Heavy-ion fusion reactions and superheavy elements

Kouichi Hagino

Tohoku University, Sendai, Japan

1. H.I. fusion reactions: why are they interesting?
2. Coupled-channels approach
3. Future perspectives: superheavy elements

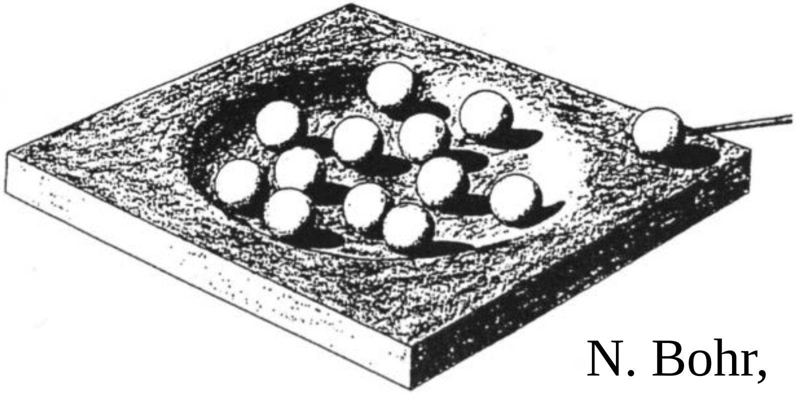
Recent review article:
K. Hagino and N. Takigawa, Prog. Theo. Phys.128 (‘12)1061.

YITP school on “Recent Progress of Nuclear Structure and Reaction Physics”, YITP, Dec. 18-22, 2017

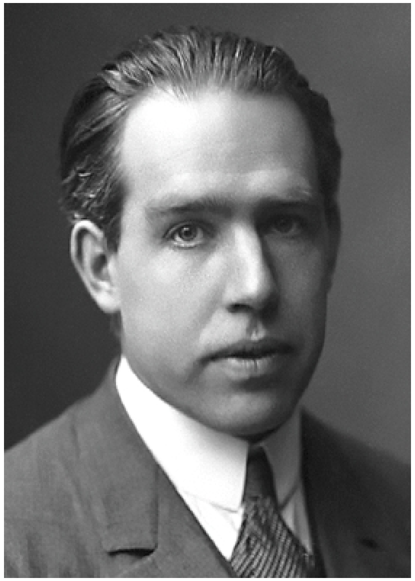
Fusion reactions: compound nucleus formation

Niels Bohr (1936)

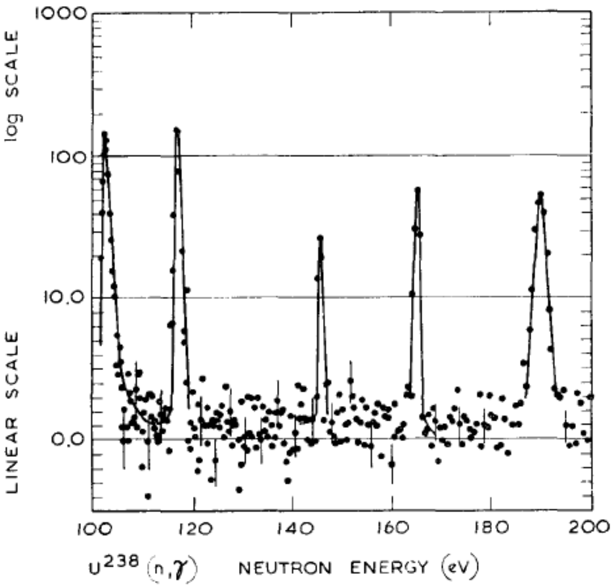
Neutron capture of nuclei → compound nucleus



N. Bohr,
Nature 137 ('36) 351



Wikipedia



cf. Experiment of Enrico Fermi (1935)
many very narrow (=long life-time)
resonances (width ~ eV)

M. Asghar et al., Nucl. Phys. 85 ('66) 305

Fusion reactions: compound nucleus formation

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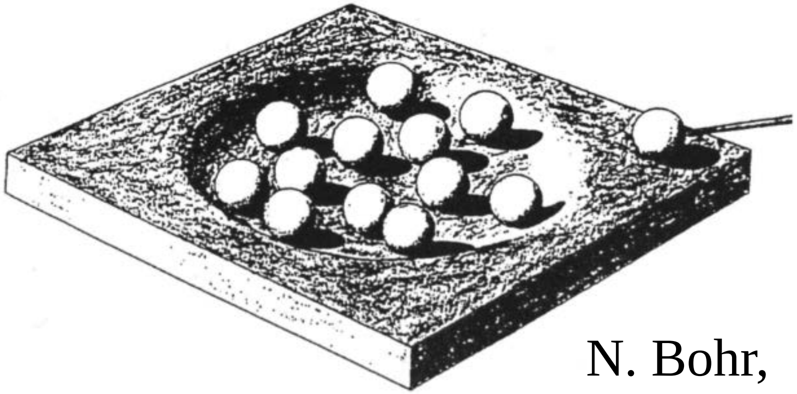
Neutron capture of nuclei → compound nucleus



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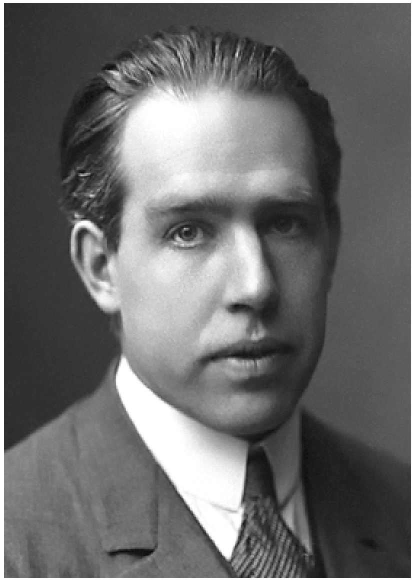
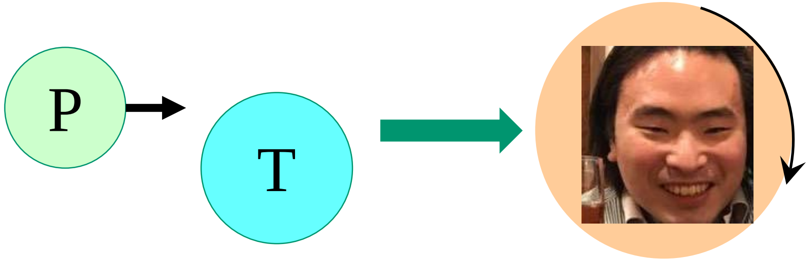


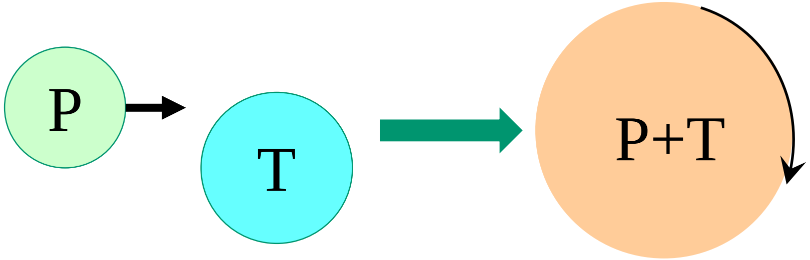
Wikipedia

forming a compound nucleus with heavy-ion reactions = H.I. fusion

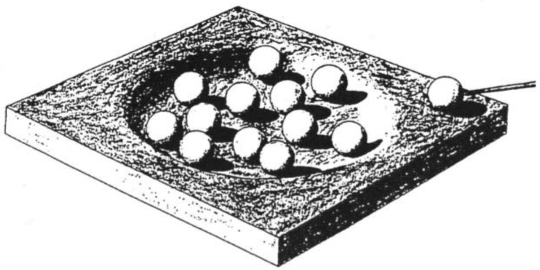


compound nucleus

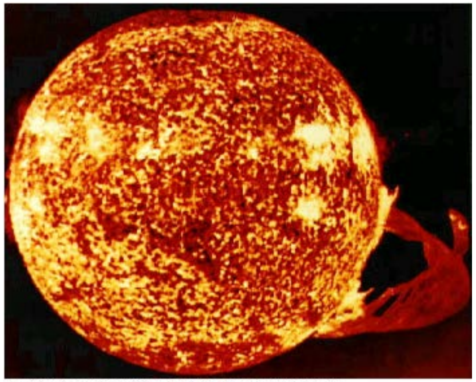
Fusion reactions: compound nucleus formation



compound nucleus

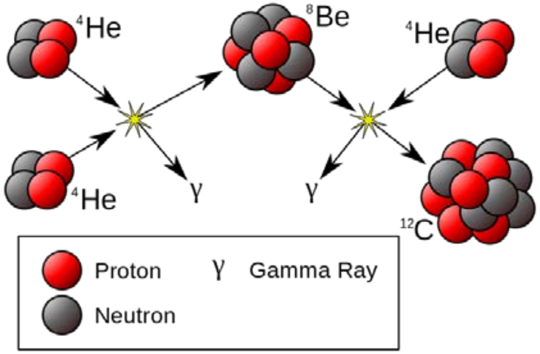


cf. Bohr ‘36

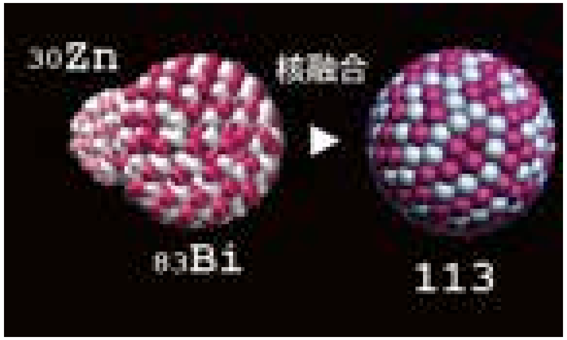


NASA, Skylab space station December 19, 1973, solar flare reaching 588 000 km off solar surface

energy production in stars (Bethe ‘39)



nucleosynthesis

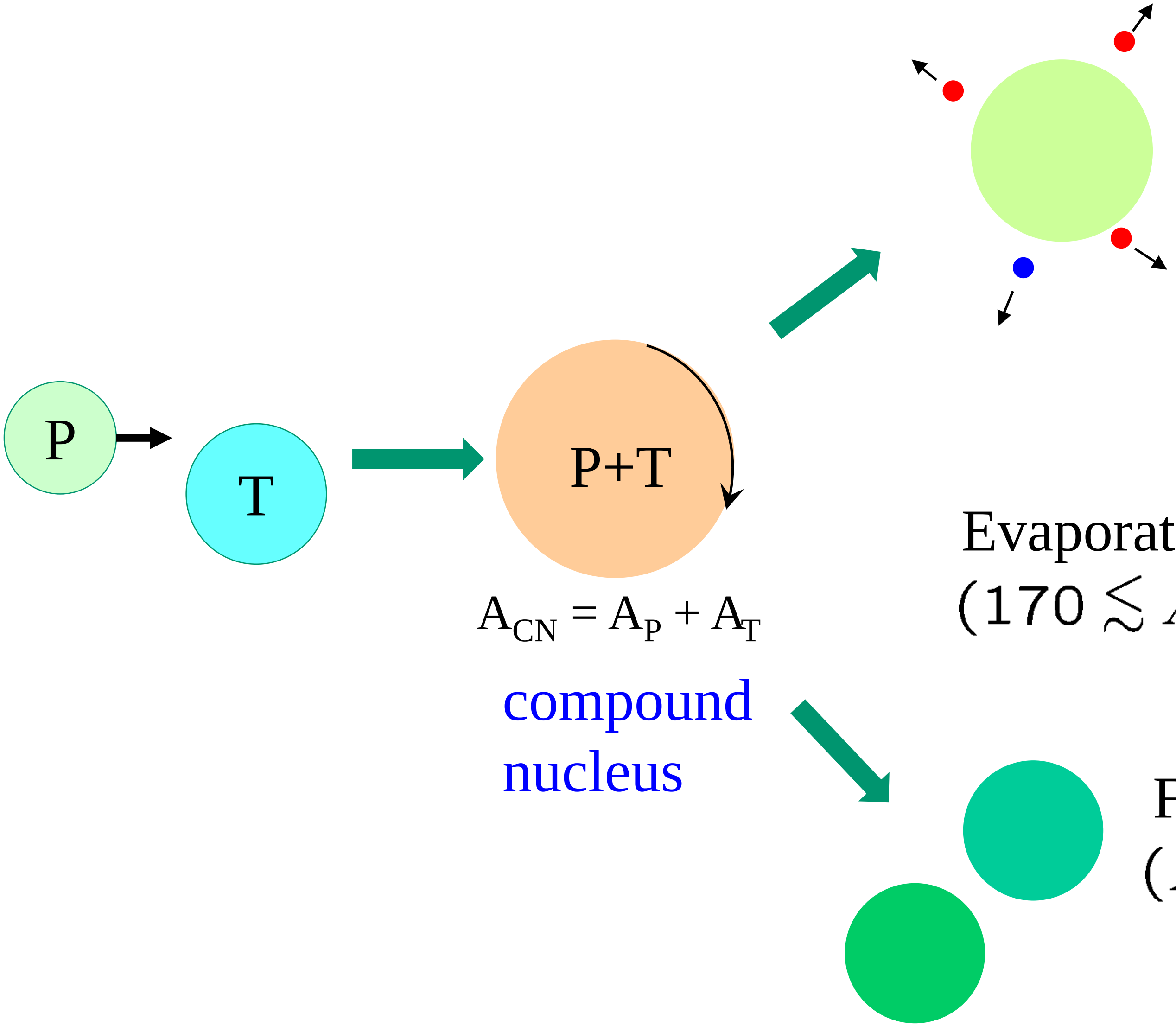


superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems with strong interaction

