Recent developments in heavy-ion fusion reactions around the Coulomb barrier

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collaborator:
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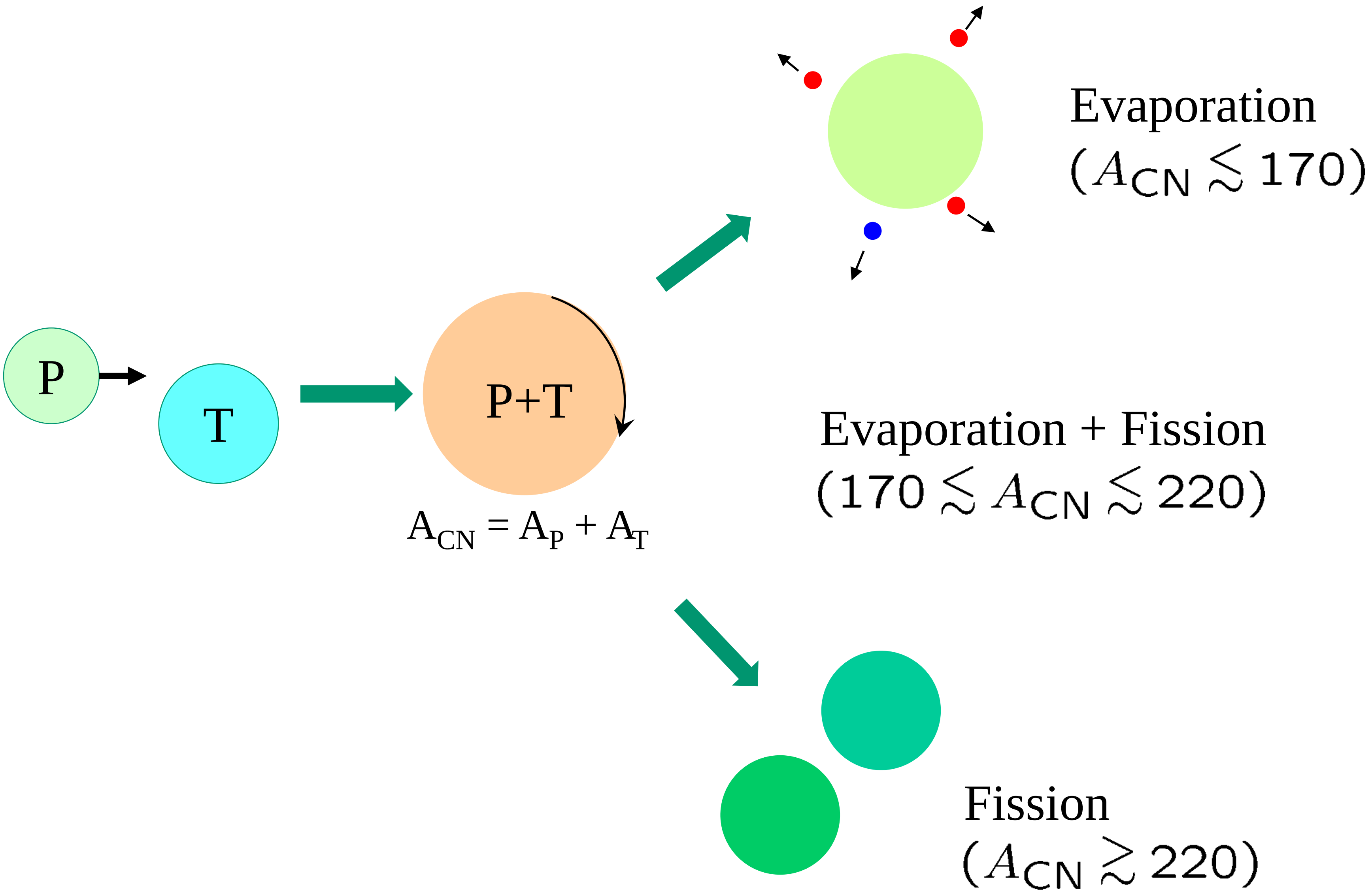
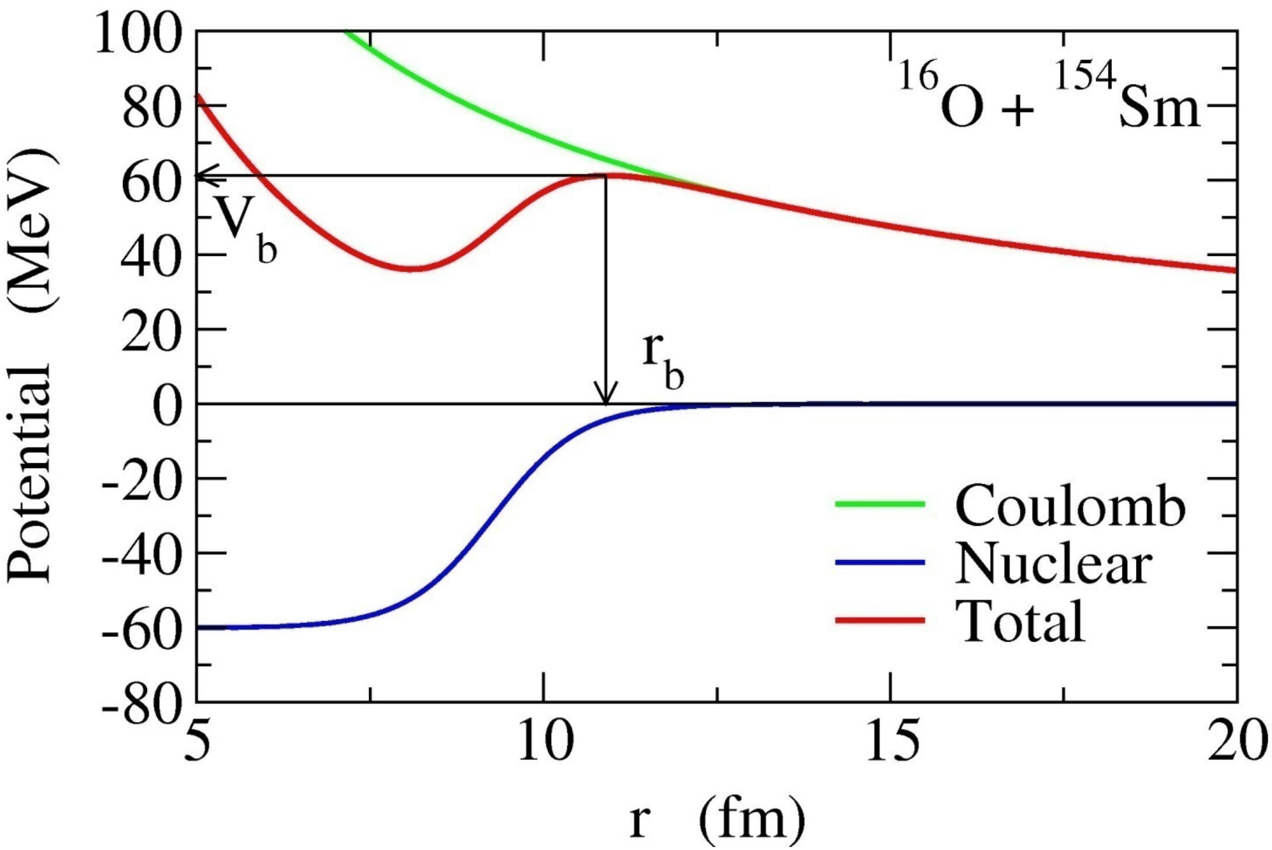
1. *Introduction: H.I. sub-barrier fusion reactions*
   - *potential model and Wong formula*
   - *coupled-channels approach*
2. *A Bayesian approach to fusion barrier distributions*
3. *C.C. calculation with "beyond-mean-field" method*
4. *Summary*

Introduction: heavy-ion fusion reactions

Fusion: compound nucleus formation



Inter-nucleus potential



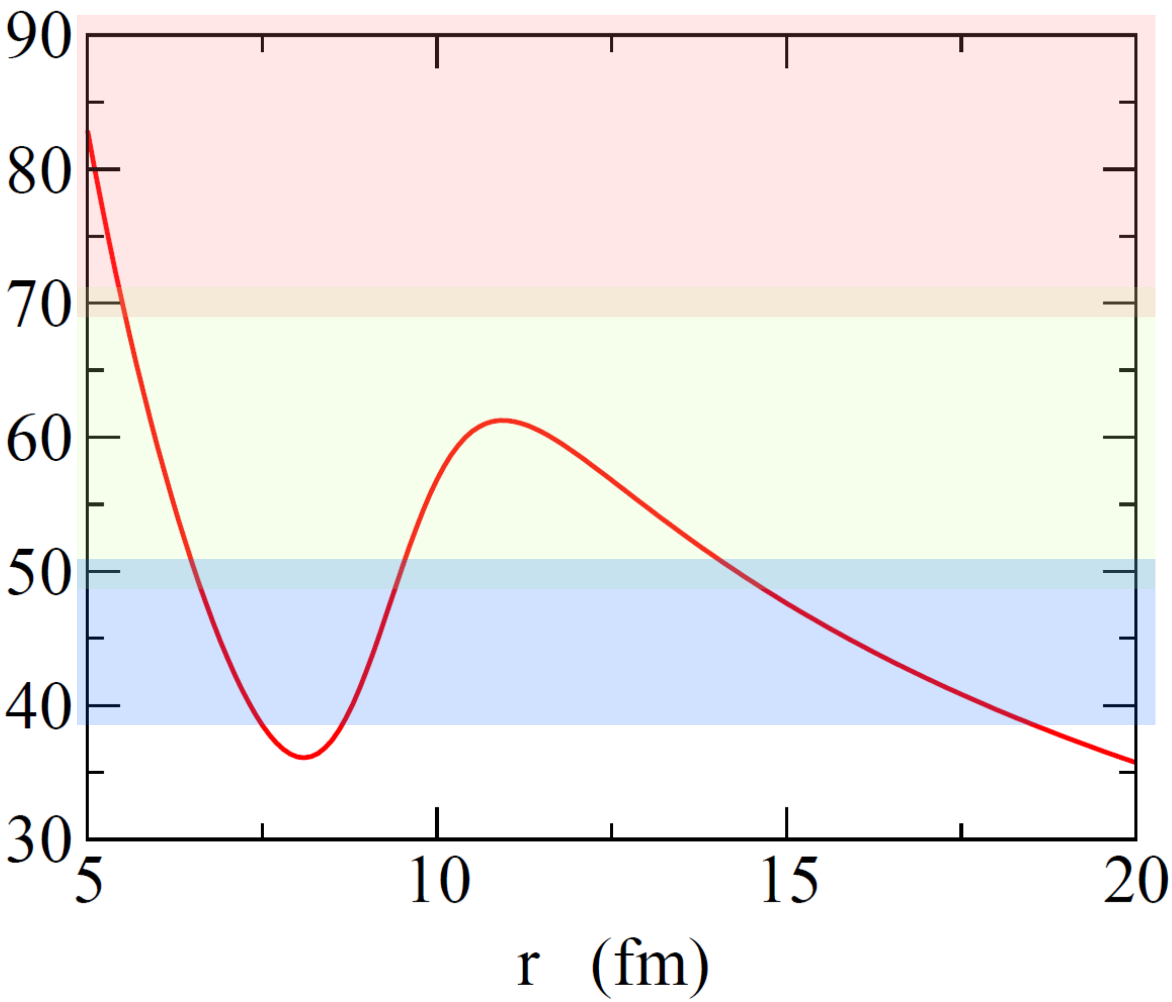
Two forces:

1. Coulomb force
   Long range,
   repulsive
2. Nuclear force
   Short range,
   attractive

Potential barrier
(Coulomb barrier)

- above barrier energies
- sub-barrier energies
- deep subbarrier energies

Energy regions

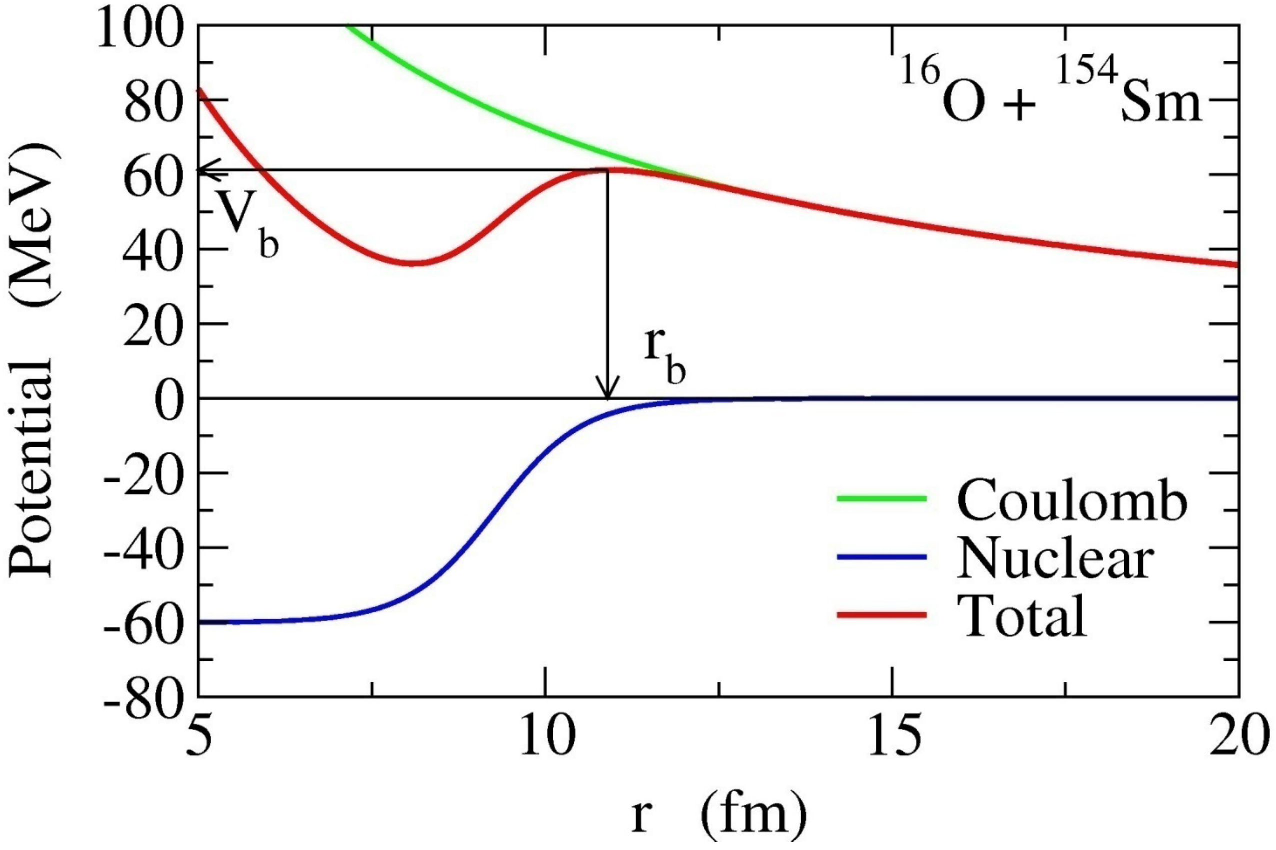


above barrier region
$(E \gtrsim V_b + 10\text{MeV})$

sub-barrier region ←
$(|E - V_b| \lesssim 10\text{MeV})$

deep sub-barrier region
$(E \lesssim V_b - 10\text{MeV})$

Potential model for fusion

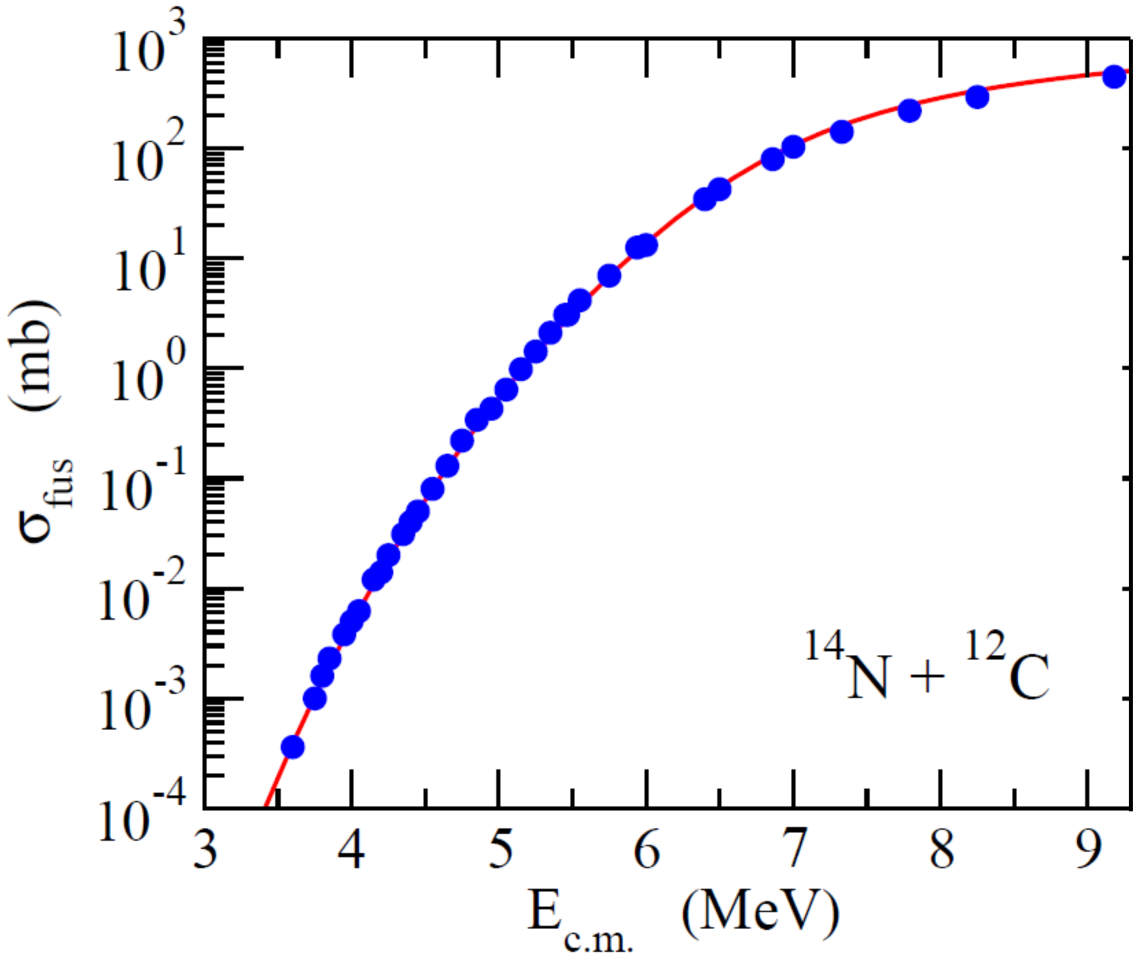


the simplest approach to fusion cross sections: potential model

$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

the simplest approach: potential model with $V(r)$ + absorption

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

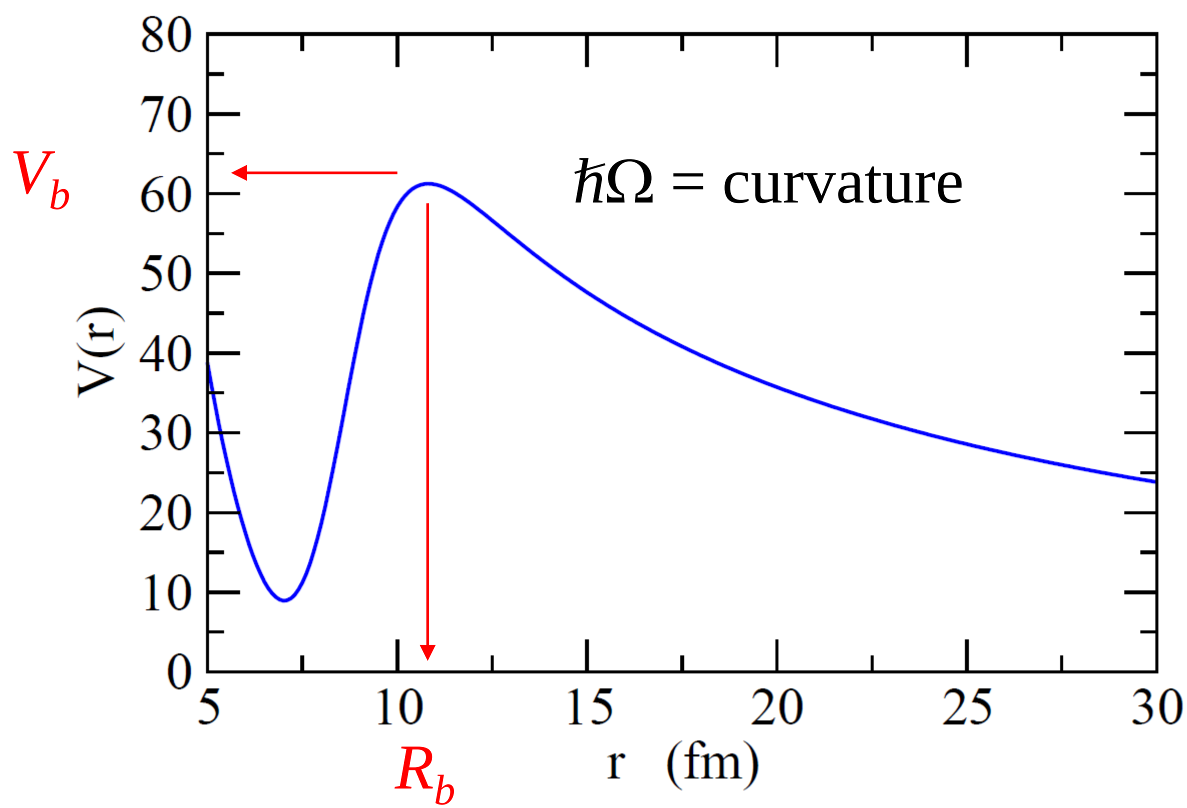


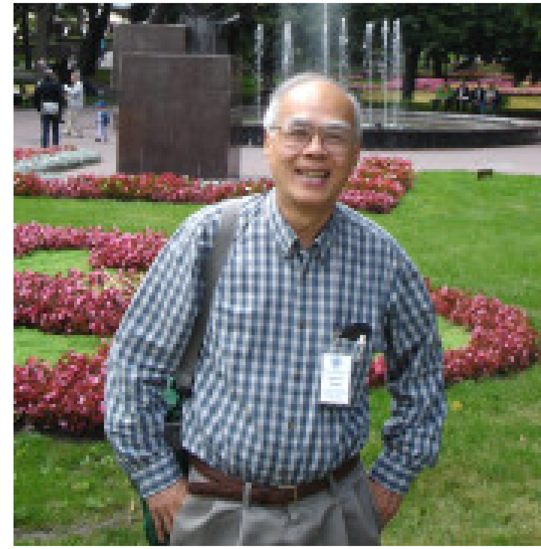
Wong's formula

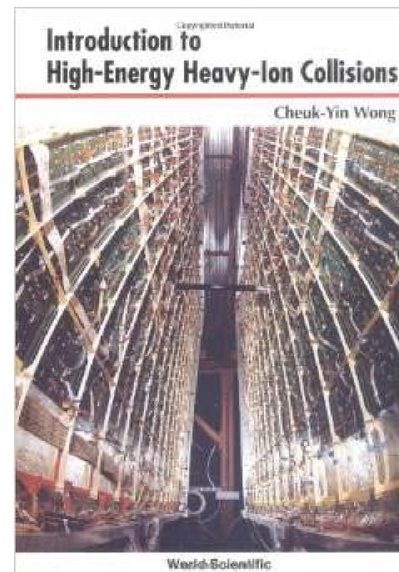
C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

