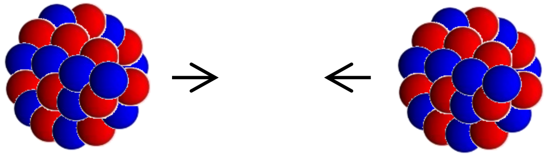
Nuclear shape dynamics in low-energy heavy-ion reactions

heavy ions = nuclei with A $\geq$ 4



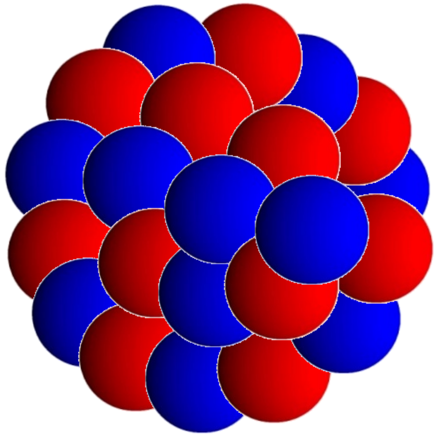
Kouichi Hagino

Kyoto University, Kyoto, Japan

1. Introduction to low-energy nuclear physics
2. Potential model: optical potentials
3. Multi-channel dynamics: coupled-channels approach
4. Probing nuclear structure in Relativistic heavy-ion collisions
5. Summary

physics colloquium, the University of Tennessee, Sep. 29, 2025

Low-energy nuclear physics

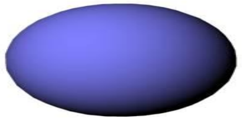


Nuclei as quantum many-body systems of nucleons

← to understand several properties in terms of nucleon d.o.f.

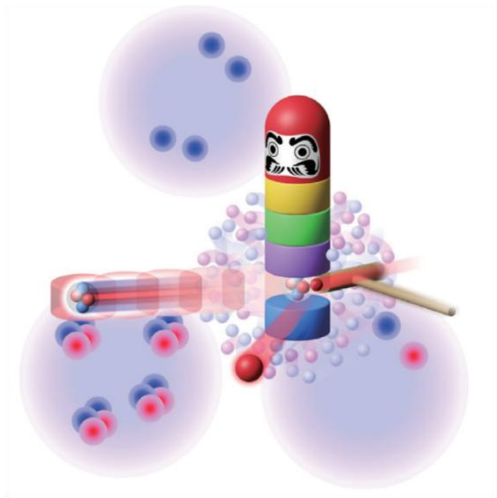
- static properties: nuclear structure

  the mass, the size, the shape....

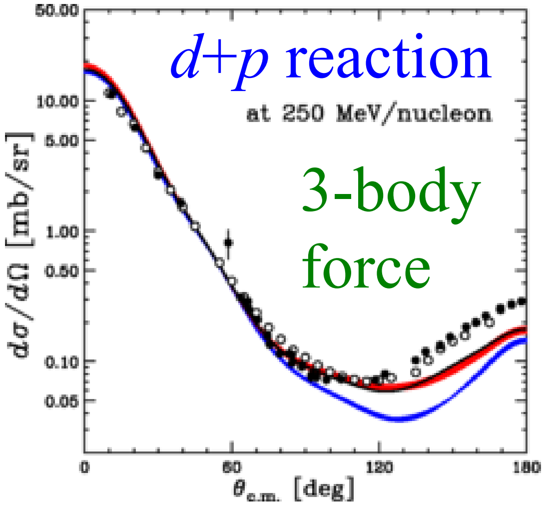


- dynamics: nuclear reactions

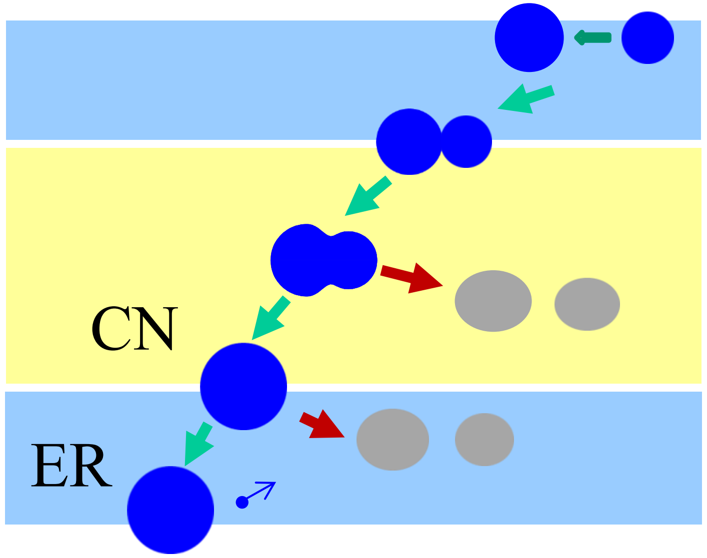
✓ Nuclear Reactions as a tool to investigate nuclear structure



knock-out reactions



K. Sekiguchi et al., PRC89 ('14) 064007

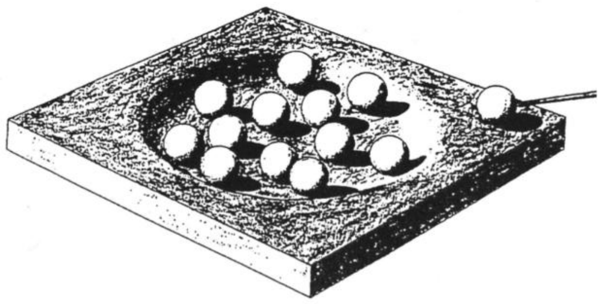


a synthesis of SHE

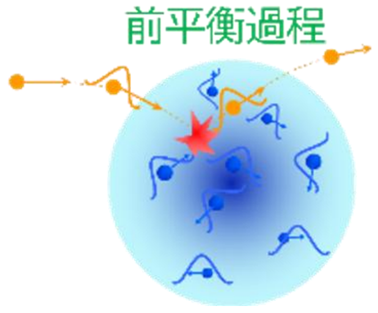
Two aspects of nuclear reactions

✓ a tool for nuclear structure

✓ reaction dynamics ⟵ this talk

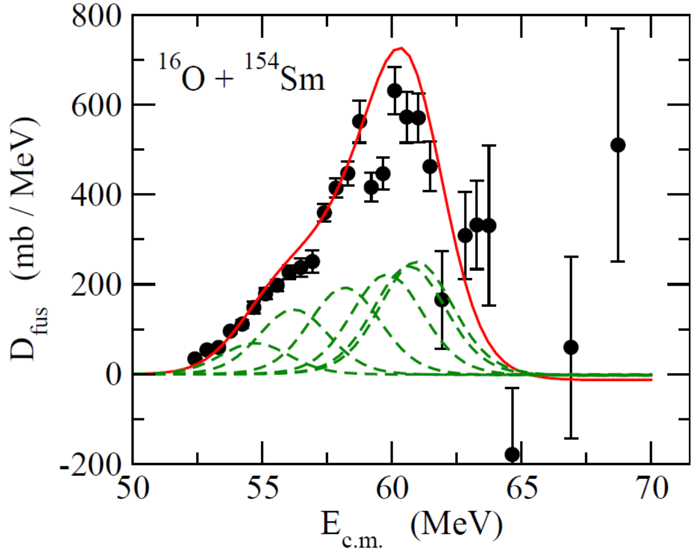


compound nucleus and quantum chaos



pre-equilibrium reactions

fusion barrier distribution



nucleus: a composite system

✓ rich reaction processes

✓ a rich interplay between nuclear structure and reaction

✓ elastic scattering

✓ inelastic scattering

✓ particle transfer reactions

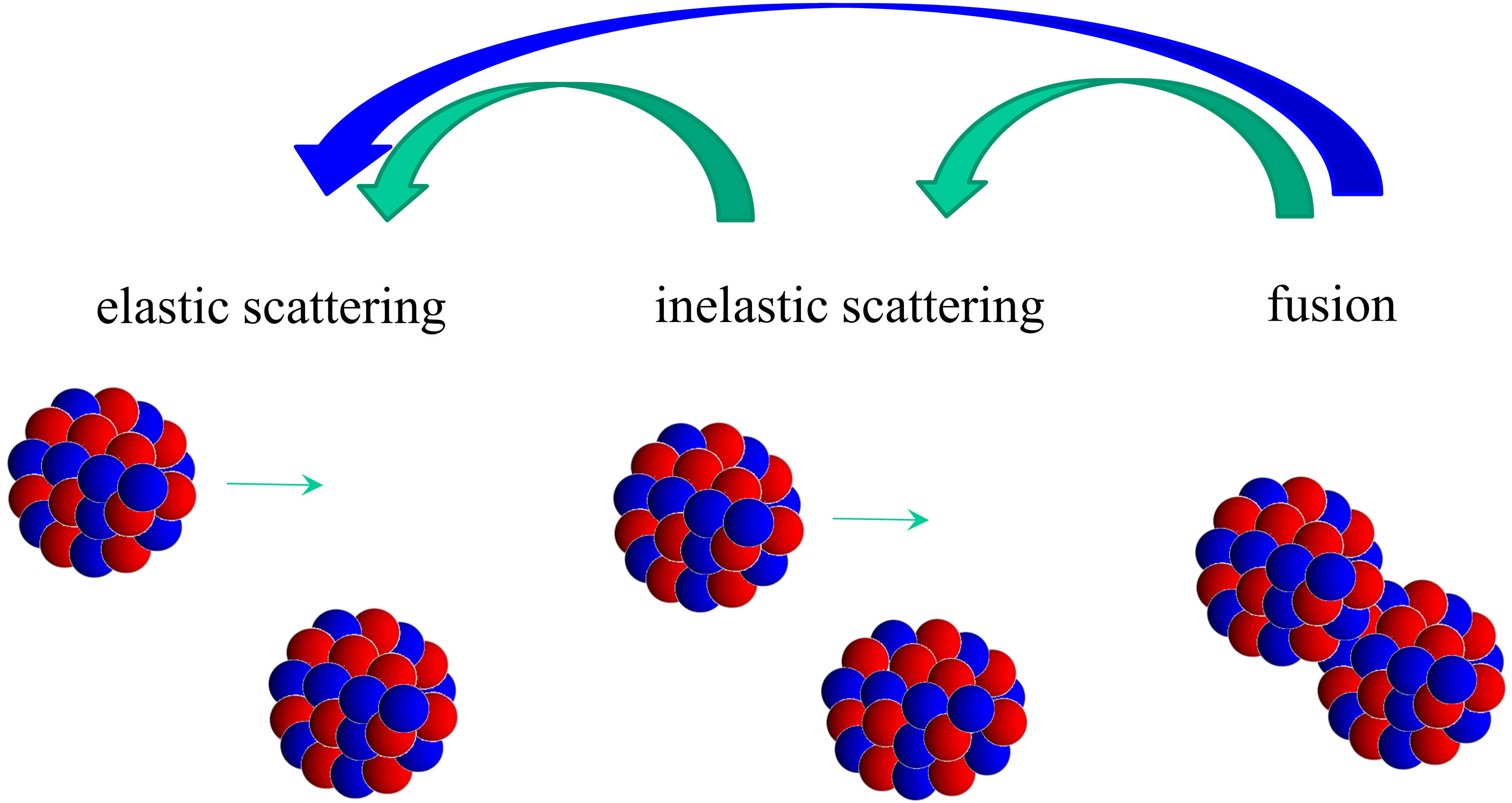
✓ fusion reactions

etc.

✓ g.s. properties (mass, size, shape....)

✓ excitations

quantum many-body dynamics (nuclear reactions)

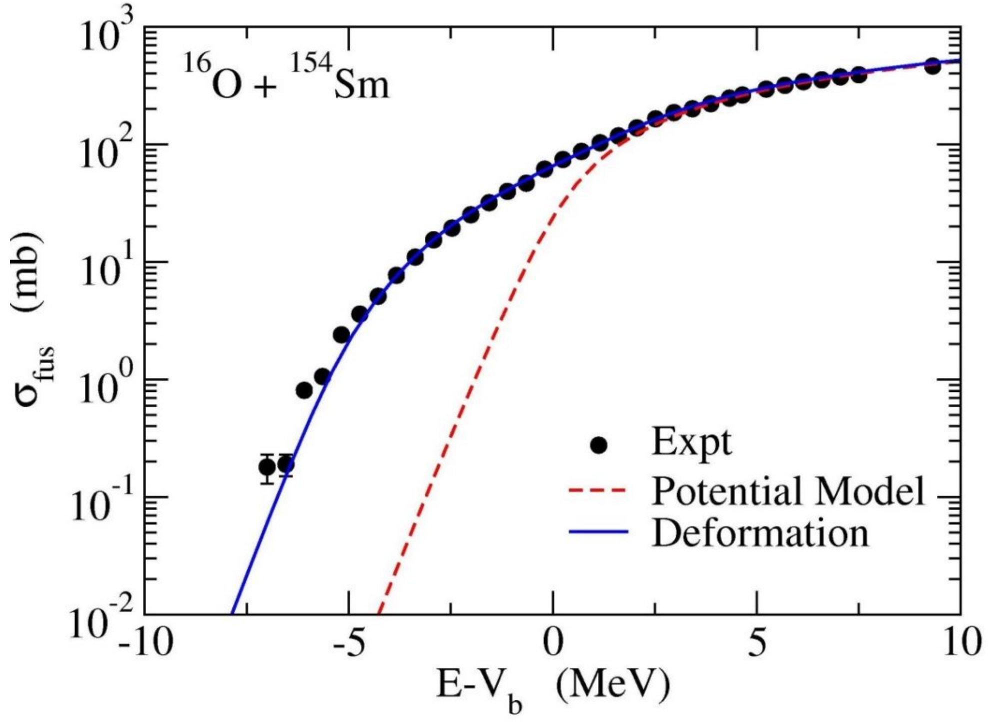


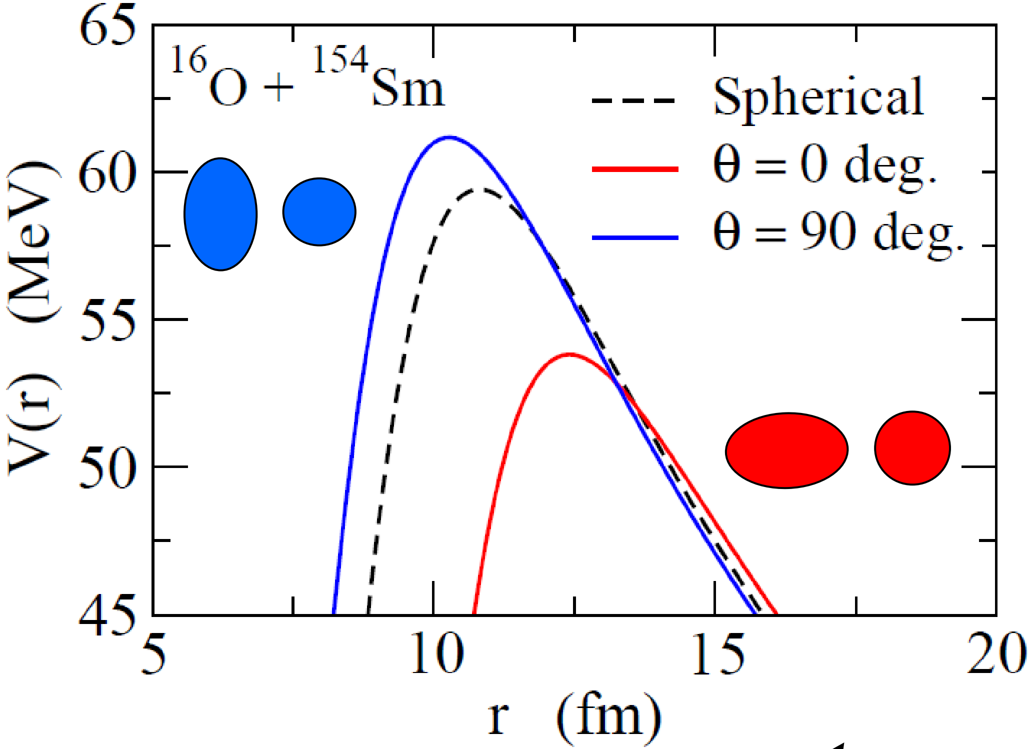
physics of nuclear reactions:
a unified description of these processes

A good example of the interplay between structure and reaction

Sub-barrier enhancement of fusion cross sections

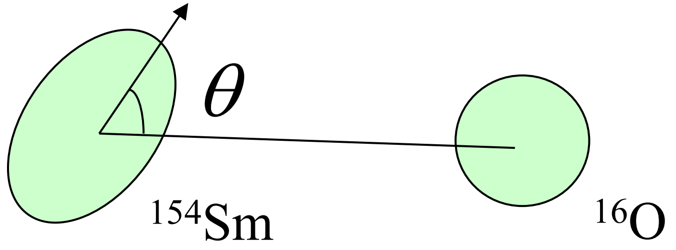
: a large impact of inelastic processes on fusion





a "snapshot" of a rotating nucleus

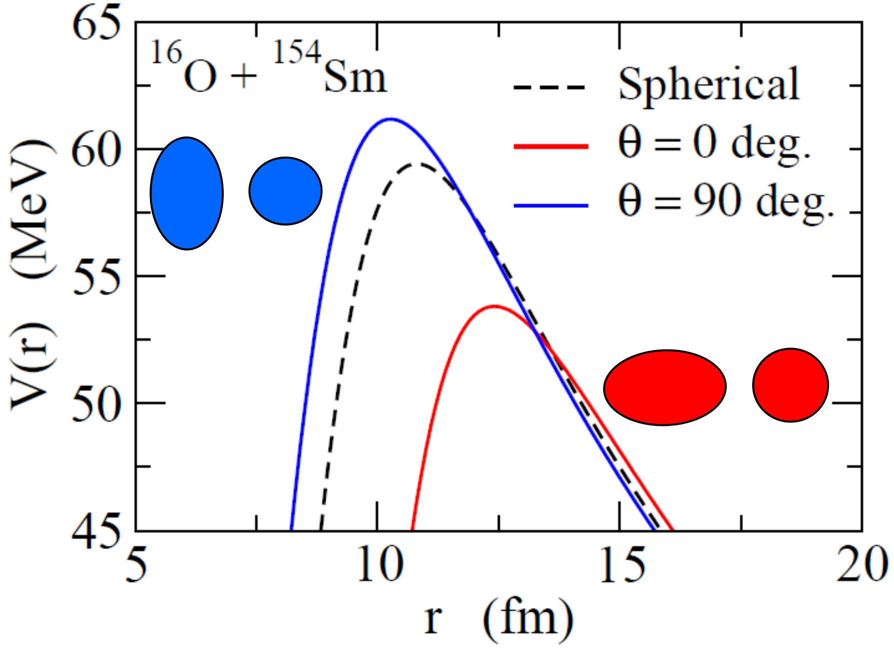
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$



K.Hagino and N. Takigawa, PTP128 (2012) 1061

Taking a snapshot of a nucleus with a "fast" nuclear reaction

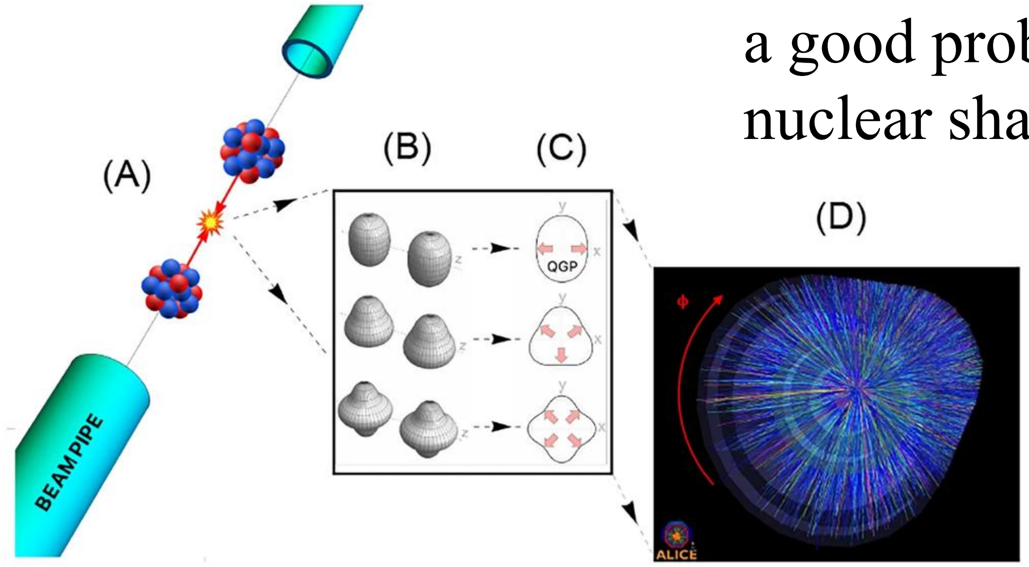
low-energy H.I. fusion reactions of a deformed nucleus



in recent years increasing interests in:

relativistic H.I. collisions with a deformed nucleus

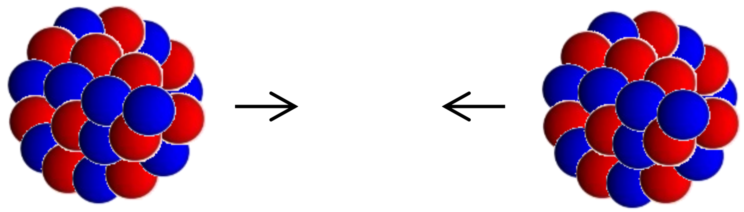
a good probe of nuclear shapes



J. Jia et al., Nucl. Sci. Tech. 35, 220 (2024)

Large similarities → intersection of **High *E*** and **Low *E*** HI collisions

Single-channel calculations



g.s. g.s.

optical potential

$$V_{\rm opt}(r) = V(r) - iW(r)$$

- elastic scattering
- absorption process

a loss of incident flux

✓ inelastic scattering
✓ transfer reaction
✓ fusion etc.

