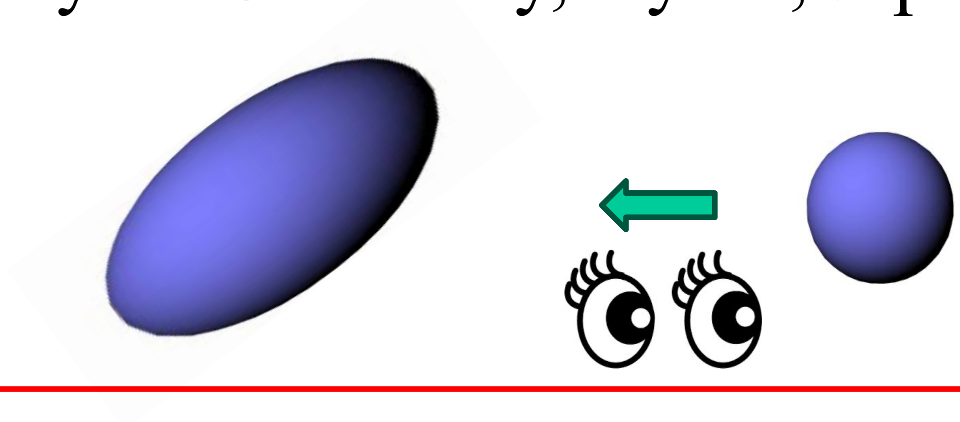
# Intersection of nuclear structure and low-energy nuclear collisions


Kouichi Hagino

Kyoto University, Kyoto, Japan



a *slow* mode
a *fast* mode

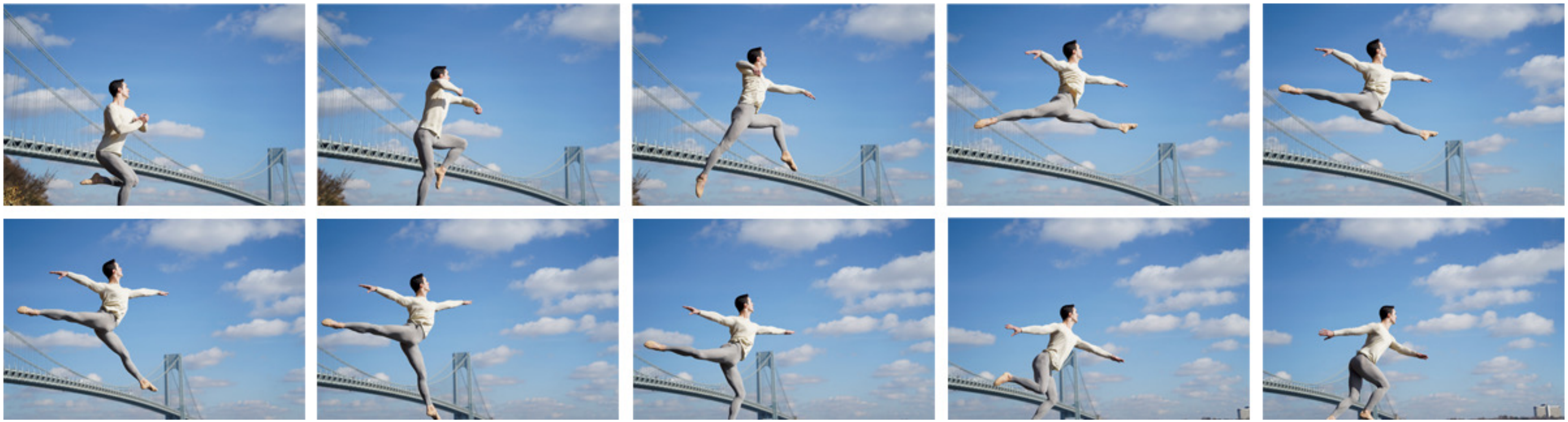
1. Introduction: overview of Low-energy Nuclear Reactions
2. Role of deformation in sub-barrier fusion reactions
3. A short comment on relativistic heavy-ion collisions
   : octupole vibration of $^{208}$Pb
4. Summary


Workshop on "Intersection of nuclear structure and high-energy nuclear collisions", Fudan University, 2025.5.12-20

## Snapshots

taking snapshots of a "slow" motion with a high-speed camera

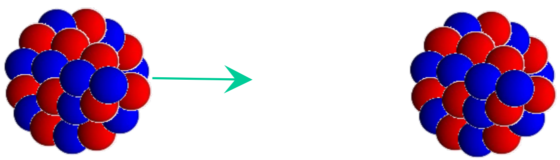
$$\tau_{\mathrm{camera}} \ll \tau_{\mathrm{motion}}$$



https://www.sony.jp/ichigan/products/ILCE-7M3/feature_3.html

(photos with a Sony camera α7III)

taking snapshots of a nucleus with a "fast" nuclear reaction



$$\tau_{\mathrm{reaction}} \ll \tau_{\mathrm{nucleus}}$$

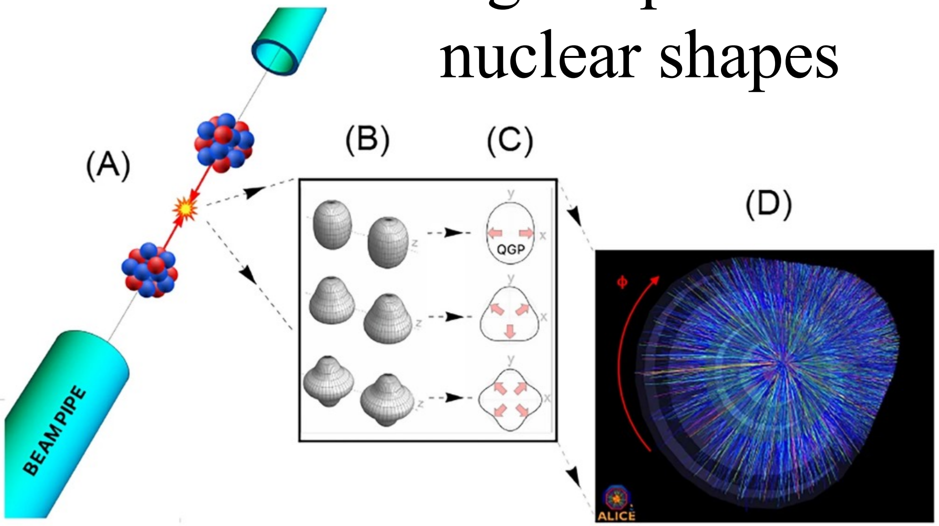
# Snapshots

taking a snapshot of a nucleus with a “fast” nuclear reaction

relativistic H.I. collisions
with a deformed nucleus

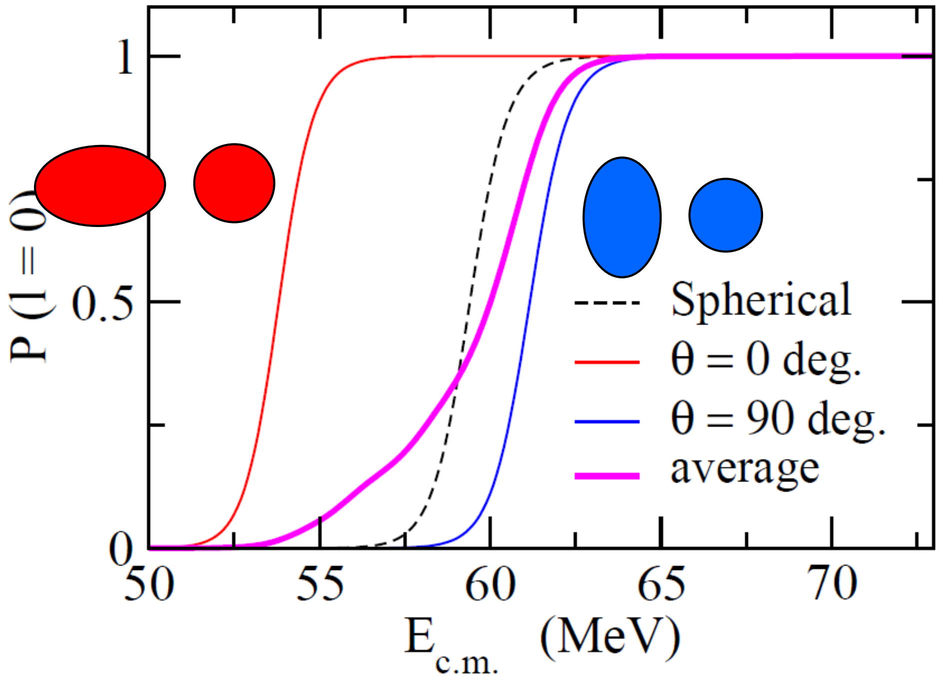
**low-energy H.I. fusion reactions
of a deformed nucleus**

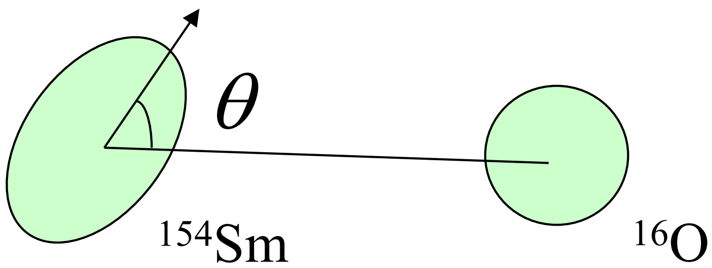
a good probe of
nuclear shapes




J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

increasing interests
in recent years



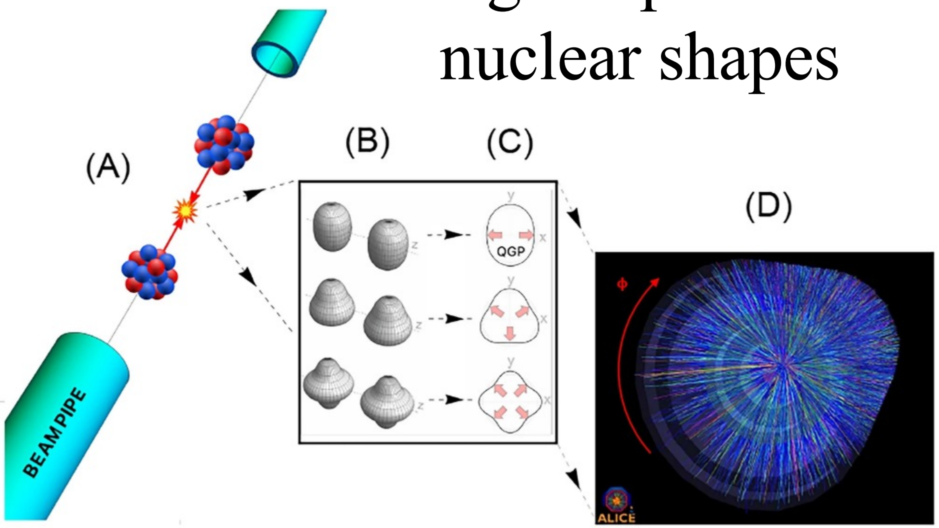
# Snapshots

taking a snapshot of a nucleus with a "fast" nuclear reaction

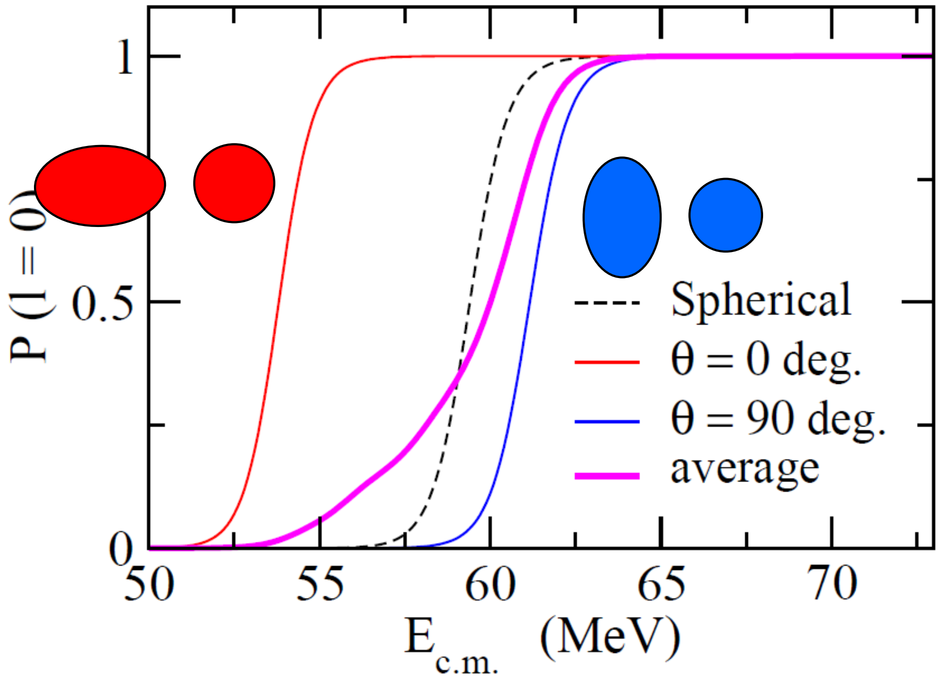
relativistic H.I. collisions
with a deformed nucleus

**low-energy H.I. fusion reactions
of a deformed nucleus**

a good probe of
nuclear shapes




J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

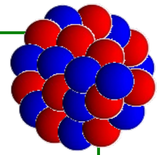



Large similarities: intersection of **High *E*** and **Low *E*** HI collisions

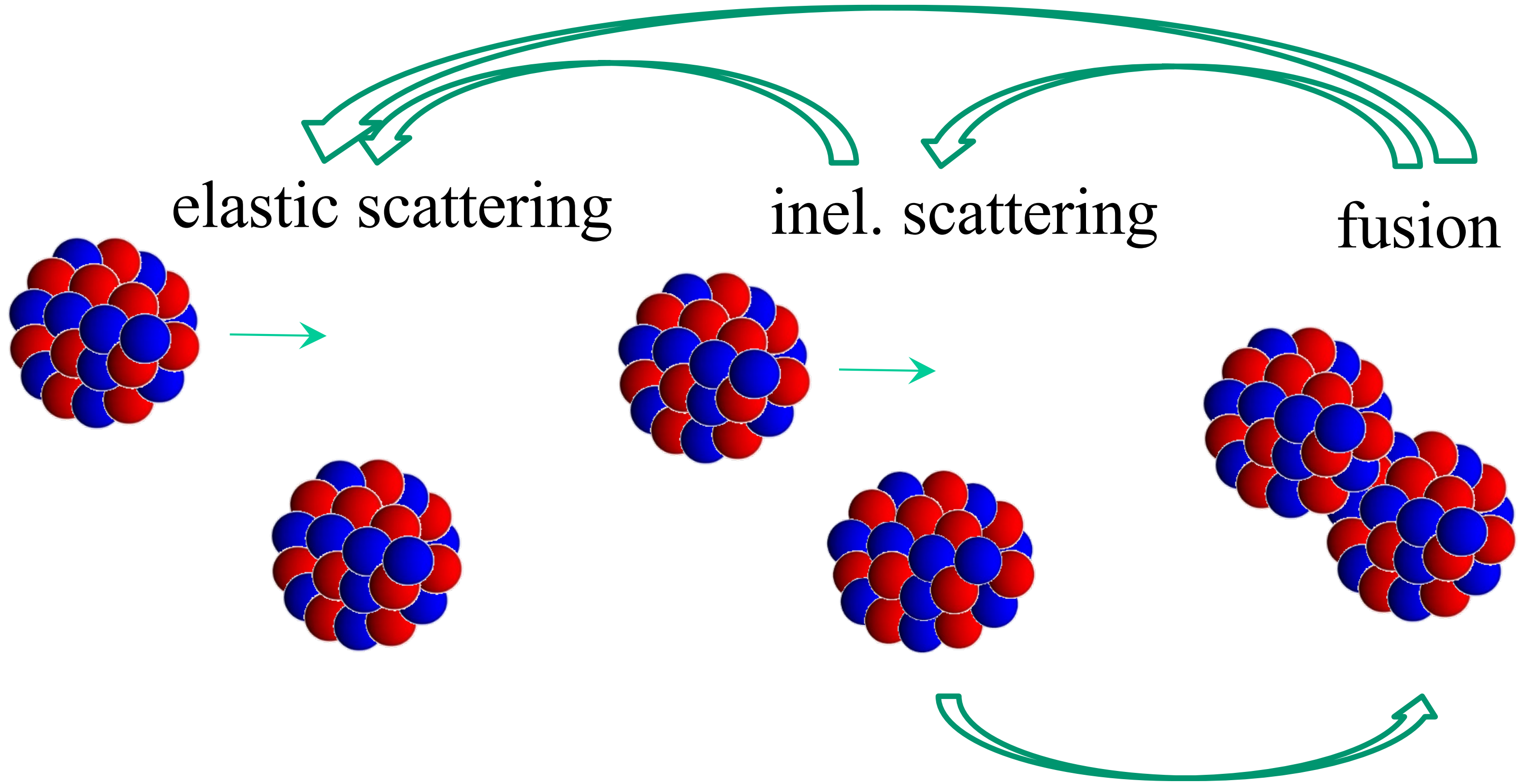
# Introduction: low-energy nuclear reactions

nucleus: a composite system

✓ various sort of reactions

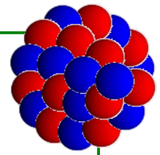


- elastic scattering
- inelastic scattering
- transfer reactions
- breakup reactions
- fusion reactions



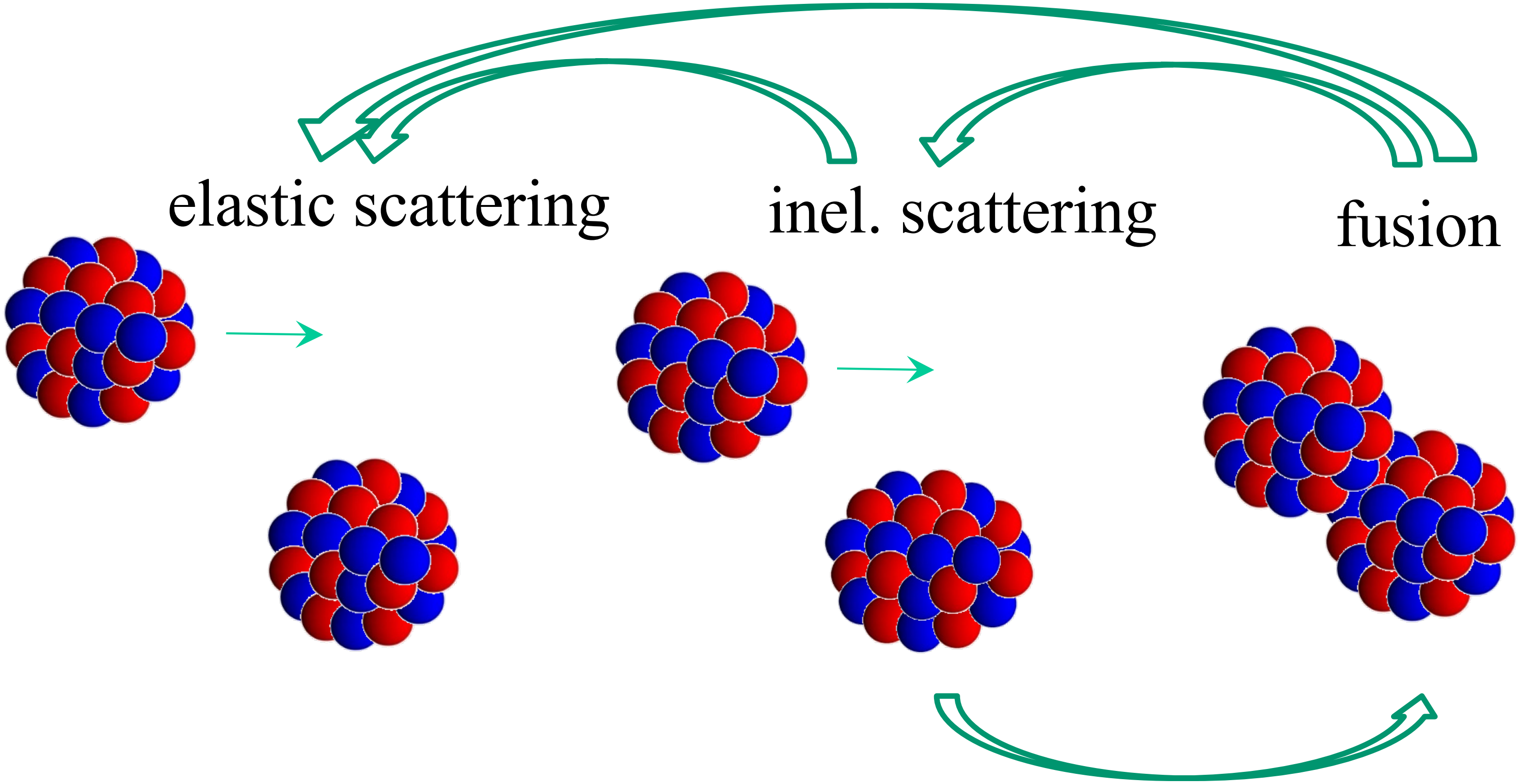
# Introduction: low-energy nuclear reactions

nucleus: a composite system
✓ various sort of reactions
✓ an interplay between nuclear structure and reaction



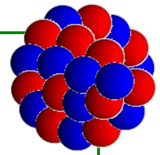
shapes, excitations, …..

