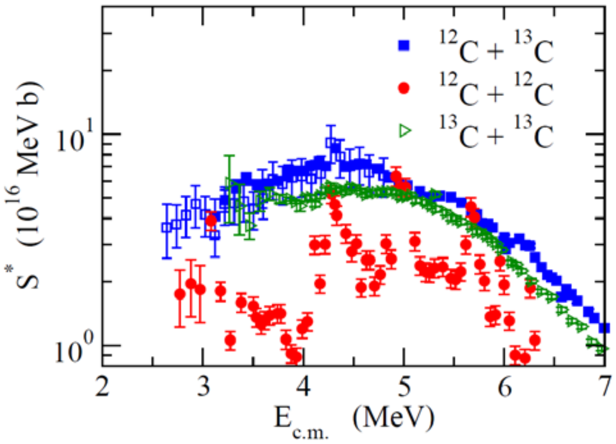
Subbarrier fusion of carbon isotopes
~ from resonance structure to fusion oscillations ~



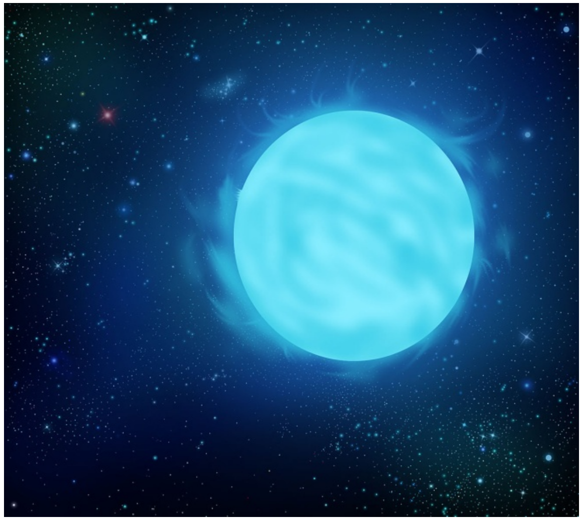
Kouichi Hagino, *Tohoku University*
Neil Rowley, *IPN Orsay*

1. *Introduction: $^{12}C + ^{12}C$ fusion*
2. *Molecular resonances at subbarrier energies*
3. *Fusion oscillations at above barrier energies*
4. *Summary*

Introduction: $^{12}C + ^{12}C$ fusion

$^{12}C+^{12}C$ fusion : a key reaction in nuclear astrophysics

Carbon burning
in massive stars



$^{12}C+^{12}C \rightarrow \alpha + ^{20}Ne$

$^{12}C+^{12}C \rightarrow p + ^{23}Na$

stellar evolution

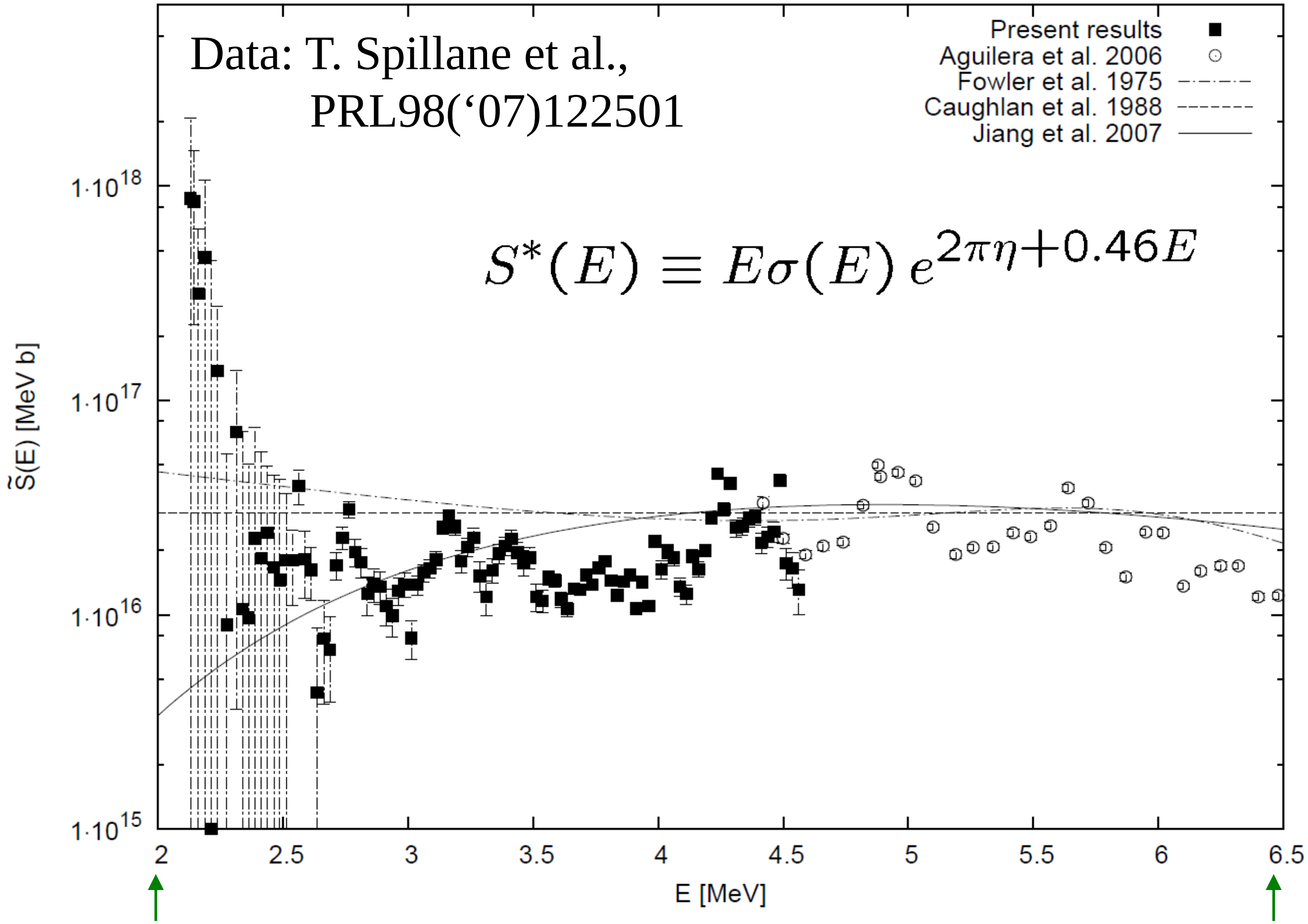
Type Ia supernovae

X-ray superburst

deep layer of the outer crust in accreting neutron stars

important to understand $^{12}C+^{12}C$ fusion at deep subbarrier energies

Experimental data at low energies



Data: T. Spillane et al.,
PRL98('07)122501

$$S^*(E) \equiv E\sigma(E)\, e^{2\pi\eta+0.46E}$$

T. Spillane et al.,
PoS (NIC X) 016

$E_G \sim$ 2 MeV (T=0.85 GK)

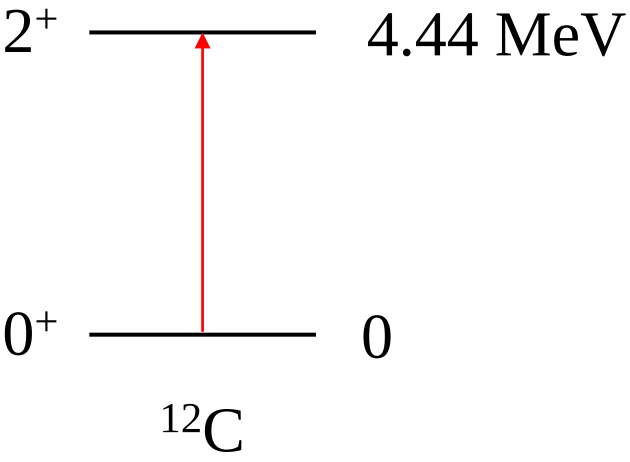
$\sim V_b$

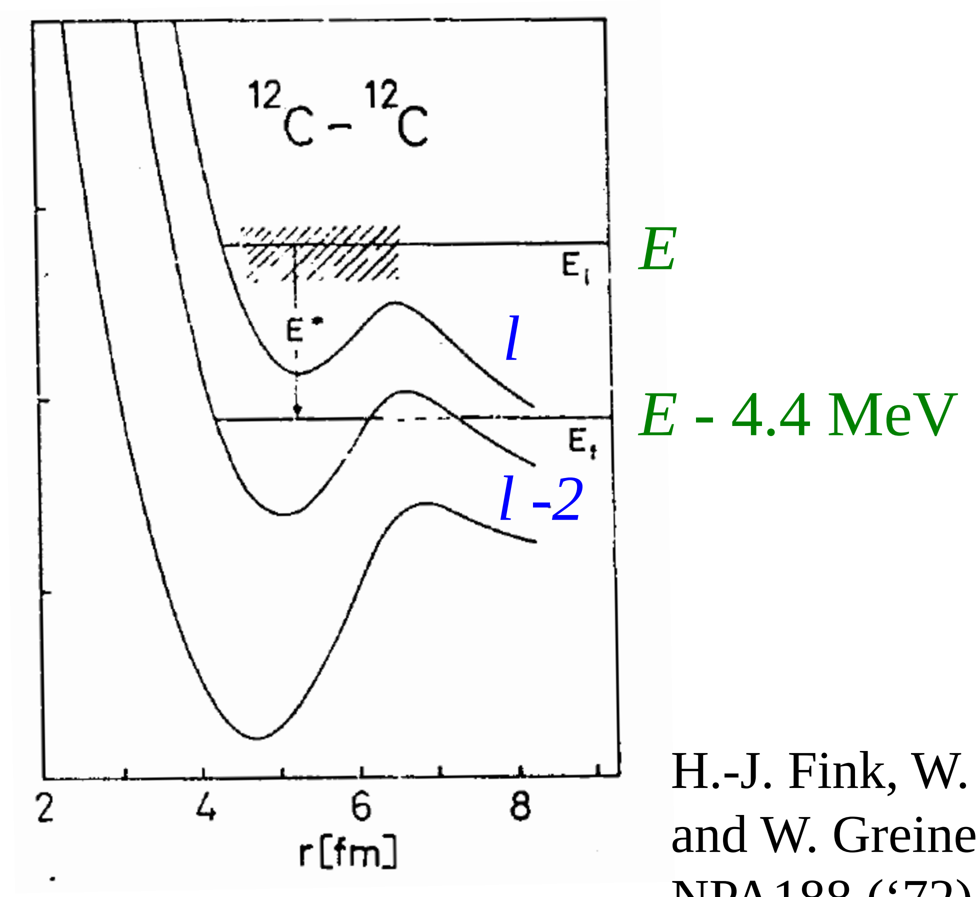
- ✓ pronounced resonance structures (narrow resonances)
  "molecular resonances"
- ✓ difficult to extrapolate down to E_G

Theoretical calculations

- Nogami-Imanishi model (B. Imanishi, PL 27B (‘68) 267, NPA125 (‘69) 33)
- Band-crossing model (Y. Kondo, T. Matsuse, Y. Abe, PTP59 (‘78)465)
- Double resonance model (W. Scheid, W. Greiner, R. Lemmer, PRL25 (‘70) 176)

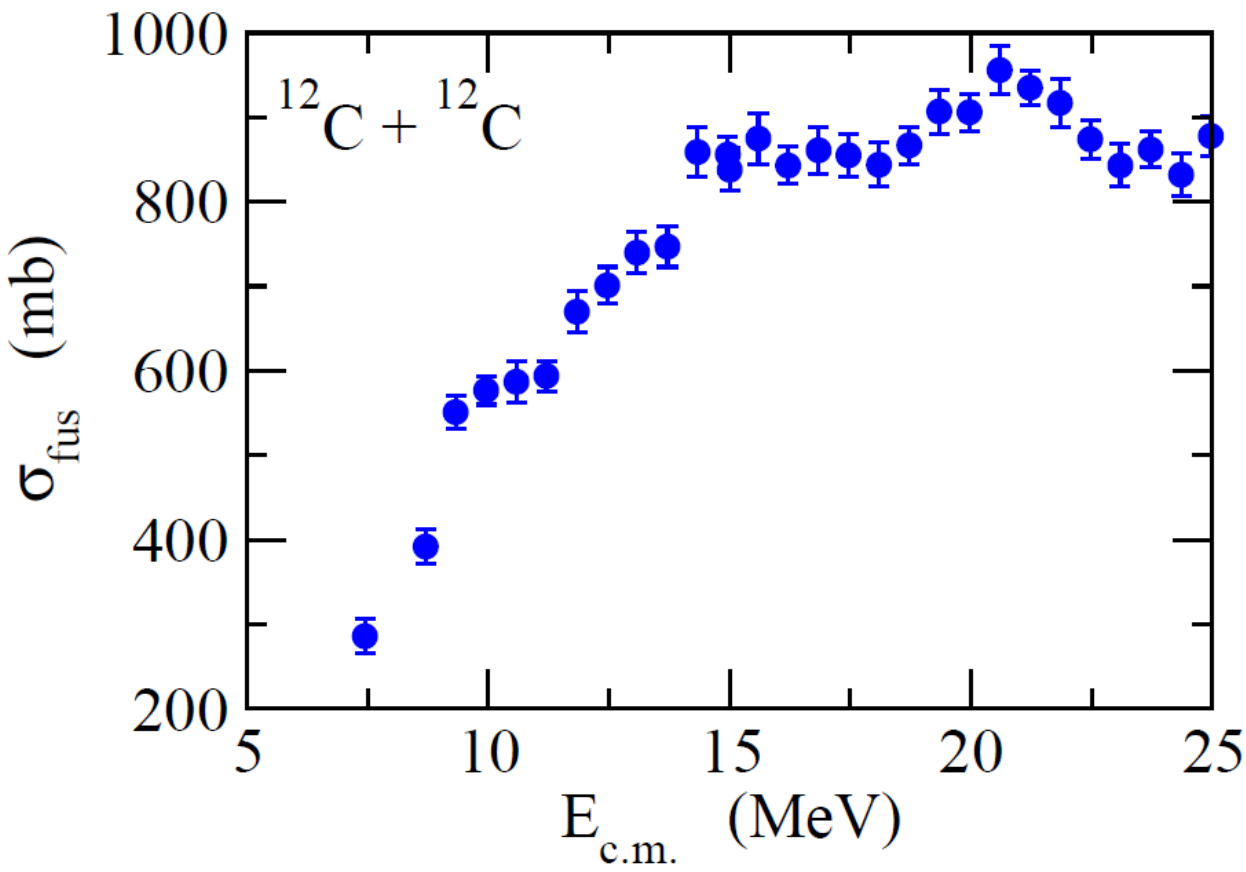
* the basic concept is all same





H.-J. Fink, W. Scheid, and W. Greiner, NPA188 (‘72) 259

Experimental data at above barrier energies



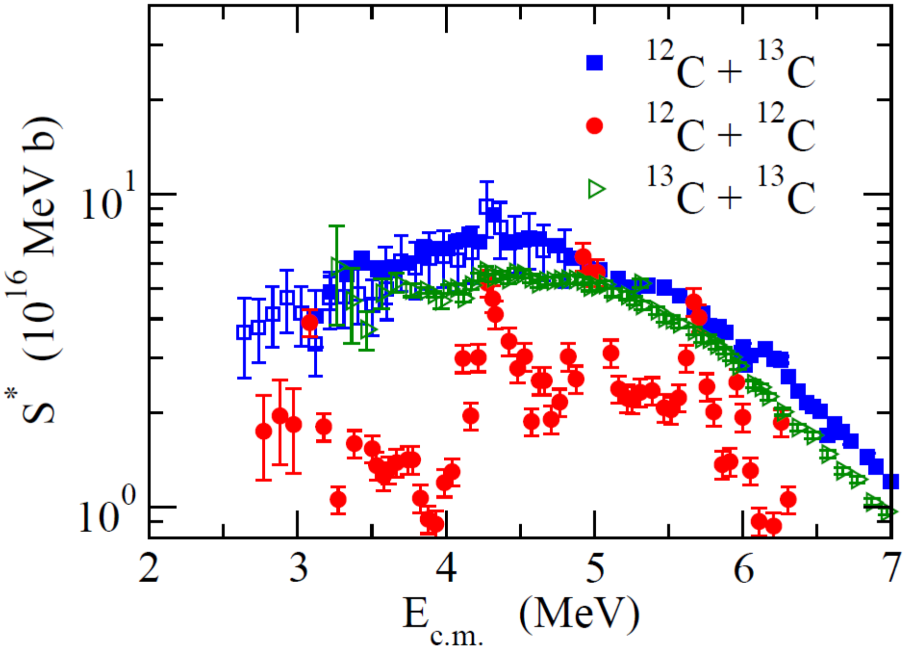
Data: D.G. Kovar et al., PRC20 (‘79) 1305