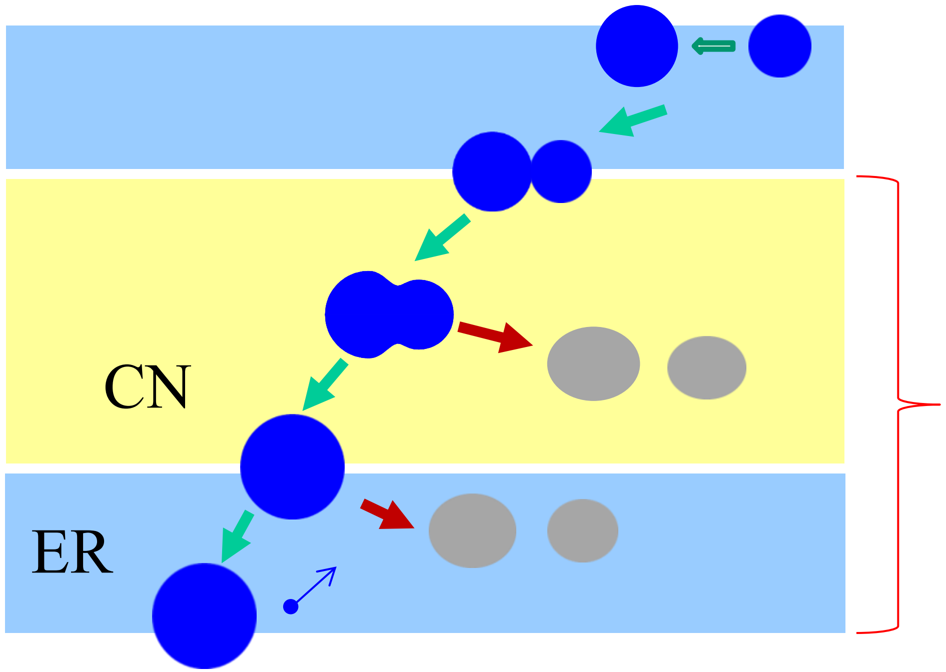
Optical model calculations

$$V_{\rm opt}(r) = V(r) - iW(r) \;\; \to \sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$

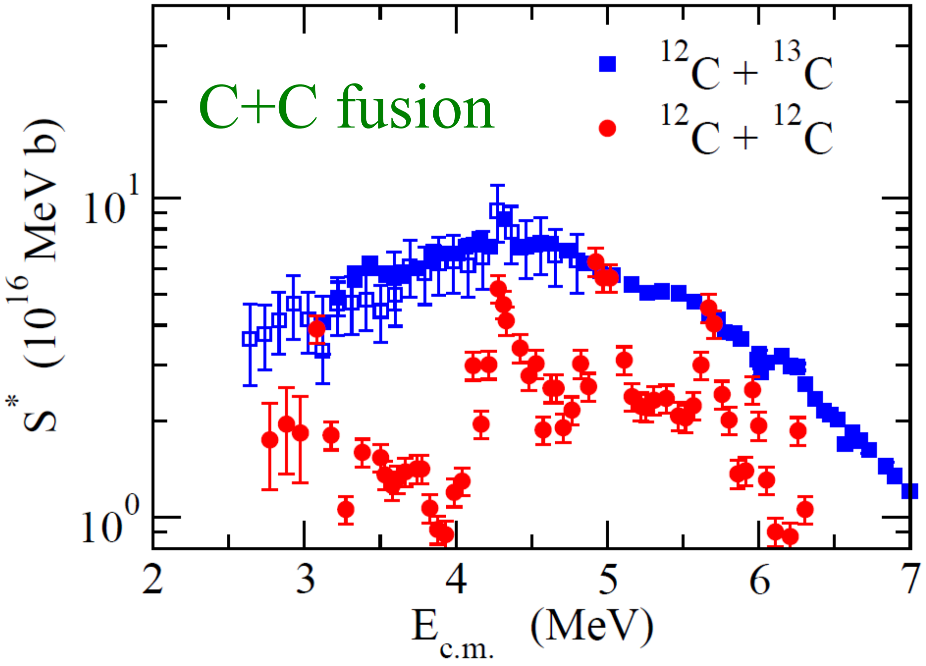
◆ no dynamics after the capture: only the absorption of elastic flux

◆ dynamics are important in several cases:

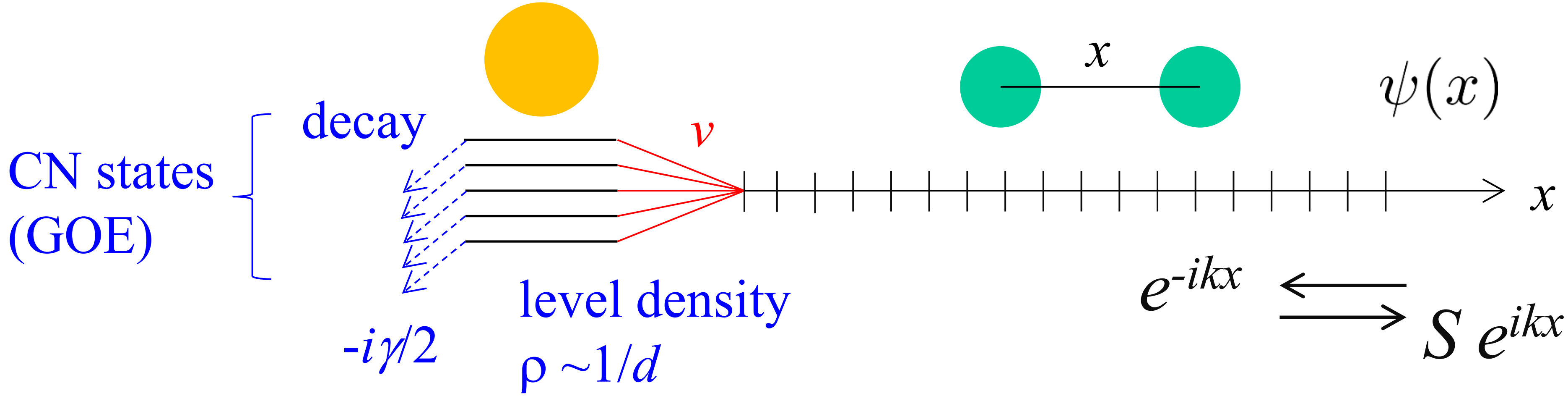
syntheses of superheavy elements

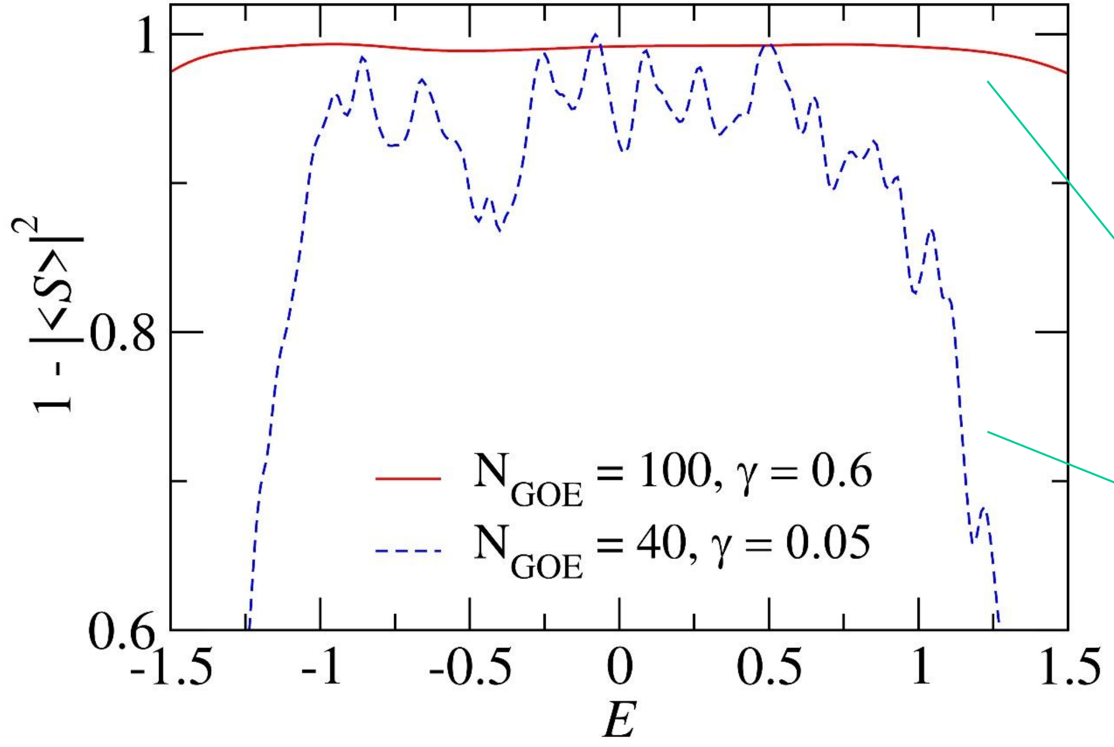


light-ion fusion reactions

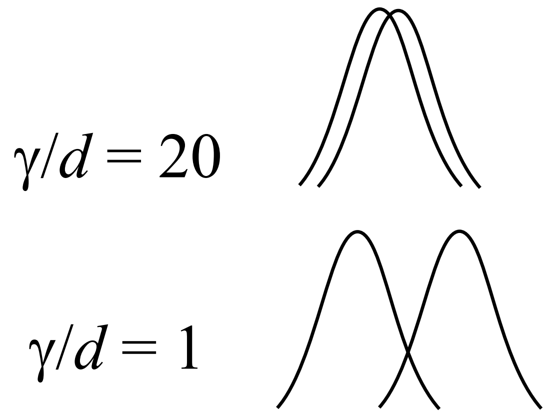


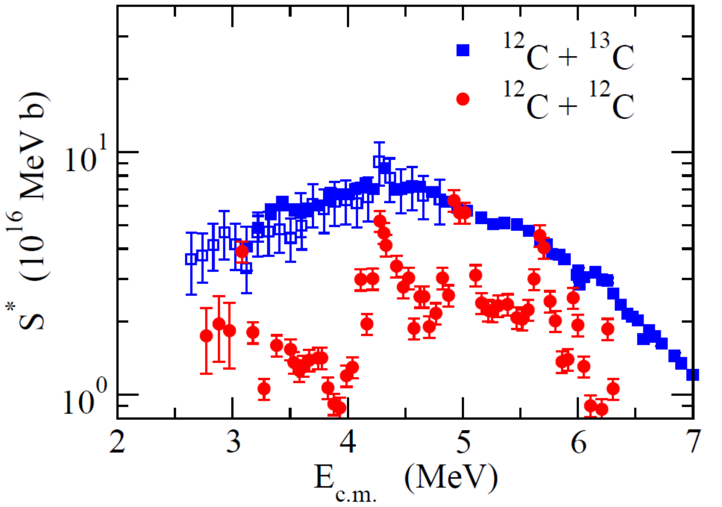
<u>a schematic model for the imaginary part</u> : a microscopic understanding of the optical potential





$S \sim 0$: strong absorption



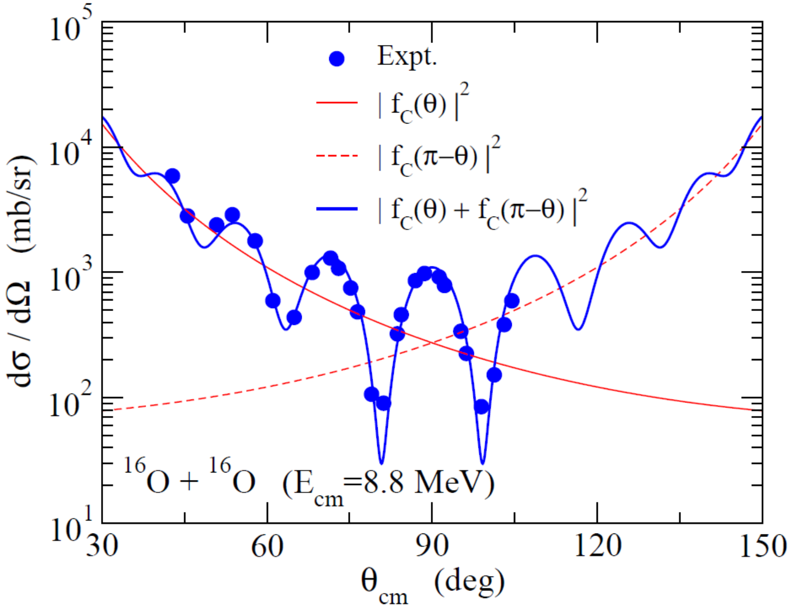


K.Hagino, Phys. Rev. C112, 034611 (2025).

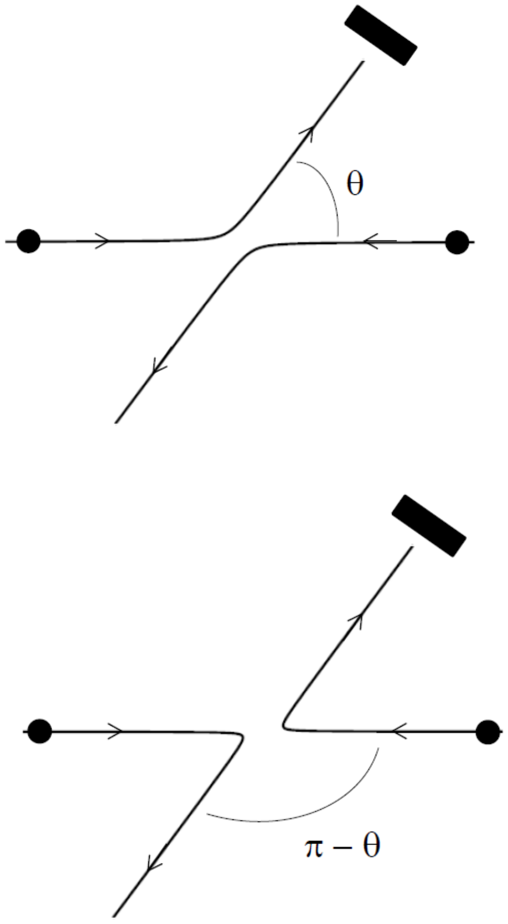
imaging quantum interference phenomena

quantum interference phenomena in heavy-ion reactions

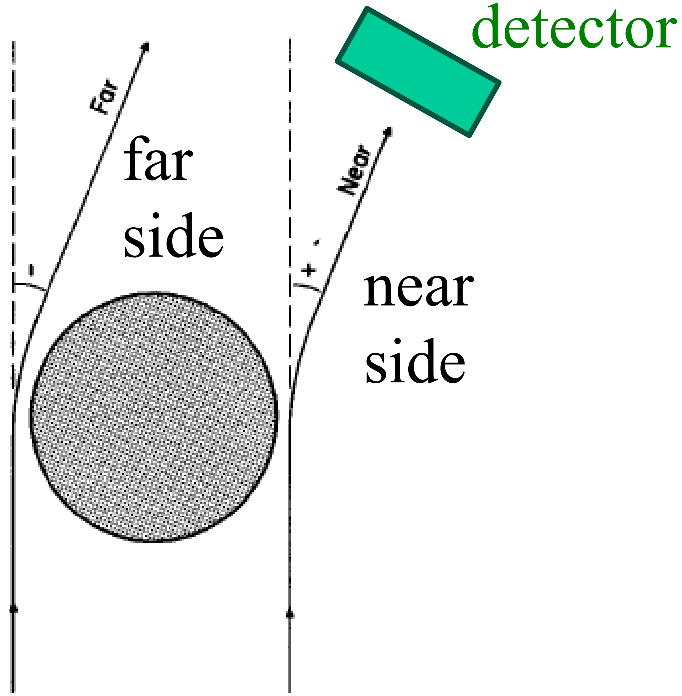
✓ Mott Scattering



expt: D.A. Bromley et al., Phys. Rev. 123 (‘61)878



✓ near side-far side interference

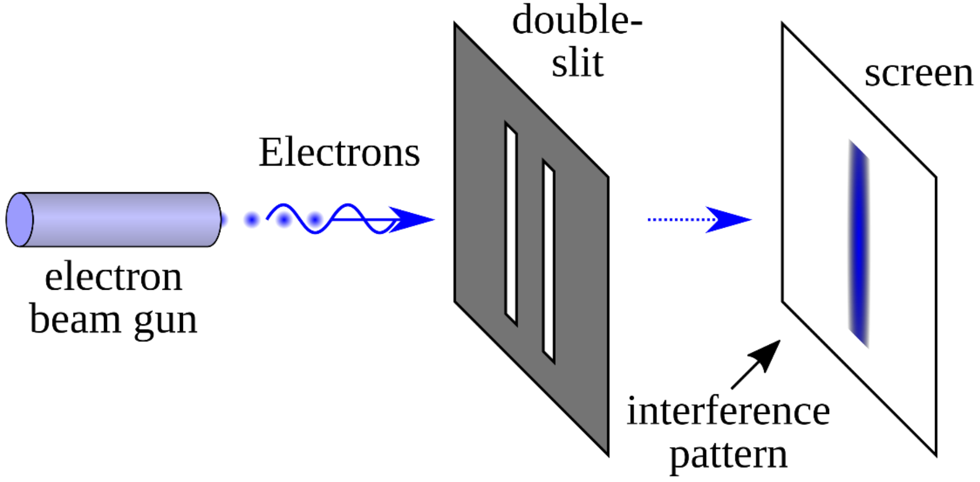


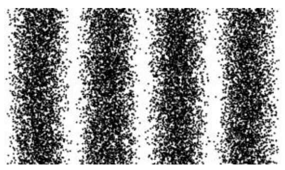
✓ nuclear-Coulomb interference

✓ barrier wave-internal wave interference

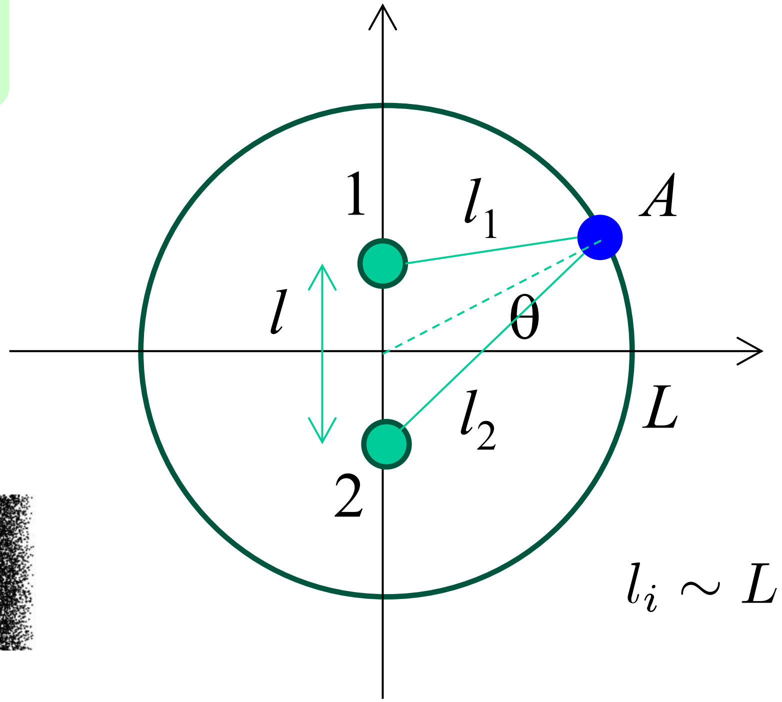
imaging quantum interference phenomena

a double slit problem





Wikipedia



$$l_i \sim L\left(1 \pm \frac{l}{2L}\sin\theta\right)$$

the amplitude at A

$$f(\theta) \quad = \quad f_1(\theta) + f_2(\theta) \rightarrow P = |f_1(\theta) + f_2(\theta)|^2$$