- elastic scattering
- inelastic scattering
- transfer reactions
- breakup reactions
- fusion reactions



# Introduction: low-energy nuclear reactions

nucleus: a composite system

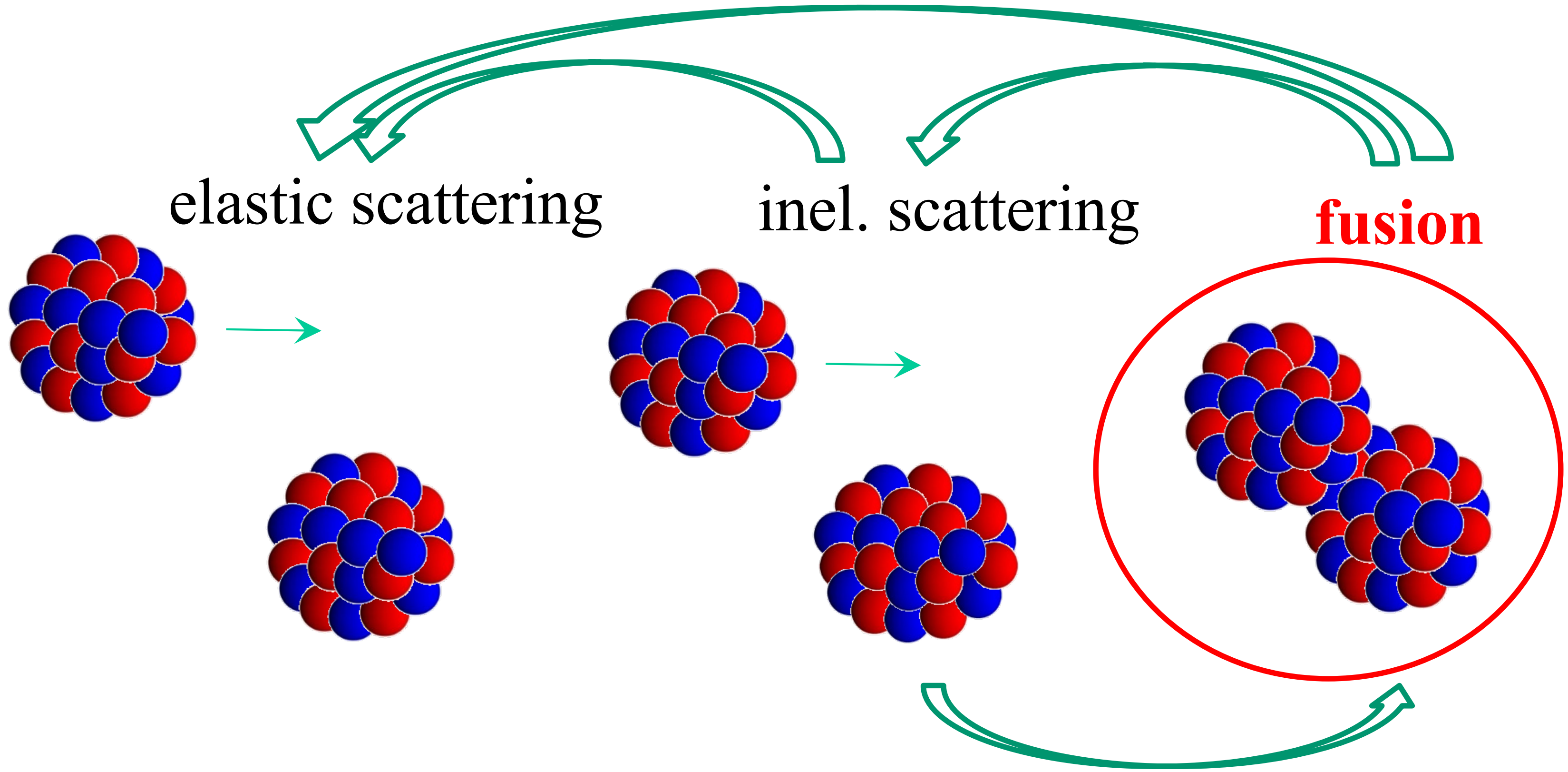


✓ various sort of reactions

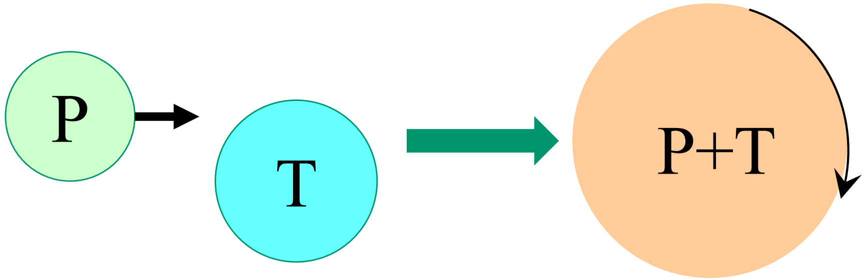
✓ an interplay between nuclear structure and reaction

shapes, excitations, …..

- elastic scattering
- inelastic scattering
- transfer reactions
- breakup reactions
- fusion reactions



# Fusion reactions: compound nucleus formation

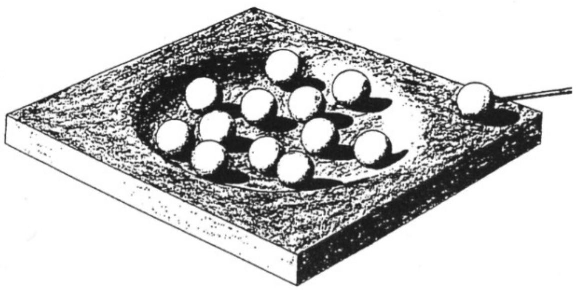


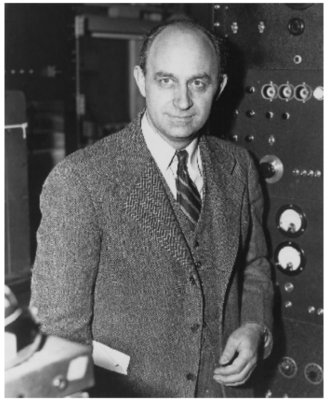

compound
nucleus

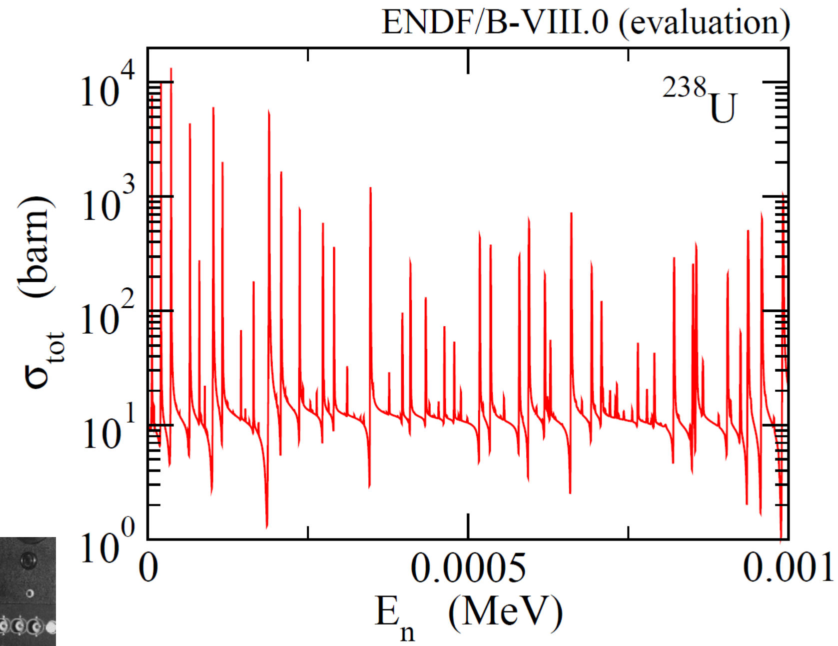
Niels Bohr (1936)

Wikipedia

neutron capture
reactions

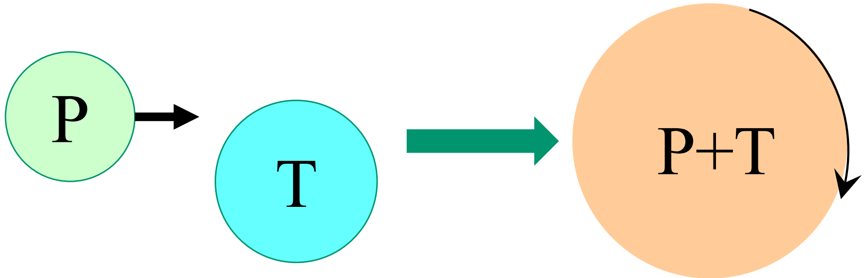




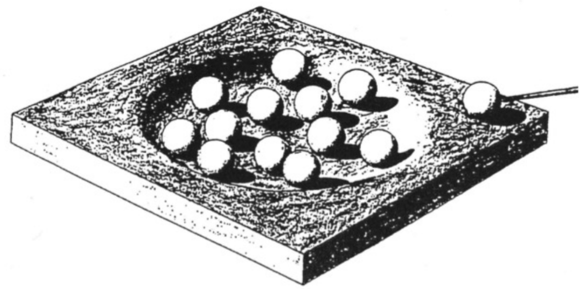



Exp: Enrico Fermi (1935)
many very narrow
resonances (width ~ eV)

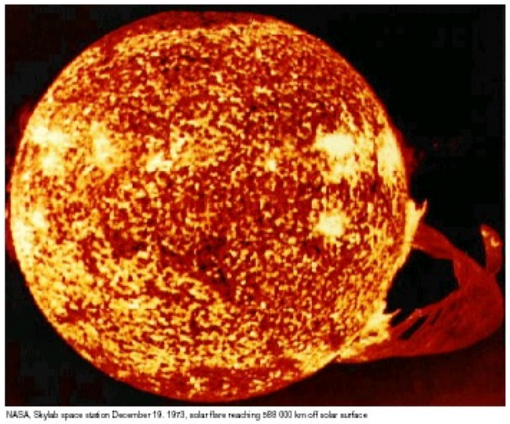
# Fusion reactions: compound nucleus formation



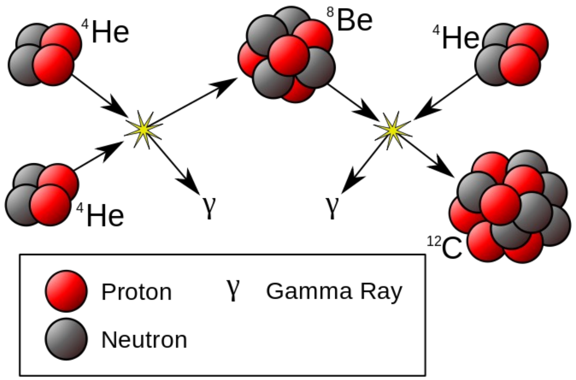

compound nucleus



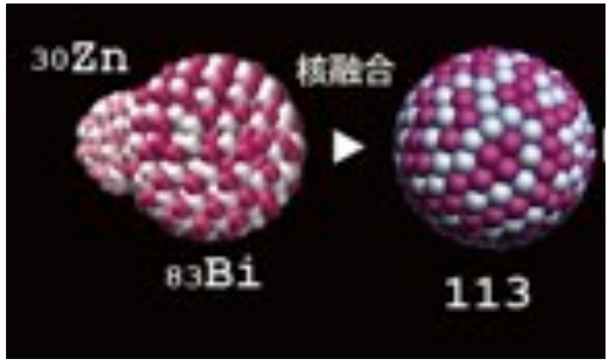
cf. N. Bohr '36



energy production in stars (Bethe '39)




nucleosynthesis

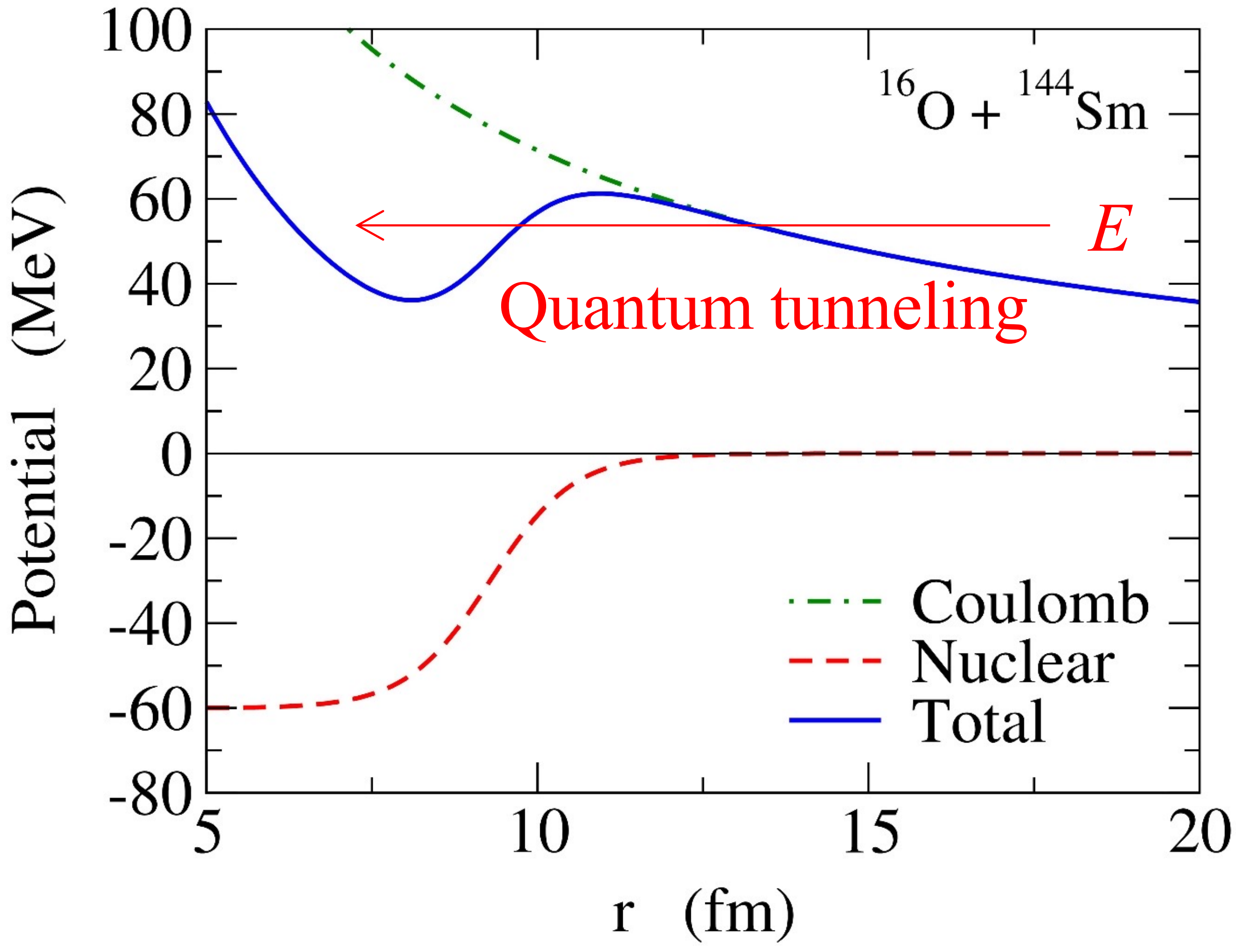



superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems with strong interaction

⟵ microscopic understanding: an ultimate goal of nuclear physics

# Coulomb barrier




1. Coulomb interaction
   long range, repulsion
2. Nuclear interaction
   short range, attraction

↓

Potential barrier
(Coulomb barrier)

Fusion: takes place by overcoming the barrier

the barrier height → defines the energy scale of a system

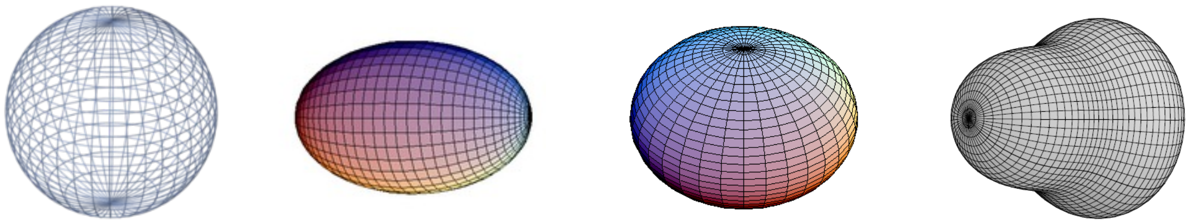
Fusion reactions at energies around the Coulomb barrier

# Low-energy heavy-ion fusion reactions and quantum tunneling

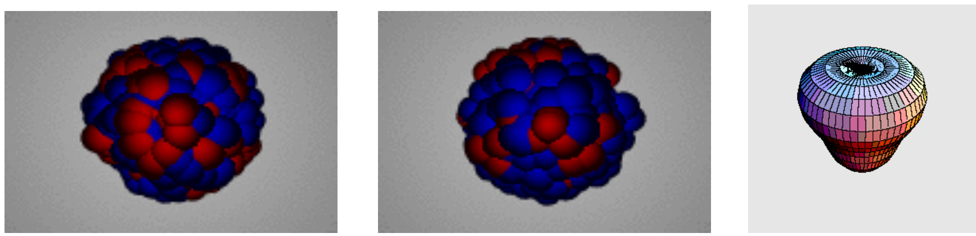
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes



