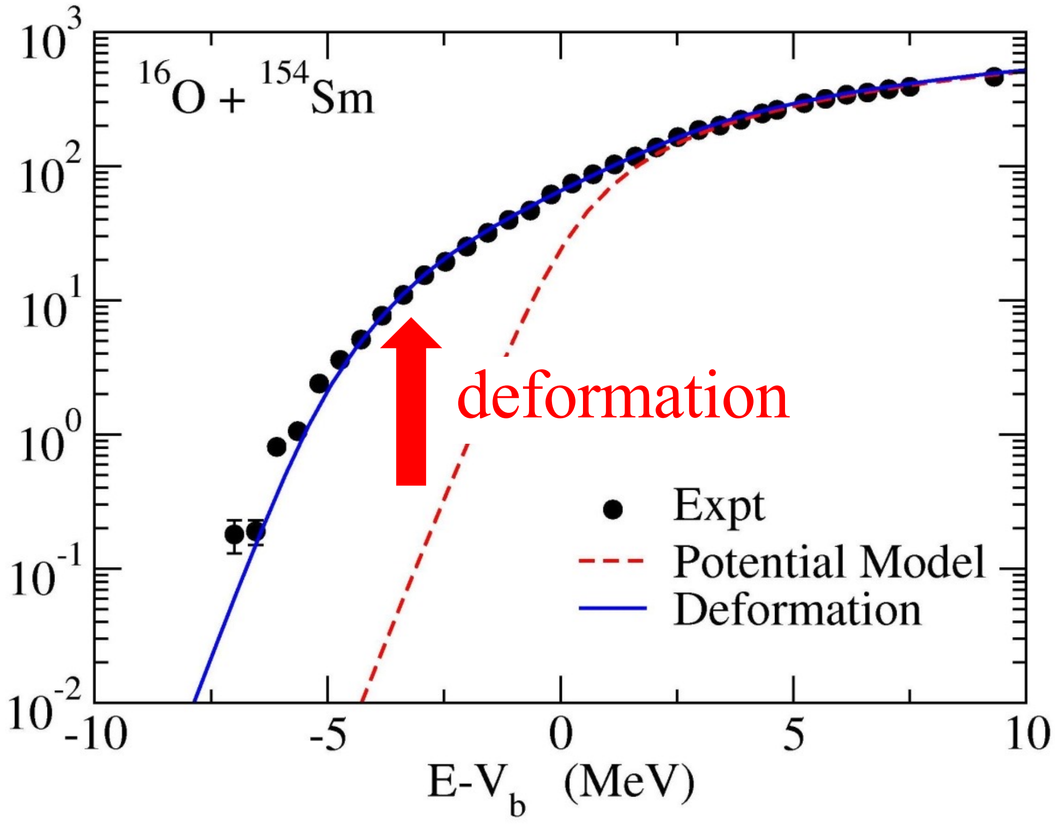


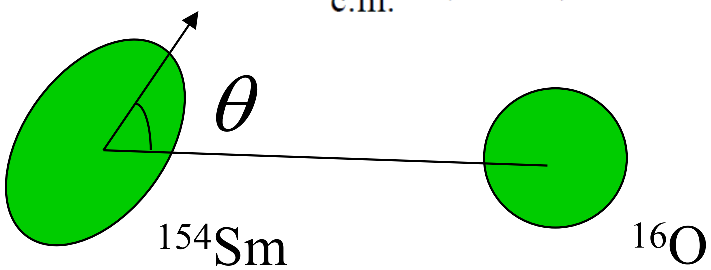
Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus



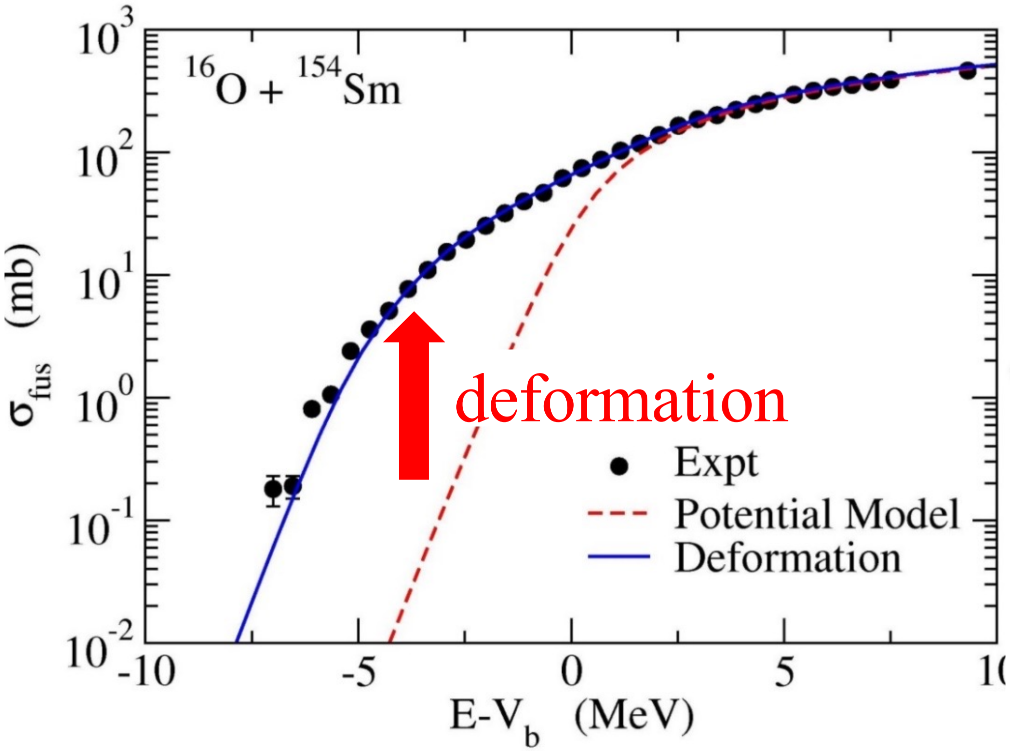




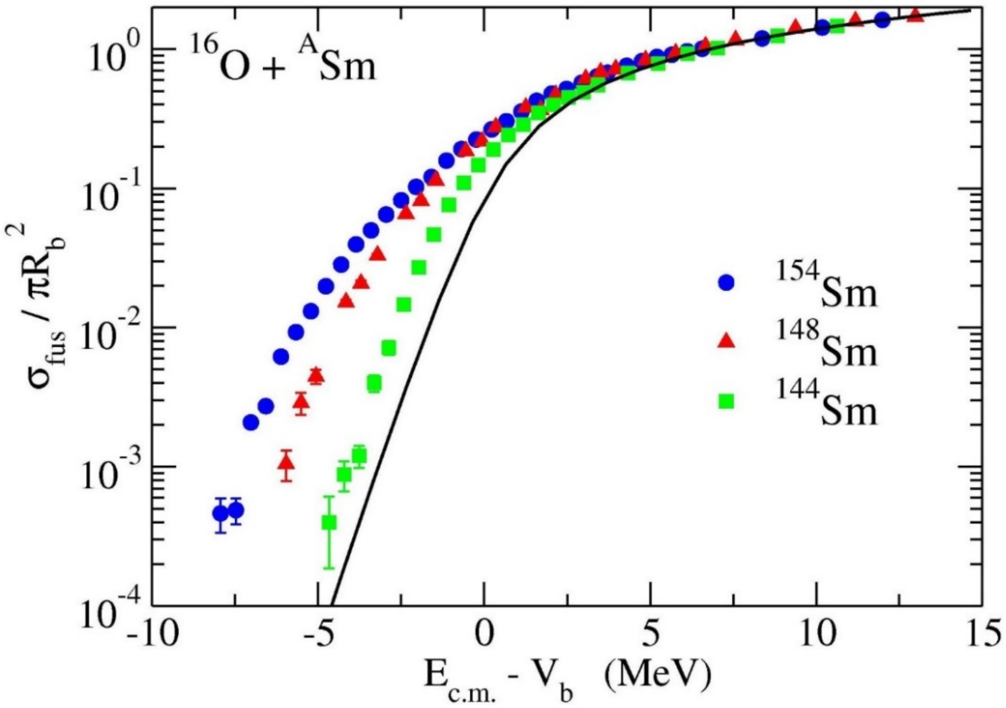


$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

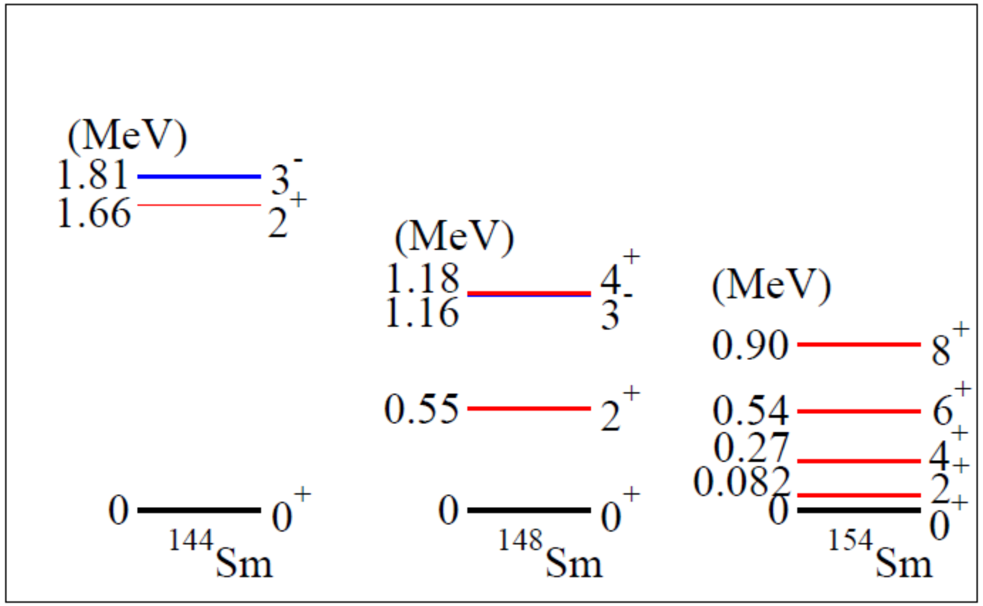
Fusion: strong interplay between nuclear structure and reaction



similar enhancement
for non-deformed nuclei



strong correlation
with nuclear spectrum
→ coupling assisted
tunneling phenomena



Fusion barrier distribution

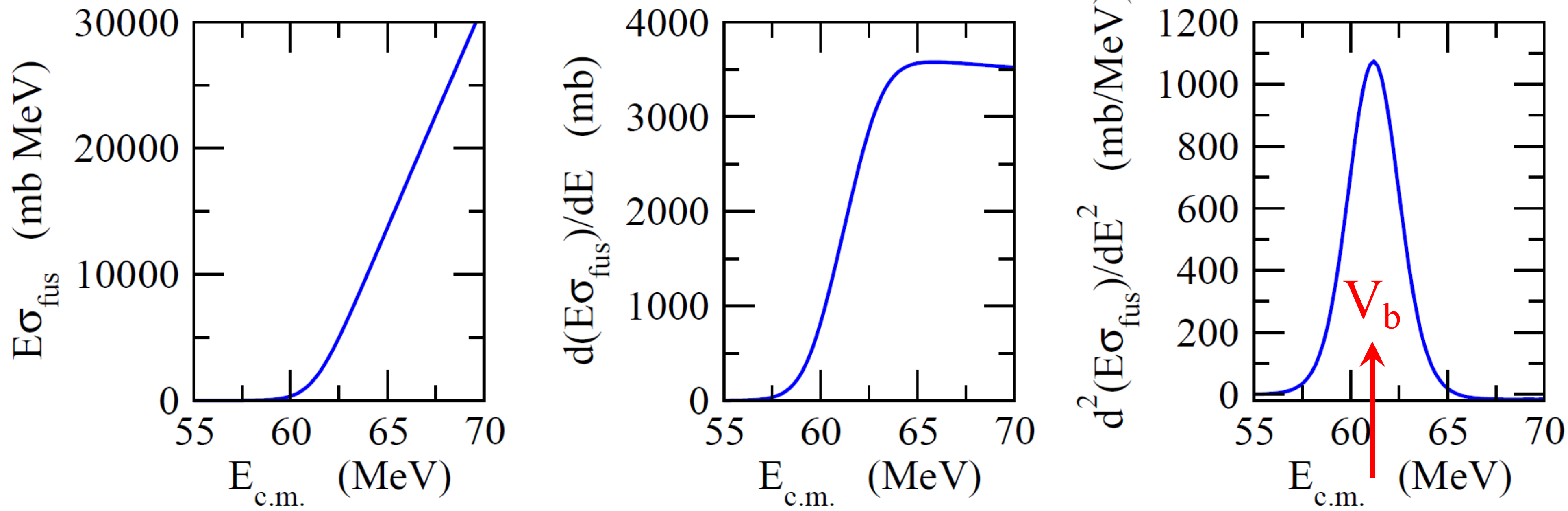
$$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 (‘91) 25

$E\sigma_{\rm fus}$

$$\frac{d(E\sigma_{\rm fus})}{dE} \sim \pi R_b^2\, P_{l=0}(E)$$

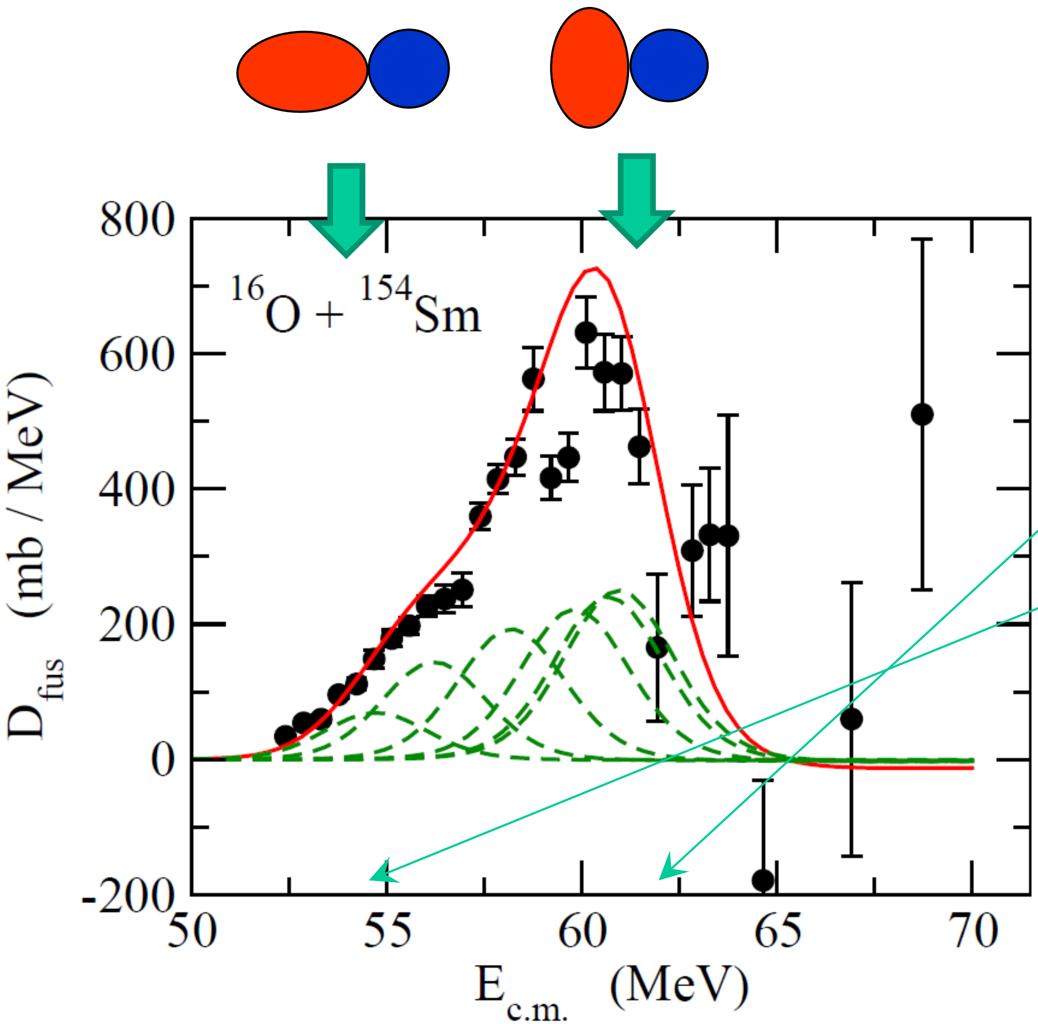
$$\frac{d^2(E\sigma_{\rm fus})}{dE^2} \sim \pi R_b^2 \cdot dP_{l=0}/dE$$