⟵ microscopic understanding: an ultimate goal of nuclear physics

Fusion reactions: compound nucleus formation



Evaporation
$(A_{\mathrm{CN}} \lesssim 170)$

n,p,α emissions
γ decay

$A_{CN} = A_P + A_T$

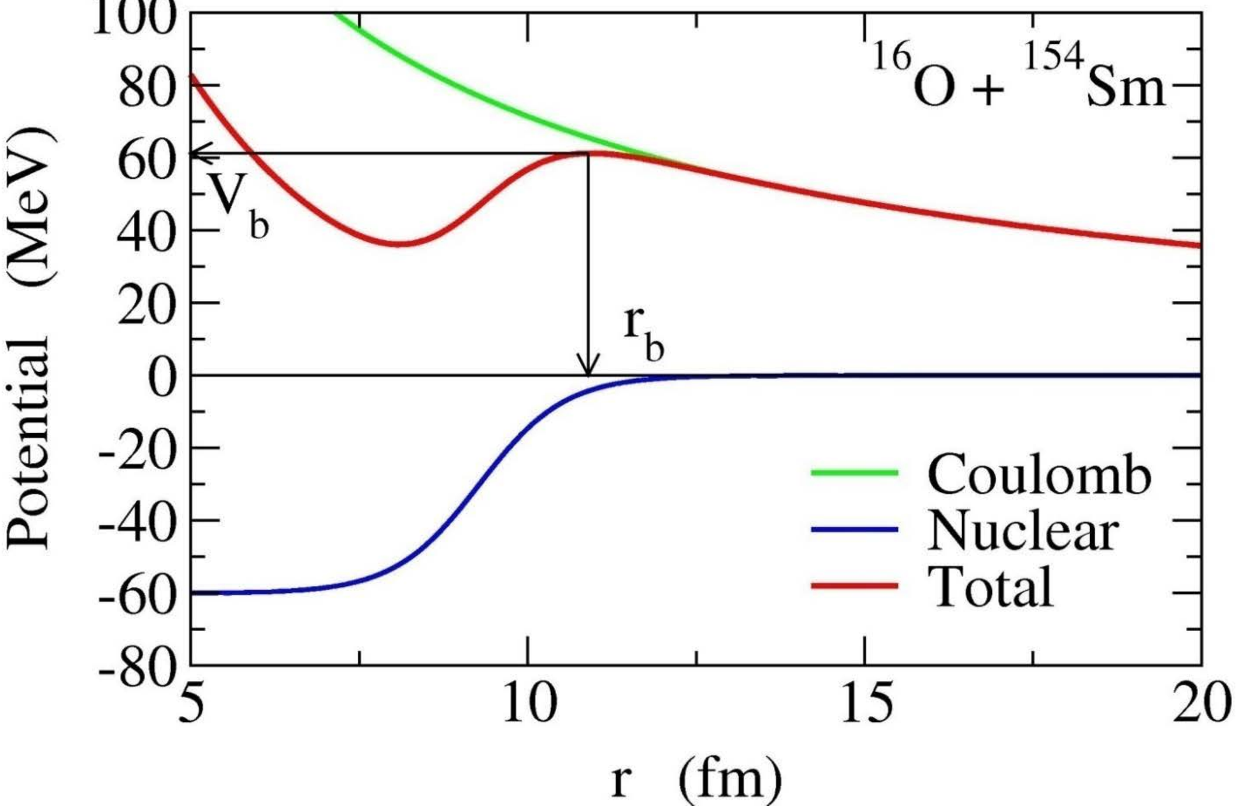
compound nucleus

Evaporation + Fission
$(170 \lesssim A_{\mathrm{CN}} \lesssim 220)$

Fission
$(A_{\mathrm{CN}} \gtrsim 220)$

cf. superheavy elements

Inter-nucleus potential



Two interactions:

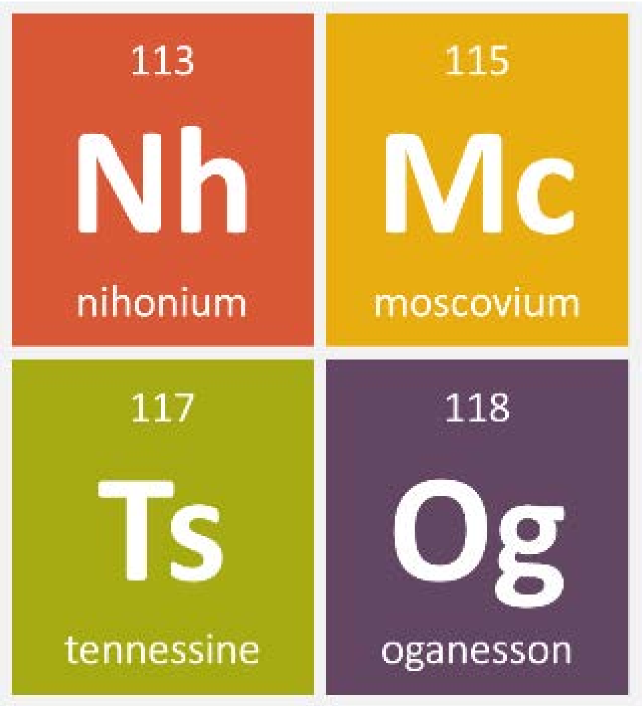
1. Coulomb force
   long range repulsion
2. Nuclear force
   short range attraction

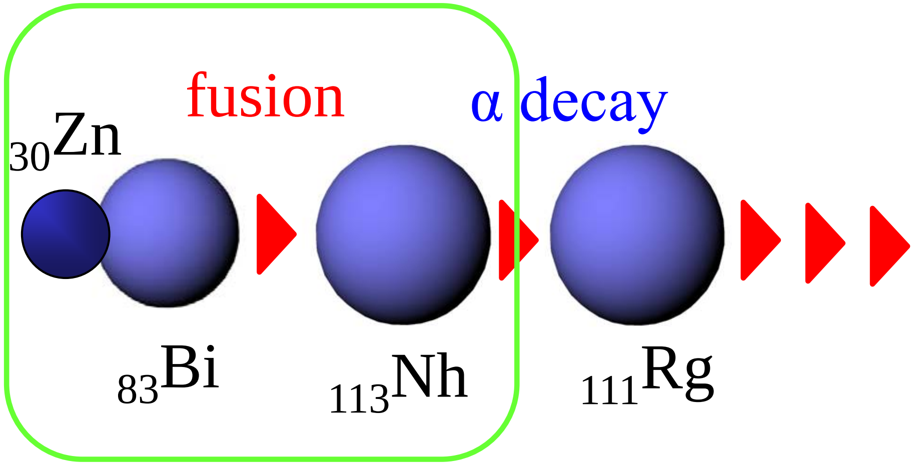
<u>potential barrier</u>
due to a cancellation
between the two
(Coulomb barrier)

- Above-barrier energies
- Sub-barrier energies
  (energies around the Coulomb barrier)
- Deep sub-barrier energies

Why sub-barrier fusion?

two obvious reasons:





superheavy elements

cf. ^{209}Bi (^{70}Zn,n) ^{278}Nh

$V_B \sim 260$ MeV

$E_{cm}^{(exp)} \sim 262$ MeV

Why sub-barrier fusion?

two obvious reasons:

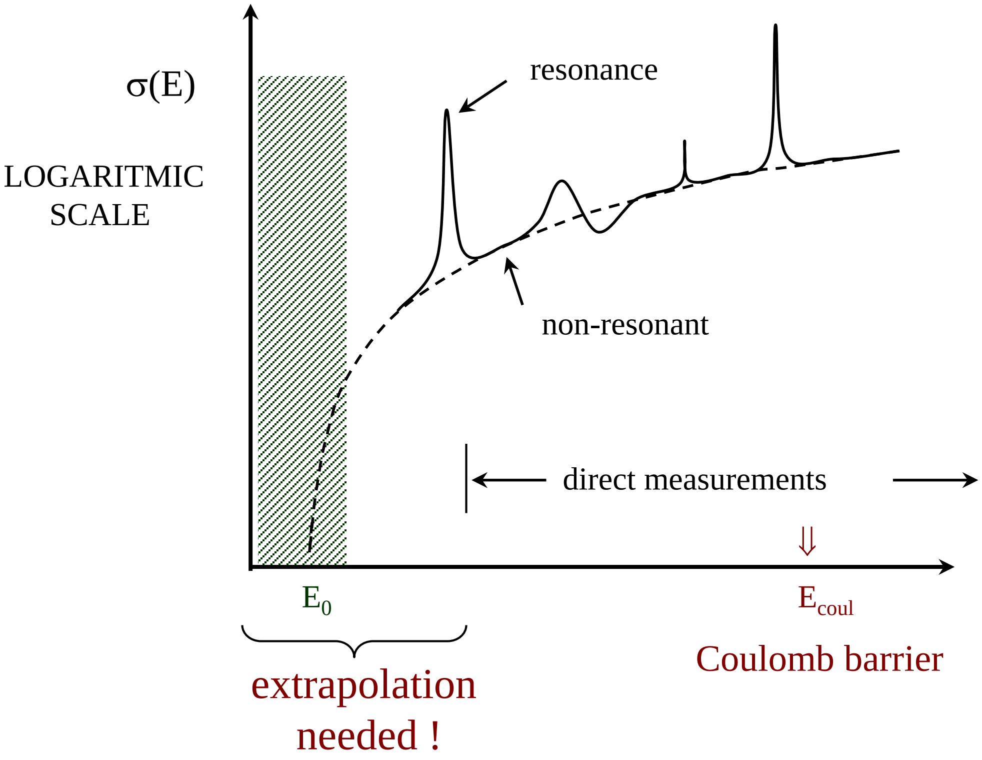
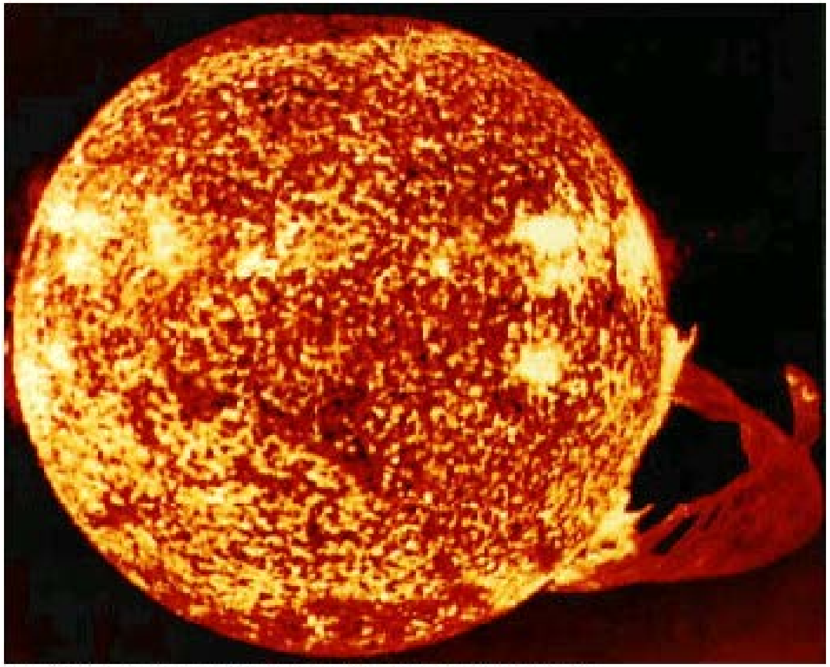


figure: M. Aliotta



NASA, Skylab space station December 19. 1973, solar flare reaching 588 000 km off solar surface

nuclear astrophysics
(nuclear fusion in stars)

cf. extrapolation of data

Why sub-barrier fusion?

two obvious reasons:

✓superheavy elements ✓ nuclear astrophysics

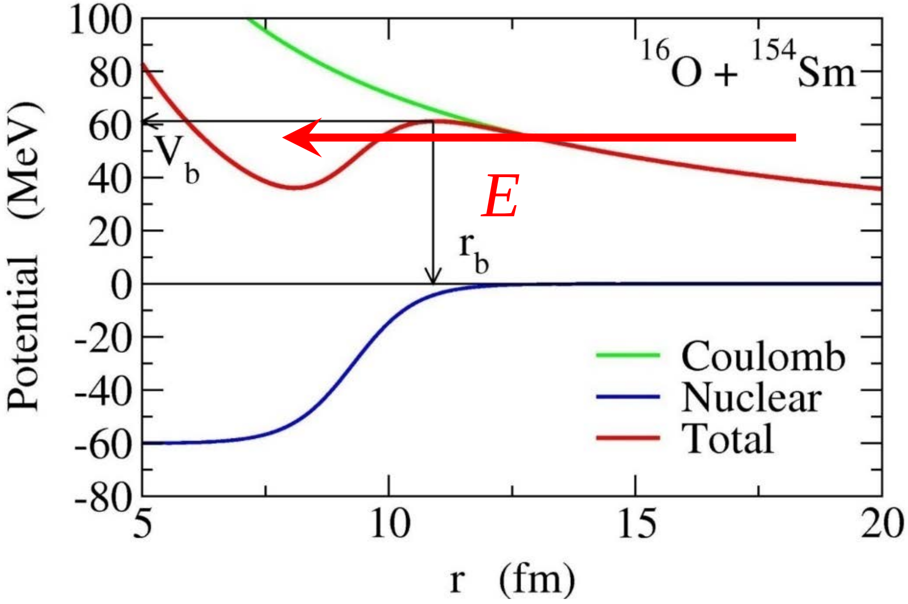
other reasons:

✓reaction dynamics

strong interplay between reaction and structure

cf. high *E* reactions: much simpler reaction mechanisms

✓many-particle tunneling



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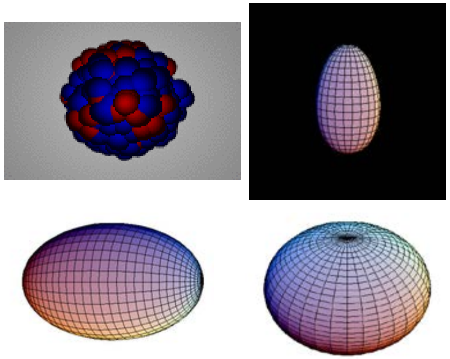


- many types of intrinsic degrees of freedom
  (several types of collective vibrations,
  deformation with several multipolarities)
- energy dependence of tunneling probability
  cf. alpha decay: fixed energy

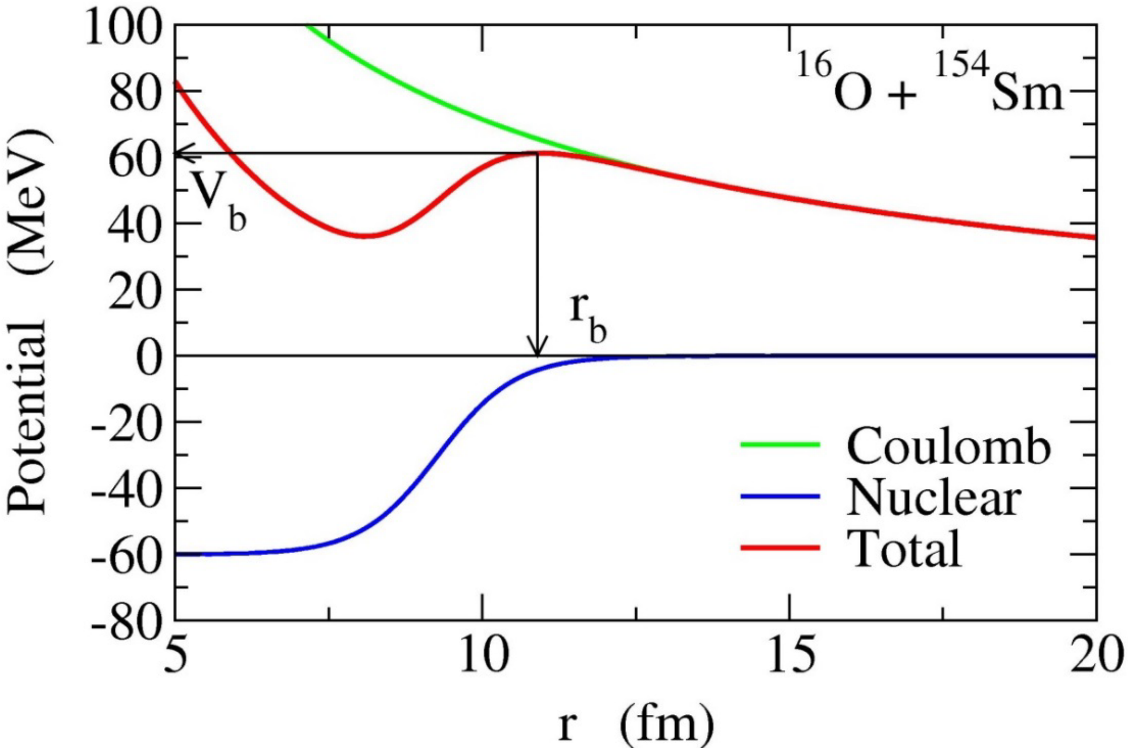
H.I. fusion reaction = an ideal playground to study quantum
tunneling with many degrees of freedom

The simplest approach to fusion: potential model

Potential model: $V(r)$ + absorption

$$\boxed{\sigma_{\text{fus}}(E)=\frac{\pi}{k^2}\sum_l (2l+1)P_l(E)}$$

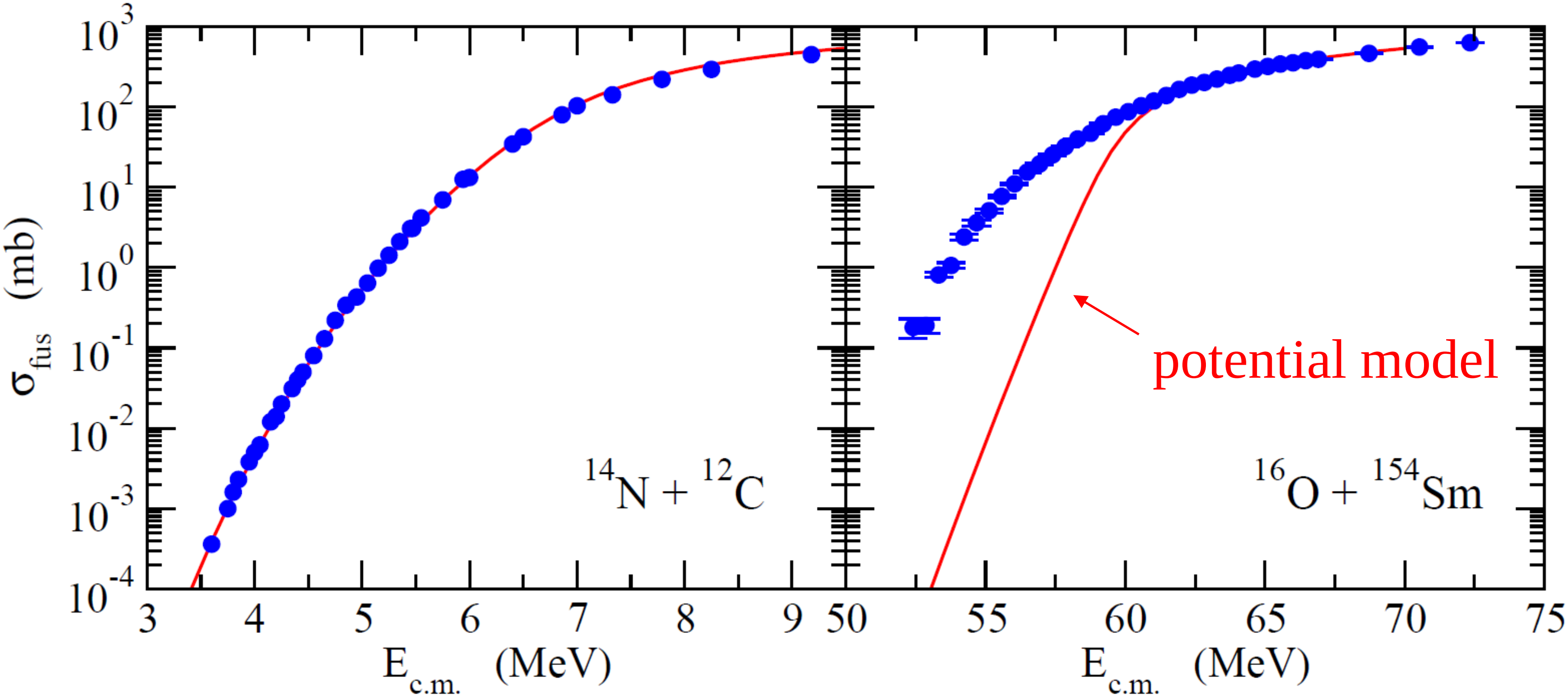
$P_l(E)$: barrier penetrability



Comparison with experimental data: large enhancement of σ_{fus}

Potential model: $V(r)$ + absorption

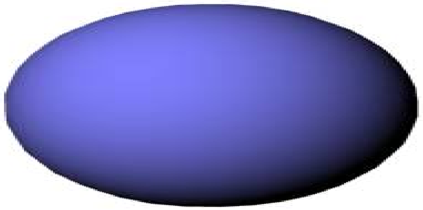
$$\boxed{\sigma_{\text{fus}} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)}$$



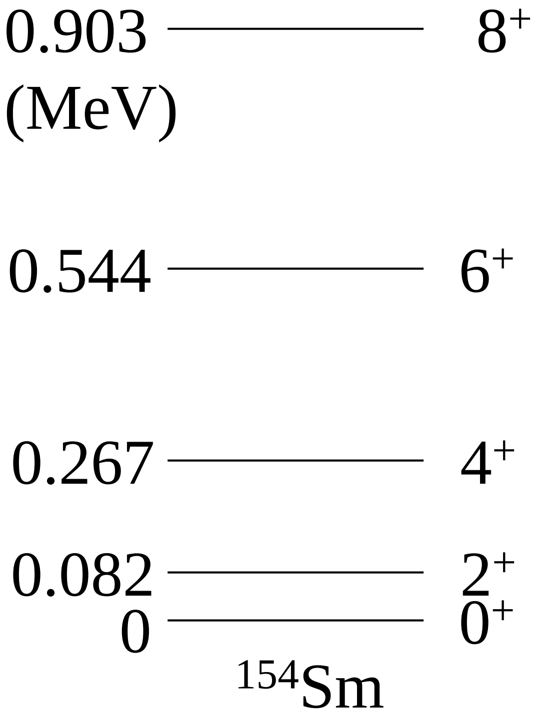
cf. seminal work:

R.G. Stokstad et al., PRL41('78) 465

^{154}Sm : a typical deformed nucleus

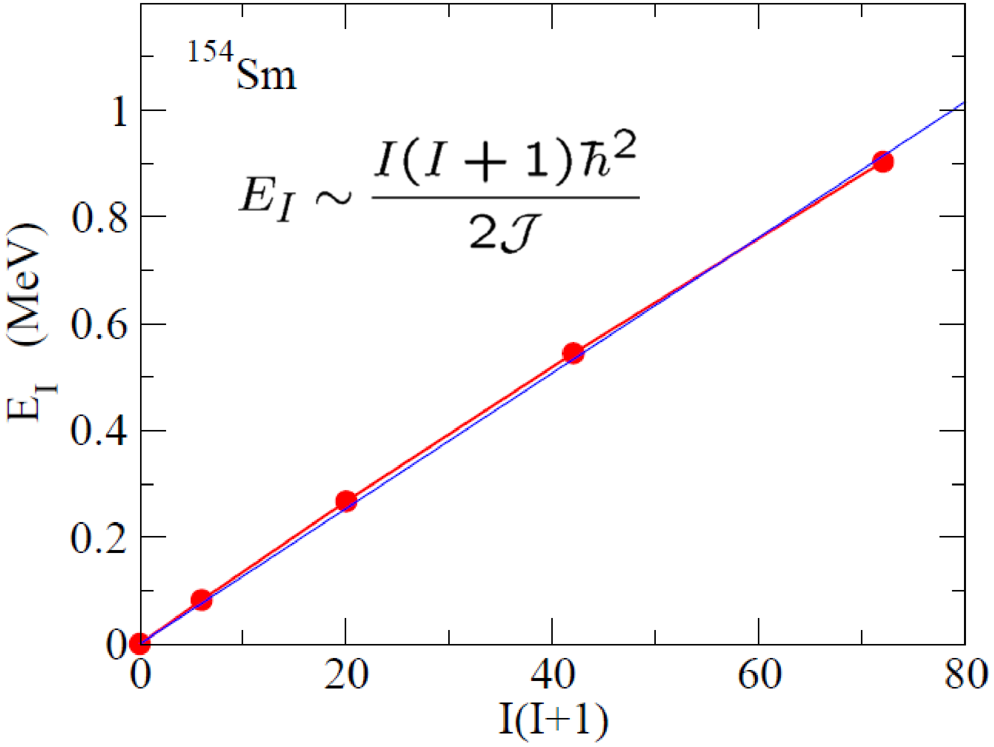


^{154}Sm

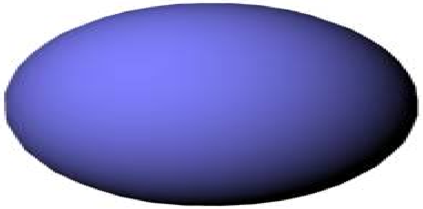


0.903 (MeV)	8^+
0.544	6^+
0.267	4^+
0.082	2^+
0	0^+

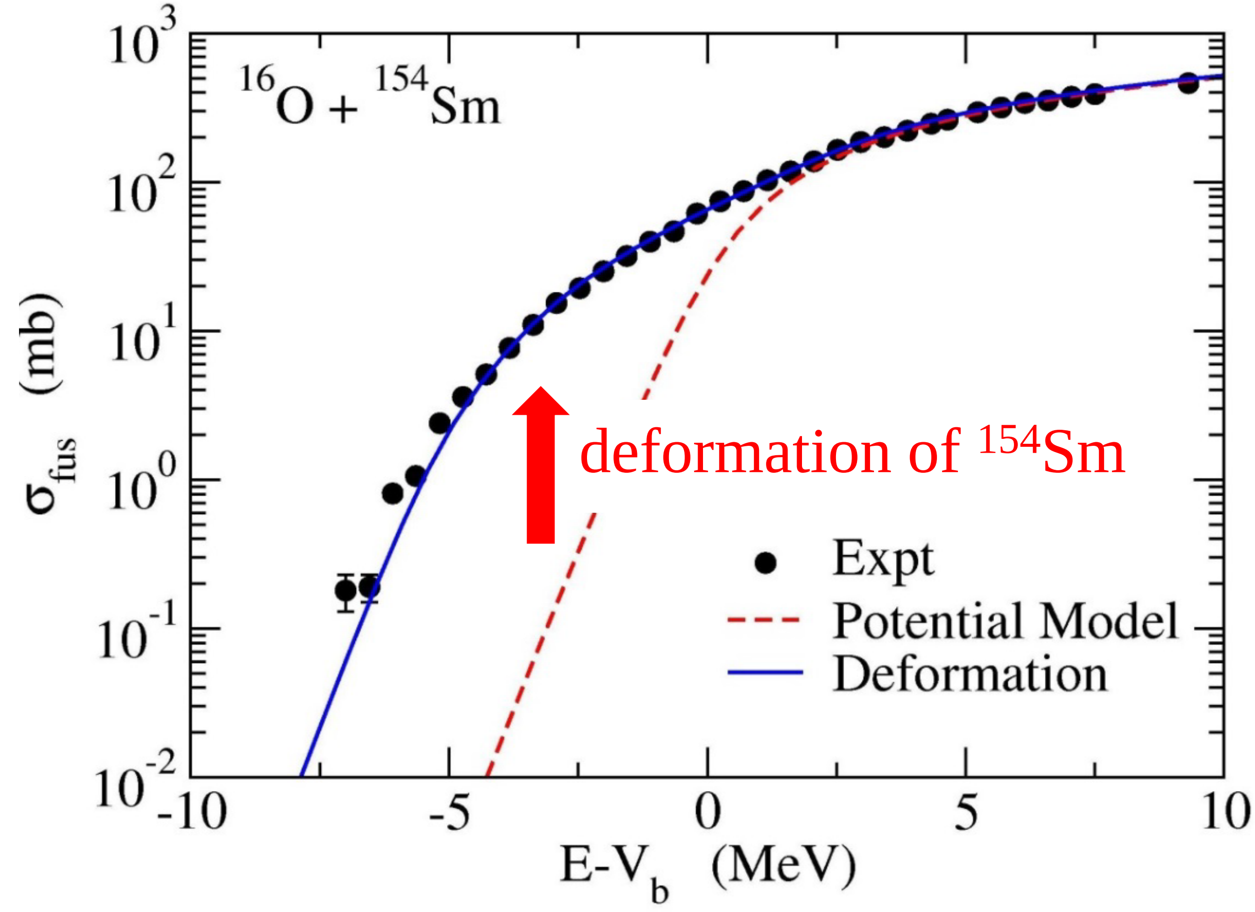
^{154}Sm



^{154}Sm : a typical deformed nucleus



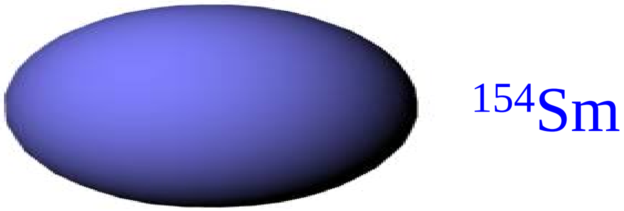
^{154}Sm

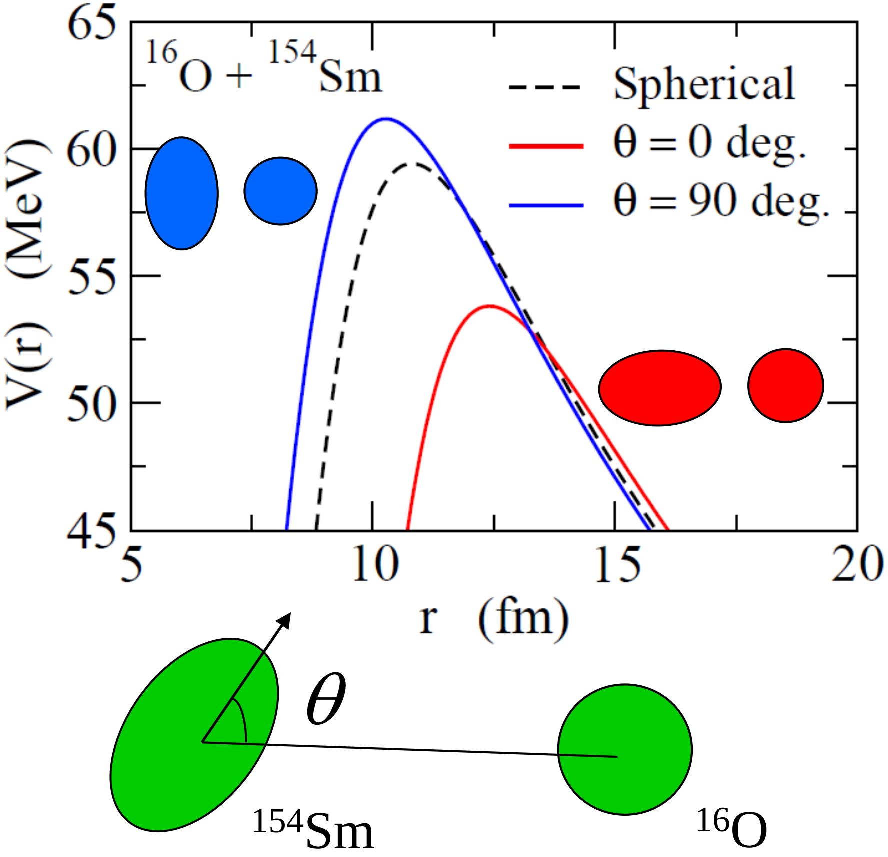


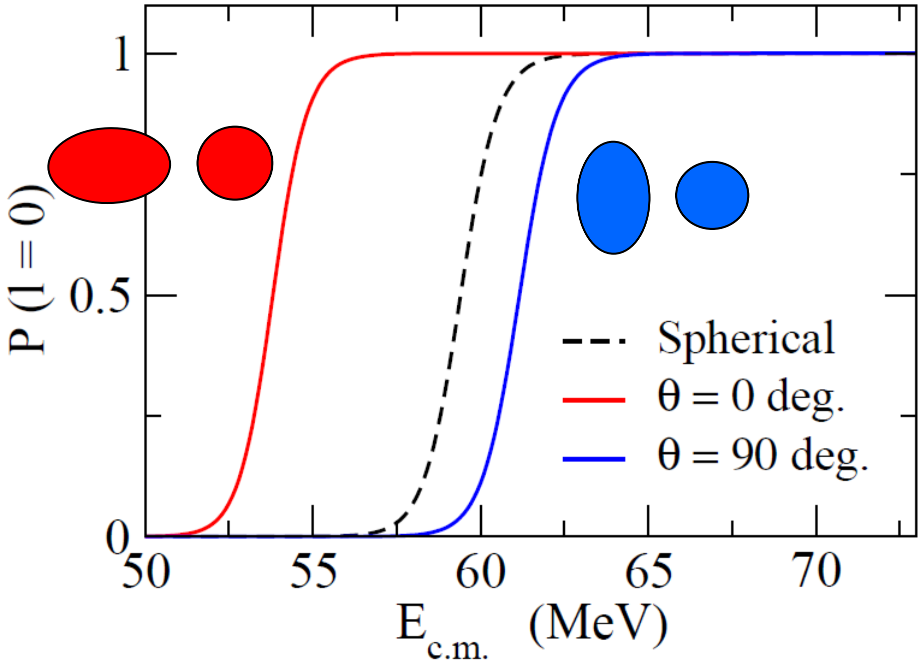
deformation of ^{154}Sm

Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus

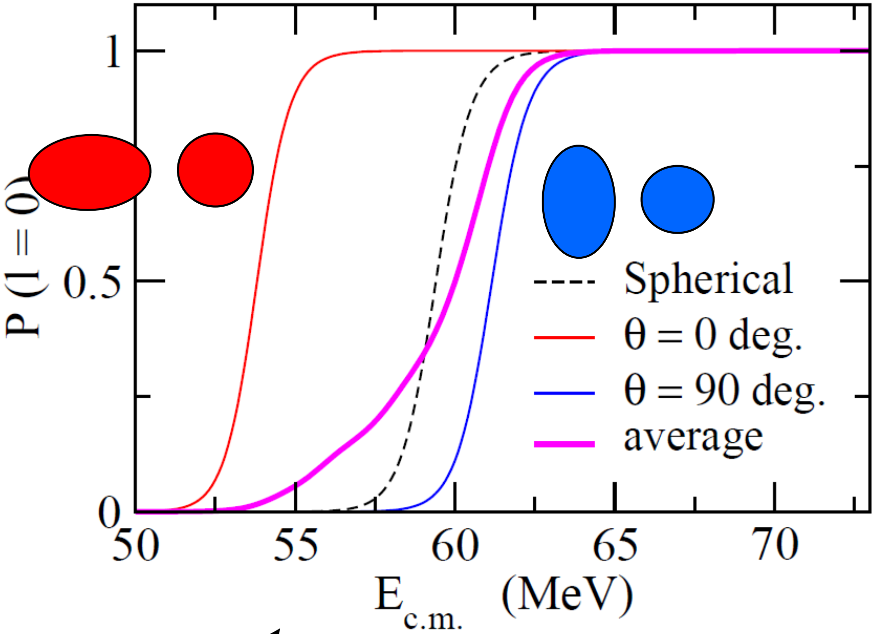


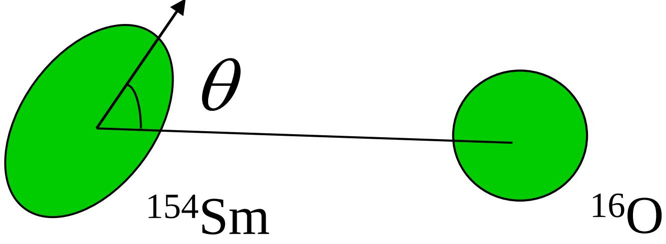




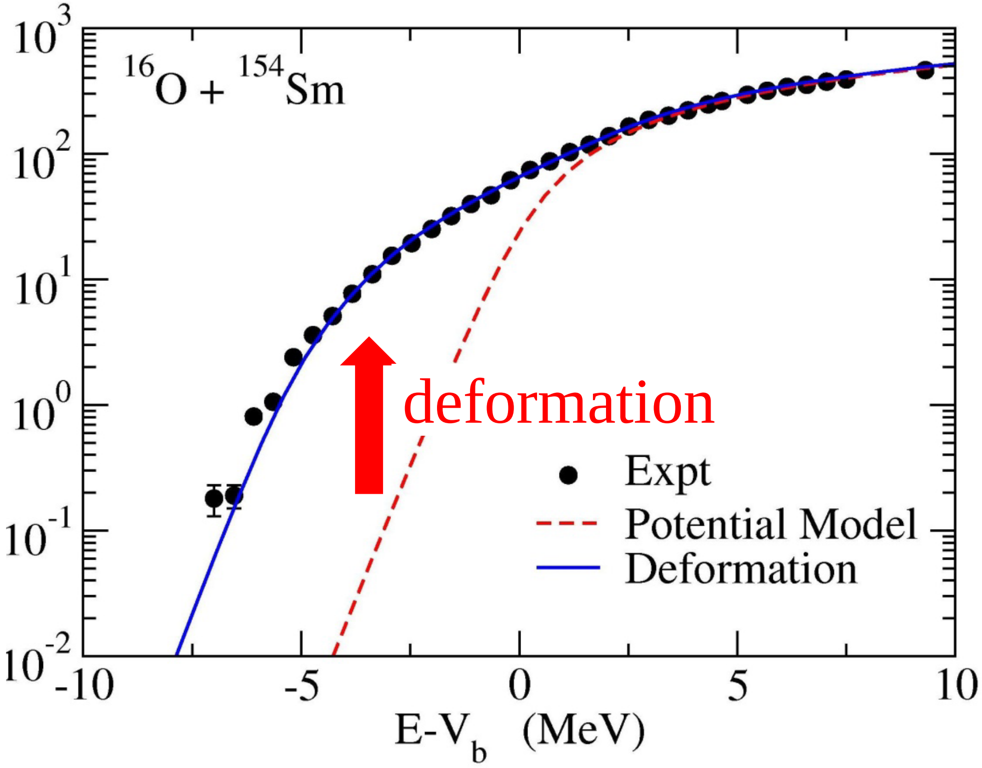
Effects of nuclear deformation

^{154}Sm : a typical deformed nucleus





$$\sigma_{\rm fus}(E)=\int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

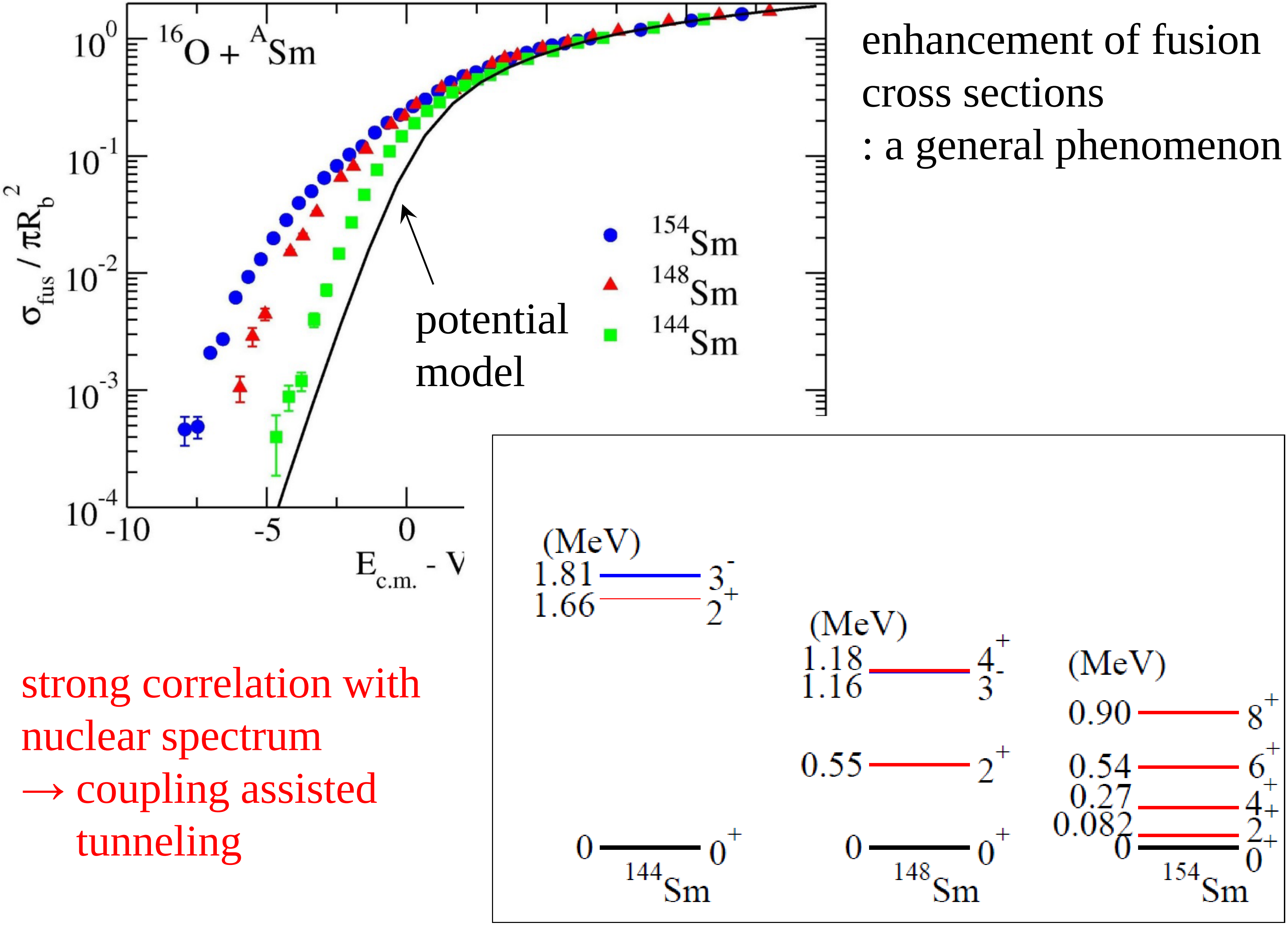


Fusion: strong interplay between nuclear structure and reaction