- several surface vibrations



https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html

several modes and adiabaticities

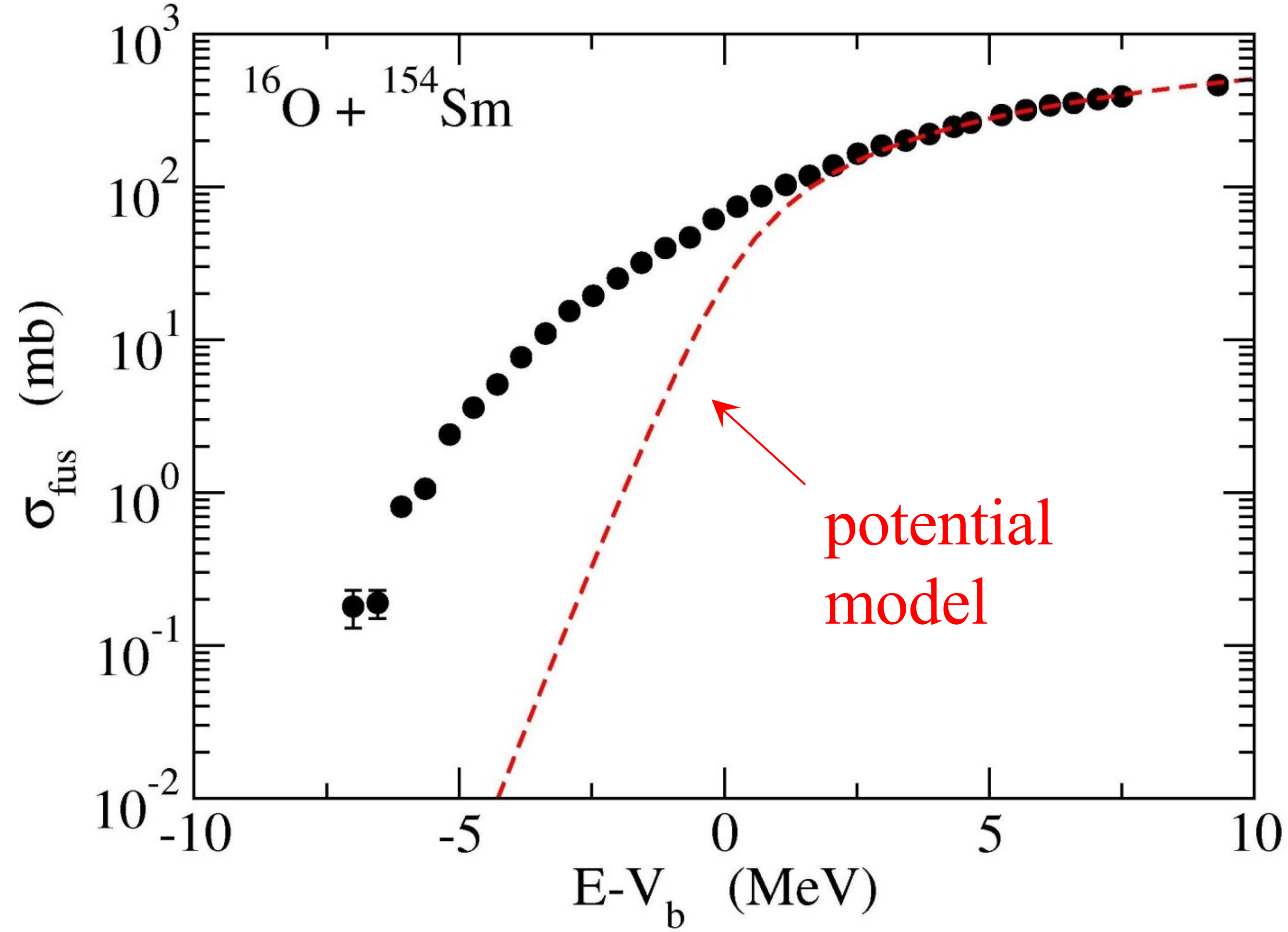
- several types of nucleon transfers

Tunneling probabilities: the exponential $E$ dependence
→ (small) nuclear structure effects are **amplified**

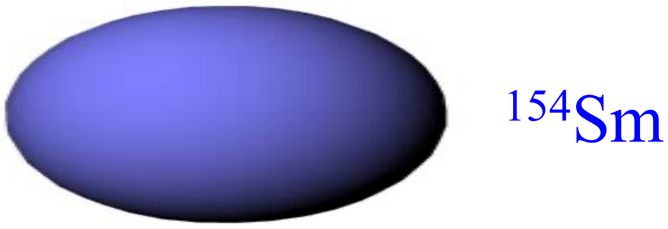
# Discovery of large sub-barrier enhancement of $\sigma_{fus}$ (~80's)

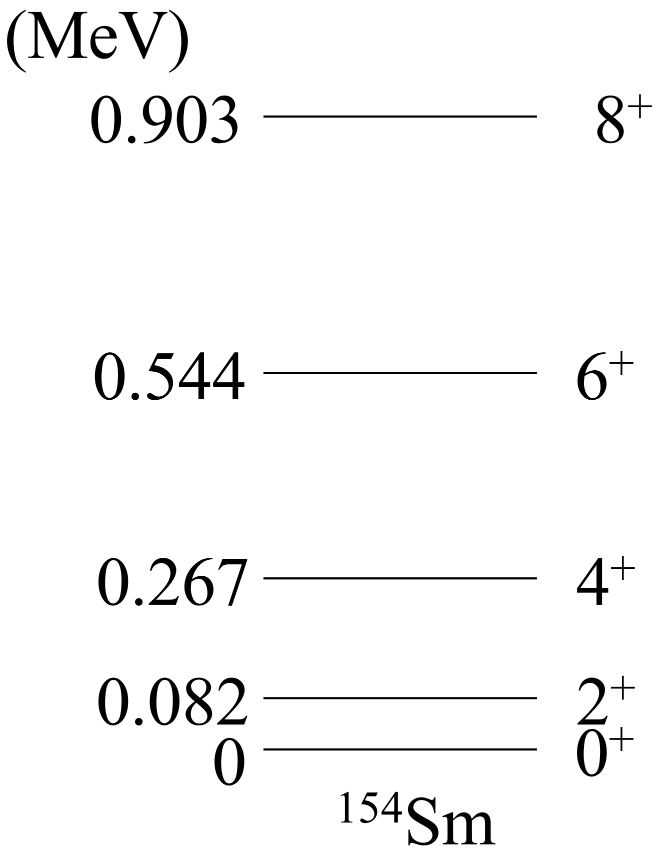
the potential model: inert nuclei (no structure)

$$\boxed{\sigma_{\mathrm{fus}} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)}$$



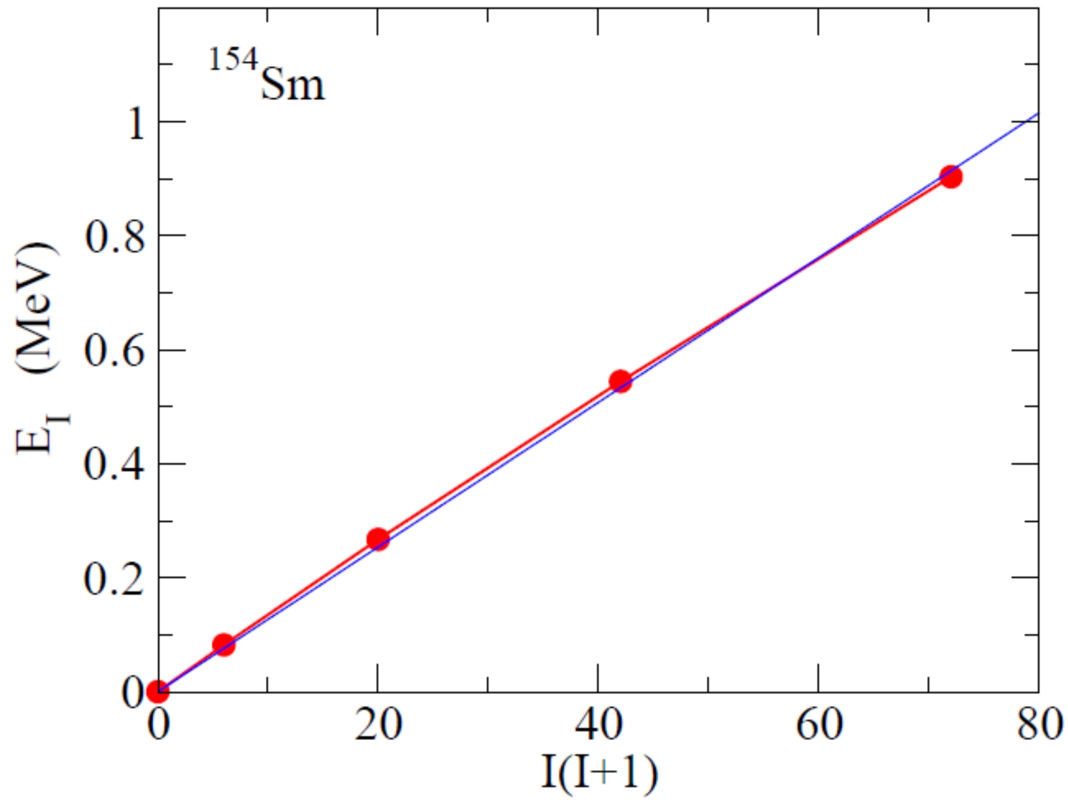
$^{154}$Sm : a typical deformed nucleus




rotational spectrum

$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

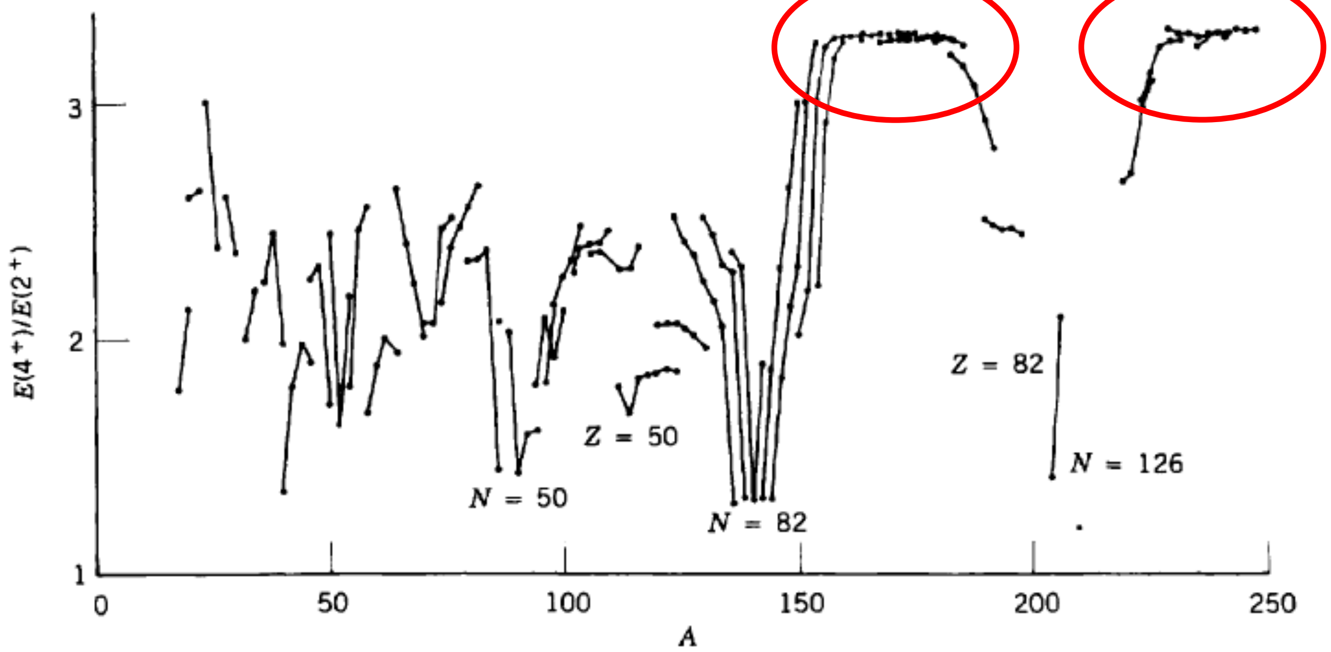



cf. a classical rigid rotor

$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

$$(I = \mathcal{J}\omega, \;\; \omega = \dot{\theta})$$

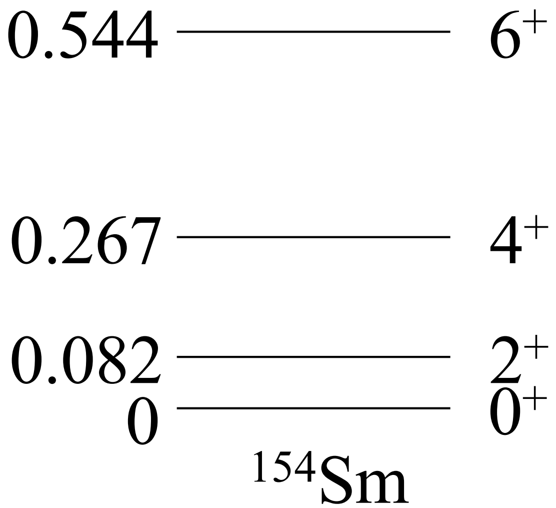
# E(4⁺)/E(2⁺) ratio

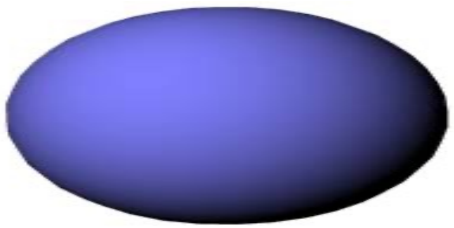



Rotor:
$E(4^+)/E(2^+) \sim 3.3$

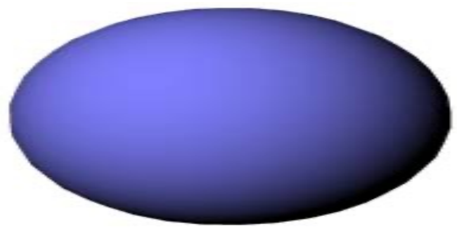
Harmonic Ocs.
$E(4^+)/E(2^+) \sim 2$

K.S. Krane, "Introductory Nuclear Physics"



## Effects of nuclear deformation



$^{154}$Sm

(MeV)

| | |
|---|---|
| 0.903 | $8^+$ |
| 0.544 | $6^+$ |
| 0.267 | $4^+$ |
| 0.082 | $2^+$ |
| 0 | $0^+$ |

$^{154}$Sm

rotational spectrum

a small rotational energy

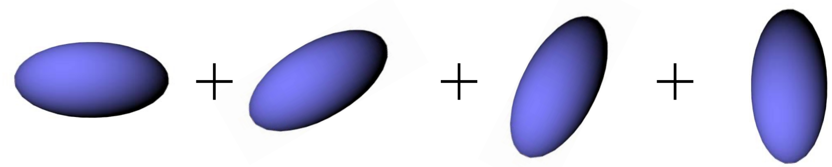
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

→ a large moment of inertia $J$

→ rotation: a slow deg. of freedom

$$E_{\rm rot} \sim E_{2^+} = 82 \text{ keV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

$\Psi_{0^+} =$

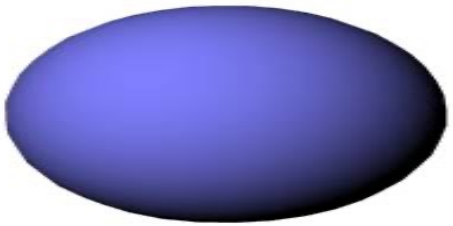


→ a spherical state in the lab. system

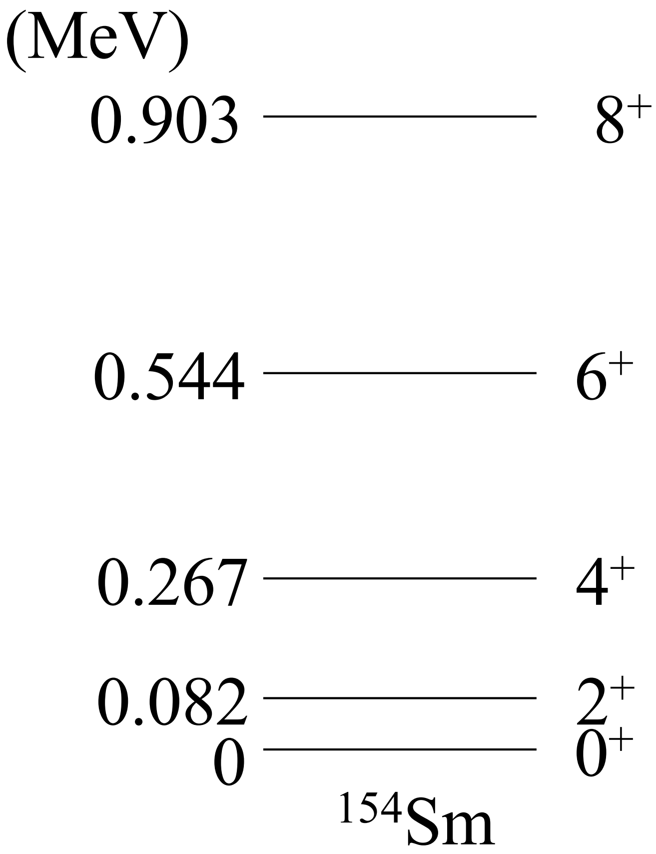
fix the orientation angle to calculate the fusion probability

"a snapshot of a rotating nucleus"

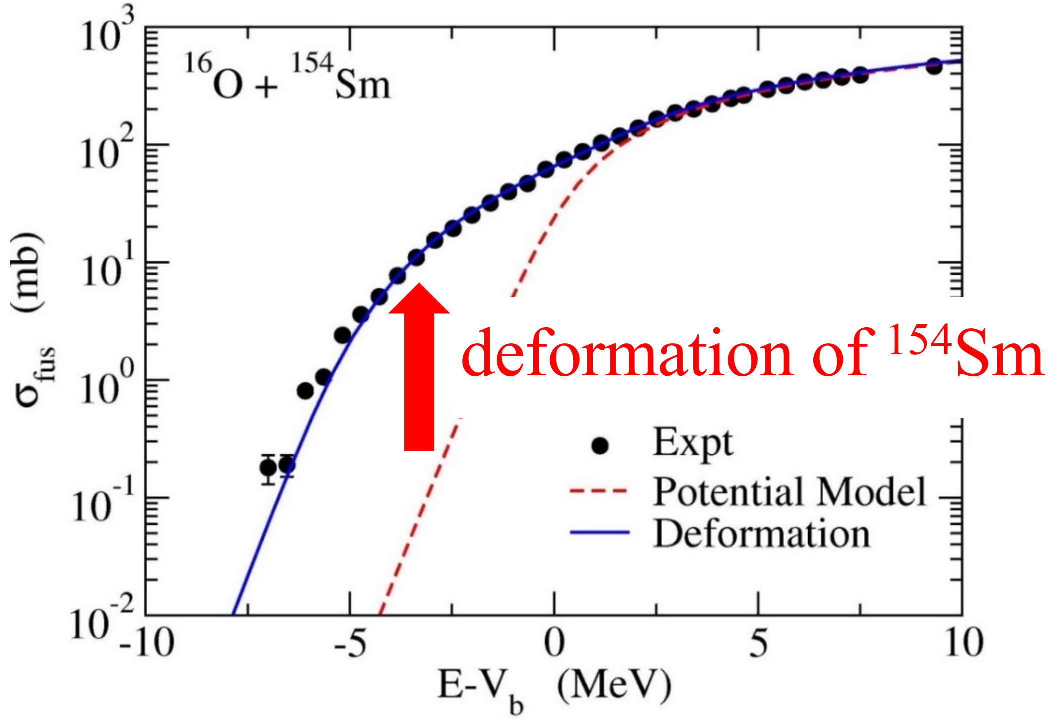
# $^{154}$Sm : a typical deformed nucleus





