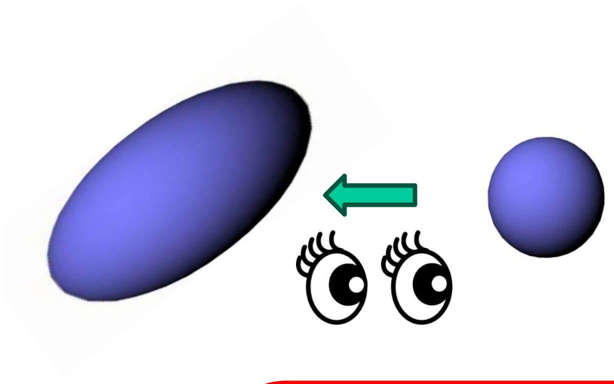
Probing nuclear structure in heavy-ion reactions
: similarities between subbarrier fusion and relativistic HIC



Kouichi Hagino

Kyoto University, Kyoto, Japan

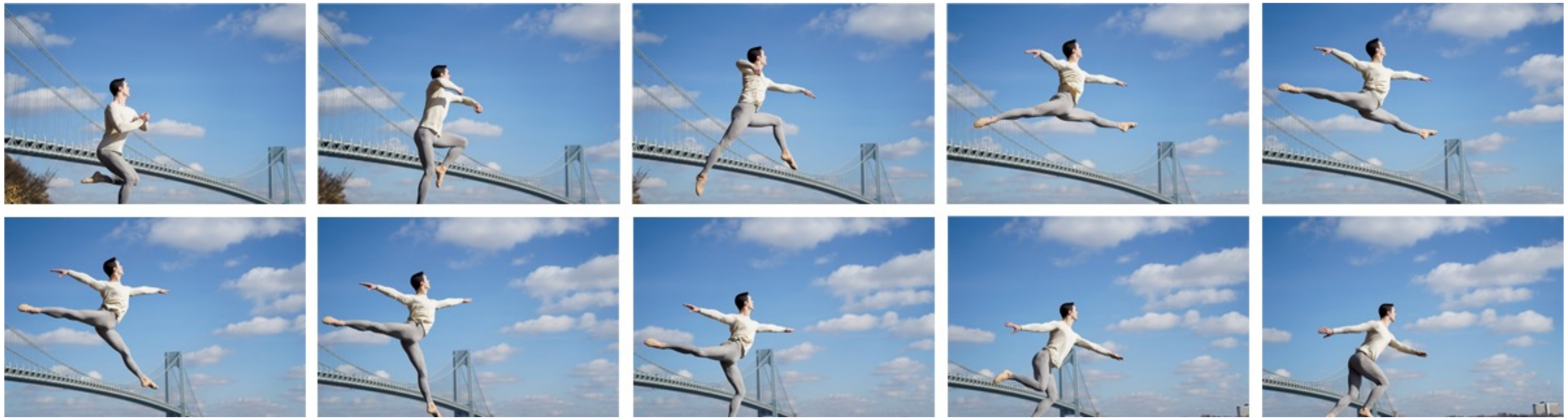
1. Introduction
2. Low-energy Nuclear Reactions: overview
3. Role of deformation in sub-barrier fusion reactions
4. Probing nuclear structure in Relativistic heavy-ion collisions
5. Summary

Workshop on "Nuclear physics across energy scales", Central China Center for Nuclear Theory, Wuhan, 2025.9. 19-21

Snapshots

taking snapshots of a “slow” motion with a high-speed camera

$$\tau_{\rm camera} \ll \tau_{\rm motion}$$

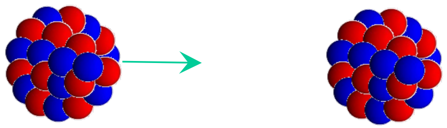


https://www.sony.jp/ichigan/products/ILCE-7M3/feature_3.html

(photos with a Sony camera α7III)

a *slow* mode
a *fast* mode

⇒ taking snapshots of a nucleus with a “fast” nuclear reaction

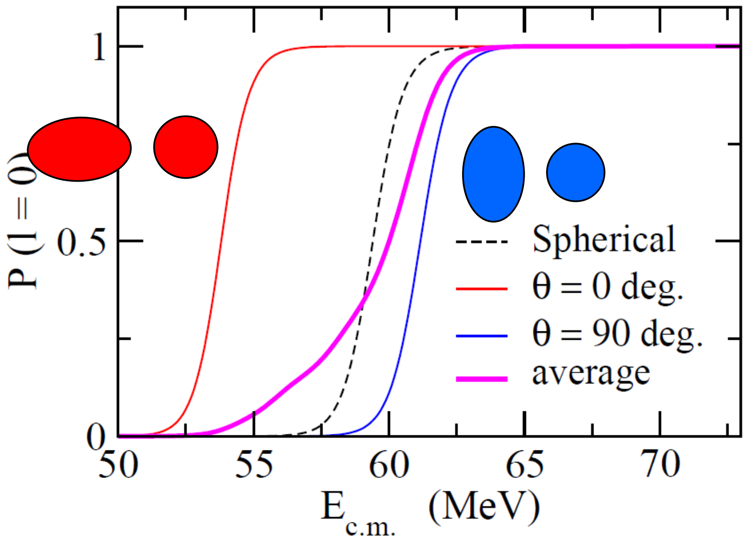


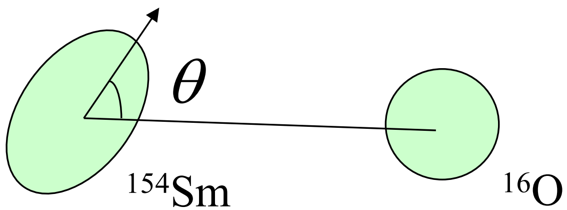
$$\tau_{\rm reaction} \ll \tau_{\rm nucleus}$$

Snapshots

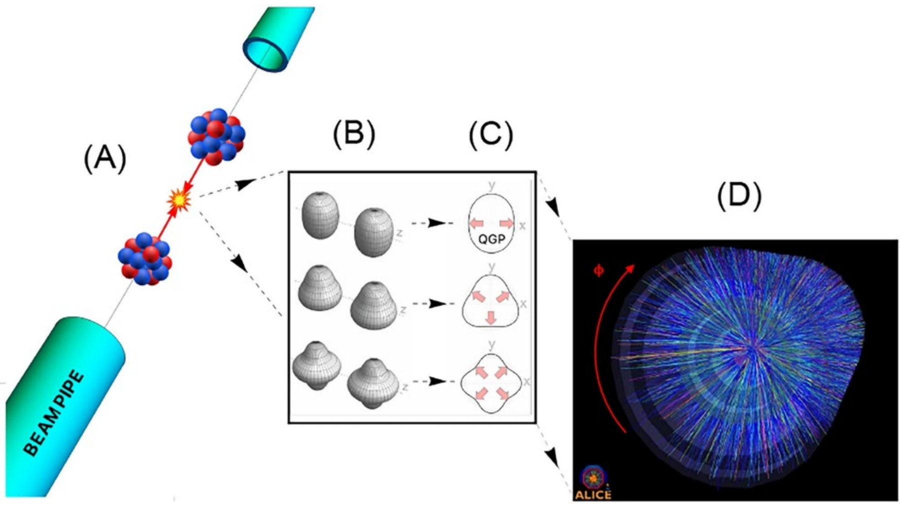
taking a snapshot of a nucleus with a “fast” nuclear reaction