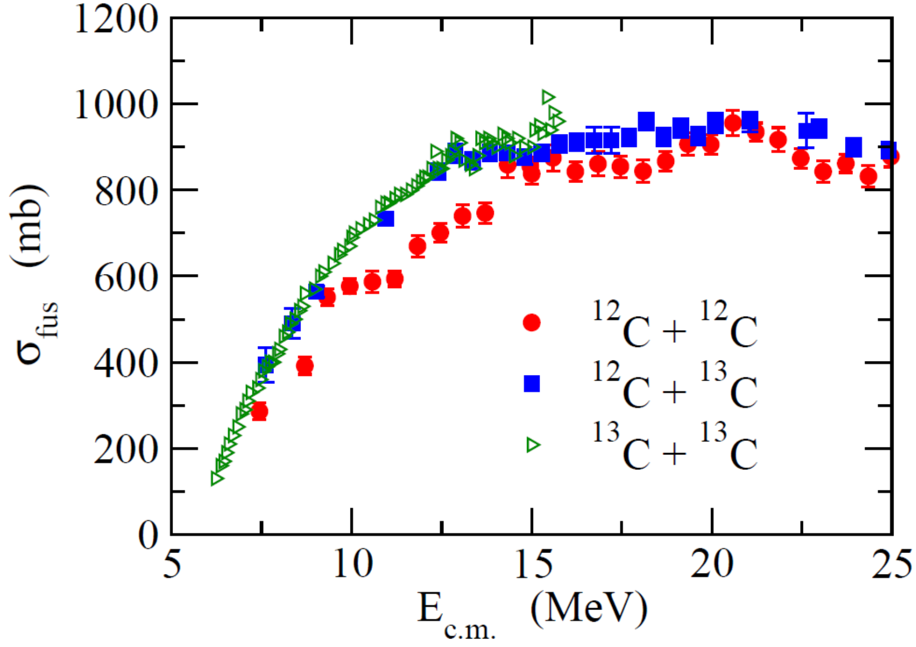
✓ fusion oscillations

← successive contributions of individual partial waves

(N. Poffe, N. Rowley, and R. Lindsay, NPA410 (‘83) 498)

Comparison with other C+C systems





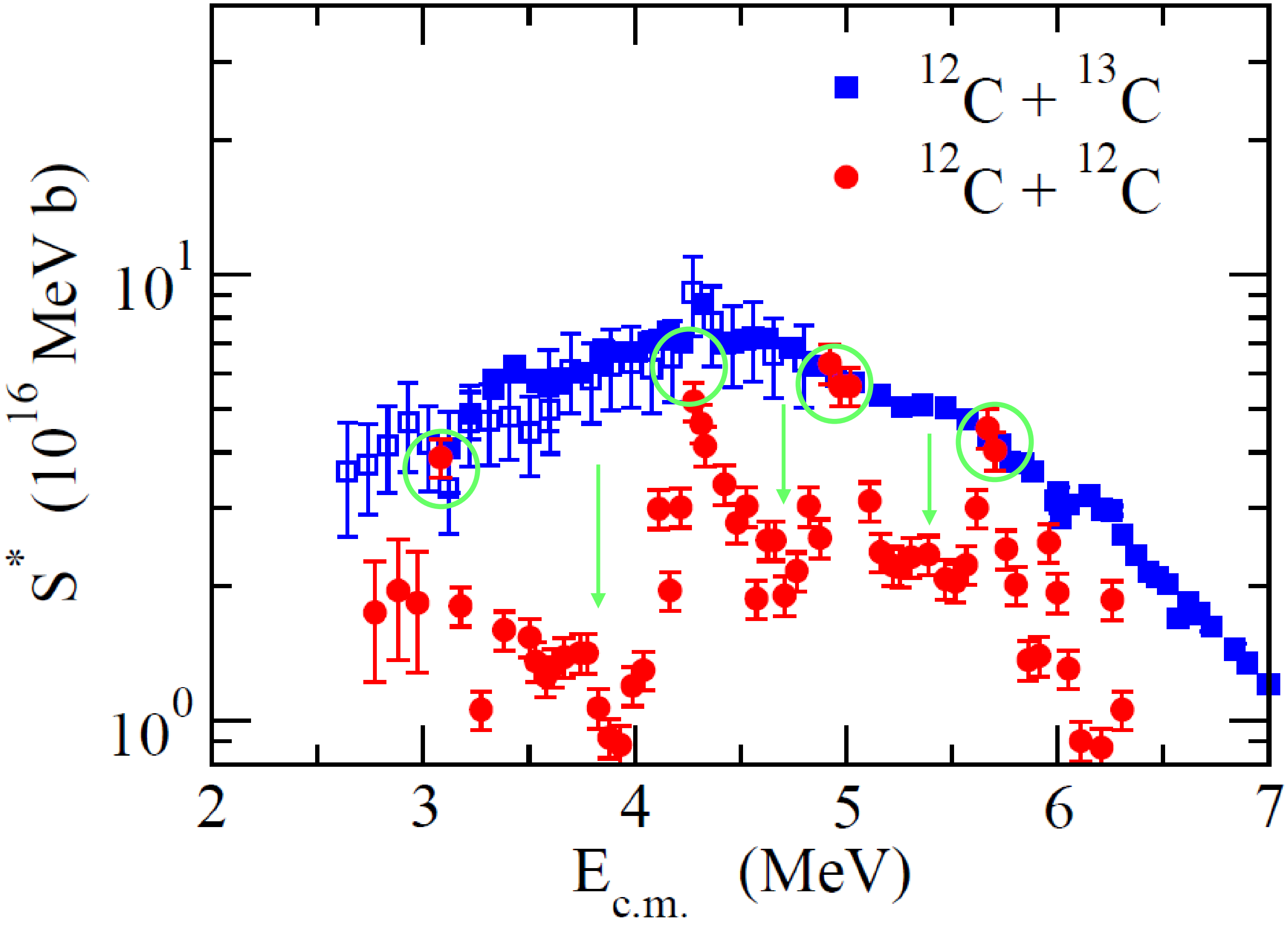
fusion cross sections for ^{12}C+^{13}C, ^{13}C+^{13}C: much less structured

How can one understand the systematics?

- from ^{12}C+^{12}C to ^{12}C+^{13}C, ^{13}C+^{13}C
  origins for the resonances/oscillations?
- from low to high energies

cf. most of the previous studies: ^{12}C+^{12}C only

Molecular resonances at subbarrier energies



M. Notani, X.D. Tang et al., PRC85('12)014607

off-resonance: fusion inhibition
on-resonance: match with $^{12}C+^{13}C$

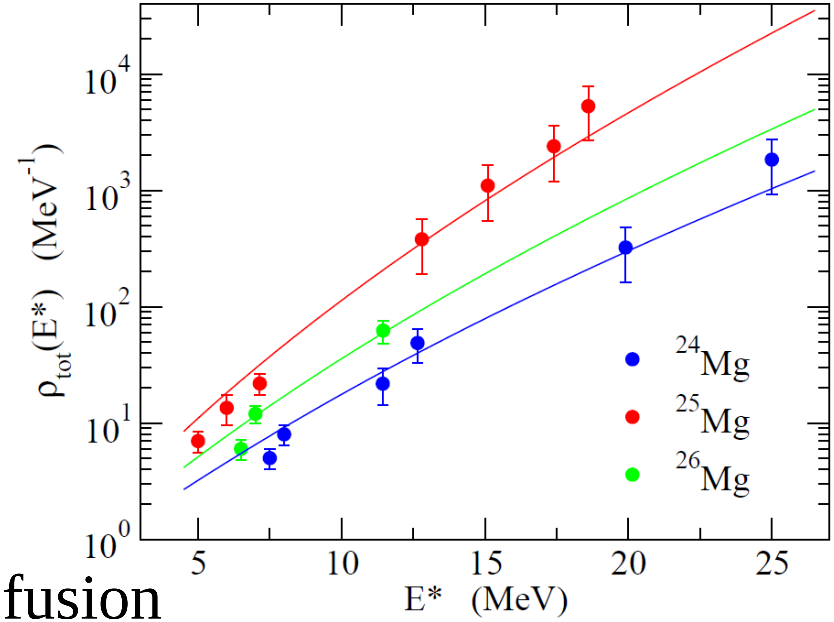
Jiang's conjecture: C.L. Jiang et al., PRL110('13)072701

properties of compound nucleus (^{24}Mg)?

^{12}C+^{12}C reaction:

- ✓ level density of ^{24}Mg : small (e-e)
- ✓ small fusion Q-value

Q = +13.9 MeV (^{12}C+^{12}C)
+16.3 MeV (^{12}C+^{13}C)
+22.5 MeV (^{13}C+^{13}C)



→ small E^* for ^{24}Mg in ^{12}C+^{12}C fusion

$$\sigma \sim \sum_J \sigma^J_{\rm cap} \underbrace{\left[1 - e^{-2\pi\Gamma_J/D_J}\right]}_{\text{large hindrance factor}}$$

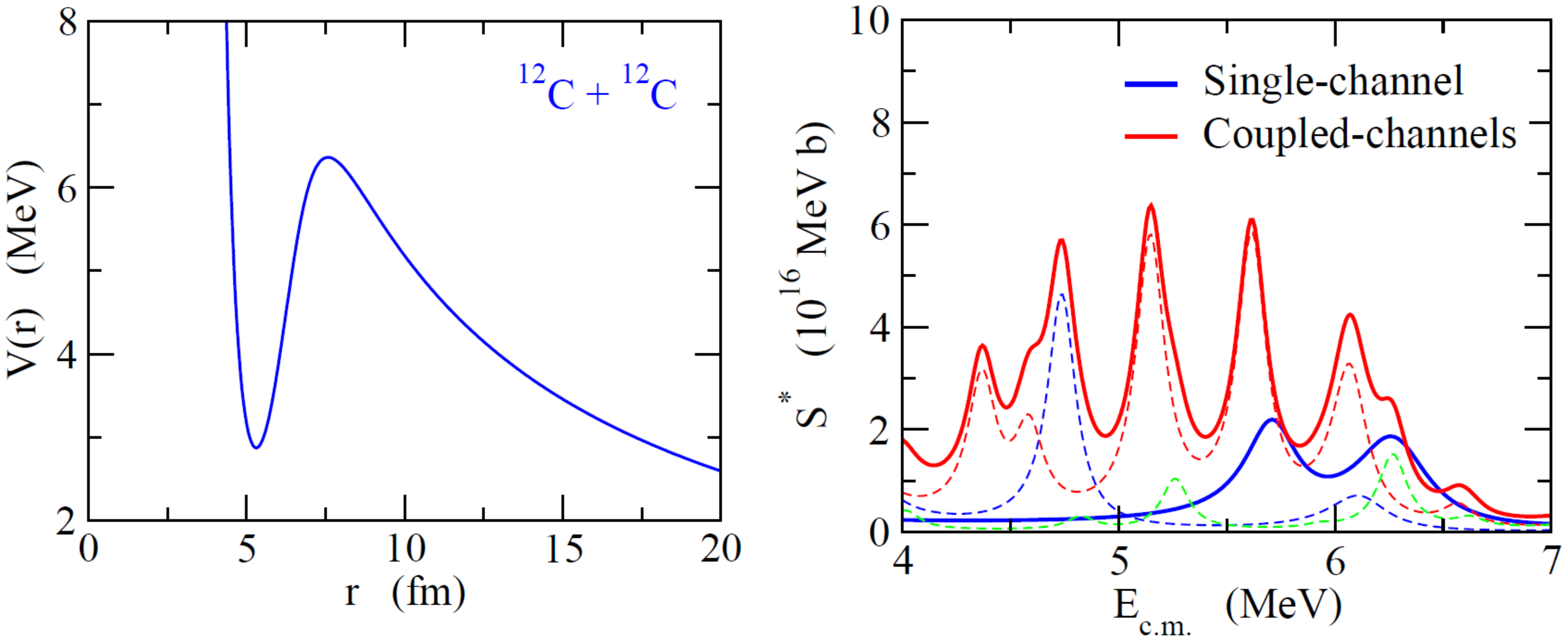
$D_J = 1/\rho_J$
Γ_J : CN width

incorporate this idea in the coupled-channels calculations?

C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP('78))

- ✓ two-range Woods-Saxon + Coulomb for the real part
- ✓ a Woods-Saxon for the imaginary part



C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP(‘78))

✓ two-range Woods-Saxon + Coulomb for the real part
✓ a Woods-Saxon for the imaginary part

$$\boxed{W(r) = -W_0 \cdot f_{\mathrm{WS}}(r) \to -w_0\, \rho_J(E^*) \cdot f_{\mathrm{WS}}(r)}$$

G. Helling, W. Scheid, W. Greiner, PL 36B (‘71) 64
H.-J. Fink, W. Scheid, W. Greiner, NPA188 (‘72) 259
J.M. Quesada, M. Lozano, G. Madurga, PLB125 (‘83) 14
M.V. Andres, Quesada, Lozano, Madurga, NPA443 (‘85) 380

✓ *E* and *J* dependent imaginary potential
✓ system dependence through ρ(*E*)

cf. Fermi’s golden rule

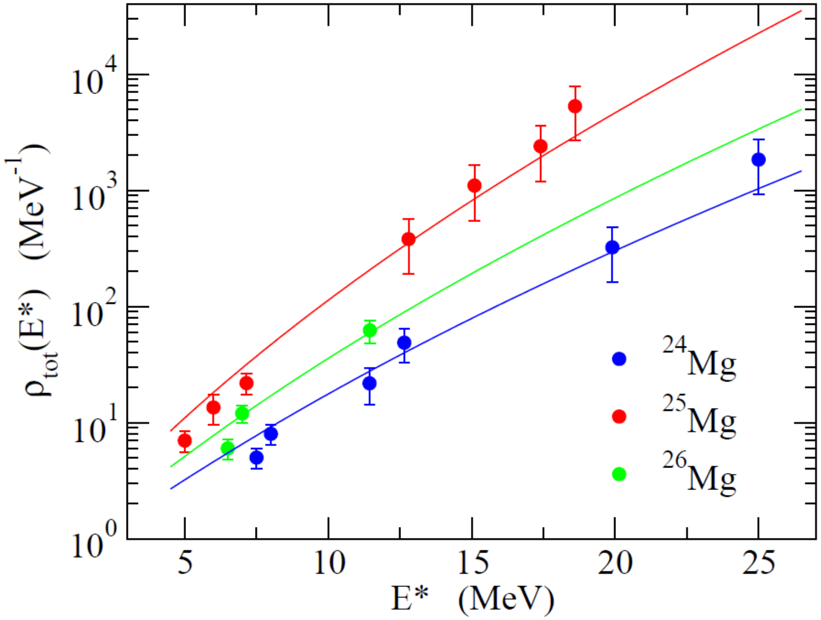
$$\frac{dw}{dt} = \frac{2\pi}{\hbar}\, |\langle \psi_{\mathrm{CN}} | V_{\mathrm{int}} | \psi_{\mathrm{elastic}} \rangle|^2 \boxed{\rho_J(E^*)}$$

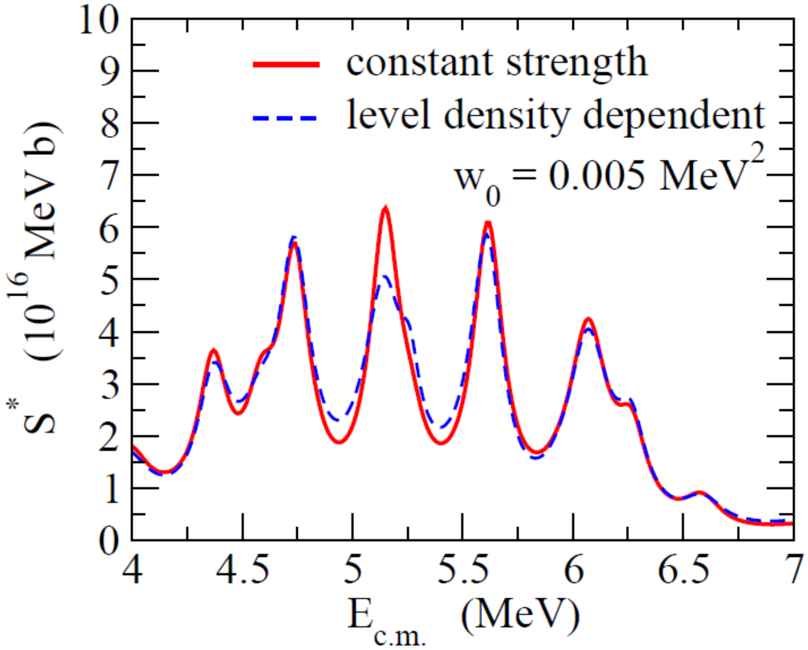
C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP('78))