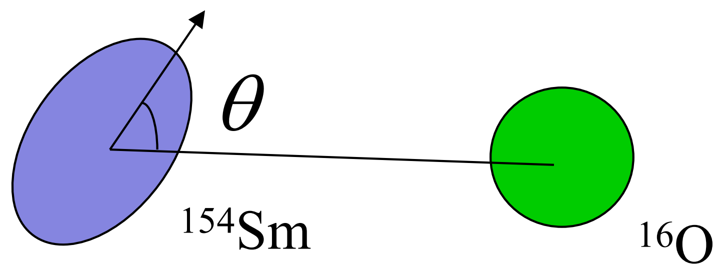



rotational spectrum



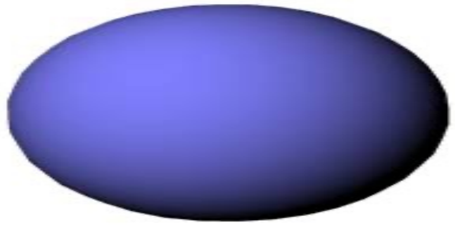

K. H. and N. Takigawa,
Prog. Theo. Phys.128 ('12)1061.

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

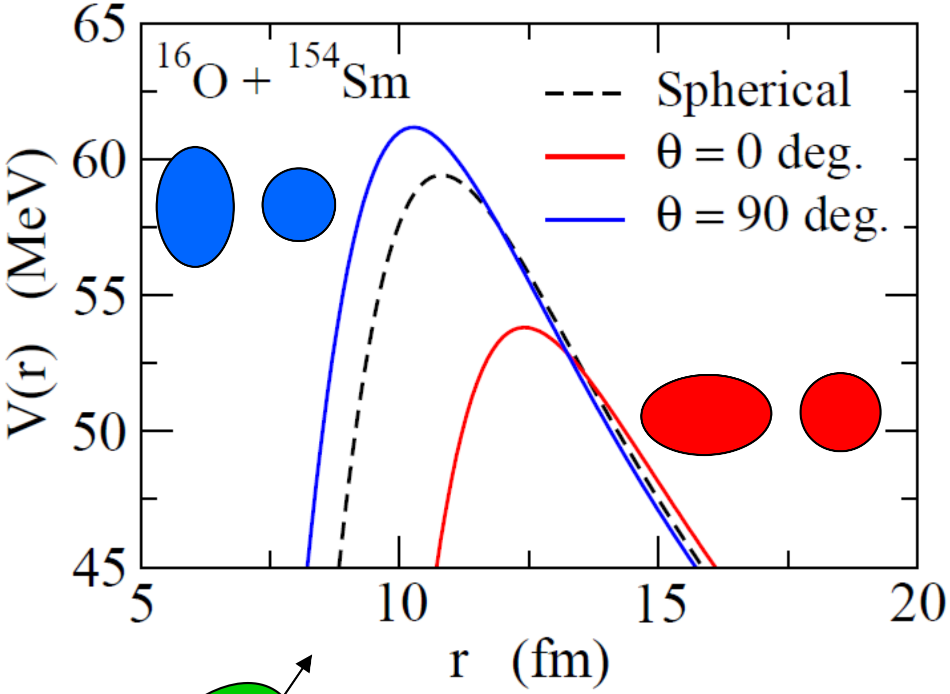


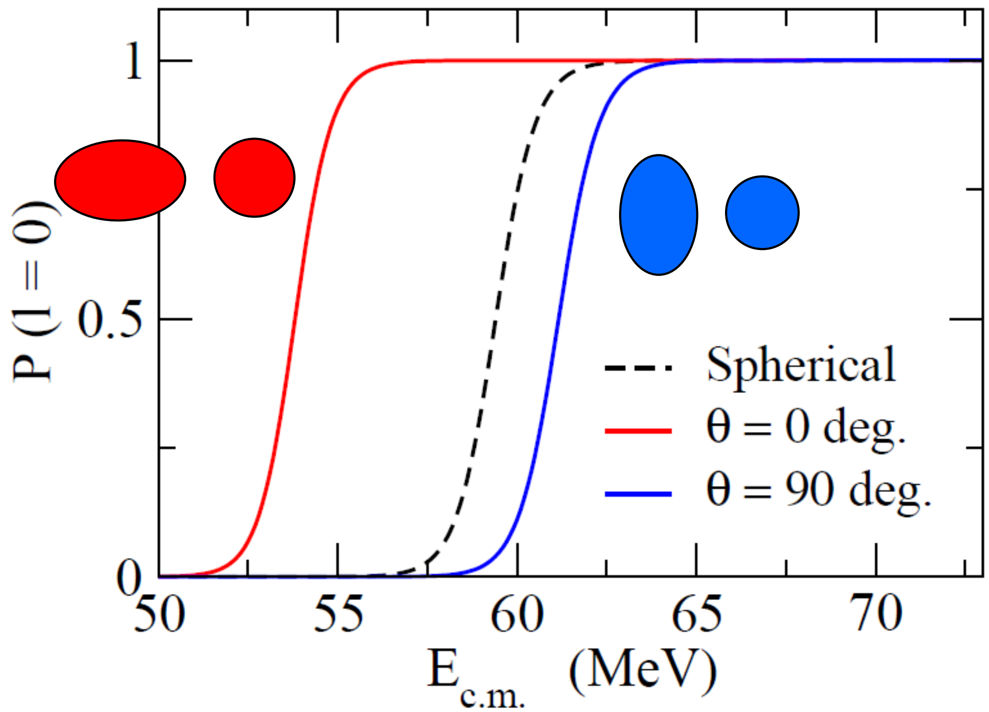
# Effects of nuclear deformation

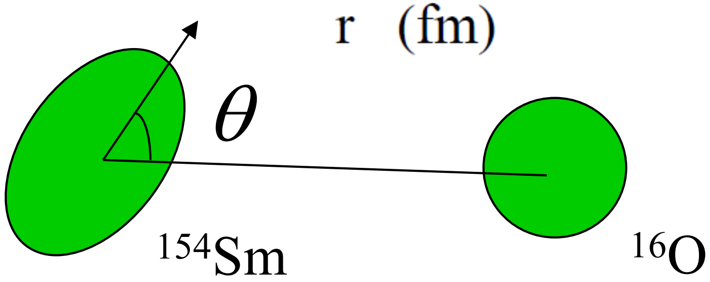
$^{154}$Sm : a typical deformed nucleus



$^{154}$Sm

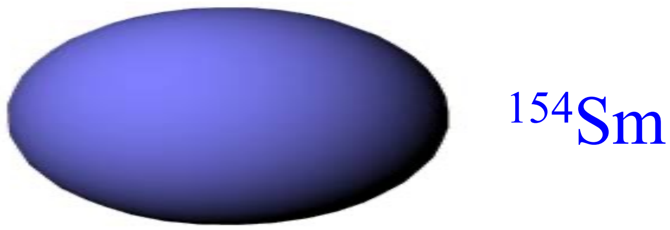


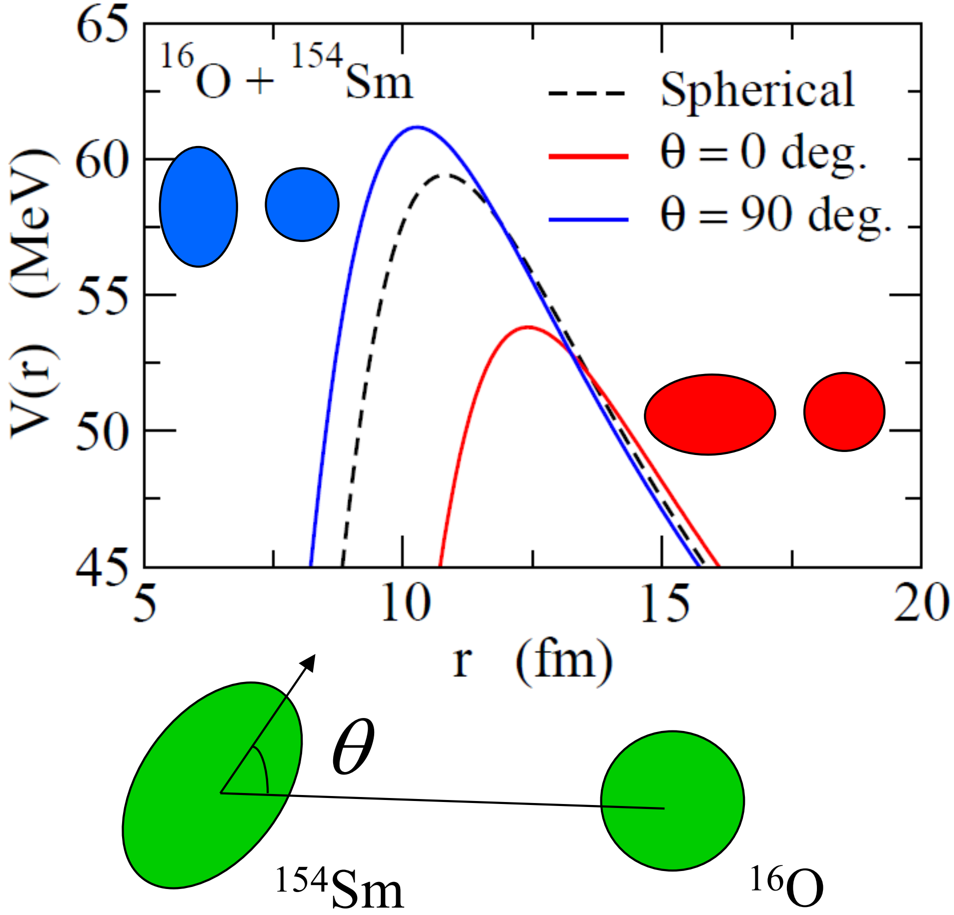



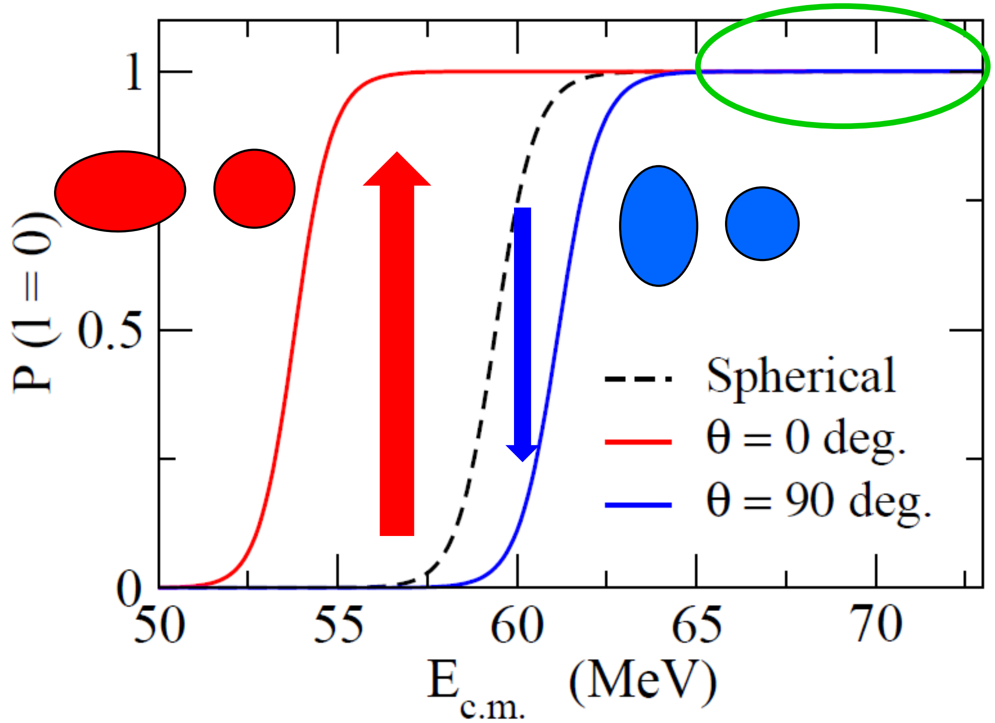



# Effects of nuclear deformation

$^{154}$Sm : a typical deformed nucleus

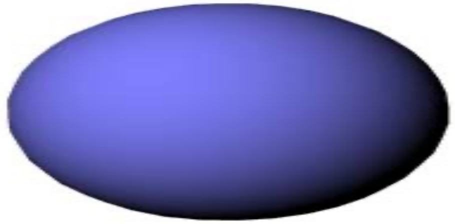


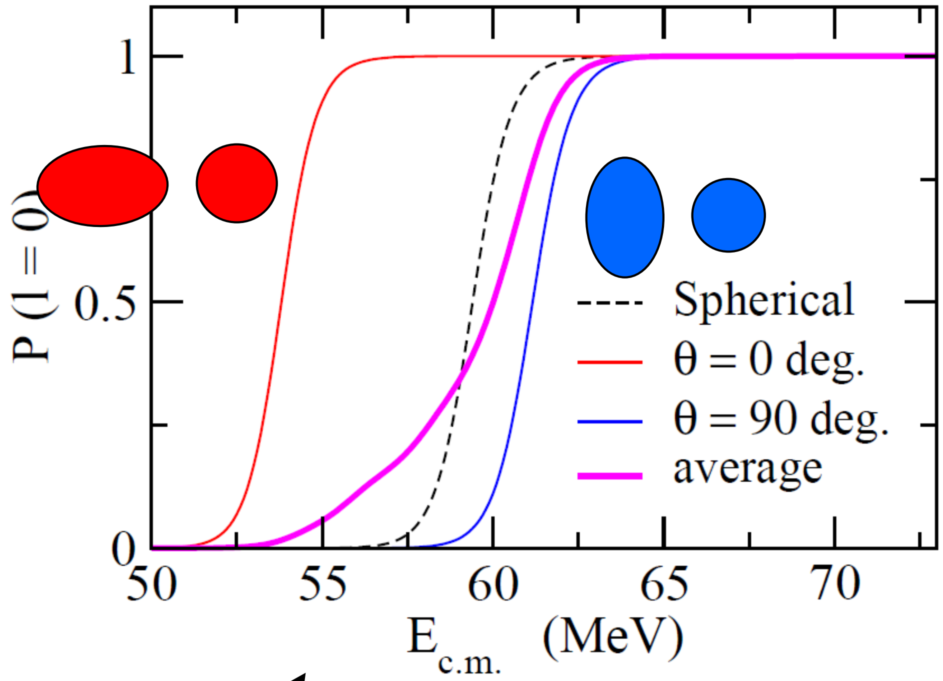



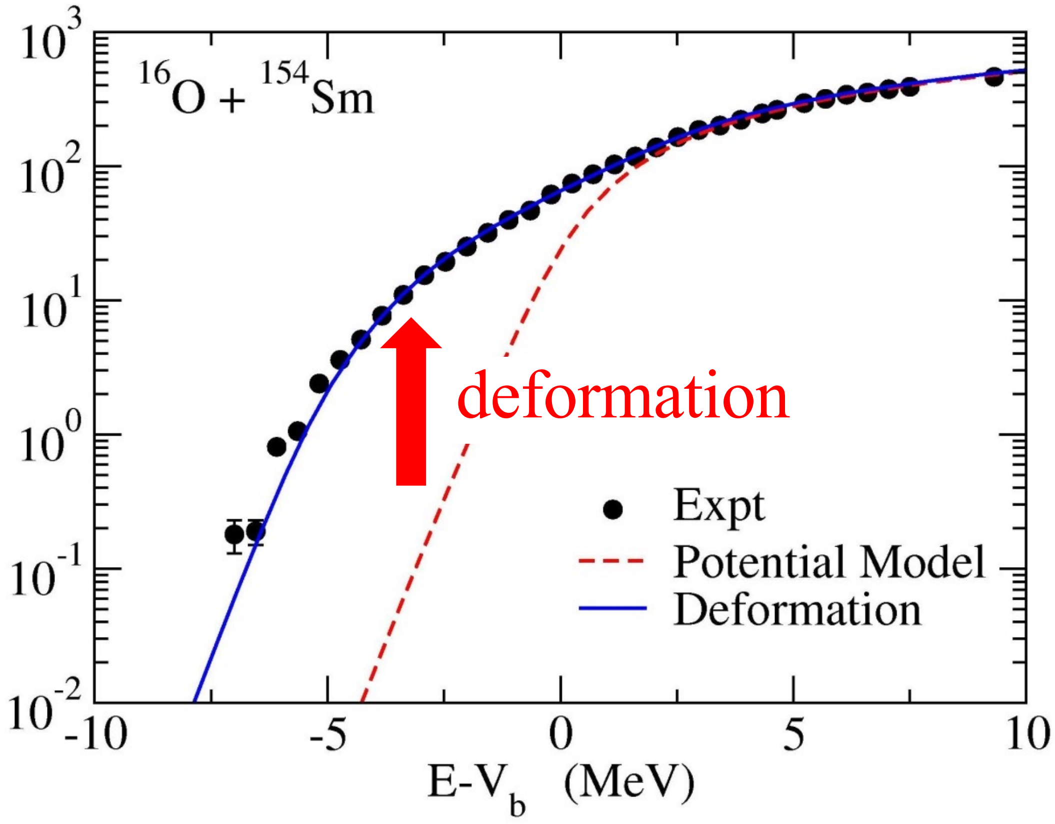
# Effects of nuclear deformation

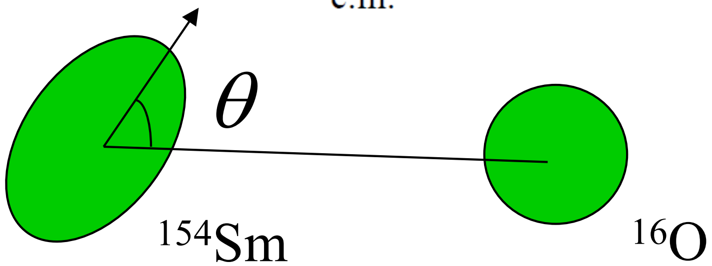
$^{154}$Sm : a typical deformed nucleus



$^{154}$Sm

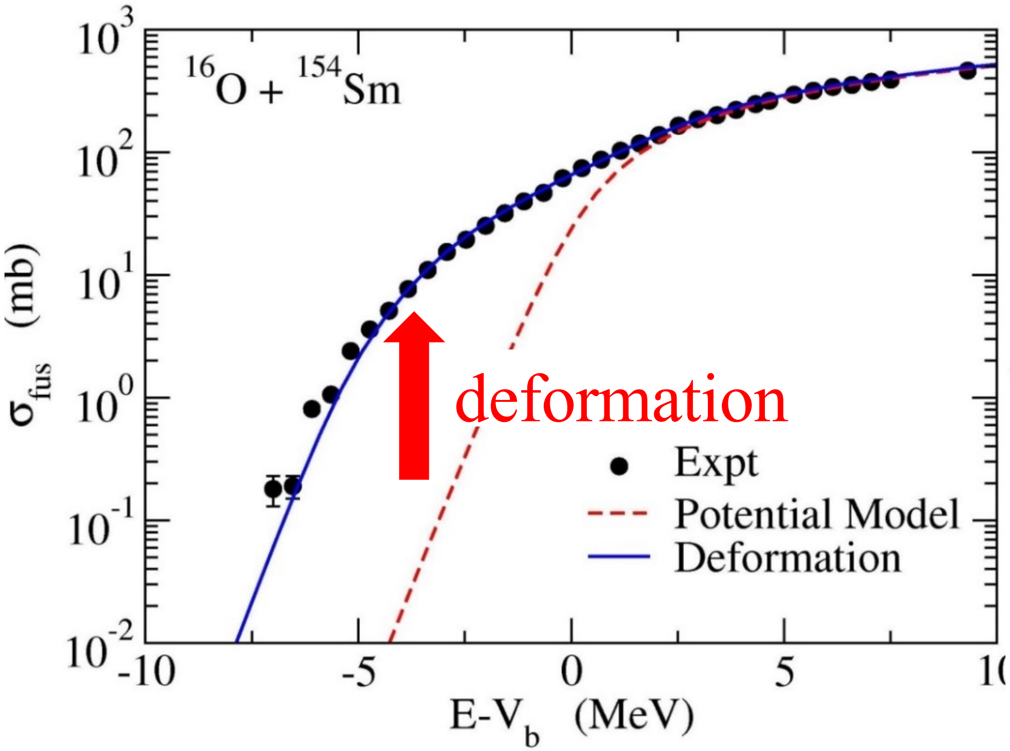


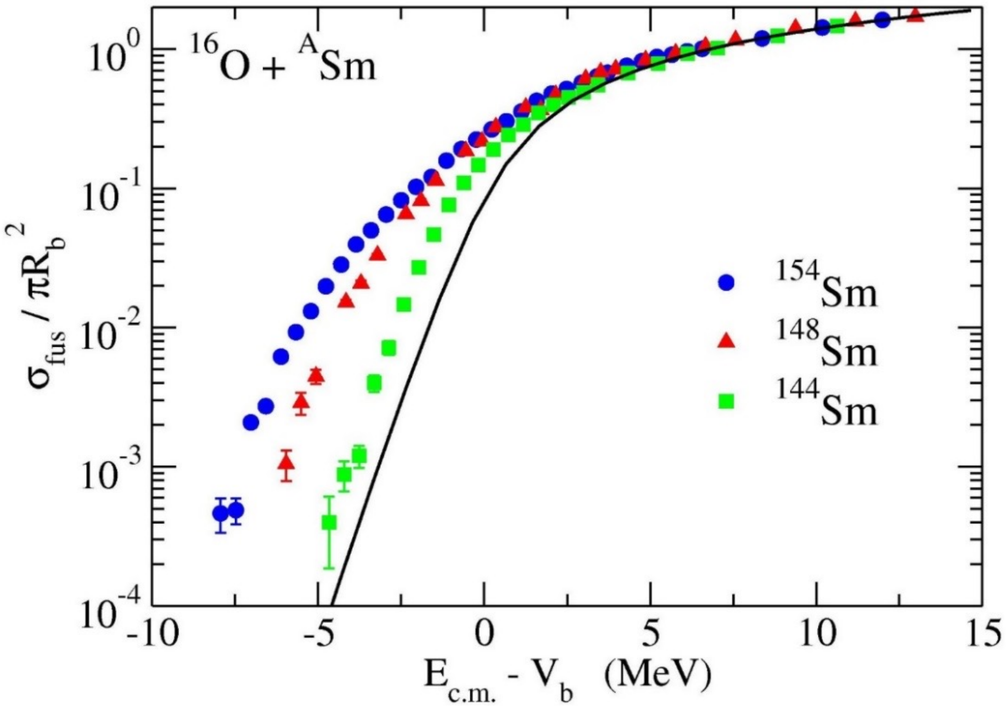




$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$

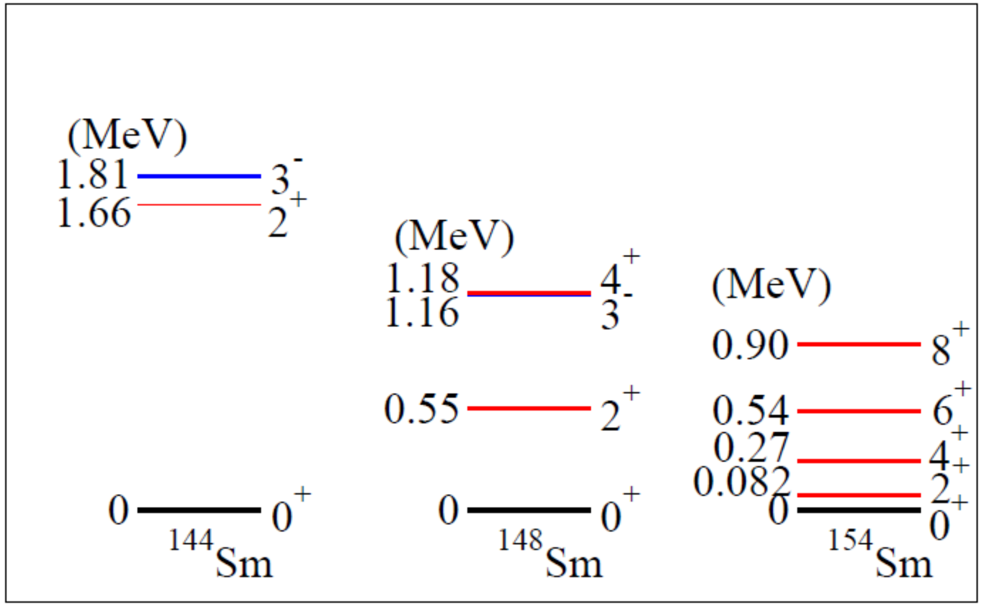
Fusion: strong interplay between nuclear structure and reaction