* Sub-barrier enhancement also for non-deformed targets:

couplings to low-lying collective excitations → coupling assisted tunneling

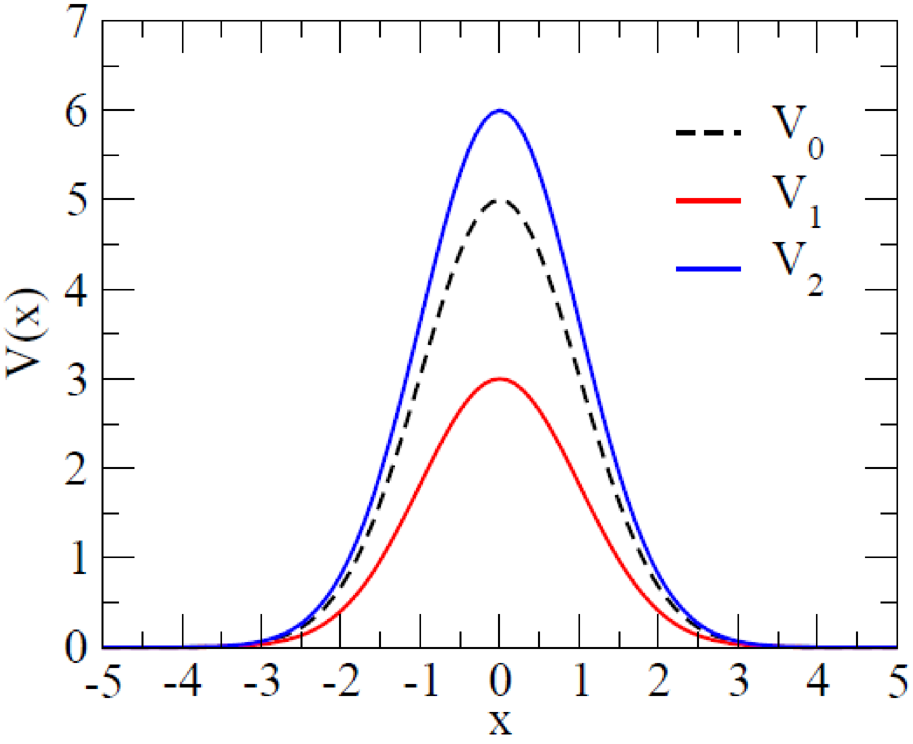
enhancement of fusion cross sections
: a general phenomenon

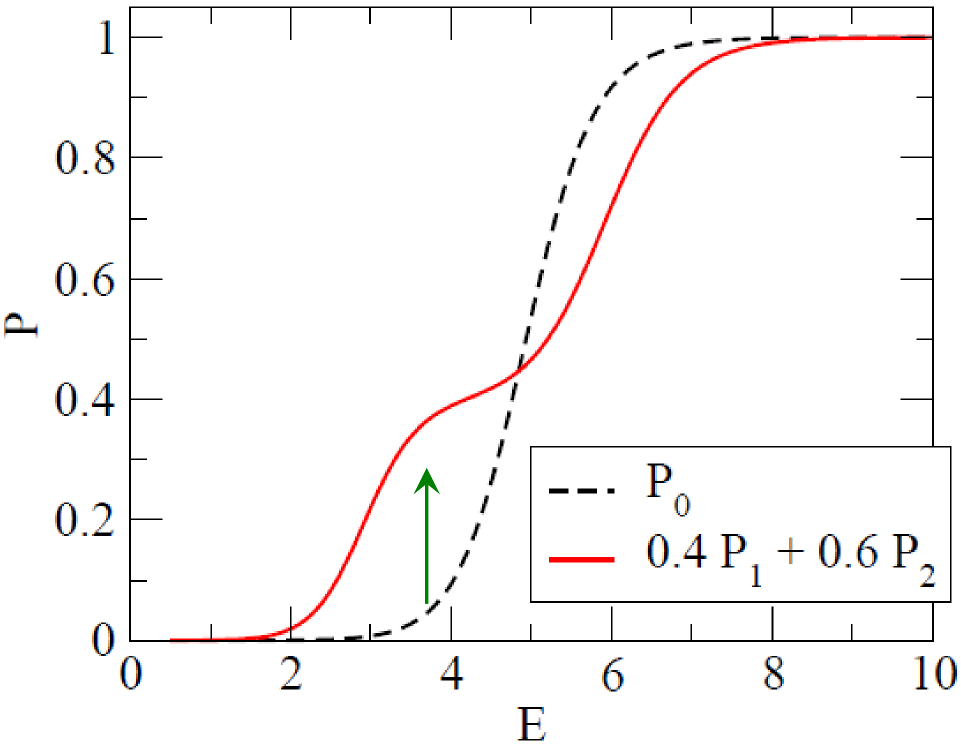


strong correlation with nuclear spectrum
→ coupling assisted tunneling

Enhancement of tunneling probability : a problem of two potential barriers

$$P(E) = P(E; V_0) \to w_1 P(E; V_1) + w_2 P(E; V_2)$$

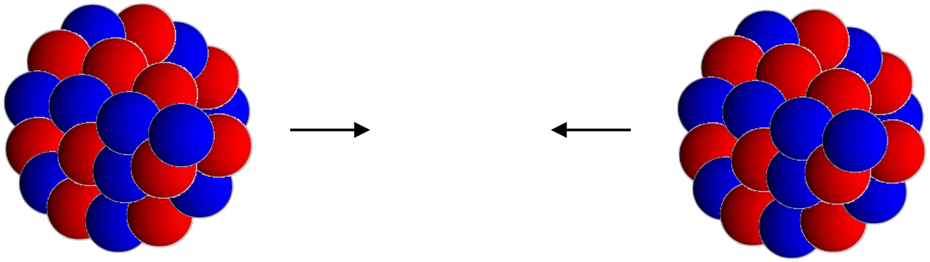




“barrier distribution” due to couplings to excited states in projectile/target nuclei

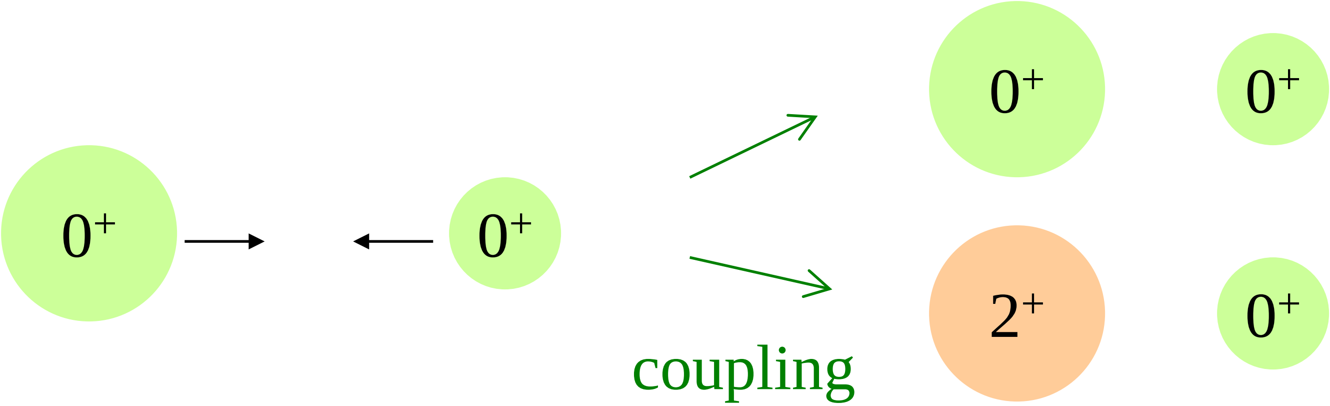
Coupled-channels method: a quantal scattering theory with excitations

many-body problem

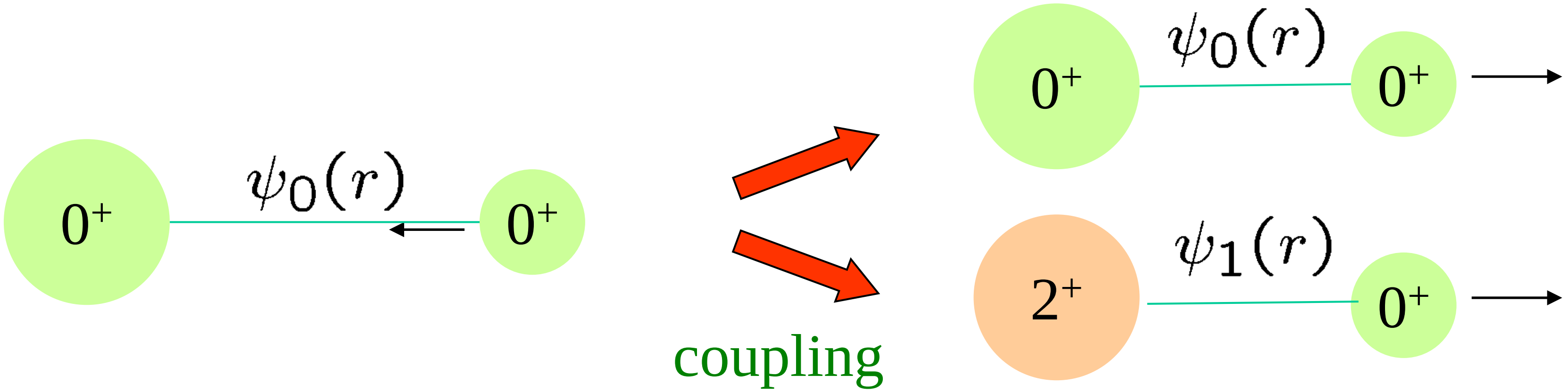


still very challenging

two-body problem, but with excitations
(coupled-channels approach)