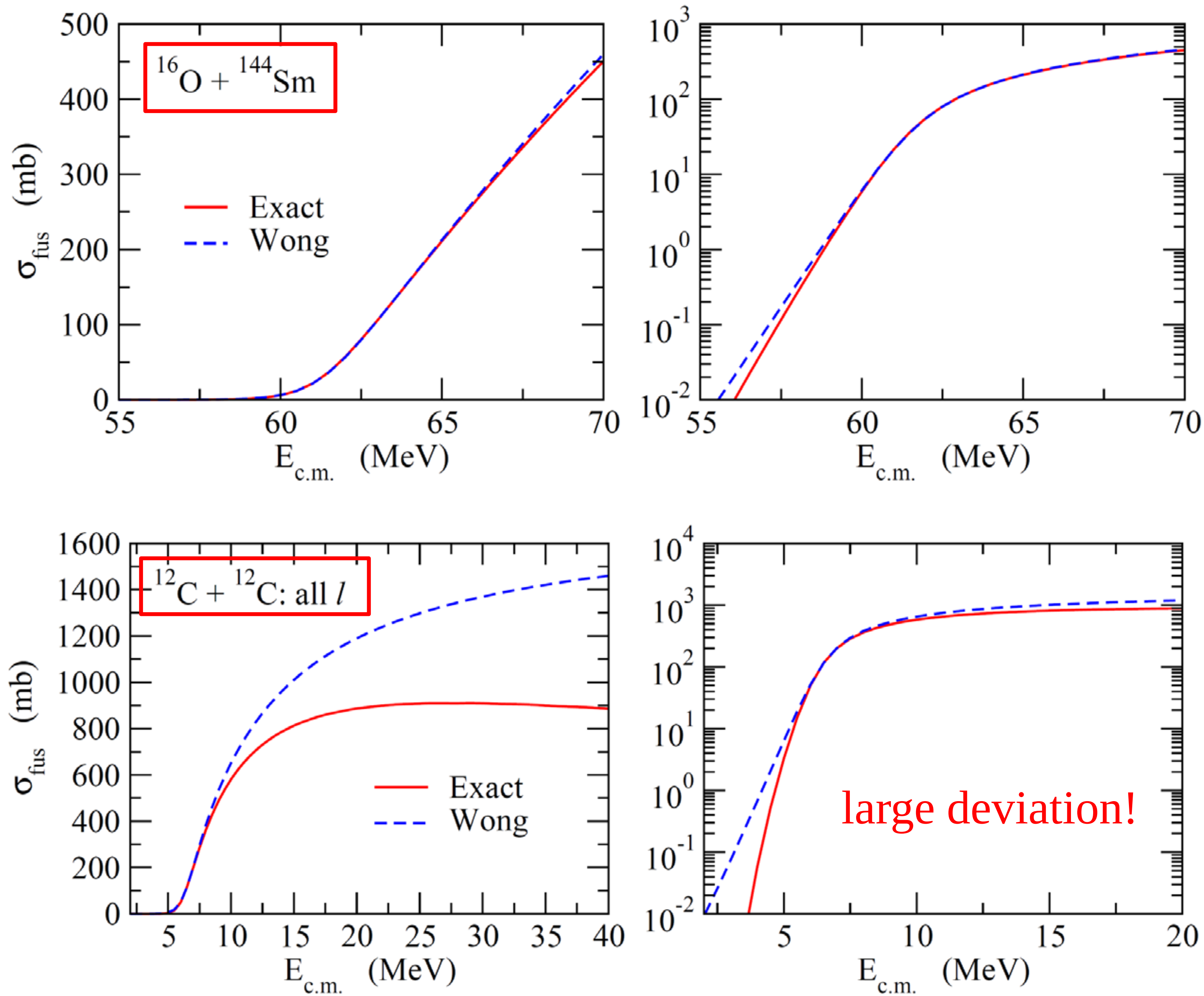
(single-channel)







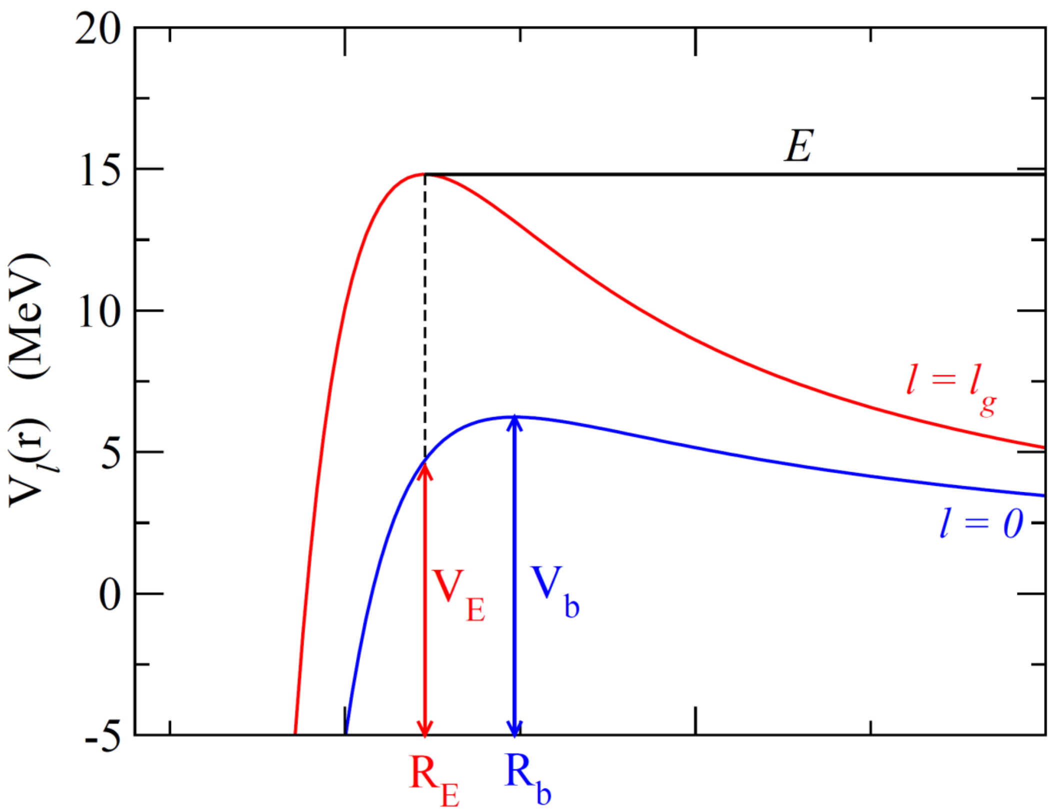
R_b, V_b, Ω_b: s-wave barrier



$^{16}O + ^{144}Sm$
Exact
Wong
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
$^{12}C + ^{12}C$: all l
large deviation!

Generalized Wong formula

N. Rowley, A. Kabir, and R. Lindsay, J. of Phys. G15('89)L269
N. Rowley and K. Hagino, PRC91 (2015) 044617



use V_b, R_b, and Ω for the grazing angular momentum, l_g

(note)

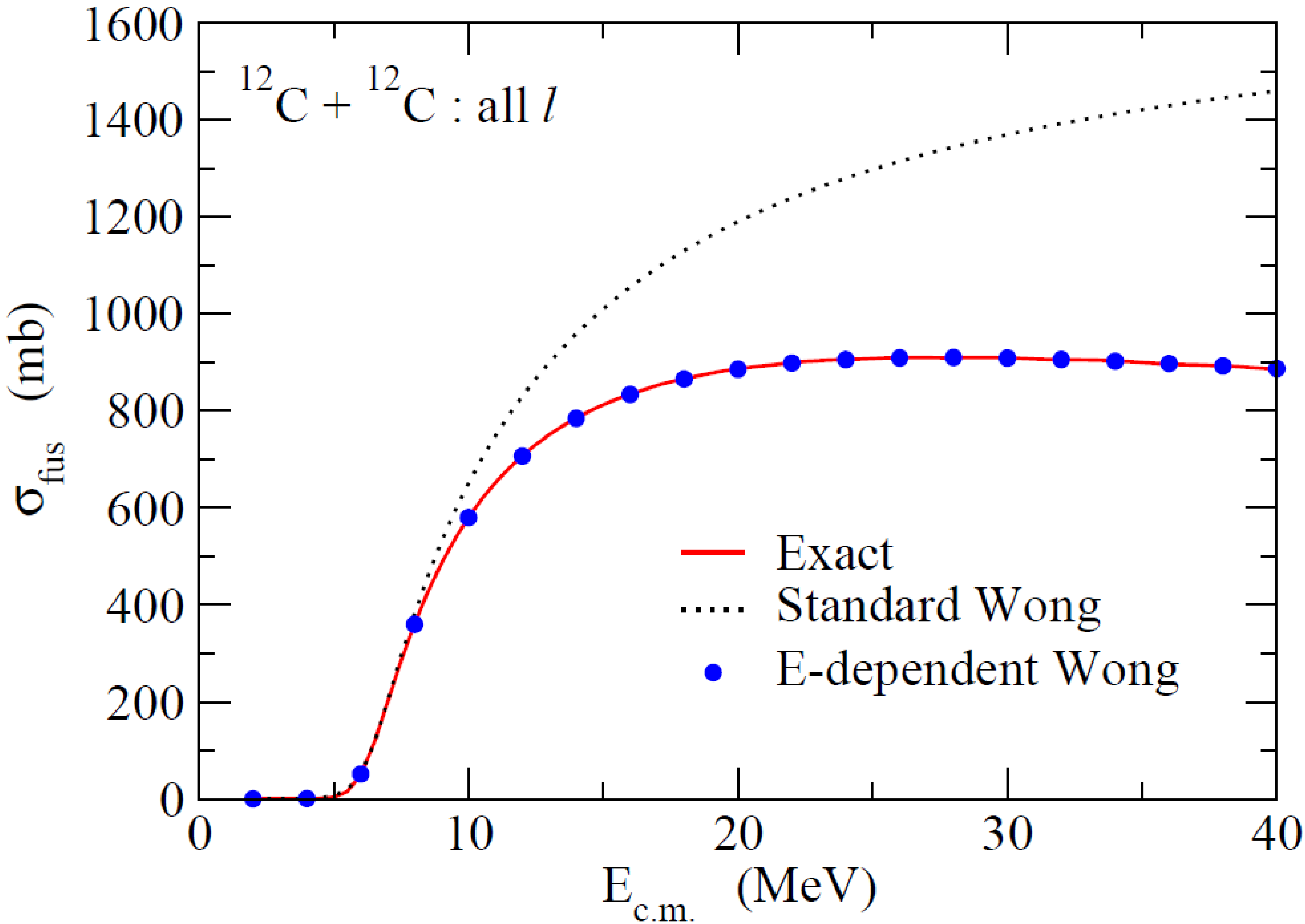
$$\begin{cases} \sigma_{\rm cl} = \pi b_g^2 \\ E = V_E + \dfrac{(kb_g)^2\hbar^2}{2\mu R_E^2} \end{cases}$$

$$\longrightarrow \sigma_{\rm cl} = \pi R_E^2 (1 - V_E/E)$$

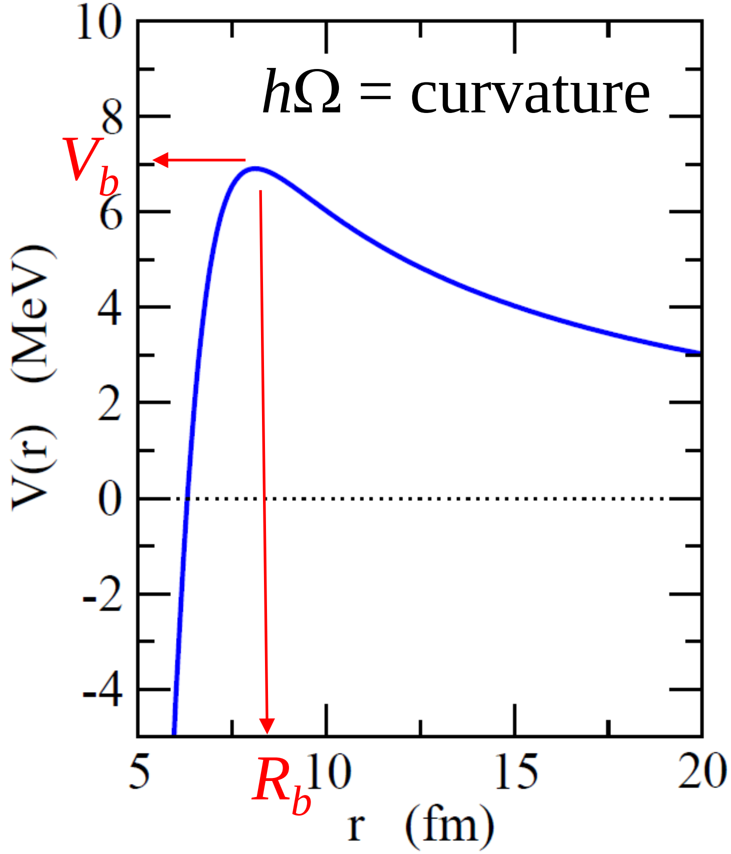
$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

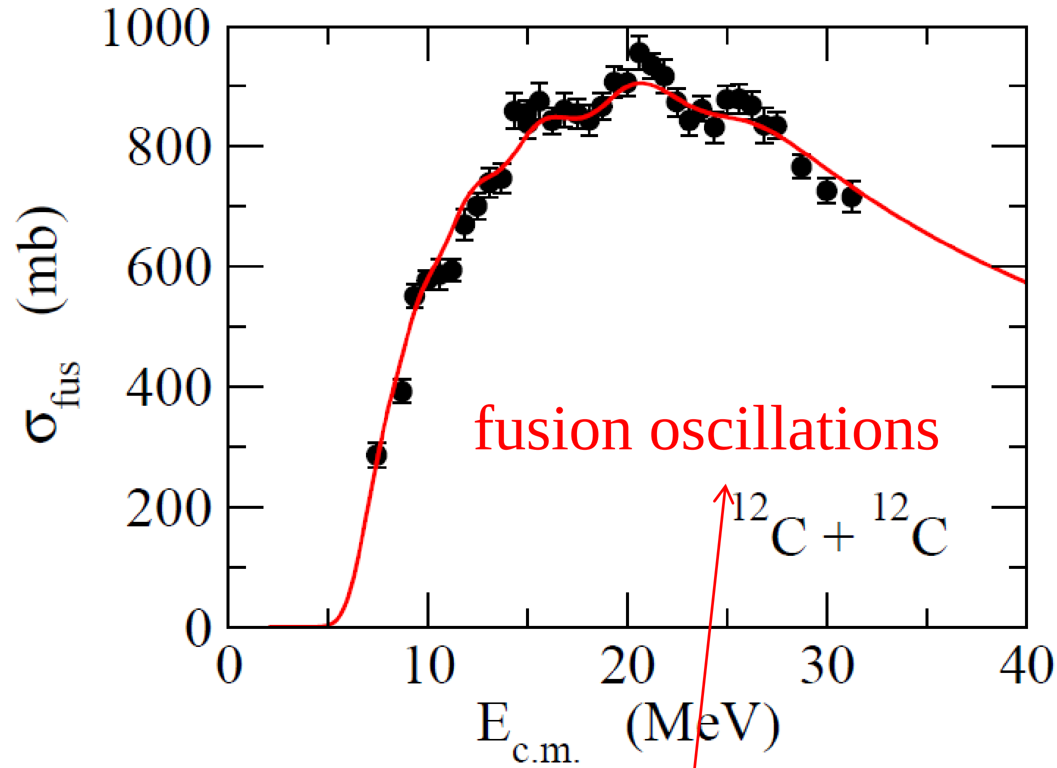
$$\Longrightarrow \sigma_{\rm fus}(E) = \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right]$$

$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega_E}{2E}\, R_E^2 \,\ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right]$$



potential model: $V(r)$ + absorption



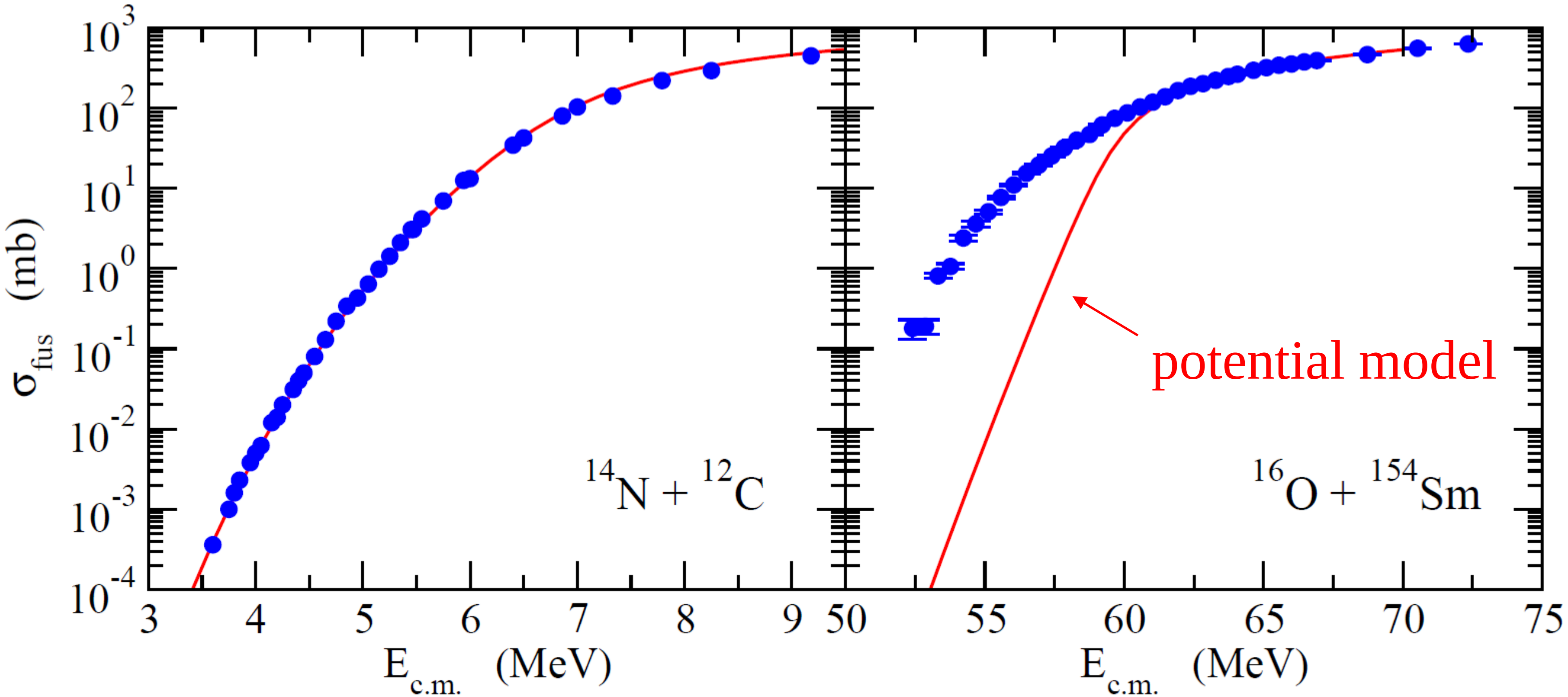


Generalized Wong formula [N. Rowley and K.H., PRC91('15)044617]

$$\sigma_{\text{fus}}(E) \sim \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right] + (osc.)$$

Discovery of large sub-barrier enhancement of σ_{fus}

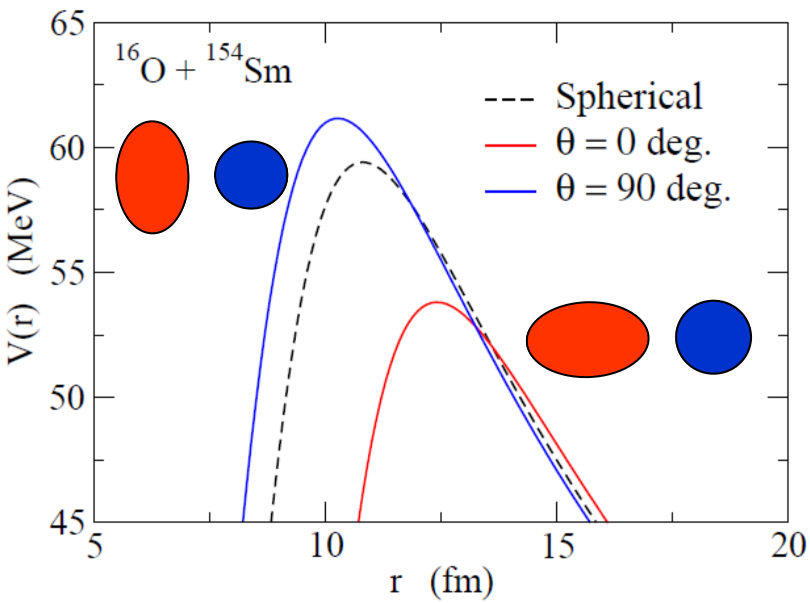
potential model: $V(r)$ + absorption

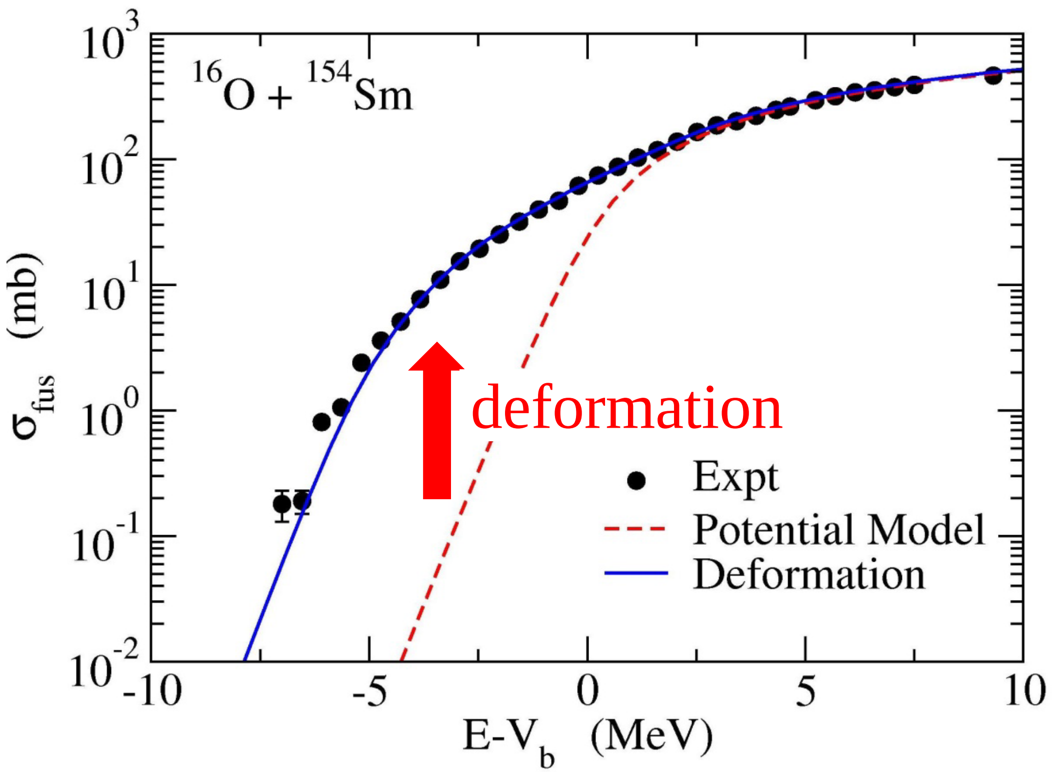


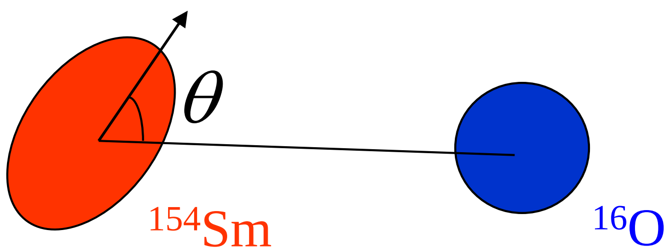
cf. seminal work:
R.G. Stokstad et al., PRL41(‘78) 465