- ✓ two-range Woods-Saxon + Coulomb for the real part
- ✓ a Woods-Saxon for the imaginary part

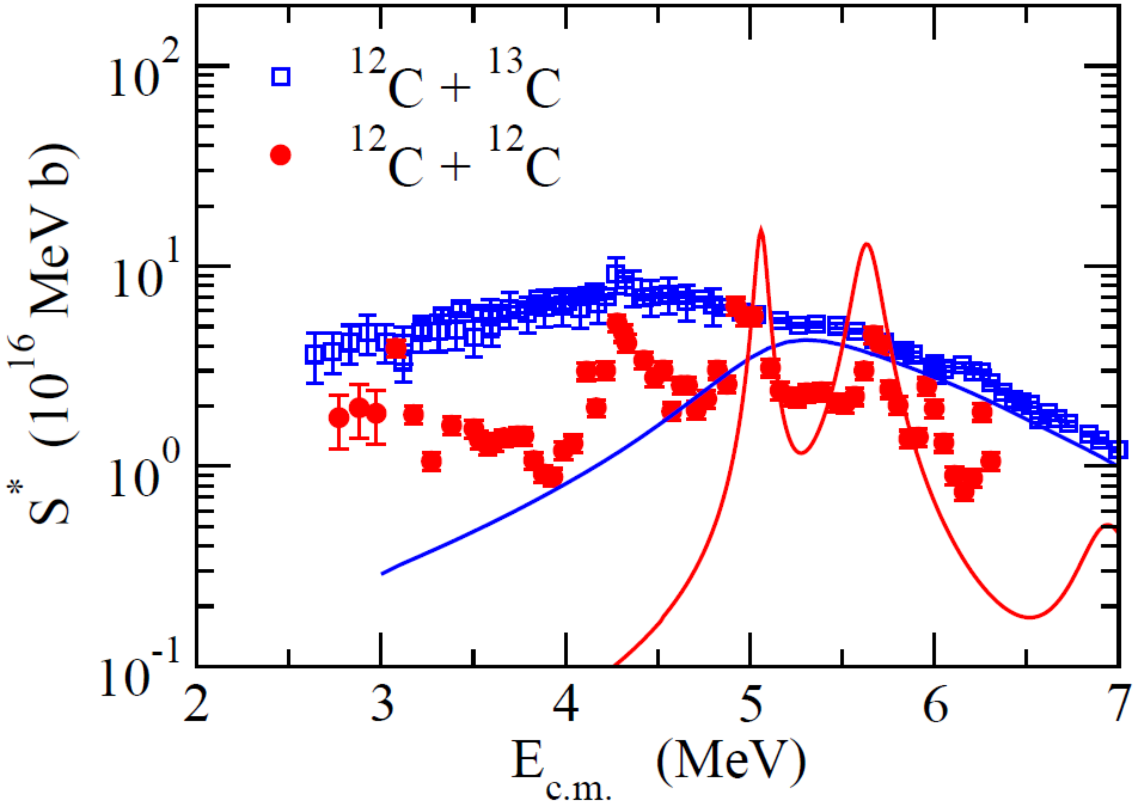
$$\boxed{W(r) = -W_0 \cdot f_{\mathrm{WS}}(r) \to -w_0\,\rho_J(E^*)\cdot f_{\mathrm{WS}}(r)}$$





$$\rho_J(E^*) = \frac{(2J+1)e^{-(J+1/2)^2/2\sigma^2}}{4\sigma^3\sqrt{2\pi}}\,\frac{\sqrt{\pi}}{12}\,\frac{e^{2\sqrt{aE^*}}}{a^{1/4}(E^*)^{5/4}} \qquad \left(\sigma^2 = 0.088\,aA^{2/3}\sqrt{\frac{E^*}{a}}\right)$$

Results of coupled-channels calculations



^{12}C (0^+, 2^+: 4.44)
^{13}C ($1/2^-$, $3/2^-$:3.68)
+ mutual excitations

✓ structured ^{12}C+^{12}C
✓ smooth ^{12}C+^{13}C

system dependence:
qualitatively reproduced

underestimate of fusion cross sections at deep subbarrier energies:
⟶ couplings to 3^- and 0_2^+ (Hoyle state)
a/o transfer channel $^{12}C(^{12}C,^{8}Be)^{16}O$?

cf. role of Hoyle state in ^{12}C+^{12}C:
M. Assuncao and P. Descouvemont, PLB723 (‘13) 355

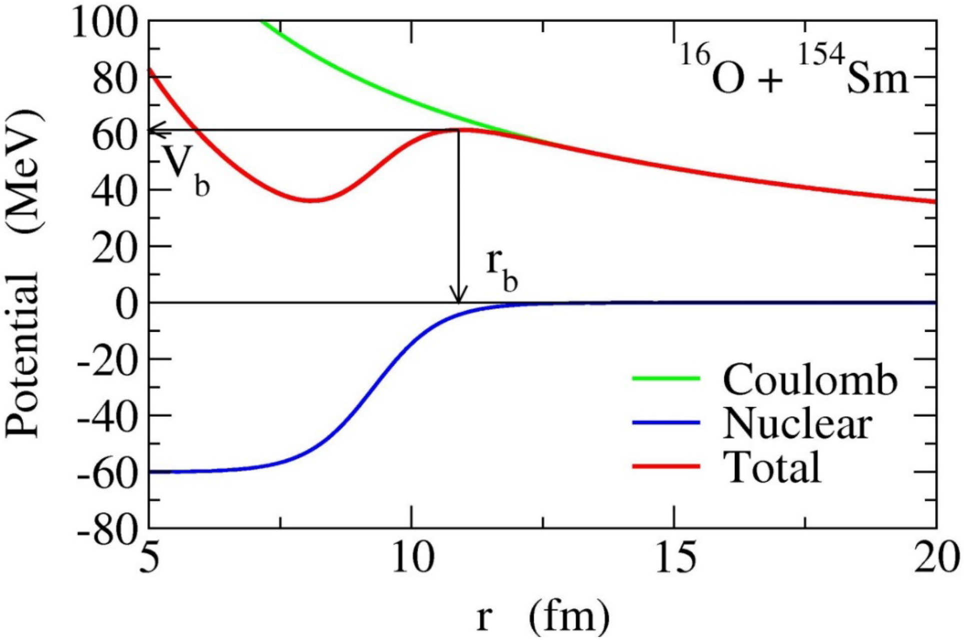
Fusion oscillations at above barrier energies

high-*E* : high level density of CN ⟶ overlapping resonances

⟶ strong absorption

$$\boxed{\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)}$$

$P_l(E)$: barrier penetrability



Wong's formula

C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

i) Approximate the Coul. barrier by a parabola: $V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$

$$\longrightarrow \qquad P_0(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$

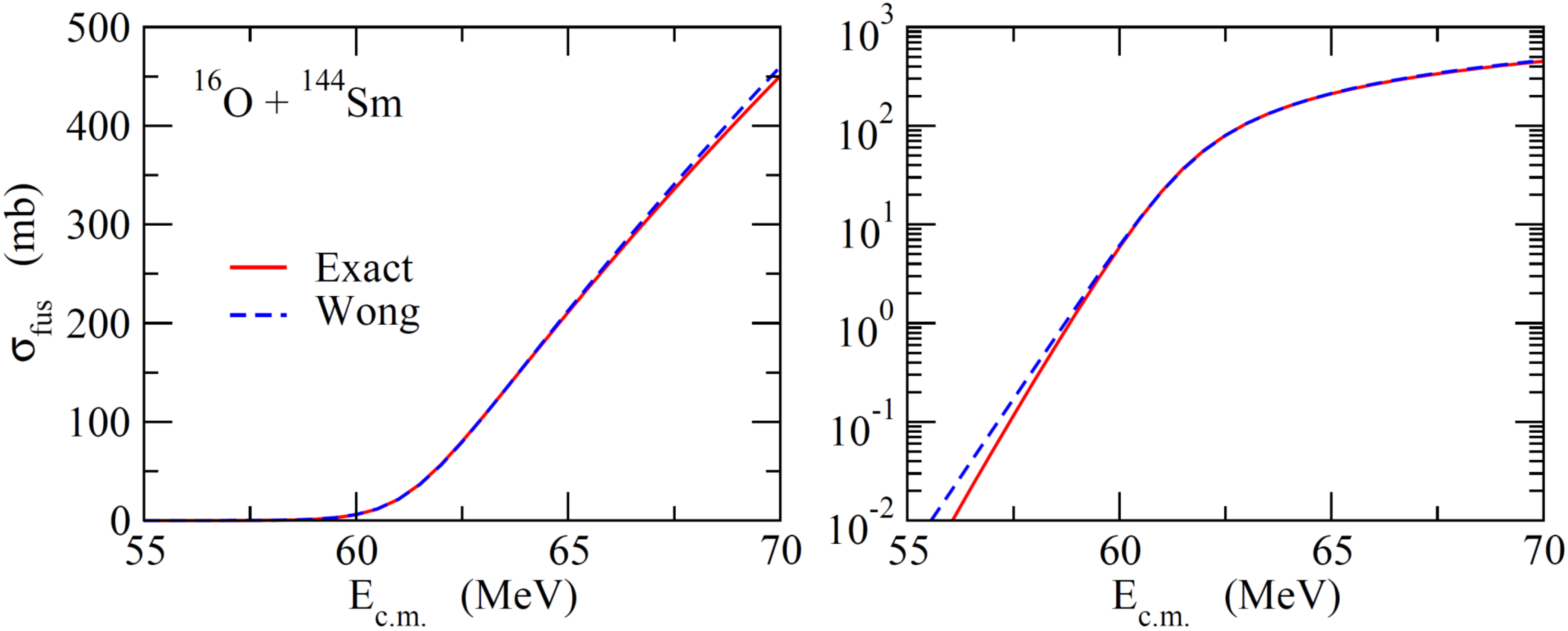
ii) *l*-independent barrier position and curvature:

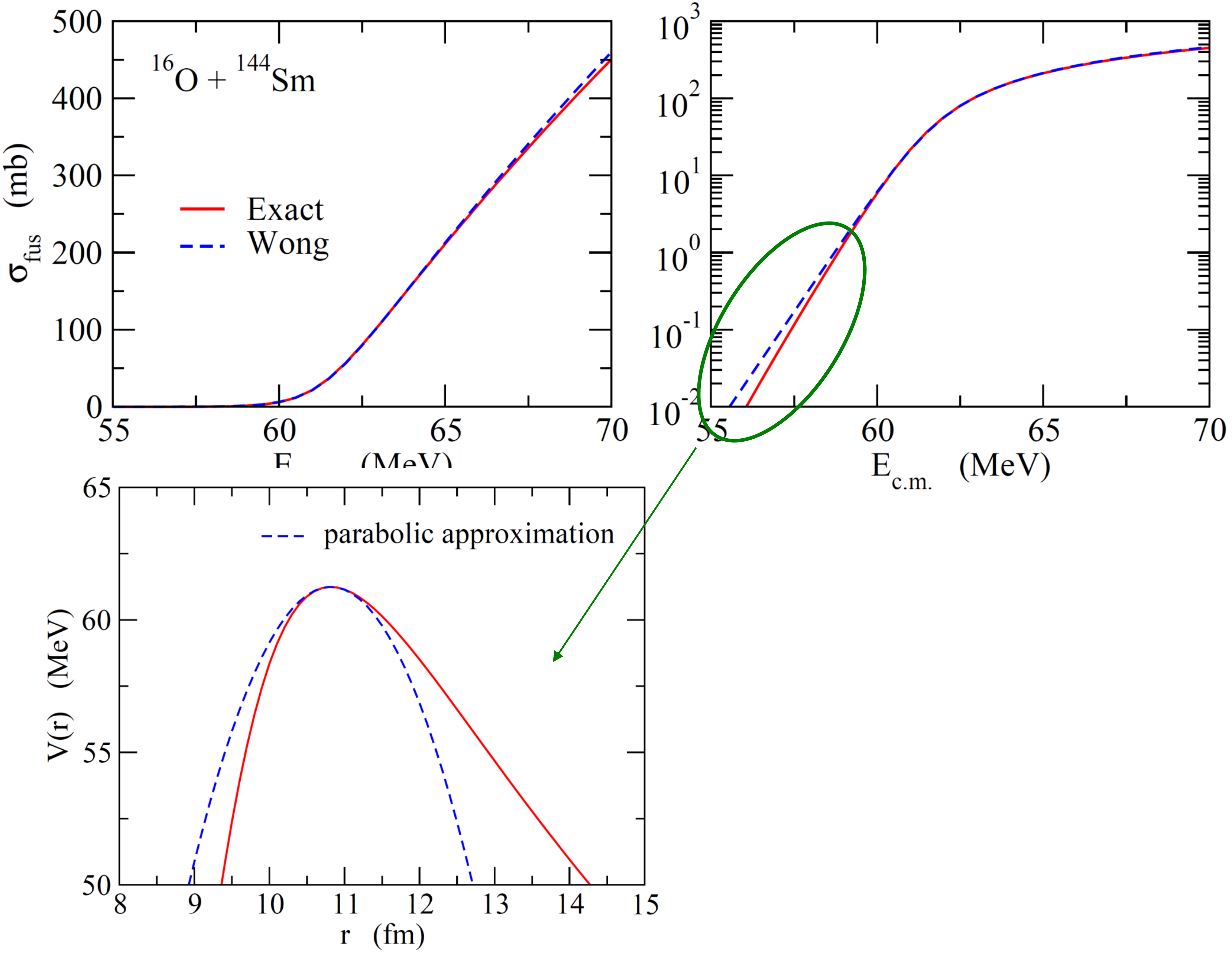
$$\longrightarrow \qquad P_l(E) \sim P_0\left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}\right)$$

iii) Replace the sum of *l* with an integral

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \rightarrow \frac{\pi}{k^2}\int dl\,(2l+1)P(l,E)$$

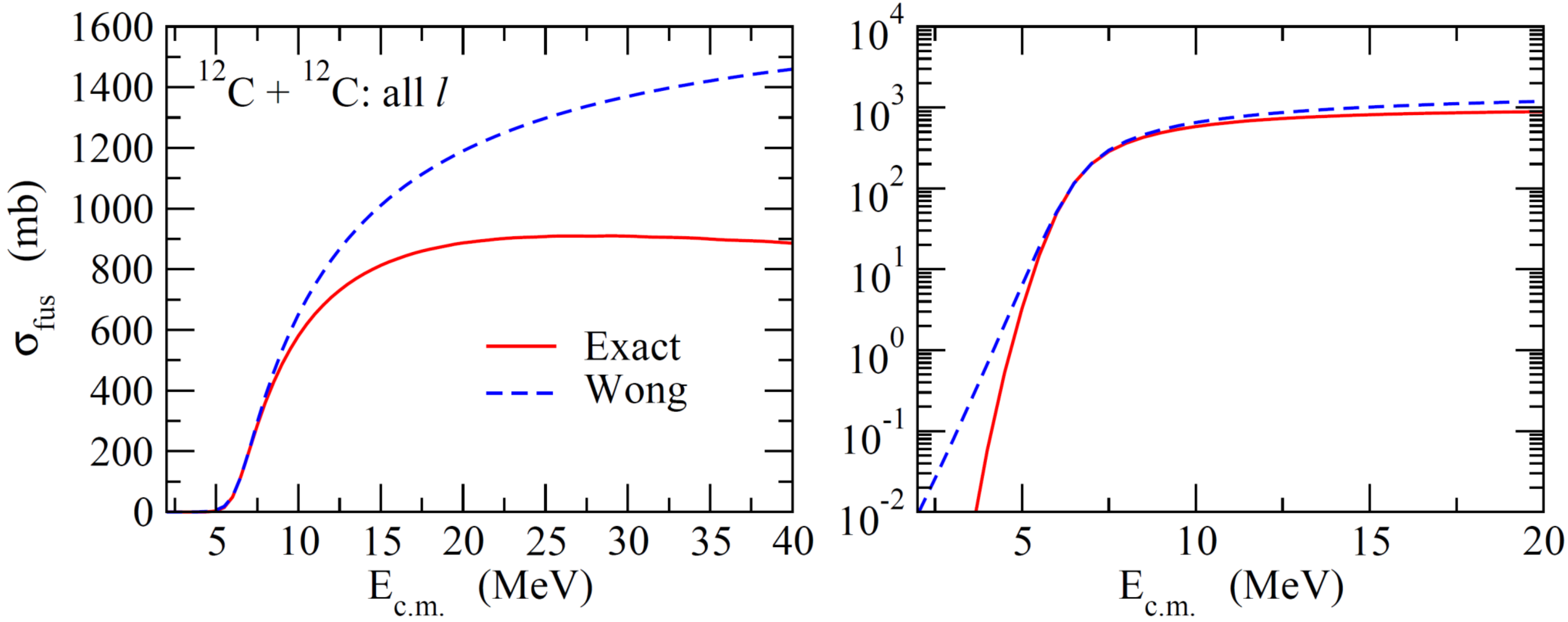
$$\boxed{\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E}R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]}$$