Coupled-channels method: a quantal scattering theory with excitations

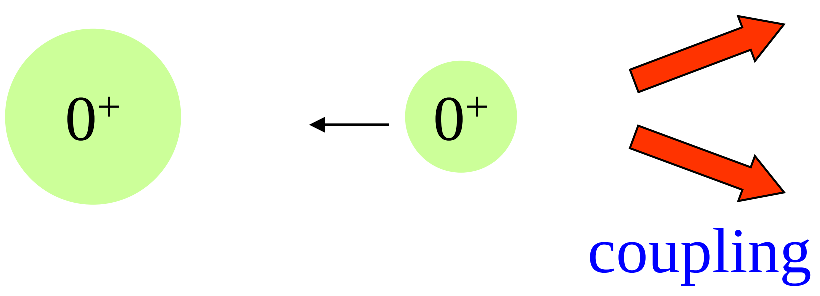


$$\left[-\frac{\hbar^2}{2\mu}\nabla^2+\overleftrightarrow{V}(r)-\overleftrightarrow{E}\right]\vec{\psi}(\boldsymbol{r})=0$$

if written down more explicitly:

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2+V_0(r)+\underset{\overset{\uparrow}{\text{excitation energy}}}{\epsilon_k}-E\right]\psi_k(\boldsymbol{r})+\sum_{k'}\langle\phi_k|\underset{\overset{\uparrow}{\text{excitation operator}}}{V_{\mathrm{coup}}}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r})=0$$

Coupled-channels method: a quantal scattering theory with excitations



coupling

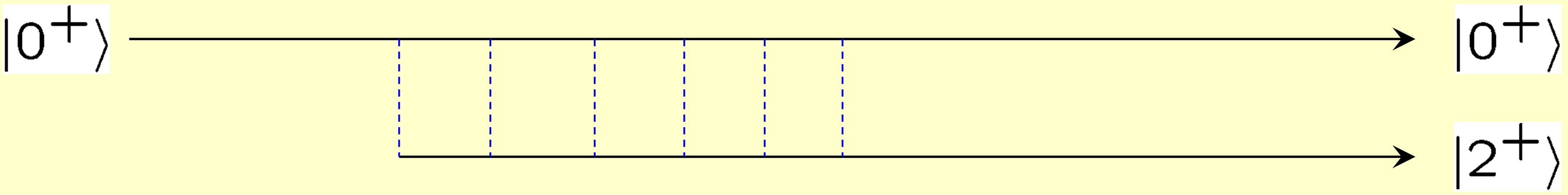
entrance channel

excited channel

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2+V_0(r)+\epsilon_k-E\right]\psi_k(\boldsymbol{r})+\sum_{k'}\langle\phi_k|V_{\rm coup}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r})=0$$

excitation energy

excitation operator



full order treatment of excitation/de-excitation dynamics during reaction

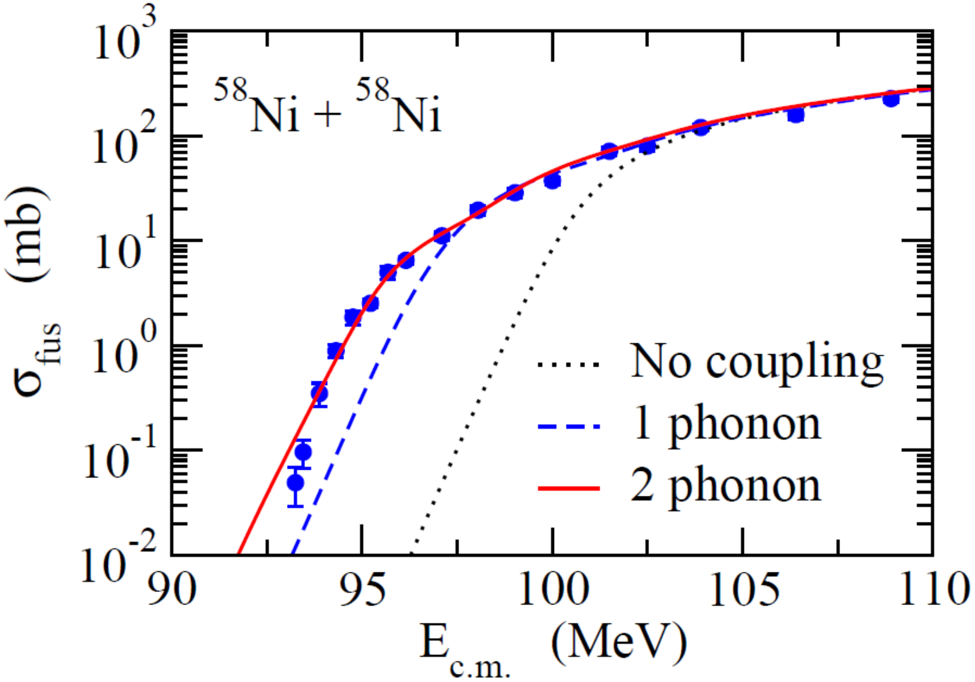
Inputs for C.C. calculations

i) Inter-nuclear potential

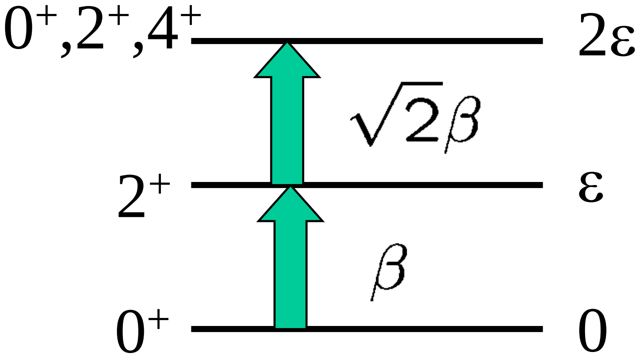
a fit to experimental data at above barrier energies

ii) Intrinsic degrees of freedom

in most of cases, (macroscopic) collective model
(rigid rotor / harmonic oscillator)



simple harmonic oscillator

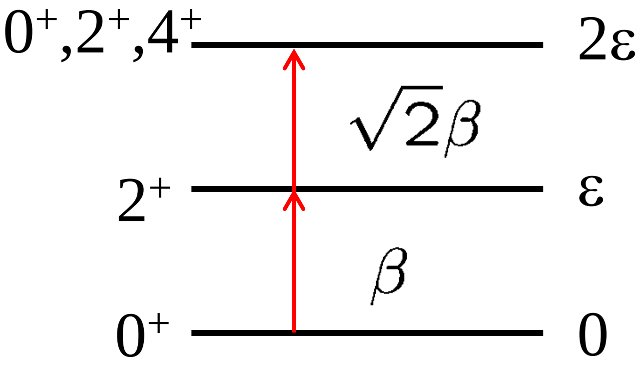


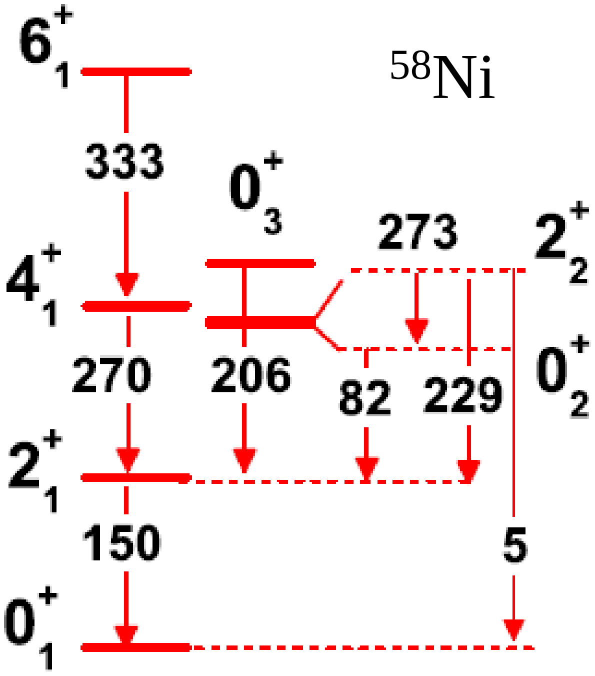
Further development: semi-microscopic modelling

K.H. and J.M. Yao, PRC91(‘15) 064606

CCFULL
+ microscopic nuclear structure
calculations
(GCM, Shell Model, IBM…..)