Effect of nuclear deformation

^{154}Sm : a deformed nucleus with $\beta_2 \sim 0.3$

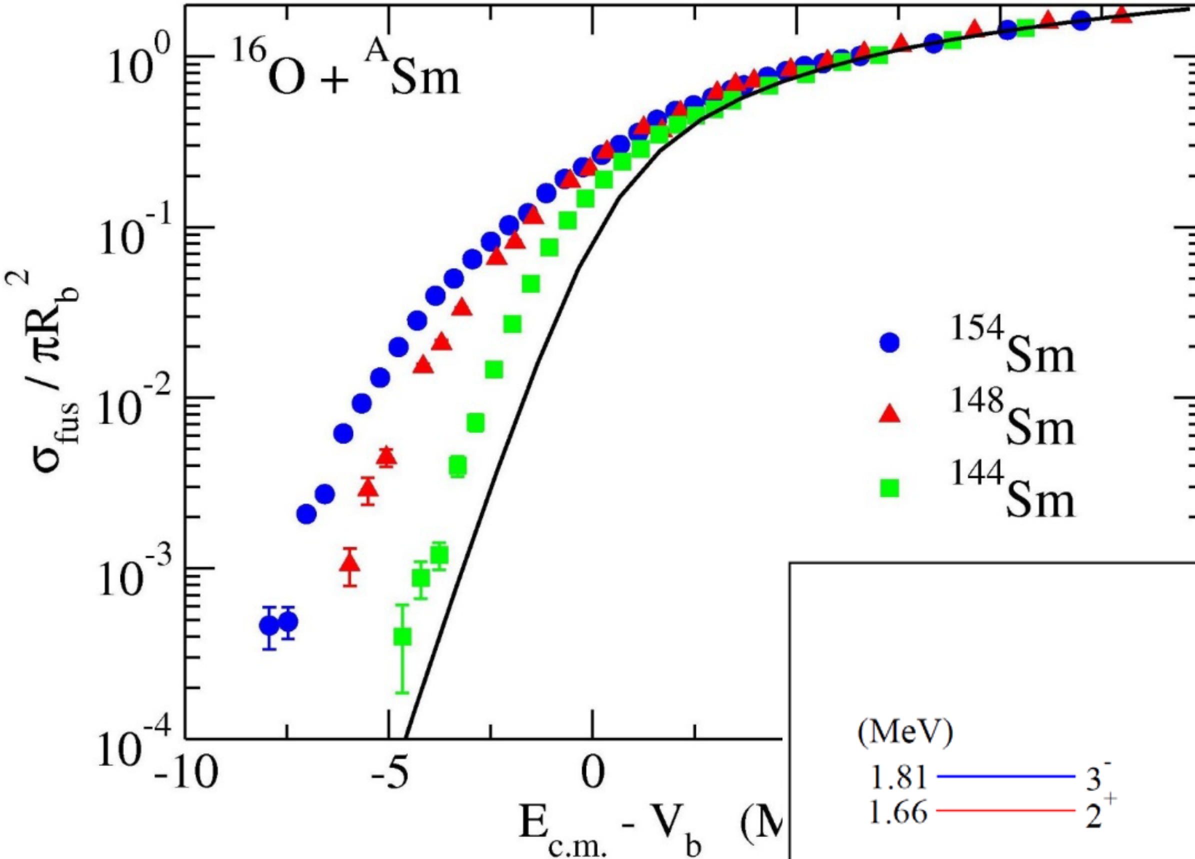


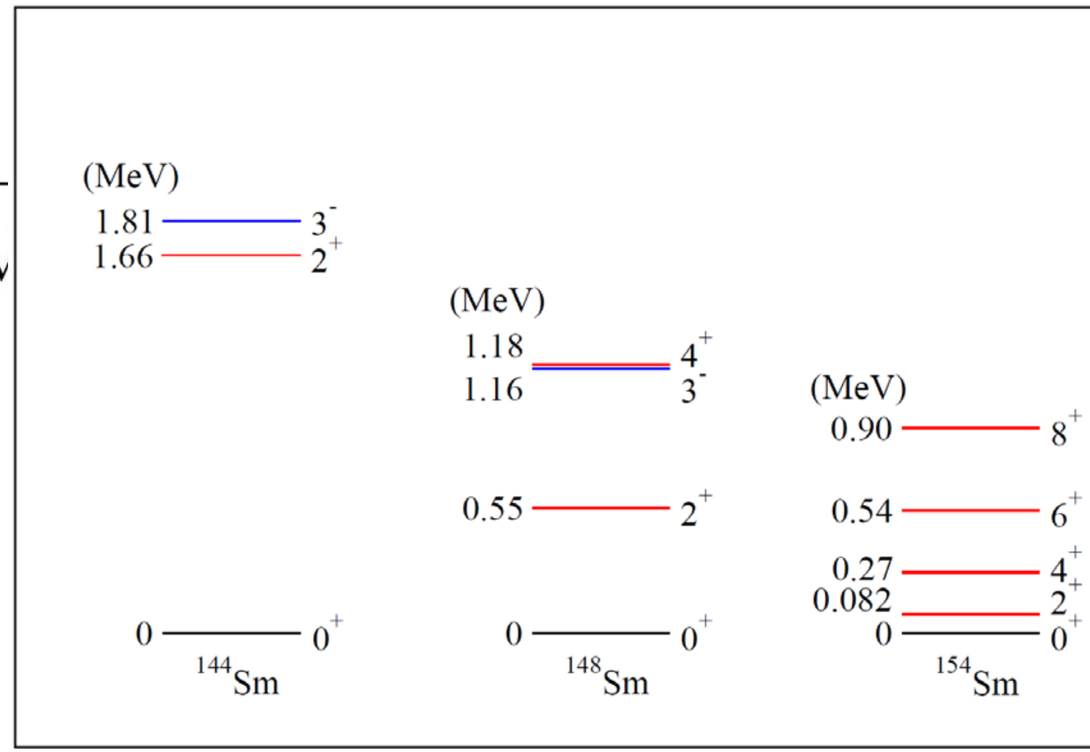




$$\sigma_{\mathrm{fus}}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\mathrm{fus}}(E;\theta)$$

Fusion: strong interplay between nuclear structure and nuclear reaction

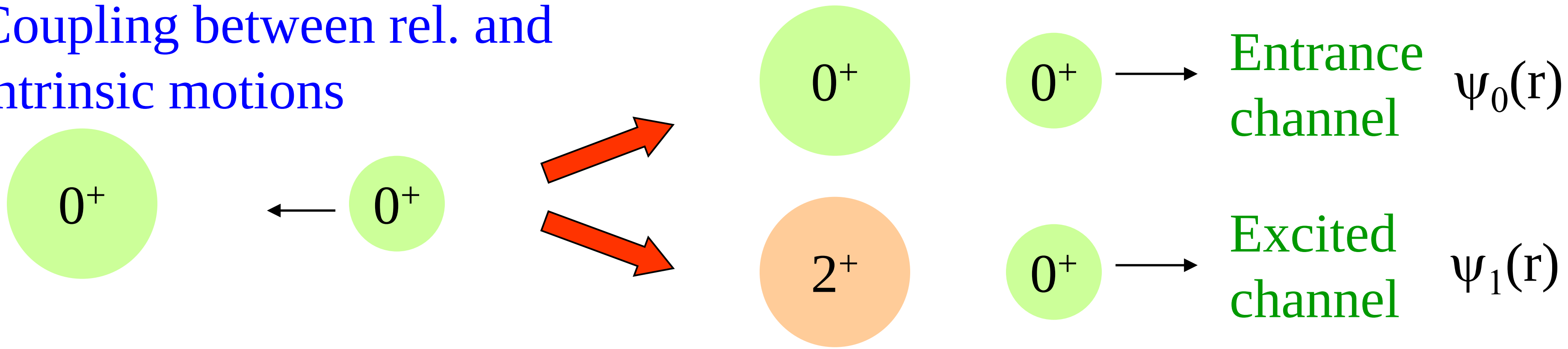


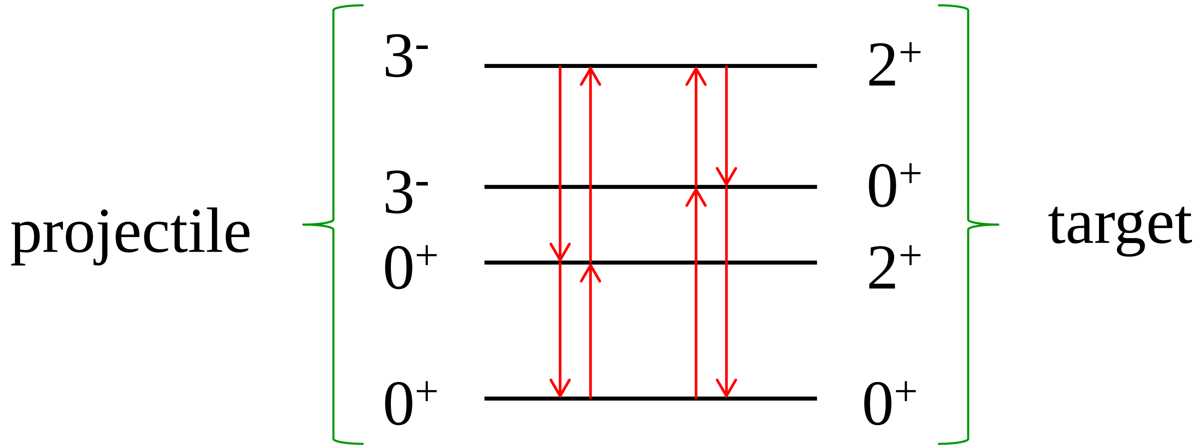


Strong target dependence
at $E < V_b$

Coupled-Channels method

Coupling between rel. and intrinsic motions





$$\Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi)$$

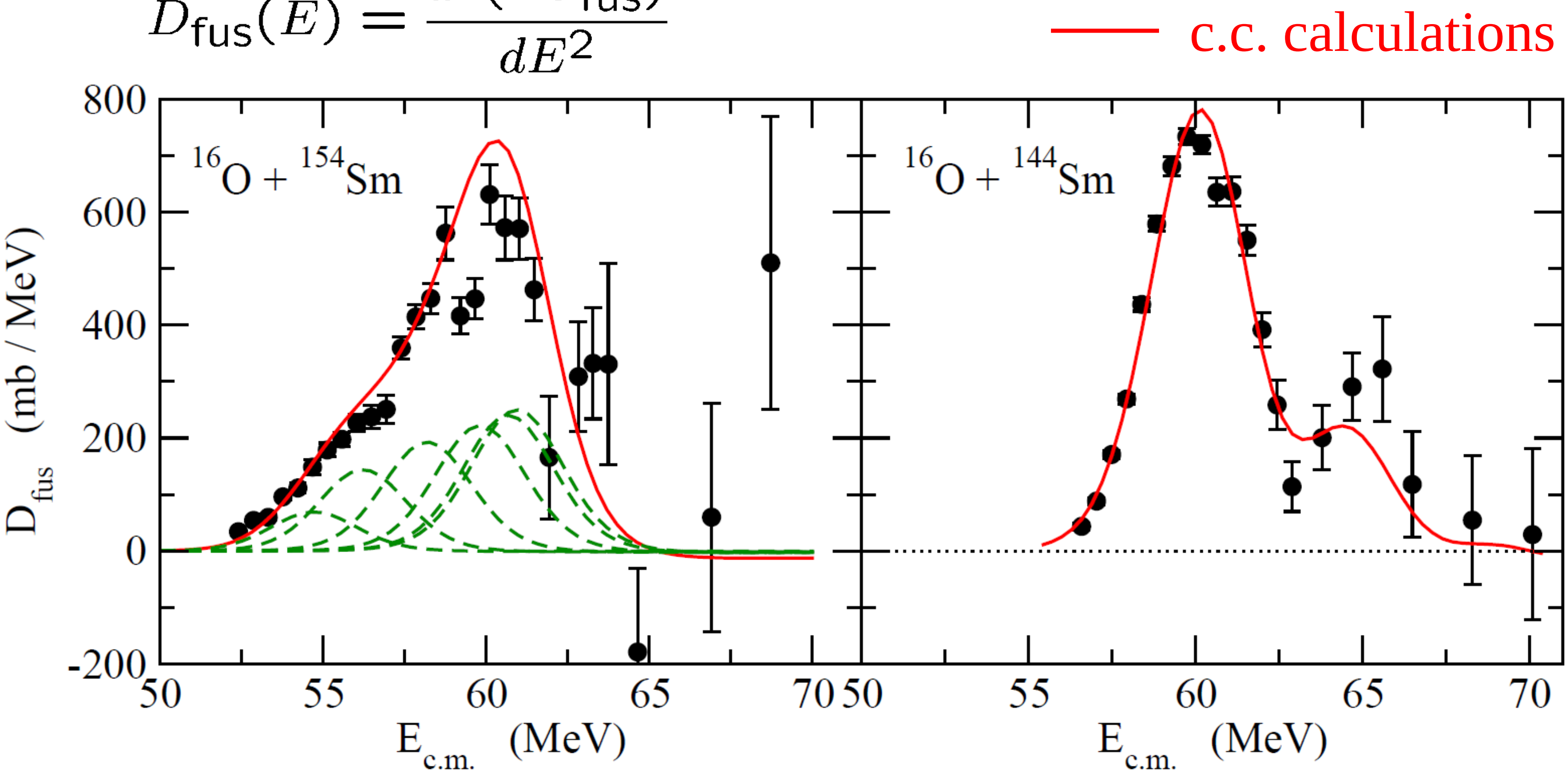
→ coupled Schroedinger equations for $\psi_k(r)$

C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143)

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254(‘91))

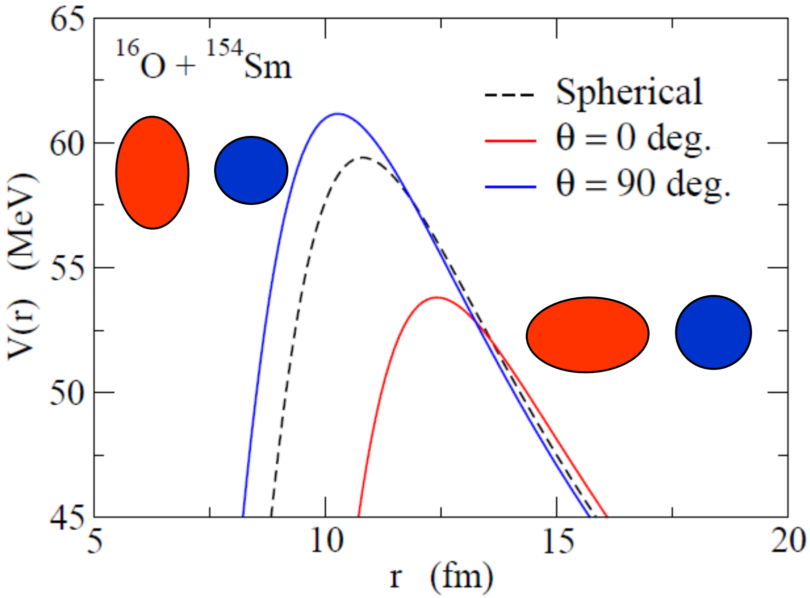
$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$



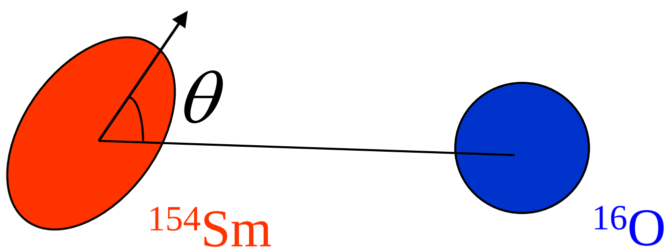
K.H., N. Takigawa, PTP128 (‘12) 1061

Effect of nuclear deformation

^{154}Sm : a deformed nucleus with $\beta_2 \sim 0.3$



deformation:
single barrier → many barriers

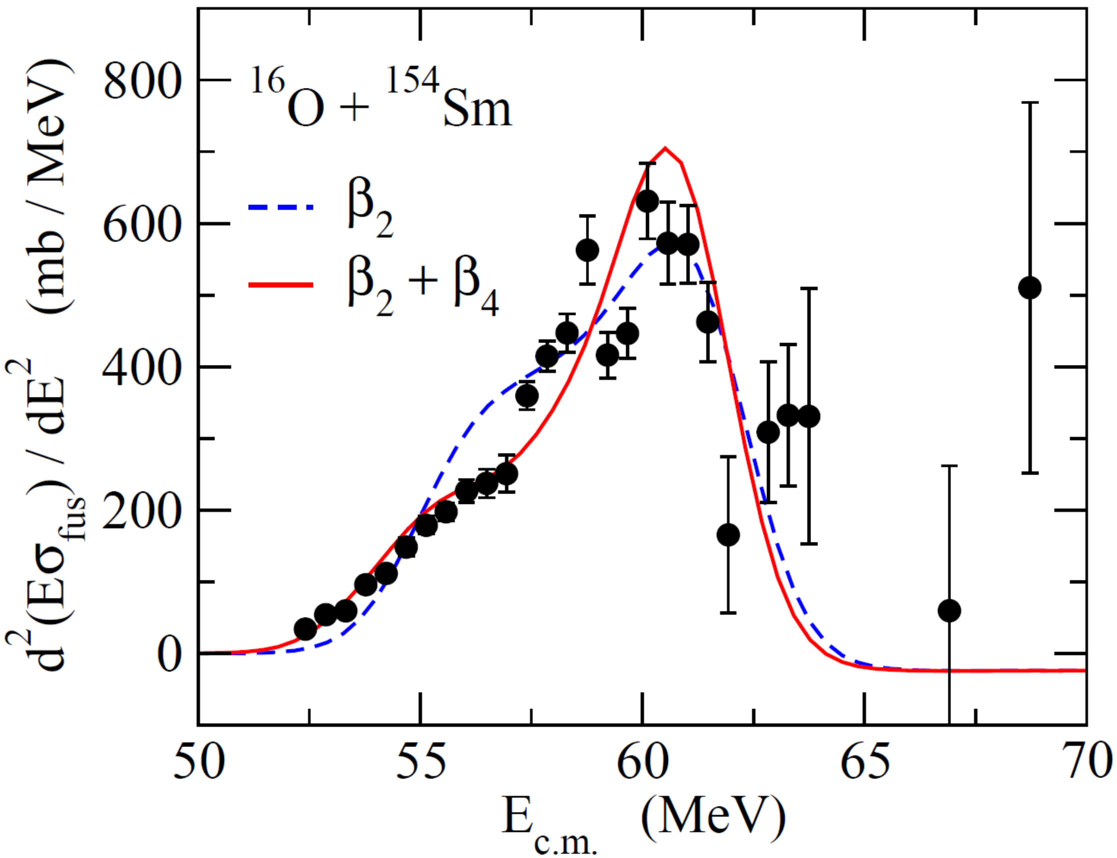


$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

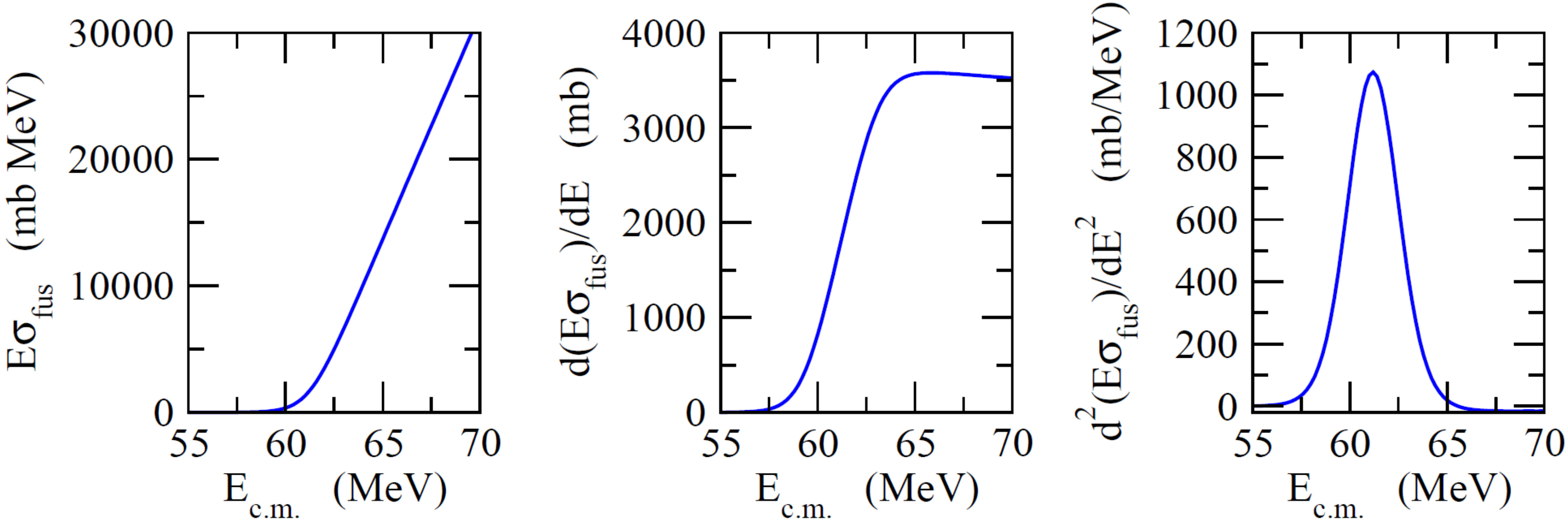
Fusion barrier distribution

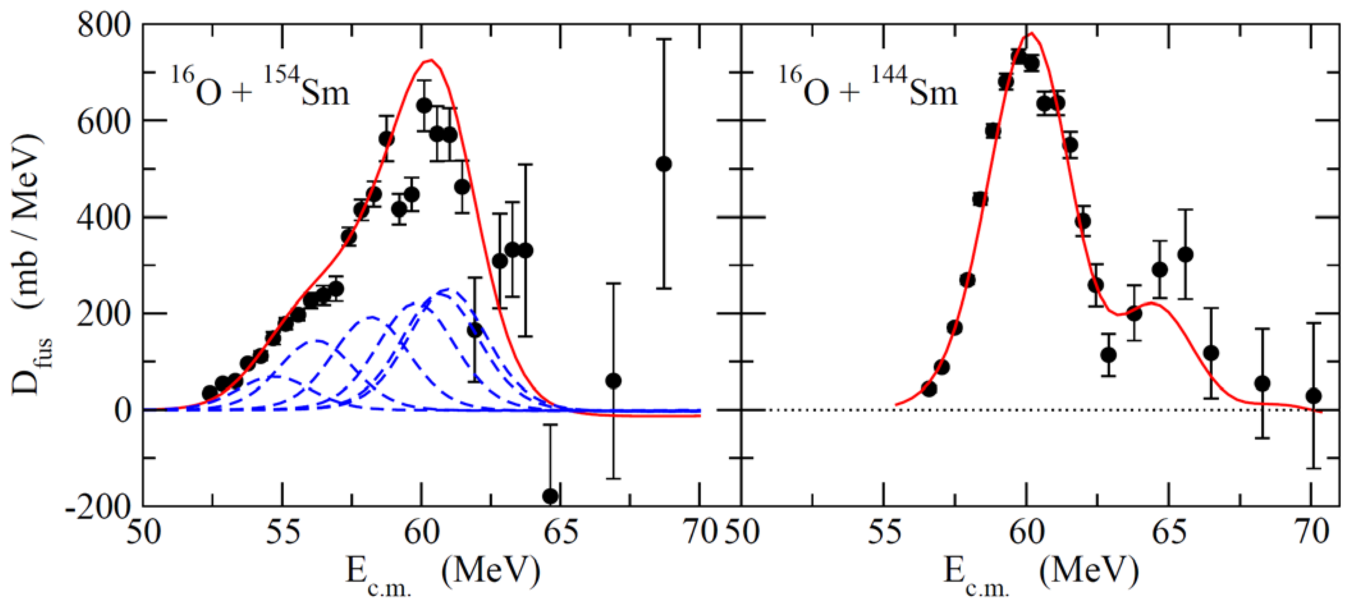
$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

- N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25
- J.X. Wei, J.R. Leigh et al., PRL67 ('91) 3368
- M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48 ('98) 401



$\sim \pi R_b^2 \, P_{l=0}(E)$ $\sim \pi R_b^2 \cdot dP_{l=0}/dE$





K.H. and N. Takigawa, PTP128 ('12) 1061

A Bayesian approach to fusion barrier distributions

K.H., PRC93 ('16) 061601(R)