low-energy H.I. fusion reactions of a deformed nucleus





relativistic H.I. collisions with a deformed nucleus

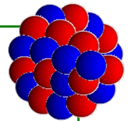


J. Jia et al., Nucl. Sci. Tech. 35, 220 (2024)

increasing interests in recent years

Introduction: low-energy nuclear reactions

nucleus: a composite system

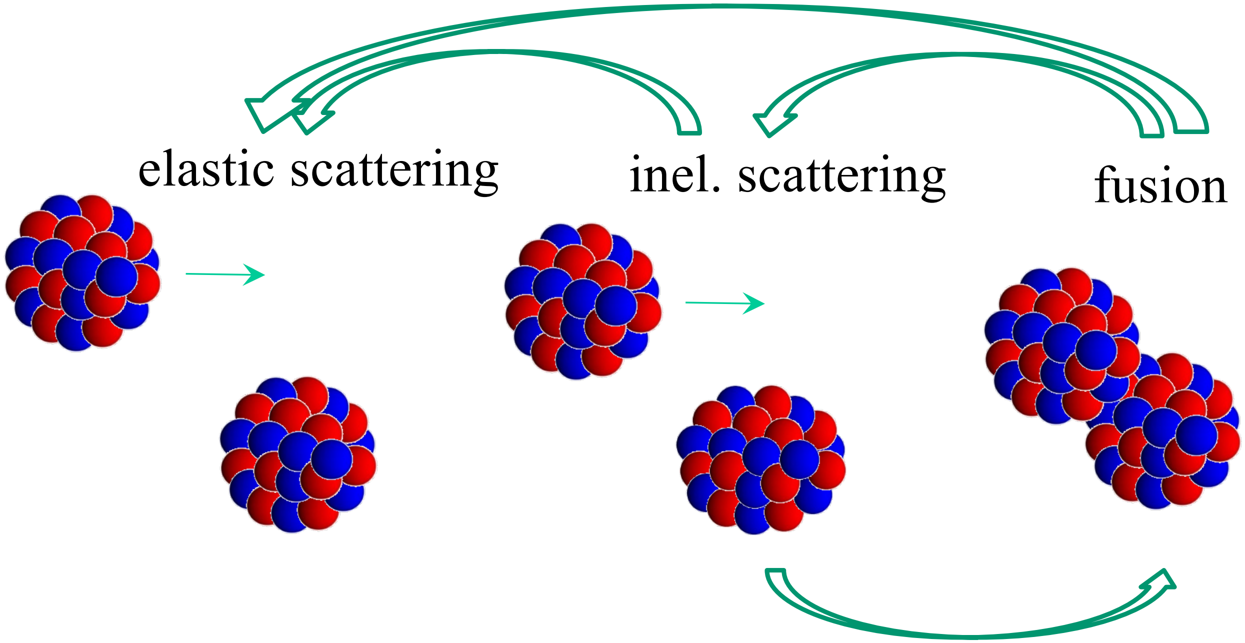


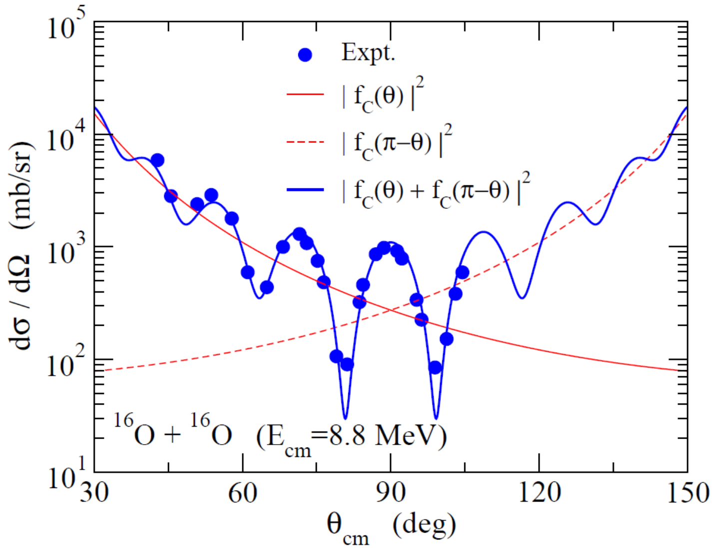
✓ various sort of reactions

✓ an interplay between nuclear structure and reaction

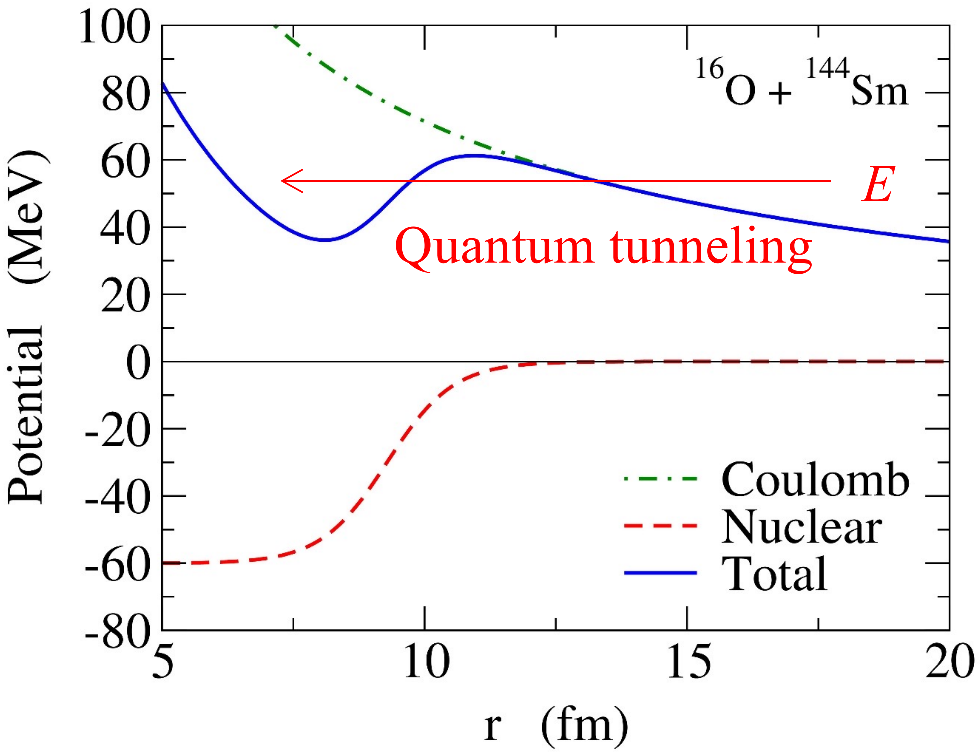
shapes, excitations, …..

- elastic scattering
- inelastic scattering
- transfer reactions
- breakup reactions
- fusion reactions





Coulomb barrier



1. Coulomb interaction
   long range, repulsion
2. Nuclear interaction
   short range, attraction

Potential barrier (Coulomb barrier)

Fusion: takes place by overcoming the barrier

the barrier height → defines the energy scale of a system

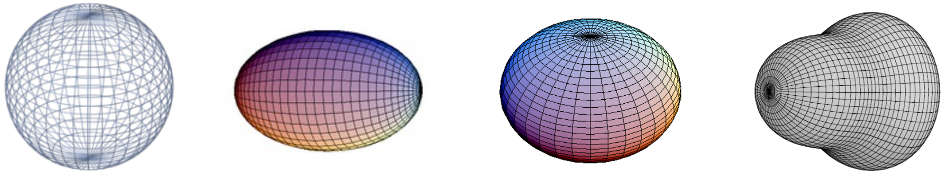
Fusion reactions at energies around the Coulomb barrier

Sub-barrier fusion reactions and quantum tunneling

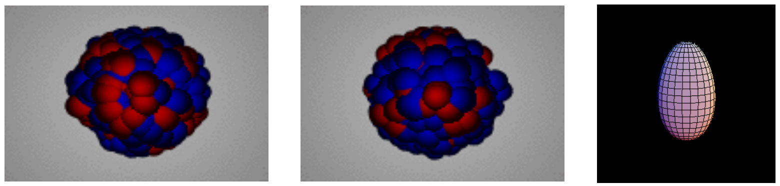
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes



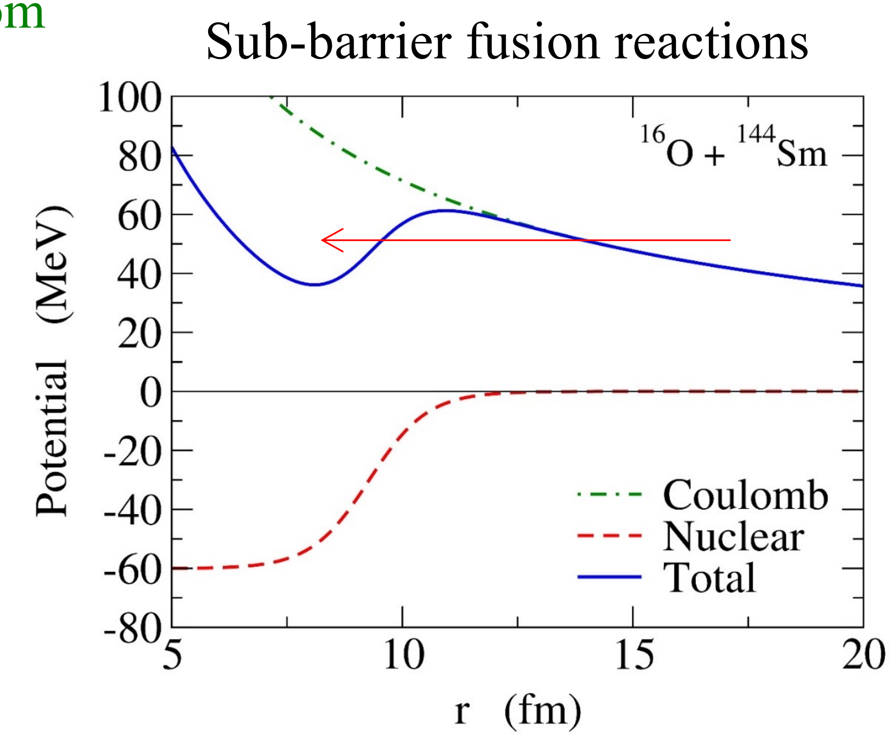
- several surface vibrations



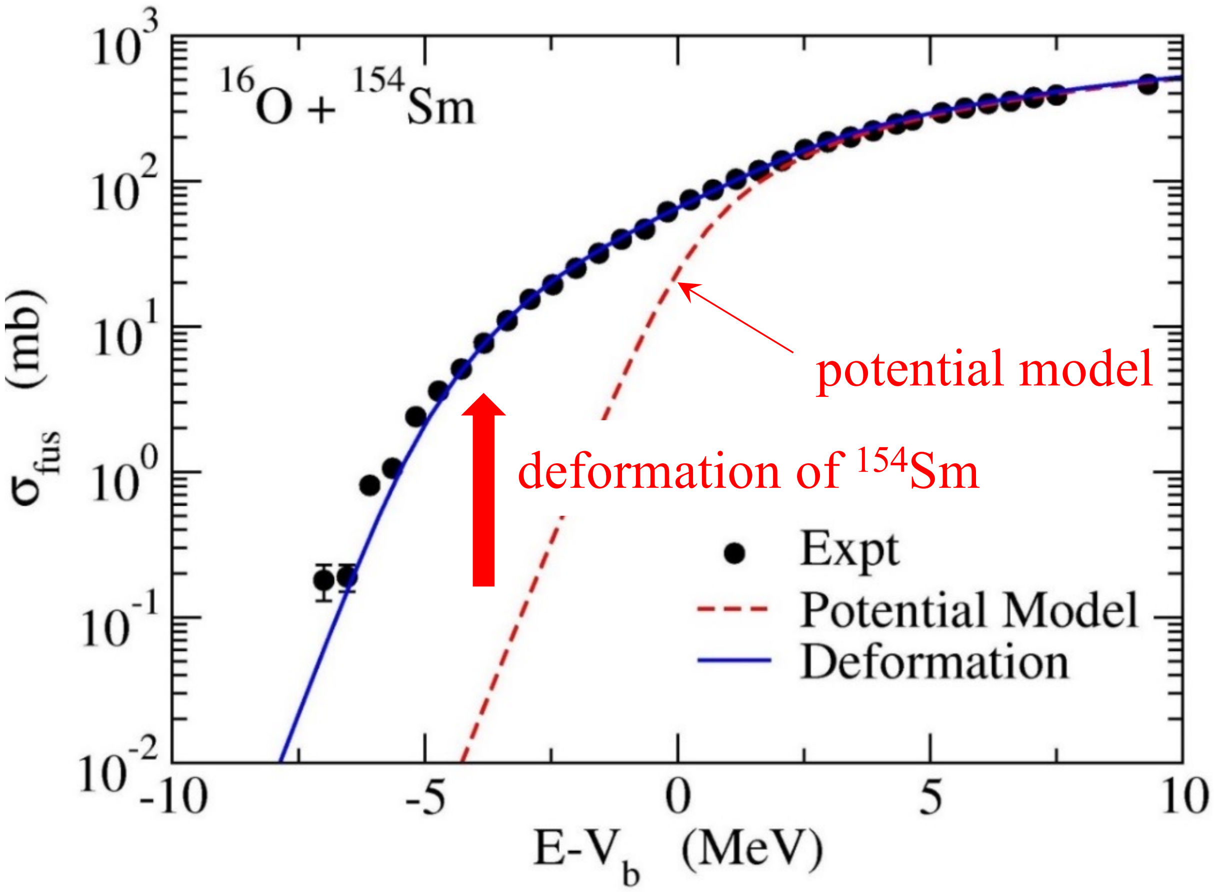
several modes and adiabaticities

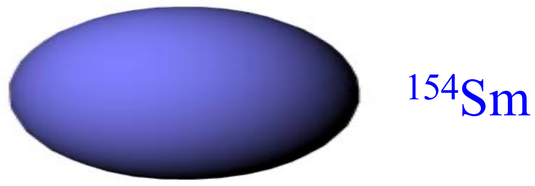
- several types of nucleon transfers

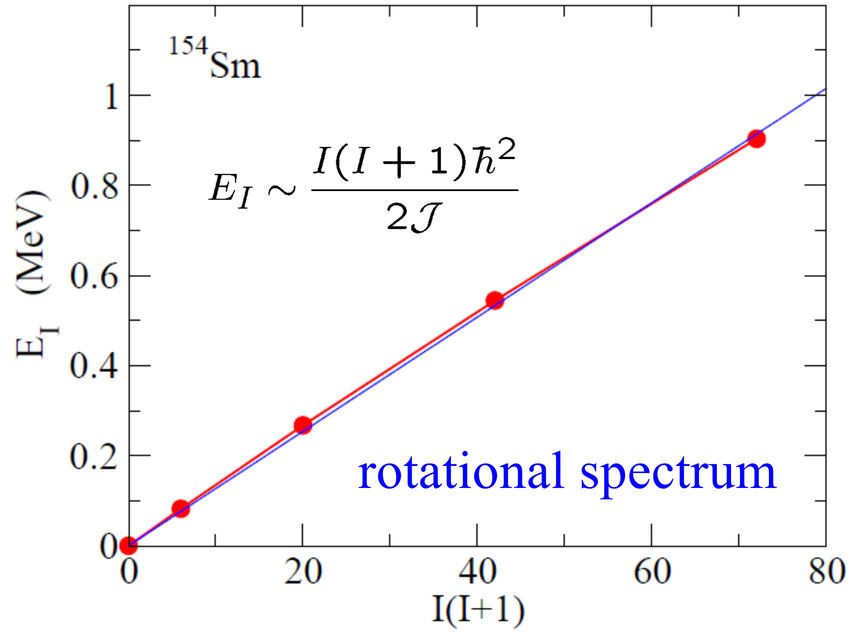
Tunneling probabilities: the exponential *E* dependence
→ nuclear structure effects are amplified



Sub-barrier fusion reactions and quantum tunneling

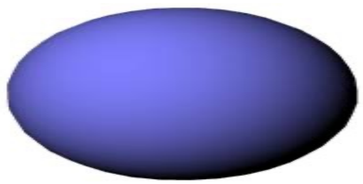




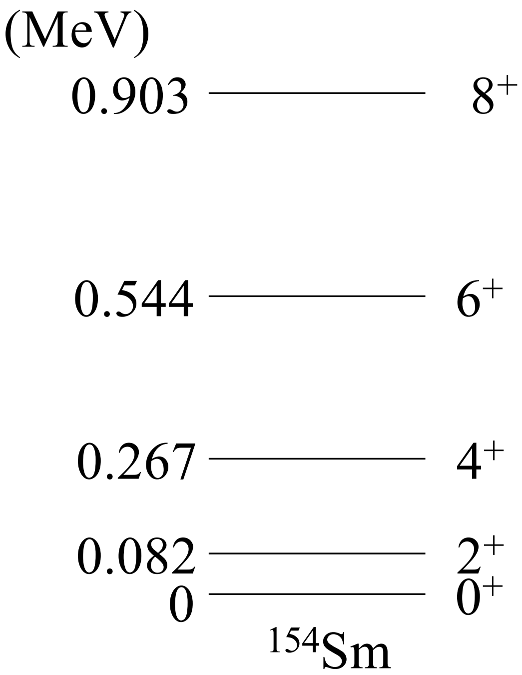


K. Hagino and N. Takigawa, Prog. Theo. Phys.128 (2012)1061.

Effects of nuclear deformation on fusion



^{154}Sm



rotational spectrum

a small rotational energy

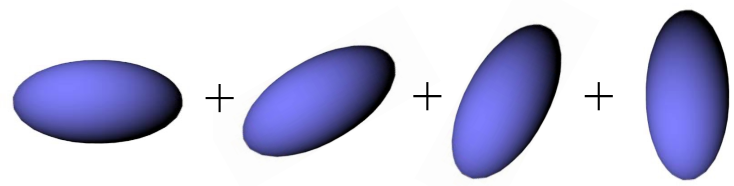
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

$\rightarrow$ a large moment of inertia J

$\rightarrow$ rotation: a slow deg. of freedom

$$\begin{aligned} E_{\rm rot} &\sim E_{2^+} = 82 \text{ keV} \\ E_{\rm tunnel} &\sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV} \end{aligned}$$

$\Psi_{0^+} =$



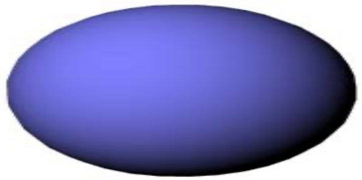
$\rightarrow$ a spherical state in the lab. system

fix the orientation angle to calculate
the fusion probability

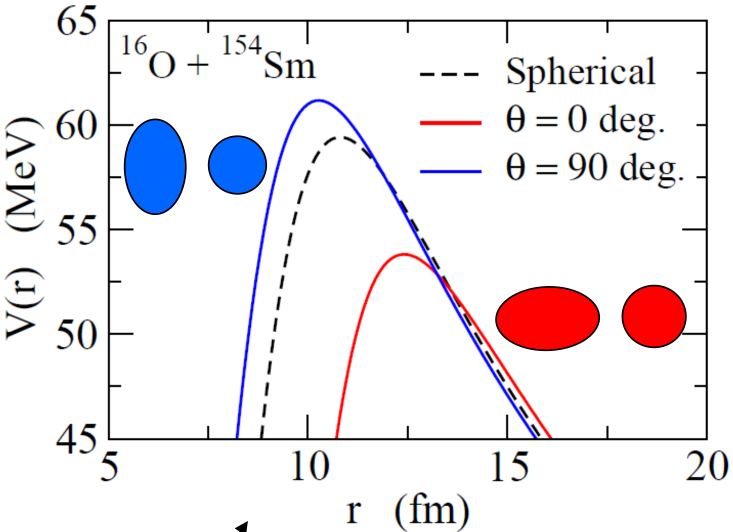
"a snapshot of a rotating nucleus"

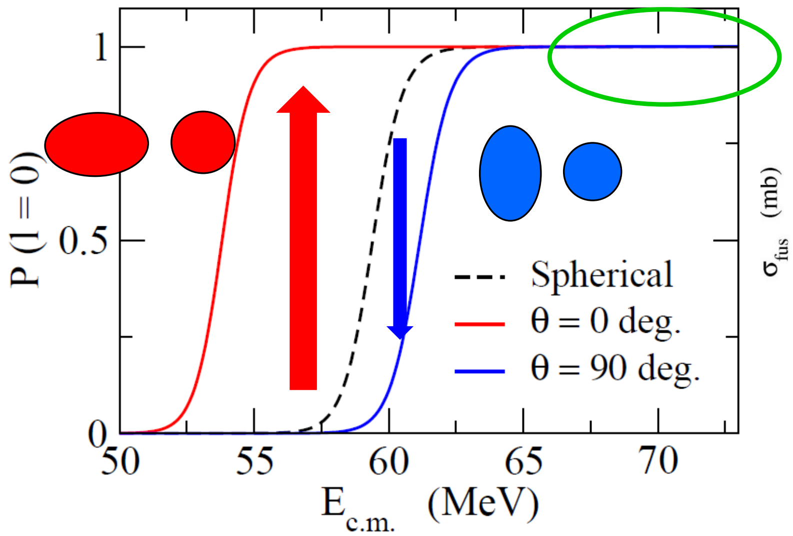
Effects of nuclear deformation on fusion