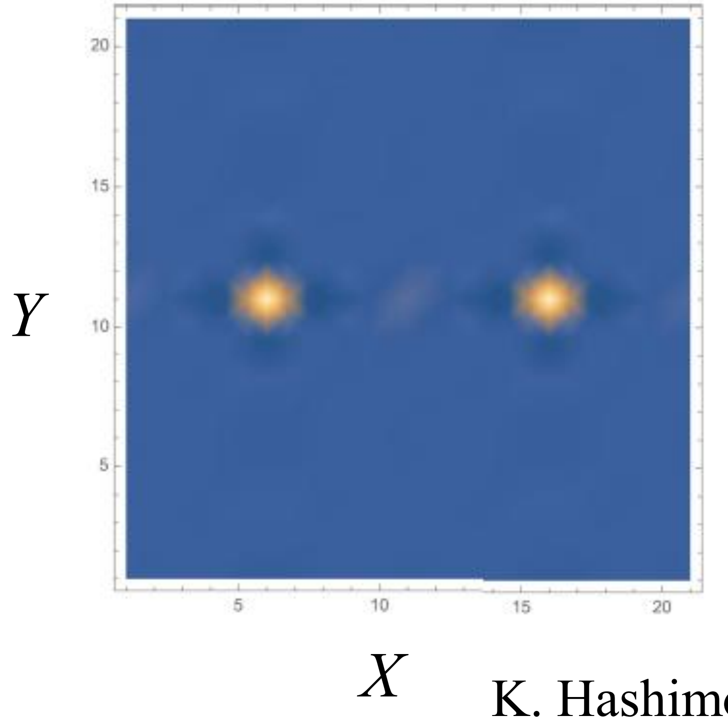
$$f_i(\theta) \quad = \quad A\sin\left(\frac{2\pi}{\lambda} l_i - \omega t\right)$$

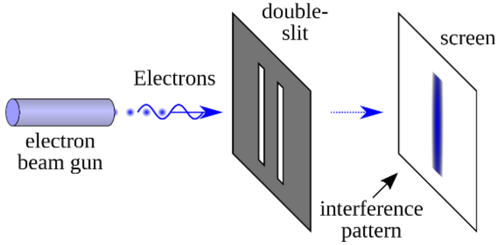
<u>the Fourier transform of *f*(θ) in a limited region:</u>

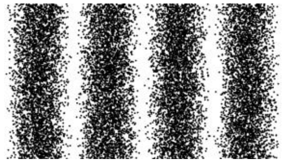
$$\Phi(X,Y) \propto \int_{\theta_2}^{\theta_2} d\theta \int_{\varphi_1}^{\varphi_2} d\varphi\, e^{ik(\theta X + \varphi Y)} f(\theta,\varphi)$$

imaging quantum interference phenomena

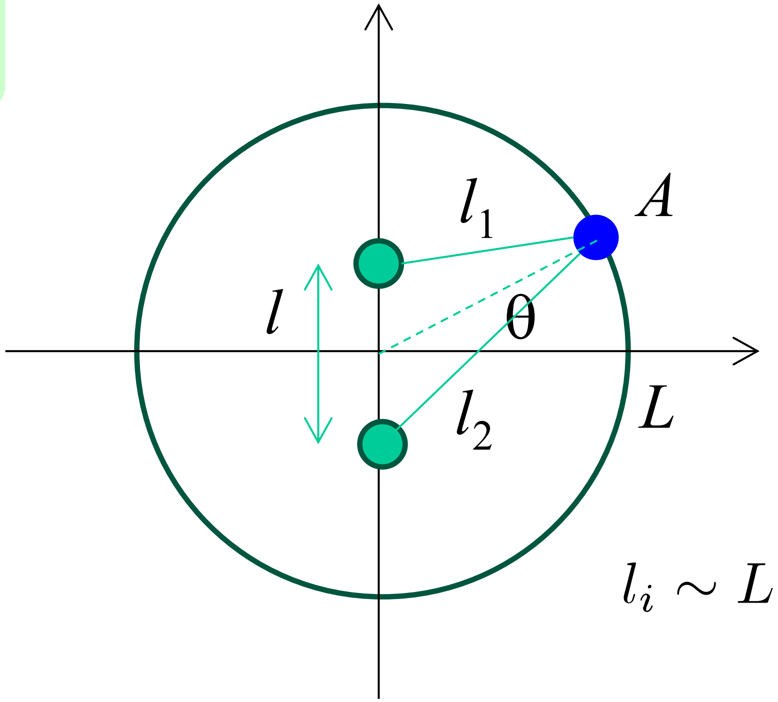
$|\Phi(X,Y)|^2$







K. Hashimoto et al.,
PTEP2023, 043B04 (2023)



$$l_i \sim L\left(1 \pm \frac{l}{2L}\sin\theta\right)$$

the amplitude at A

$$f(\theta) \;=\; f_1(\theta) + f_2(\theta) \to P = |f_1(\theta) + f_2(\theta)|^2$$

$$f_i(\theta) \;=\; A\sin\left(\frac{2\pi}{\lambda} l_i - \omega t\right)$$

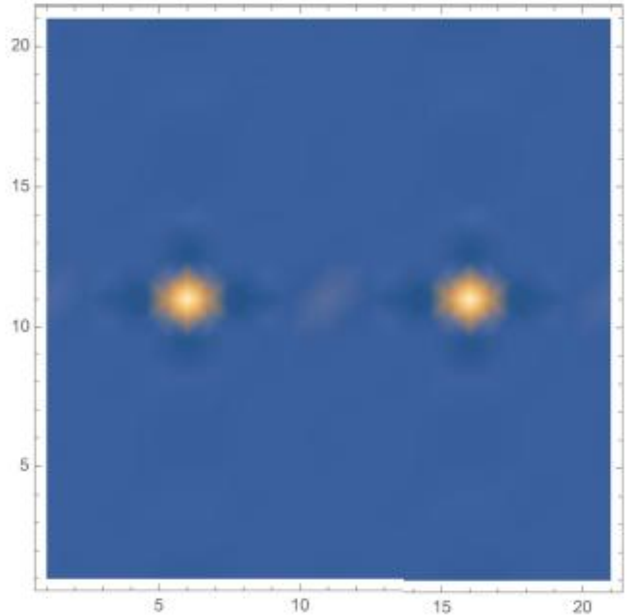
the Fourier transform of $f(\theta)$ in a limited region:

$$\Phi(X,Y) \propto \int_{\theta_2}^{\theta_2} d\theta \int_{\varphi_1}^{\varphi_2} d\varphi\, e^{ik(\theta X + \varphi Y)} f(\theta,\varphi)$$

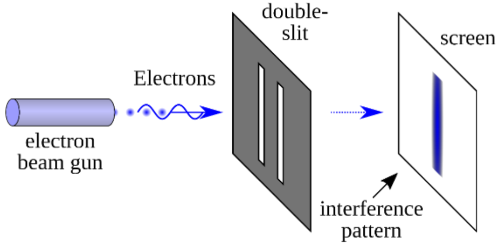
imaging quantum interference phenomena

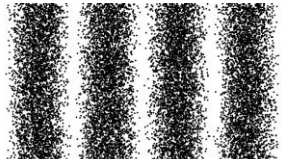
$|\Phi(X,Y)|^2$

Y



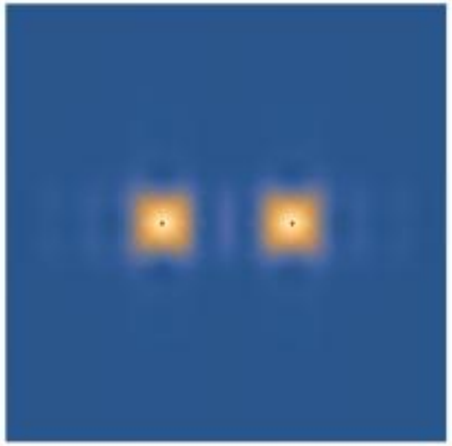
X



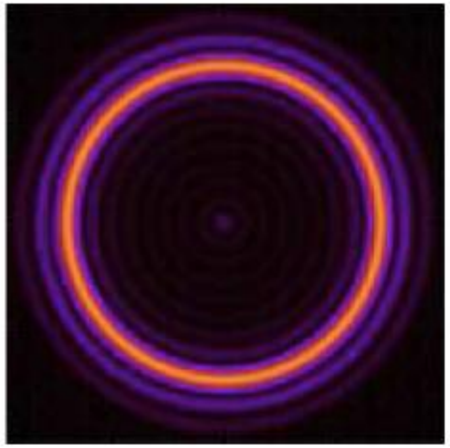


similar

Other applications in particle physics



scattering of string



imaging black holes through AdS/CFT

K. Hashimoto, Y. Matsuo, and T. Yoda, PTEP2023, 043B04 (2023) : “String is a double slit”

K. Hashimoto, S. Kinoshita, and K. Murata, PRL123, 031602 (2019), PRD101, 066018 (2020)

imaging quantum interference phenomena

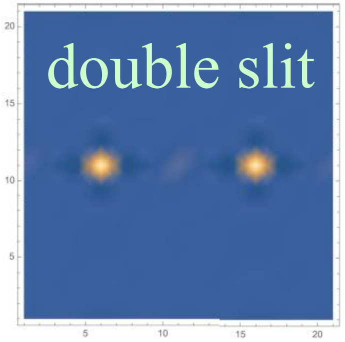
K. Hagino and T. Yoda, PLB848, 138326 (2024).
K. Heo and K.Hagino, PRC111, 034612 (2025).

Application to nuclear reactions:

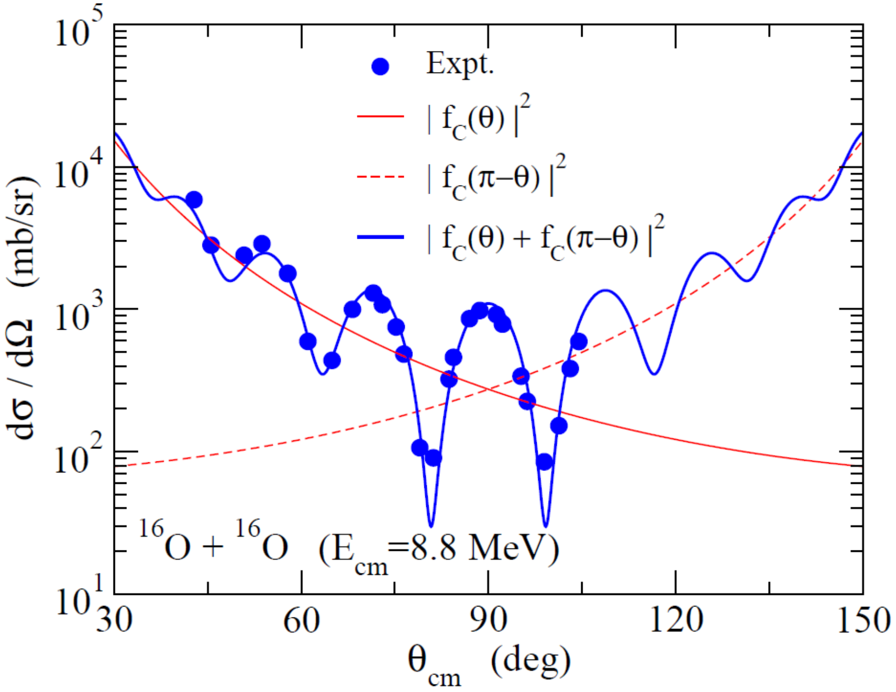
$$V_{\rm opt}(r) = V(r) - iW(r) \longrightarrow f(\theta) \longrightarrow \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

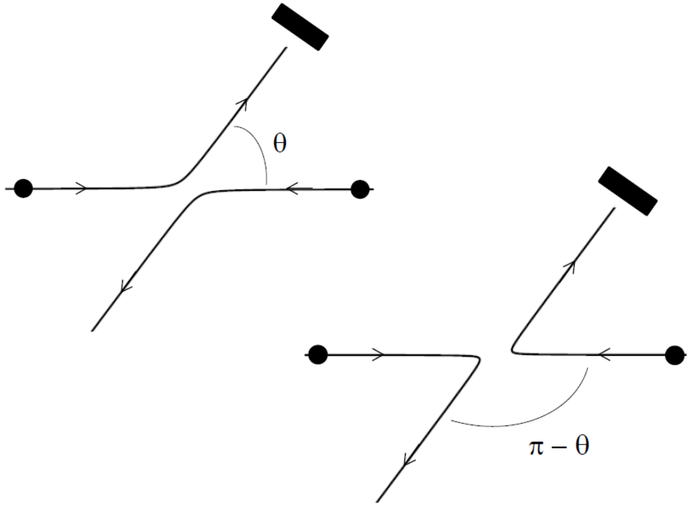
F.T.

$$\Phi(X,Y) \rightarrow |\Phi(X,Y)|^2$$



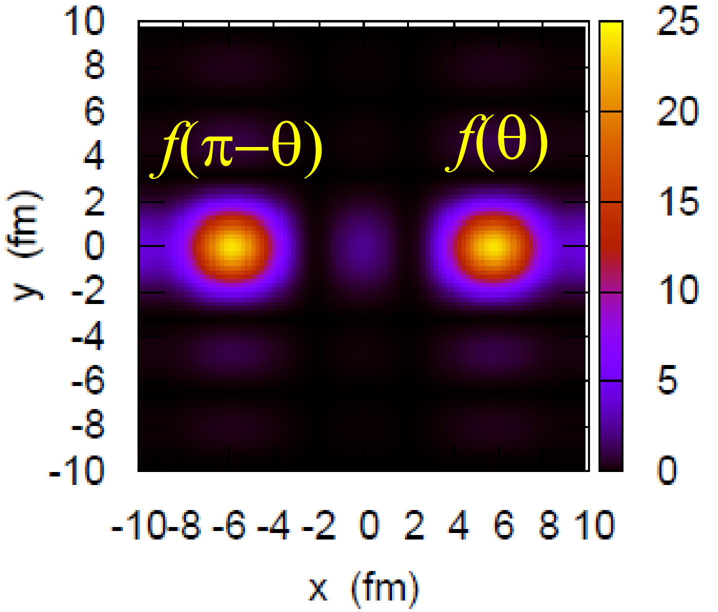
^{16}O+^{16}O at E = 8.8 MeV





$$\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi-\theta)|^2$$

θ_0=90 deg., $\Delta\theta$=30 deg.



imaging quantum interference phenomena

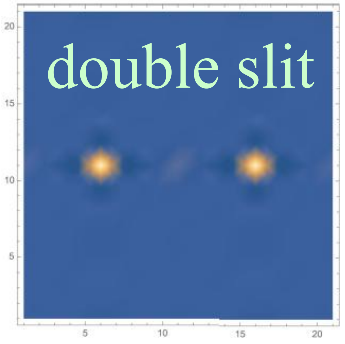
K. Hagino and T. Yoda, PLB848, 138326 (2024).
K. Heo and K.Hagino, PRC111, 034612 (2025).