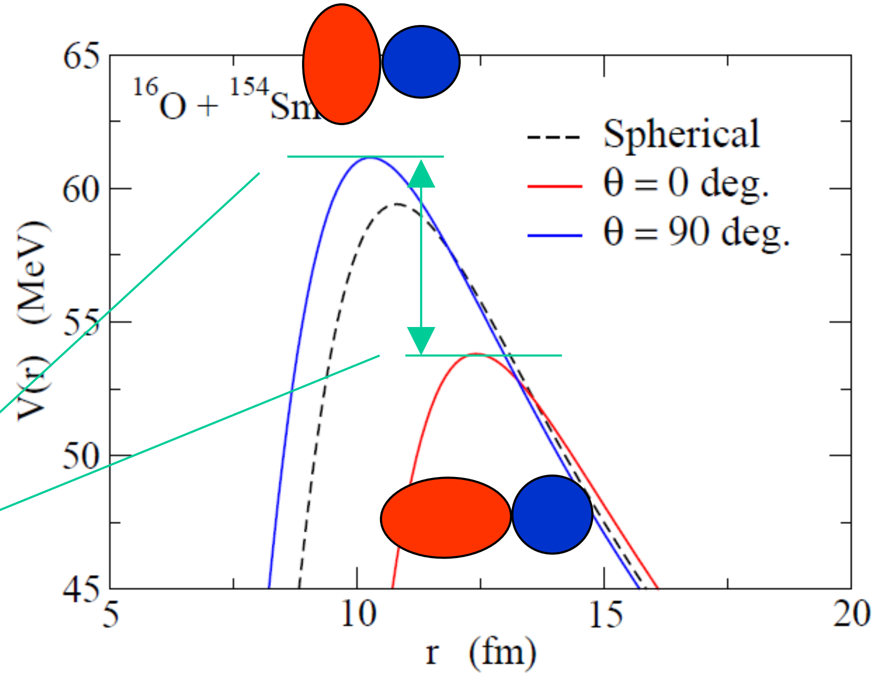


K.H. and N. Takigawa, PTP128 (‘12) 1061

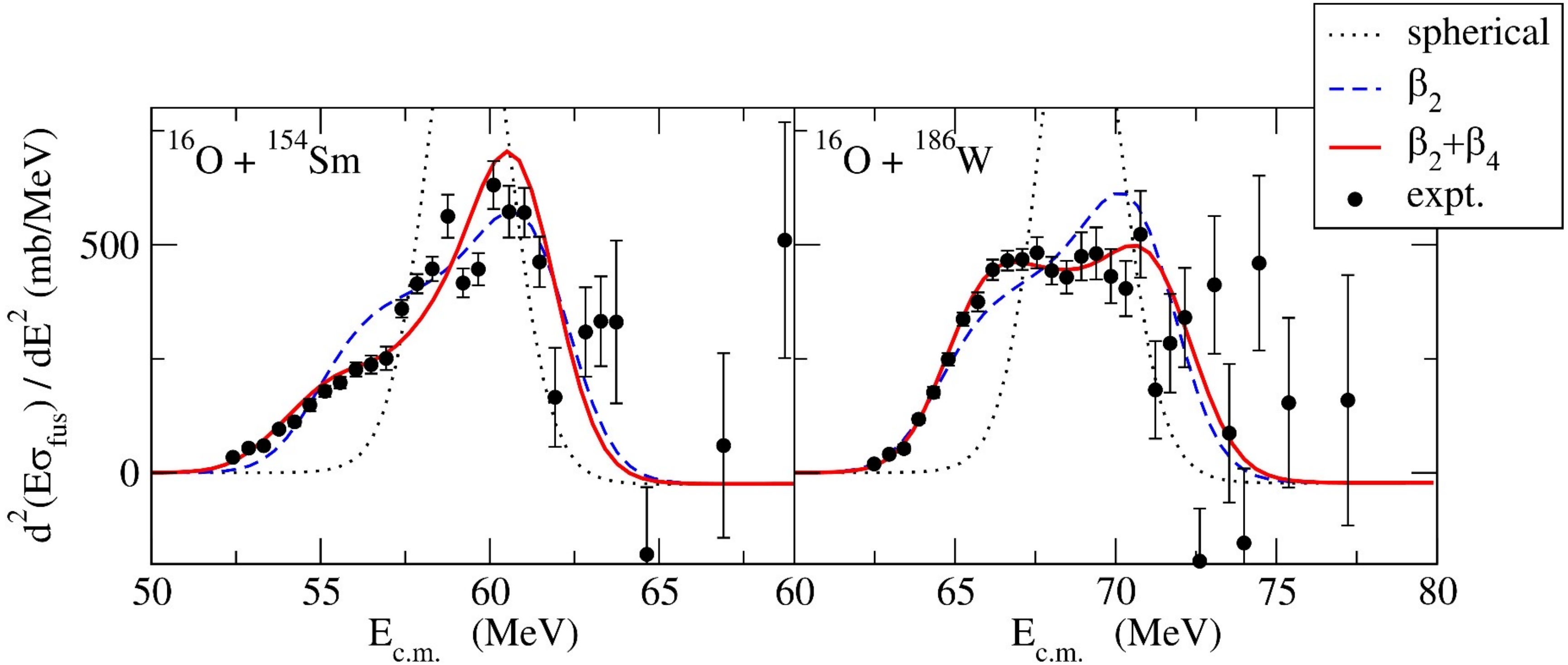
✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2} \sim \pi R_b^2 \frac{dP_{l=0}}{dE}$$





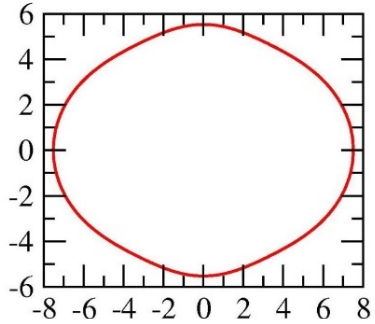
Data: J.R. Leigh et al.,
PRC52 ('95) 3151



$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

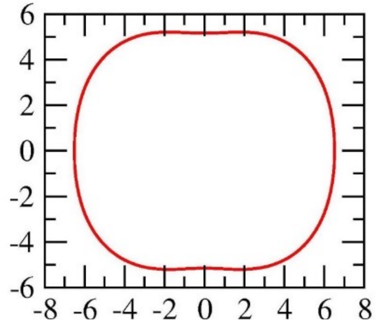
$\beta_2 = 0.33$

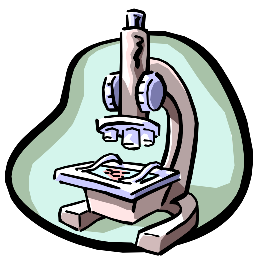
$\beta_4 = +0.05$



$\beta_2 = 0.29$

$\beta_4 = -0.03$





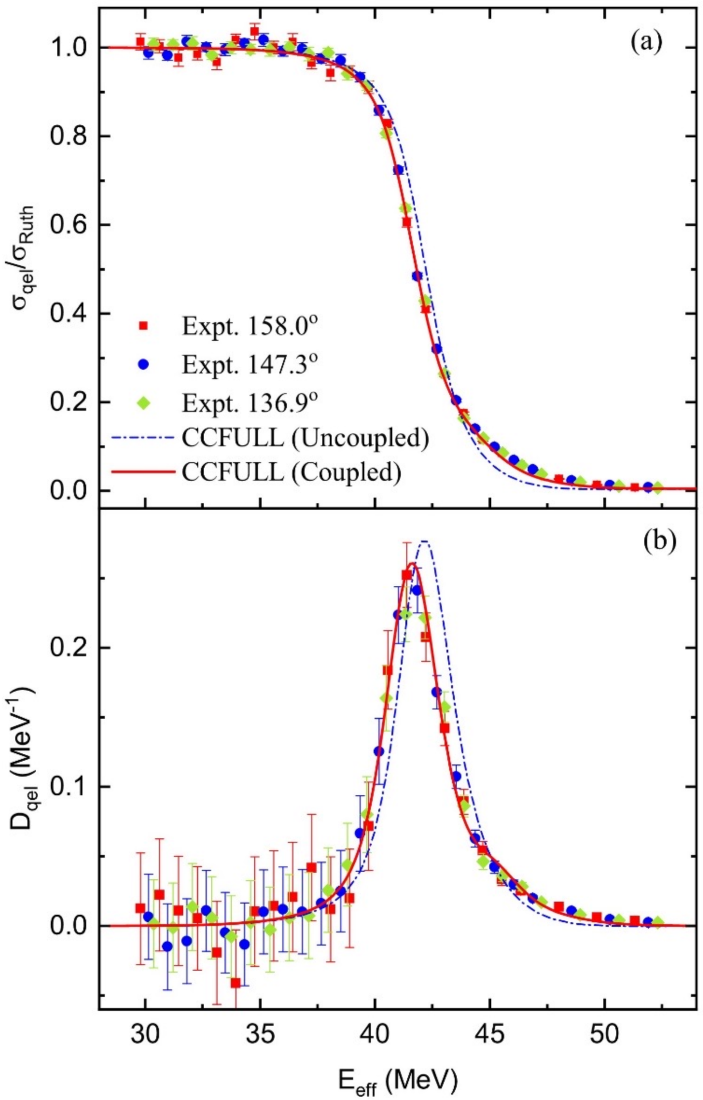
sensitive to the sign of β_4!