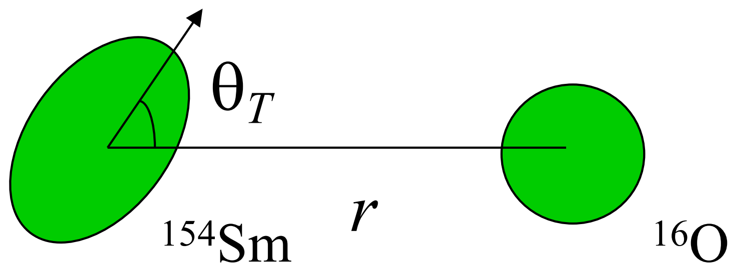

similar enhancement
for non-deformed nuclei



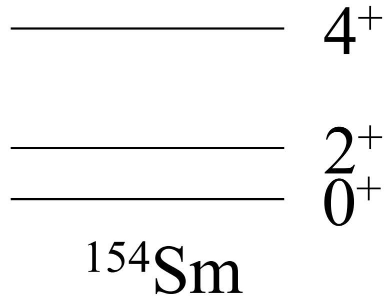

strong correlation
with nuclear spectrum
→ coupling assisted
tunneling phenomena



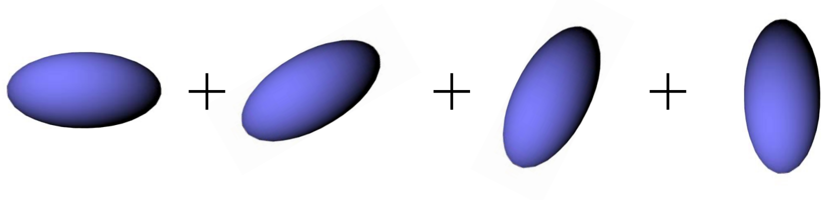
# Adiabatic approximation in low-energy nuclear reactions




potential:
$V(r) \rightarrow V(r, \theta_T)$




$\Psi_{0+} =$



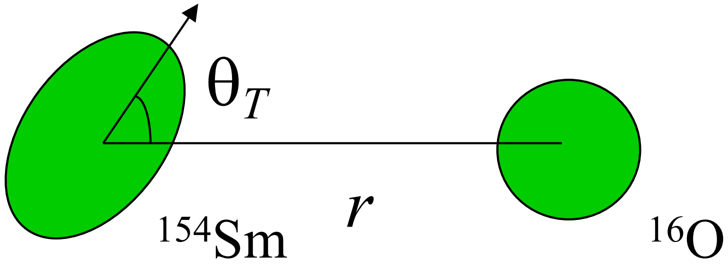

Fusion cross sections:

$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\text{fus}}[E; V(r,\theta_T)]$$
$$= 2\pi \int_{-1}^{1} d(\cos\theta_T)\underbrace{|Y_{00}(\theta_T)|^2}_{\text{ground state wave function}}\sigma_{\text{fus}}[E; V(r,\theta_T)]$$

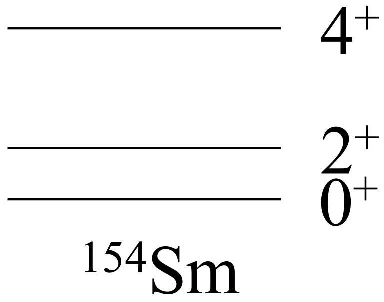
Elastic scattering:

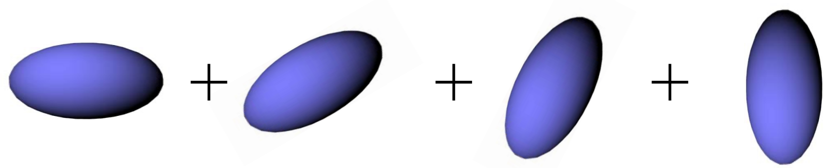
$$\frac{d\sigma_{\text{el}}}{d\Omega} = |f(\theta)|^2; \qquad f(\theta) = \int_0^1 d(\cos\theta_T) f_{\text{el}}[\theta; V(r,\theta_T)]$$

# Adiabatic approximation in low-energy nuclear reactions




potential:
$V(r) \rightarrow V(r, \theta_T)$




$\Psi_{0^+} =$ 

$$\sigma_{\mathrm{fus}}(E) = 2\pi \int_{-1}^{1} d(\cos\theta_T) \underline{|Y_{00}(\theta_T)|^2} \sigma_{\mathrm{fus}}[E; V(r, \theta_T)]$$

ground state wave function

→ NB: this does not mean that a nucleus is not excited
during reactions

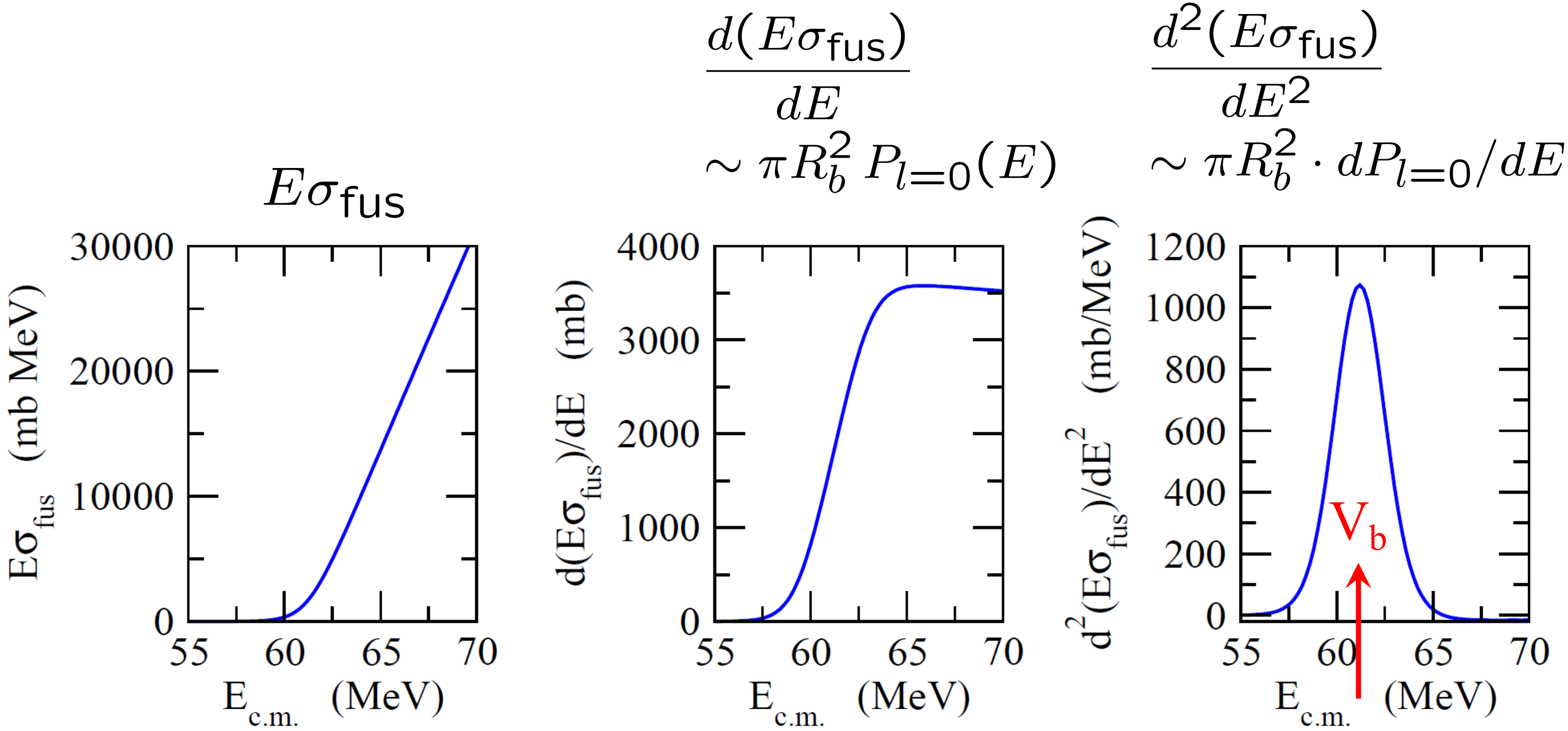
fixed orientation: totally uncertain of the angular momentum

$$|\theta_T\rangle = \sum_{I=0}^{\infty} \langle \theta_T | Y_{I0} \rangle | Y_{I0} \rangle$$

# Fusion barrier distribution

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

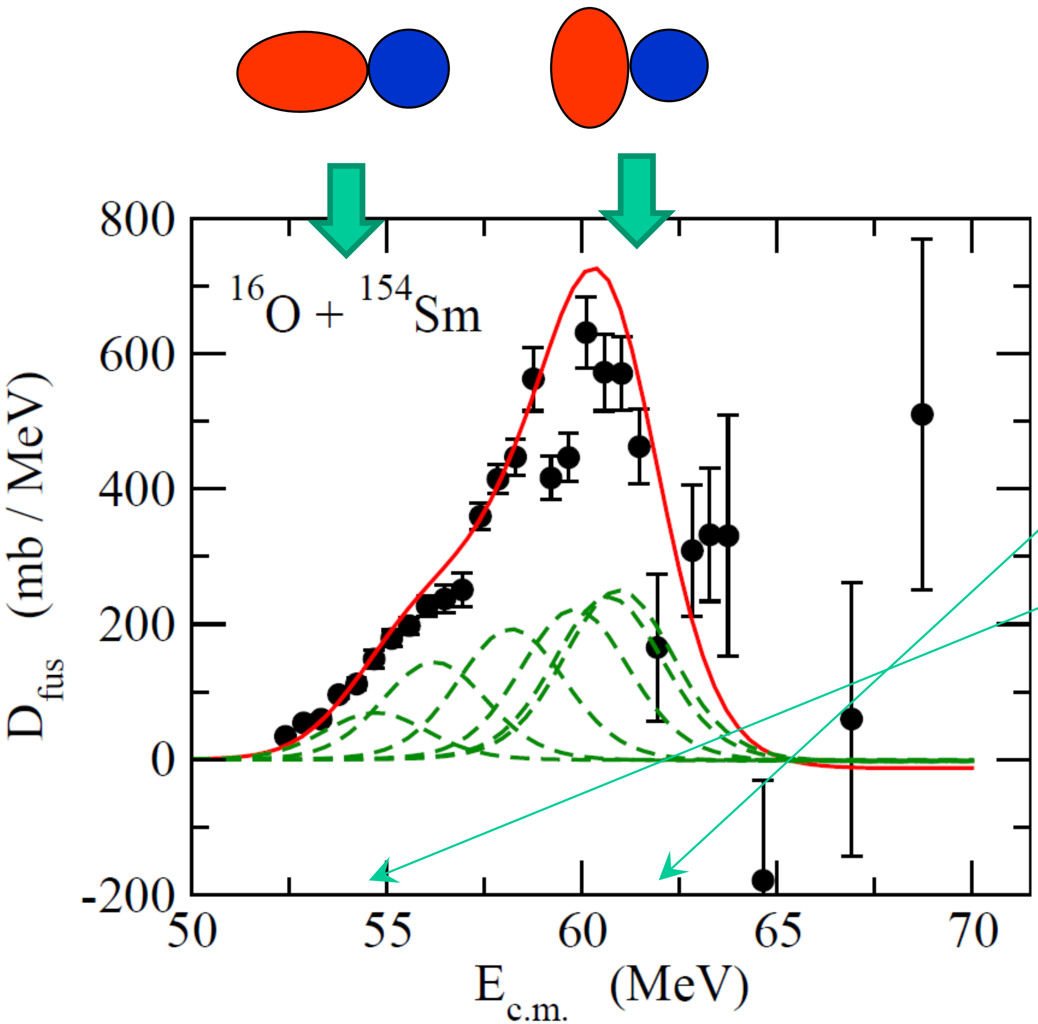
N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

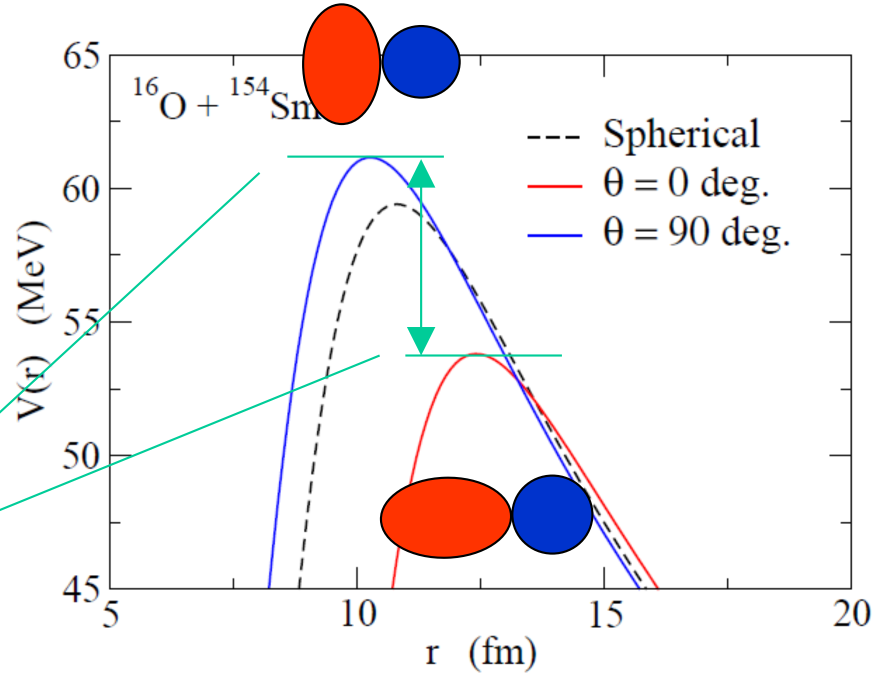



K.H. and N. Takigawa, PTP128 ('12) 1061

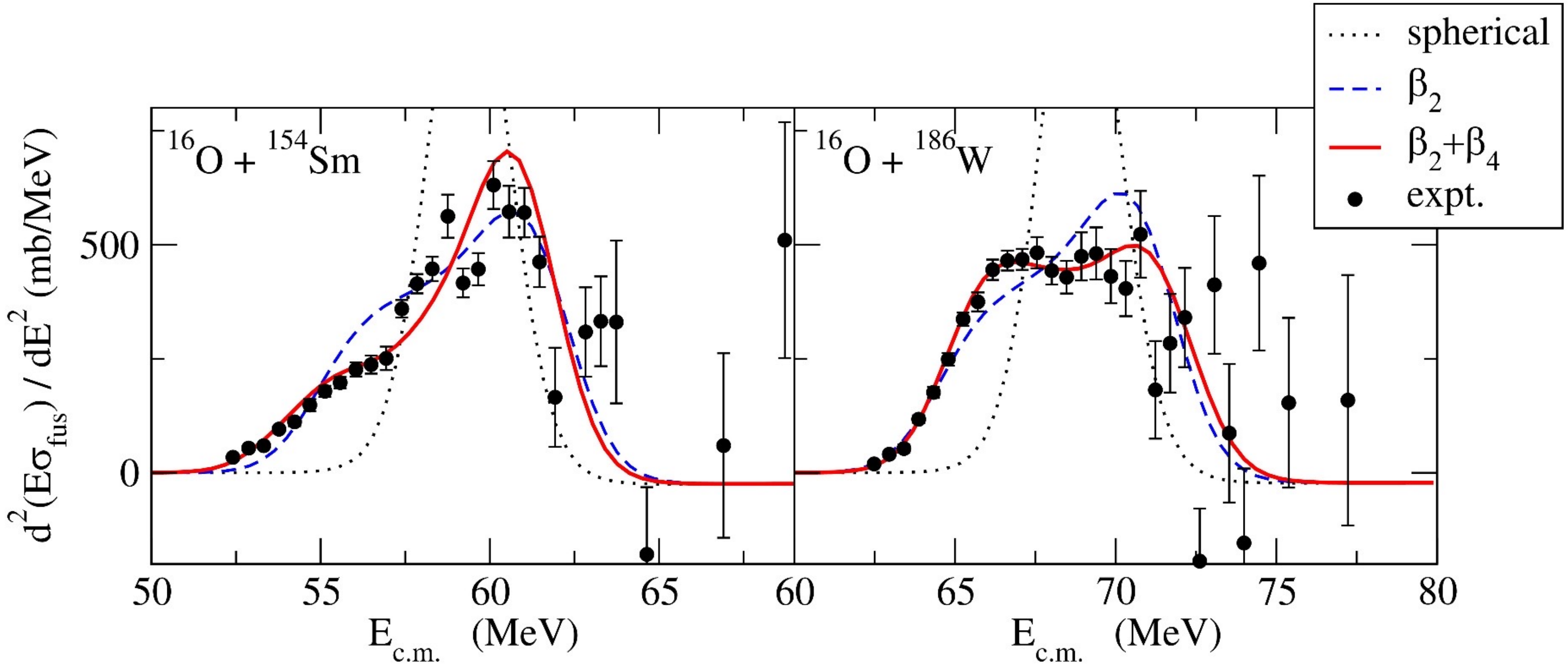
✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2} \sim \pi R_b^2 \frac{dP_{l=0}}{dE}$$



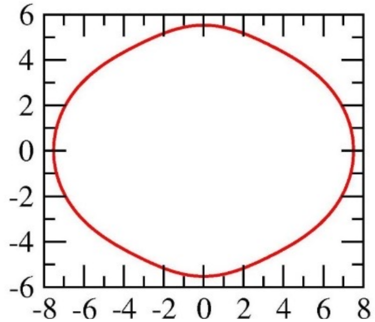

Data: J.R. Leigh et al.,
PRC52 ('95) 3151

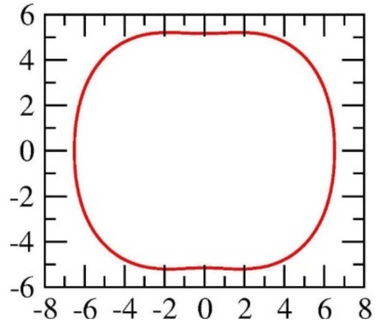


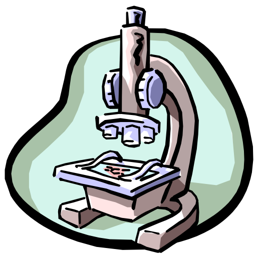

$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

$$\beta_2 = 0.33 \qquad \beta_2 = 0.29$$

$$\beta_4 = +0.05 \qquad \beta_4 = -0.03$$



sensitive to the sign of $\beta_4$!