Application to nuclear reactions:

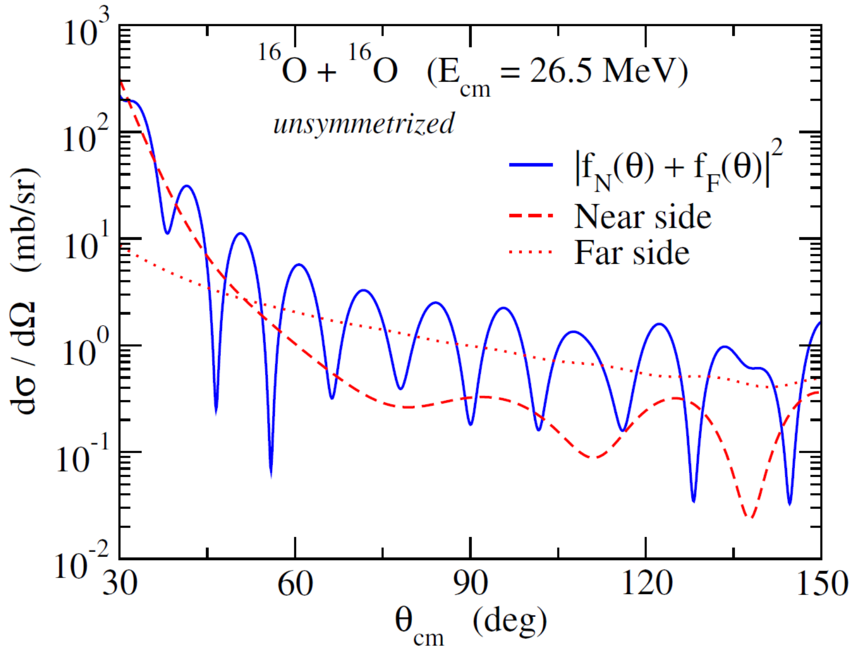
$$V_{\text{opt}}(r) = V(r) - iW(r) \longrightarrow f(\theta) \longrightarrow \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

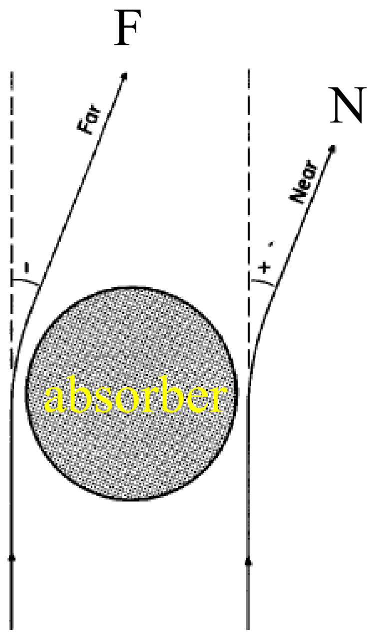
F.T.

$$\Phi(X,Y) \to |\Phi(X,Y)|^2$$

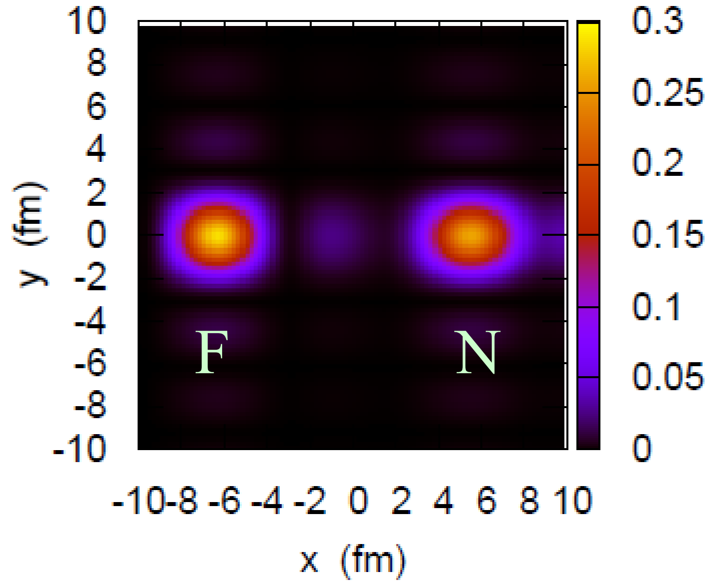


^{16}O+^{16}O at E = 26.5 MeV



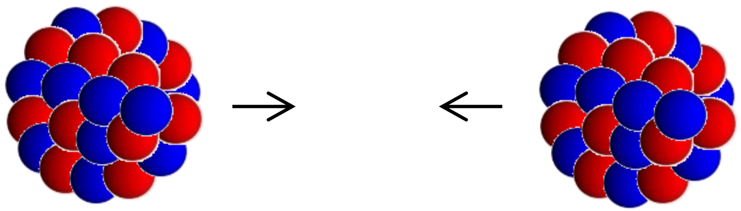


θ_0=55 deg., $\Delta\theta$=15 deg.

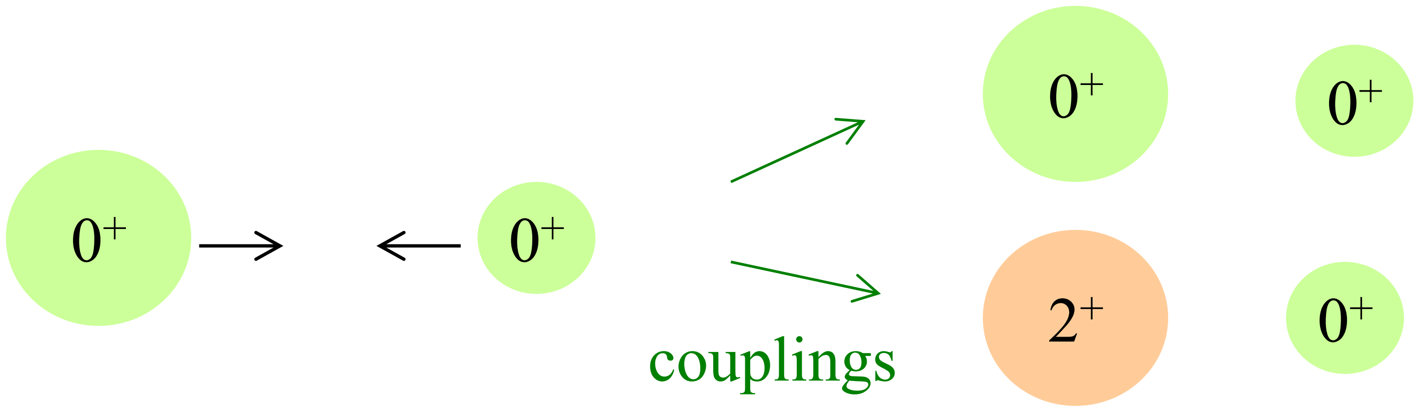


$$f(\theta) = f_N(\theta) + f_F(\theta)$$

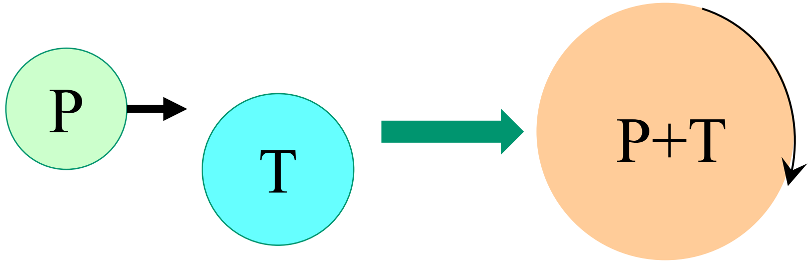
Multi-channel dynamics



coupled-channels approach



Fusion reactions: compound nucleus formation

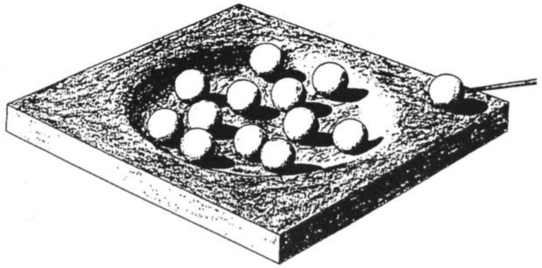


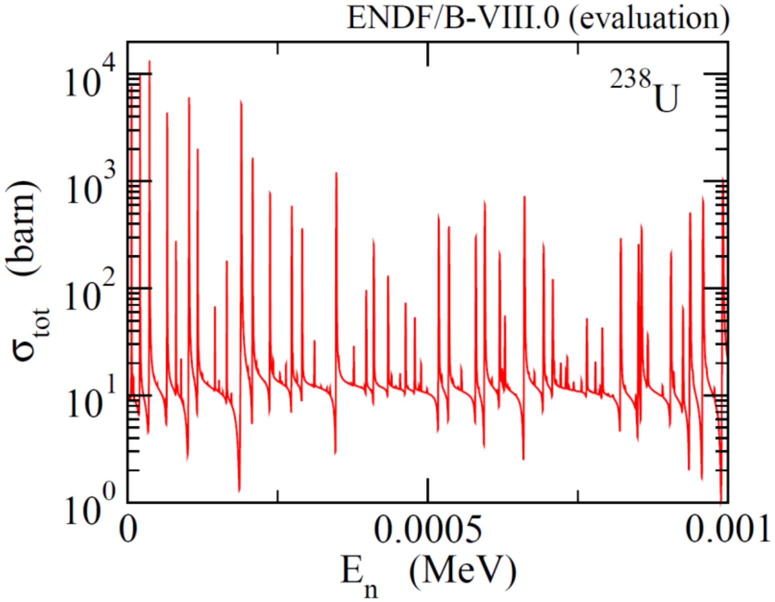
compound nucleus

Niels Bohr (1936)

Wikipedia

neutron capture reactions

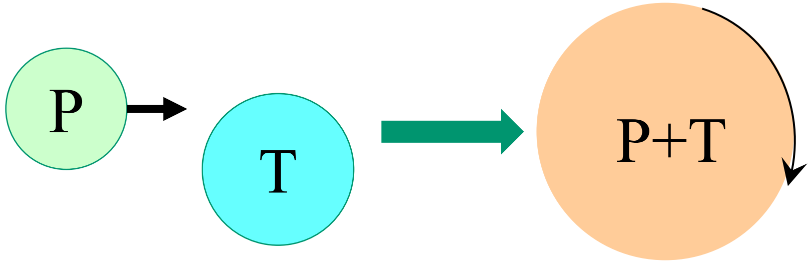




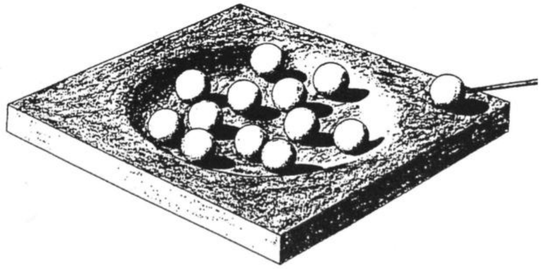
Exp: Enrico Fermi (1935)

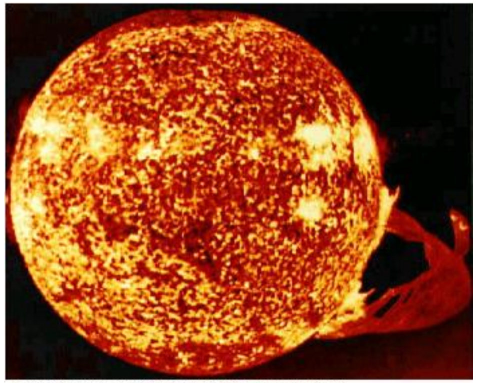
many very narrow resonances (width ~ eV)

Fusion reactions: compound nucleus formation

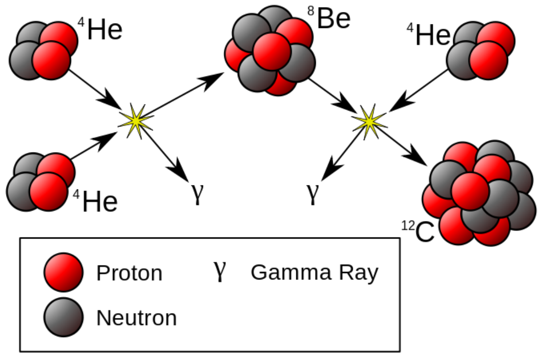


compound
nucleus

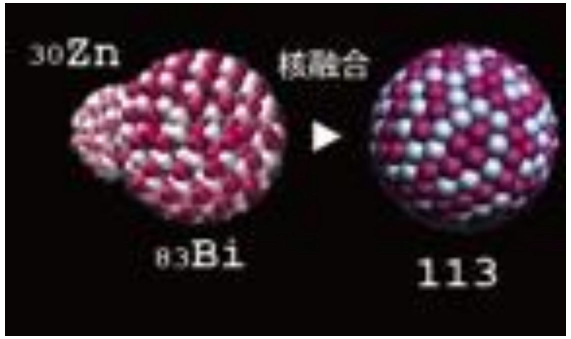




energy production
in stars (Bethe '39)



nucleosynthesis

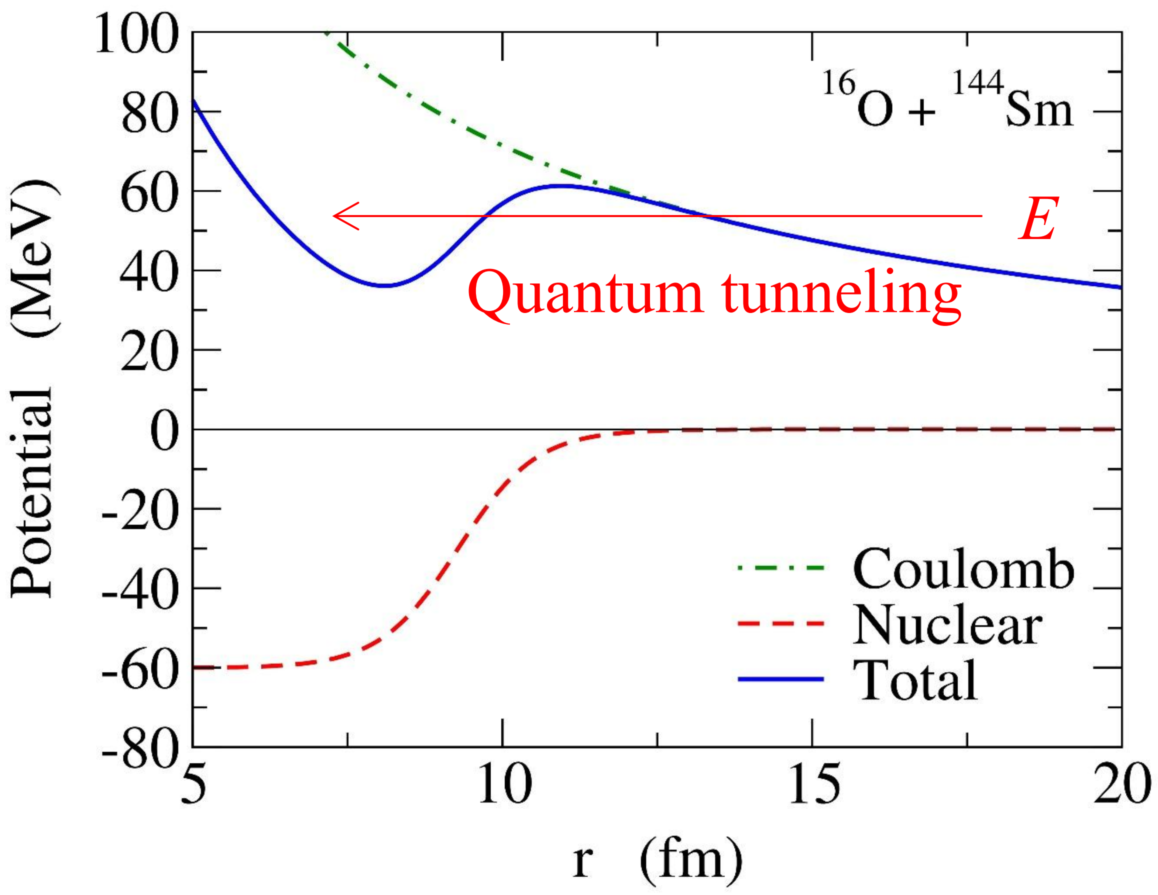


superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems

← microscopic understanding: an ultimate goal of nuclear physics

Coulomb barrier



1. Coulomb interaction

   long range, repulsion

2. Nuclear interaction

   short range, attraction

Potential barrier (Coulomb barrier)

Fusion: takes place by overcoming the barrier

the barrier height → defines the energy scale of a system

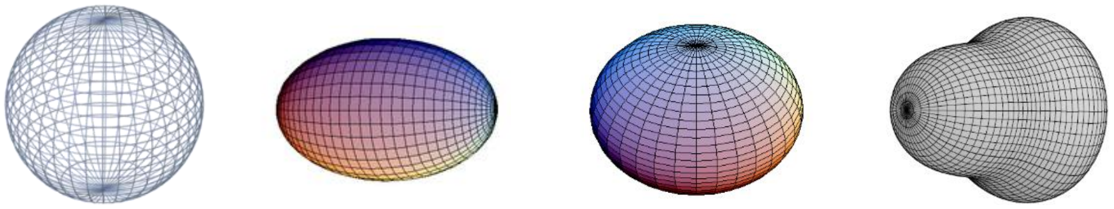
Fusion reactions at energies around the Coulomb barrier

Sub-barrier fusion reactions and quantum tunneling

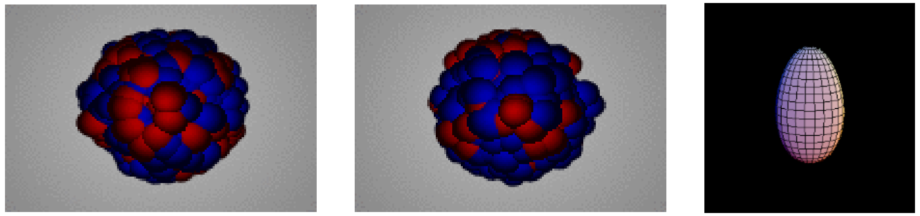
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes

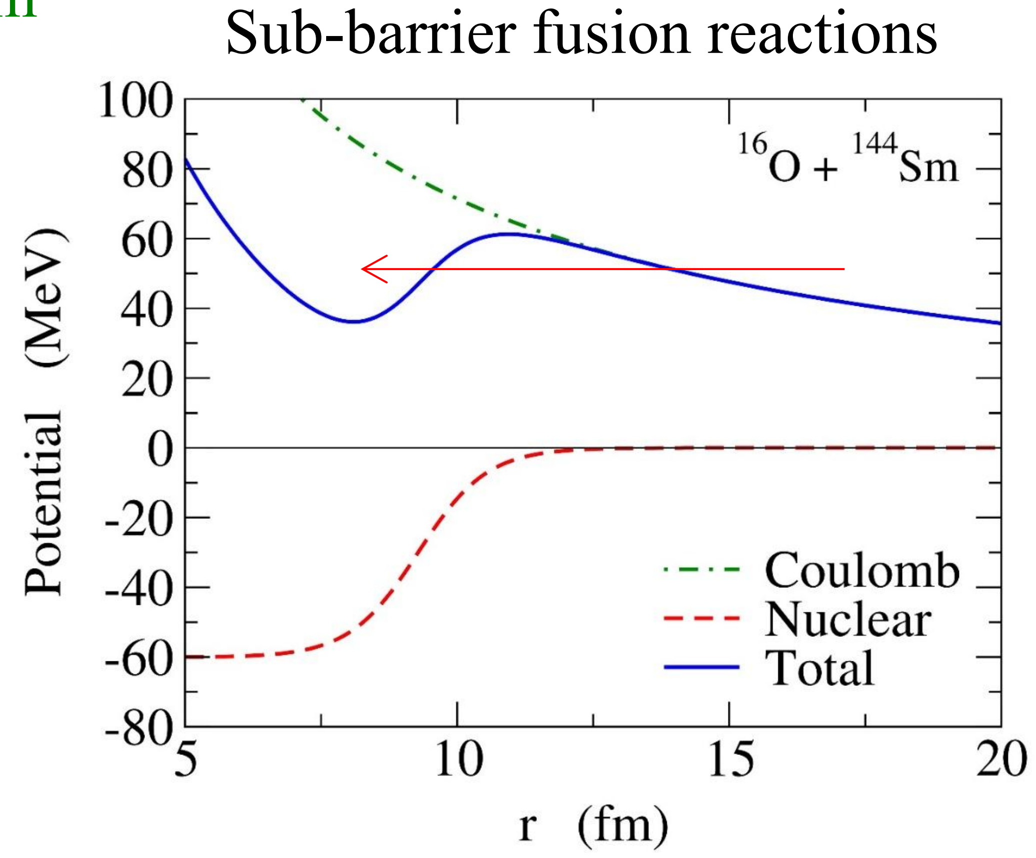


- several surface vibrations



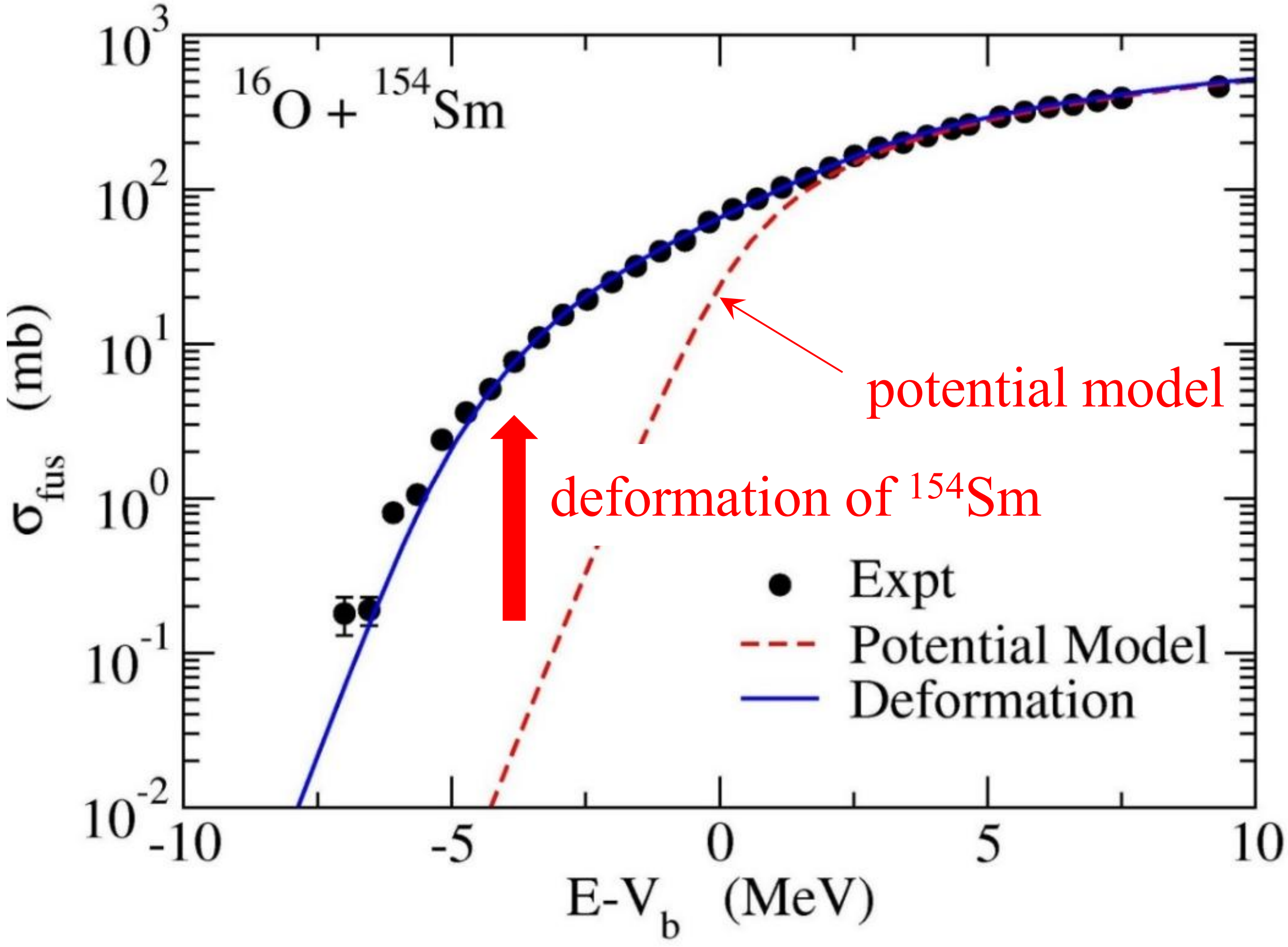
several modes and adiabaticities

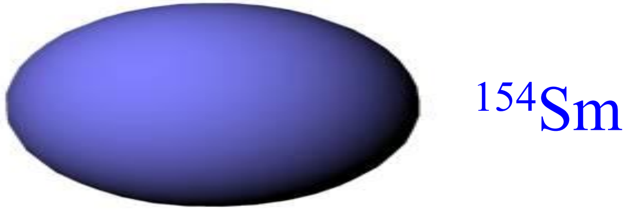
- several types of nucleon transfers

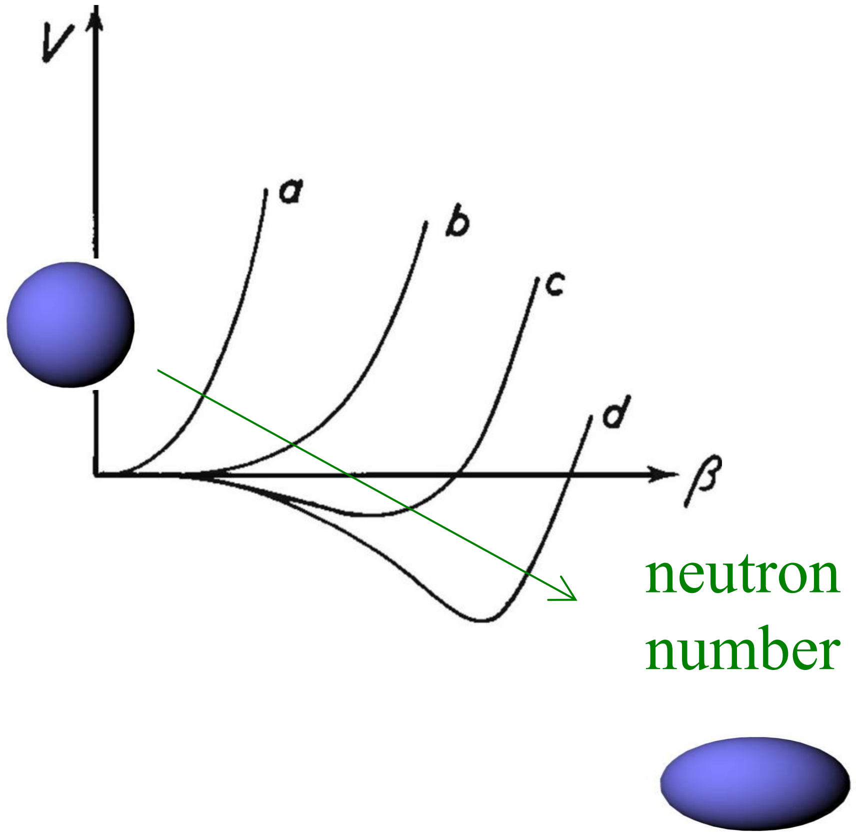


Tunneling probabilities: the exponential E dependence
→ nuclear structure effects are amplified

Sub-barrier fusion reactions and quantum tunneling

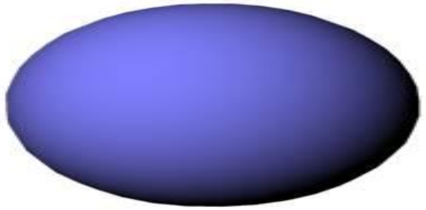




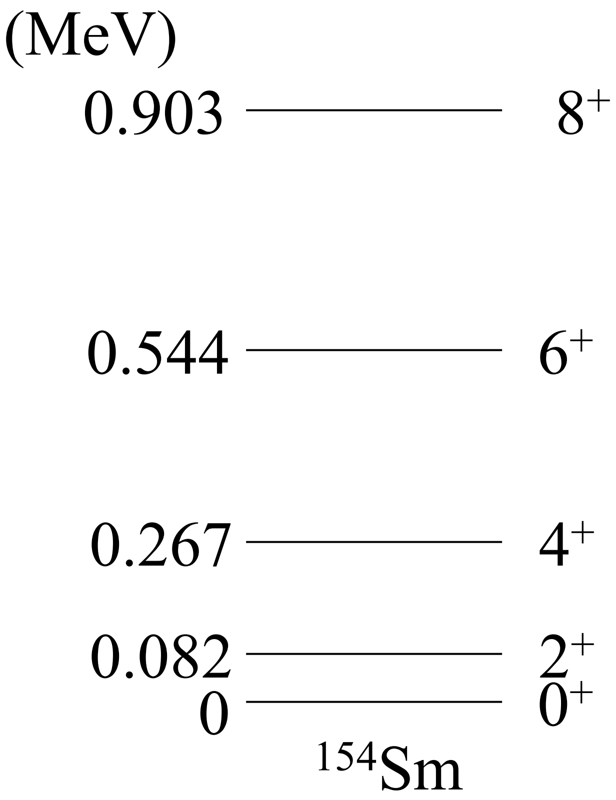


K. Hagino and N. Takigawa, Prog. Theo. Phys.128 (2012)1061.

Effects of nuclear deformation on fusion

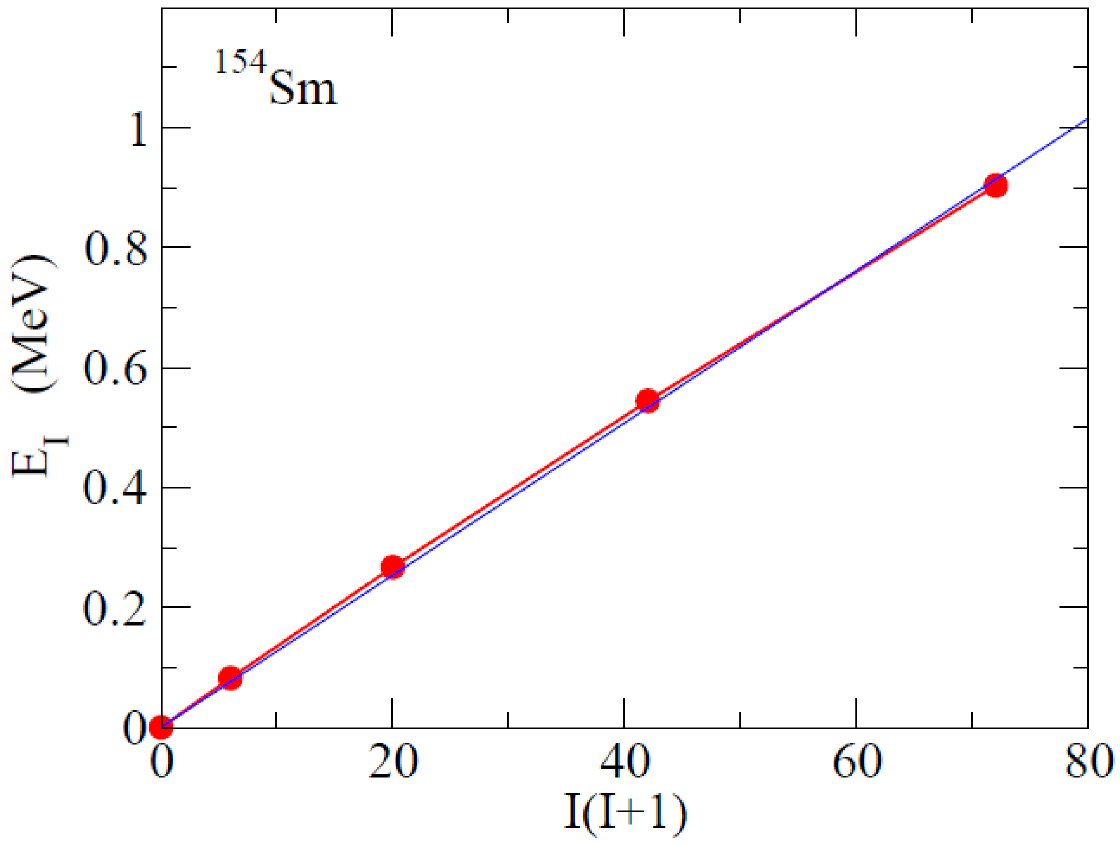


^{154}Sm

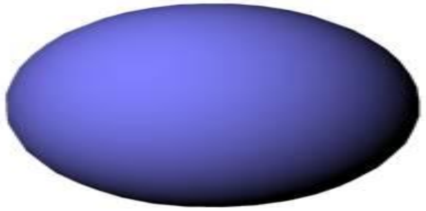


rotational spectrum

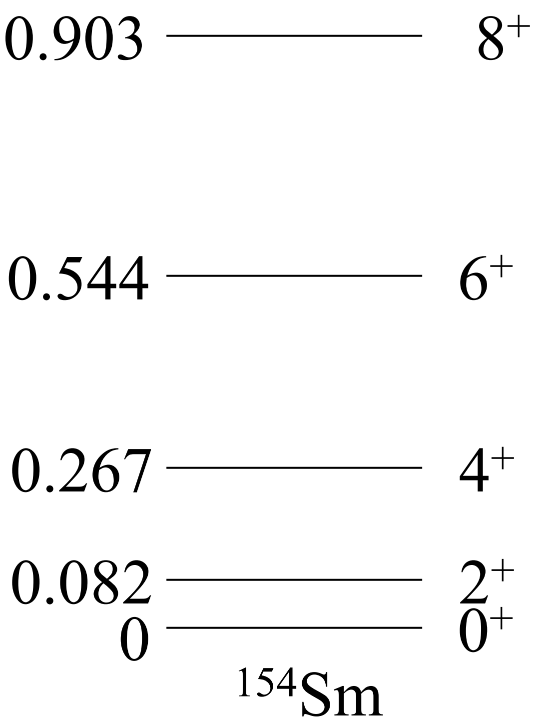
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$



Effects of nuclear deformation on fusion



^{154}Sm



rotational spectrum

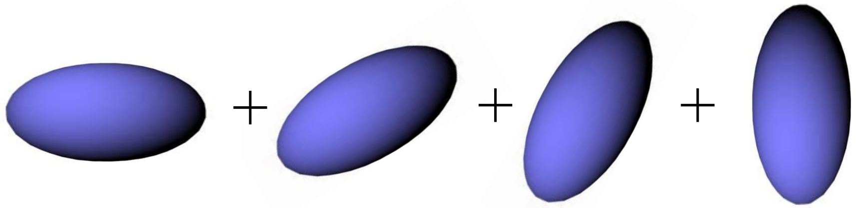
a small rotational energy

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

→ a large moment of inertia J

→ rotation: a slow deg. of freedom

$$E_{\rm rot} \sim E_{2^+} = 82 \text{ keV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

$\Psi_{0^+} =$ 

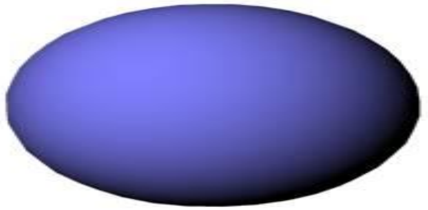
→ a spherical state in the lab. system

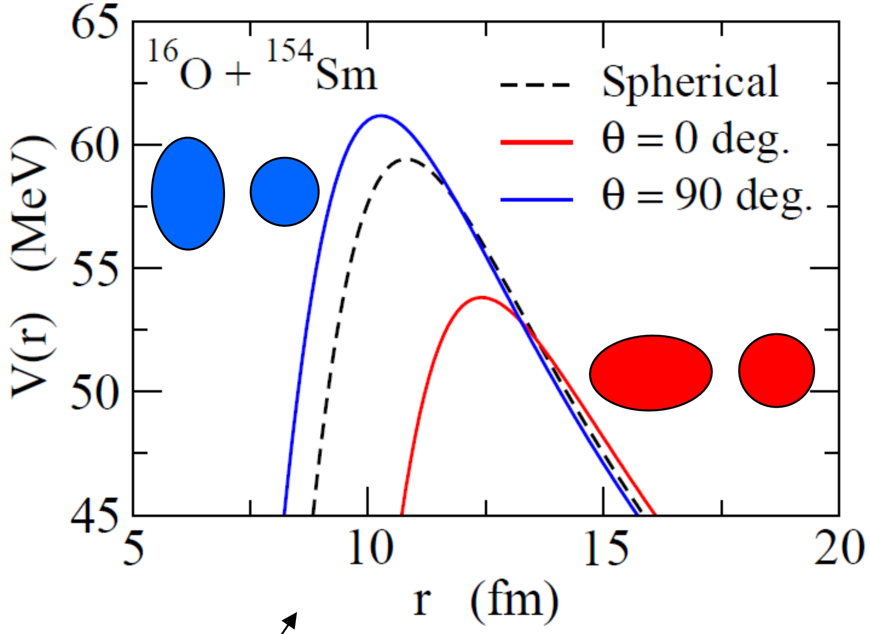
fix the orientation angle to calculate the fusion probability

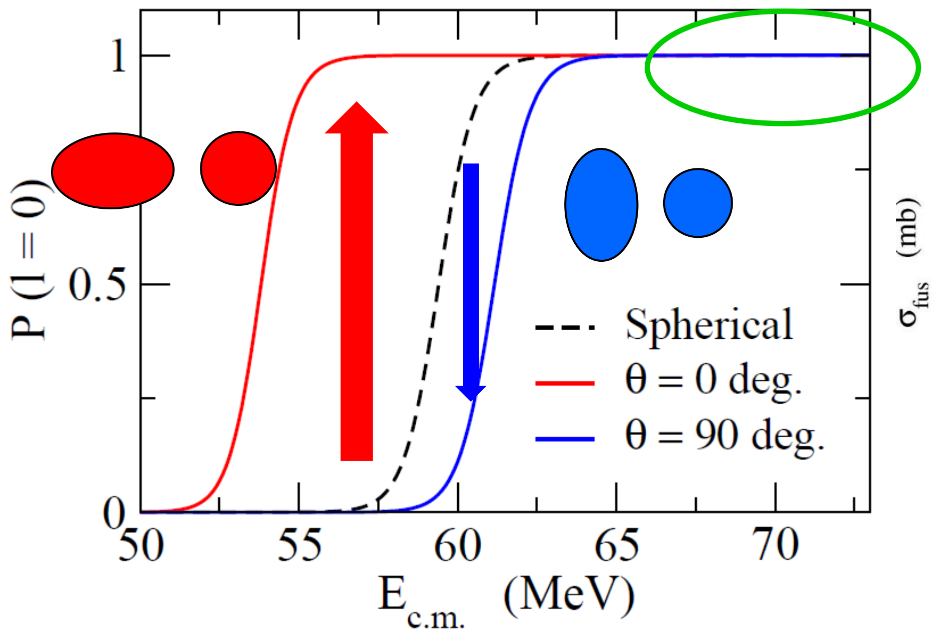
"a <u>snapshot</u> of a rotating nucleus"

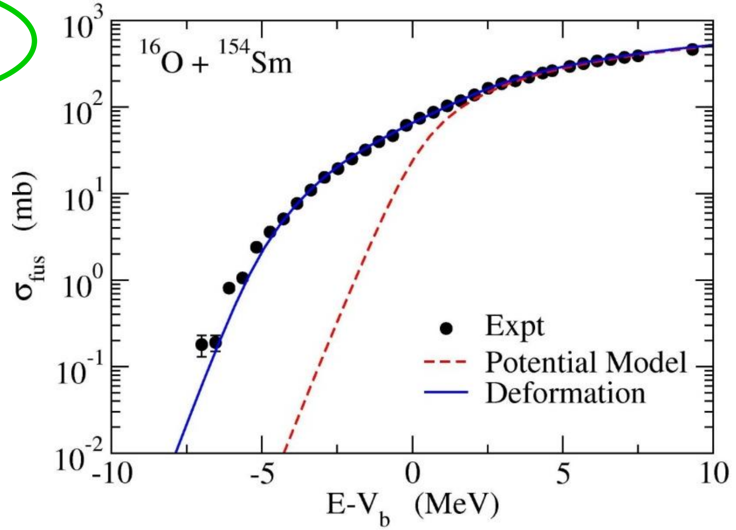
Effects of nuclear deformation on fusion