→ Fusion as a quantum tunneling microscope for nuclei

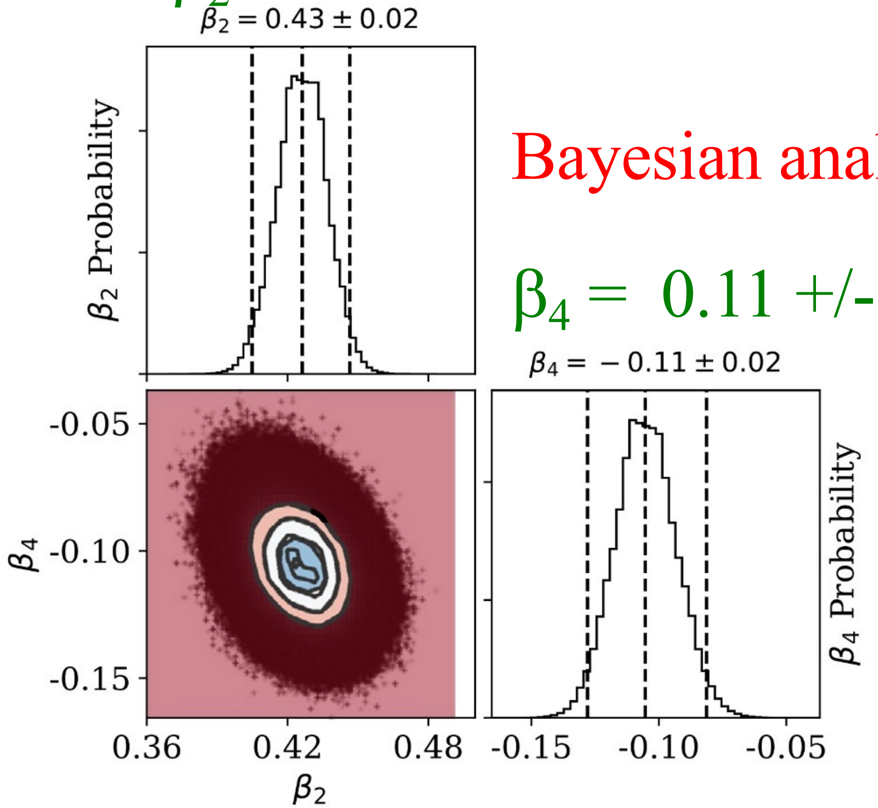
Determination of β_4 of ^{24}Mg with quasi-elastic barrier distributions

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).

^{24}Mg + ^{90}Zr



$\beta_2 = 0.43$ +/- 0.02



Bayesian analysis

$\beta_4 = 0.11$ +/- 0.02

high precision determination of β_4

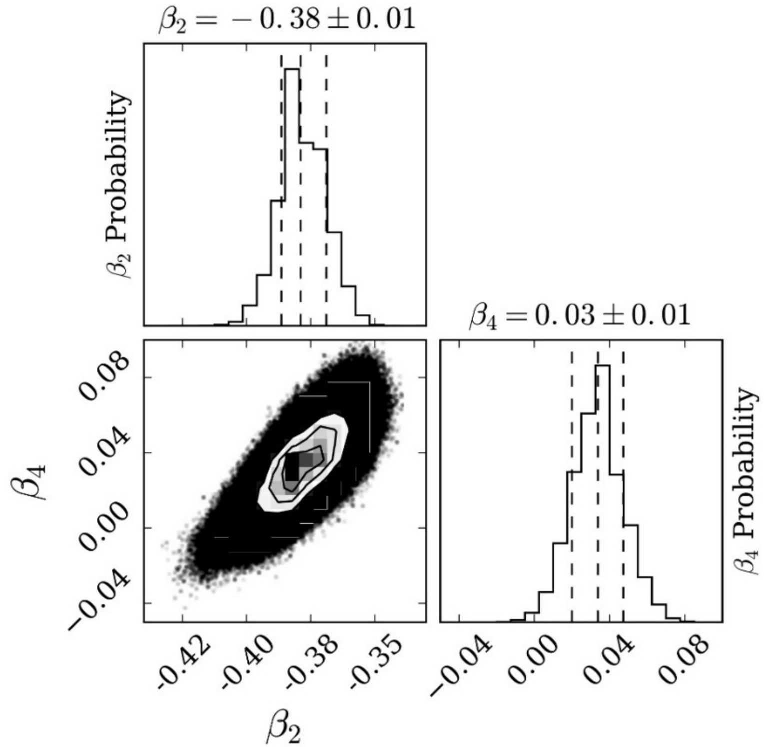
→ for the first time

cf. (p,p'): $\beta_4 = -0.05$ +/- 0.08

R. De Swiniarski et al., PRL23, 317 (1969)

Emulator for multi-channel scattering

a similar study for ^{28}Si
with ^{28}Si+^{90}Zr



Y.K. Gupta, V.B. Katariya, G.K. Prajapati, K.Hagino et al., PLB845, 138120 (2023).

← needs to repeat many calculations with different (β_2,β_4)

an emulator to speed-up the calculations

cf. a recent review:

T. Duguet et al.,
Rev. Mod. Phys. 96, 031002 (2024)

Emulator for multi-channel scattering

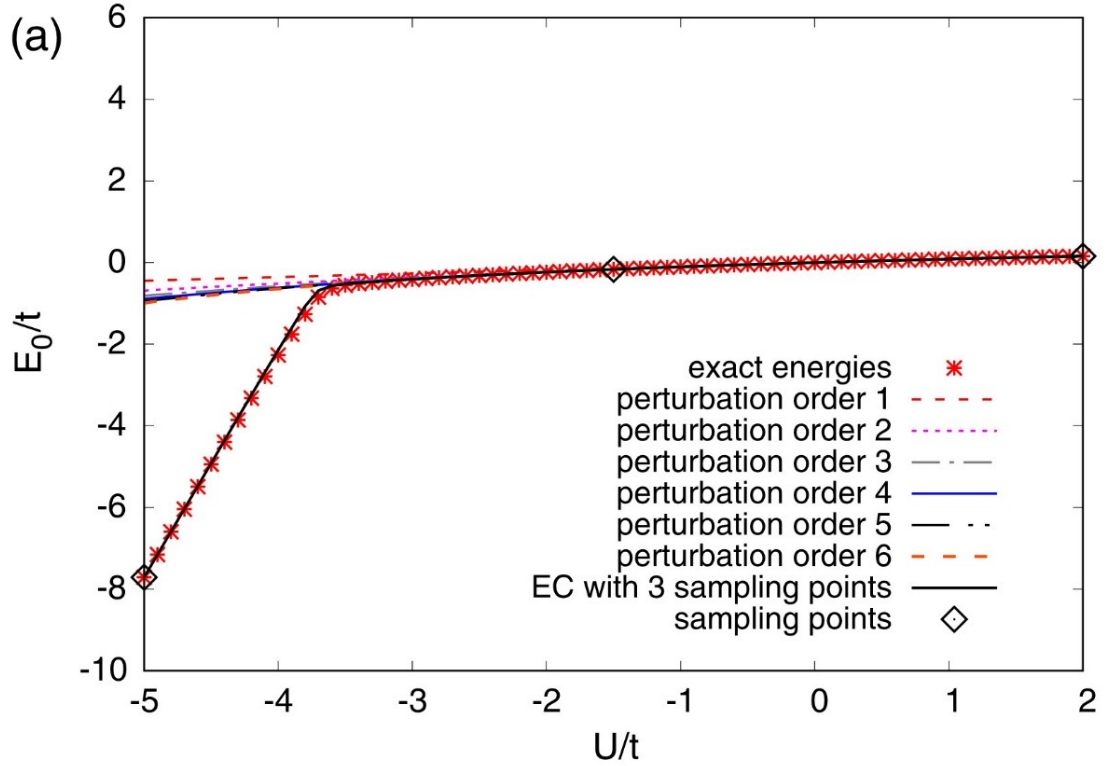
eigenvector continuation for a bound state

$$\Psi(\theta) = \sum_{i=1}^{N} c_i \Psi(\theta_i)$$

$$H(\theta)|\Psi(\theta)\rangle = E(\theta)|\Psi(\theta)\rangle$$

Bose-Hubbard model

$$H \sim -t \sum_{n,n'} a^\dagger(n')a(n) + \frac{U}{2}\sum_n [a^\dagger(n)a(n)]^2$$



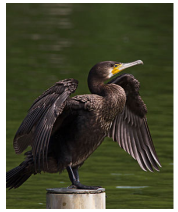
D. Frame, R. He, I. Ipsen, Daniel Lee, Dean Lee, and E. Rrapaj, PRL121, 032501 (2018)

Emulator for multi-channel scattering

✓ eigenvector continuation for a bound state

$$\Psi(\theta) = \sum_{i=1}^{N} c_i \Psi(\theta_i) \qquad H(\theta)|\Psi(\theta)\rangle = E(\theta)|\Psi(\theta)\rangle$$

✓ extension to scattering problems



鵜

사다새 제

K. Hagino, S. Yoshida, M. Kimura, and K. Uzawa,
in preparation

$$\Psi_E(x,\theta) = \sum_{i=1}^{N} c_i \Psi_E(x,\theta_i)$$

$$H(\theta)|\Psi_E(\theta)\rangle = E|\Psi_E(\theta)\rangle$$

$$|\Psi_E(\theta)\rangle \to S(E;\theta)$$

θ: parameters of a nucleus-nucleus potential

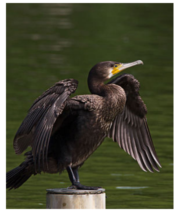
$$S(E,\theta) = \sum_{i=1}^{N} c_i S(E,\theta_i)$$