→ Fusion as a quantum tunneling microscope for nuclei

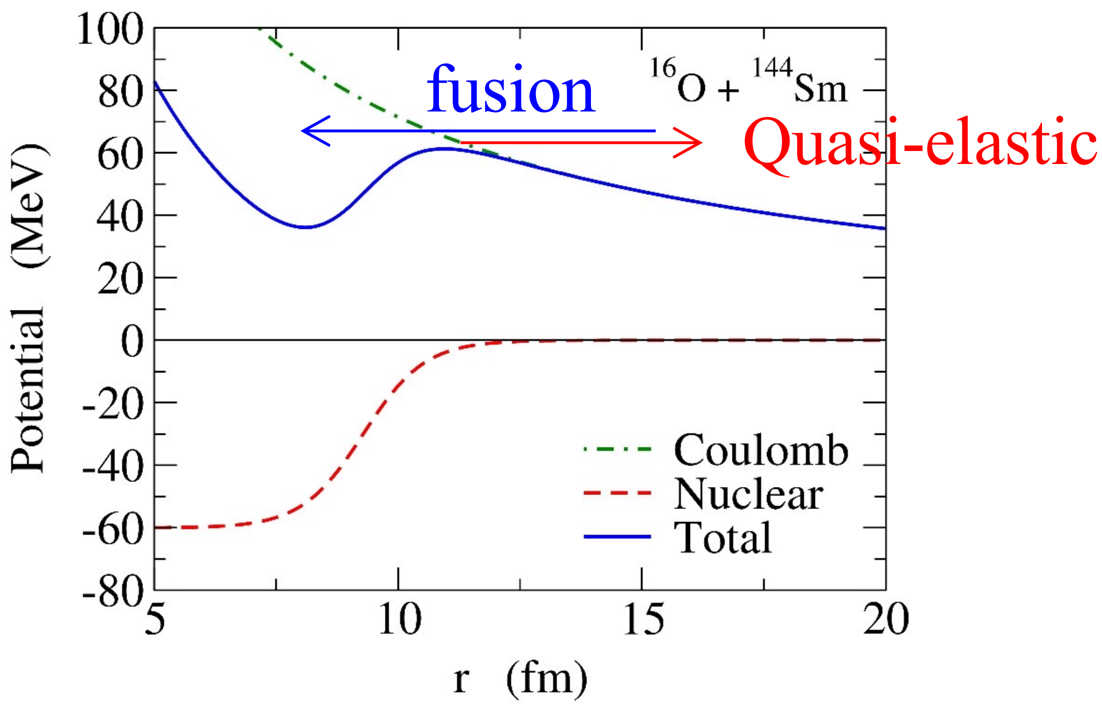
# Quasi-elastic barrier distribution

$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}\right)$$

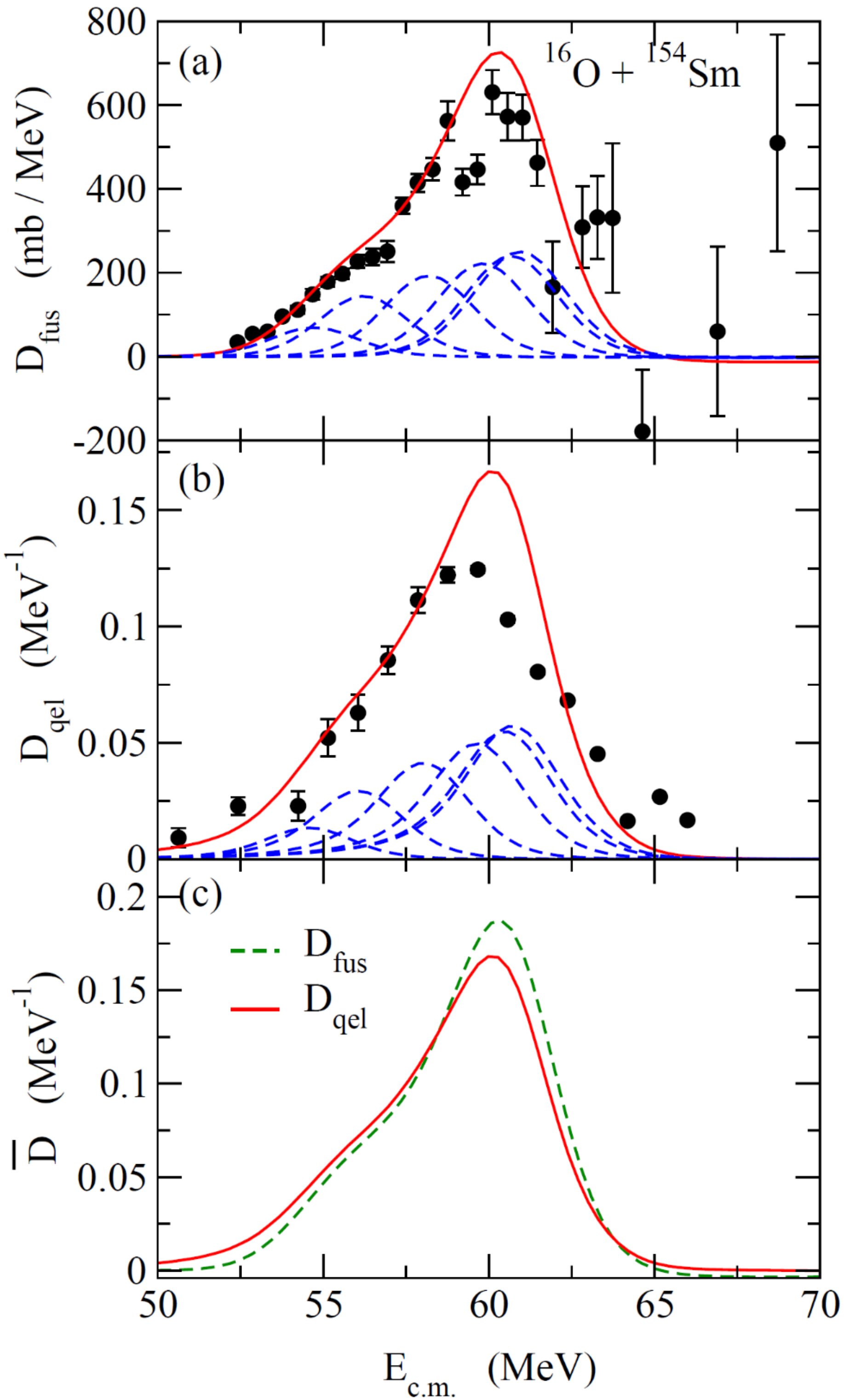
H. Timmers et al., NPA584('95)190

## Quasi-elastic scattering:

A sum of all the reaction processes other than fusion
(elastic + inelastic + transfer + ……)




$$P_{l=0}(E) = 1 - R_{l=0}(E) \sim 1 - \frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}$$




$D_{fus}$ and $D_{qel}$: behave similarly to each other

cf. Application to reactions relevant to SHE

$^{48}Ca + ^{248}Cm \rightarrow {}^{296}_{116}Lv^*$

T. Tanaka et al.,
JPSJ 87 ('18) 014201
PRL124 ('20) 052502

$^{51}V + ^{248}Cm \rightarrow {}^{299}119^*$

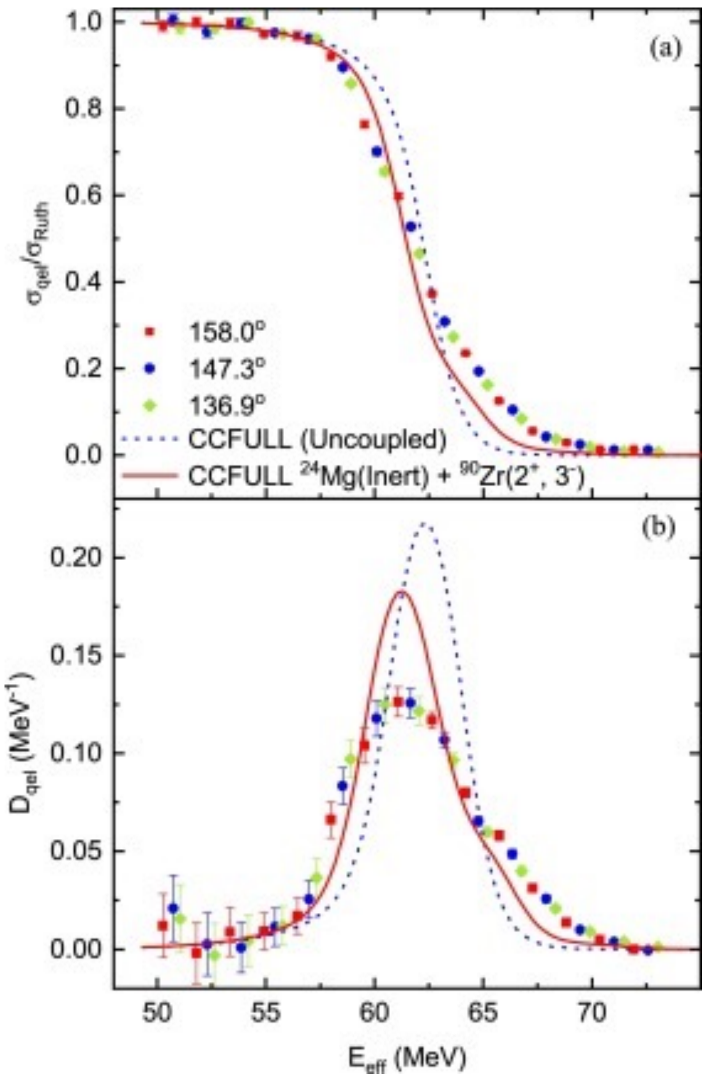
M. Tanaka et al.,
JPSJ 91 ('22) 084201

K.H. and N. Rowley, PRC69('04)054610

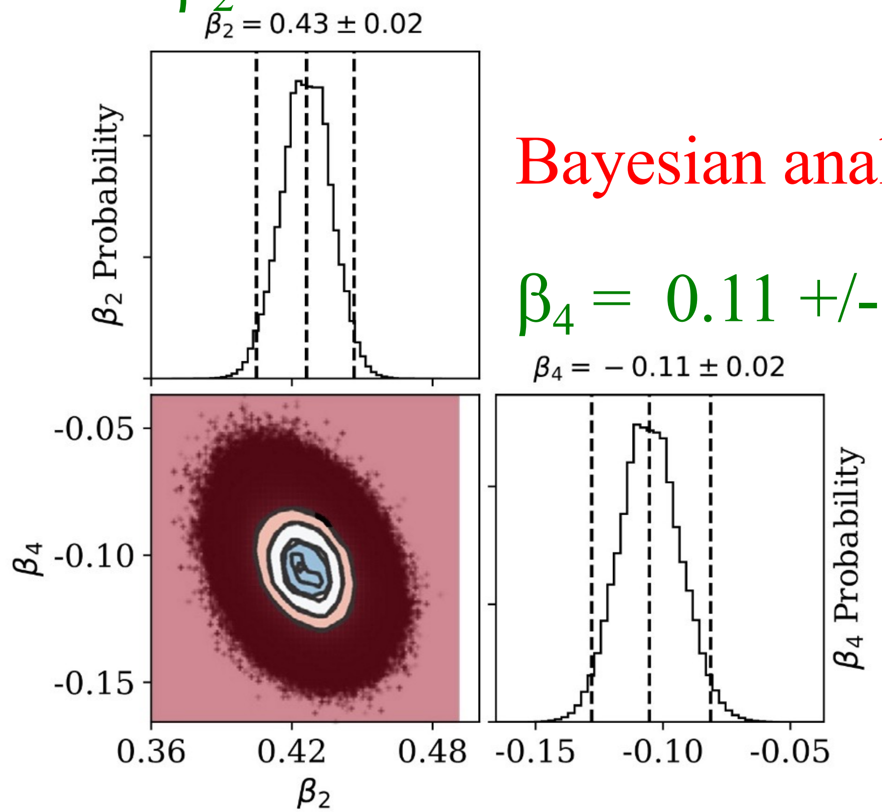
# Determination of $\beta_4$ of $^{24}$Mg with quasi-elastic barrier distributions

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).

$^{24}$Mg + $^{90}$Zr




$\beta_2$ = 0.43 +/- 0.02




Bayesian analysis

$\beta_4$ = 0.11 +/- 0.02

high precision determination of $\beta_4$
→ for the first time

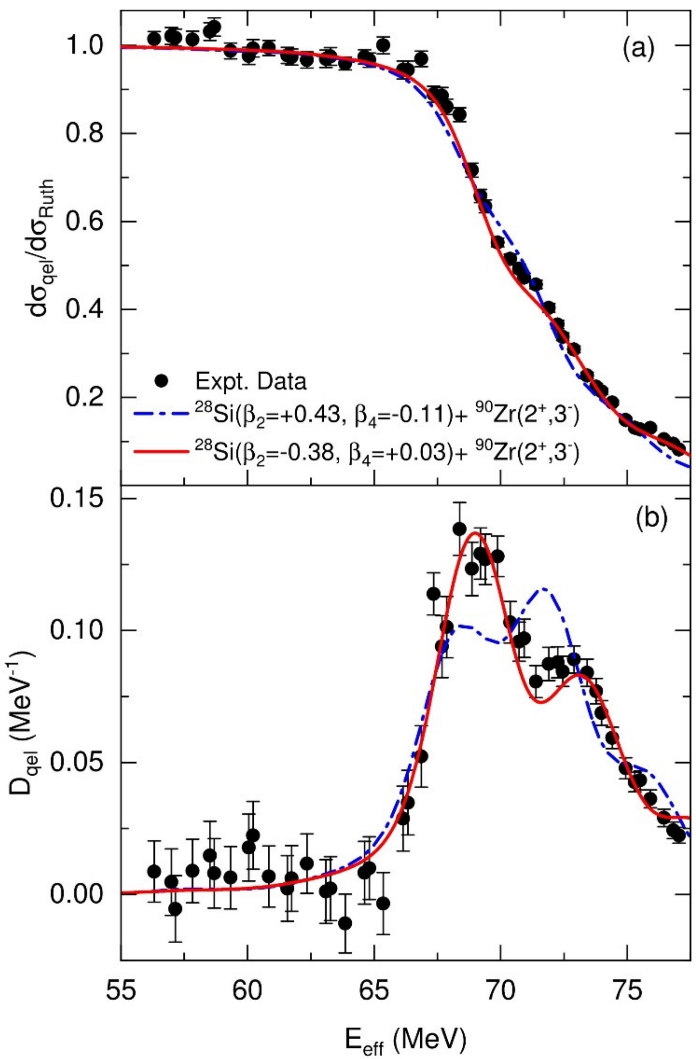
cf. (p,p'): $\beta_4$ = -0.05 +/- 0.08

R. De Swiniarski et al., PRL23, 317 (1969)

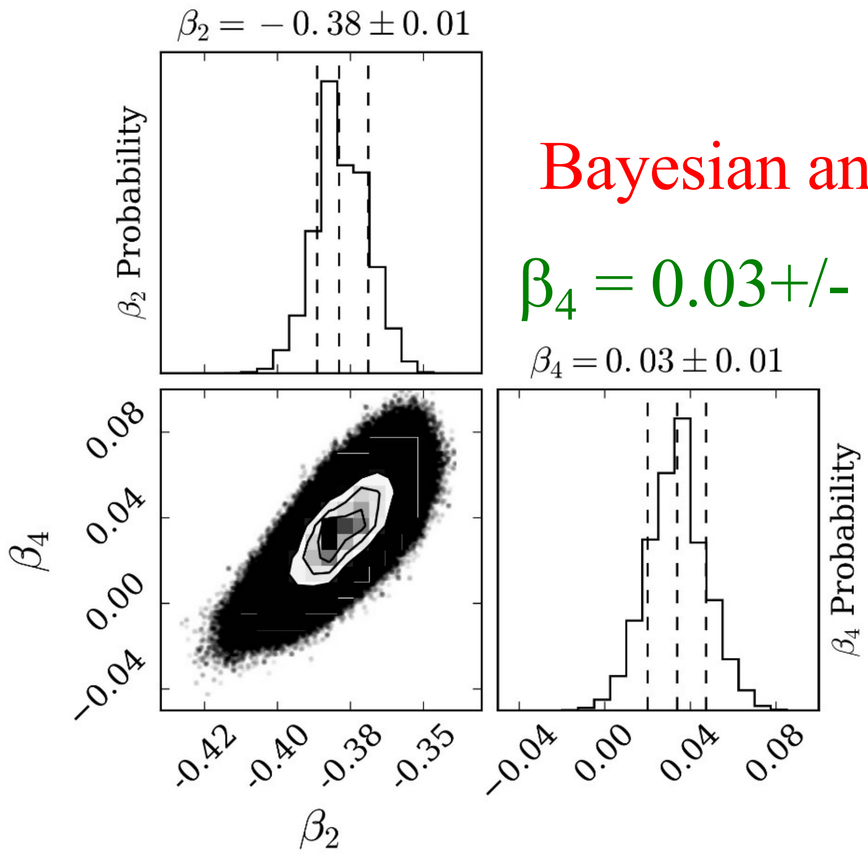
# Determination of $\beta_4$ of $^{28}$Si with quasi-elastic scattering

Y.K. Gupta, V.B. Katariya, G.K. Prajapati, K.H., et al., PLB845, 138120 (2023).