simple harmonic
oscillator

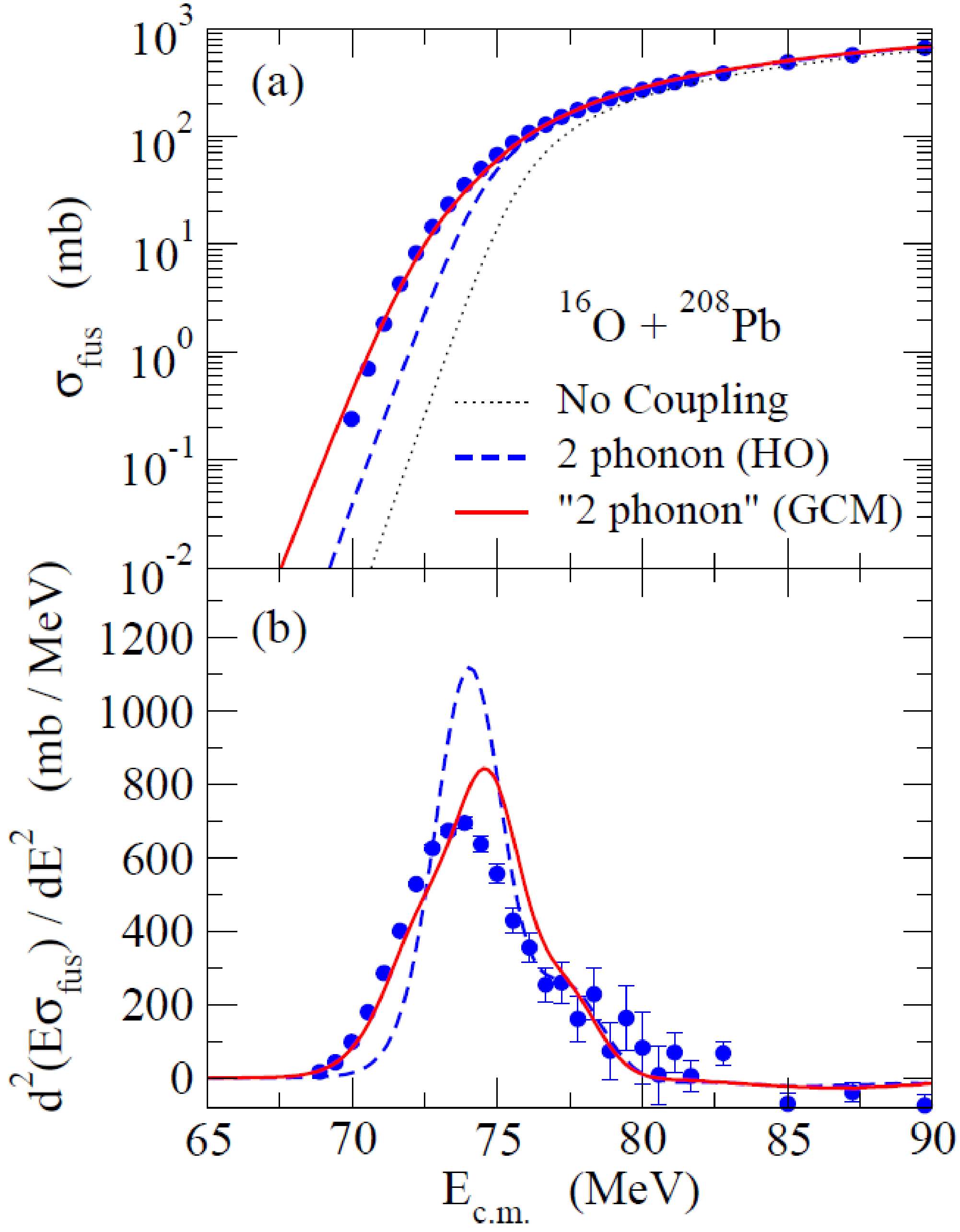


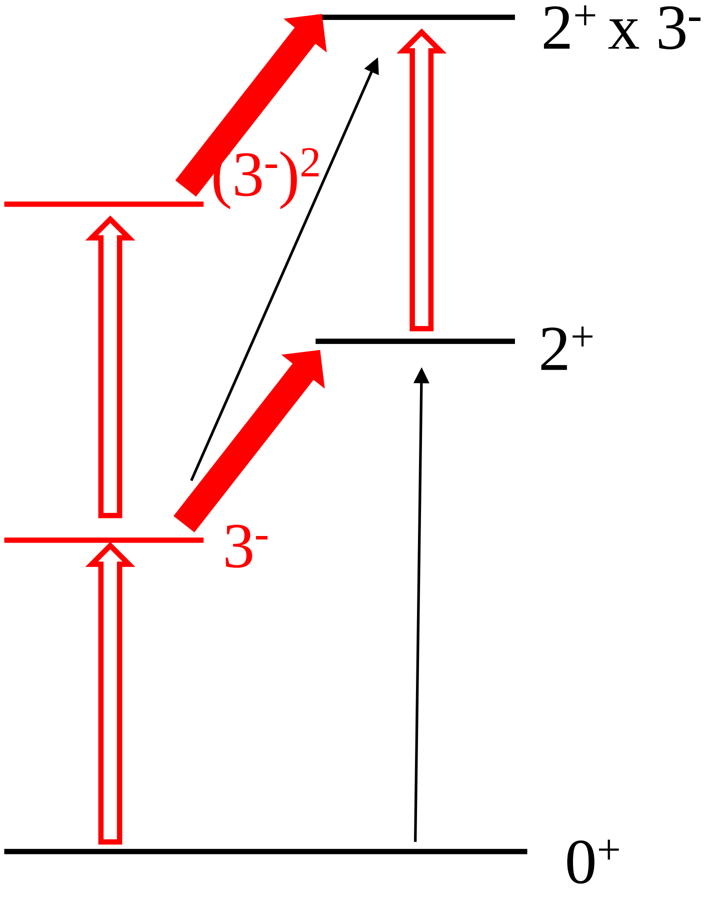


relativistic MF + GCM

anharmonicity of phonon spectra

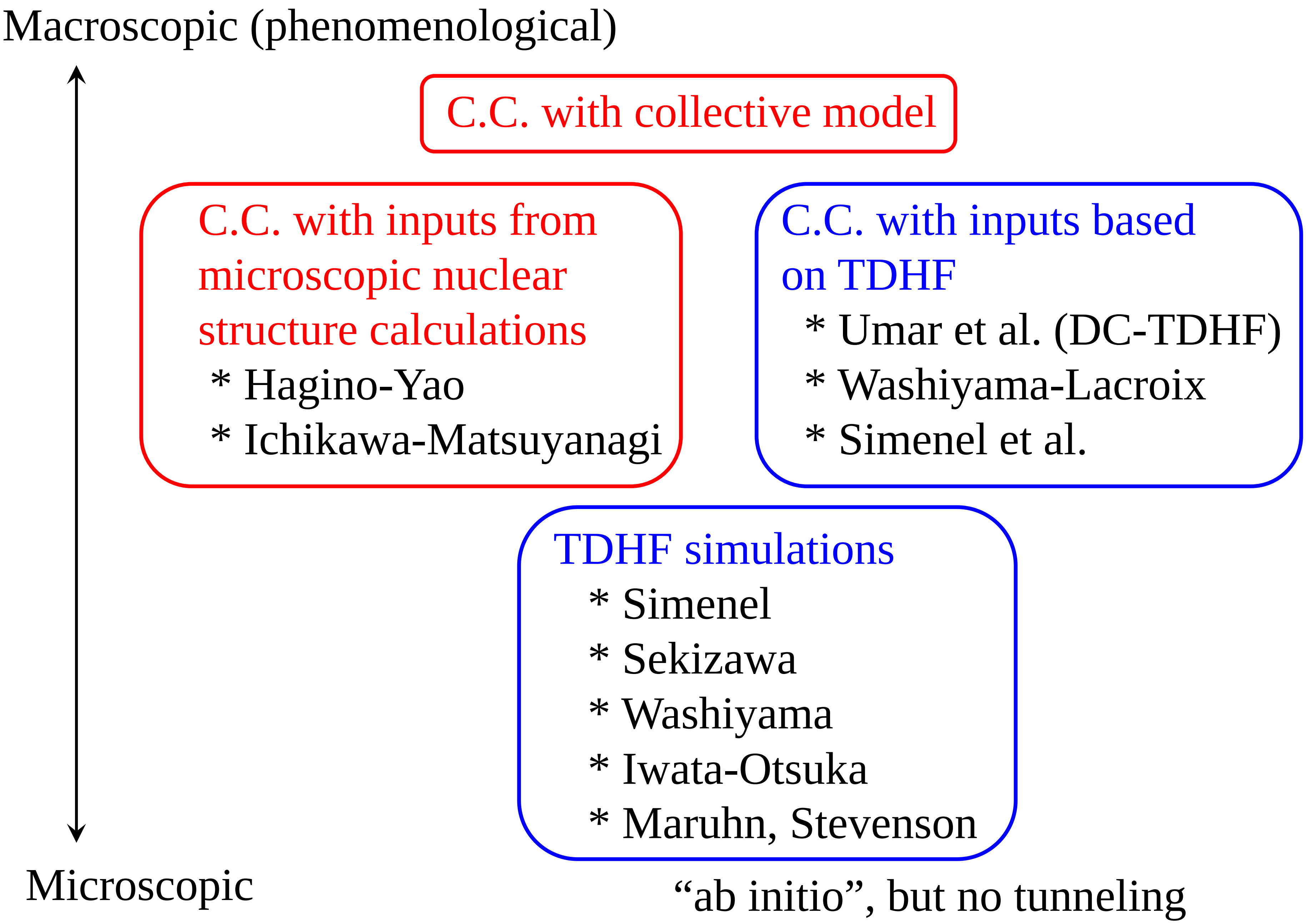
CCFULL with RMF+GCM





J.M. Yao and K.H.,
PRC94 (‘16) 11303(R)

From phenomenological approach to microscopic approach



Macroscopic (phenomenological)

C.C. with collective model

C.C. with inputs from microscopic nuclear structure calculations

- * Hagino-Yao
- * Ichikawa-Matsuyanagi

C.C. with inputs based on TDHF

- * Umar et al. (DC-TDHF)
- * Washiyama-Lacroix
- * Simenel et al.

TDHF simulations

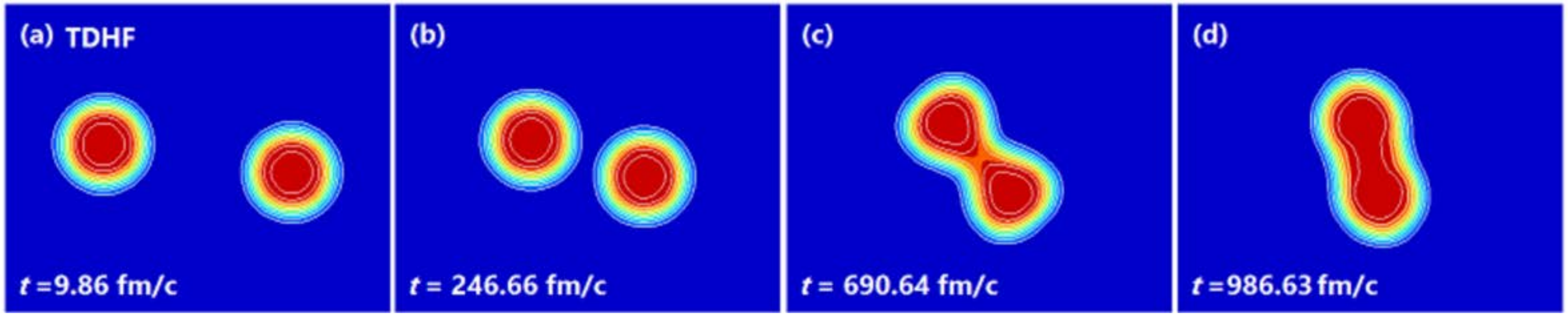
- * Simenel
- * Sekizawa
- * Washiyama
- * Iwata-Otsuka
- * Maruhn, Stevenson

Microscopic

“ab initio”, but no tunneling

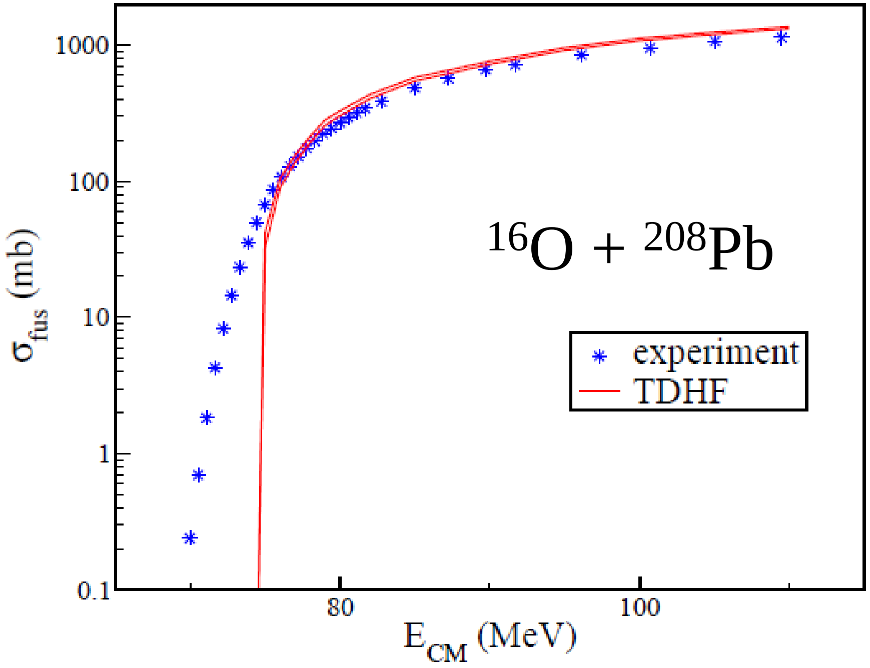
TDHF simulation

TDHF = Time Dependent Hartree-Fock



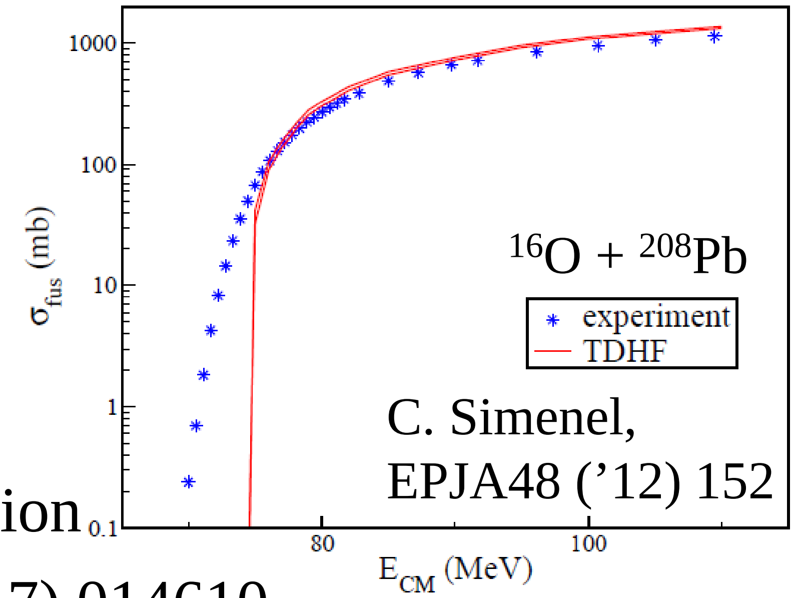
S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 (‘15) 020056

“ab-initio”, but no tunneling



C. Simenel,
EPJA48 (’12) 152

TDHF simulation



C. Simenel,
EPJA48 ('12) 152

"ab-initio", but no tunneling

✓ DC-TDHF (Umar, Oberacker, Maruhn)