^{16}O + ^{144}Sm
Exact
Wong
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
0
100
200
300
400
500
55
60
65
70
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}



^{16}O + ^{144}Sm
σ_{fus} (mb)
500
400
300
200
100
0
55
60
65
70
Exact
Wong
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
55
60
65
70
$E_{c.m.}$ (MeV)
parabolic approximation
V(r) (MeV)
65
60
55
50
8
9
10
11
12
13
14
15
r (fm)

Wong formula for light heavy-ion fusion



Wong formula:

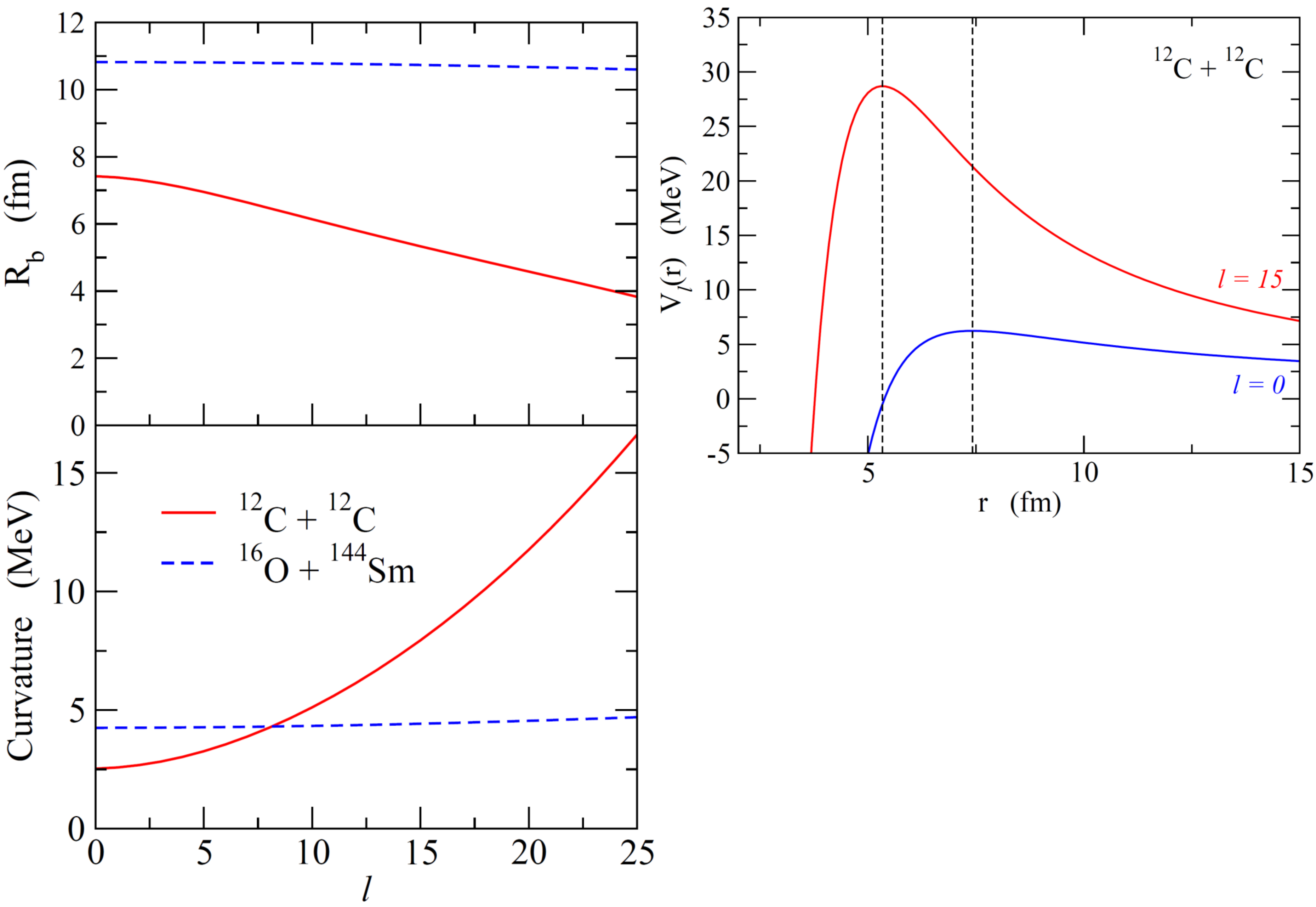
i) Approximate the Coul. barrier by a parabola

ii) *l*-independent barrier position and curvature ←

iii) Replace the sum of *l* with an integral

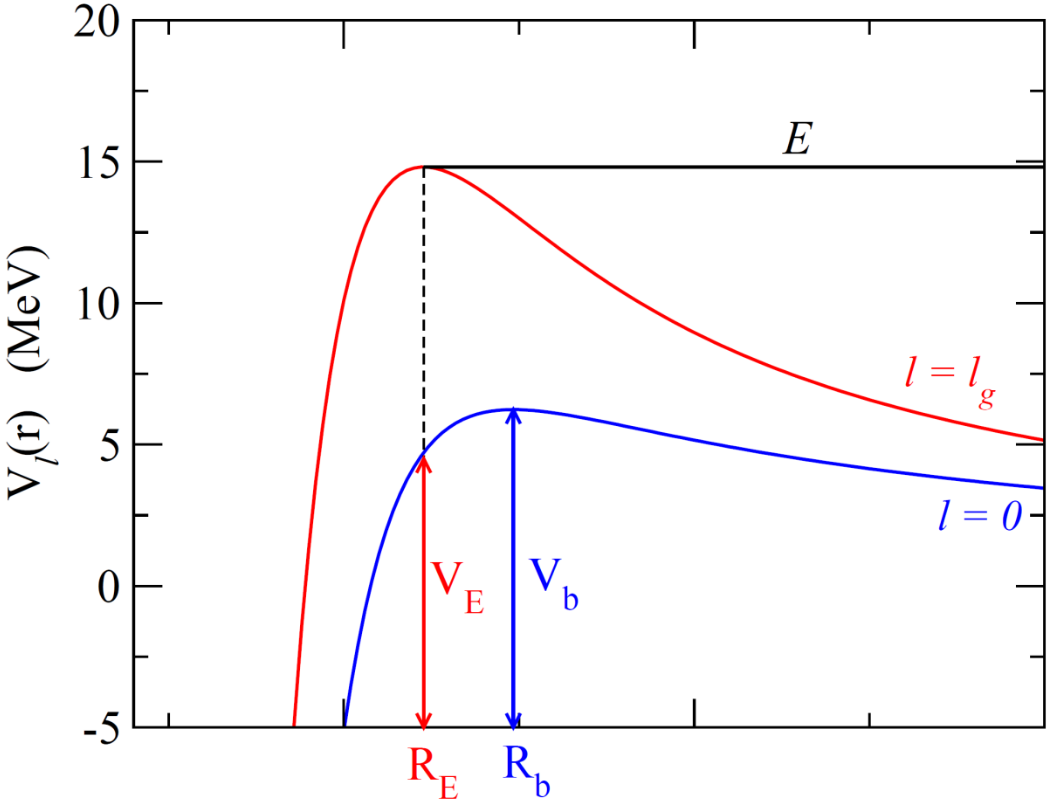
$$V_{\mathrm{cent}}(r) = \frac{l(l+1)\hbar^2}{2\mu r^2}$$

small


R_b (fm)
0
2
4
6
8
10
12
Curvature (MeV)
0
5
10
15
$^{12}C + ^{12}C$
$^{16}O + ^{144}Sm$
0
5
10
15
20
25
l
$V_l(r)$ (MeV)
-5
0
5
10
15
20
25
30
35
$^{12}C + ^{12}C$
$l = 15$
$l = 0$
5
10
15
r (fm)

E-dependent Wong formula

N. Rowley, A. Kabir, and R. Lindsay, J. of Phys. G15(‘89)L269
N. Rowley and K. Hagino, in preparation



use V_b, R_b, and Ω
for the grazing angular
momentum, l_g

(note)

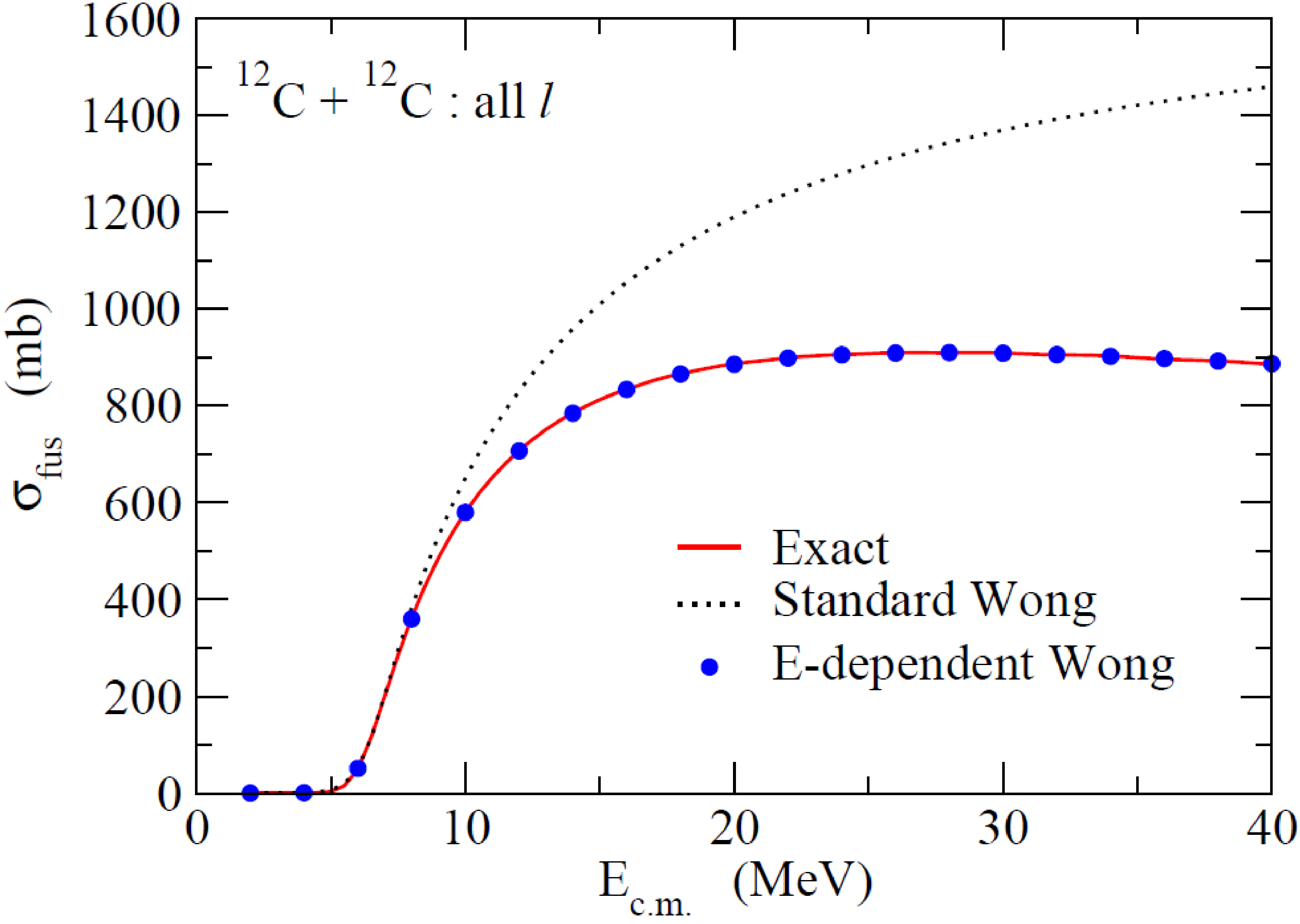
$$\left\{\begin{array}{lcl}\sigma_{\rm cl} & = & \pi b_g^2 \\ E & = & V_E + \dfrac{(kb_g)^2\hbar^2}{2\mu R_E^2}\end{array}\right.$$

$$\longrightarrow \sigma_{\rm cl} = \pi R_E^2(1 - V_E/E)$$

$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

$$\Longrightarrow \boxed{\sigma_{\rm fus}(E) = \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right]}$$

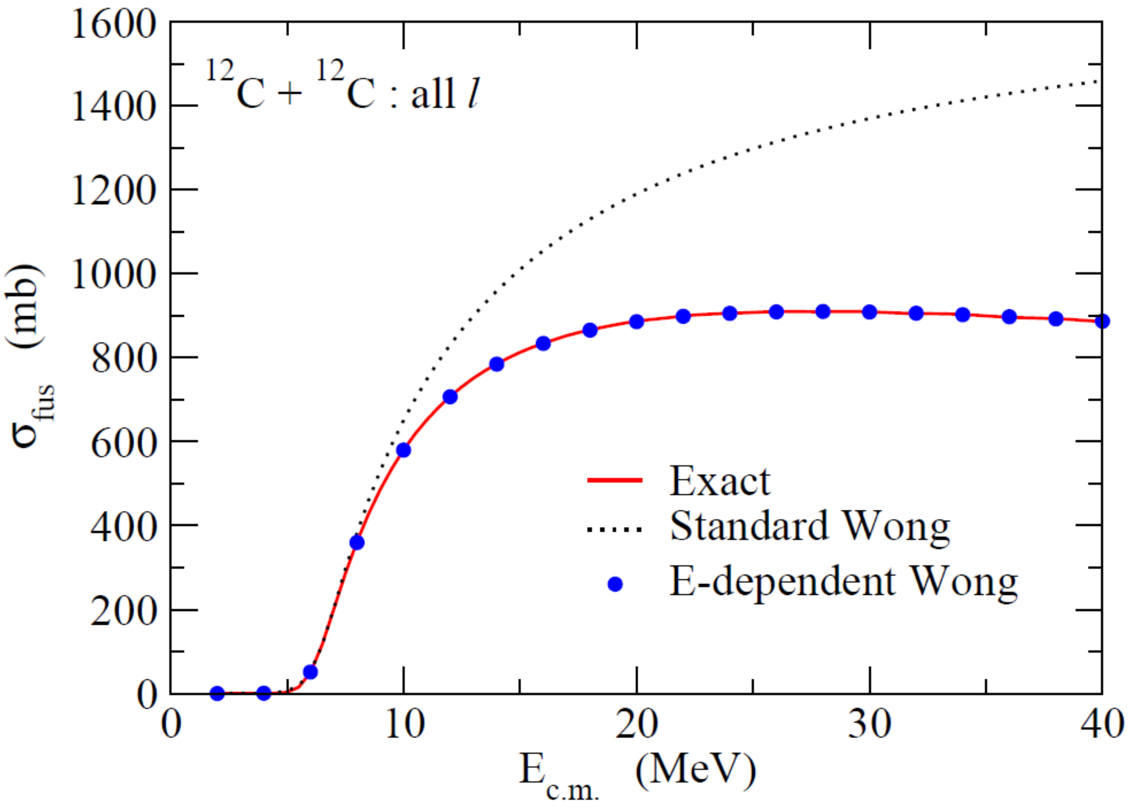
$$\sigma_{\mathrm{fus}}(E) = \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right]$$