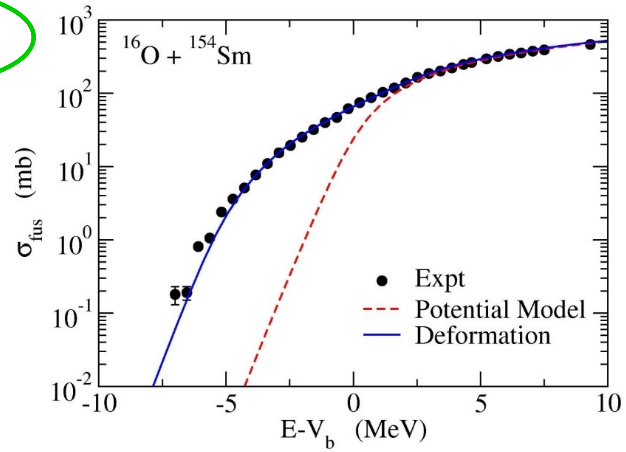
^{154}Sm : a typical deformed nucleus

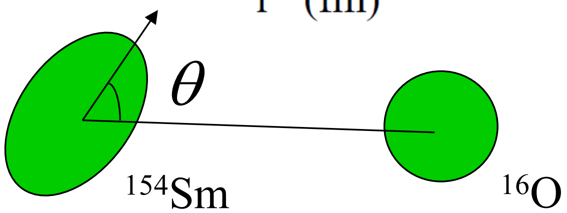


^{154}Sm



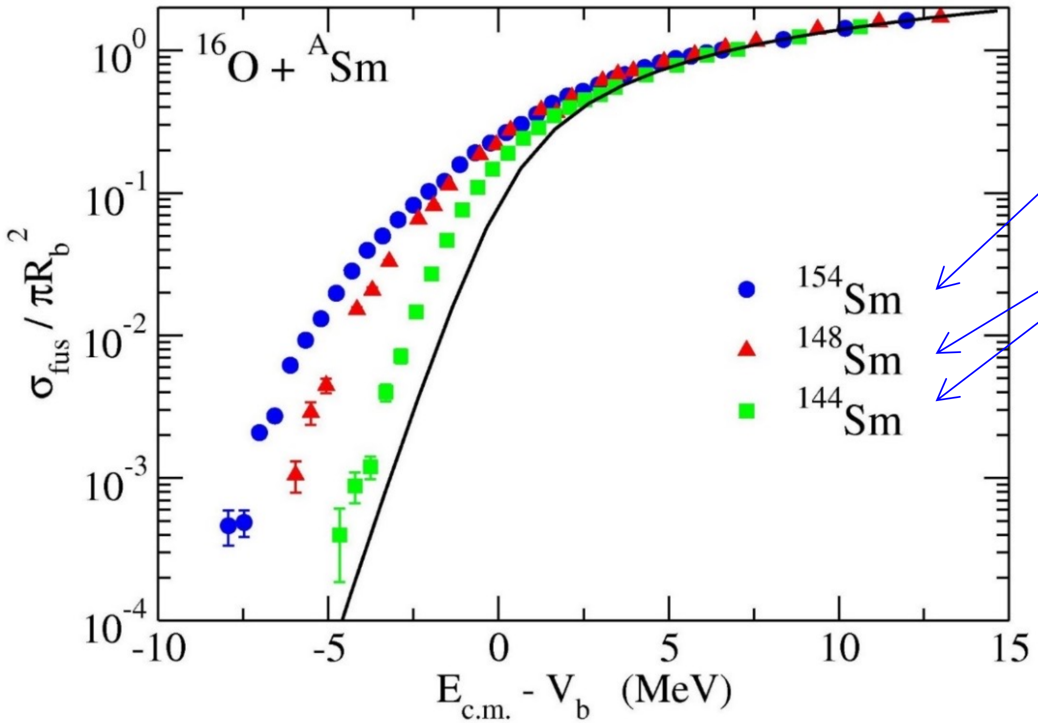






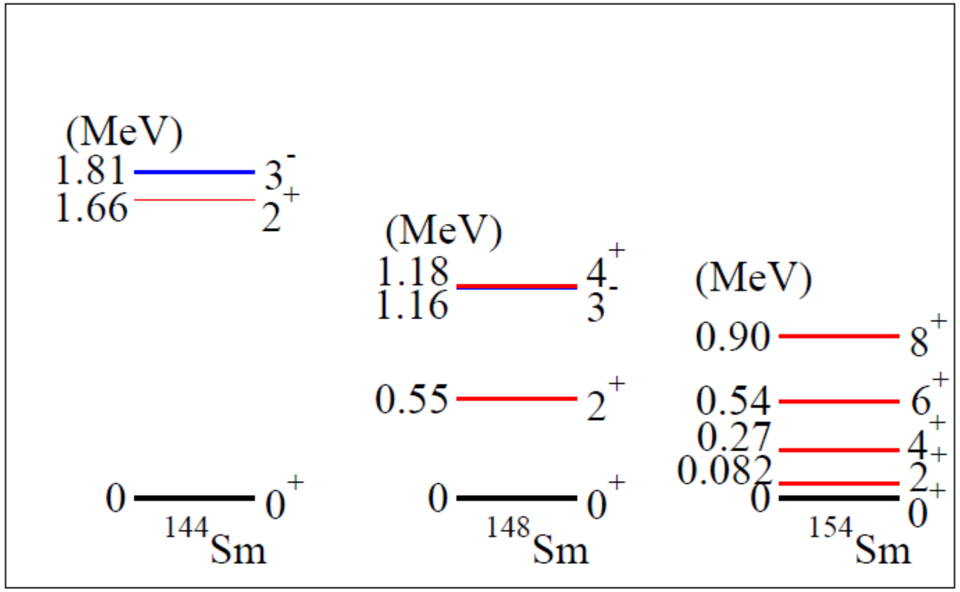
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$

Fusion: strong interplay between nuclear structure and reaction similar enhancement for non-deformed nuclei



static deformation

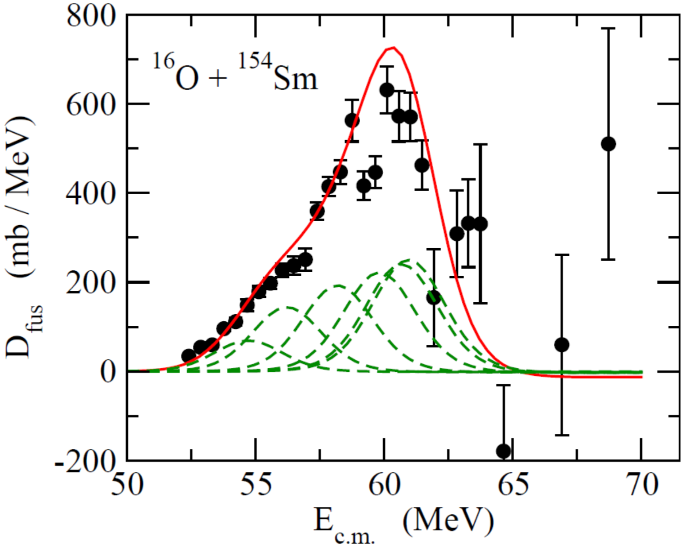
spherical, but with vibration

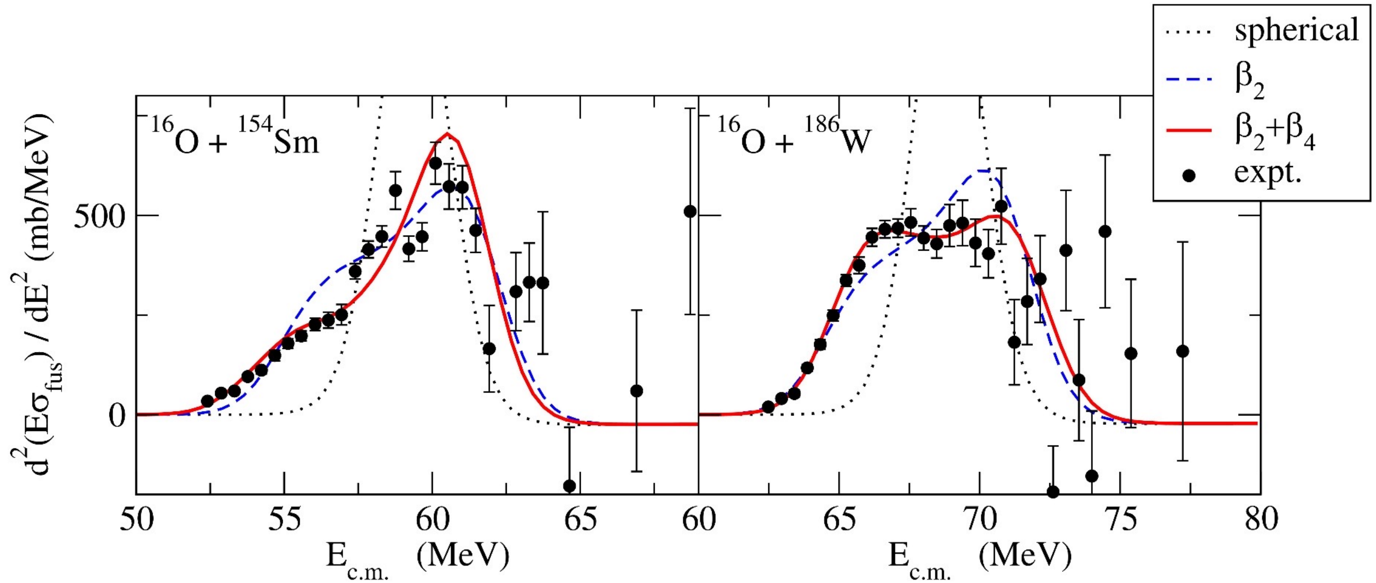


<u>Fusion barrier distribution</u>

$$\boxed{D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

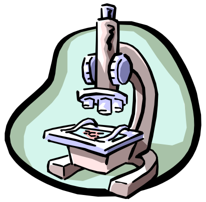


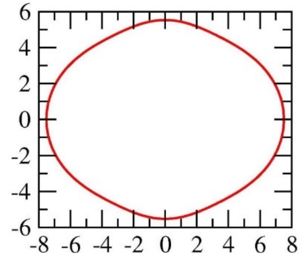


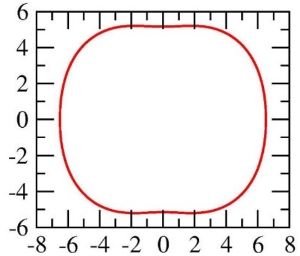
$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