$c_i \leftarrow$ the Kohn variational principle

Emulator for multi-channel scattering

extension to scattering problems





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사다새 제

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in preparation

1D two-channel problem:

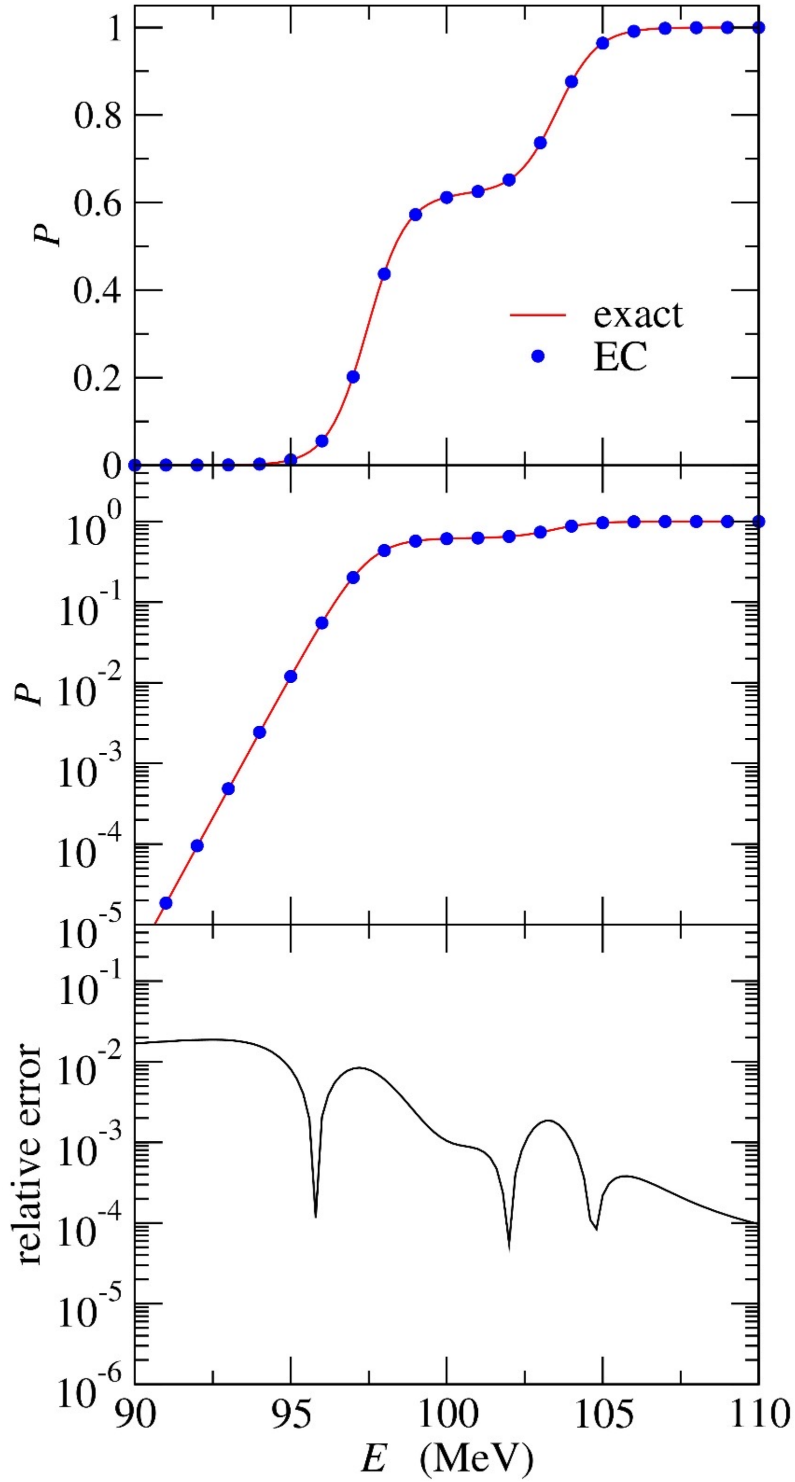
$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\begin{pmatrix}\phi_1(x)\\ \phi_2(x)\end{pmatrix}+\begin{pmatrix}V(x) & F(x)\\ F(x) & V(x)+\epsilon\end{pmatrix}\begin{pmatrix}\phi_1(x)\\ \phi_2(x)\end{pmatrix}=E\begin{pmatrix}\phi_1(x)\\ \phi_2(x)\end{pmatrix}$$

Gaussian barrier:

$$\begin{aligned} V(x) &= V_0\, e^{-x^2/2s^2} \\ F(x) &= F_0\, e^{-x^2/2s_f^2} \end{aligned}$$

$$\begin{aligned} \phi_s(x) &\to e^{ik_sx}\delta_{s,1}+R_se^{-ik_sx} \qquad (x\to-\infty), \\ &\to T_se^{ik_sx} \qquad (x\to\infty), \end{aligned}$$

$$P=|T_1|^2+\frac{k_2}{k_1}|T_2|^2=1-|R_1|^2-\frac{k_2}{k_1}|R_2|^2.$$



$$\begin{pmatrix} V(x) & F(x) \\ F(x) & V(x)+\epsilon \end{pmatrix} \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

$$\begin{aligned} V(x) &= V_0\, e^{-x^2/2s^2} \\ F(x) &= F_0\, e^{-x^2/2s_f^2} \end{aligned}$$

$V_0 = 100$ MeV, $s = s_f = 3$ fm,
$F_0 = 3$ MeV

EC:

$$\Psi_E(x,\theta) = \sum_{i=1}^{N} c_i \Psi_E(x,\theta_i)$$

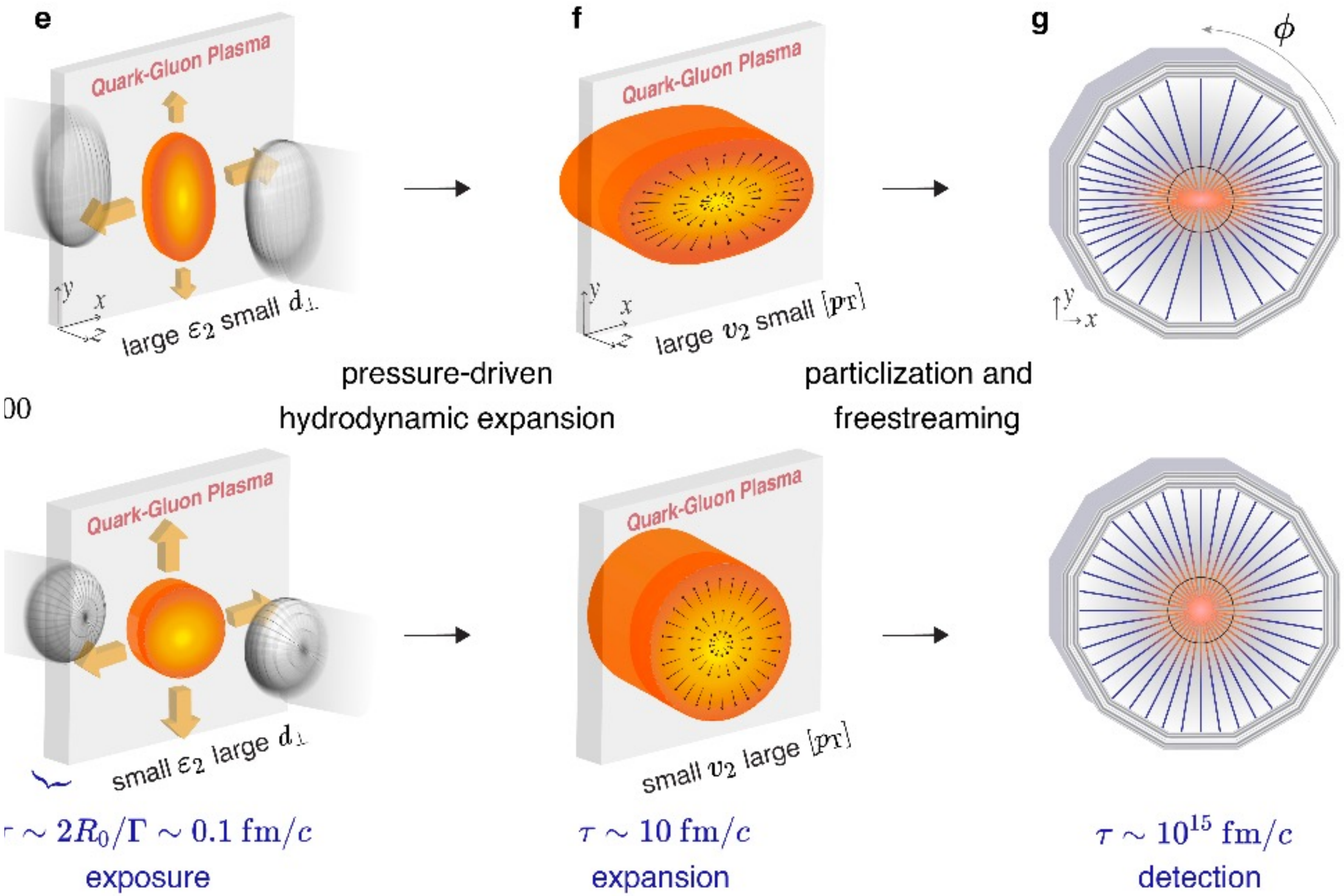
$F_0 = 1.5, 2.0, 2.5, 3.5, 4.5$ MeV

varying the coupling strength
between the channels 1 and 2

K. Hagino, S. Yoshida, M. Kimura,
and K. Uzawa, in preparation

Probing nuclear shapes in Rel. H.I. collisions

the initial shape of QGP **expansion** **detection**



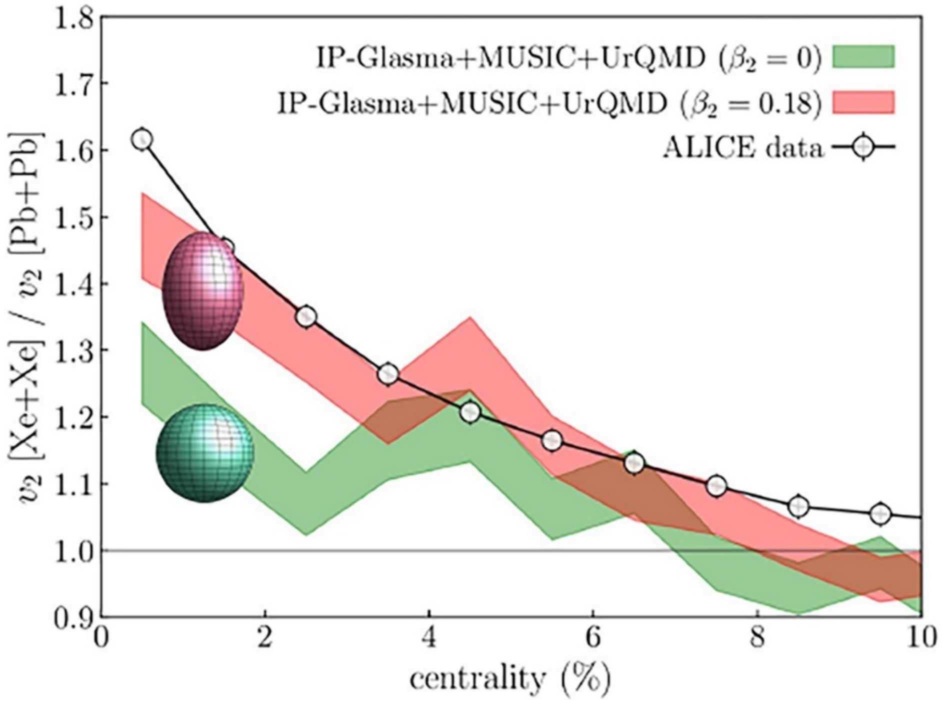
M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)

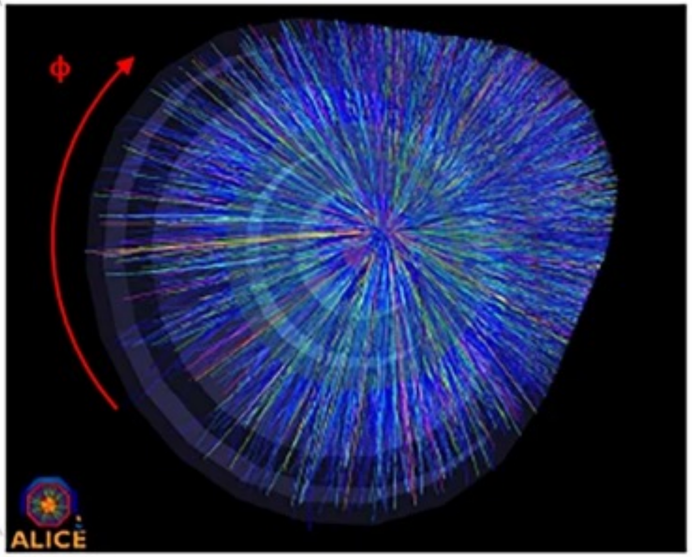
Probing nuclear shapes in Rel. H.I. collisions

flow:
the final N-distribution

$$\frac{1}{N}\frac{dN}{d\phi} = \frac{1}{2\pi}\left[1 + 2\sum_n v_n \cos n(\phi - \Psi_n)\right]$$

elliptic（楕円）flow ν_2





the ratio of ^{129}Xe+^{129}Xe to ^{208}Pb+^{208}Pb
→ quadrupole deformation of ^{129}Xe

J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

Probing nuclear shapes in Rel. H.I. collisions

flow:
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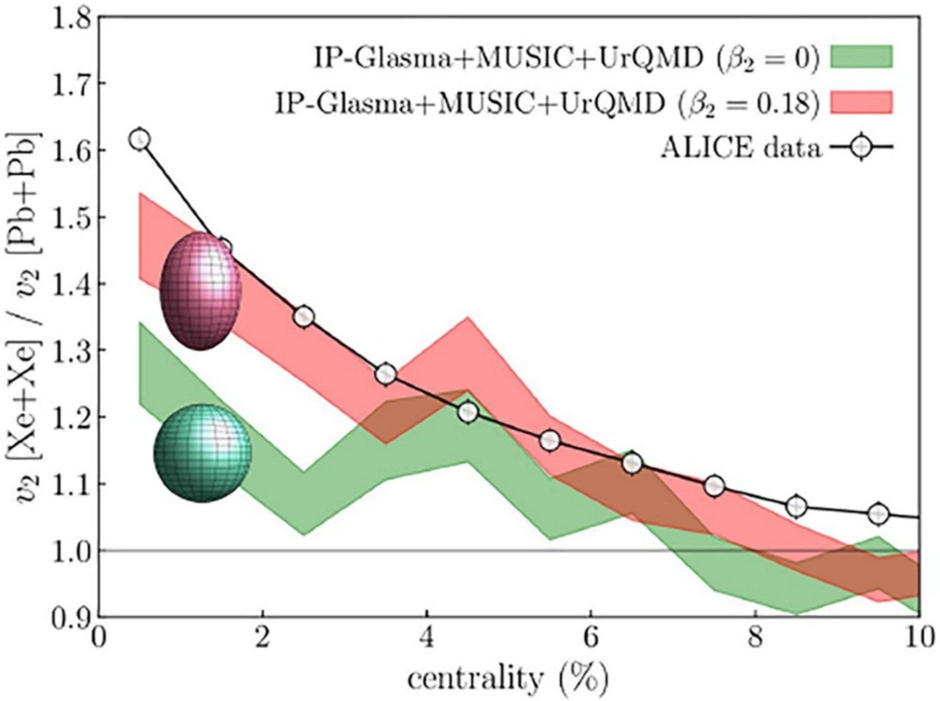
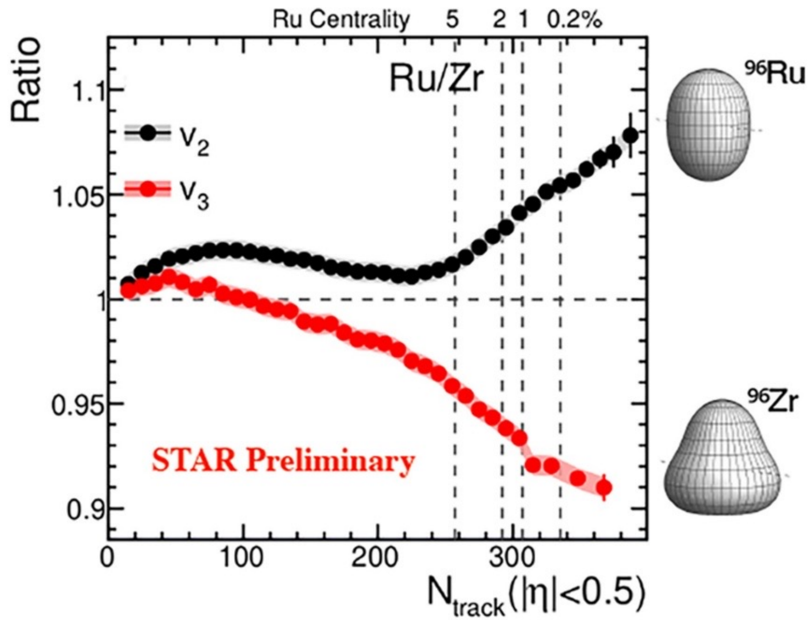




the ratio of ^{96}Ru+^{96}Ru to ^{96}Zr+^{96}Zr
→ octupole deformation of ^{96}Zr

the ratio of ^{129}Xe+^{129}Xe to ^{208}Pb+^{208}Pb
→ quadrupole deformation of ^{129}Xe