$^{28}$Si + $^{90}$Zr




$\beta_2$ = -0.38 +/- 0.01




Bayesian analysis

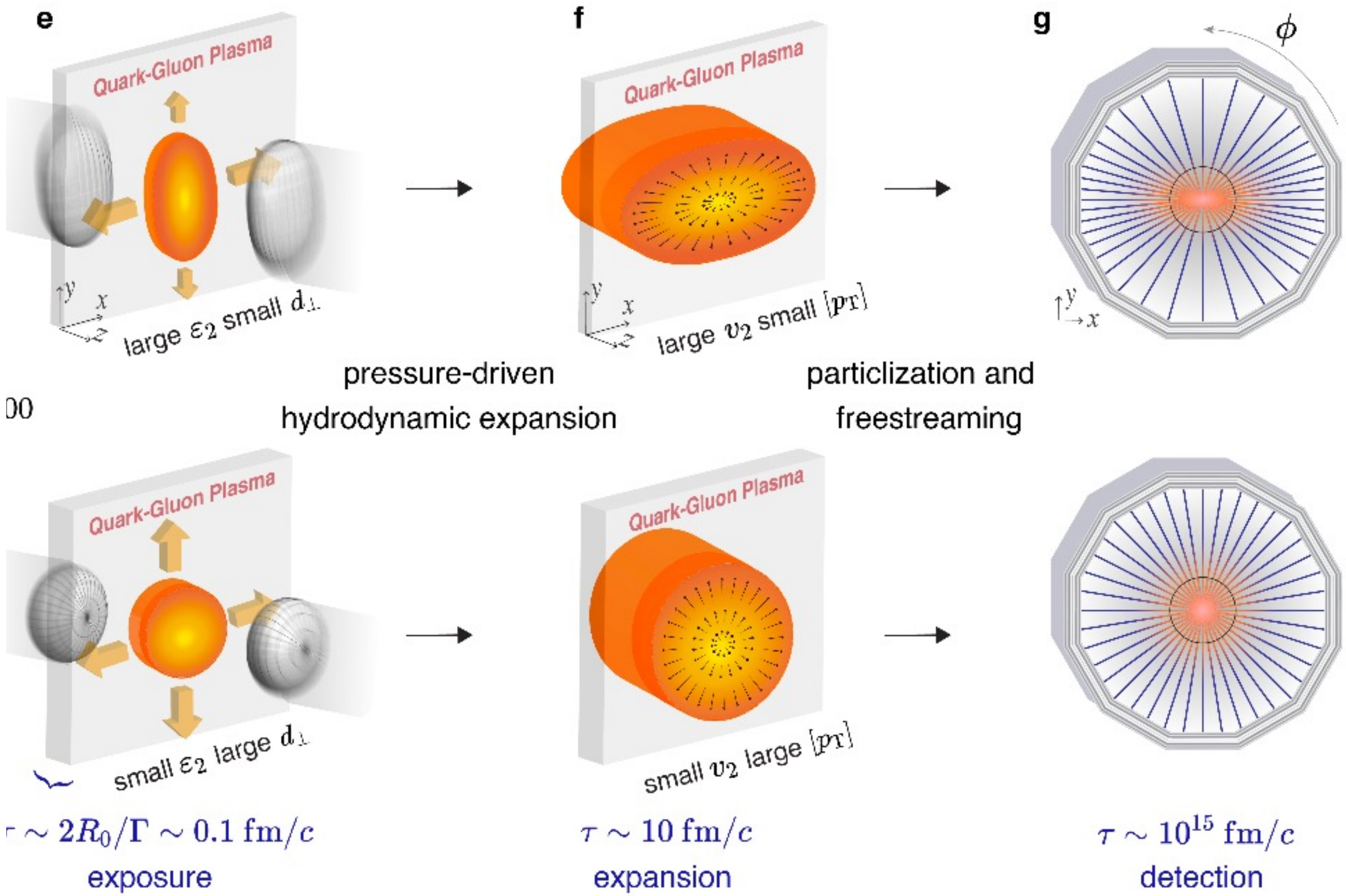
$\beta_4$ = 0.03+/- 0.01

cf. (n,n'): $\beta_4$ = 0.20 +/- 0.05

G. Haouat et al., PRC30, 1795 (1984)

# Probing nuclear shapes in Rel. H.I. collisions

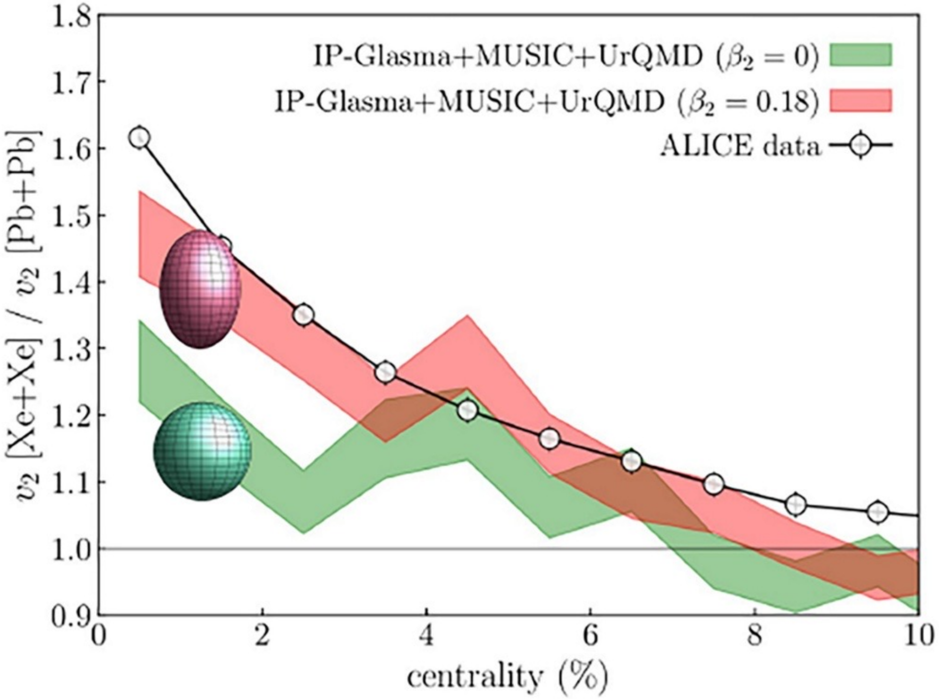
**the initial shape of QGP** **expansion** **detection**

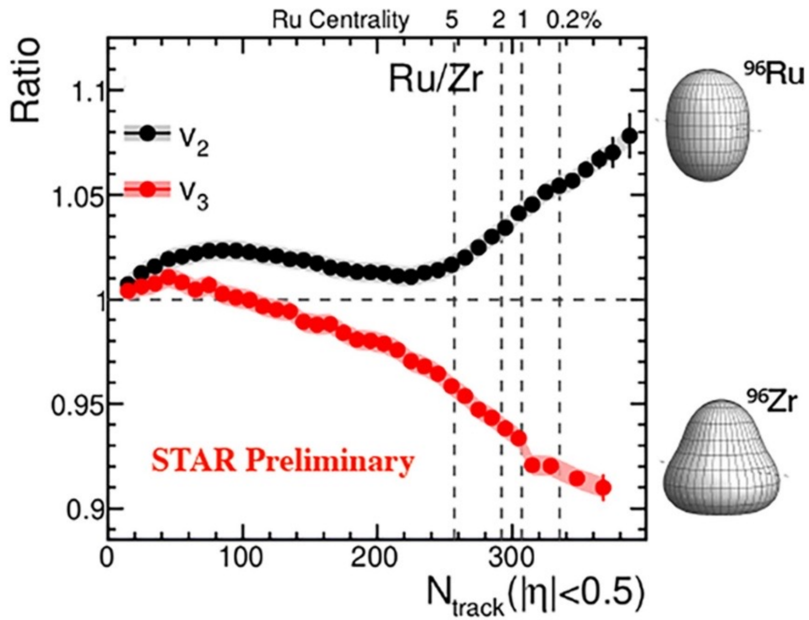



M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)

# Probing nuclear shapes in Rel. H.I. collisions

This has opened up a strong intersection between nuclear structure and relativistic HI collisions ← topics of this workshop



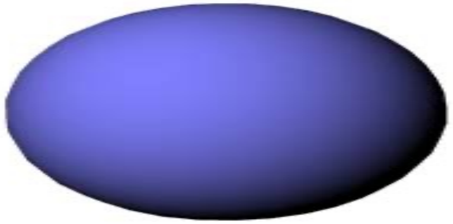




J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

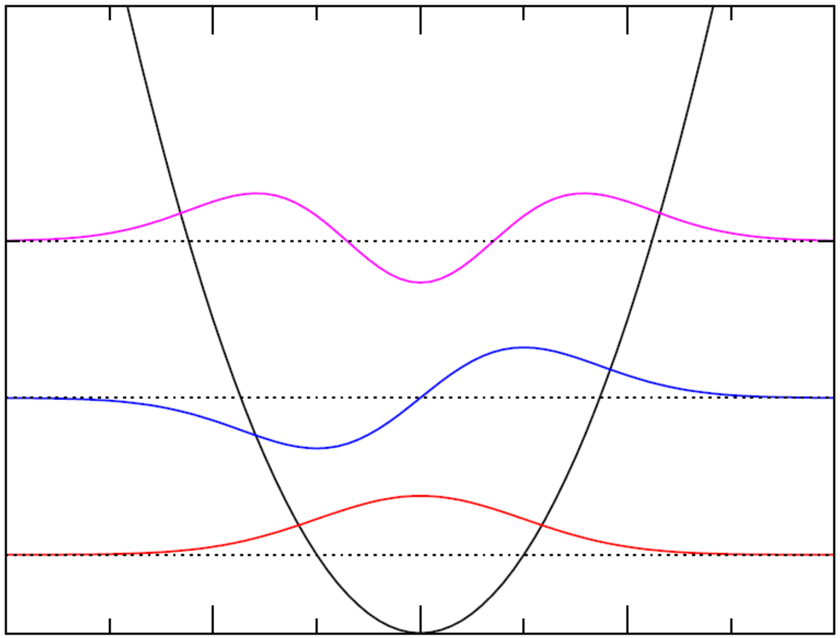
- ✓ quadrupole deformation $\beta_2$
- ✓ octupole deformation $\beta_3$
- ✓ triaxial deformation $\gamma$
- ✓ cluster structure

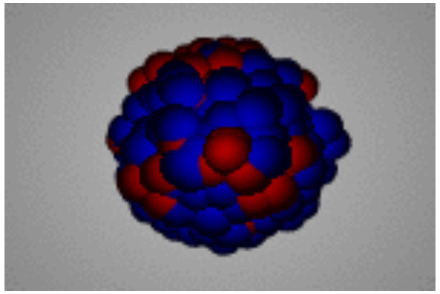
# Probing nuclear shapes in Rel. H.I. collisions

So far, the main focus has been on a static deformation of a nucleus



There also exist several dynamical deformations of a nucleus





$\langle \beta \rangle = 0$

but fluctuates around β=0

$\to \langle \beta^2 \rangle \neq 0$

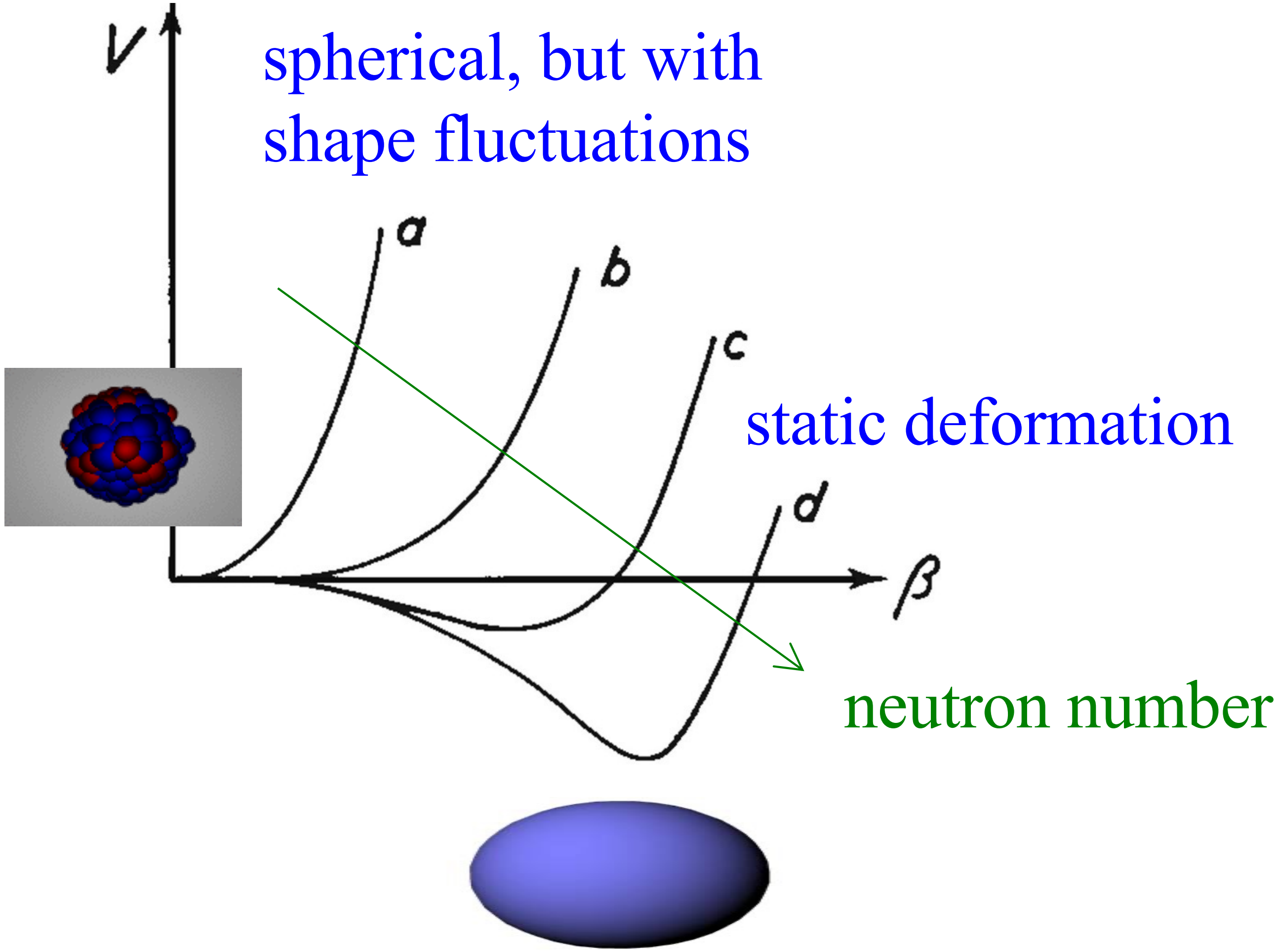
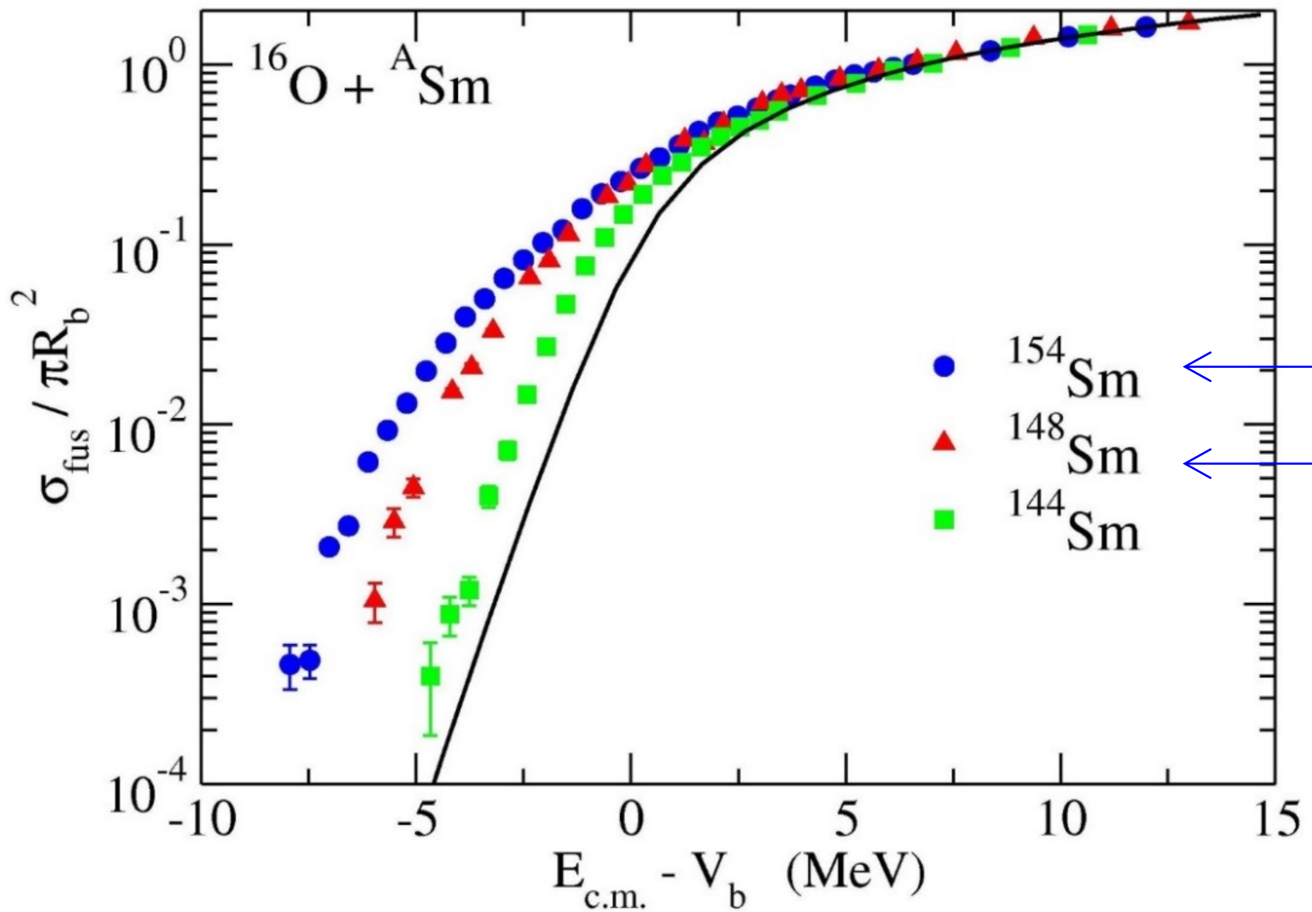



figure:
B.R. Mottelson,
Nobel Lecture

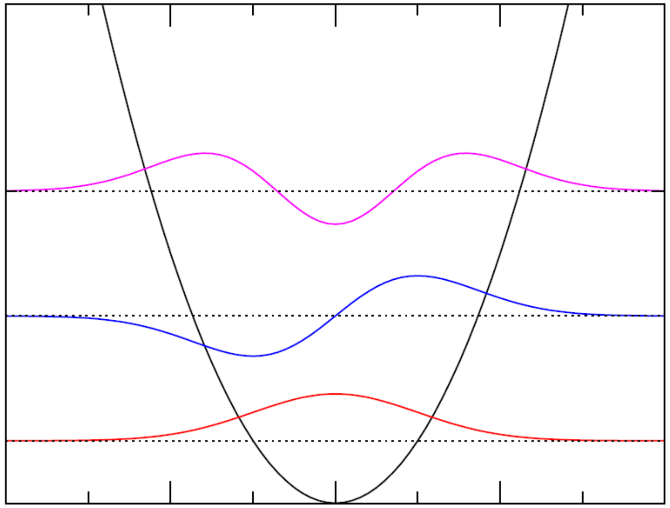
Surface vibrations of a spherical nucleus can still significantly affect H.I. sub-barrier fusion reactions

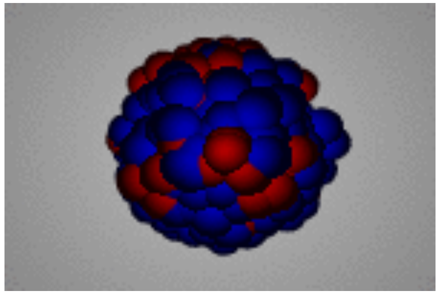



static deformation

spherical, but with vibration

# Probing nuclear shapes in Rel. H.I. collisions





$$\langle \beta \rangle = 0$$

but fluctuates around β=0

the adiabatic approximation for vibrations:

H. Esbensen, Nucl. Phys. A352, 147 (1981)

**FUSION AND ZERO-POINT MOTIONS**

H. ESBENSEN

*Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

Received 14 July 1980

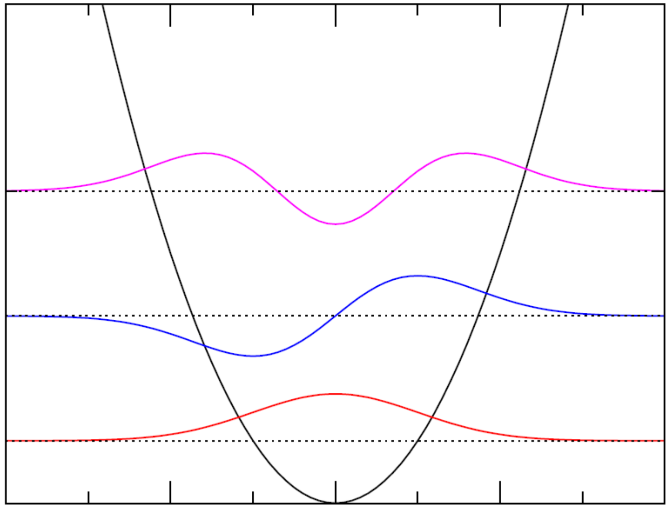
$$\sigma_{\rm fus}(E) \sim \int d\beta\, w(\beta)\sigma_0(E;\beta)$$

$$w(\beta) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\beta^2/2\sigma^2}$$

$$\sqrt{\langle \beta_\lambda^2 \rangle} = \frac{4\pi}{3ZR^\lambda}\sqrt{\frac{B(E\lambda : 0^+ \to \lambda^\pi)}{e^2}}$$

K.H. and N. Takigawa, PTP128 (‘12) 1061

# Probing nuclear shapes in Rel. H.I. collisions



In most of the cases, the vibrational motion is not slow for fusion:

$$E_{\rm vib} \sim 2 \text{ MeV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