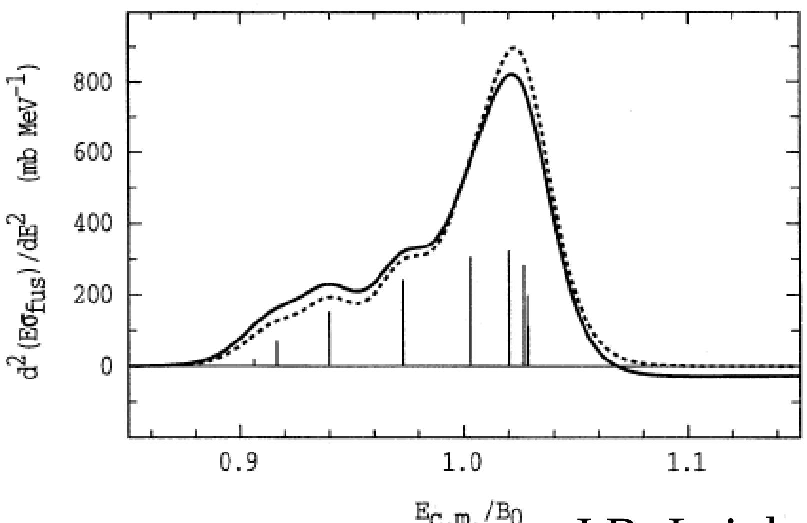
Fusion barrier distributions

➢ Coupled-channels analyses

✓ a standard approach
✓ need to know the nature of collective excitations

➢ Direct fit to experimental data

$$D_{\mathrm{fus}}(E) = \sum_k w_k D_0(E; B_k, R_k, \hbar\Omega_k)$$



✓ phenomenological
✓ no need to know the nature of coll. excitations
✓ quick and convenient way
✓ the number of barriers? (over-fitting problem)

J.R. Leigh et al., PRC52 ('95) 3151

Bayesian spectrum deconvolution

K. Nagata, S. Sugita, and M. Okada, Neural Networks 28 ('12) 82

- ✓ data set: $D_{exp} = \{E_i, d_i, \delta d_i\}$ (i = 1 ~ M)
- ✓ fitting function: $D_{\mathrm{fit}}(E;\tilde{\boldsymbol{\theta}},K) = \sum_{k=1}^{K} w_k\,\phi_k(E;\boldsymbol{\theta}_k)$

Bayes theorem

$$P(K|D_{\mathrm{exp}}) = \frac{P(D_{\mathrm{exp}}|K)P(K)}{P(D_{\mathrm{exp}})}$$

$$\propto P(D_{\mathrm{exp}}|K) = \int d\tilde{\boldsymbol{\theta}}\, e^{-\chi^2(\tilde{\boldsymbol{\theta}},K)/2} P(\tilde{\boldsymbol{\theta}})$$

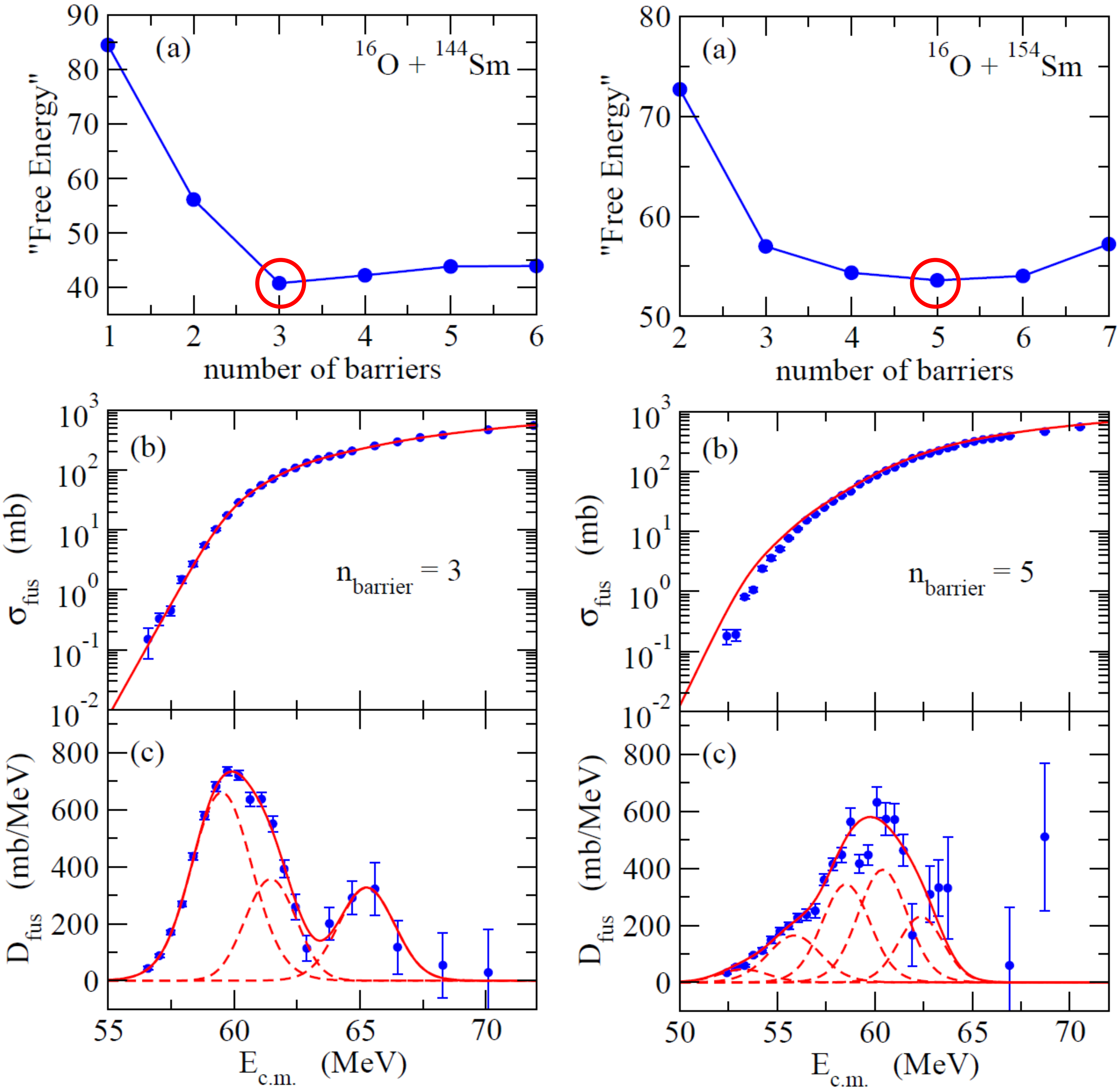
$$\chi^2(\tilde{\boldsymbol{\theta}},K) = \sum_{i=1}^{M}\left(\frac{d_i - D_{\mathrm{fit}}(E_i;\tilde{\boldsymbol{\theta}},K)}{\delta d_i}\right)^2$$

most probable value of K: maximize $Z(K) = \int d\tilde{\boldsymbol{\theta}}\, e^{-\chi^2(\tilde{\boldsymbol{\theta}},K)/2} P(\tilde{\boldsymbol{\theta}})$

(high dim. integral → MC method)

or equivalently, minimize the "Free Energy" $F(K) = -\ln Z(K)$

→ optimize the other parameters for a given value of K

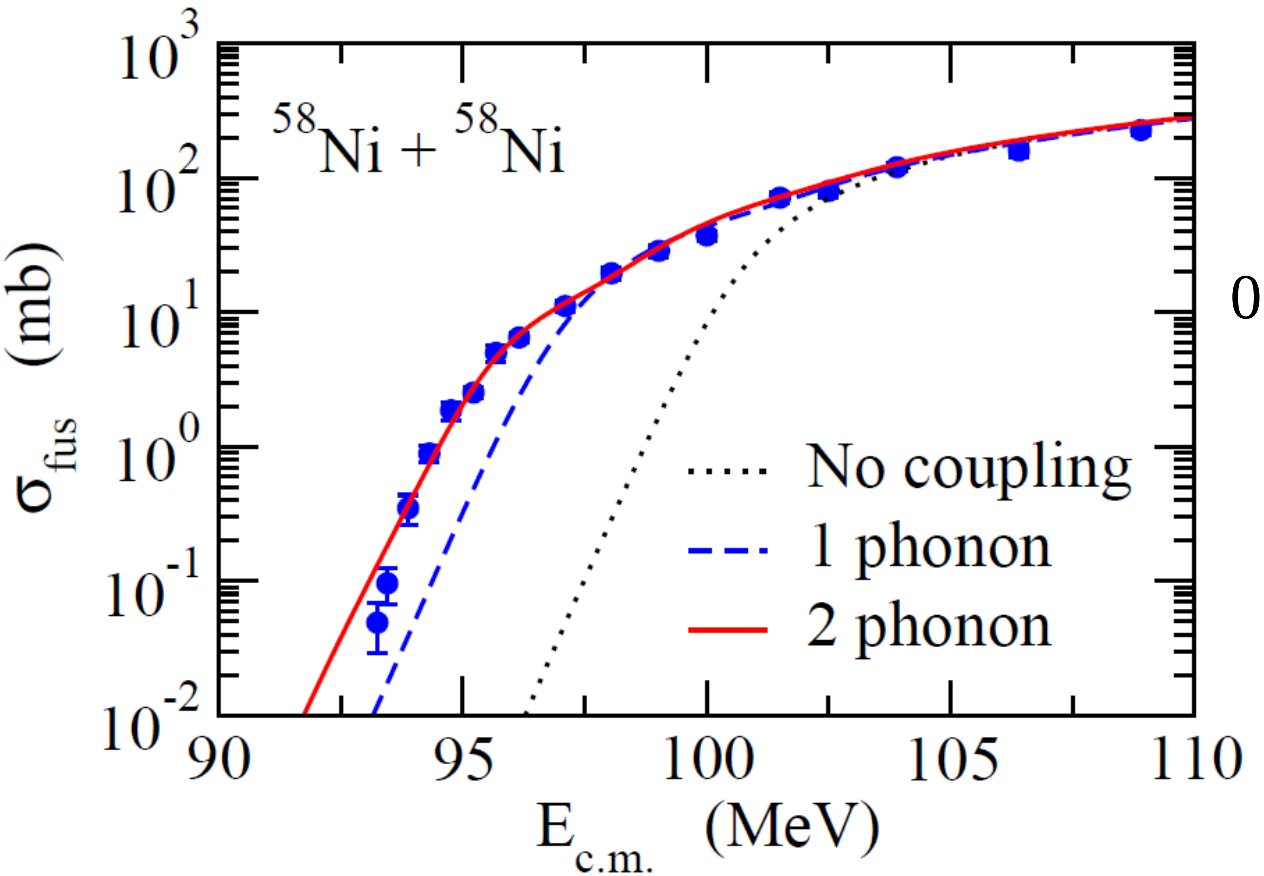


K.H., PRC93 ('16) 061601(R)

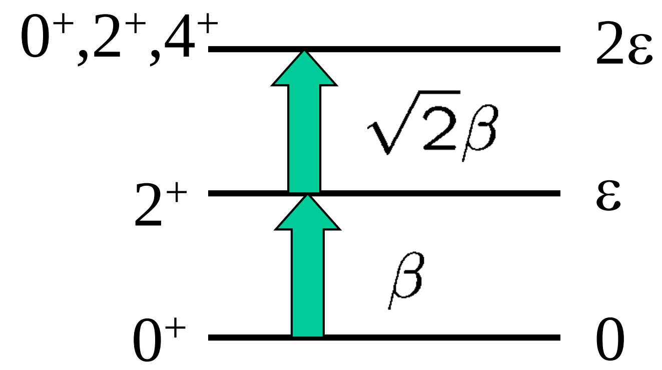
Semi-microscopic modeling of sub-barrier fusion

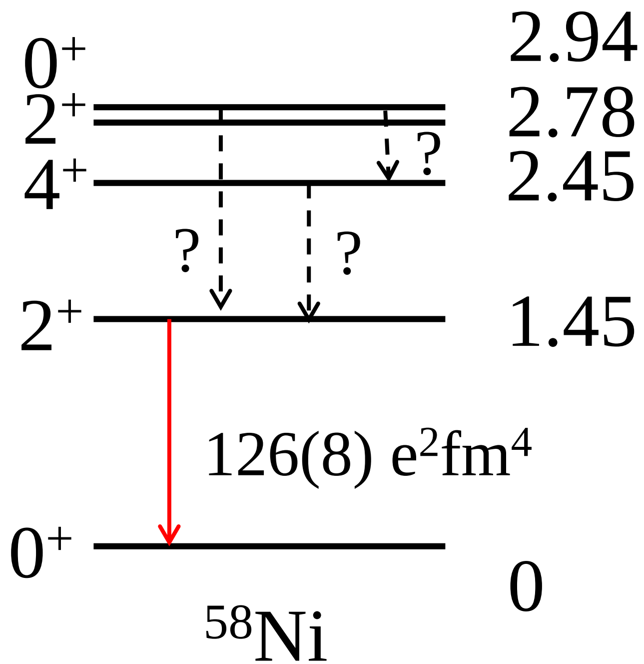
K.H. and J.M. Yao, PRC91('15) 064606

multi-phonon excitations



simple harmonic oscillator

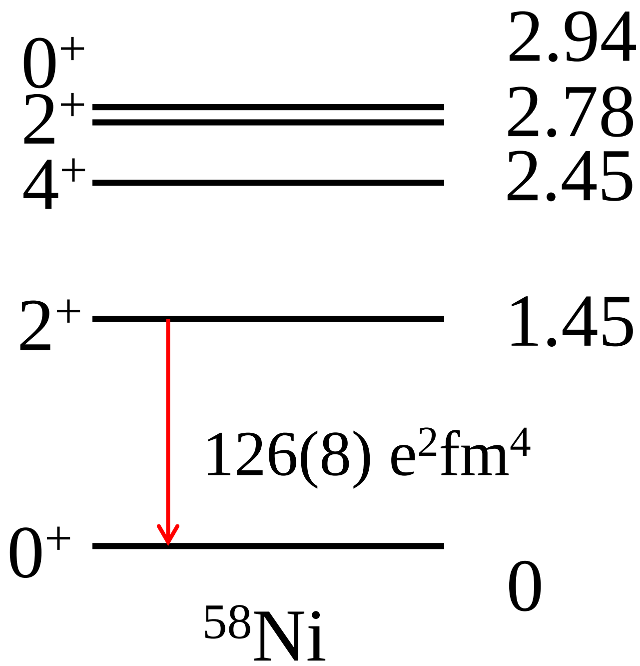




Simple harmonic oscillator
: justifiable?

$Q(2_1^+) = -10 +/- 6\ e\mathrm{fm}^2$

Anharmonic vibrations



- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

$$|JM\rangle = \int d\beta\, f_J(\beta) \hat{P}^J_{M0} |\Phi(\beta)\rangle$$

✓ MF + ang. mom. projection

+ particle number projection

+ generator coordinate method (GCM)

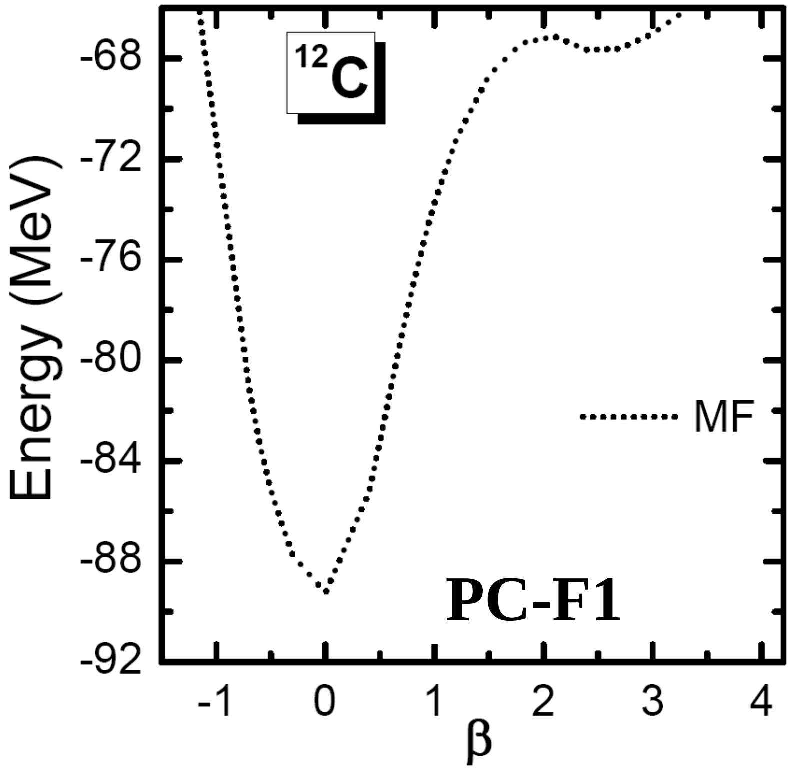
$Q(2_1^+)$ = -10 +/- 6 $e$fm^2

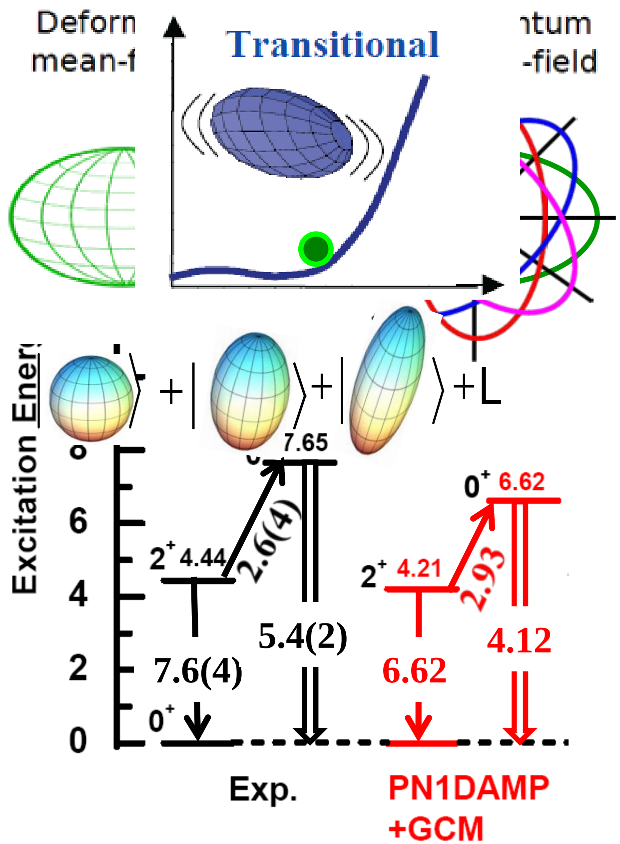
M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 (‘03) 121

J.M. Yao et al., PRC89 (‘14) 054306

Beyond MF: Illustration with ^{12}C : (GCM+PNP+AMP)

$$|\Phi_{IM_I}\rangle = \sum_\beta F^I(\beta)\hat{P}^I_{M_I K}\hat{P}^N\hat{P}^Z|\varphi(\beta)\rangle$$

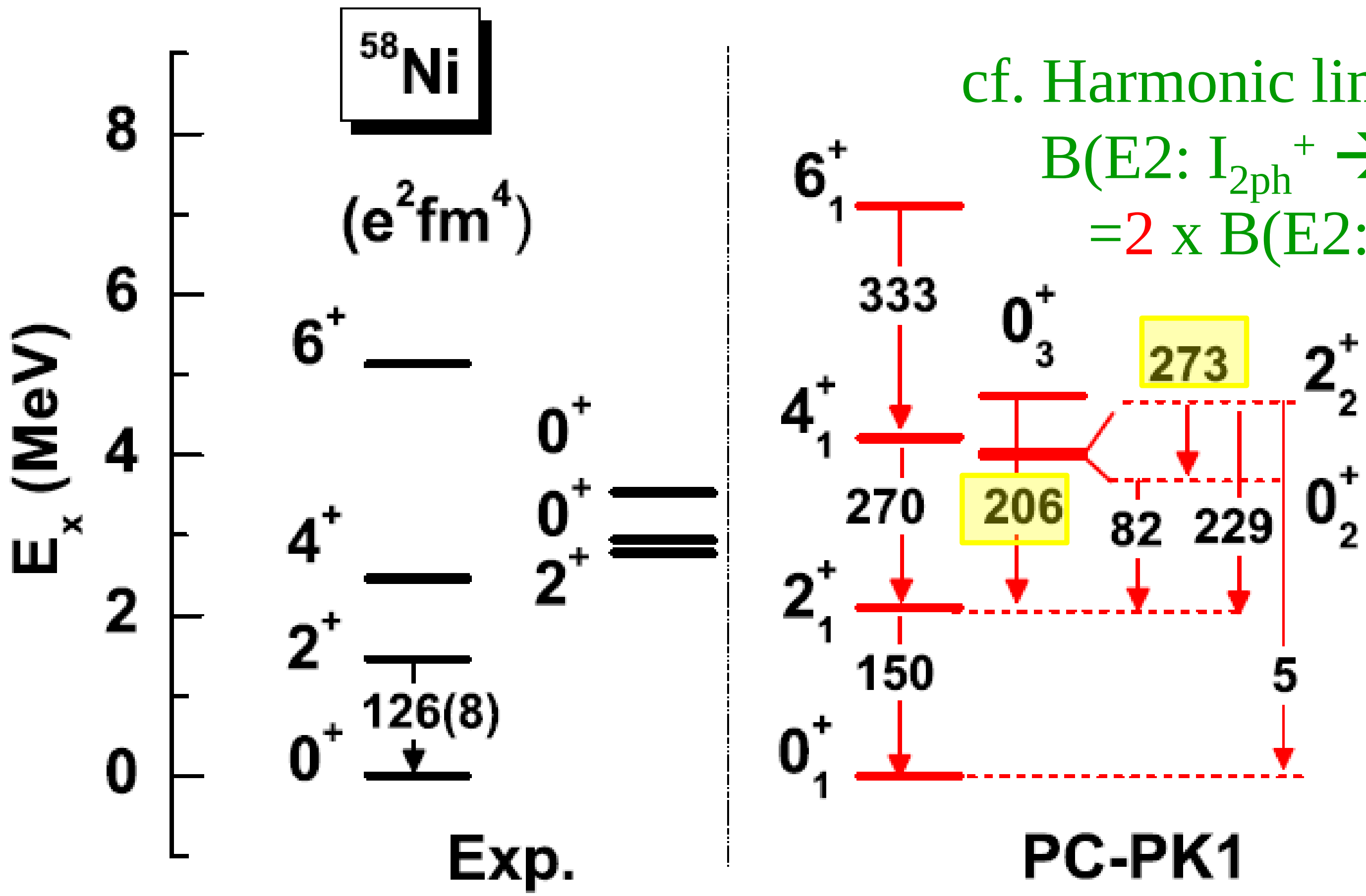




➢ Low-lying spectrum is reproduced rather well.

Recent beyond-MF (MR-DFT) calculations for ^{58}Ni

K.H. and J.M. Yao, PRC91 (‘15) 064606
J.M. Yao, M. Bender, and P.-H. Heenen, PRC91 (‘15) 024301



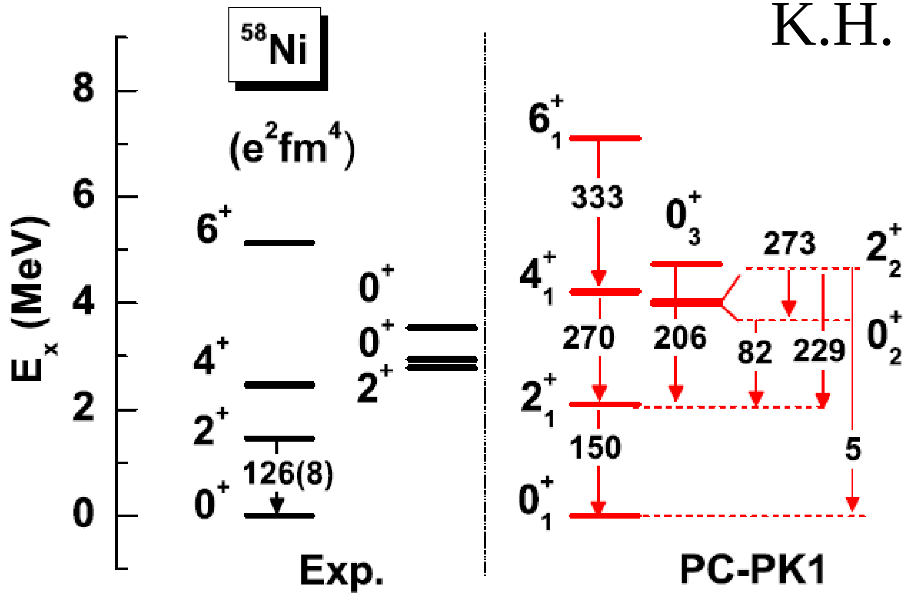
cf. Harmonic limit:
$B(E2: I_{2ph}^+ \rightarrow 2_1^+)$
$= 2 \times B(E2: 2_1^+ \rightarrow 0_1^+)$

- ✓ A large fragmentation of $(2^+ \times 2^+)_{J=0}$
- ✓ A strong transition from 2_2^+ to 0_2^+

→ effects on sub-barrier fusion?