J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

Probing nuclear shapes in Rel. H.I. collisions

flow:

the final N-distribution

$$\frac{1}{N}\frac{dN}{d\phi} = \frac{1}{2\pi}\left[1 + 2\sum_n v_n \cos n(\phi - \Psi_n)\right]$$

other examples:

✓ triaxial (γ) deformation

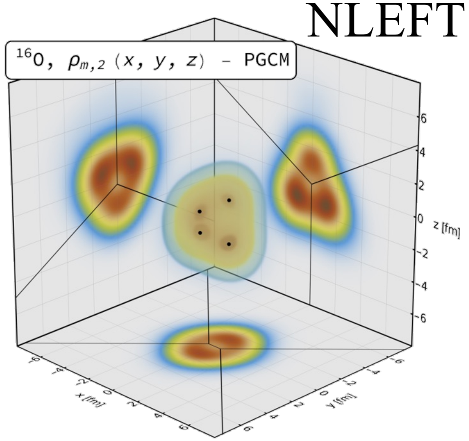


- G. Aad et al., PRC107, 054910 (2023)
- STAR collaboration, Nature 635, 67 (2024)

✓ α cluster

- ^{16}O+^{16}O: ALICE, preliminary
- Y. Wang et al., PRC109, L051904 (2024)

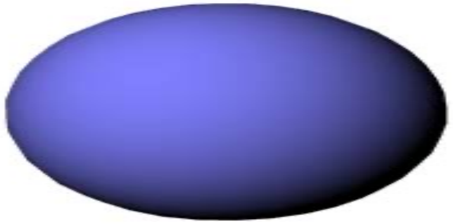
NLEFT



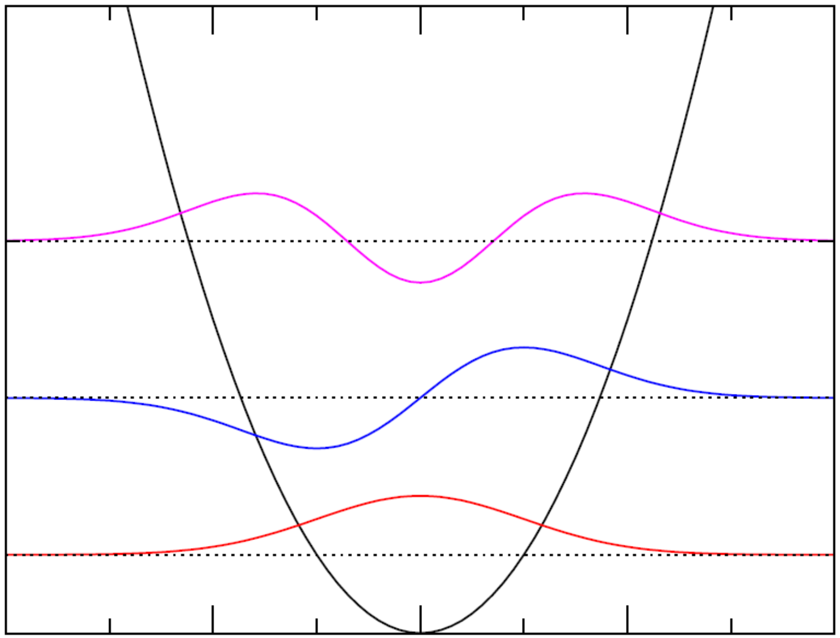
G. Giacalone et al., arXiv: 2402.05995

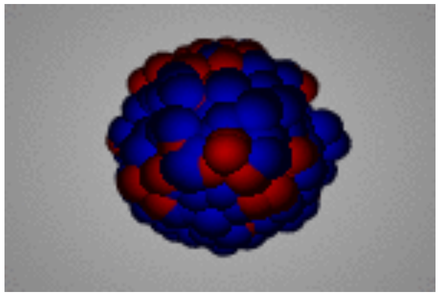
Probing nuclear shapes in Rel. H.I. collisions

So far, the focus has been only on a static deformation of a nucleus



There also exist several dynamical deformations of a nucleus





$$\langle \beta \rangle = 0$$

but fluctuates around β=0

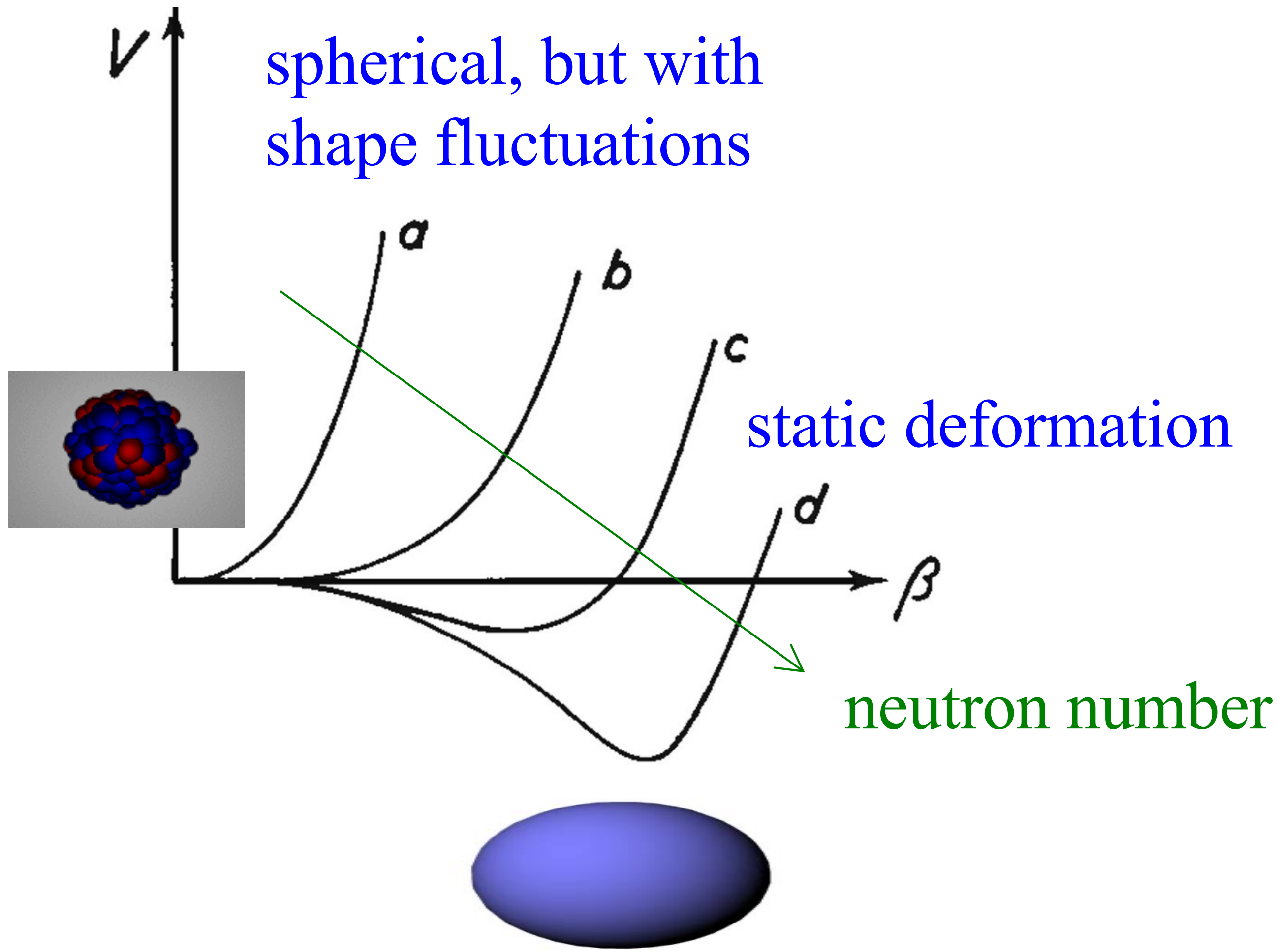
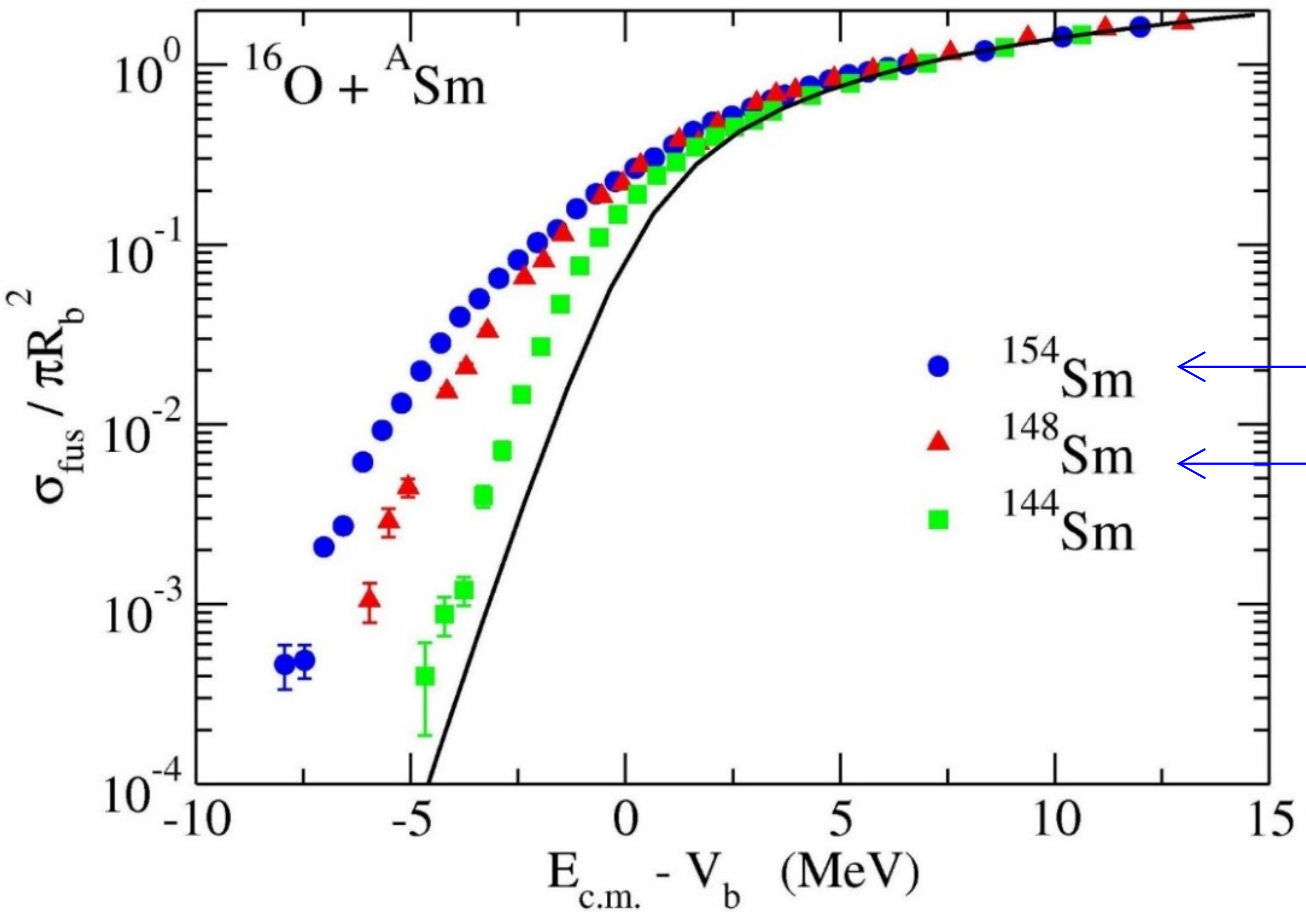