$\beta_2 = 0.33$ $\beta_2 = 0.29$

$\beta_4 = +0.05$ $\beta_4 = -0.03$







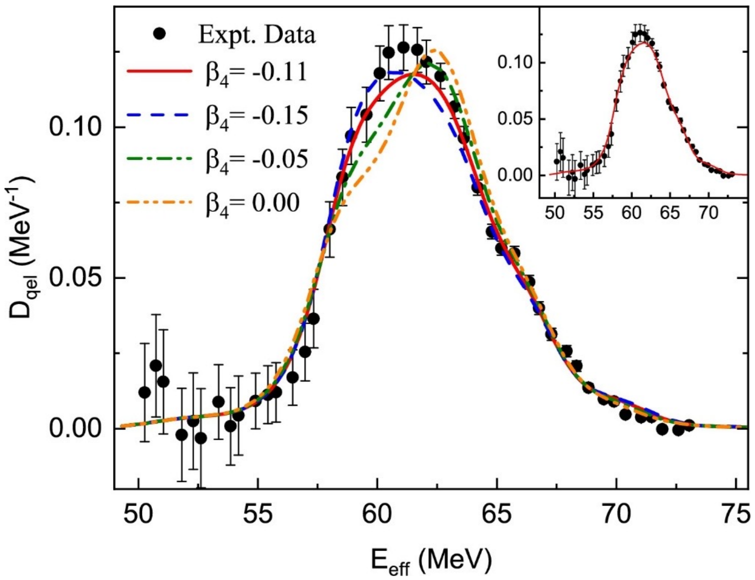
sensitive to the sign of β_4!

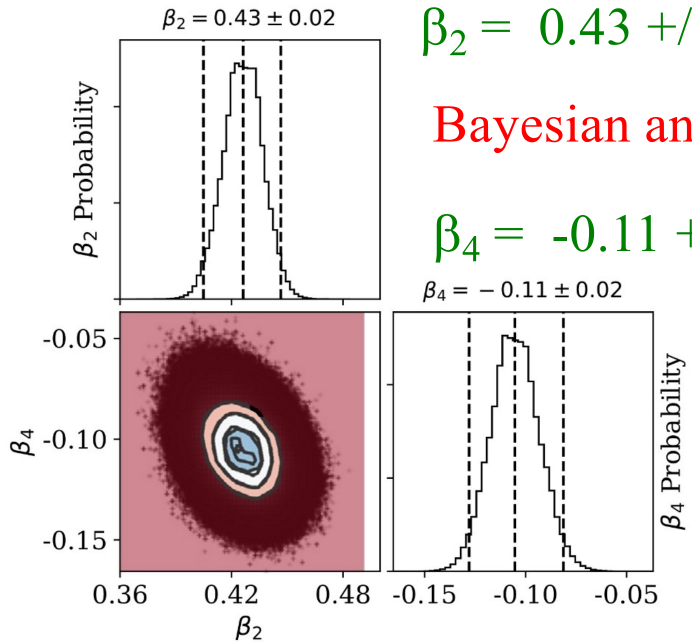
→ Fusion as a quantum tunneling microscope for nuclei

Determination of β_4 of ^{24}Mg and ^{28}Si with quasi-elastic barrier distributions

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).
Y.K. Gupta et al., Phys. Lett. B845, 138120 (2023).

^{24}Mg + ^{90}Zr





$\beta_2 =$ 0.43 +/- 0.02

Bayesian analysis

$\beta_4 =$ -0.11 +/- 0.02

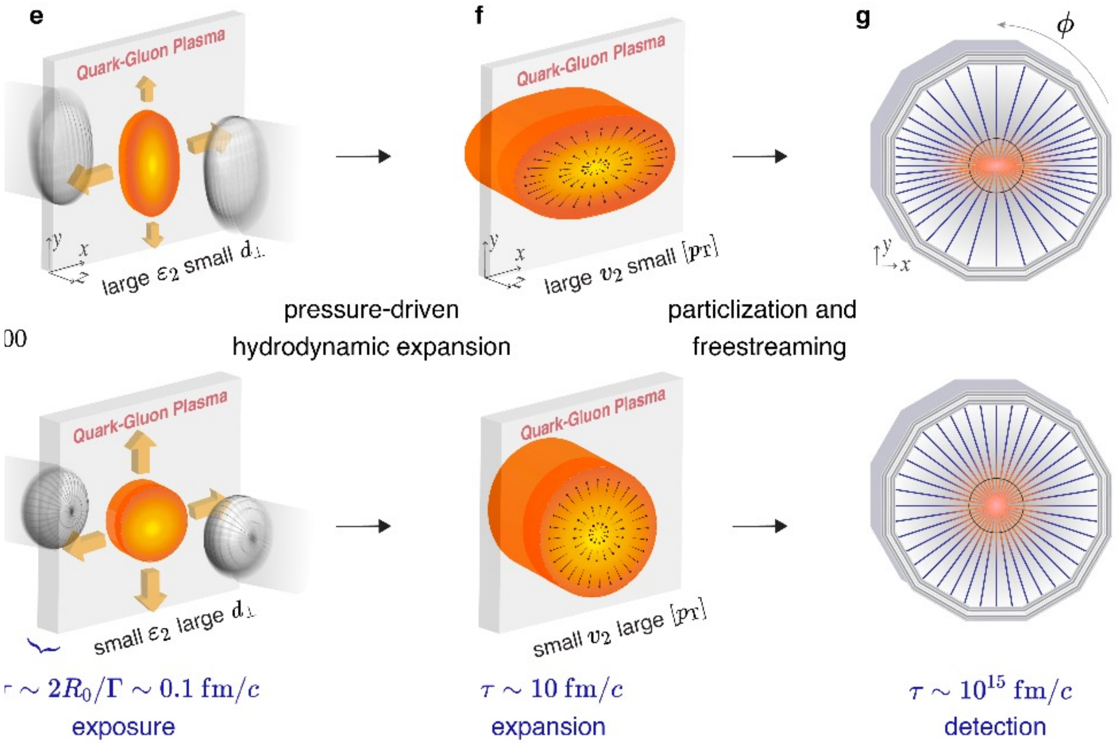
high precision determination of β_4
→ for the first time

cf. Construction of an emulator:
K. Hagino, Z. Liao, S. Yoshida, M. Kimura, and K. Uzawa, Phys. Rev. C112, 024618 (2025).

cf. (p,p'): β_4 = -0.05 +/- 0.08
R. De Swiniarski et al., PRL23, 317 (1969)

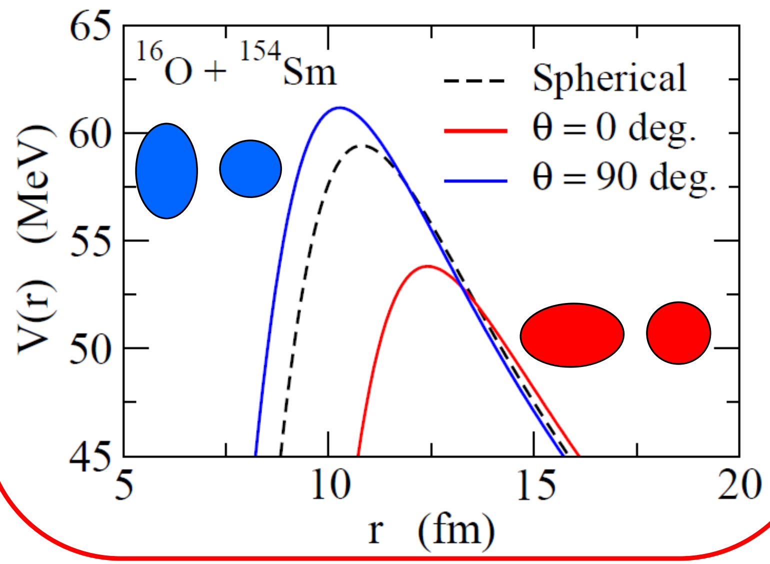
Probing nuclear shapes in Relativistic Heavy-Ion collisions

the initial shape of QGP **expansion** **detection**



M.I. Abdulhamid et al. (STAR collaboration)
Nature 635, 67 (2024)

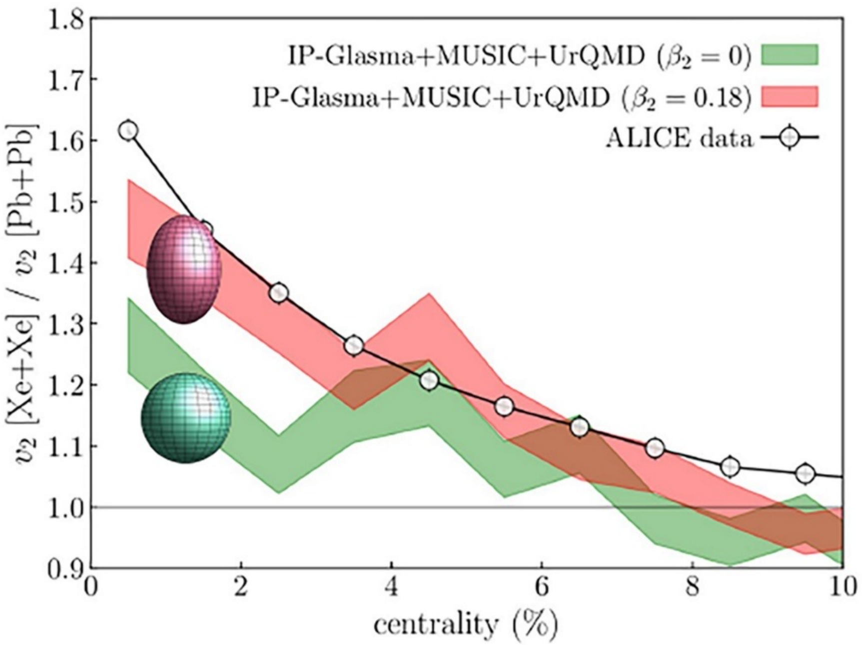
large similarities to low-energy H.I. fusion reactions of deformed nuclei