$\rightarrow$ but this can be very slow for rel. H.I. collisions!

the adiabatic approximation for vibrations:

H. Esbensen, Nucl. Phys. A352, 147 (1981)

**FUSION AND ZERO-POINT MOTIONS**

H. ESBENSEN

*Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

Received 14 July 1980

$$\sigma_{\rm fus}(E) \sim \int d\beta\, w(\beta)\sigma_0(E;\beta)$$

$$w(\beta) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\beta^2/2\sigma^2}$$

$$\sqrt{\langle\beta_\lambda^2\rangle} = \frac{4\pi}{3ZR^\lambda}\sqrt{\frac{B(E\lambda : 0^+ \to \lambda^\pi)}{e^2}}$$

K.H. and N. Takigawa, PTP128 ('12) 1061

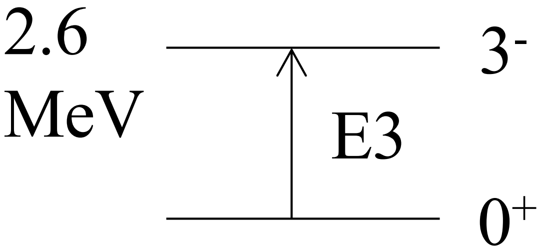
# Probing nuclear shapes in Rel. H.I. collisions

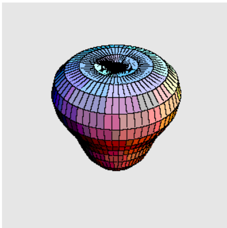
Very recent preprint: D. Xu et al., arXiv: 2504.19644

**A "breathing" octupole $^{208}$Pb nucleus: resolving the elliptical-to-triangular azimuthal anisotropy puzzle in ultracentral relativistic heavy ion collisions**

Duoduo Xu,[1] Shujun Zhao,[1] Hao-jie Xu,[2,3,*] Wenbin Zhao,[4,5,†] Huichao Song,[1,6,7,‡] and Fuqiang Wang[8,§]

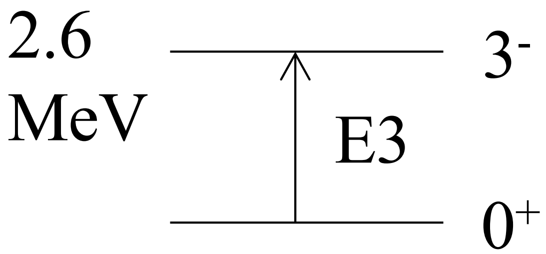
octupole vibration of $^{208}$Pb

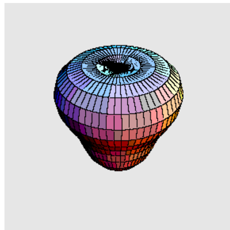


https://web-docs.gsi.de/~wolle
/TELEKOLLEG /KERN/index-s.html

# What is the nature of the $3_1^-$ state in $^{208}$Pb?



https://web-docs.gsi.de/~wolle
/TELEKOLLEG /KERN/index-s.html

Linear response theory:

$V_{\rm ext}(t)$ on to the spherical g.s.

$$\rho_0(r) \to \rho_0(r) + c(t)\delta\rho(r)Y_{30}(\theta)$$

$^{208}_{82}$Pb$_{126}$: a double magic nucleus

$\longrightarrow$ spherical

$$\langle\beta_3\rangle = 0$$

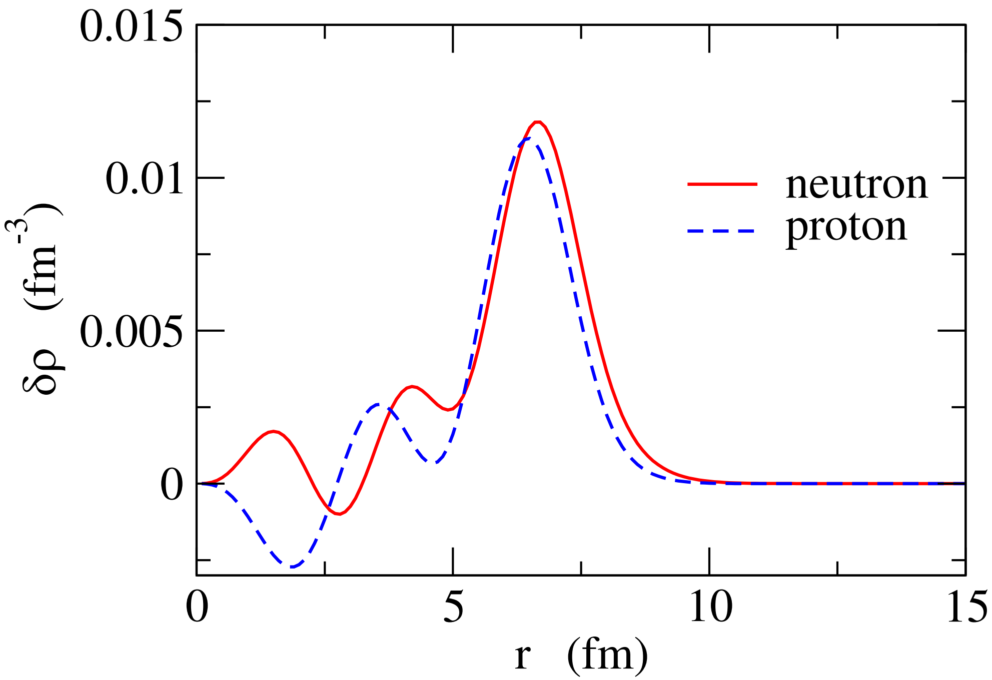
but, the shape can fluctuate

$\longrightarrow$ octupole vibration

$$\langle(\beta_3)^2\rangle \neq 0$$

from the measured B(E3)

$$\sqrt{\langle\beta_3^2\rangle} = \frac{4\pi}{3ZR^3}\sqrt{\frac{B(E3:0^+\to 3^-)}{e^2}} \sim 0.16$$




✓ surface peaked

✓ n and p in phase (isoscalar)

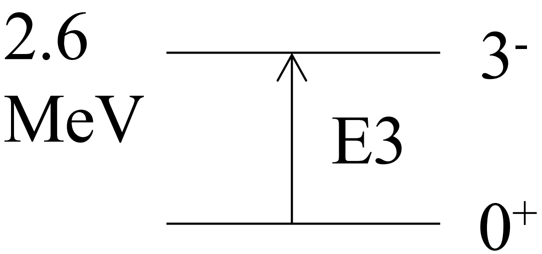
# Probing nuclear shapes in Rel. H.I. collisions

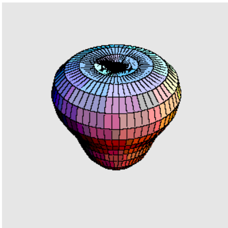
Very recent preprint: D. Xu et al., arXiv: 2504.19644

**A "breathing" octupole $^{208}$Pb nucleus: resolving the elliptical-to-triangular azimuthal anisotropy puzzle in ultracentral relativistic heavy ion collisions**

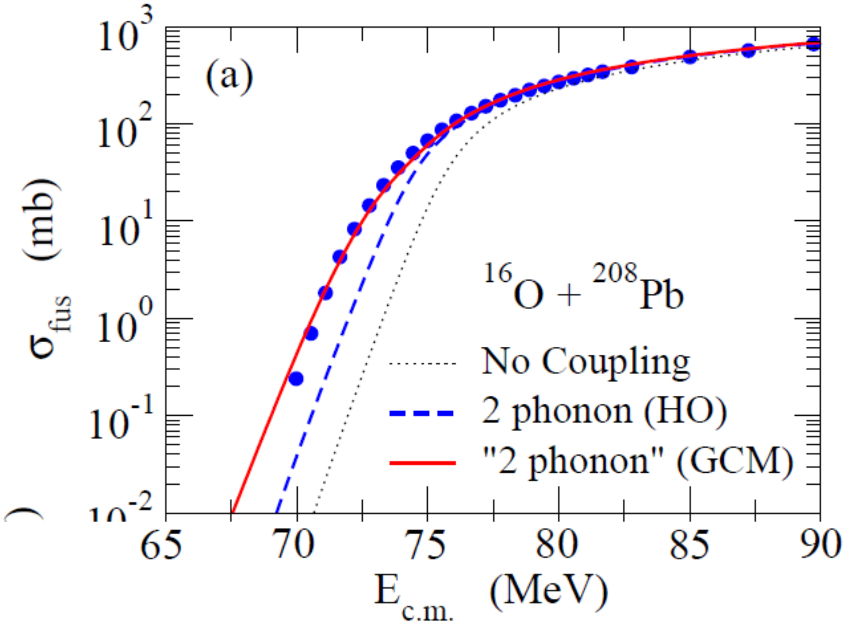
Duoduo Xu,[1] Shujun Zhao,[1] Hao-jie Xu,[2,3,*] Wenbin Zhao,[4,5,†] Huichao Song,[1,6,7,‡] and Fuqiang Wang[8,§]

octupole vibration of $^{208}$Pb



https://web-docs.gsi.de/~wolle /TELEKOLLEG /KERN/index-s.html




$^{208}$Pb: a double magic nucleus

→ spherical, but oct. vib.

from the measured B(E3)

$$\rightarrow \sqrt{\langle \beta_3^2 \rangle} \sim 0.16$$

cf. this effect has been well-known in HI fusion reactions

J.M. Yao and K. Hagino, PRC94 ('16) 11303(R)

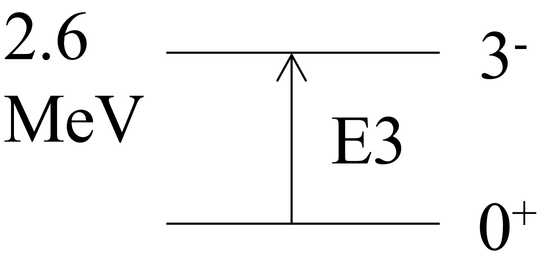
# Probing nuclear shapes in Rel. H.I. collisions

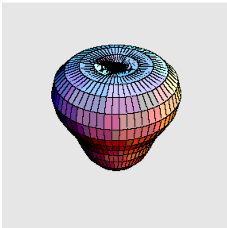
Very recent preprint: D. Xu et al., arXiv: 2504.19644

**A "breathing" octupole $^{208}$Pb nucleus: resolving the elliptical-to-triangular azimuthal anisotropy puzzle in ultracentral relativistic heavy ion collisions**

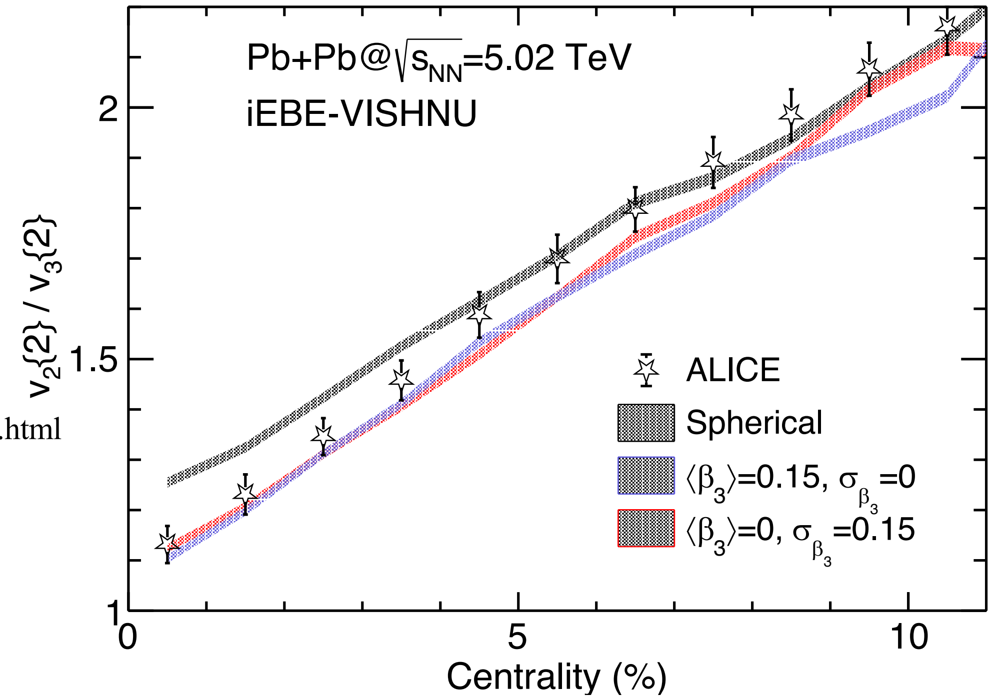
Duoduo Xu,[1] Shujun Zhao,[1] Hao-jie Xu,[2,3,*] Wenbin Zhao,[4,5,†] Huichao Song,[1,6,7,‡] and Fuqiang Wang[8,§]

octupole vibration of $^{208}$Pb



https://web-docs.gsi.de/~wolle
/TELEKOLLEG /KERN/index-s.html




$^{208}$Pb: a double magic nucleus

$\rightarrow$ spherical, but oct. vib.

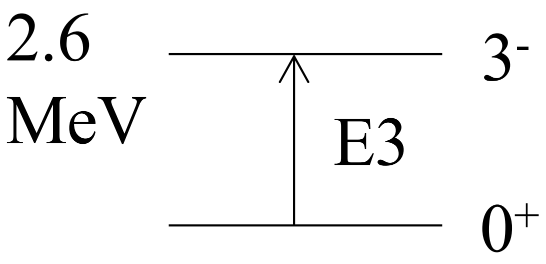
from the measured B(E3)

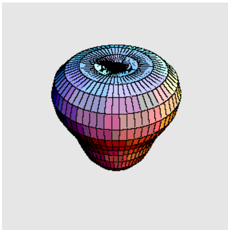
$$\rightarrow \sqrt{\langle\beta_3^2\rangle} \sim 0.16$$

# Probing nuclear shapes in Rel. H.I. collisions

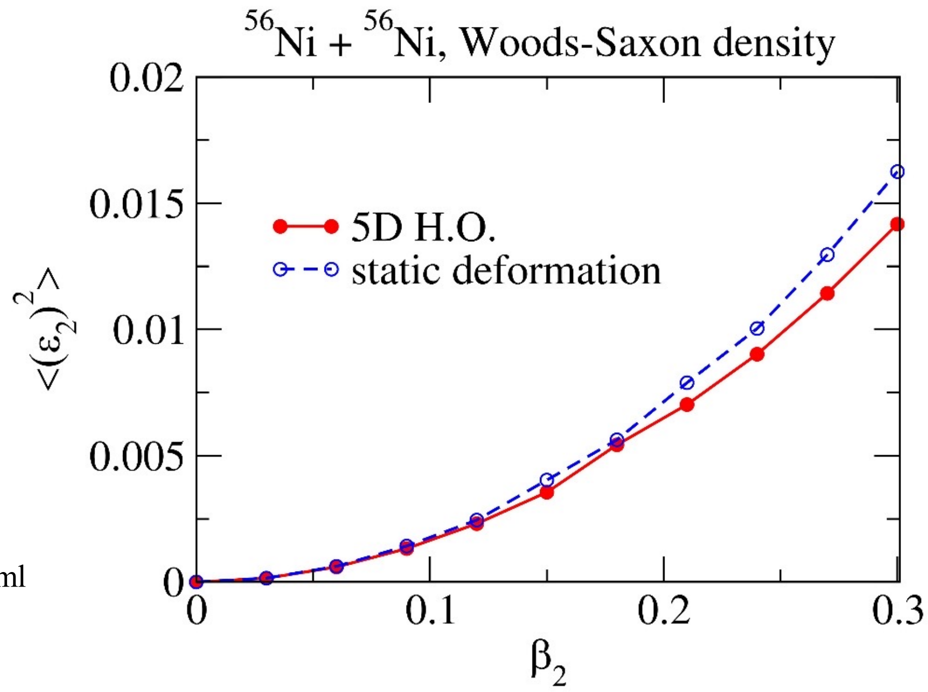
Duoduo Xu et al., arXiv: 2504.19644

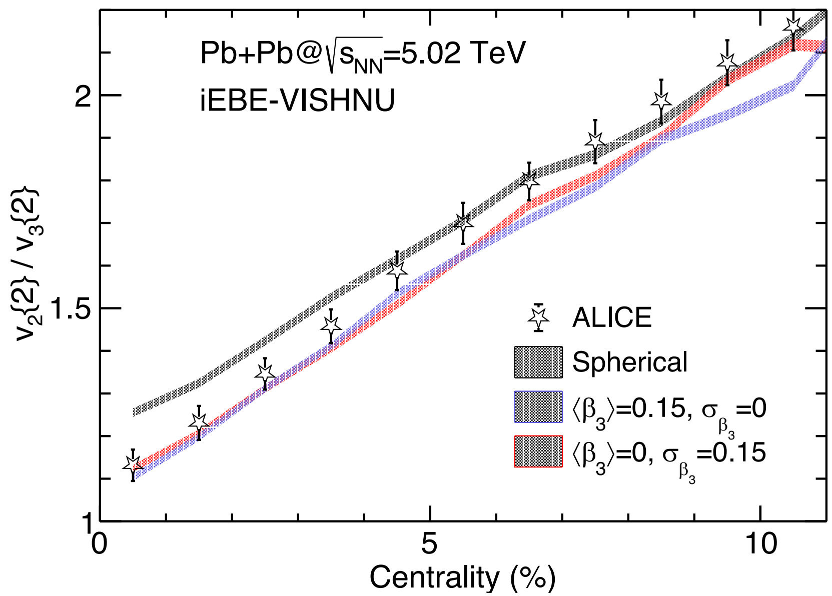
octupole vibration of $^{208}$Pb



https://web-docs.gsi.de/~wolle
/TELEKOLLEG /KERN/index-s.html




eccentricity parameter:

$$\epsilon_2 e^{2i\Phi} = -\frac{\int d\boldsymbol{r}\, r^2 \sin^2\theta\, e^{2i\phi} \sum_i \rho_i(\boldsymbol{r}, \boldsymbol{\alpha}_i)}{\int d\boldsymbol{r}\, r^2 \sin^2\theta\, \sum_i \rho_i(\boldsymbol{r}, \boldsymbol{\alpha}_i)}$$

K. Hagino and M. Kitazawa,
in preparation

# Summary

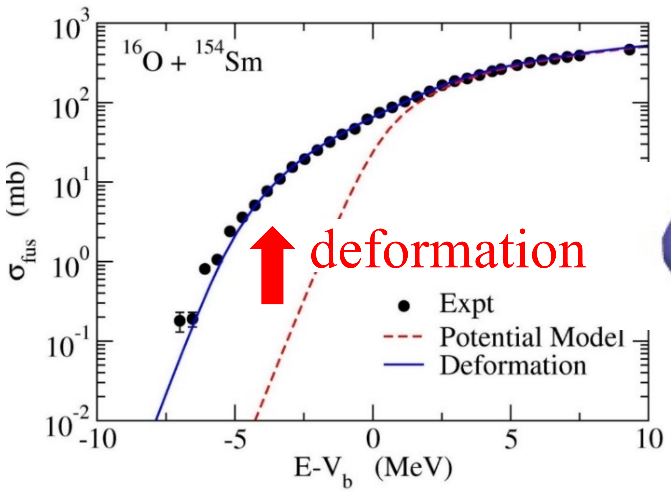
## Heavy-ion fusion reactions around the Coulomb barrier

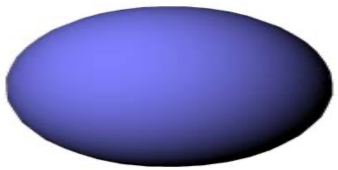
✓Strong interplay between nuclear structure and reaction
✓Quantum tunneling with various intrinsic degrees of freedom
✓Role of deformation in sub-barrier enhancement

$\longrightarrow$ a snapshot of the rotational motion

$\downarrow$

amplified



## Relativistic H.I. Collisions: fast collisions $\rightarrow$ a snapshot of a nucleus

A tool to probe nuclear deformations: $\beta_2$, $\gamma$, $\beta_3$ deformations, cluster

$\longrightarrow$ so far, mainly static deformation

: surface vibrations of a spherical nucleus?