Continuum approximation

Wong formula:

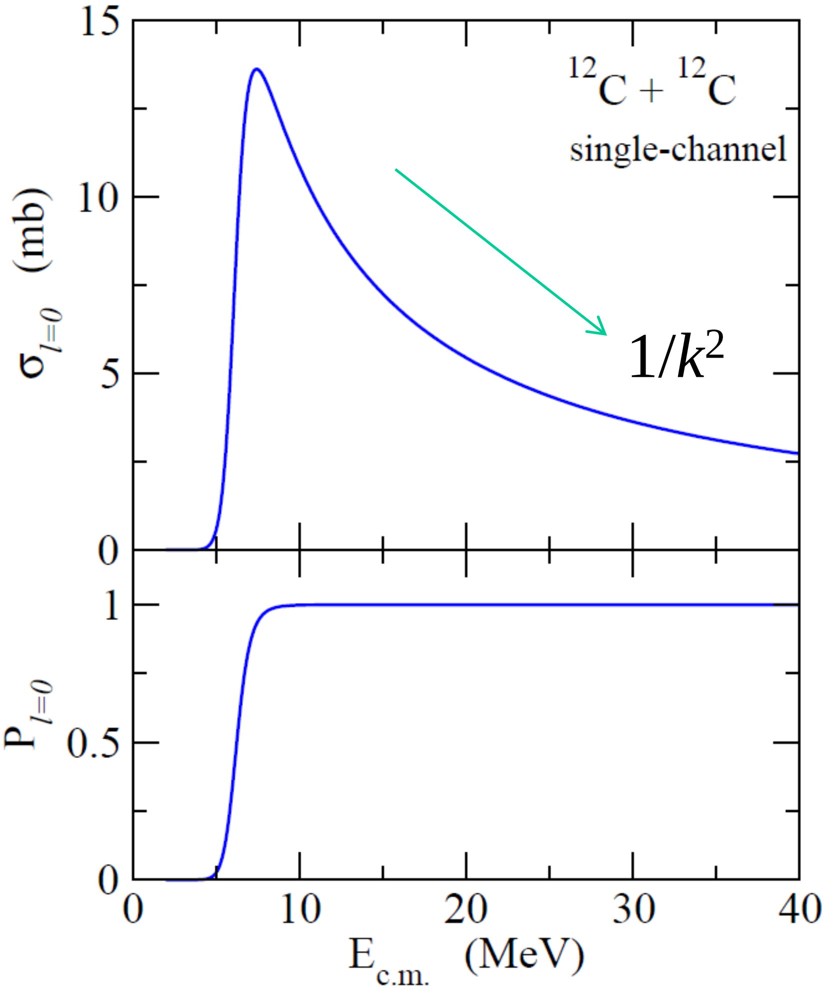
$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \to \frac{\pi}{k^2}\int dl\,(2l+1)P(l,E)$$

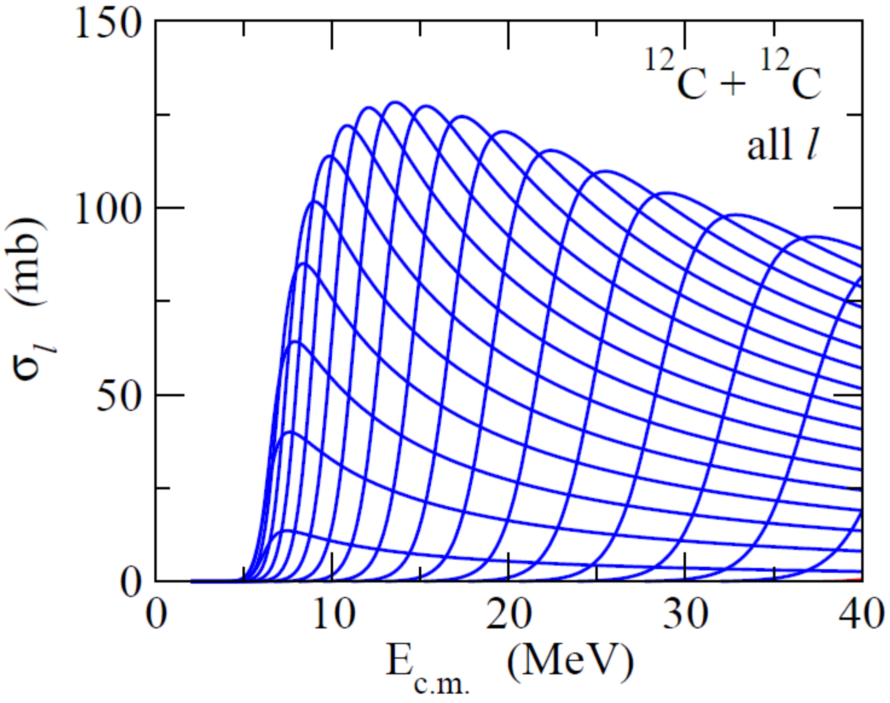


the continuum approximation: appears very good but…..

Fusion oscillations at above barrier energies

$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$





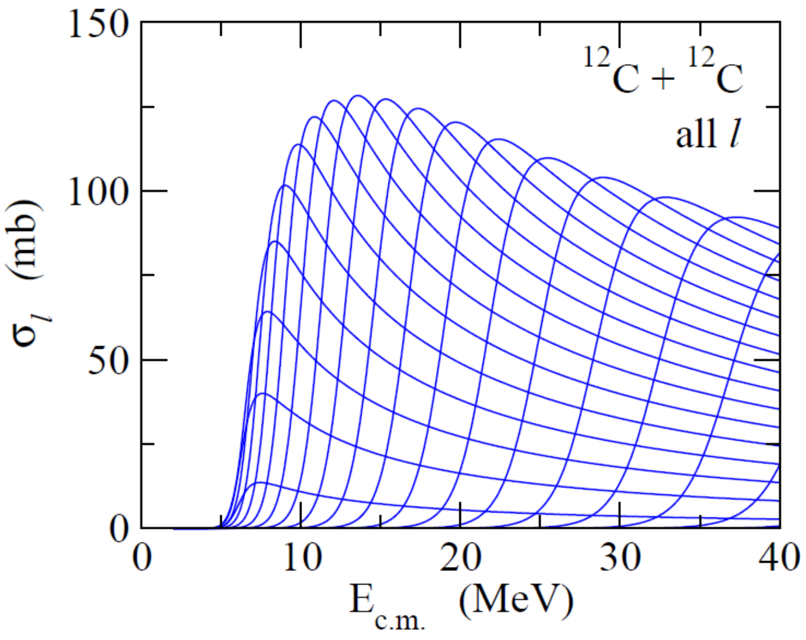
discrete *l*-sum

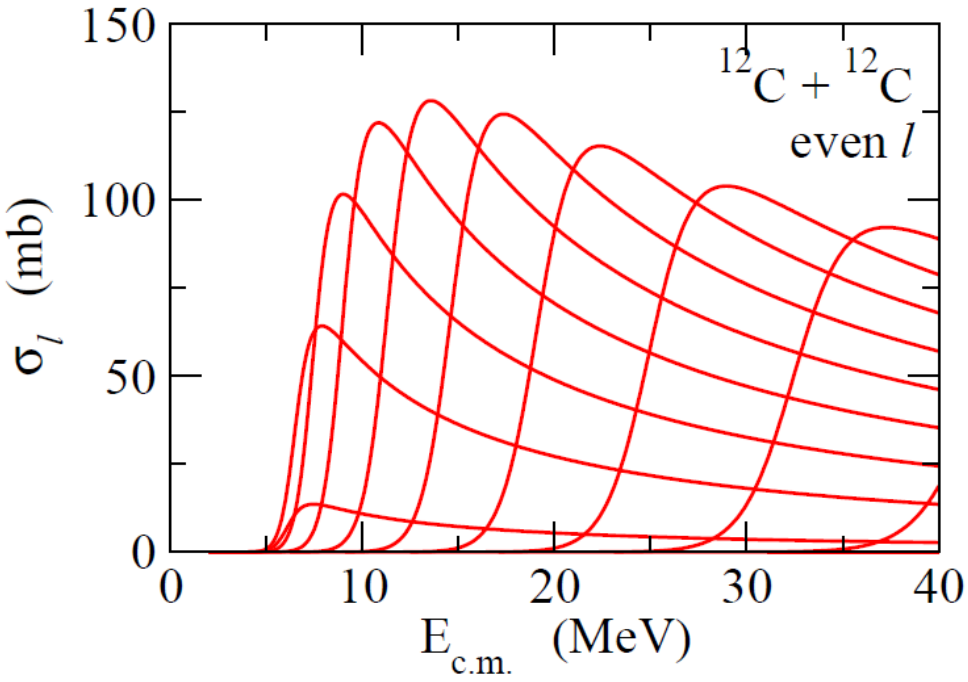
⟶ (oscillatory) structure in σ_{fus}

effect of symmetrization: fusion oscillations in light symmetric systems

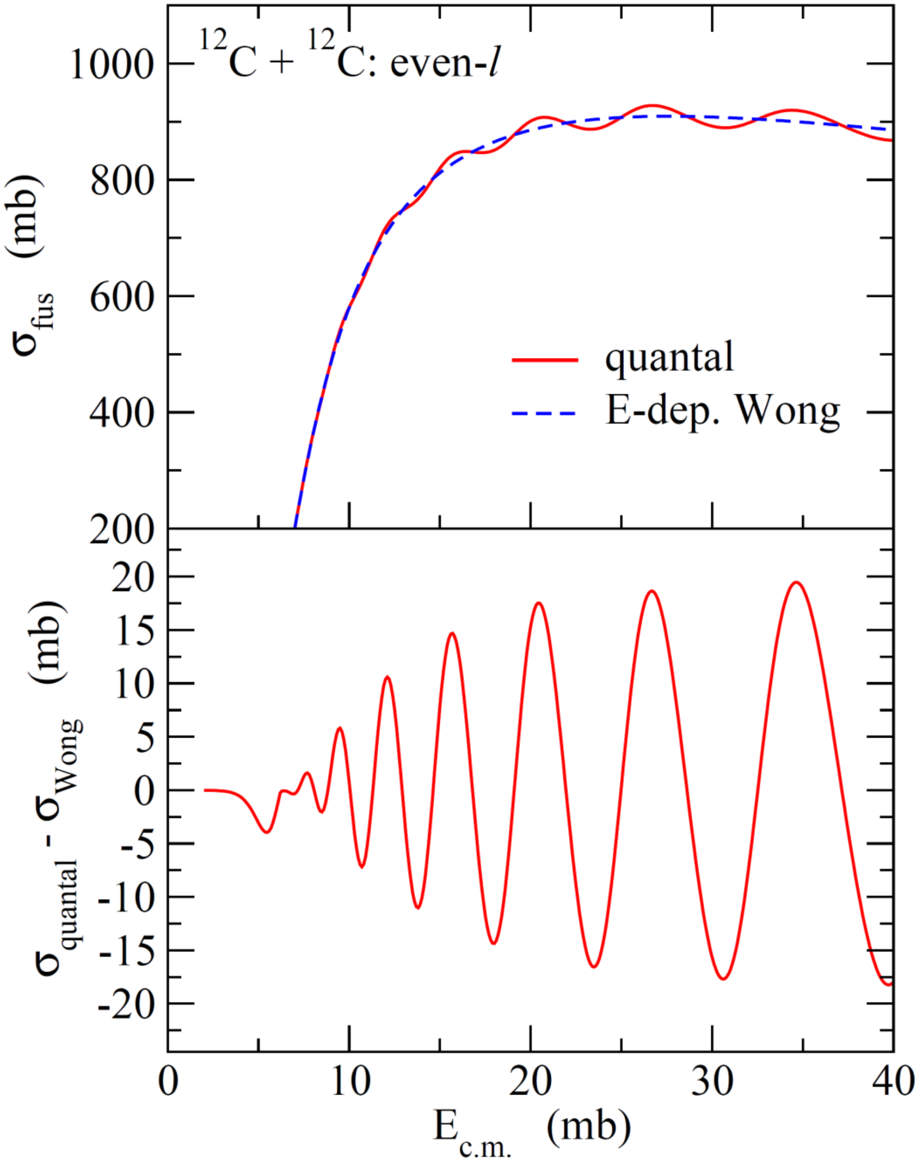
fusion of identical spin-zero bosons: wf has to be symmetric

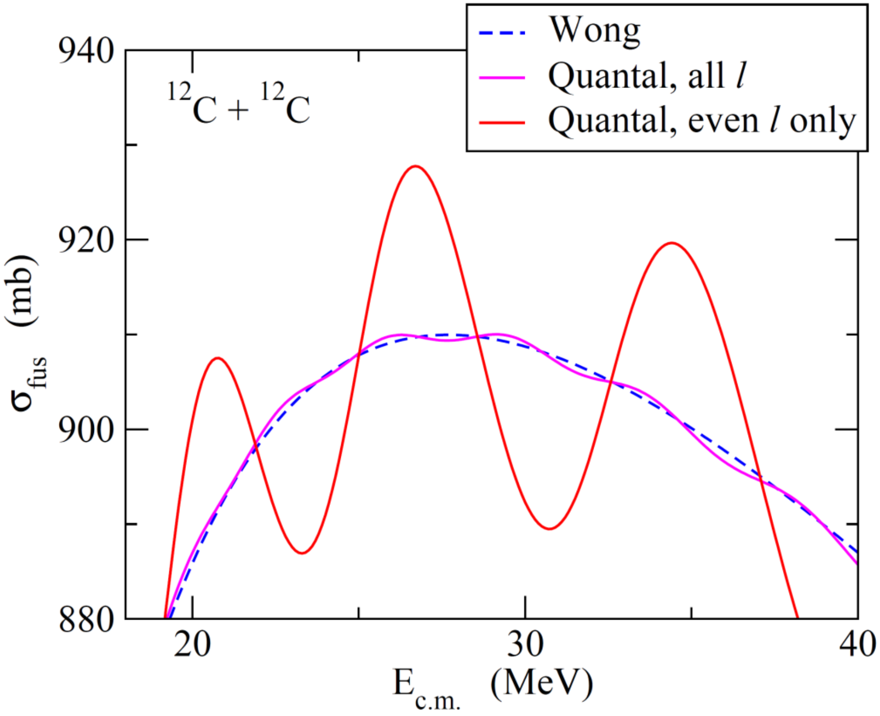
$$\sigma_{\mathsf{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \rightarrow \frac{\pi}{k^2}\sum_l (1+(-)^l)(2l+1)P_l(E)$$

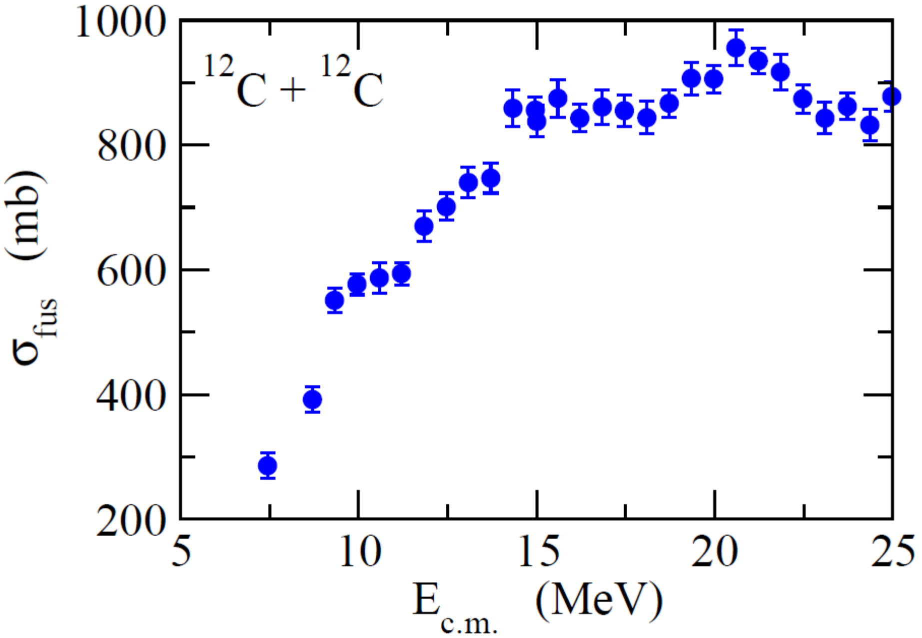




- ✓the angular mom. is quantized in units of 2-hbar
- ✓a larger amplitude of fusion oscillations