Semi-microscopic coupled-channels model for sub-barrier fusion

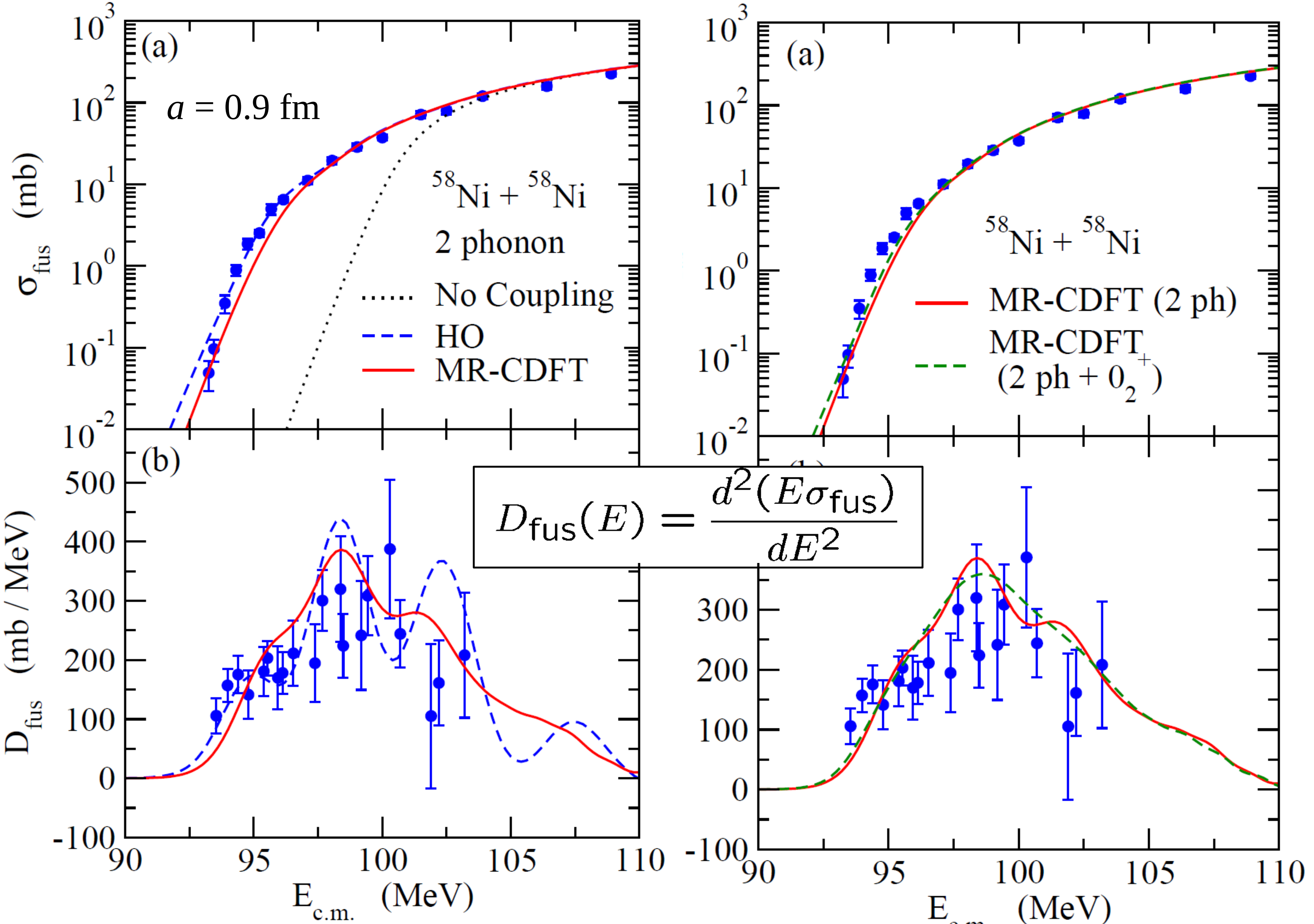
K.H. and J.M. Yao, PRC91 ('15) 064606



microscopic multi-pole operator

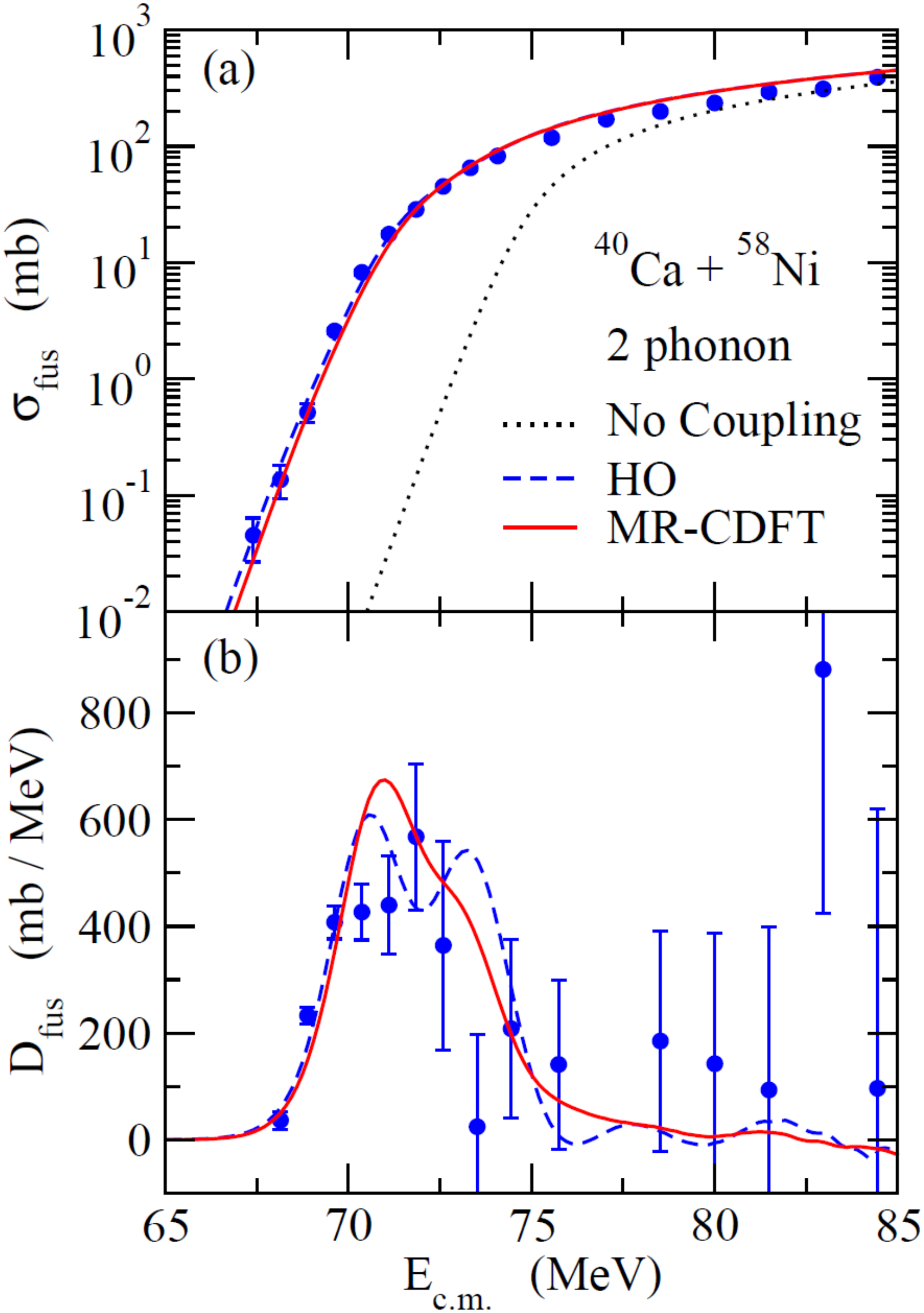
$$\checkmark \quad V_{\rm coup} \sim -R_T \frac{dV_N}{dr} \alpha_\lambda \cdot Y_\lambda(\hat{\boldsymbol{r}}) \to -R_T \frac{dV_N}{dr} Q_\lambda \cdot Y_\lambda(\hat{\boldsymbol{r}})$$

- ✓ *M*(E2) from MR-DFT calculation ← among higher members of phonon states
- ✓ scale to the empirical B(E2; $2_1^+ \to 0_1^+$)
- ✓ still use a phenomenological potential
- ✓ use the experimental values for E_x
- ✓ β_N and β_C from M_n/M_p for each transition
- ✓ axial symmetry (no 3^+ state)



$$D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

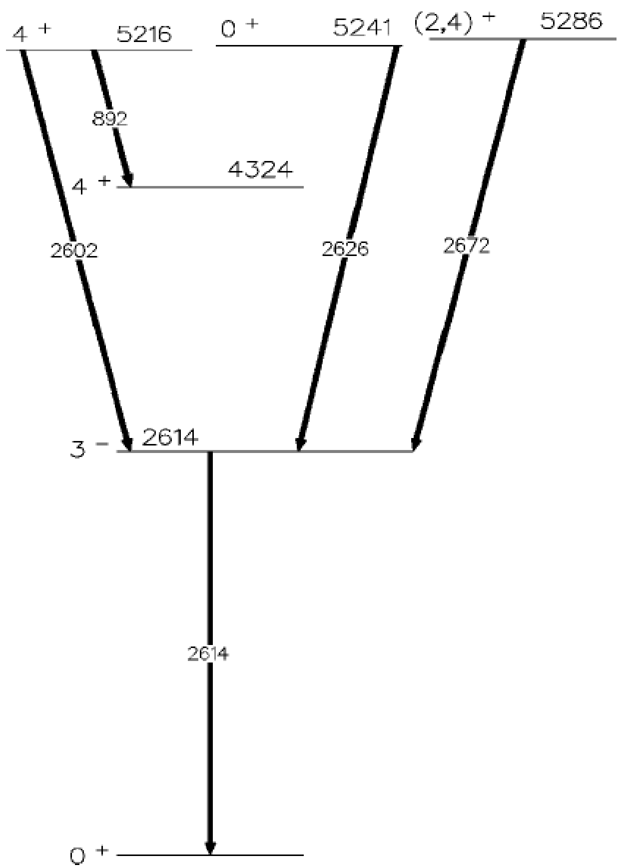
K.H. and J.M. Yao, PRC91 ('15) 064606

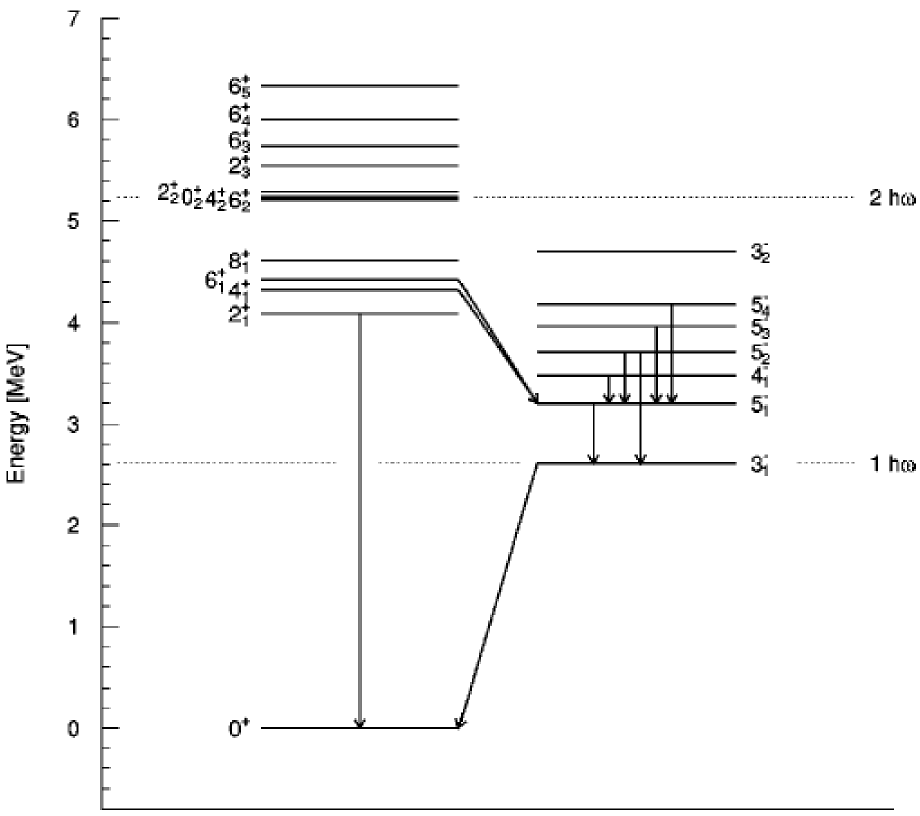


Experimental data:
D. Bourgin, S. Courtin et al.,
PRC90('14)044601.

Application to $^{16}O + ^{208}Pb$ fusion reaction

double-octupole phonon states in ^{208}Pb

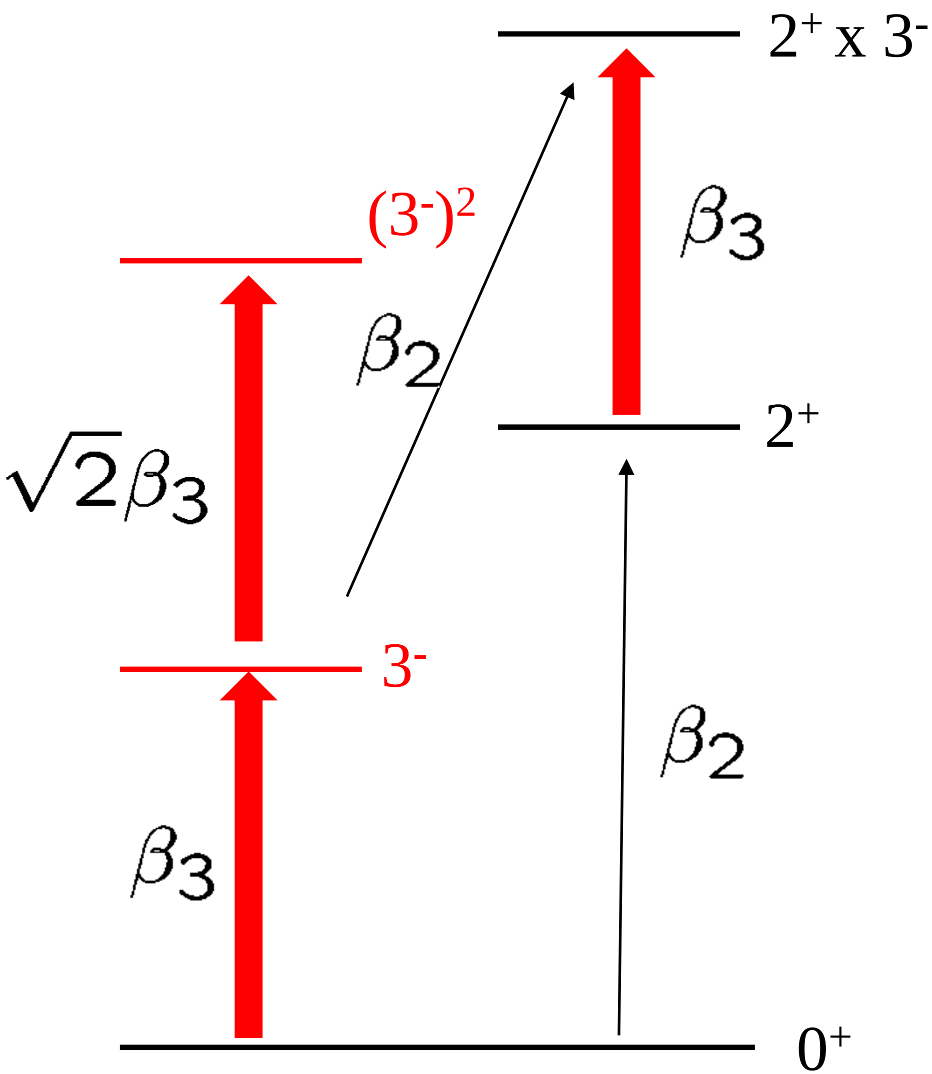


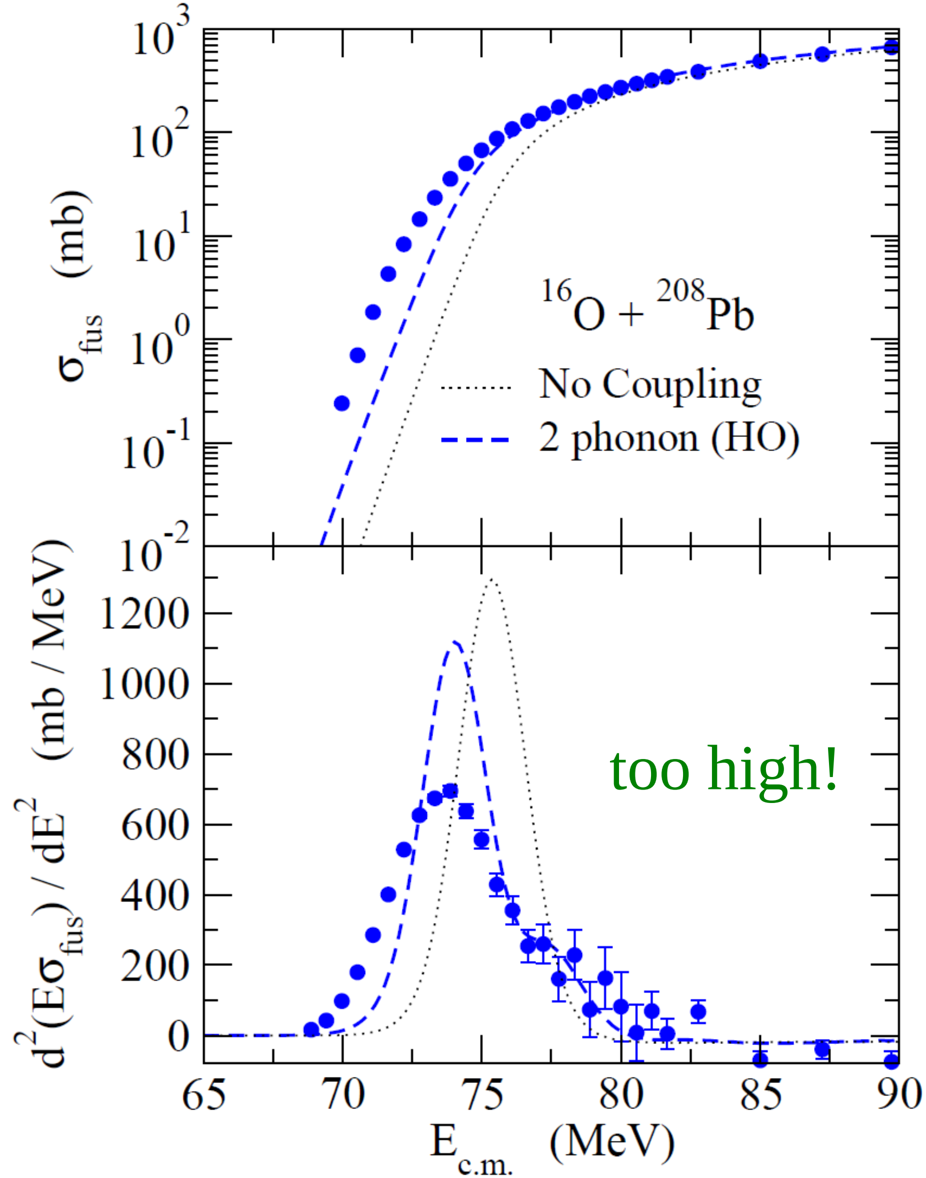


M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 (‘98) R2085

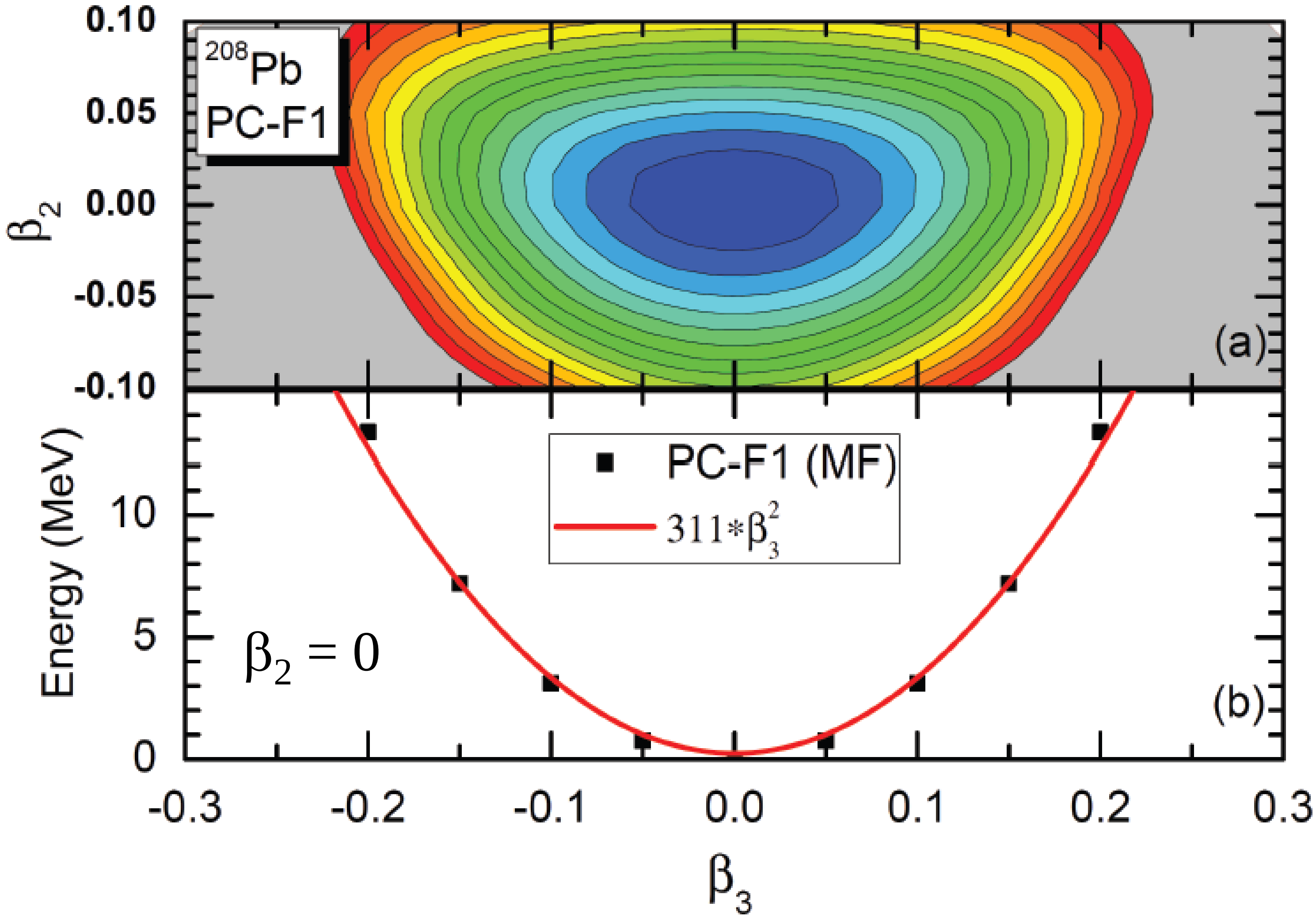
K. Vetter, A.O. Macchiavelli et al., PRC58 (‘98) R2631