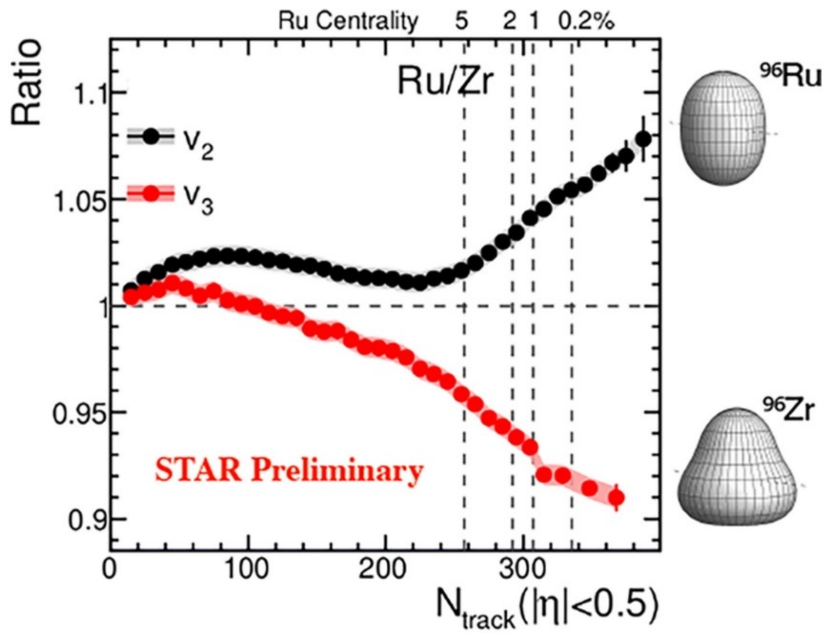


→ an intersection of **High *E*** and **Low *E*** HI collisions

Probing nuclear shapes in Relativistic Heavy-Ion collisions

This has opened up a strong intersection between nuclear structure and relativistic HI collisions ← topics of this workshop



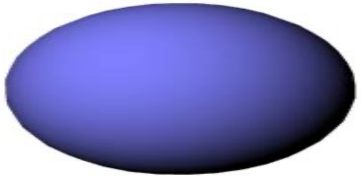


- ✓ quadrupole deformation β_2
- ✓ octupole deformation β_3
- ✓ triaxial deformation γ
- ✓ cluster structure

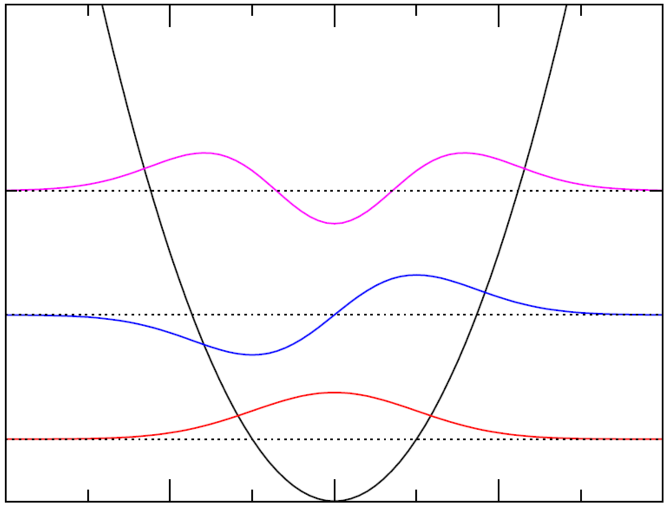
J. Jia et al.,
Nucl. Sci. Tech. 35, 220 (2024)

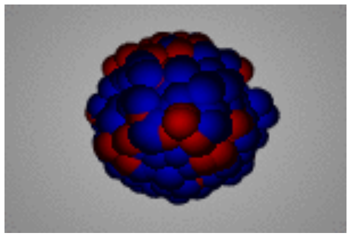
Probing nuclear shapes in Relativistic Heavy-Ion collisions

So far, the focus has been mainly on a static deformation of a deformed nucleus



There also exist several dynamical deformations of a spherical nucleus





$\langle \beta \rangle = 0$

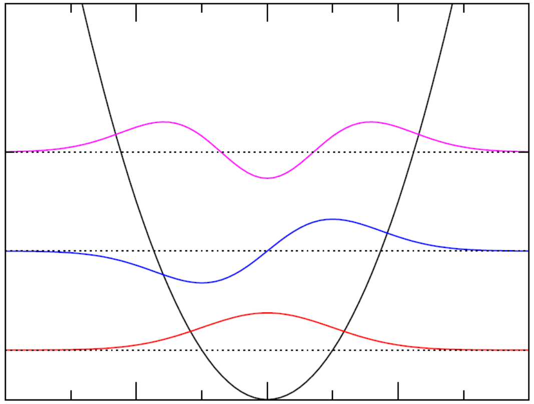
but fluctuates around β=0
(zero-point motion)

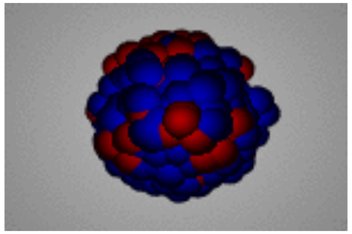
cf. 1-dim. H.O.

$$\phi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$$

$$\to \langle x \rangle = 0, \quad \langle x^2 \rangle = 1/2\alpha$$

Probing nuclear shapes in Relativistic Heavy-Ion collisions





$$\langle \beta \rangle = 0$$

but fluctuates around β=0

In most of the cases, the vibrational motion is not slow for fusion:

$$E_{\rm vib} \sim 2 \text{ MeV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

→ but this can be very slow in rel. H.I. collisions!

the adiabatic approximation for vibrations:

H. Esbensen, Nucl. Phys. A352, 147 (1981)

FUSION AND ZERO-POINT MOTIONS

H. ESBENSEN

Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 14 July 1980

$$\sigma_{\rm fus}(E) \sim \int d\beta\, w(\beta)\sigma_0(E;\beta)$$

$$w(\beta) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\beta^2/2\sigma^2}$$

$$\sqrt{\langle \beta_\lambda^2 \rangle} = \frac{4\pi}{3ZR^\lambda}\sqrt{\frac{B(E\lambda)\uparrow}{e^2}}$$

Probing nuclear shapes in Relativistic Heavy-Ion collisions

during a collision, a nucleus can be excited to the 3^- state

dynamical deformation parameter

ε_3 3^-

$$\hat{T}_{\rm E3} \propto \alpha_{3\mu} = \frac{\beta_3}{\sqrt{7}}(a^\dagger_{3\mu} + (-1)^\mu a_{3-\mu})$$

0^+

$$\rho(\boldsymbol{r},\{\alpha_{\lambda\mu}\}) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a}}; \quad R(\theta,\phi) = R_0\left(1+\sum_{\lambda,\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})\right)$$

If $E_{\rm inc} \gg \varepsilon_3 \rightarrow$ the adiabatic approximation

the eigen-channels

0^+, 3^-

$$|\nu_i\rangle = \alpha_i|0^+\rangle + \beta_i|3^-\rangle$$

$$\rightarrow |0^+\rangle = \sum_i \gamma_i\,|\nu_i\rangle$$

$$\sigma = \sum_i \underbrace{|\gamma_i|^2}_{=w_i}\sigma_i$$

for H.O.

$$w(\beta) \propto e^{-\beta^2/2\sigma^2}$$

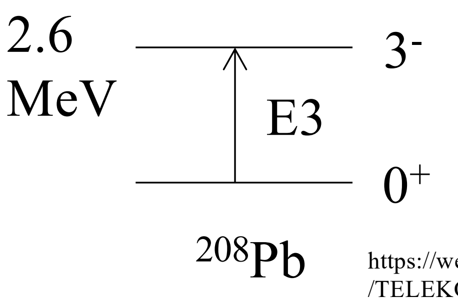
Probing nuclear shapes in Relativistic Heavy-Ion collisions

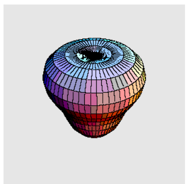
Very recent preprint: D. Xu et al., arXiv: 2504.19644

A "breathing" octupole ^{208}Pb nucleus: resolving the elliptical-to-triangular azimuthal anisotropy puzzle in ultracentral relativistic heavy ion collisions