^{12}C + ^{12}C: even-l
quantal
E-dep. Wong
σ_{fus} (mb)
1000
800
600
400
200
$\sigma_{quantal} - \sigma_{Wong}$ (mb)
20
15
10
5
0
-5
-10
-15
-20
0
10
20
30
40
$E_{c.m.}$ (mb)


Wong
Quantal, all l
Quantal, even l only
^{12}C + ^{12}C
σ_{fus} (mb)
940
920
900
880
20
30
40
$E_{c.m.}$ (MeV)


^{12}C + ^{12}C
σ_{fus} (mb)
1000
800
600
400
200
5
10
15
20
25
$E_{c.m.}$ (MeV)

Analytic formula for fusion oscillations

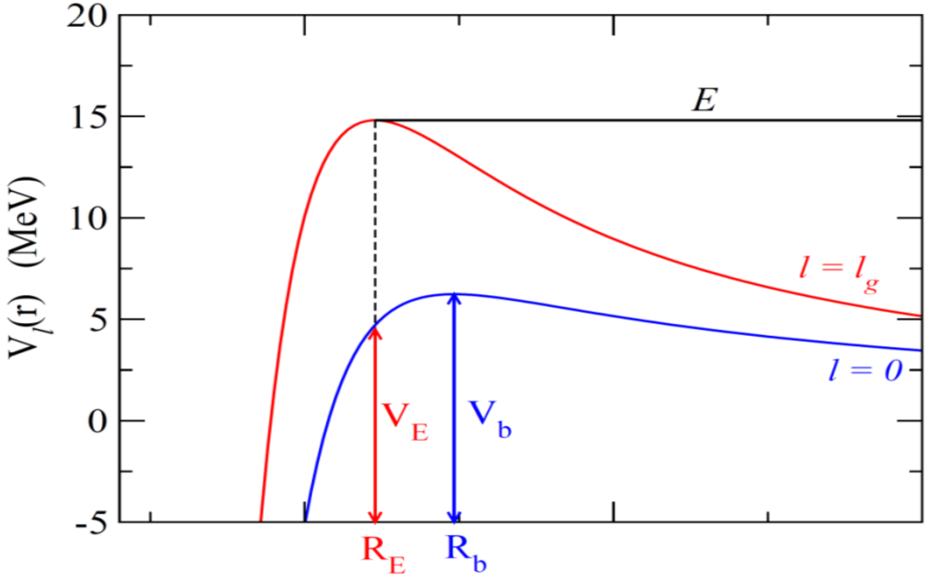
N. Poffe, N. Rowley, and R. Lindsay, Nucl. Phys. A410 (‘83) 498
N. Rowley and K. Hagino, in preparation

$$\sigma_{\rm fus}(E) \;=\; \frac{\pi}{k^2}\sum_l (1\pm(-)^l)(2l+1)P_l(E)$$

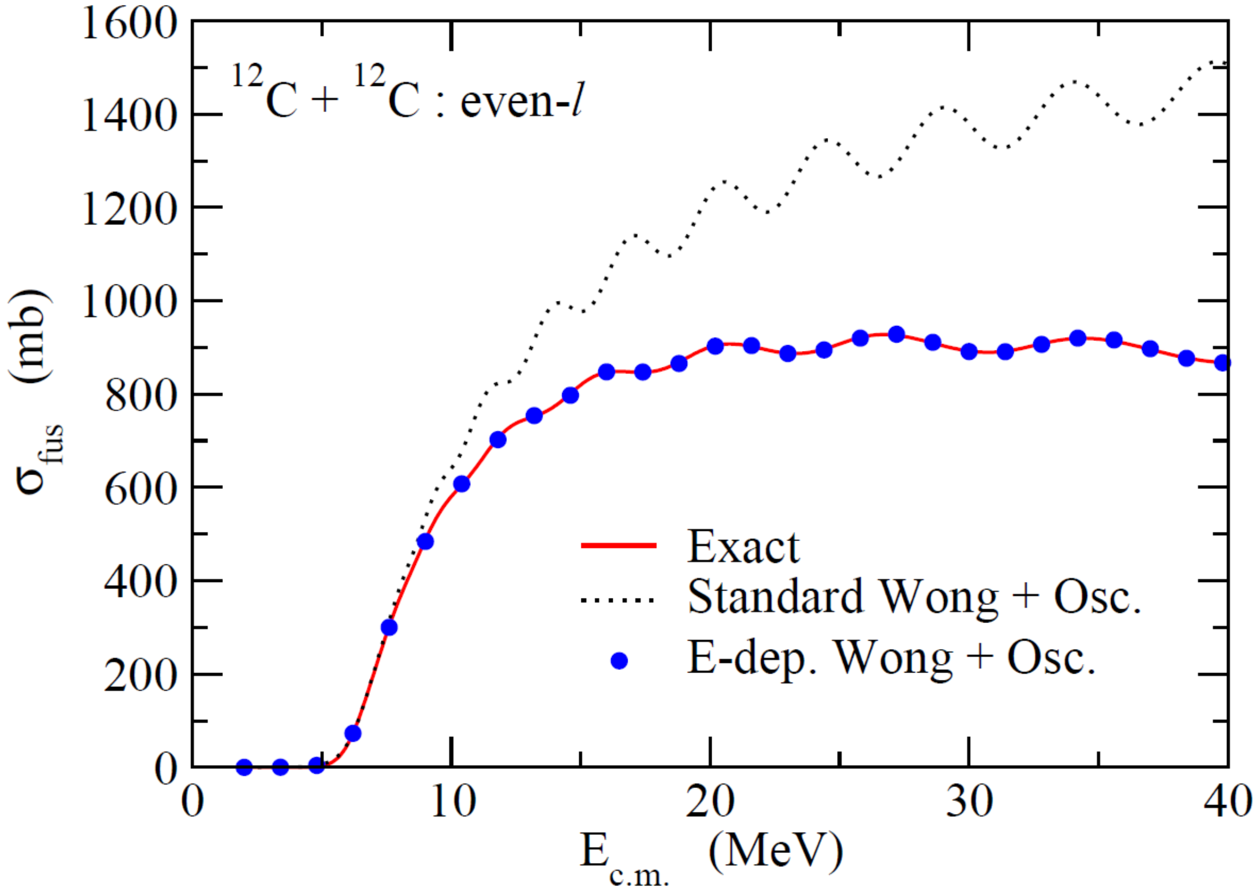
$$\boxed{\sim \sigma_{\rm E-Wong} \pm 2\pi R_E^2\,\frac{\hbar\Omega_E}{E}\,e^{-\xi}\,\sin(\pi l_g)}$$

← Poisson sum formula

$$\xi = \pi\cdot\frac{\hbar\Omega_E}{2l_g+1}\cdot\frac{\mu R_E^2}{\hbar^2}$$



$$\sigma_{\rm osc}(E) = 2\pi R_b^2 \frac{\hbar\Omega}{E} e^{-\xi} \sin(\pi l_g), \qquad \xi = \pi \cdot \frac{\hbar\Omega}{2l_g+1} \cdot \frac{\mu R_b^2}{\hbar^2}$$



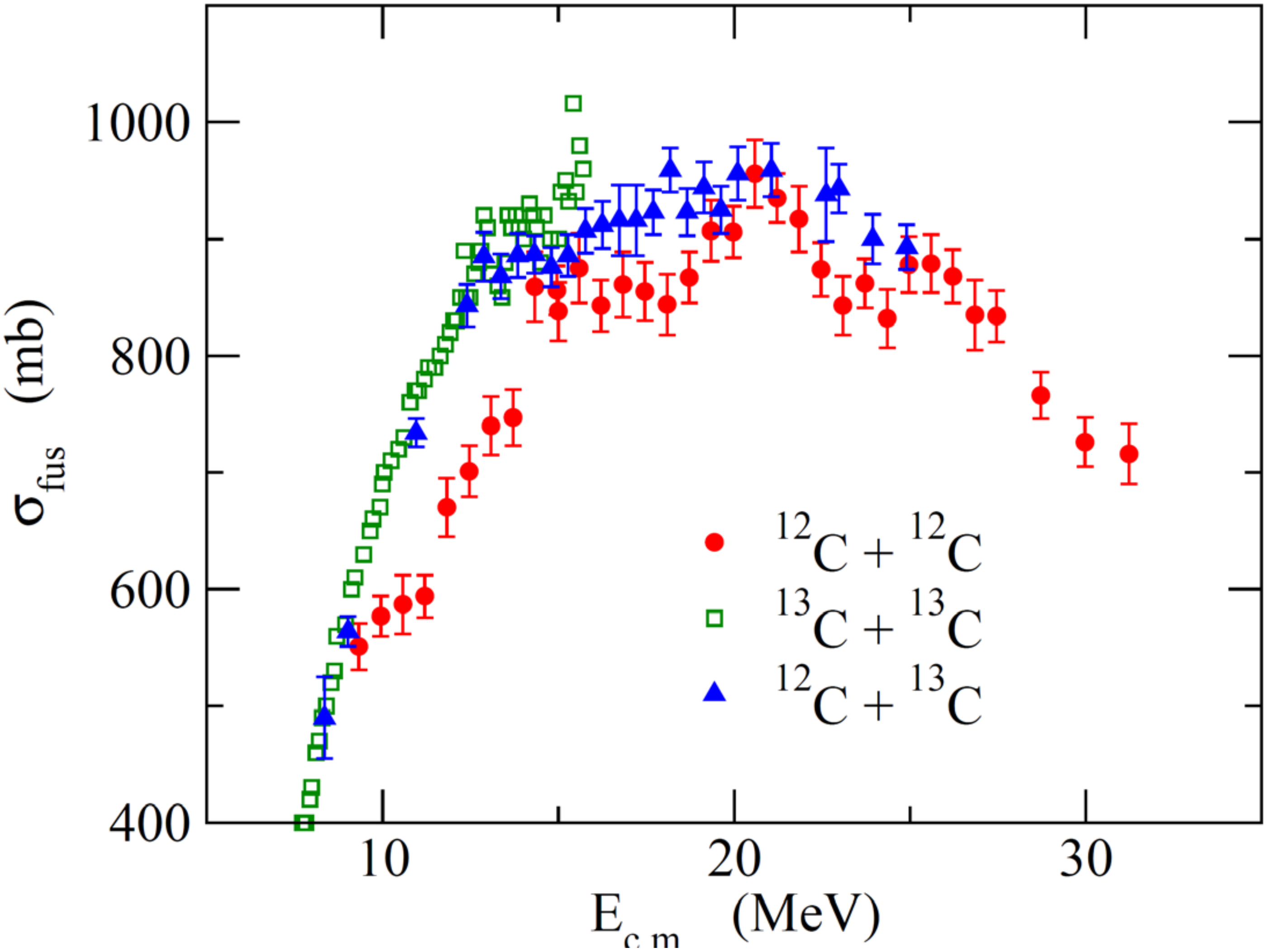
(note)

$$\frac{|\sigma_{\rm osc}|}{\sigma_{\rm Wong}} \sim \frac{2\hbar\Omega}{E - V_b} \cdot e^{-\xi} \quad \curvearrowright \quad 2l_g + 1 \gg \pi\hbar\Omega \cdot \frac{\mu R_b^2}{\hbar^2}$$

in order for the osc. to be visible

$\longrightarrow$ light symmetric systems

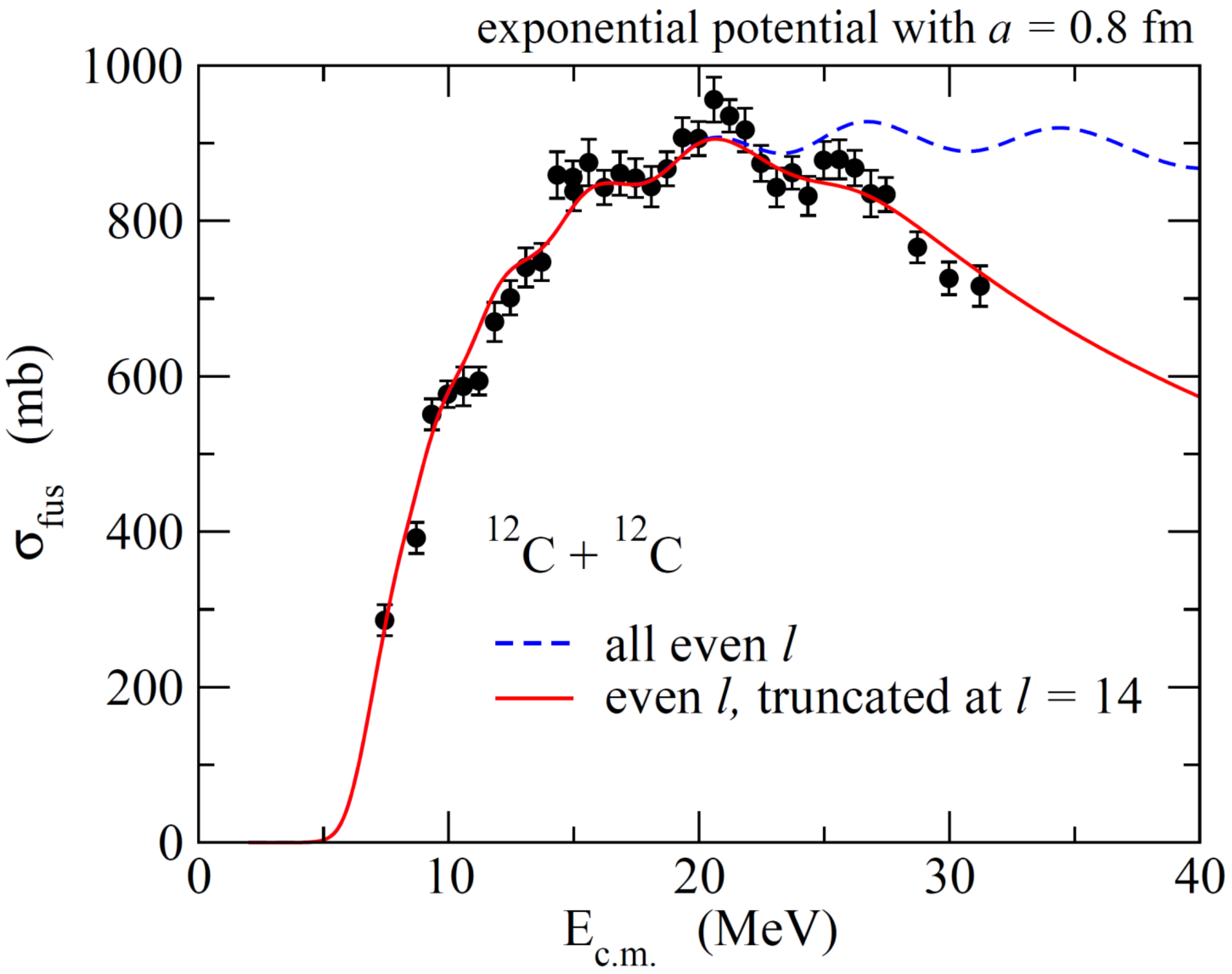
Comparison with experimental data



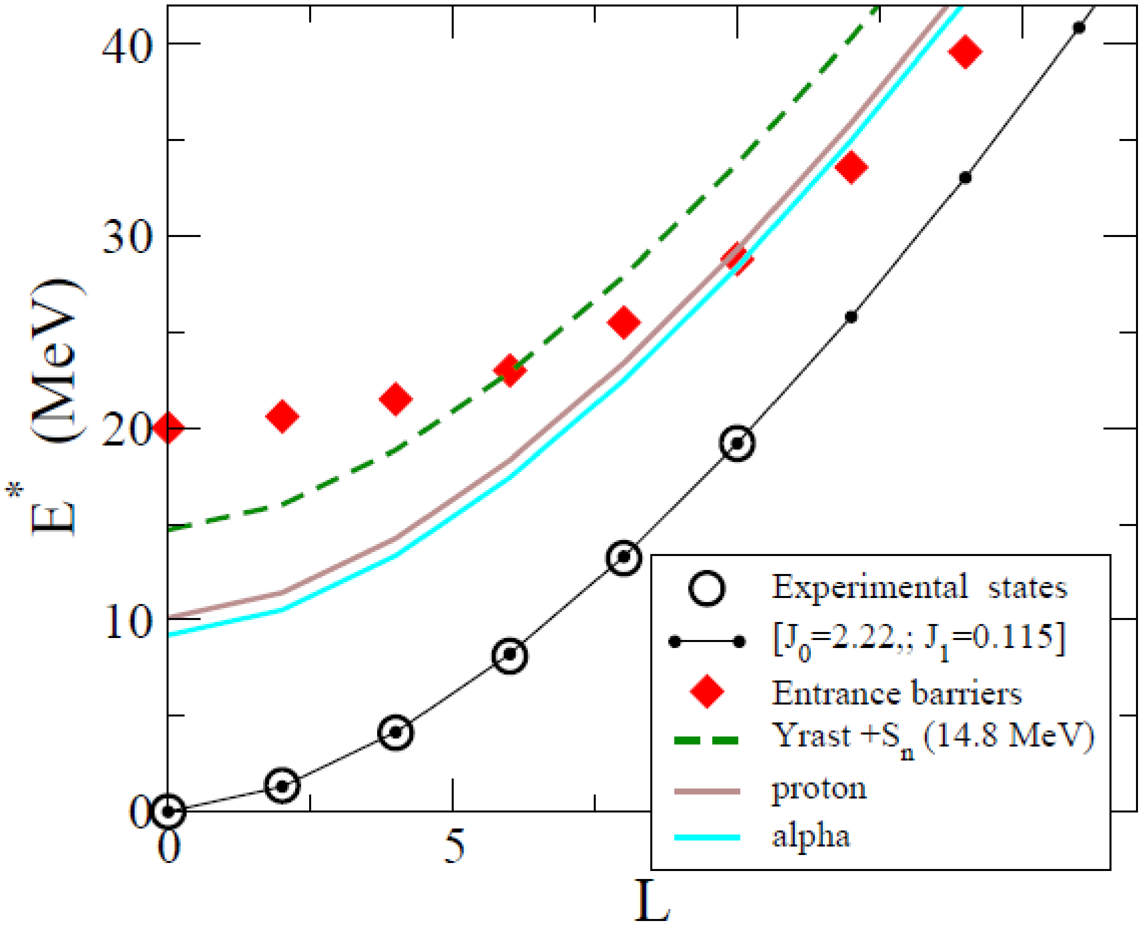
analyses with single-channel calculations

i) Comparison with the experimental data: $^{12}C + ^{12}C$

$^{12}C_{g.s.}$: 0^+ $\longrightarrow$ the relative w.f. has to be spatially symmetric