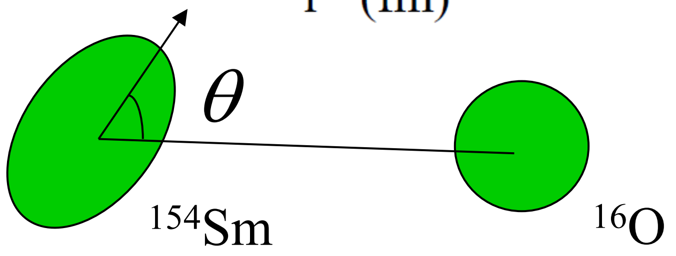
^{154}Sm : a typical deformed nucleus











$$\sigma_{\mathrm{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\mathrm{fus}}(E;\theta)$$

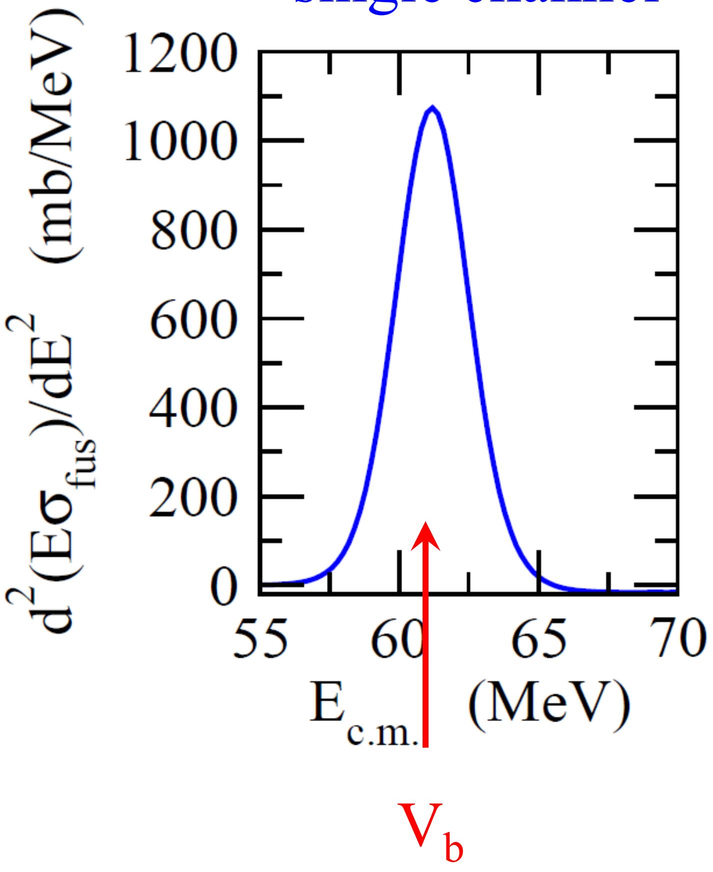
Fusion: strong interplay between nuclear structure and reaction

Fusion barrier distribution

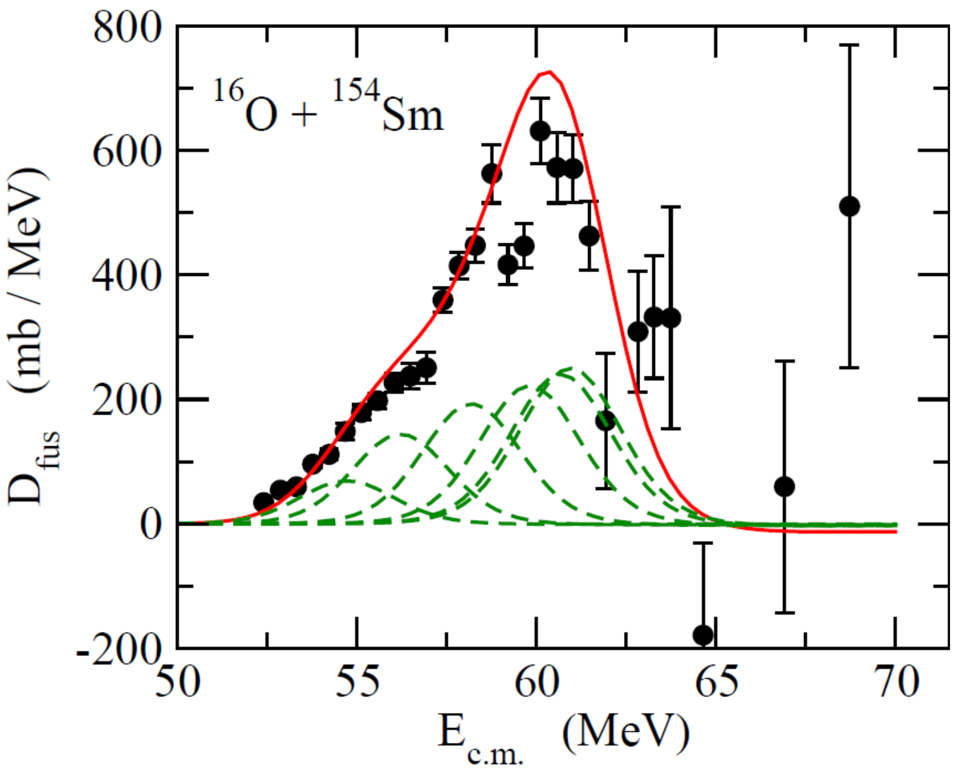
$$\boxed{D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2}} \quad \propto \frac{dP_{l=0}}{dE}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

single channel



with multi-barriers

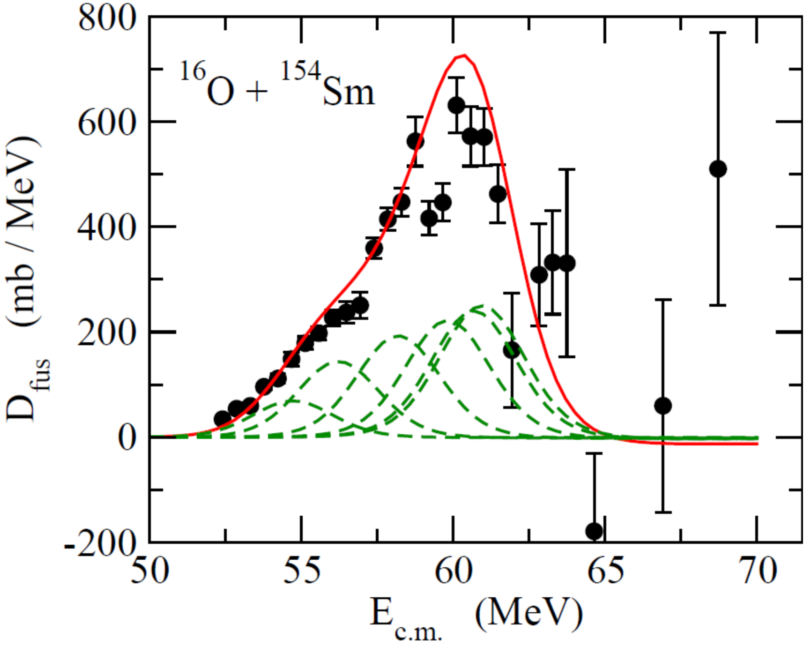


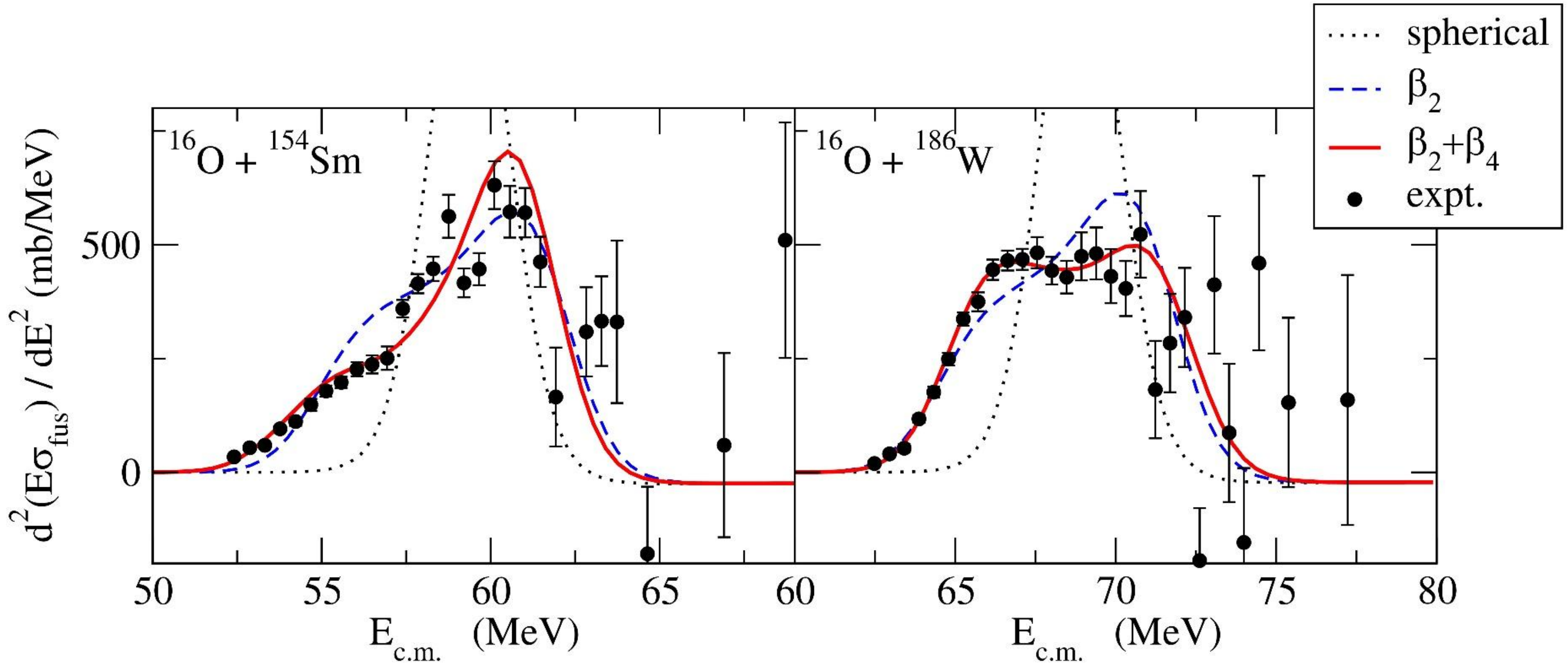
K.H. and N. Takigawa, PTP128 ('12) 1061

Fusion barrier distribution

$$\boxed{D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2}}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

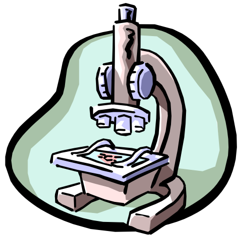


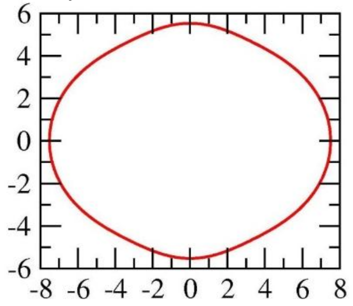


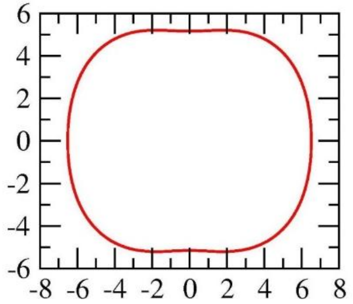
$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

$$\beta_2 = 0.33 \qquad \beta_2 = 0.29$$

$$\beta_4 = +0.05 \qquad \beta_4 = -0.03$$







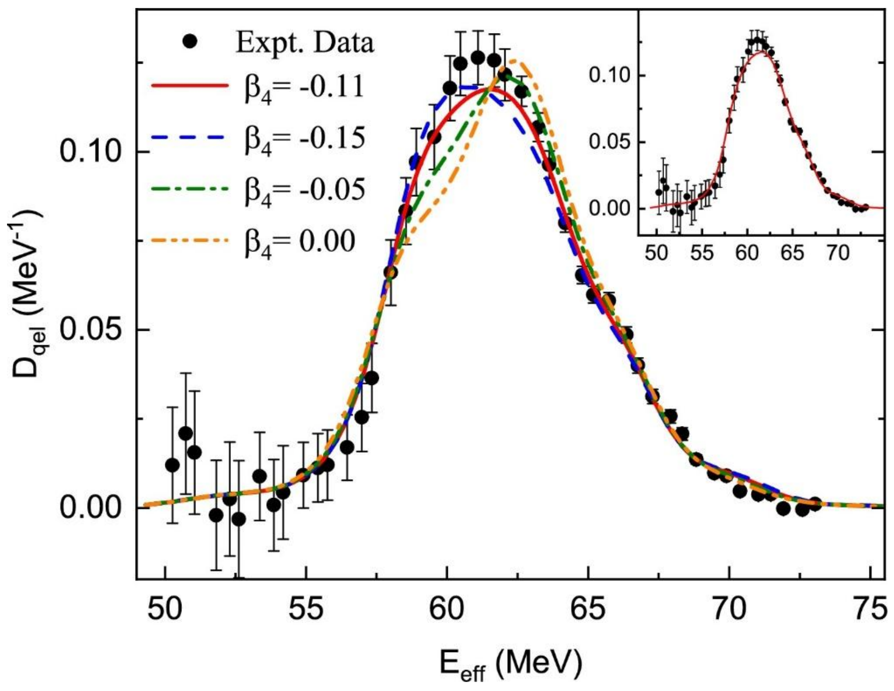
sensitive to the sign of β_4!

→ Fusion as a quantum tunneling microscope for nuclei

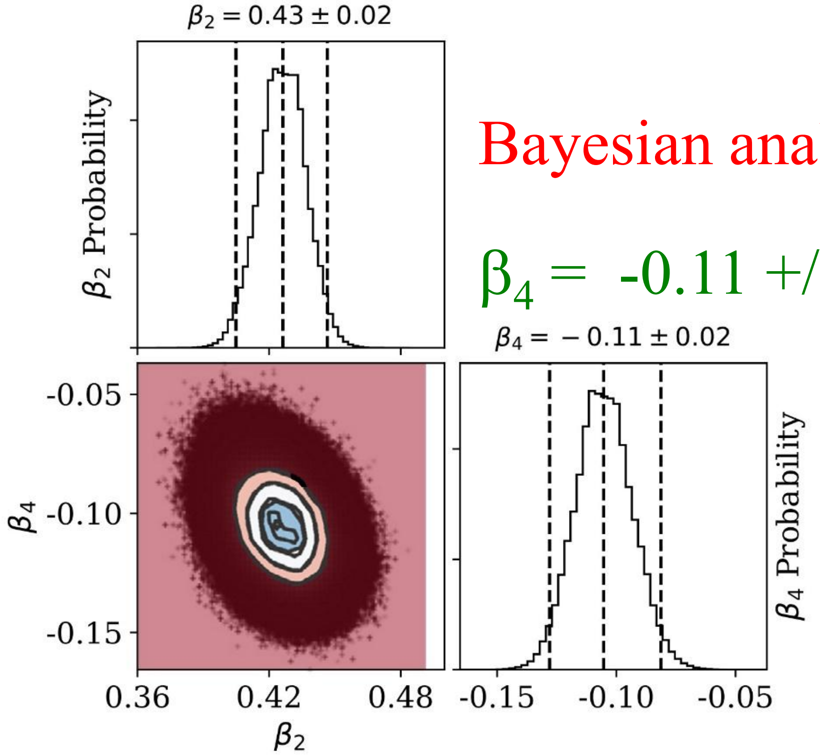
Determination of β_4 of ^{24}Mg with quasi-elastic barrier distributions

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).

^{24}Mg + ^{90}Zr



β_2 = 0.43 +/- 0.02



Bayesian analysis

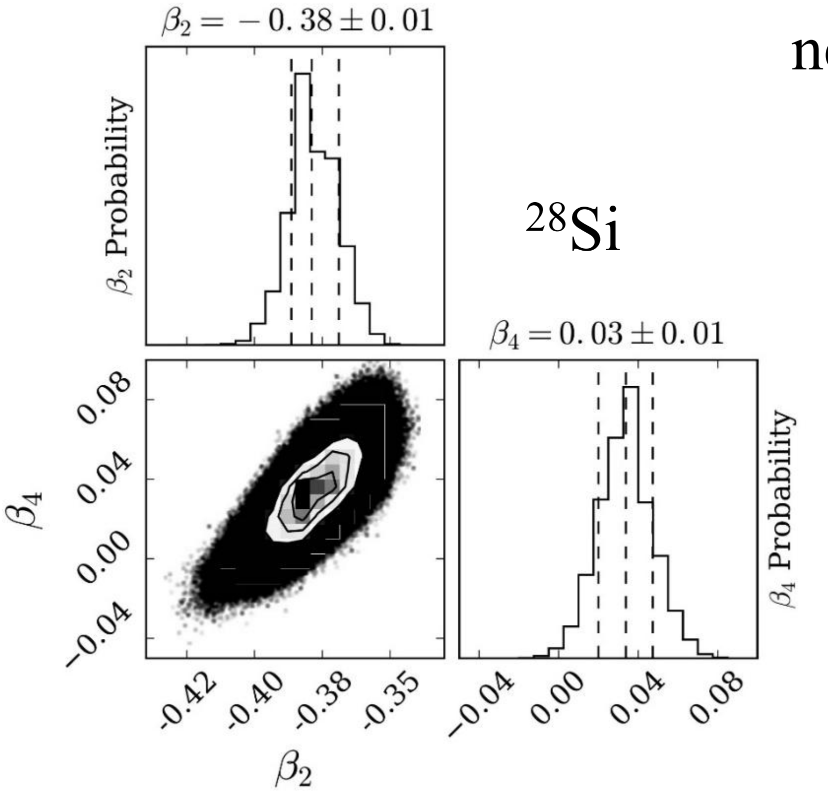
β_4 = -0.11 +/- 0.02

high precision determination of β_4
→ for the first time

cf. (p,p'): β_4 = -0.05 +/- 0.08

R. De Swiniarski et al., PRL23, 317 (1969)

Emulator for multi-channel scattering



^{28}Si

Y.K. Gupta et al., PLB845, 138120 (2023).

needs to repeat many calculations with different (β_2,β_4)

$\rightarrow$ an emulator to speed-up the calculations

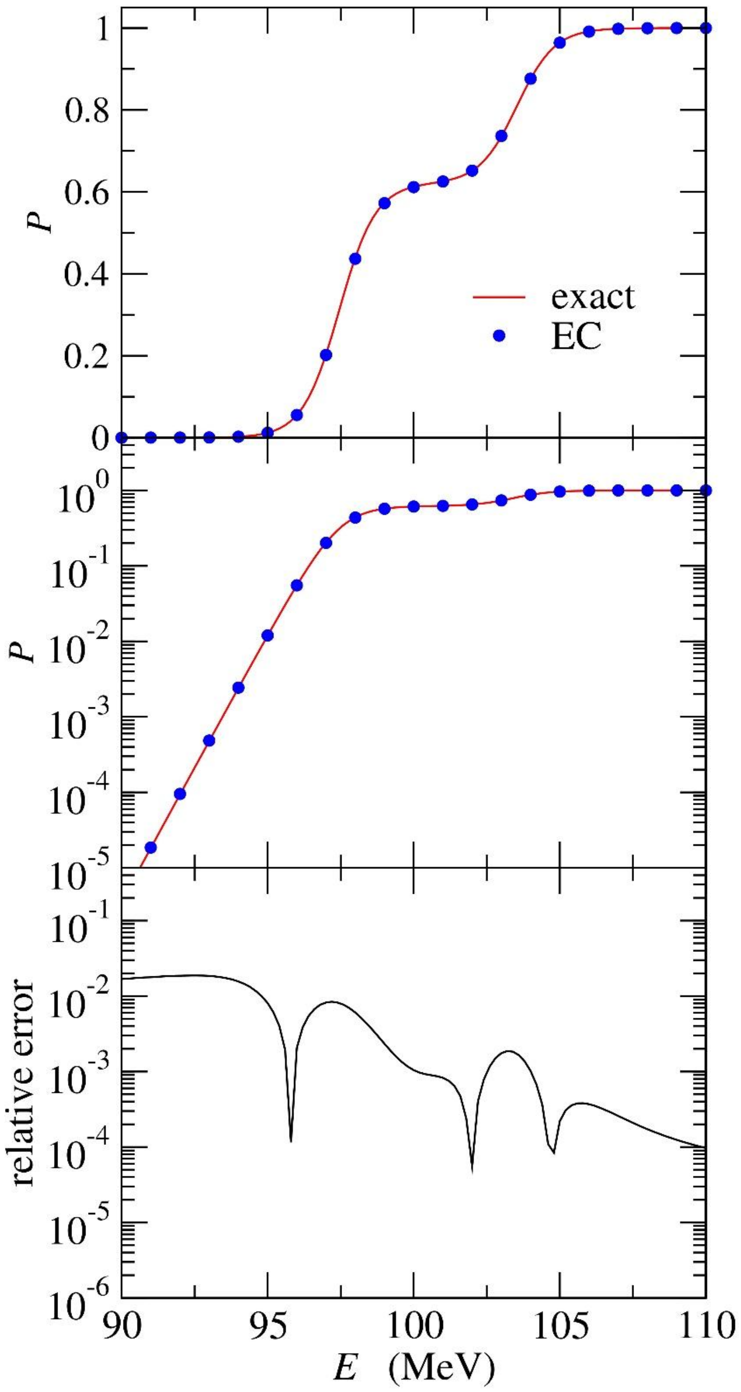
Eigenvector continuation

- bound state problems:

$$H(\theta)|\Psi(\theta)\rangle = E(\theta)|\Psi(\theta)\rangle \qquad \Psi(\theta) = \sum_{i=1}^{N} c_i \Psi(\theta_i)$$

T. Duguet et al., Rev. Mod. Phys. 96, 031002 (2024)

- Extension to scattering problems:
  - R. Furnstahl et al., PLB809, 135719 (2020)
  - C. Drisshler et al., PLB823, 136777 (2021)
  - J. Liu, J. Lei, and Z. Ren, PLB858, 139070 (2024)
  - K. Hagino, Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa, Phys. Rev. C112, 024618 (2025).