figure:
B.R. Mottelson,
Nobel Lecture

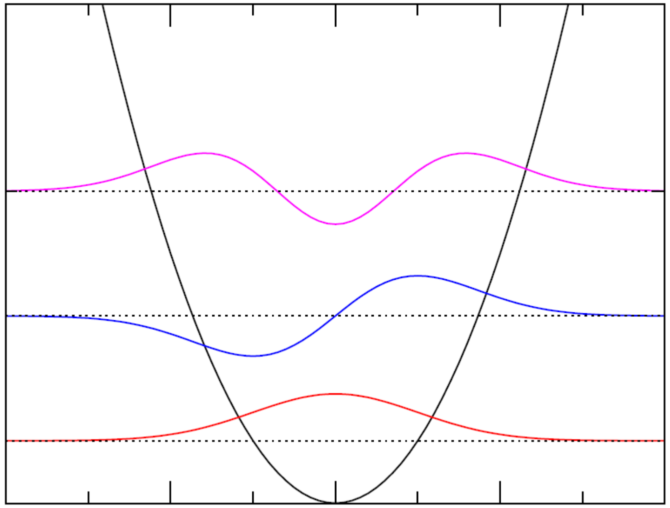
Surface vibrations of a spherical nucleus can still significantly affect H.I. sub-barrier fusion reactions

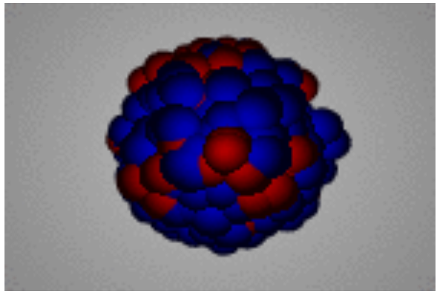


static deformation

spherical, but with vibration

Probing nuclear shapes in Rel. H.I. collisions





$$\langle \beta \rangle = 0$$

but fluctuates around β=0

the adiabatic approximation for vibrations:

Nuclear Physics **A352** (1981) 147 – 156; © *North-Holland Publishing Co., Amsterdam*

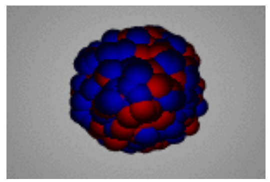
FUSION AND ZERO-POINT MOTIONS

H. ESBENSEN

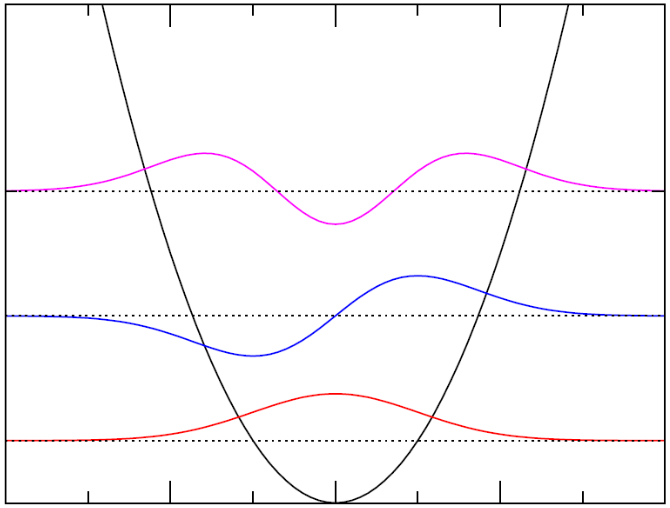
Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 14 July 1980

$$\sigma_{\rm fus}(E) \sim \int dx\, e^{-\alpha x^2} \sigma_0(E; x)$$



Probing nuclear shapes in Rel. H.I. collisions



In most of the cases, the vibrational motion is not slow for fusion:

$$E_{\rm vib} \sim 2 \text{ MeV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

→ but this can be very slow for rel. H.I. collisions!

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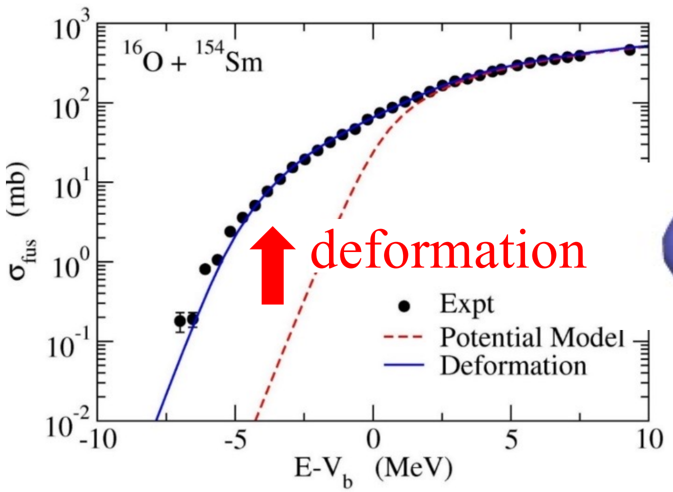
Summary

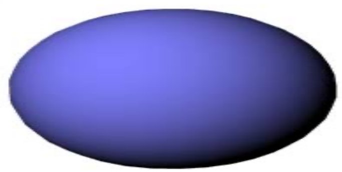
<u>Heavy-ion fusion reactions around the Coulomb barrier</u>

✓Strong interplay between nuclear structure and reaction
✓Quantum tunneling with various intrinsic degrees of freedom
✓Role of deformation in sub-barrier enhancement

→ a snapshot of the rotational motion

↓ amplified





<u>Relativistic H.I. Collisions: fast collisions → a snapshot of a nucleus</u>

A tool to probe nuclear deformations: β_2, γ, β_3 deformations, cluster

→ so far, only static deformation

: surface vibrations of a spherical nucleus?