Application to $^{16}O + {}^{208}Pb$ fusion reaction



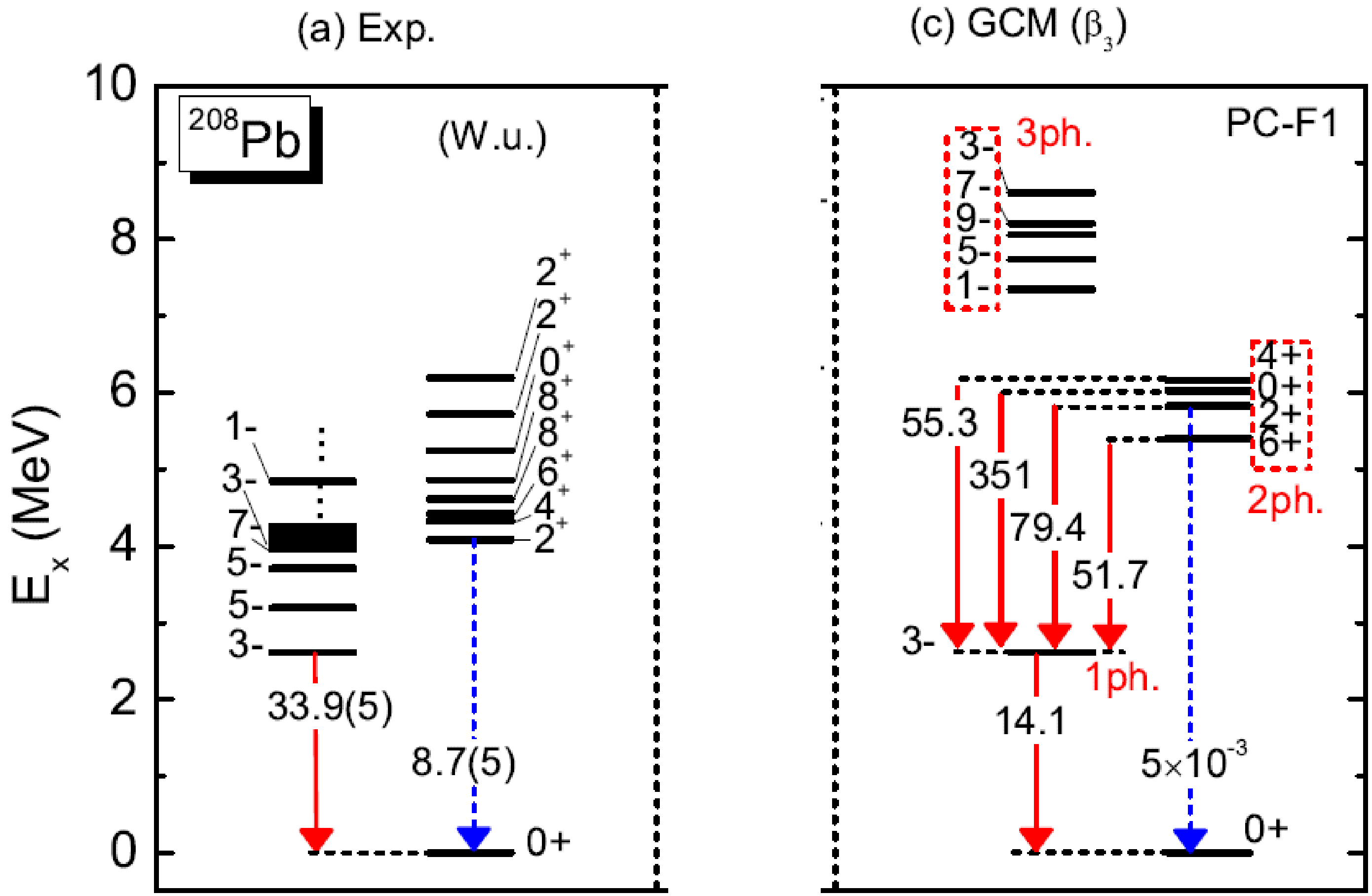


cf. C.R. Morton et al., PRC60('99) 044608 potential energy surface of ^{208}Pb (RMF with PC-F1)



J.M. Yao and K.H., PRC94 (‘16) 11303(R)

Expt. data $\qquad$ β_2=0, fluctuation in β_3



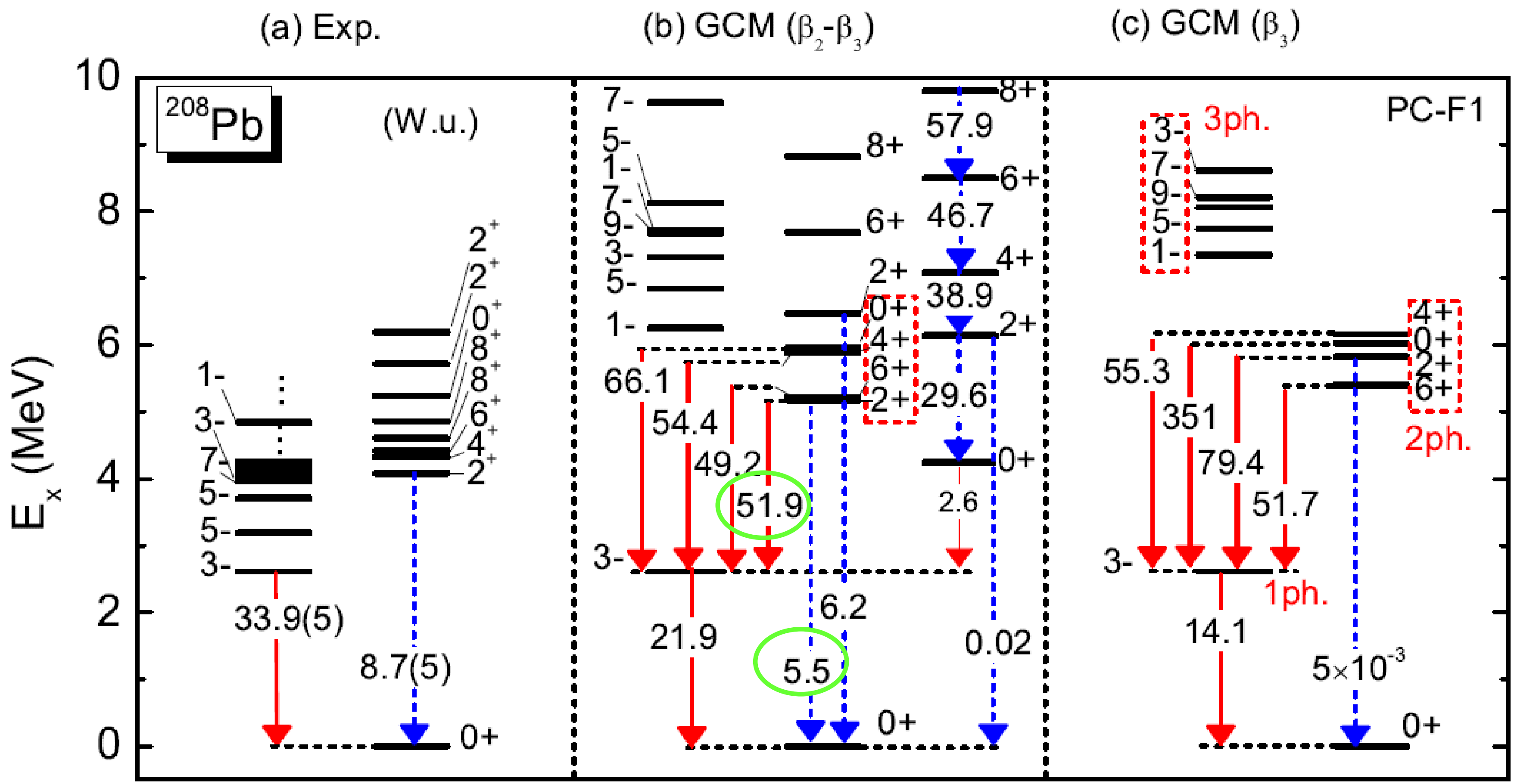
- $E_{2ph} \sim E_{1ph}$
- large anharmonicity in B(E3);

  cf. H.O.: $B(E3: I_{2ph} \rightarrow 3_1^-) = 2\, B(E3: 3_1^- \rightarrow \text{g.s.})$
- underestimate B(E3) (and B(E2))

expt. data

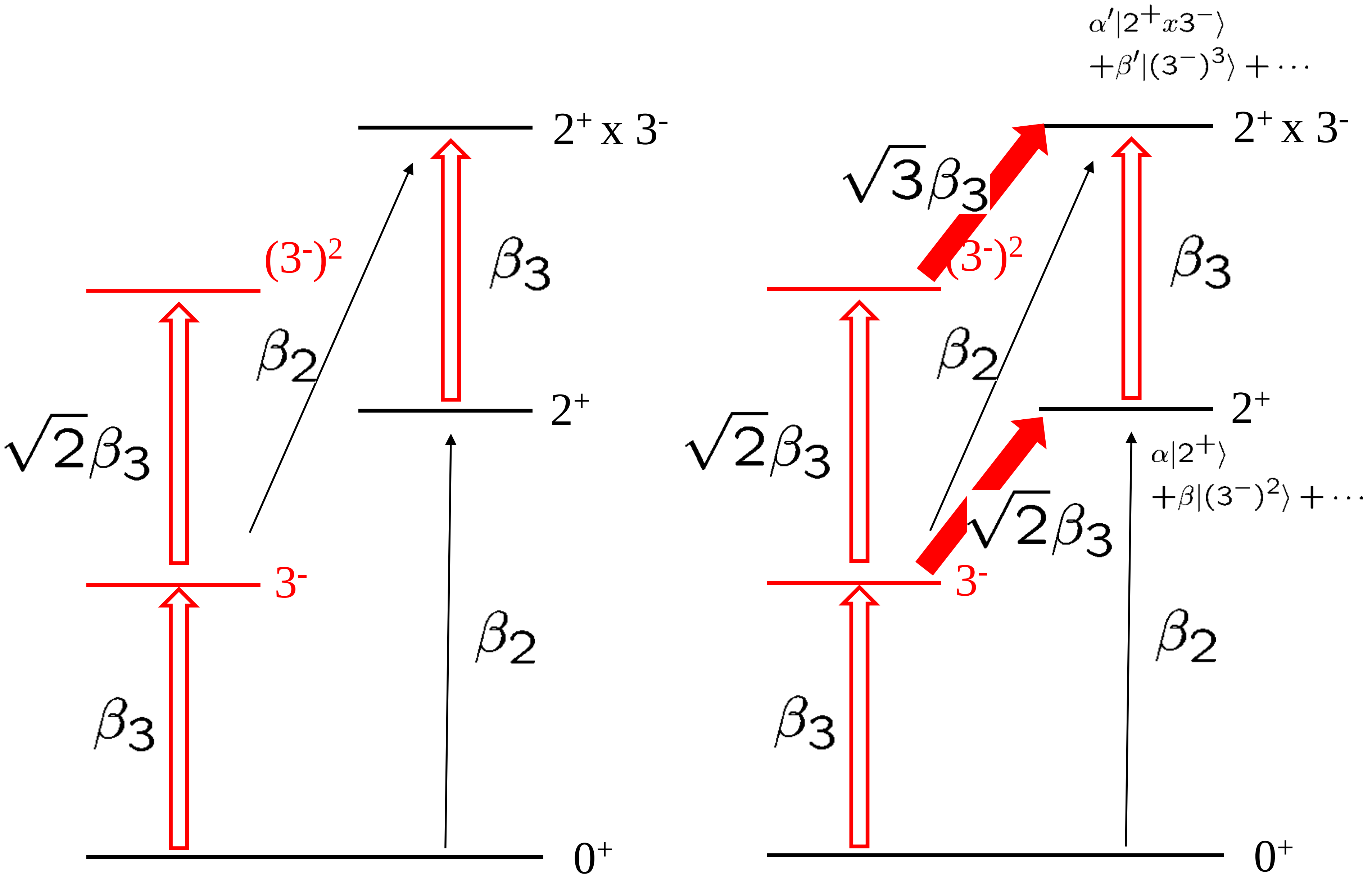
fluctuation both in β_3 and β_2

fluctuation in β_3 frozen at β_2=0



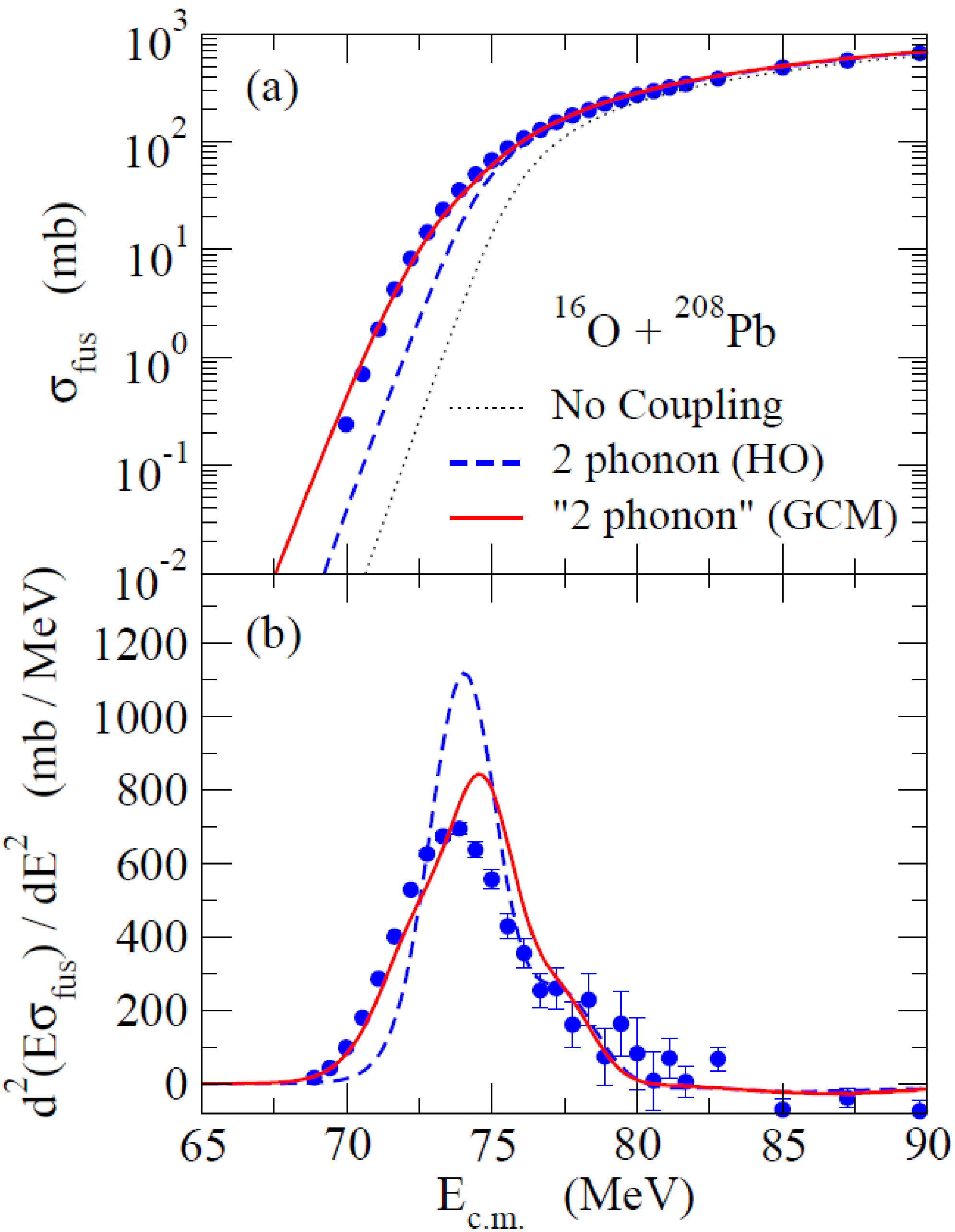
2_1^+ state: strong coupling both to g.s. and 3_1^-

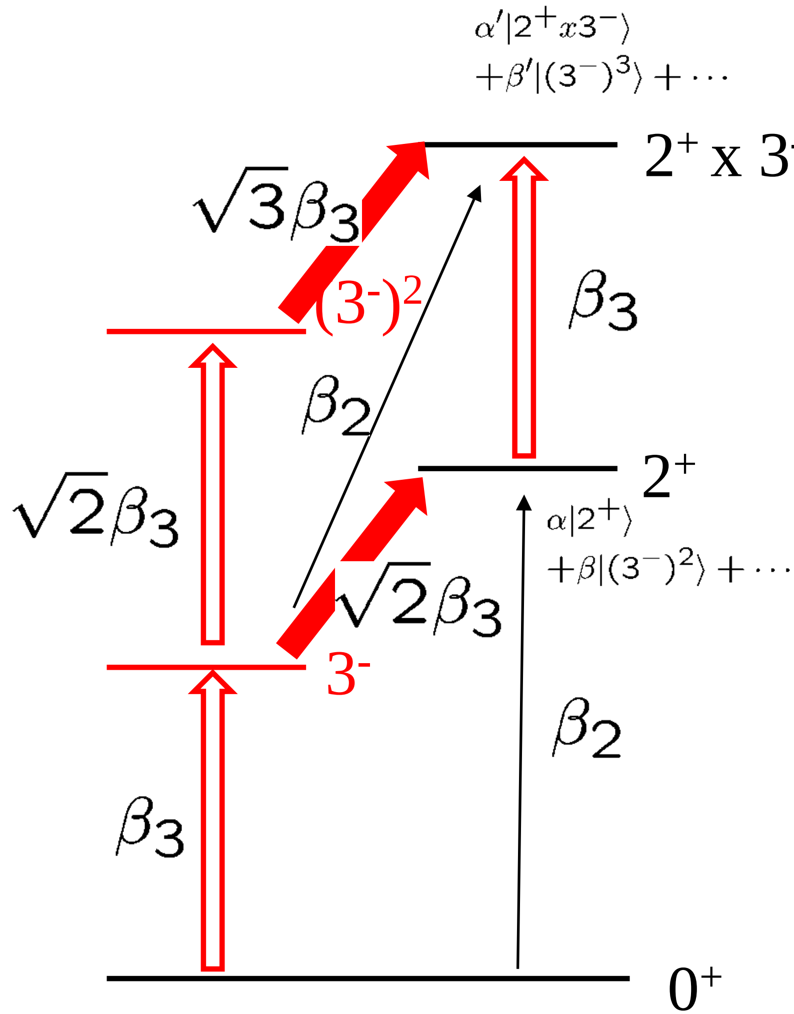
$$\longrightarrow \quad |2_1^+\rangle = \alpha|2^+\rangle_{\rm HO} + \beta|[3^- \otimes 3^-]^{(I=2)}\rangle_{\rm HO} + \cdots$$



Harmonic Oscillator

Anharmonicity





J.M. Yao and K.H., PRC94 ('16) 11303(R)

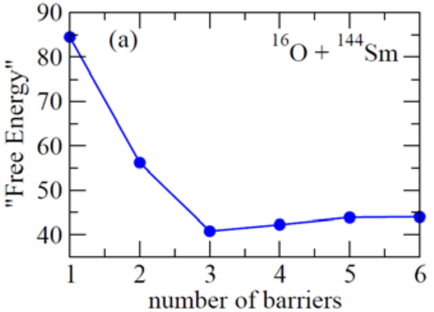
Summary

Heavy-ion subbarrier fusion reactions

✓strong interplay between reaction and structure
cf. fusion barrier distributions

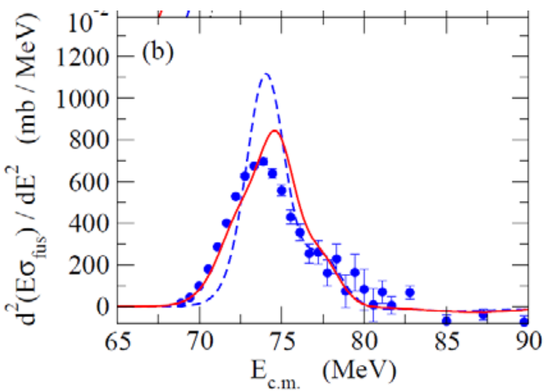
➢A Bayesian approach to fusion barrier distributions

✓ a quick and convenient way to analyze data
✓ determination of the number of barriers



➢C.C. calculations with MR-DFT method

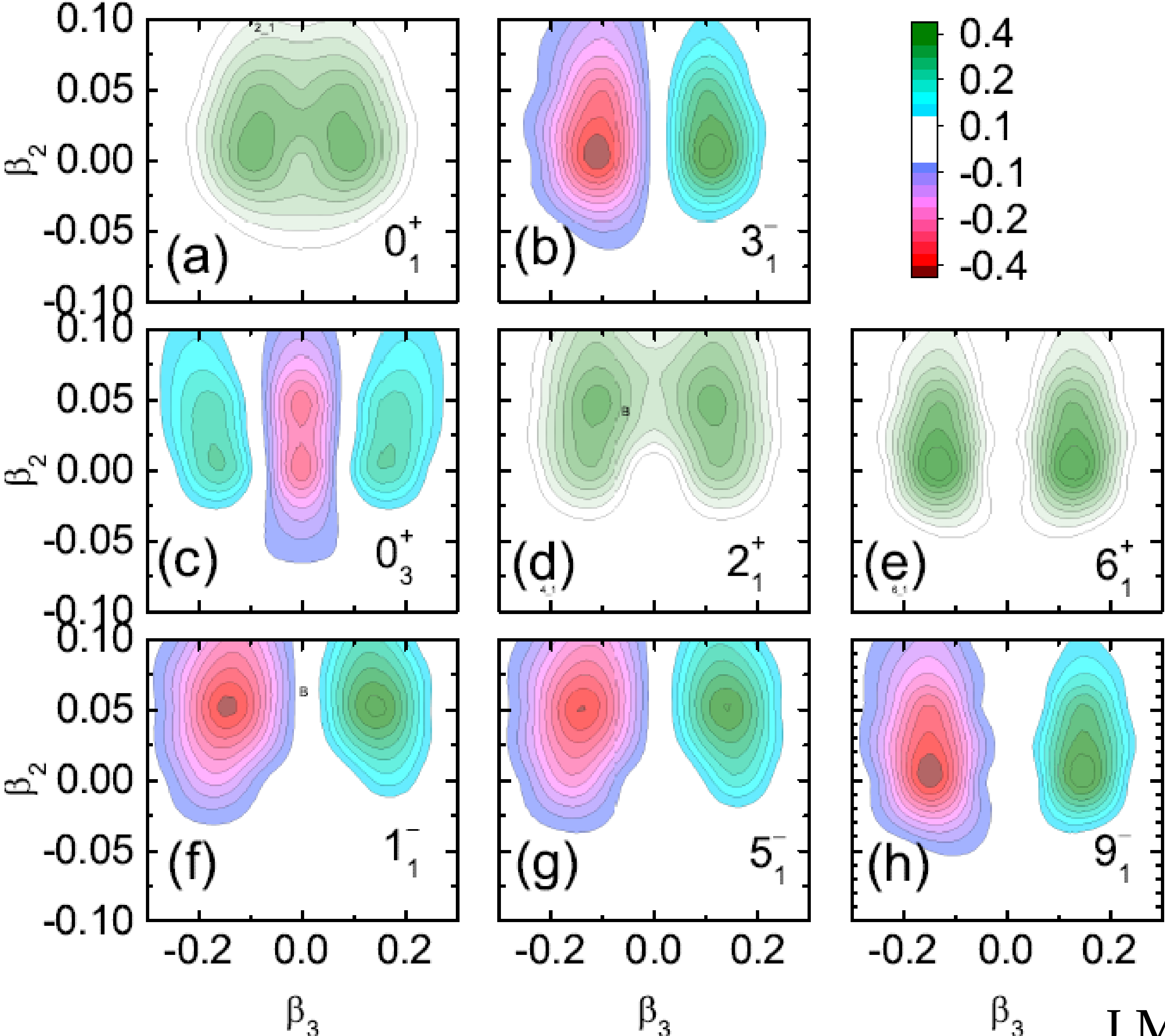
✓ anharmonicity
✓ truncation of phonon states
✓ octupole vibrations: $^{16}O + ^{208}Pb$