1D two-channel problem:

$$H = \begin{pmatrix} V(x) & F(x) \\ F(x) & V(x)+\epsilon \end{pmatrix}$$

$$\begin{aligned} V(x) &= V_0\, e^{-x^2/2s^2} \\ F(x) &= F_0\, e^{-x^2/2s_f^2} \end{aligned}$$

V_0 = 100 MeV, s=s_f=3 fm,
F_0 = 3 MeV

$$\Psi_E(x, F_0) = \sum_{i=1-5} c_i \Psi_E(x, F_{0i})$$

F_{0i} = 1.5, 2.0, 2.5, 3.5, 4.5 MeV

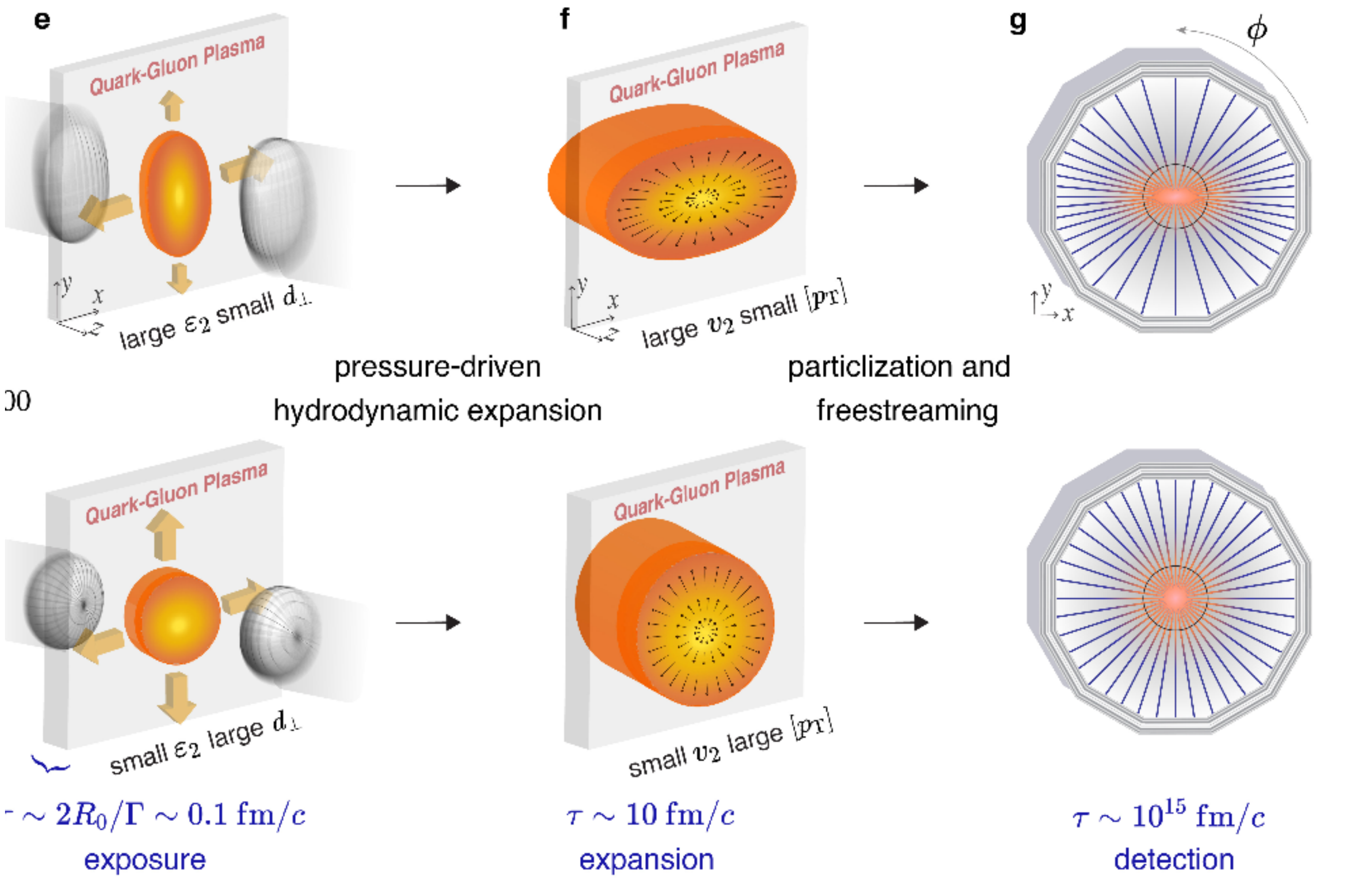
to simulate F_0 = 3 MeV

EC: the discrete basis method + Kohn variation principle

cf. K.H. and G.F. Bertsch, PRC110, 054610 (2024)

K. H., Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa,
Phys. Rev. C112, 024618 (2025).

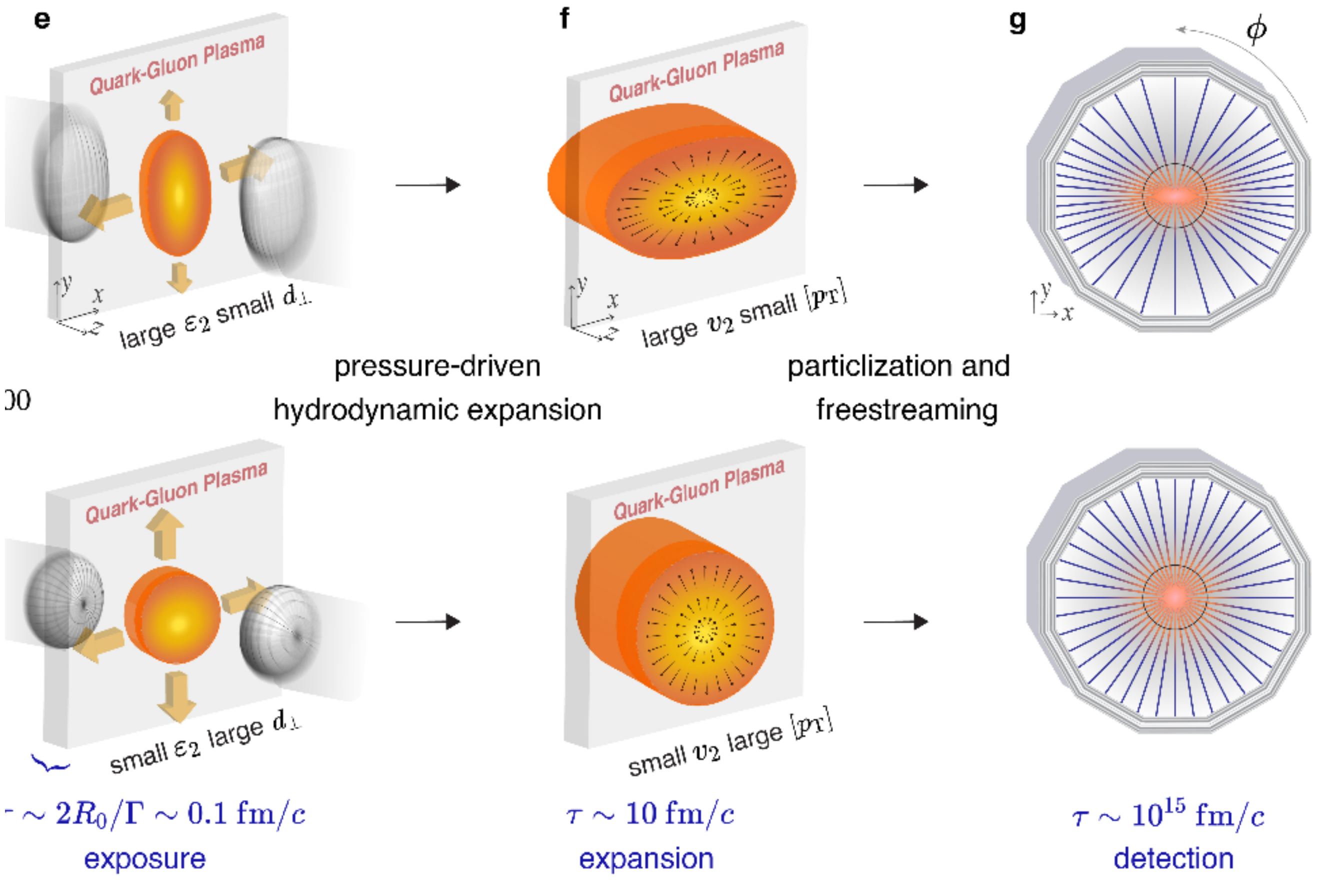
Relativistic Heavy-Ion collisions



M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)

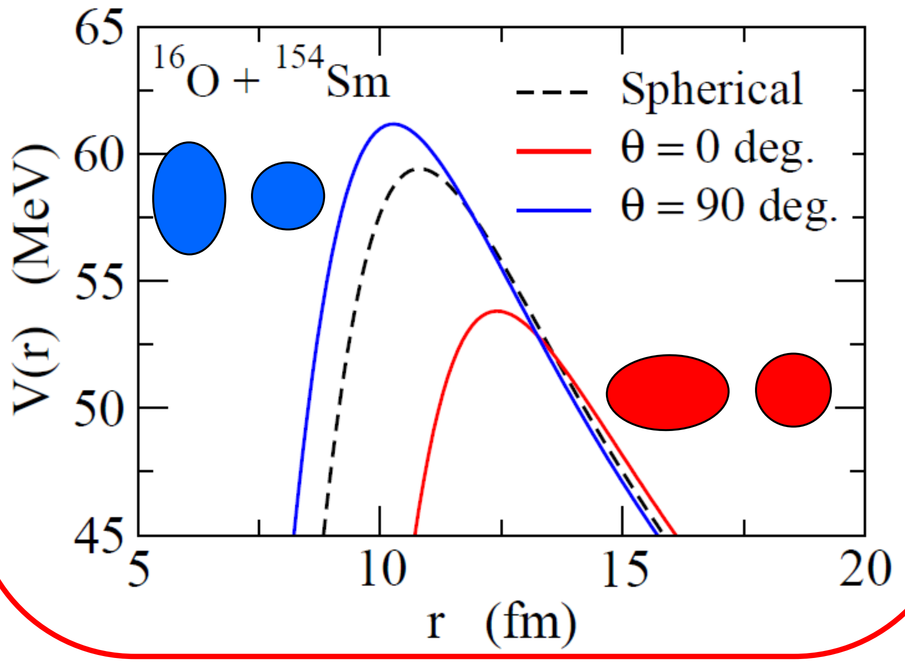
Probing nuclear shapes in Relativistic Heavy-Ion collisions

the initial shape of QGP **expansion** **detection**



M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)

large similarities to low-energy H.I. fusion reactions of deformed nuclei



→ an intersection of **High *E*** and **Low *E*** H.I. collisions

Taking a snapshot of a nucleus with a "fast" nuclear reaction

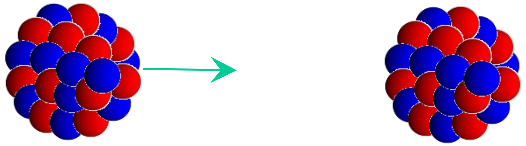
taking snapshots of a "slow" motion with a high-speed camera

$$\tau_{\mathrm{camera}} \ll \tau_{\mathrm{motion}}$$

https://www.sony.jp/ichigan/products/ILCE-7M3/feature_3.html

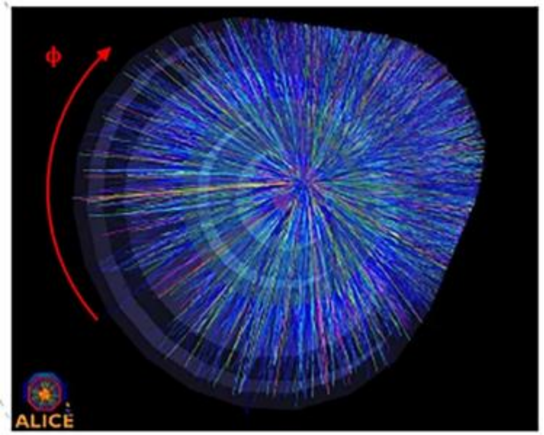
(photos with a Sony camera α7III)

⇒ taking snapshots of a nucleus with a "fast" nuclear reaction



$$\tau_{\mathrm{reaction}} \ll \tau_{\mathrm{nucleus}}$$

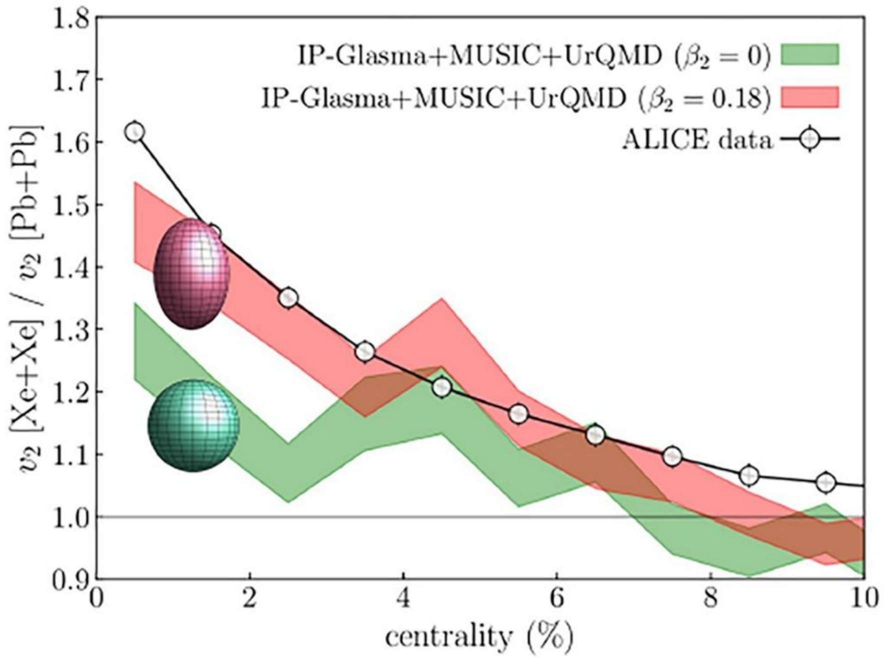
Probing nuclear shapes in Rel. H.I. collisions



flow:

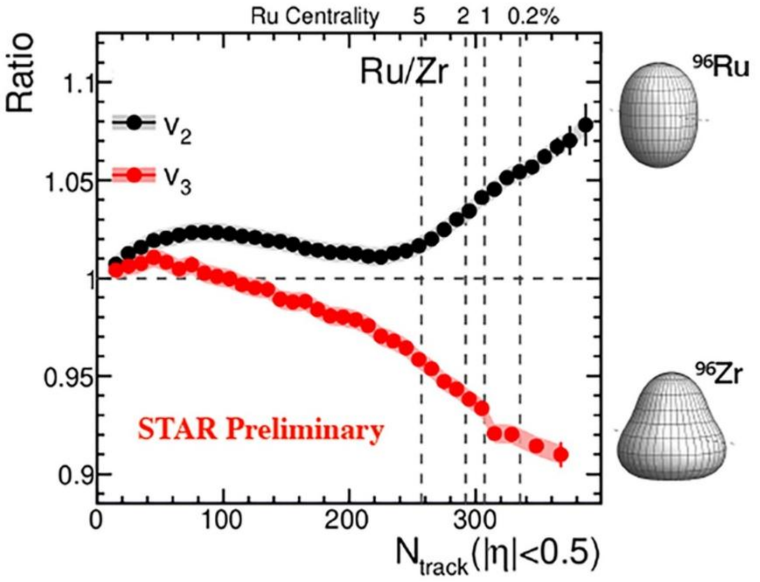
the final N-distribution $$\frac{1}{N}\frac{dN}{d\phi} = \frac{1}{2\pi}\left[1 + 2\sum_n v_n \cos n(\phi - \Psi_n)\right]$$

elliptic flow ν_2



the ratio of ^{129}Xe+^{129}Xe to ^{208}Pb+^{208}Pb

→ quadrupole deformation of ^{129}Xe



the ratio of ^{96}Ru+^{96}Ru to ^{96}Zr+^{96}Zr

→ octupole deformation of ^{96}Zr

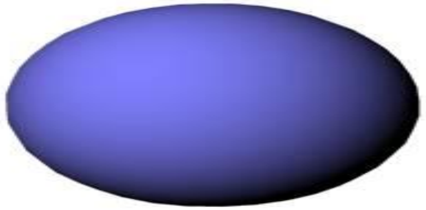
other examples:

- ✓ γ deformation
- ✓ α cluster

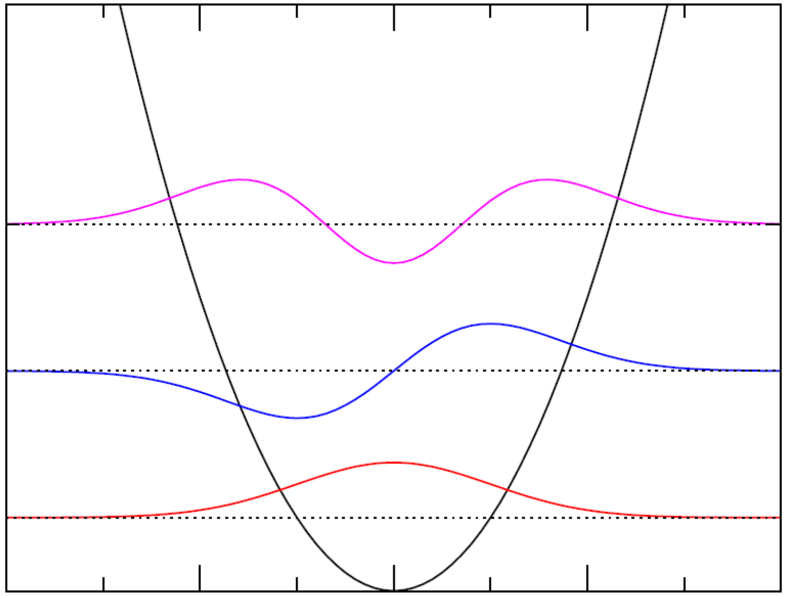
J. Jia et al., Nucl. Sci. Tech. 35, 220 (2024)

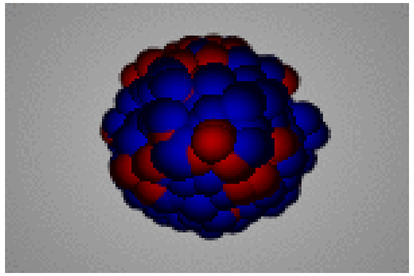
Probing nuclear shapes in Relativistic Heavy-Ion collisions

So far, the focus has been mainly on a static deformation of a deformed nucleus



There also exist several dynamical deformations of a spherical nucleus





$\langle\beta\rangle = 0$

but fluctuates around β=0
(zero-point motion)

cf. 1-dim. H.O.