Barriers and Yrast line for ^{24}Mg

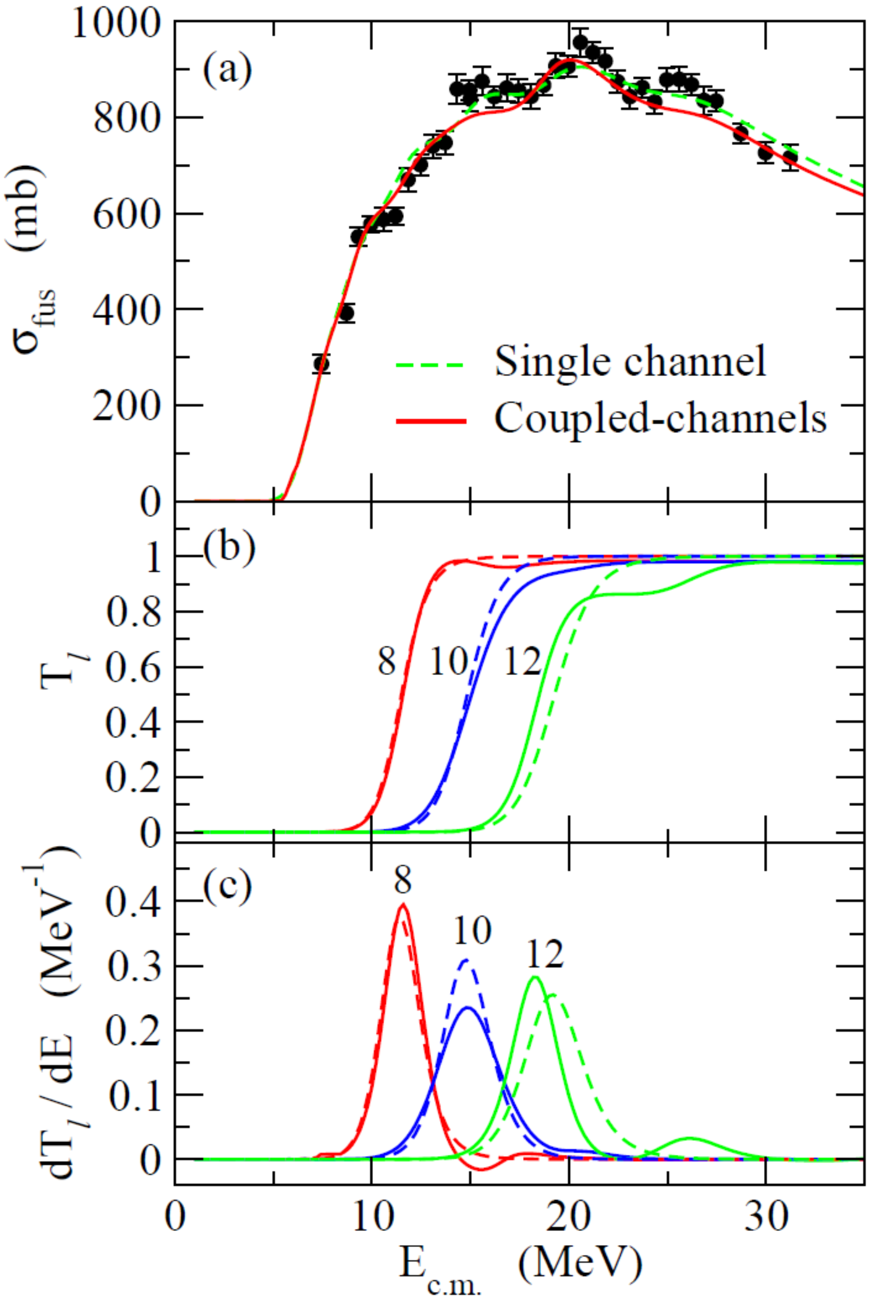


$S_n = 16.5$ MeV, $S_p = 11.69$ MeV

→ high *l*: particle evaporation inhibited

fission a/o γ-ray

Role of channel couplings



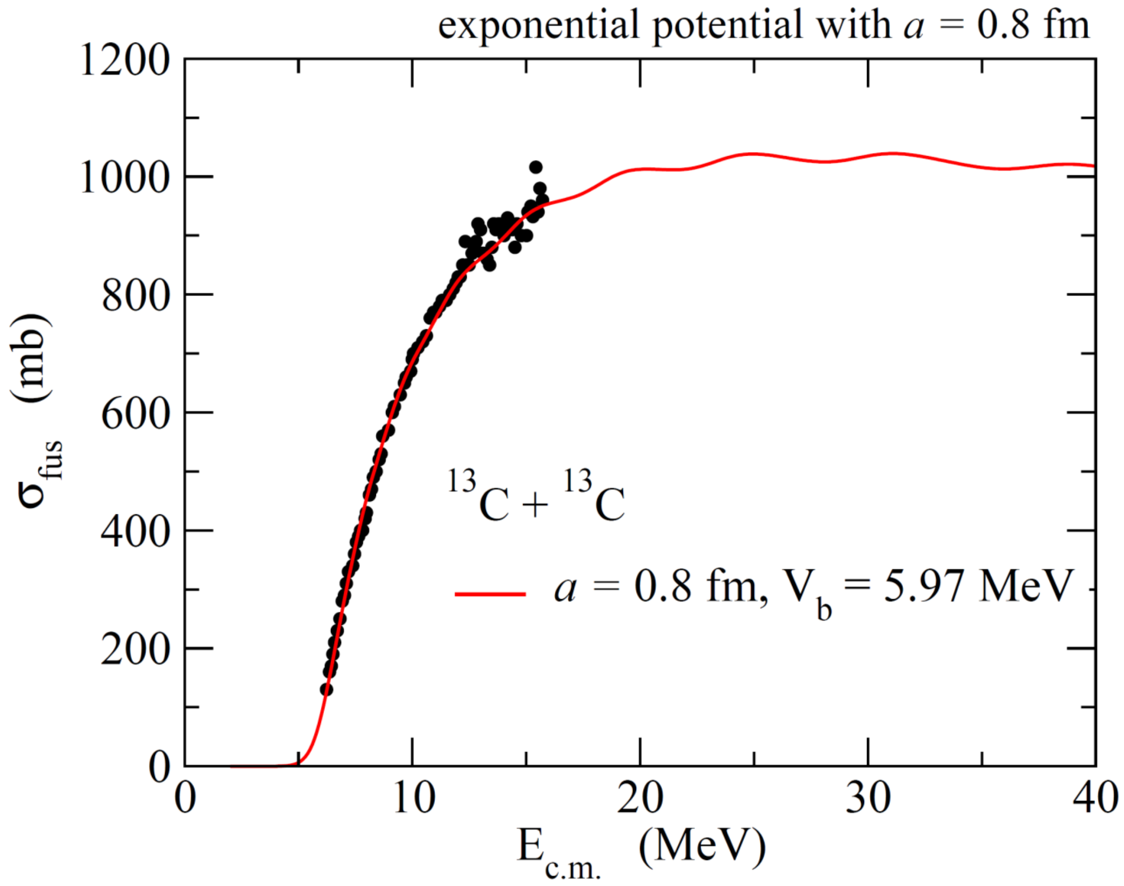
The main features of oscillations (the peak energies and the phase) : not affected much

ii) $^{13}\mathrm{C} + {}^{13}\mathrm{C}$

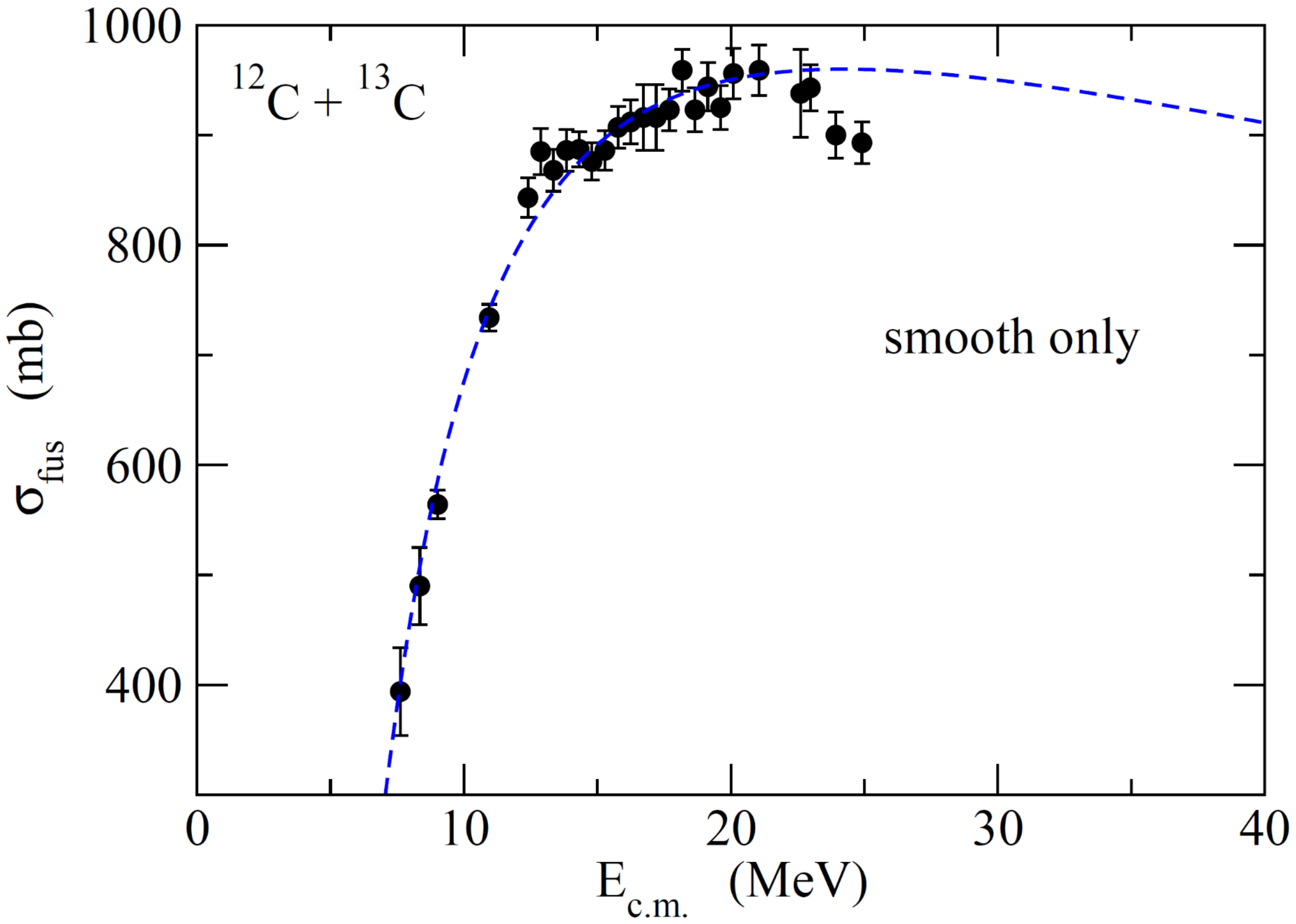
$^{13}\mathrm{C}_{\mathrm{g.s.}}$: $1/2^{-}$ ⟶ the relative w.f. has to be spatially symmetric for $S = 0$
spatially anti-symmetric for $S = 1$

$$\sum_l \to \frac{1}{4}\sum_l \left(1 + (-1)^l\right) + \frac{3}{4}\sum_l \left(1 - (-1)^l\right)$$

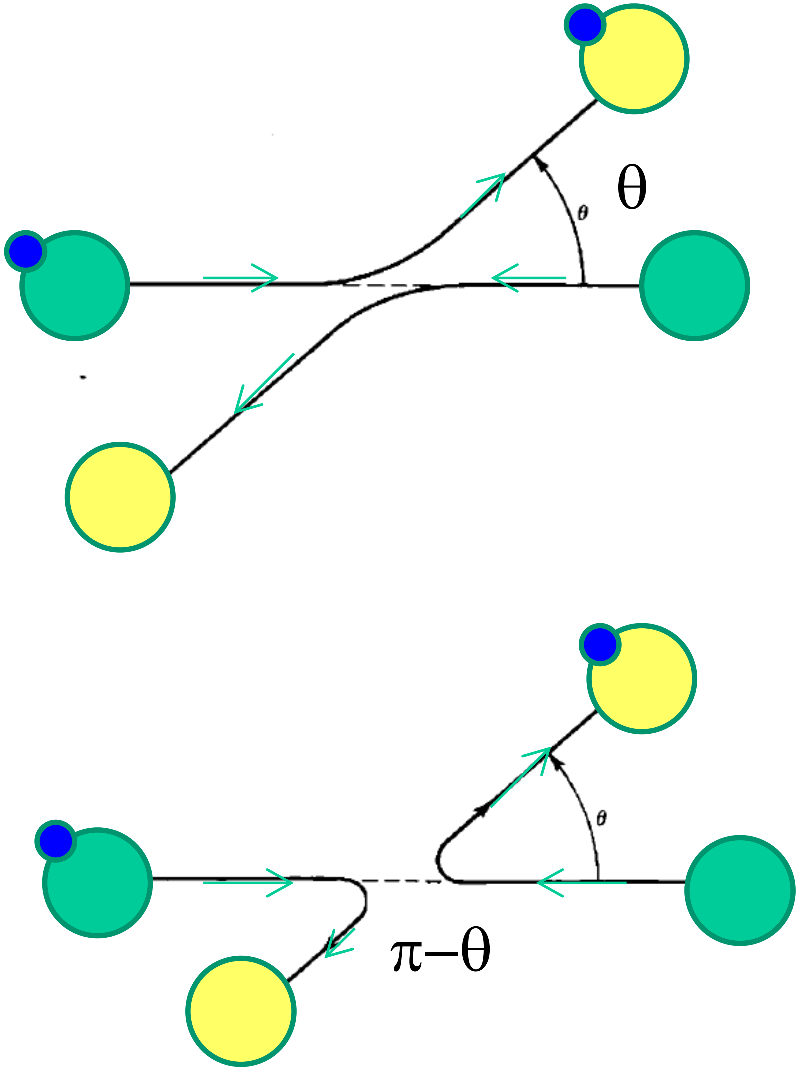
$$\boxed{\sigma_{\mathrm{osc}} = \frac{1}{2}\sigma_{\mathrm{osc}}(\mathrm{odd} - l)}$$



iii) $^{12}C + ^{13}C$


$^{12}C + ^{13}C$
smooth only
1000
800
600
400
0
10
20
30
40
σ_{fus} (mb)
$E_{c.m.}$ (MeV)

role of elastic transfer



elastic scattering

$f_{\mathrm{el}}(\theta)$

indistinguishable

transfer

$f_{\mathrm{trans}}(\pi-\theta)$

$$\boxed{f(\theta) \rightarrow f_{\mathrm{el}}(\theta) + f_{\mathrm{trans}}(\pi-\theta)}$$