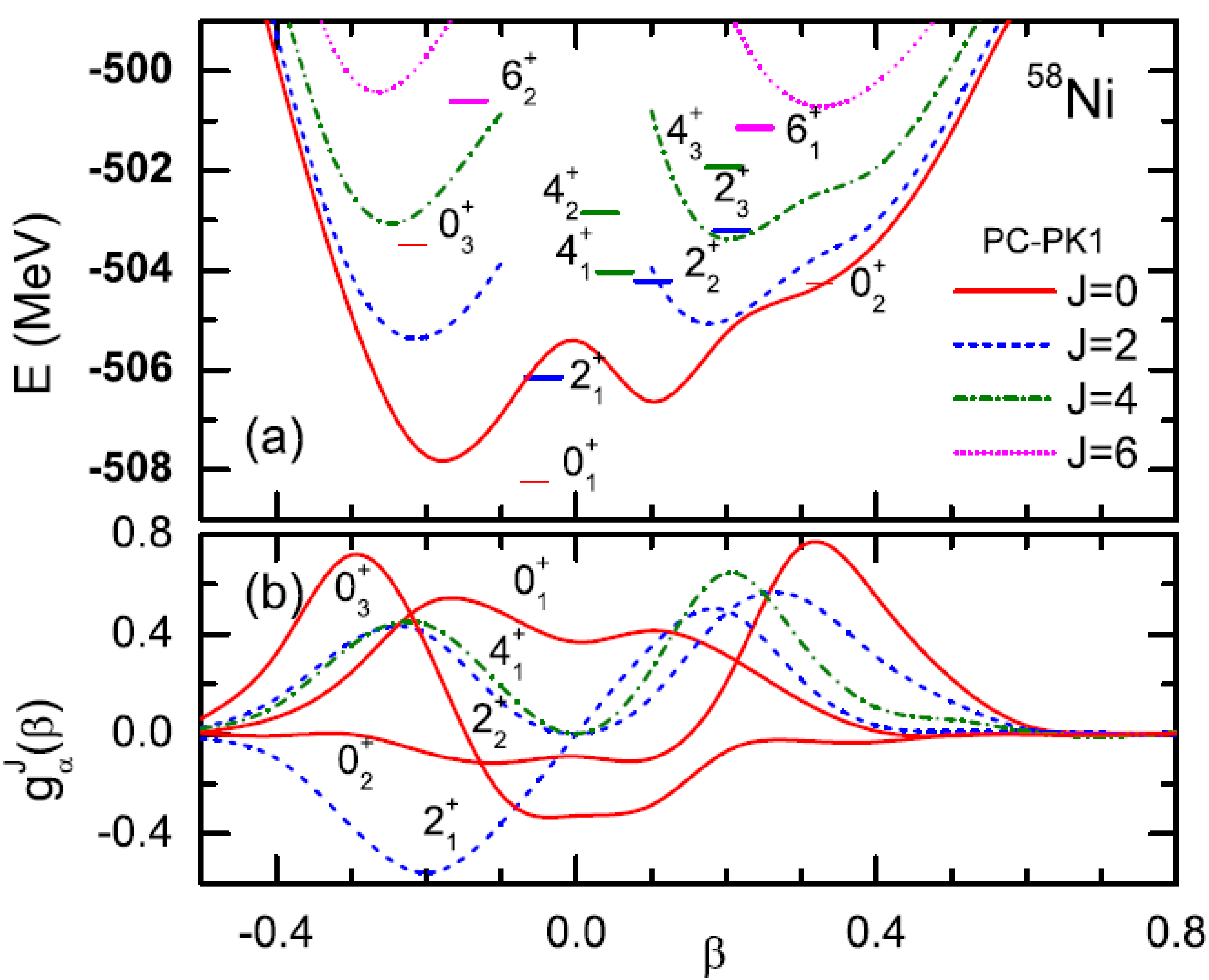
more flexibility:

- application to transitional nuclei
- a good guidance to a Q-moment of excited states

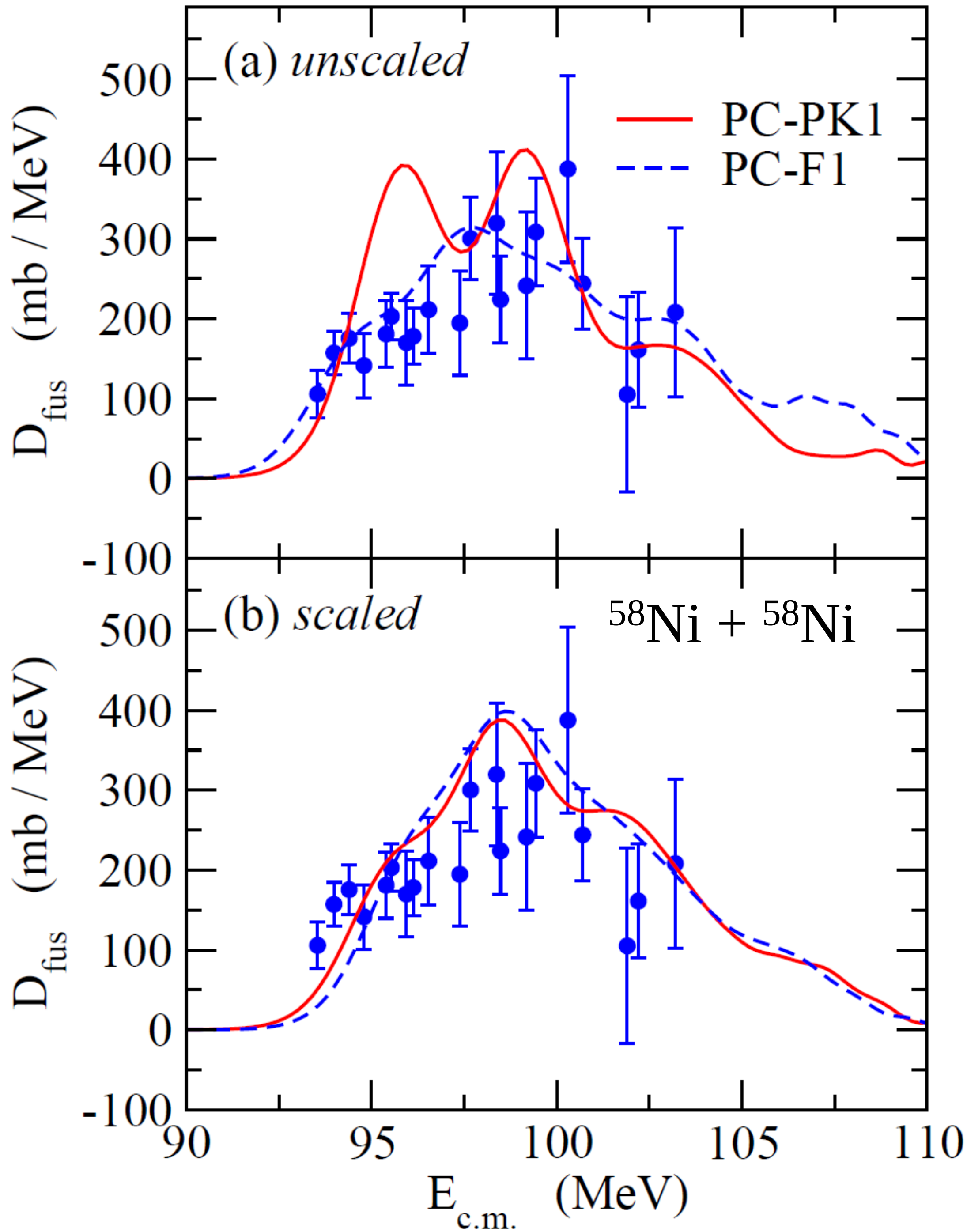
C.C. with shell model?

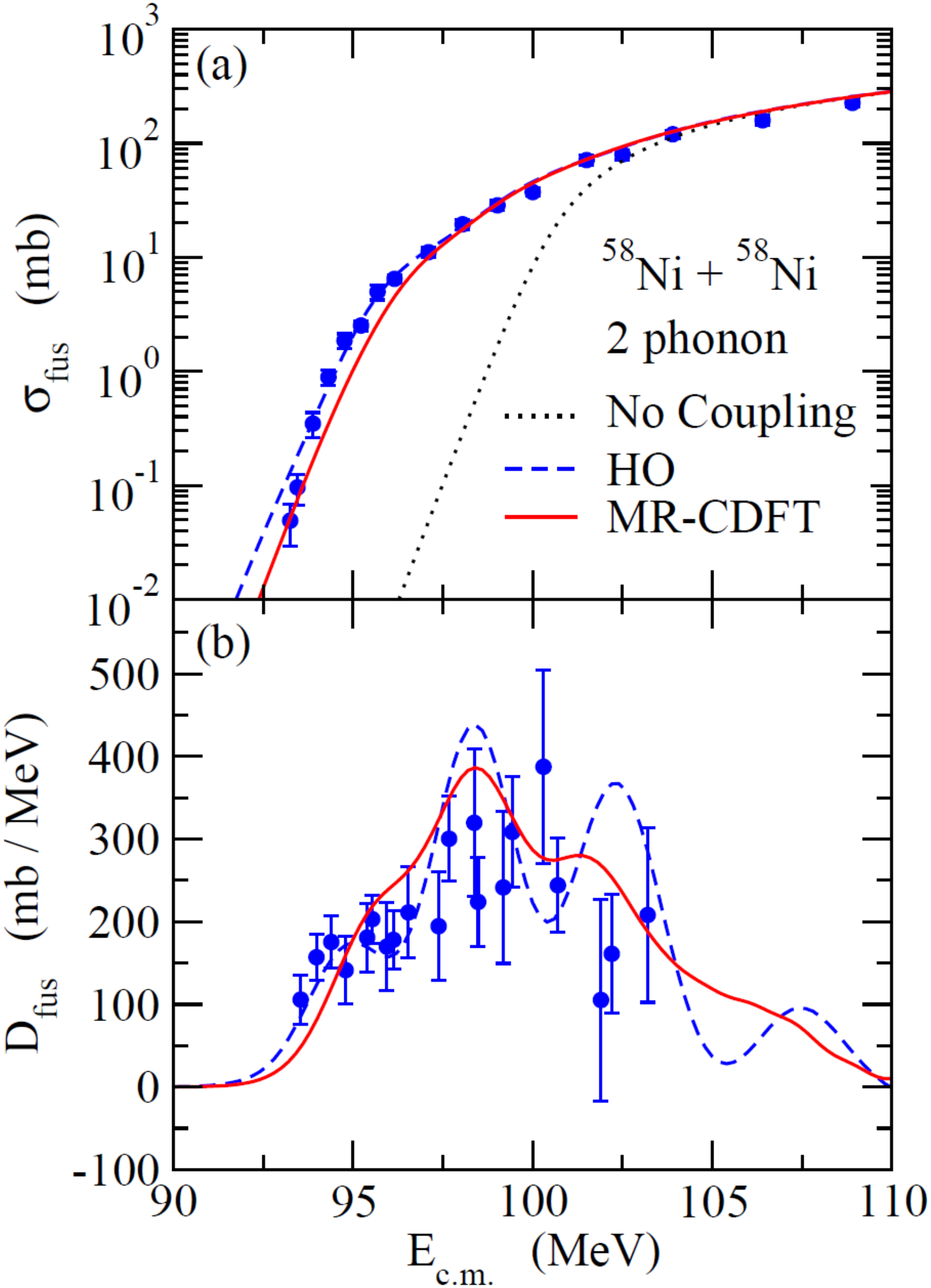


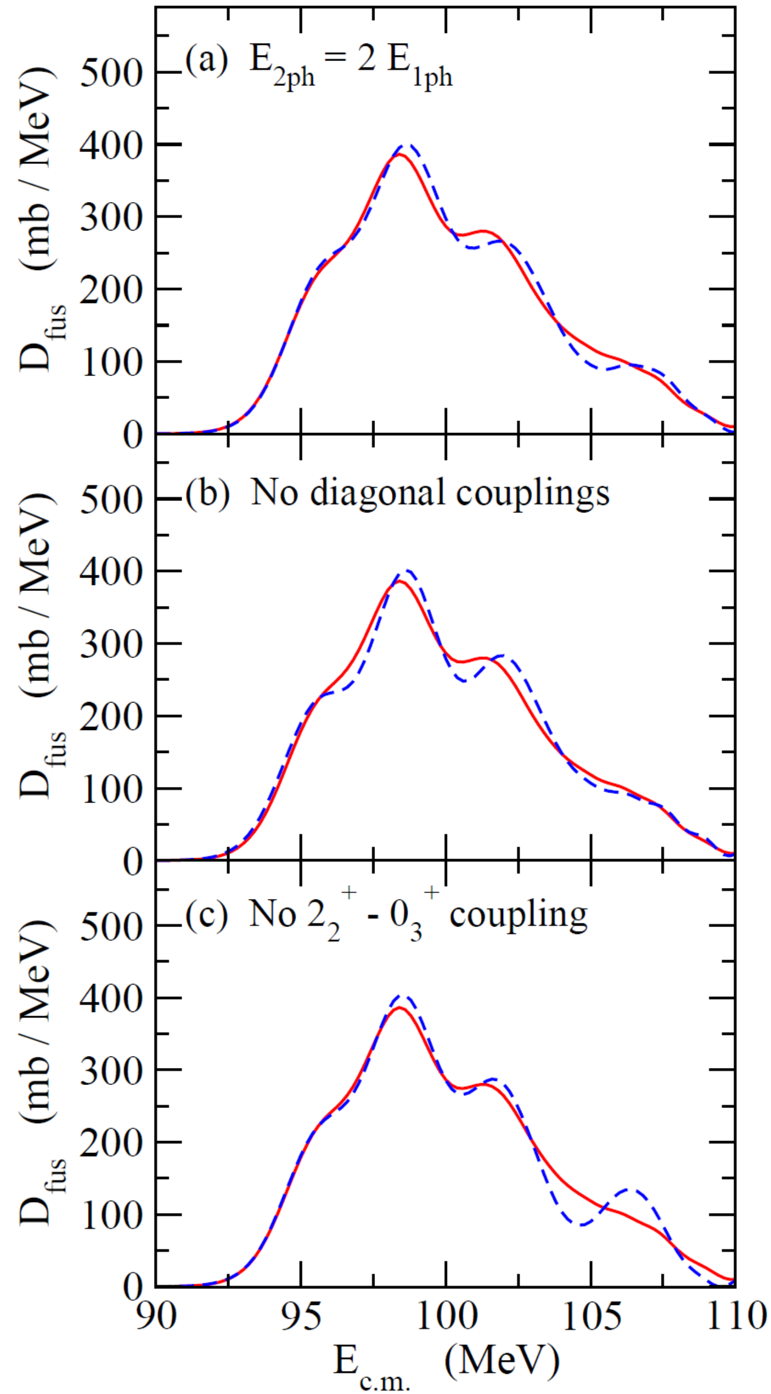
J.M. Yao and K.H., submitted (2016)

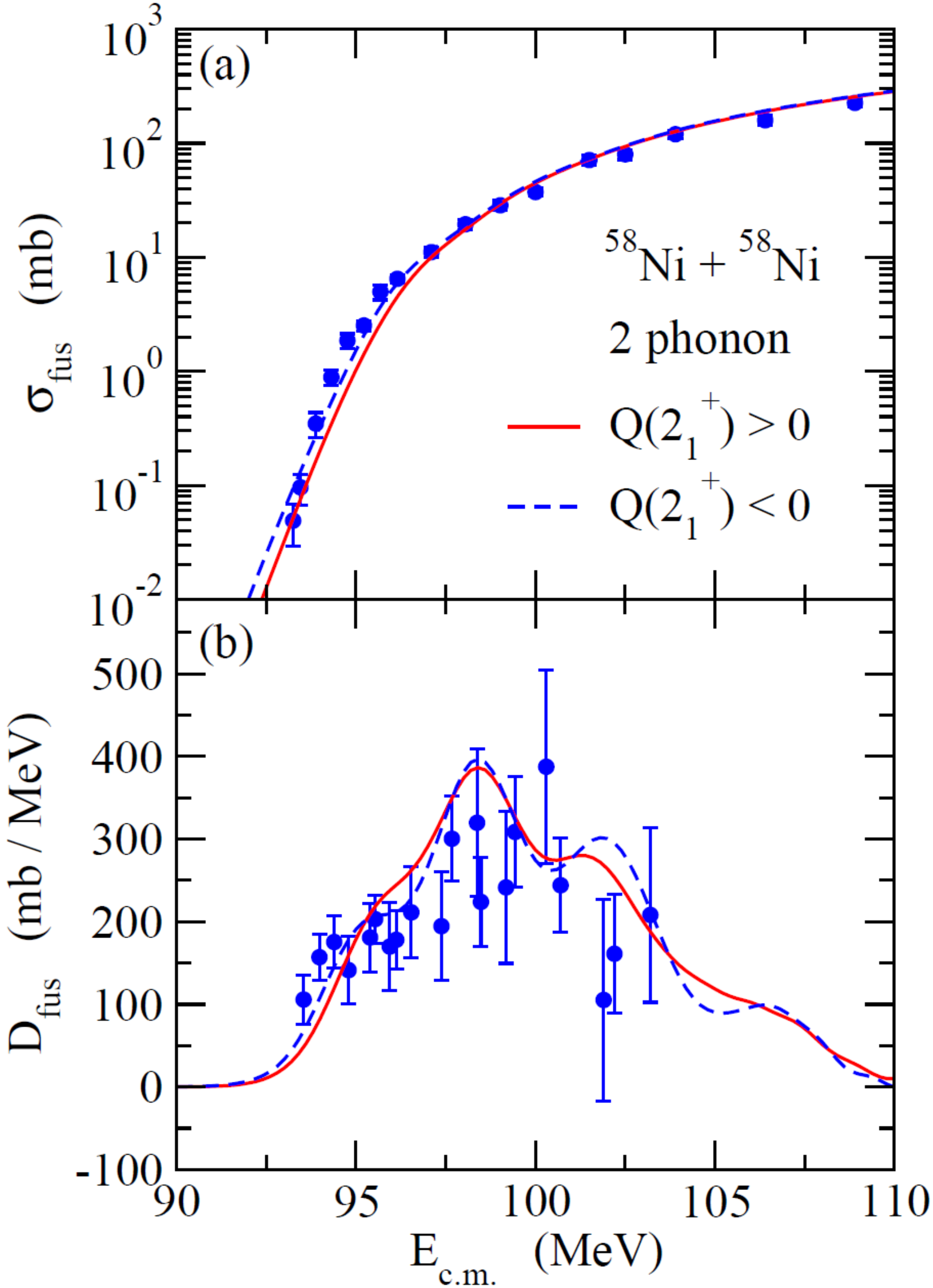


58Ni
PC-PK1
J=0
J=2
J=4
J=6
(a)
(b)
E (MeV)
-500
-502
-504
-506
-508
0.8
0.4
0.0
-0.4
-0.4
0.0
0.4
0.8
β
$g^J_\alpha(\beta)$
6^+_2
6^+_1
4^+_3
4^+_2
4^+_1
2^+_3
2^+_2
2^+_1
0^+_3
0^+_2
0^+_1

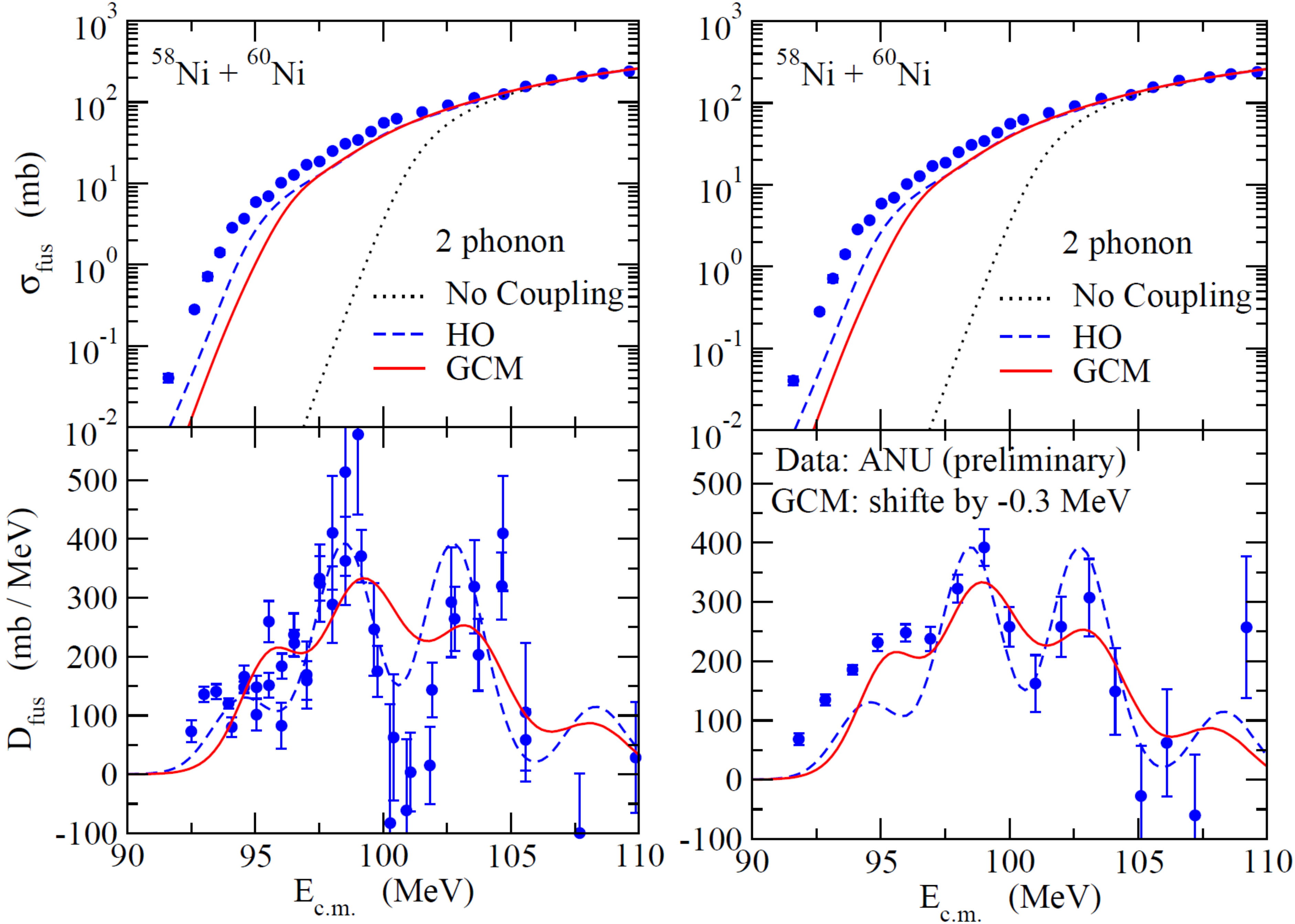

(a) *unscaled*
PC-PK1
PC-F1
(b) *scaled*
^{58}Ni + ^{58}Ni
D_{fus} (mb / MeV)
$E_{c.m.}$ (MeV)
500
400
300
200
100
0
-100
90
95
100
105
110


(a)
^{58}Ni + ^{58}Ni
2 phonon
No Coupling
HO
MR-CDFT
σ_{fus} (mb)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
(b)
D_{fus} (mb / MeV)
500
400
300
200
100
0
-100
90
95
100
105
110
$E_{c.m.}$ (MeV)

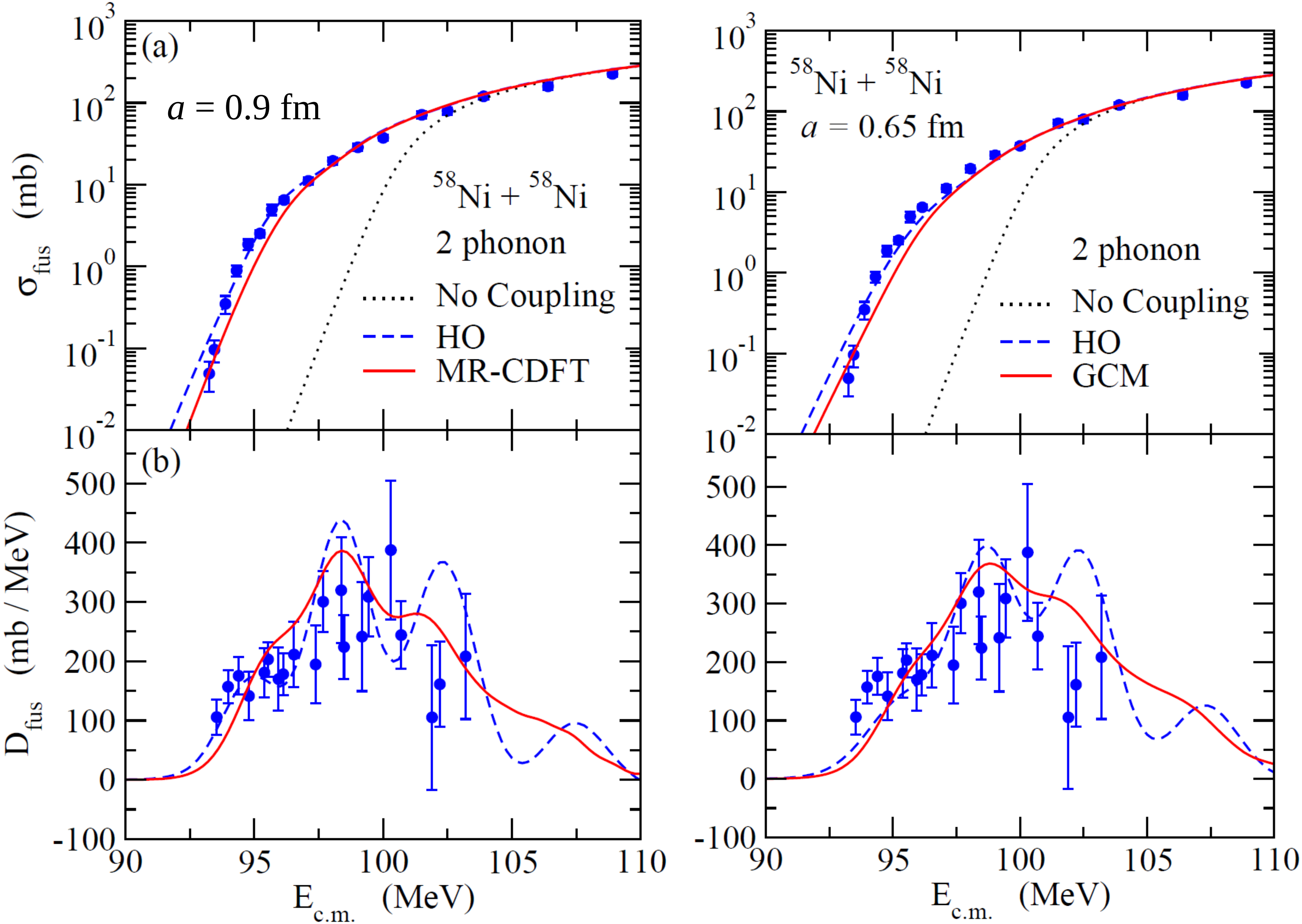

(a) E_{2ph} = 2 E_{1ph}
(b) No diagonal couplings
(c) No 2_2^+ - 0_3^+ coupling
D_{fus} (mb / MeV)
500
400
300
200
100
0
90
95
100
105
110
$E_{c.m.}$ (MeV)