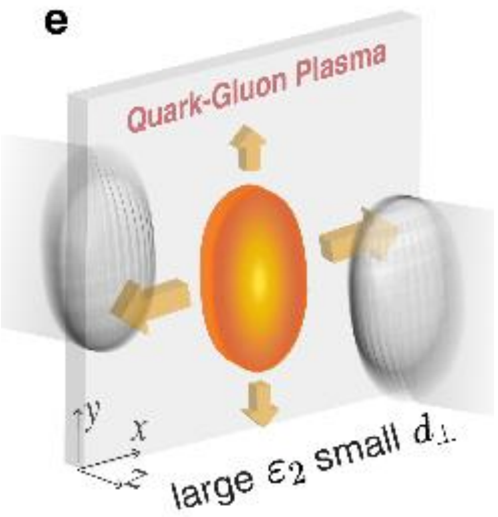
$$\phi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$$

$$\to \langle x\rangle = 0, \quad \langle x^2\rangle = 1/2\alpha$$

a typical example:
2_1^+ at 1.45 MeV in ^{58}Ni

Probing nuclear shapes in Relativistic H.I. collisions

eccentricity parameter: deformation parameter of $\rho_z(x,y) \equiv \int_{-\infty}^{\infty} dz\, \rho(\boldsymbol{r})$



$$\boxed{\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\int d\boldsymbol{r}\, r_\perp^2\, e^{2i\phi}\rho(\boldsymbol{r},\{\alpha_{2\mu}\})}{\int d\boldsymbol{r}\, r_\perp^2\, \rho(\boldsymbol{r},\{\alpha_{2\mu}\})} = -\frac{\langle (x-iy)^2\rangle}{\langle x^2+y^2\rangle}}$$

deformed Woods-Saxon density

$$\rho(\boldsymbol{r},\{\alpha_{\lambda\mu}\}) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a}}; \qquad R(\theta,\phi) = R_0\left(1+\sum_{\lambda,\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})\right)$$

surface vibration

$$H = \frac{1}{2}\sum_{\lambda,\mu}\left(B_\lambda|\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda|\alpha_{\lambda\mu}|^2\right)$$

$$\langle|\epsilon_n|^2\rangle \propto \int\left(\prod_{\lambda,\mu} d\alpha_{\lambda\mu}\, \boxed{e^{-\alpha^2_{\lambda\mu}/2\sigma^2_\lambda}}\right)|\epsilon_n(\{\alpha_{\lambda\mu}\})|^2$$

static deformation (axial symmetry)

$$\alpha_{\lambda\mu} = \beta_\lambda D^\lambda_{0\mu}(\Omega)$$

$$\langle|\epsilon_n|^2\rangle = \int\frac{d\Omega}{8\pi^2}|\epsilon_n(\Omega)|^2$$

Are these distinguishable?

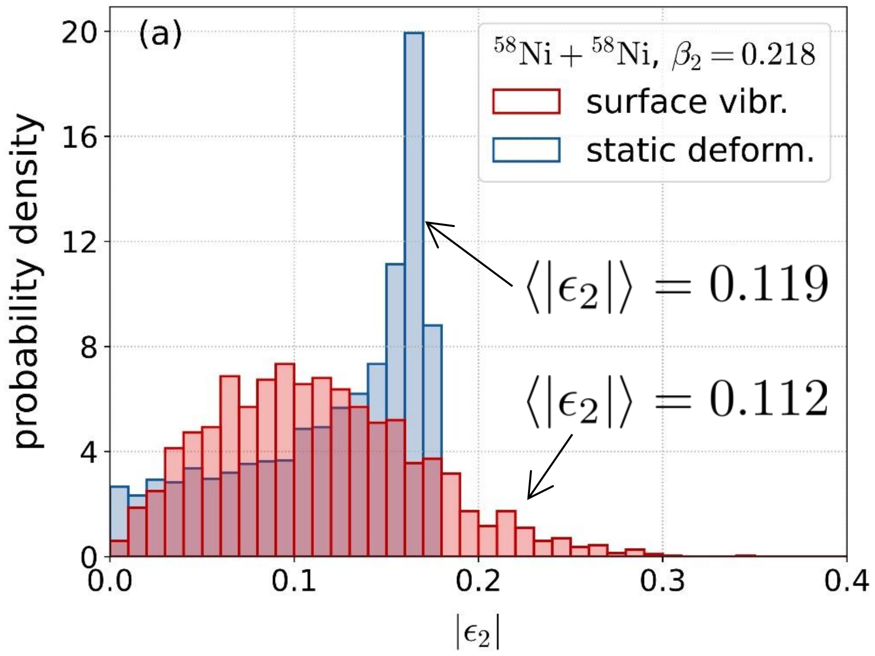
Probing nuclear shapes in Relativistic H.I. collisions

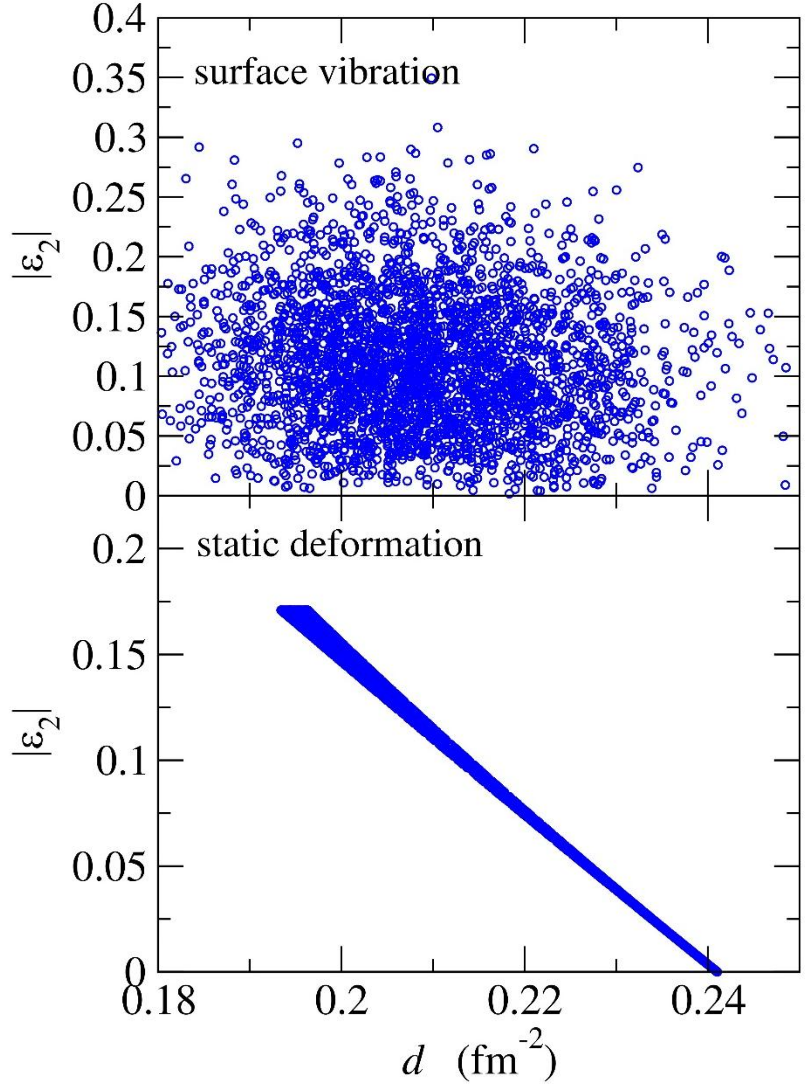
$$\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\langle (x-iy)^2\rangle}{\langle x^2+y^2\rangle}$$

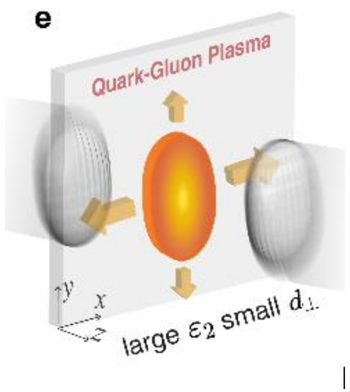
^{58}Ni+^{58}Ni scattering ($\beta_2 \sim 0.218$)

$$\begin{cases} \langle|\epsilon_n|^2\rangle \propto \int \left(\prod_{\lambda,\mu} d\alpha_{\lambda\mu}\, e^{-\alpha_{\lambda\mu}^2/2\sigma_\lambda^2}\right) |\epsilon_n(\{\alpha_{\lambda\mu}\})|^2 & \text{(vib.)} \\ \langle|\epsilon_n|^2\rangle = \int \frac{d\Omega}{8\pi^2}\, |\epsilon_n(\Omega)|^2 & \text{(static def.)} \end{cases}$$

Monte Carlo sampling (3000 samples)







the inverse area

$$d \equiv \frac{1}{\sqrt{\langle x^2\rangle\langle y^2\rangle}}$$

$$\leftrightarrow \quad \bar{p}_T$$

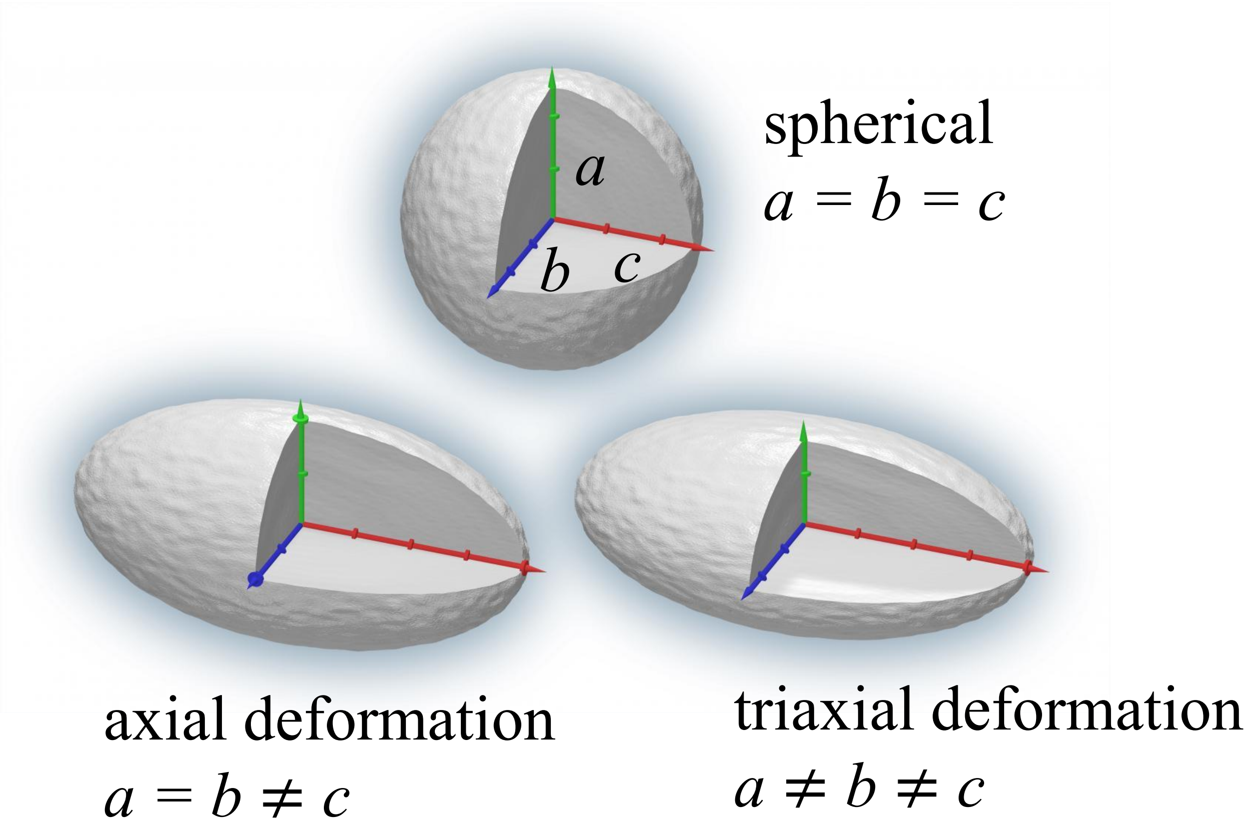
K. Hagino and M. Kitazawa, Phys. Rev. C, in press.

Probing nuclear shapes in Relativistic H.I. collisions

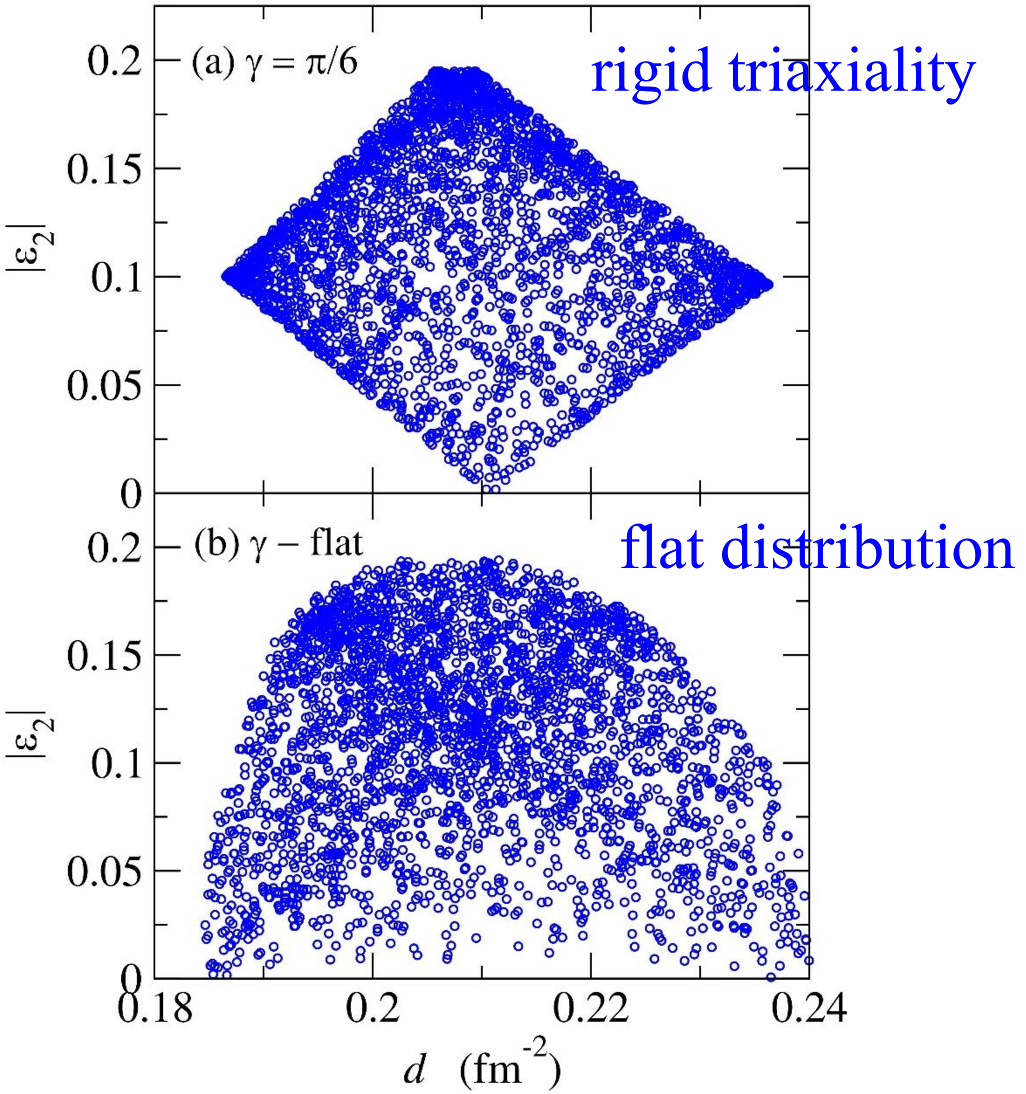
$$\frac{\Delta a}{\Delta c} = -\frac{1}{2}(1-\sqrt{3}\tan\gamma)$$
$$\frac{\Delta b}{\Delta c} = -\frac{1}{2}(1+\sqrt{3}\tan\gamma)$$

^{58}Ni+^{58}Ni scattering ($\beta_2 \sim 0.218$)

non-axial deformation (triaxiality)

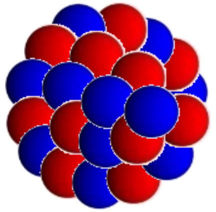


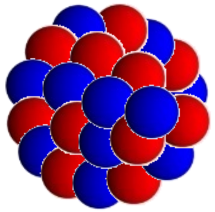
https://phys.org/news/
2016-10-lightest-superdeformed-triaxial-atomic-nucleus.html



K. Hagino and M. Kitazawa, Phys. Rev. C, in press.

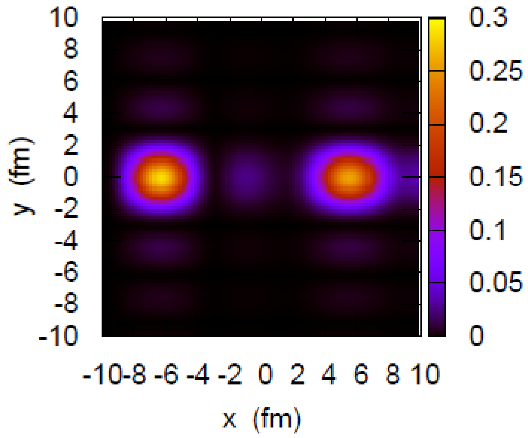
Summary





✓ Nuclear Reaction Dynamics

- a microscopic approach to absorption due to CN formation
- imaging elastic scattering ← Fourier transform of $f(\theta)$
- an emulator for multi-channel reactions



✓ Similarities between low-*E* H.I. fusion and Relativistic H.I. Collisions

→ a snapshot of a nucleus

A tool to probe nuclear deformations

→ surface vibrations of a spherical nucleus