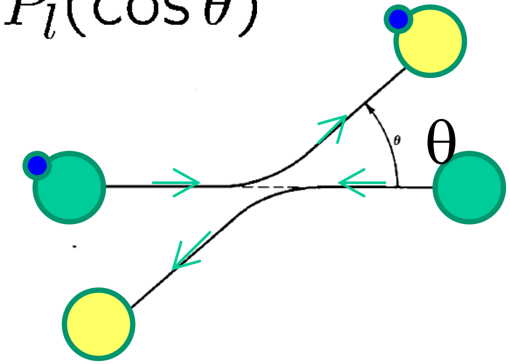
role of elastic transfer

$$\boxed{f(\theta) \to f_{\mathrm{el}}(\theta) + f_{\mathrm{trans}}(\pi-\theta)}$$

$$f_{\mathrm{el}}(\theta) = \sum_l (2l+1)\frac{S_l^{\mathrm{el}}-1}{2ik}P_l(\cos\theta)$$

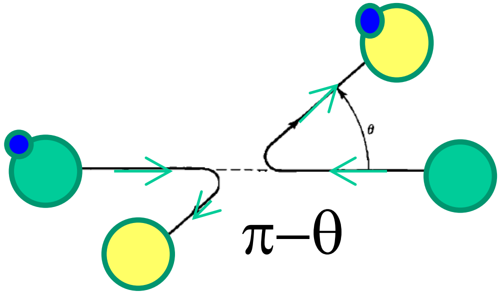
$$f_{\mathrm{trans}}(\pi-\theta) = \sum_l (2l+1)\frac{S_l^{\mathrm{trans}}}{2ik}\underline{P_l(\cos(\pi-\theta))}$$

$$= (-)^l P_l(\cos\theta)$$



$$\boxed{S_l^{\mathrm{eff}} = S_l^{\mathrm{el}} + (-1)^l\, S_l^{\mathrm{trans}}}$$

if $S_l^{\mathrm{trans}} \sim \alpha\dfrac{\partial S_l^{\mathrm{el}}}{\partial l}$



$$\boxed{S_l^{\mathrm{eff}} = S^{\mathrm{el}}(l + (-1)^l\alpha)}$$

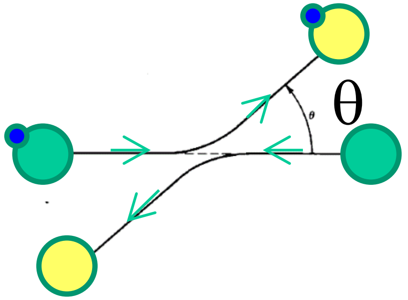
role of elastic transfer

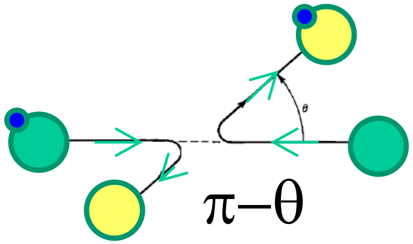
$$\boxed{S_l^{\mathrm{eff}} = S^{\mathrm{el}}(l + (-1)^l \alpha)}$$

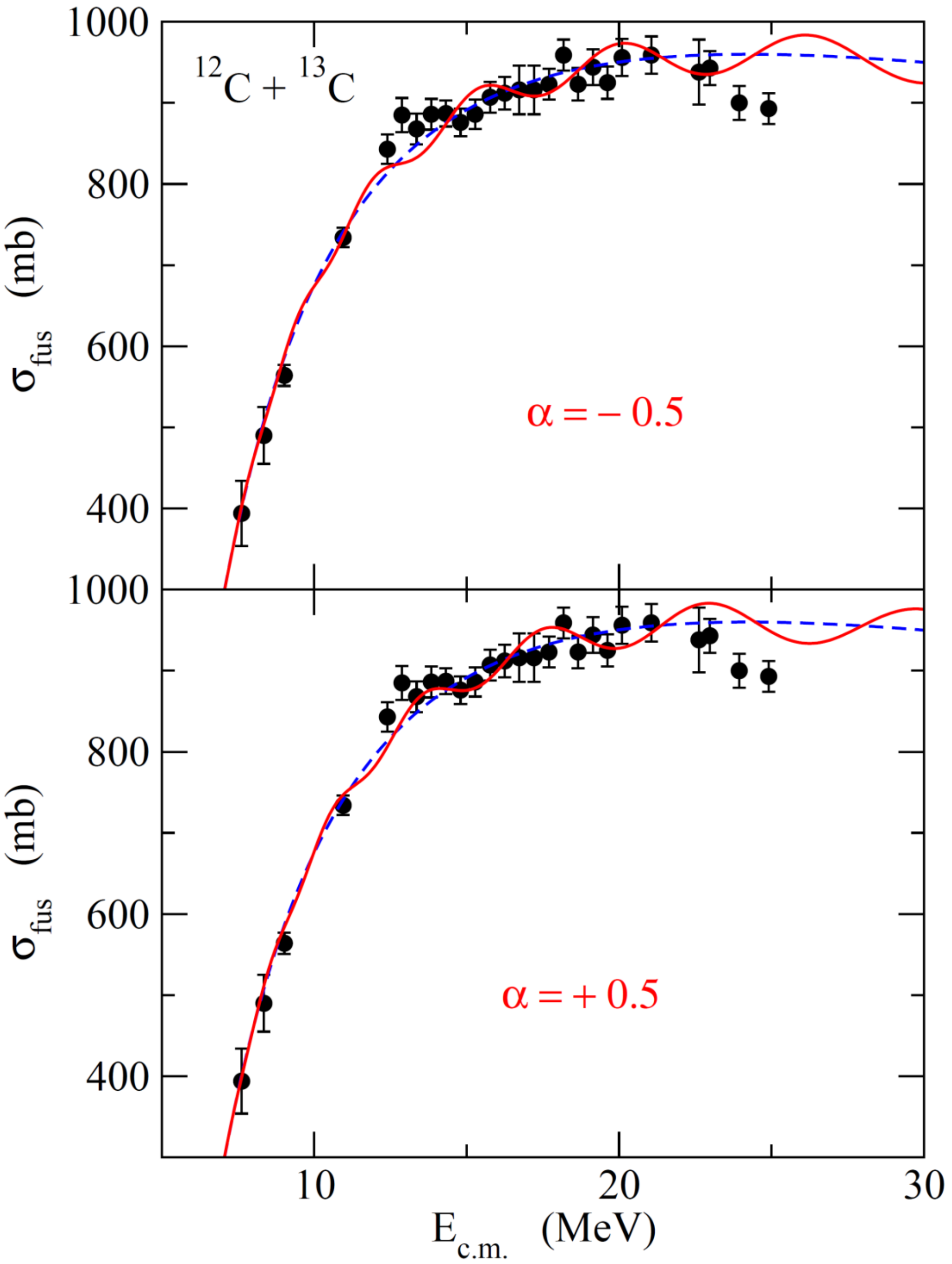
$$\sigma_{\mathrm{osc}}(E) = \pm 2\pi R_b^2 \frac{\hbar\Omega}{E} e^{-\xi} \sin(\pi l_g),$$

$$\begin{aligned} \sin(\pi l_g) &\rightarrow [\sin(\pi(l_g + \alpha)) - \sin(\pi(l_g - \alpha))]/2 \\ &= \cos(\pi l_g)\sin(\pi\alpha) \end{aligned}$$

$$\boxed{\sigma_{\mathrm{osc}}(E) = 2\pi R_b^2 \frac{\hbar\Omega}{E} e^{-\xi} \cos(\pi l_g)\sin(\pi\alpha)}$$







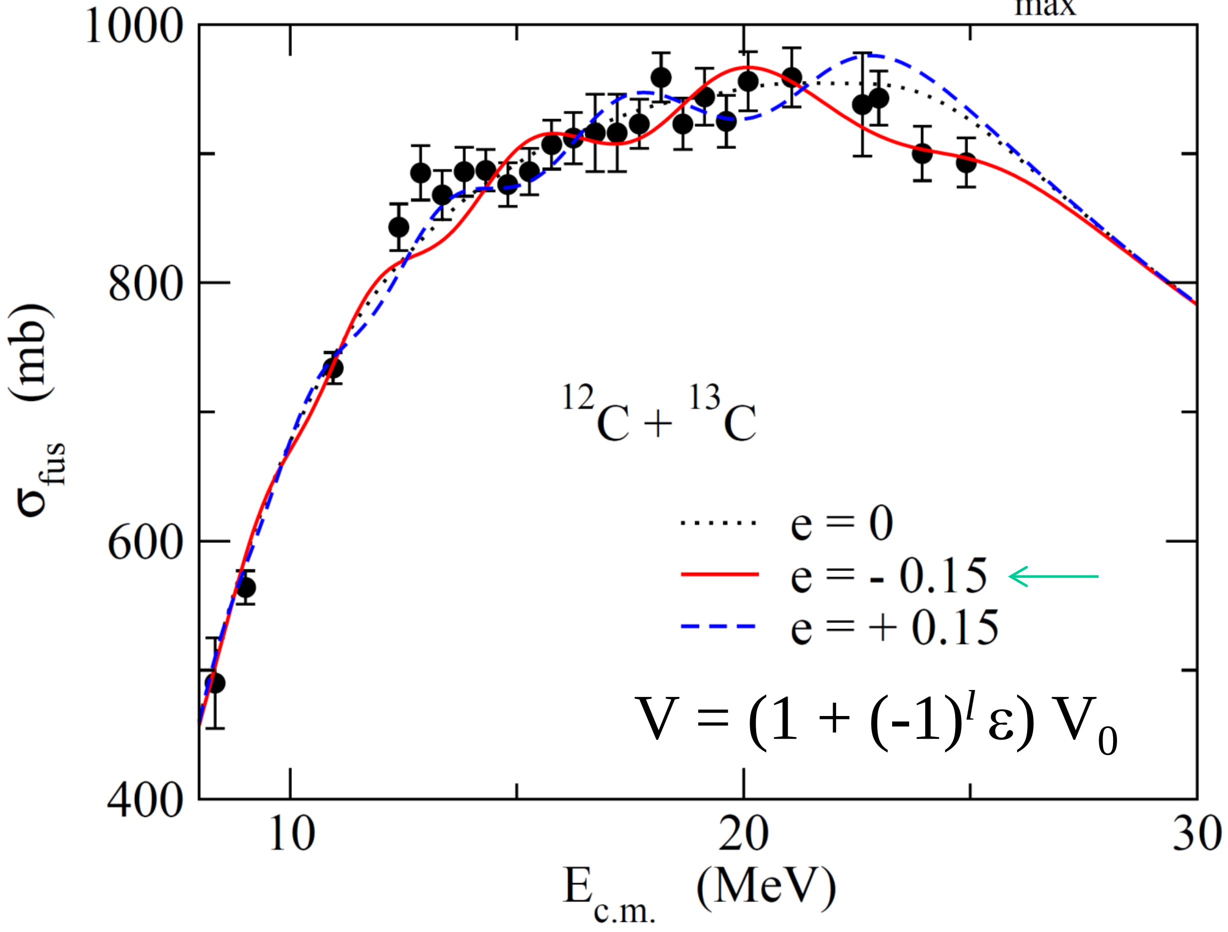
exponential potential with *a* = 0.9 fm

parity-dependent potential

- ✓W. von Oertzen and H.G. Bohlen, Phys. Rep. 19C(‘75) 1
- ✓A. Vitturi and C.H. Dasso, Nucl. Phys. A458 (‘86) 157
- ✓A. Kabir, M.W. Kermode and N. Rowley, Nucl. Phys. A481(‘88) 94

cf. $^{12}C + ^{16}O$

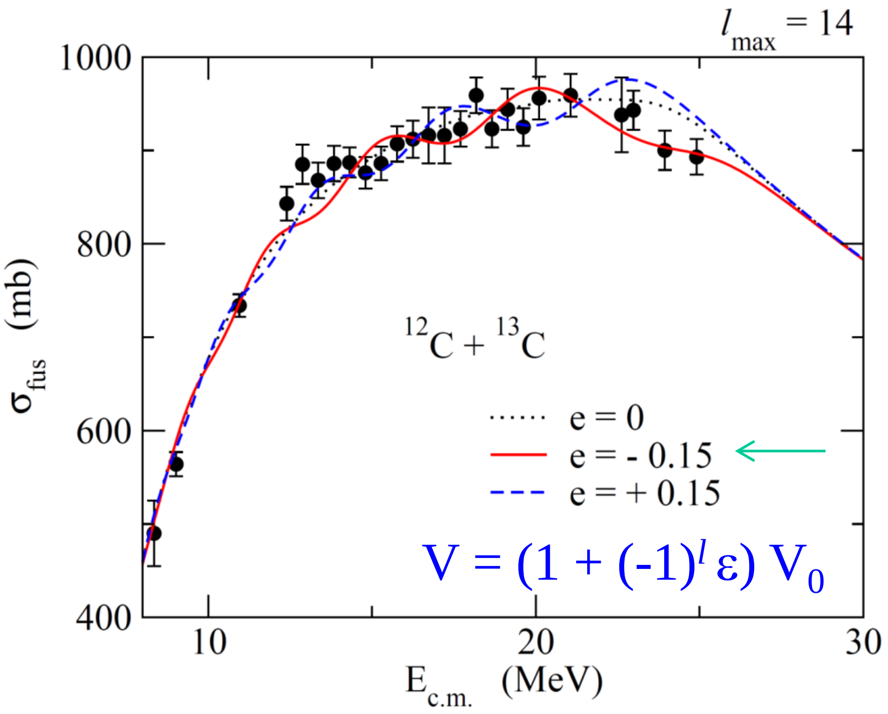
$l_{max} = 14$



$$V = (1 + (-1)^{l}\,\varepsilon)\,V_0$$

exponential potential with *a* = 0.9 fm

parity-dependent potential



$V = (1 + (-1)^l \varepsilon)\, V_0$

$\varepsilon < 0$

↕

a smaller *V*

↕

a higer barrier for even-*l*

cf. $\text{sign}(V_+ - V_-) = \varepsilon V_0 = -\varepsilon$

Baye’s simple rule: ← RGM with two-center HO shell model

D. Baye, J. Deenen, and Y. Salmon, Nucl. Phys. A289(‘77) 511
D. Baye, Nucl. Phys. A460 (‘86)581

$$\boxed{\text{sign}(V_+ - V_-) = -(-)^{A_<} \prod_{i:valence} \pi_i}$$

(nuclear potential)

for $^{12}C+^{13}C(p_{1/2})$:

Summary

sub-barrier fusion of C+C systems

- Molecular resonances at subbarrier energies

$^{12}\mathrm{C} + {}^{12}\mathrm{C}$: well pronounced resonance structure

$^{13}\mathrm{C} + {}^{13}\mathrm{C}$, $^{12}\mathrm{C} + {}^{13}\mathrm{C}$: rather smooth

← CN ^{24}Mg: low level density (low Q-value, e-e nucleus)

cf. Jiang's conjecture

- Fusion oscillations: successive contribution of discrete centrifugal barariers

$$\left.\begin{array}{l} {}^{12}\mathrm{C}(0^+) + {}^{12}\mathrm{C}(0^+) \\ {}^{13}\mathrm{C}(1/2^-) + {}^{13}\mathrm{C}(1/2^-) \end{array}\right\}\ \text{symmetrization of relative wave function}$$

$^{12}\mathrm{C} + {}^{13}\mathrm{C}$ --- elastic transfer

cf. $^{14}\mathrm{C} + {}^{14}\mathrm{C}$: R.M. Freeman, C. Beck et al., PRC24 ('81) 2390

- analytic formula for fusion oscillations

← parabolic approximation