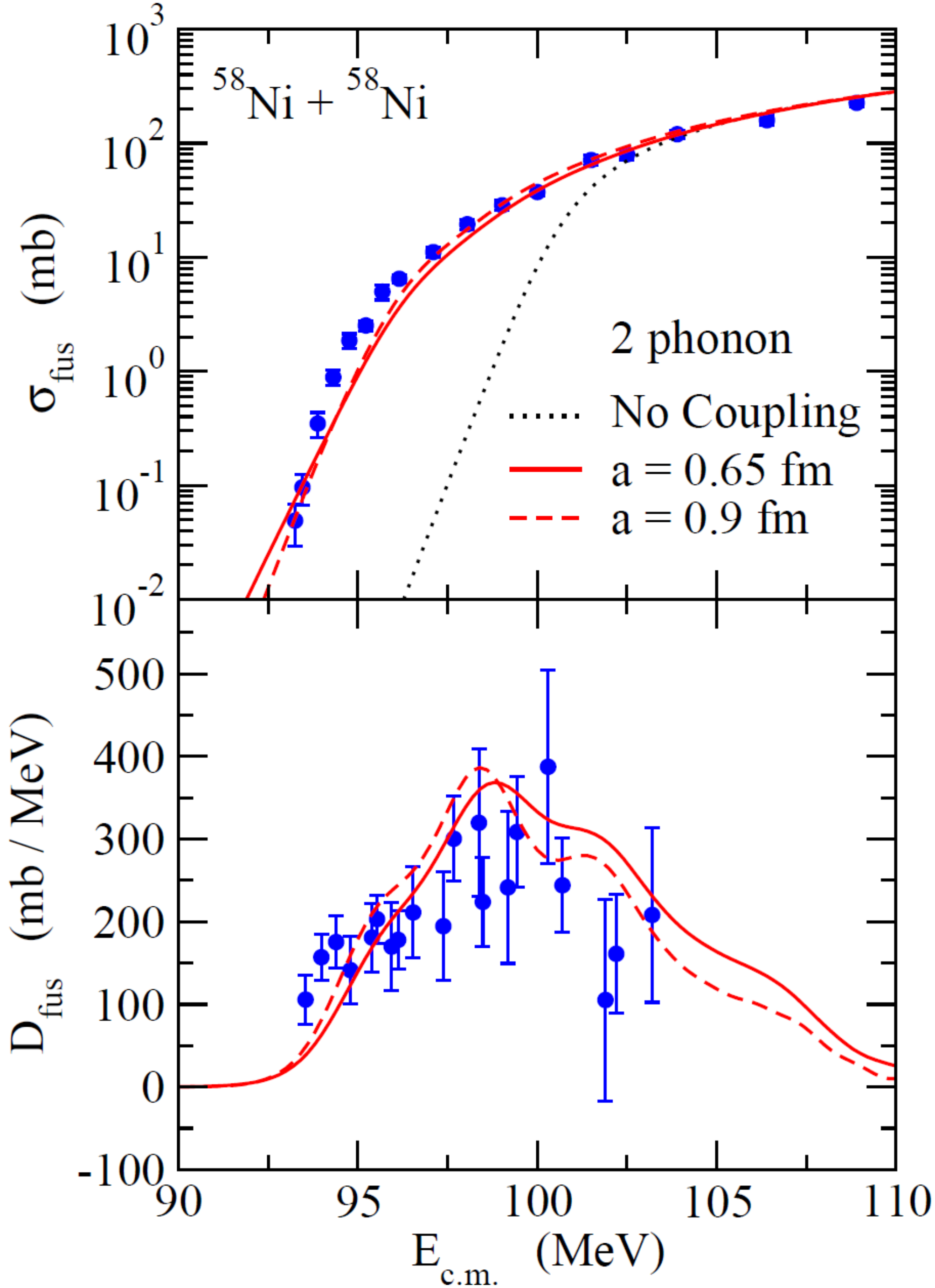
(a)
^{58}Ni + ^{58}Ni
2 phonon
Q(2$_1^+$) > 0
Q(2$_1^+$) < 0
σ_{fus} (mb)
(b)
D_{fus} (mb / MeV)
$E_{c.m.}$ (MeV)



^{58}Ni + ^{60}Ni
2 phonon
No Coupling
HO
GCM
σ_{fus} (mb)
D_{fus} (mb / MeV)
$E_{c.m.}$ (MeV)
Data: ANU (preliminary)
GCM: shifte by -0.3 MeV



K.H. and J.M. Yao, PRC91 ('15) 064606

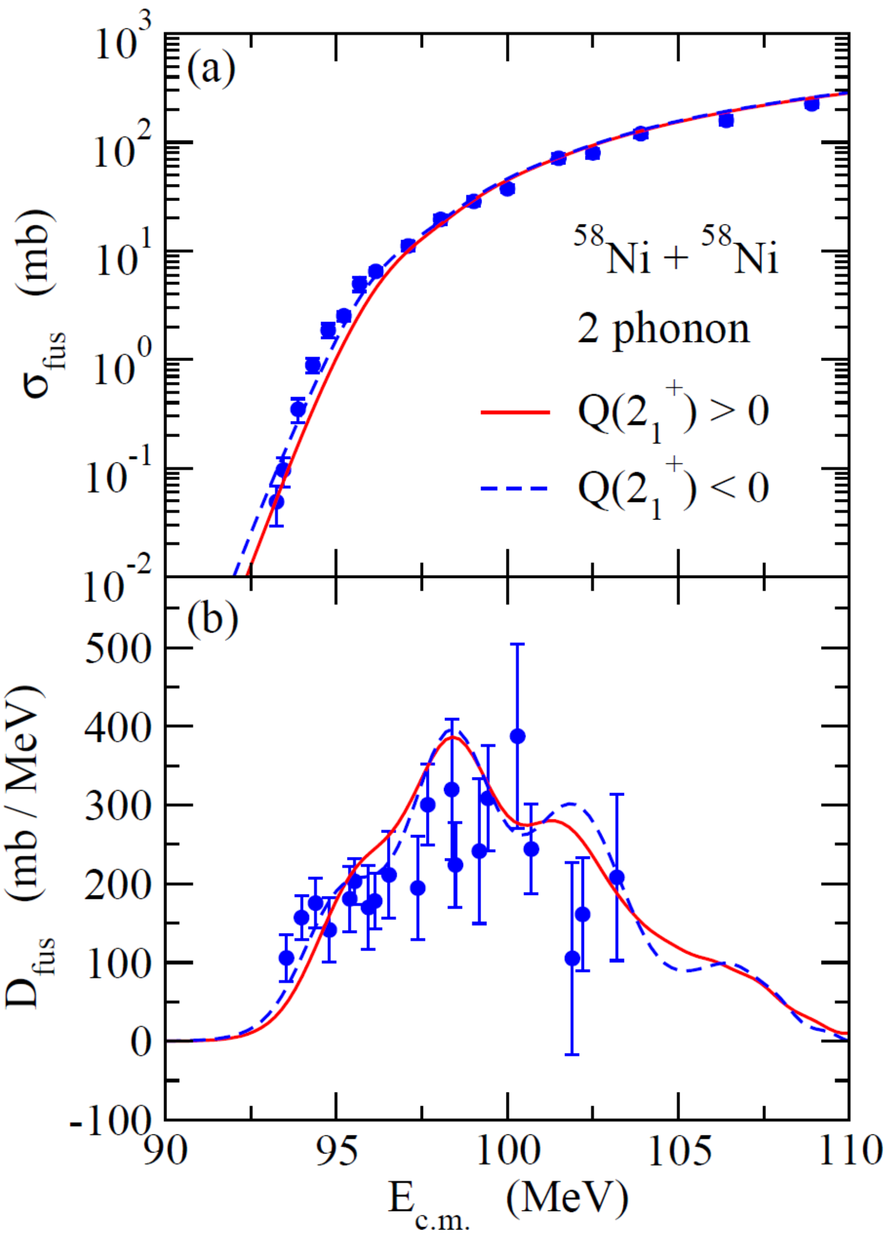


58Ni + 58Ni
2 phonon
No Coupling
a = 0.65 fm
a = 0.9 fm
σfus (mb)
Dfus (mb / MeV)
Ec.m. (MeV)

TABLE I. One-phonon excitation energies $\hbar\omega_\lambda$, β_λ values, and the associated standard deviation of surface amplitudes $\sqrt{2}\sigma_\lambda$ used in the coupled channels calculations for reactions between different nickel isotopes. Average values have been used for the reaction $^{58}Ni+^{64}Ni$.

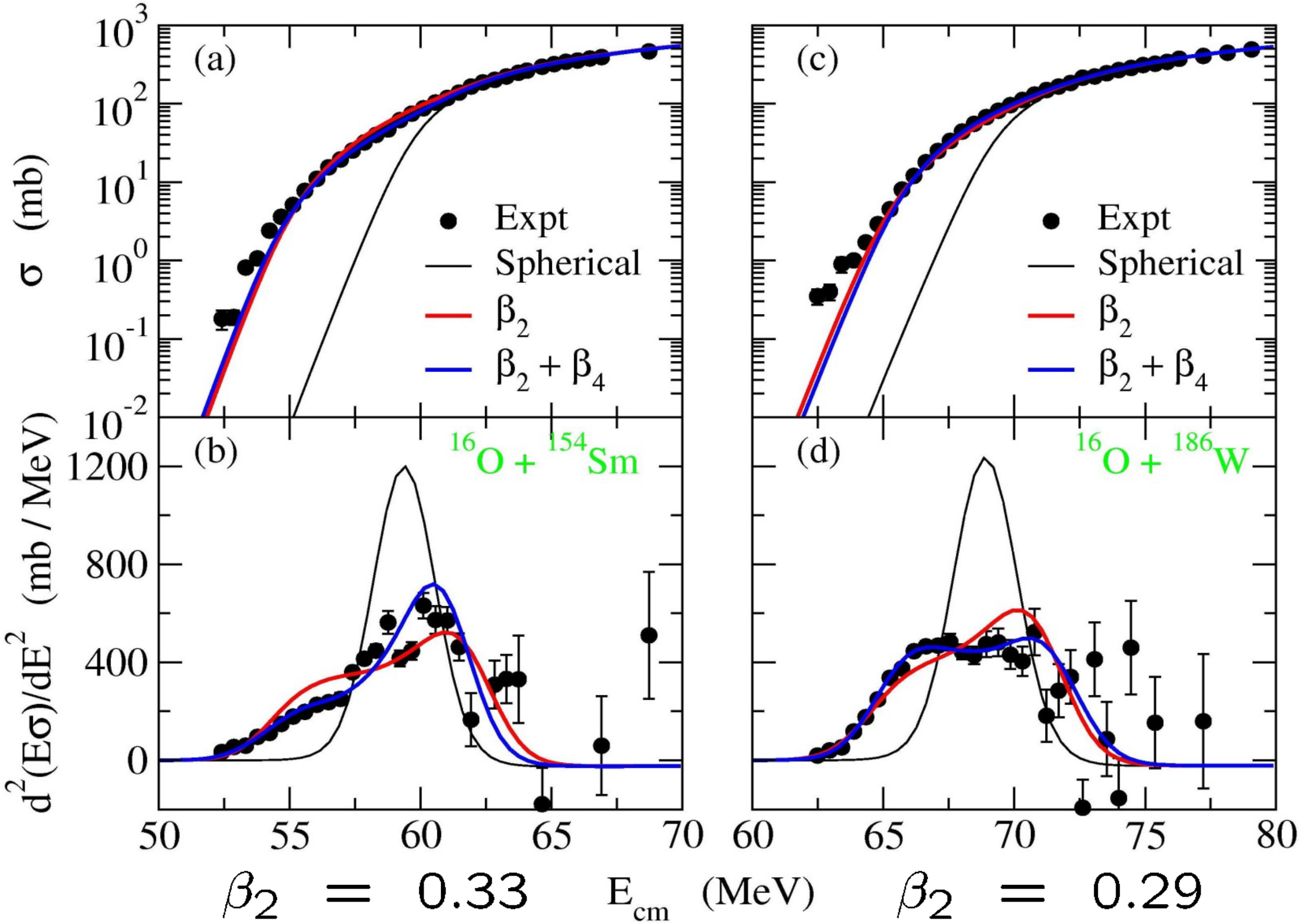
Reaction	λ	$\hbar\omega_\lambda$ (MeV)	β_λ	$\sqrt{2}\sigma_\lambda$ (fm)	ΔR (fm)[a]
$^{58}Ni+^{58}Ni$	2^+	1.45	0.187	0.337	0.00[b]
	3^-	4.47	0.20	0.368	
$^{64}Ni+^{64}Ni$	2^+	1.34	0.19	0.355	0.20
	3^-	3.56	0.18	0.336	
$^{58}Ni+^{64}Ni$	2^+	1.4		0.346	0.14
	3^-	4.0		0.352	

Role of Q-moment of the first 2^+ state



cf. $Q_{exp}(2_1^+) = -10$ +/- 6 $e$fm^2

P.M.S. Lesser et al.,
NPA223 ('74) 563.

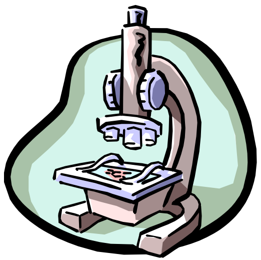


$\beta_2 = 0.33$ $\beta_2 = 0.29$

$\beta_4 = +0.05$ $\beta_4 = -0.03$

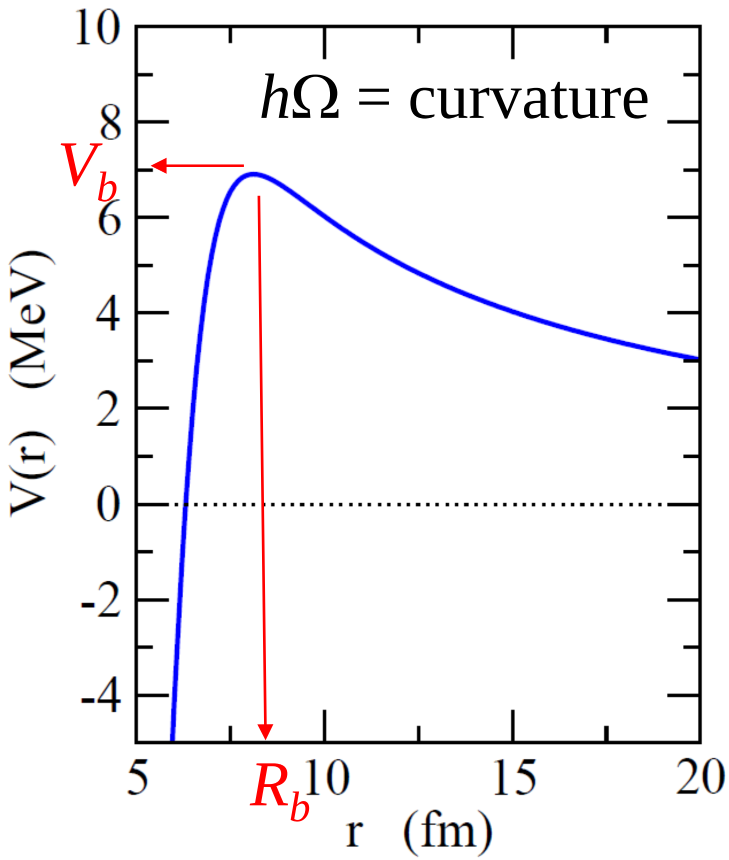
M. Dasgupta et al.,
Annu. Rev. Nucl. Part.
Sci. 48('98)401

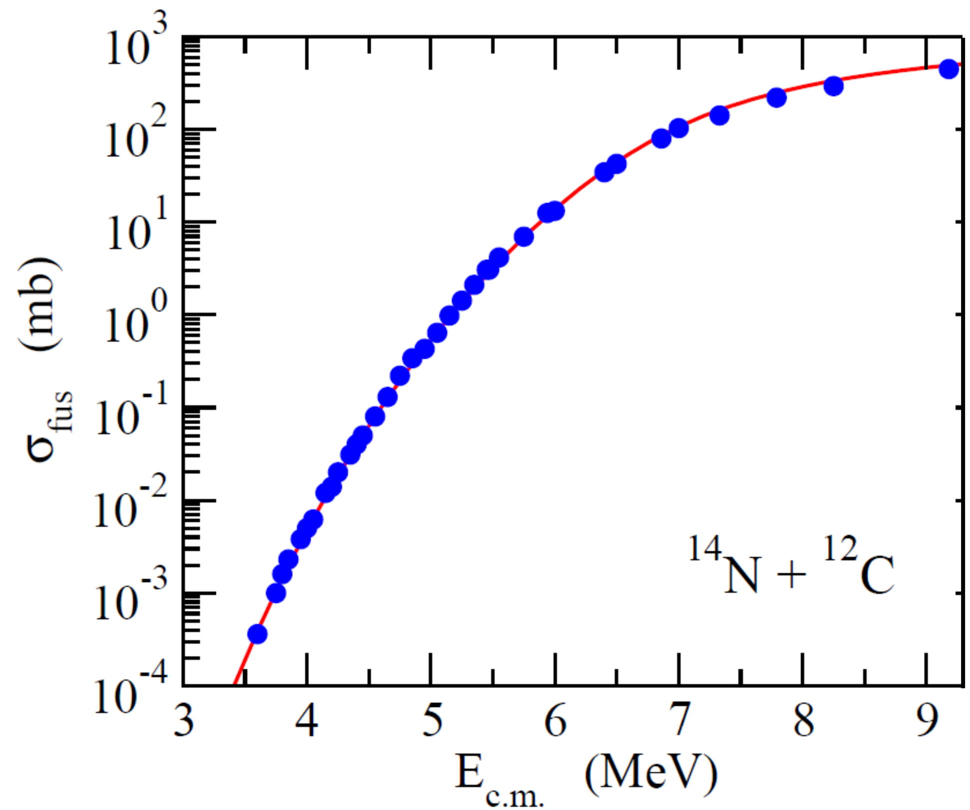
Fusion barrier distribution:
sensitive to small effects such as β_4



the simplest approach: potential model with $V(r)$ + absorption

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$





- Wong formula [C.Y. Wong, PRL31 ('73)766]

$$\sigma_{\text{fus}}(E) \sim \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

TABLE I. Optimum values for the fitting parameters obtained with the Bayesian spectral deconvolution. K is the number of barriers and w_k is the weight factor for each barrier. B_k, R_k, and $\hbar\Omega_k$ are the height, position, and curvature of each barrier, respectively.

System	K	B_k (MeV)	$w_k R_k^2$ (fm^2)	$\hbar\Omega_k$ (MeV)
$^{16}\mathrm{O}+^{144}\mathrm{Sm}$	3	59.5 ± 0.0789	64.3 ± 6.06	3.58 ± 0.149
		61.5 ± 0.153	28.6 ± 6.53	2.34 ± 0.506
		65.3 ± 0.251	29.0 ± 4.63	3.00 ± 0.338
$^{16}\mathrm{O}+^{154}\mathrm{Sm}$	5	53.3 ± 1.10	4.76 ± 2.87	3.96 ± 0.589
		55.9 ± 0.467	18.3 ± 3.16	4.40 ± 0.438
		58.5 ± 0.758	34.7 ± 19.4	3.77 ± 0.786
		60.4 ± 0.833	40.7 ± 21.4	3.90 ± 0.939
		62.4 ± 0.751	21.4 ± 10.7	3.25 ± 0.866

Bayesian spectrum deconvolution

K. Nagata, S. Sugita, and M. Okada, Neural Networks 28 ('12) 82

- ✓ data set: $D_{exp} = \{E_i, d_i, \delta d_i\}$ (i = 1 ~ M)
- ✓ fitting function: $D_{\mathrm{fit}}(E; \tilde{\boldsymbol{\theta}}, K) = \sum_{k=1}^{K} w_k\, \phi_k(E; \boldsymbol{\theta}_k)$

assumption: the data $d_i = D_{fit} + \delta d_i$

$$\longrightarrow \quad P(d_i|E_i, \tilde{\boldsymbol{\theta}}, K) = \frac{1}{\sqrt{2\pi(\delta d_i)^2}} \exp\left(-\frac{(d_i - D_{\mathrm{fit}}(E_i; \tilde{\boldsymbol{\theta}}, K))^2}{2(\delta d_i)^2}\right)$$

$$\longrightarrow \quad P(D_{\mathrm{exp}}|\tilde{\boldsymbol{\theta}}, K) = \prod_{i=1}^{M} P(d_i|E_i, \tilde{\boldsymbol{\theta}}, K) \boxed{\propto e^{-\chi^2(\tilde{\boldsymbol{\theta}}, K)/2}}$$

$$\longrightarrow \quad P(D_{\mathrm{exp}}|K) = \int d\tilde{\boldsymbol{\theta}}\, P(D_{\mathrm{exp}}|\tilde{\boldsymbol{\theta}}, K) P(\tilde{\boldsymbol{\theta}})$$

Bayes theorem

$$P(K|D_{\mathrm{exp}}) = \frac{P(D_{\mathrm{exp}}|K)P(K)}{P(D_{\mathrm{exp}})} \propto P(D_{\mathrm{exp}}|K)$$

most probable value of K: maximize

$$Z(K) = \int d\tilde{\boldsymbol{\theta}}\, e^{-\chi^2(\tilde{\boldsymbol{\theta}}, K)/2} P(\tilde{\boldsymbol{\theta}})$$

Bayesian spectrum deconvolution

K. Nagata, S. Sugita, and M. Okada, Neural Networks 28 ('12) 82

- ✓ data set: $D_{exp} = \{E_i, d_i, \delta d_i\}$ (i = 1 ~ M)
- ✓ fitting function: $D_{\text{fit}}(E;\tilde{\boldsymbol{\theta}},K) = \sum_{k=1}^{K} w_k\,\phi_k(E;\boldsymbol{\theta}_k)$

assumption: the data $d_i = D_{fit} + \delta d_i$

→ $$P(D_{\text{exp}}|K) \propto \int d\tilde{\boldsymbol{\theta}}\, e^{-\chi^2(\tilde{\boldsymbol{\theta}},K)} P(\tilde{\boldsymbol{\theta}})$$

Bayes theorem

$$P(K|D_{\text{exp}}) = \frac{P(D_{\text{exp}}|K)P(K)}{P(D_{\text{exp}})} \propto P(D_{\text{exp}}|K)$$

most probable value of K: maximize

$$Z(K) = \int d\tilde{\boldsymbol{\theta}}\, e^{-\chi^2(\tilde{\boldsymbol{\theta}},K)/2} P(\tilde{\boldsymbol{\theta}})$$

(high dim. integral → MC method)

or equivalently, minimize the "Free Energy" $F(K) = -\ln Z(K)$

→ optimize the other parameters for a given value of K