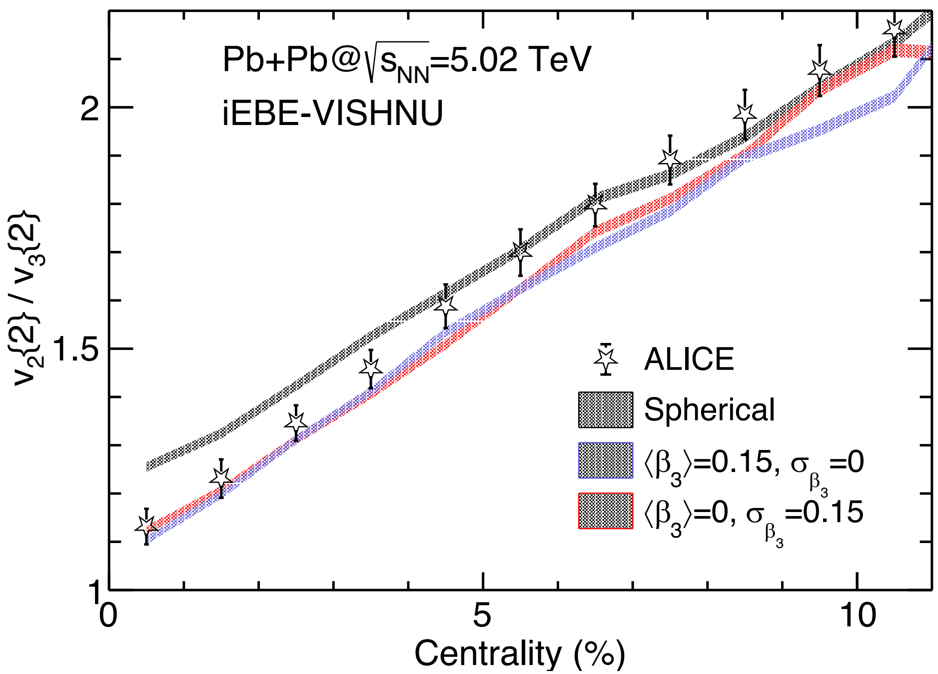
Duoduo Xu,[1] Shujun Zhao,[1] Hao-jie Xu,[2,3,*] Wenbin Zhao,[4,5,†] Huichao Song,[1,6,7,‡] and Fuqiang Wang[8,§]

octupole vibration of ^{208}Pb

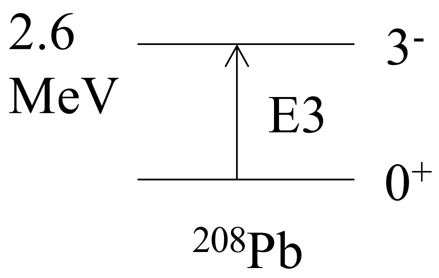


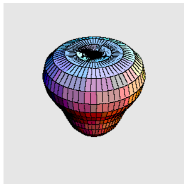


https://web-docs.gsi.de/~wolle
/TELEKOLLEG /KERN/index-s.html



What is the nature of the 3_1^- state in ^{208}Pb?





https://web-docs.gsi.de/~wolle /TELEKOLLEG /KERN/index-s.html

$^{208}_{82}\mathrm{Pb}_{126}$: a double magic nucleus

$\rightarrow$ spherical $\langle \beta_3 \rangle = 0$

but, the shape can fluctuate

$\rightarrow$ octupole vibration $\langle (\beta_3)^2 \rangle \neq 0$

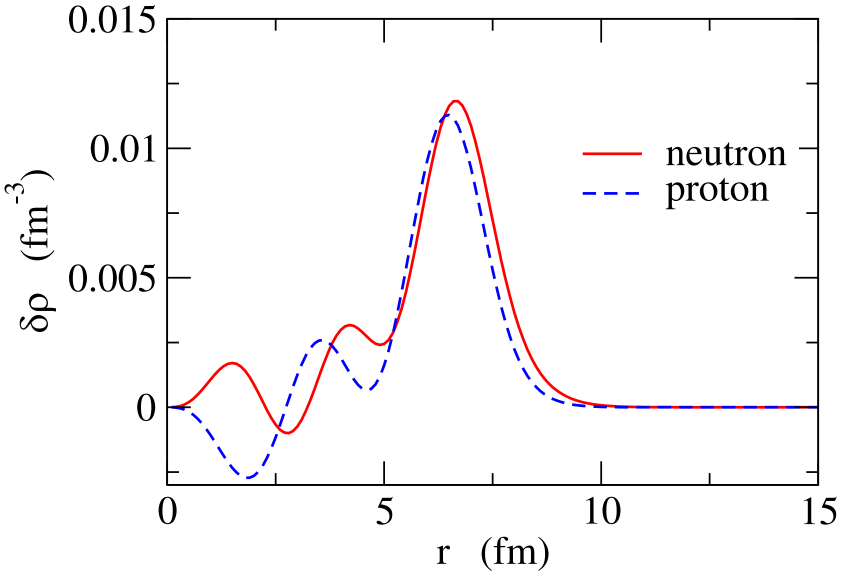
from the measured B(E3)

$$\sqrt{\langle \beta_3^2 \rangle} = \frac{4\pi}{3ZR^3}\sqrt{\frac{B(E3:0^+ \rightarrow 3^-)}{e^2}} \sim 0.144$$

Linear response theory:

$V_{\rm ext}(t)$ on to the spherical g.s.

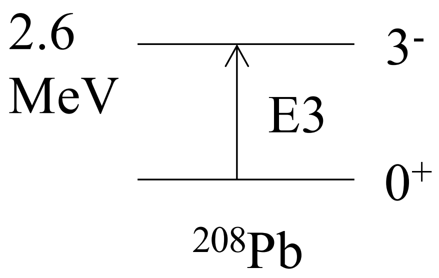
$$\rho_0(r) \rightarrow \rho_0(r) + c(t)\delta\rho(r)Y_{30}(\theta)$$

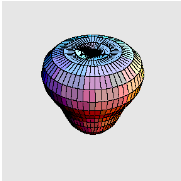


✓ surface peaked ($\rightarrow$ surface vibration)

✓ n and p in phase (isoscalar)

What is the nature of the 3_1^- state in ^{208}Pb?





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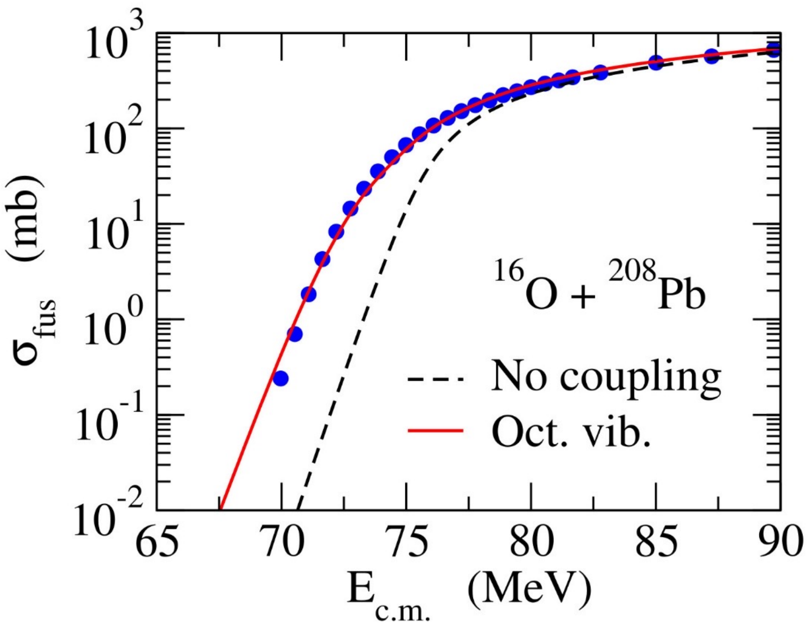
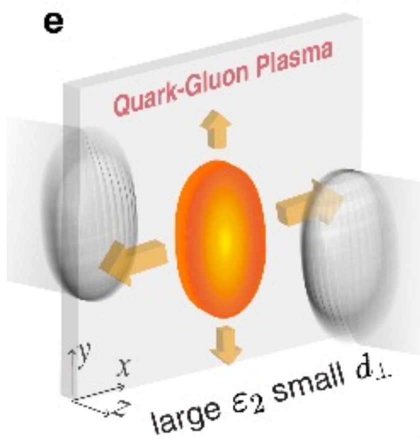
The effect of oct. vib. of ^{208}Pb has been well-known in sub-barrier fusion reactions



J.M. Yao and K. Hagino, PRC94 ('16) 11303(R)

$\rightarrow$ an application of a similar formalism to relativistic heavy-ion collisions!

Probing nuclear shapes in Relativistic H.I. collisions



eccentricity parameter: deformation parameter of $\rho_z(x,y) \equiv \int_{-\infty}^{\infty} dz\, \rho(\boldsymbol{r})$

$$\boxed{\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\int d\boldsymbol{r}\, r_\perp^2\, e^{2i\phi}\rho(\boldsymbol{r},\{\alpha_{2\mu}\})}{\int d\boldsymbol{r}\, r_\perp^2\, \rho(\boldsymbol{r},\{\alpha_{2\mu}\})} = -\frac{\langle (x-iy)^2\rangle}{\langle x^2+y^2\rangle}}$$

deformed Woods-Saxon density

$$\rho(\boldsymbol{r},\{\alpha_{\lambda\mu}\}) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a}}; \qquad R(\theta,\phi) = R_0\left(1+\sum_{\lambda,\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})\right)$$

surface vibration

$$H = \frac{1}{2}\sum_{\lambda,\mu}\left(B_\lambda|\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda|\alpha_{\lambda\mu}|^2\right)$$

$$\langle|\epsilon_n|^2\rangle \propto \int \left(\prod_{\lambda,\mu} d\alpha_{\lambda\mu}\, \boxed{e^{-\alpha^2_{\lambda\mu}/2\sigma^2_\lambda}}\right)|\epsilon_n(\{\alpha_{\lambda\mu}\})|^2$$

static deformation (axial symmetry)

$$\alpha_{\lambda\mu} = \beta_\lambda D^\lambda_{0\mu}(\Omega)$$

$$\langle|\epsilon_n|^2\rangle = \int \frac{d\Omega}{8\pi^2}\,|\epsilon_n(\Omega)|^2$$

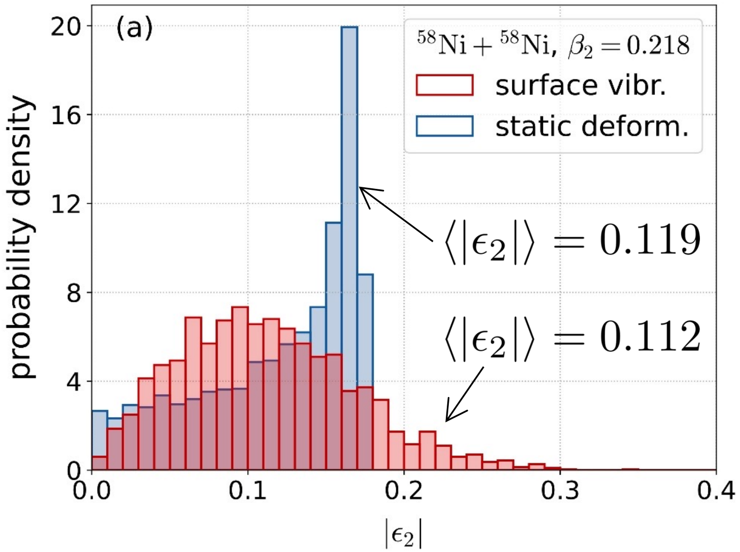
Probing nuclear shapes in Relativistic H.I. collisions

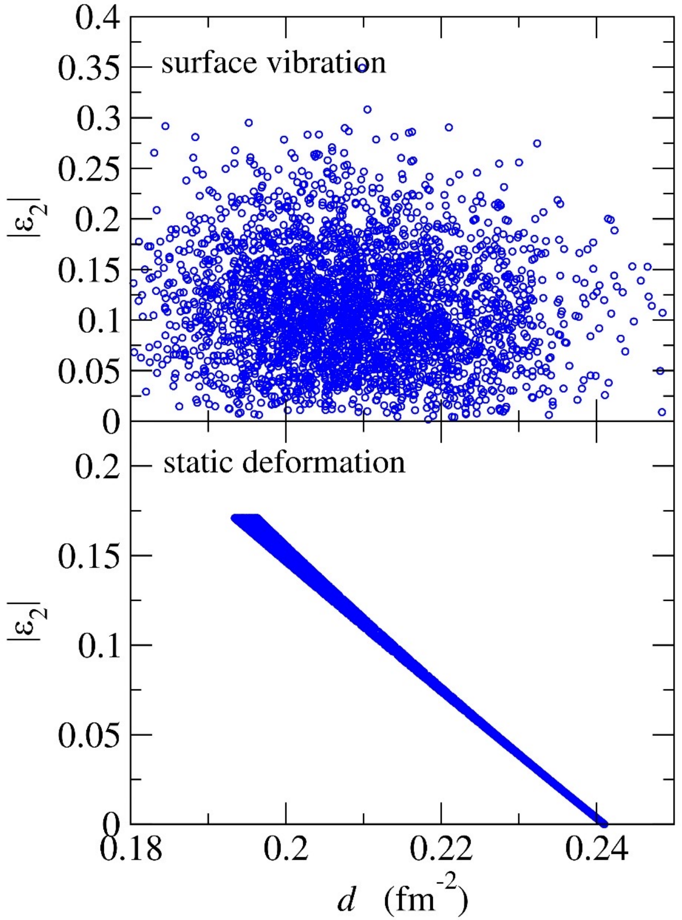
$$\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\langle (x-iy)^2\rangle}{\langle x^2+y^2\rangle}$$

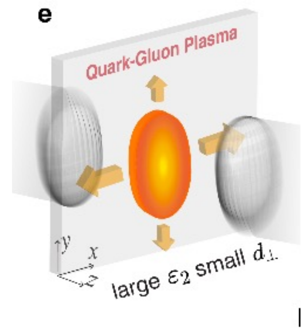
^{58}Ni+^{58}Ni scattering ($\beta_2 \sim 0.218$)

$$\begin{cases} \langle |\epsilon_n|^2\rangle \propto \int \left(\prod_{\lambda,\mu} d\alpha_{\lambda\mu}\, e^{-\alpha_{\lambda\mu}^2/2\sigma_\lambda^2}\right) |\epsilon_n(\{\alpha_{\lambda\mu}\})|^2 & \text{(vib.)} \\ \langle |\epsilon_n|^2\rangle = \int \frac{d\Omega}{8\pi^2}\, |\epsilon_n(\Omega)|^2 & \text{(static def.)} \end{cases}$$

Monte Carlo sampling (3000 samples)







the inverse area

$$d \equiv \frac{1}{\sqrt{\langle x^2\rangle\langle y^2\rangle}} \leftrightarrow \bar{p}_T$$

K. Hagino and M. Kitazawa, arXiv: 2508.05125

Probing nuclear shapes in Relativistic H.I. collisions

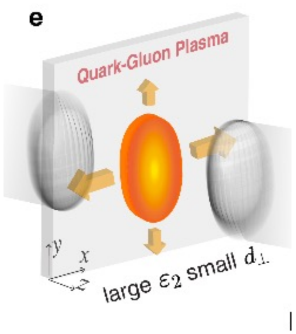
^{58}Ni+^{58}Ni scattering ($\beta_2 \sim 0.218$)

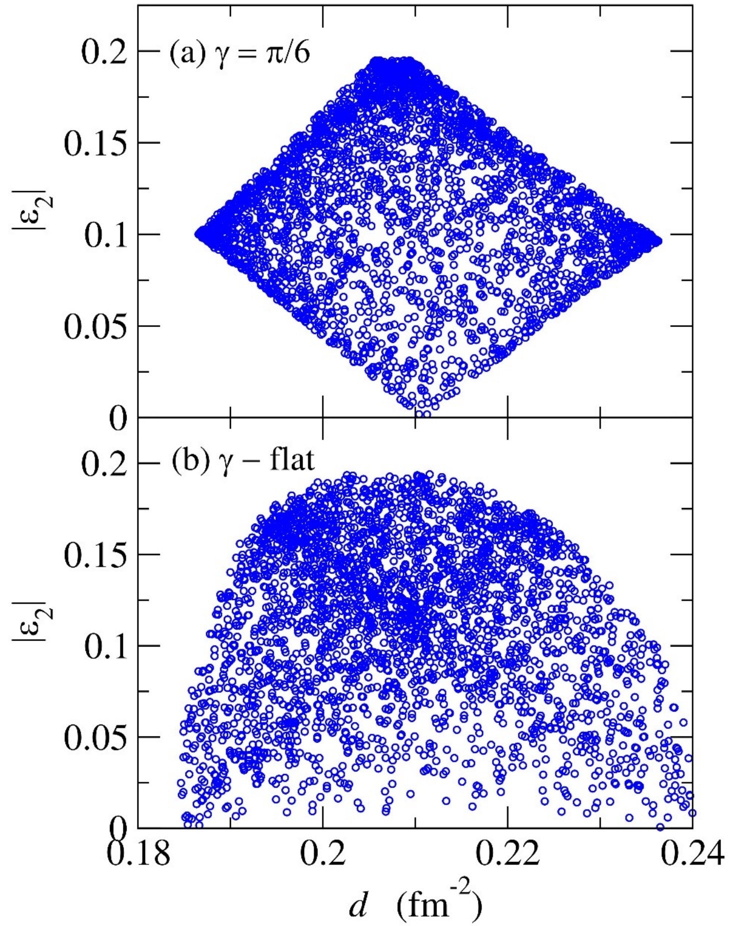
triaxiality

$$\alpha_{2\mu} = D^2_{0\mu}(\Omega)\beta_2 \cos\gamma + \frac{1}{\sqrt{2}}\left(D^2_{2\mu}(\Omega) + D^2_{-2\mu}(\Omega)\right)\beta_2 \sin\gamma$$

$$\langle|\epsilon_n|^2\rangle = \int \frac{d\Omega}{8\pi^2}\,|\epsilon_n(\Omega)|^2$$

$$\epsilon_2(\{\alpha_{2\mu}\}) = -\frac{\langle(x-iy)^2\rangle}{\langle x^2+y^2\rangle}$$

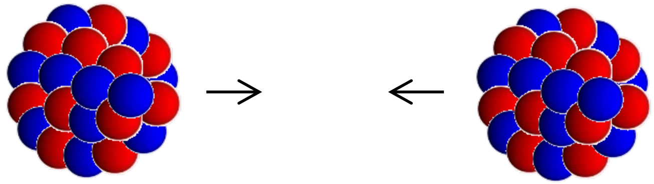




cf. L. Liu, C. Zhang et al., arxiV: 2509.09376.

K. Hagino and M. Kitazawa, arXiv: 2508.05125

Summary



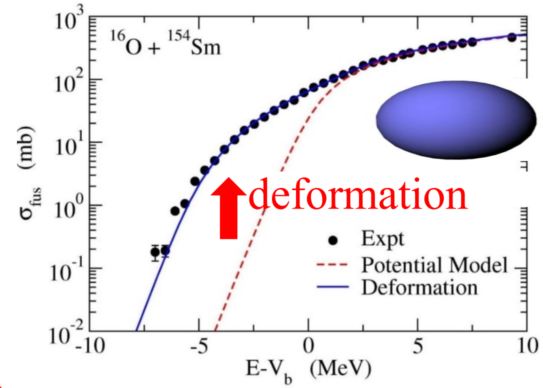
Heavy-ion fusion reactions around the Coulomb barrier

✓Strong interplay between nuclear structure and reaction
✓Quantum tunneling with various intrinsic degrees of freedom
✓Role of deformation in sub-barrier enhancement

→ a snapshot of the rotational motion

↓

amplified



✓ Similarities between low-*E* H.I. fusion and Relativistic H.I. Collisions

→ a snapshot of a nucleus

A tool to probe nuclear deformations

→ surface vibrations of a spherical nucleus