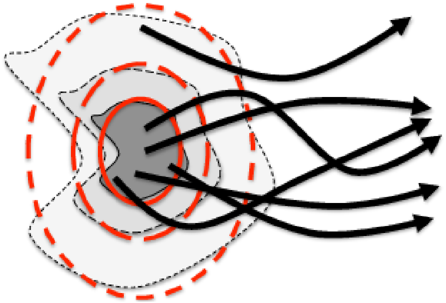
➤Beyond mean-field approximation

✓ Collective Hamiltonian and requantization

K. Wen and T. Nakatsukasa, PRC96 ('17) 014610

✓ Time-dependent Generator Coordinate Method (TD-GCM)

$$|\Psi(t)\rangle = \int dq\, f(q,t)|\Phi_q(t)\rangle$$



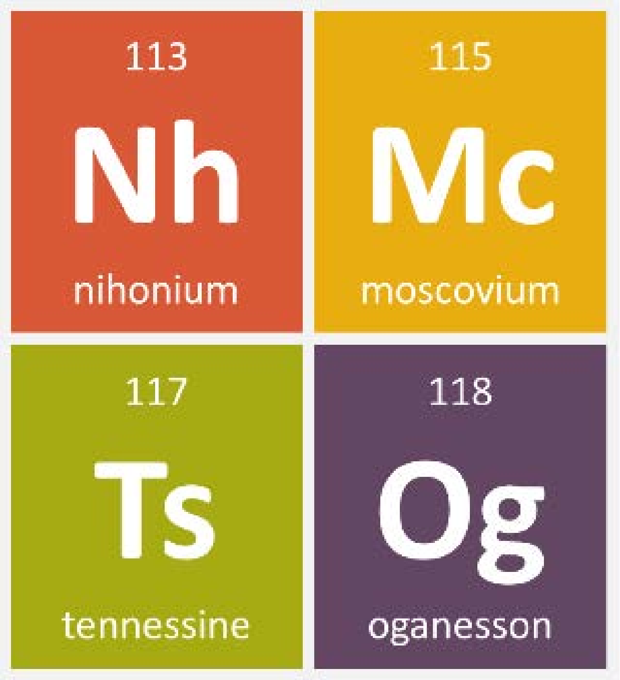
a linear superposition of many
TDHF trajectories (Slater determinants)

cf. Stochastic mean-field method
B. Yilmaz et al.,
PRC90 ('14) 054617

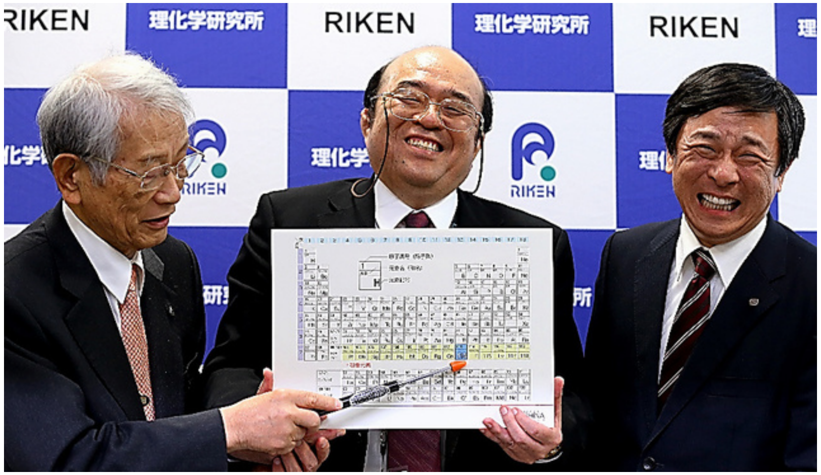
an important future direction

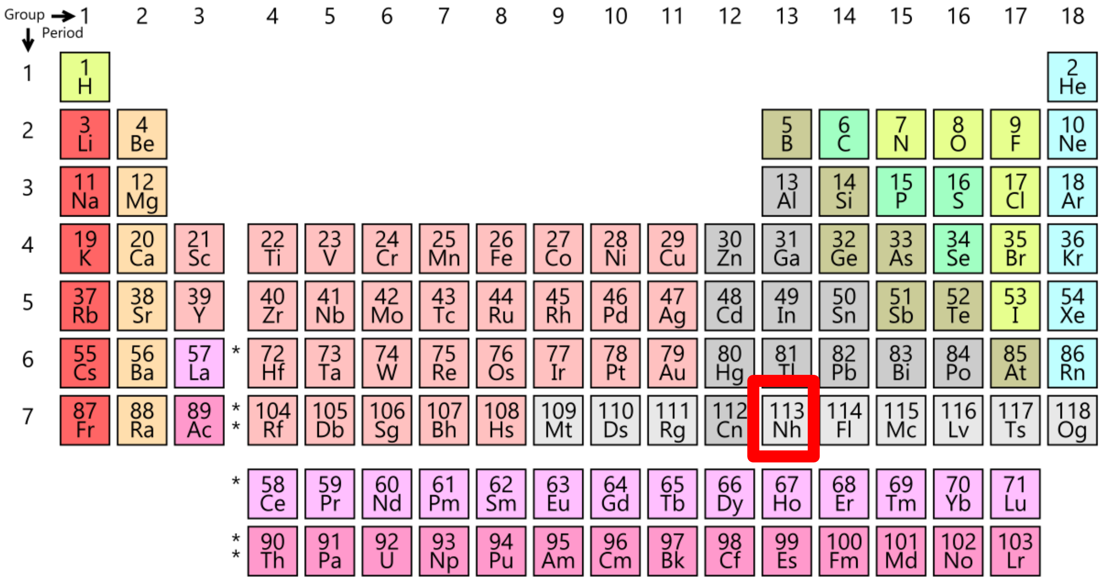
Fusion reactions for SHE

the element 113: Nh



November, 2016



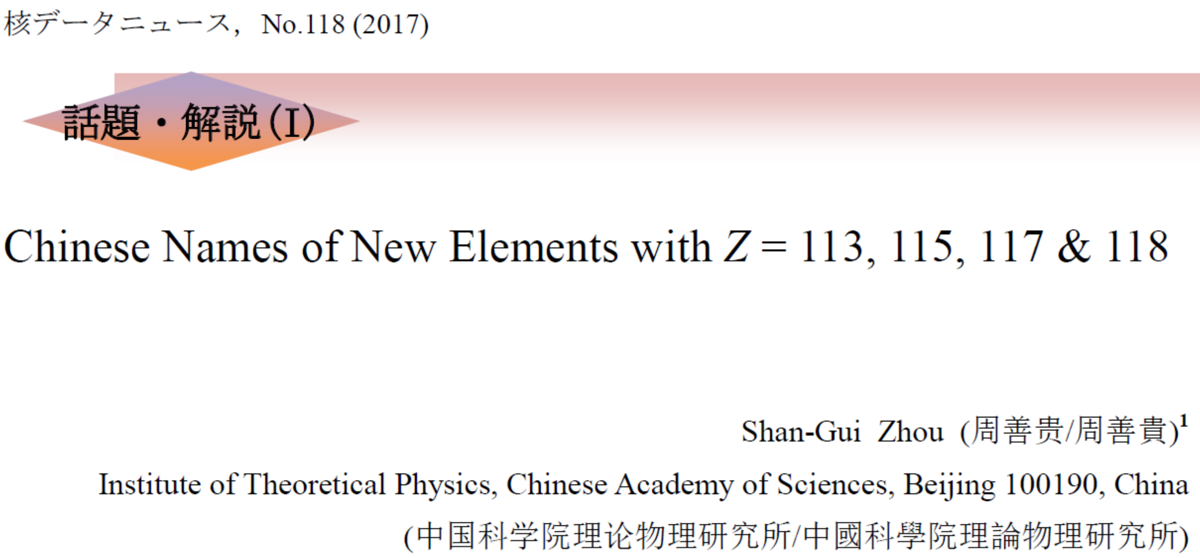


Group → / ↓ Period	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc		22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Wikipedia

Fusion reactions for SHE

the element 113: Nh


核データニュース，No.118 (2017)

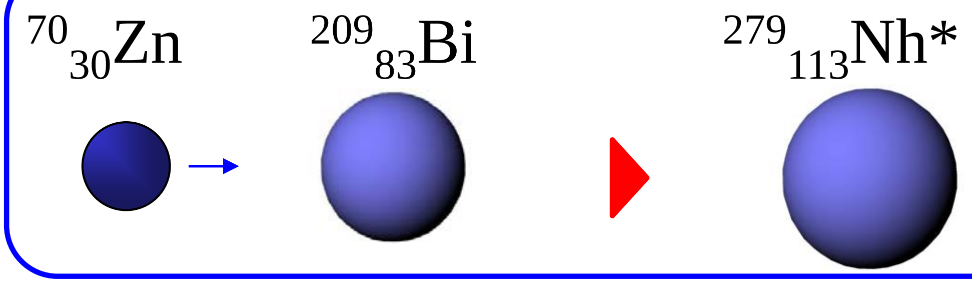
話題・解説 (I)

Chinese Names of New Elements with *Z* = 113, 115, 117 & 118

Shan-Gui Zhou (周善贵/周善貴)[1]

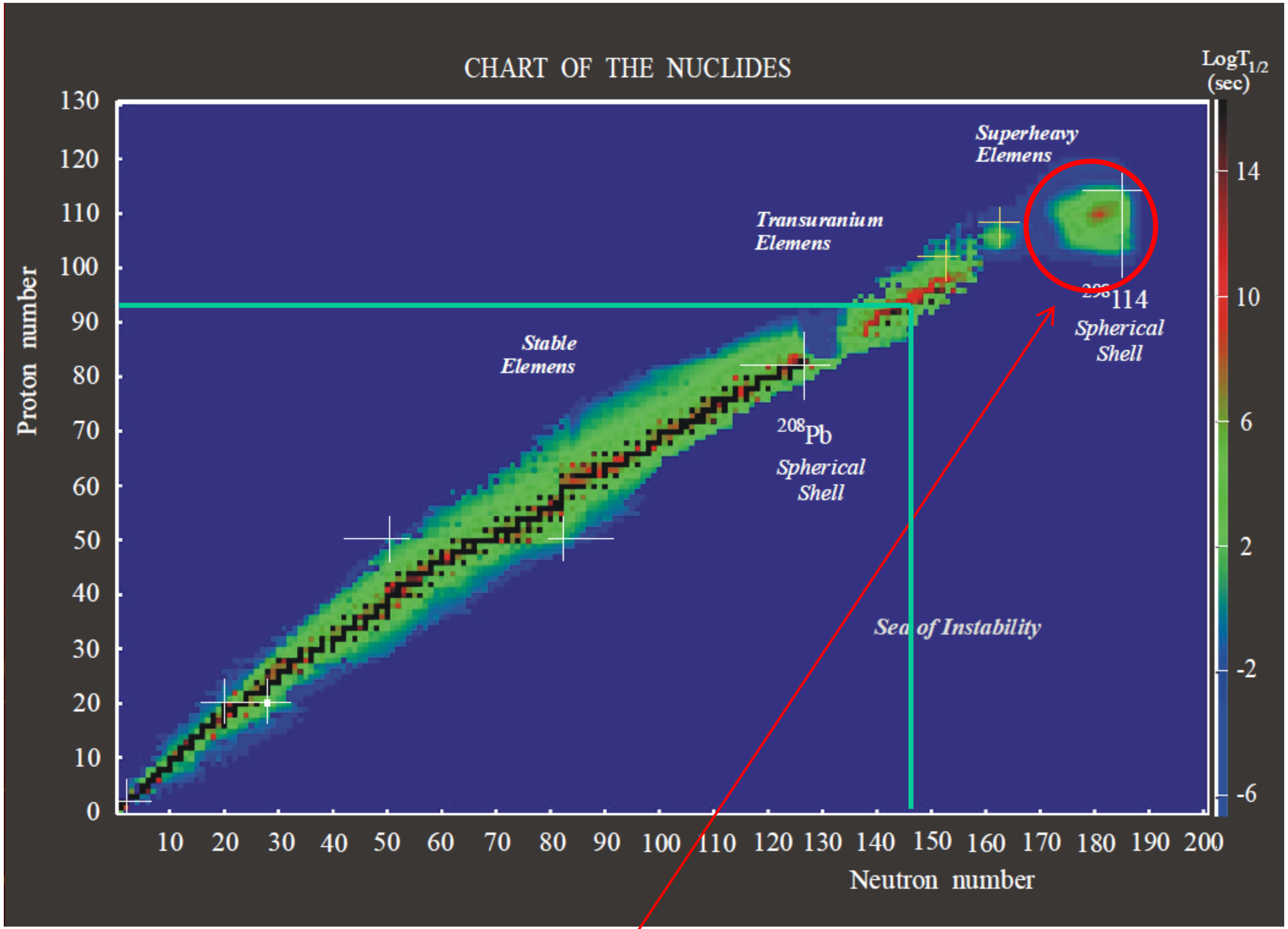
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
(中国科学院理论物理研究所/中國科學院理論物理研究所)

November, 2016



Heavy-ion fusion reaction

Future perspectives: Superheavy elements

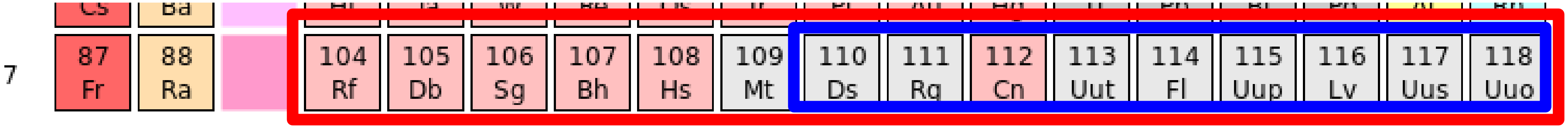


island of stability around Z=114, N=184

Yuri Oganessian

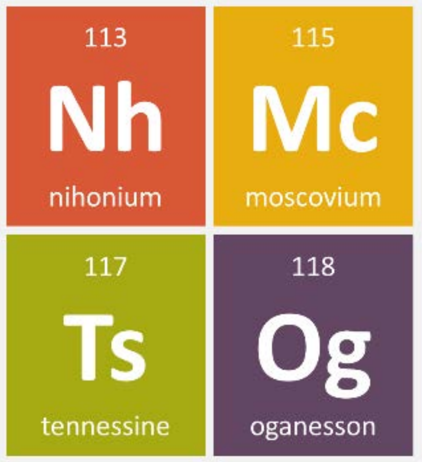
W.D. Myers and W.J. Swiatecki (1966), A. Sobiczewski et al. (1966)

who is she?



7	87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo

Z=110	Darmstadtium (Ds)	1994 Germany
Z=111	Roentgenium (Rg)	1994 Germany
Z=112	Copernicium (Cn)	1996 Germany
Z=113	**Nihonium (Nh)**	2003 Russia / 2004 Japan
Z=114	Flerovium (Fl)	1999 Russia
Z=115	**Moscovium (Mc)**	2003 Russia
Z=116	Livermoriun (Lv)	2000 Russia
Z=117	**Tennessine (Ts)**	2010 Russia
Z=118	**Oganesson (Og)**	2002 Russia

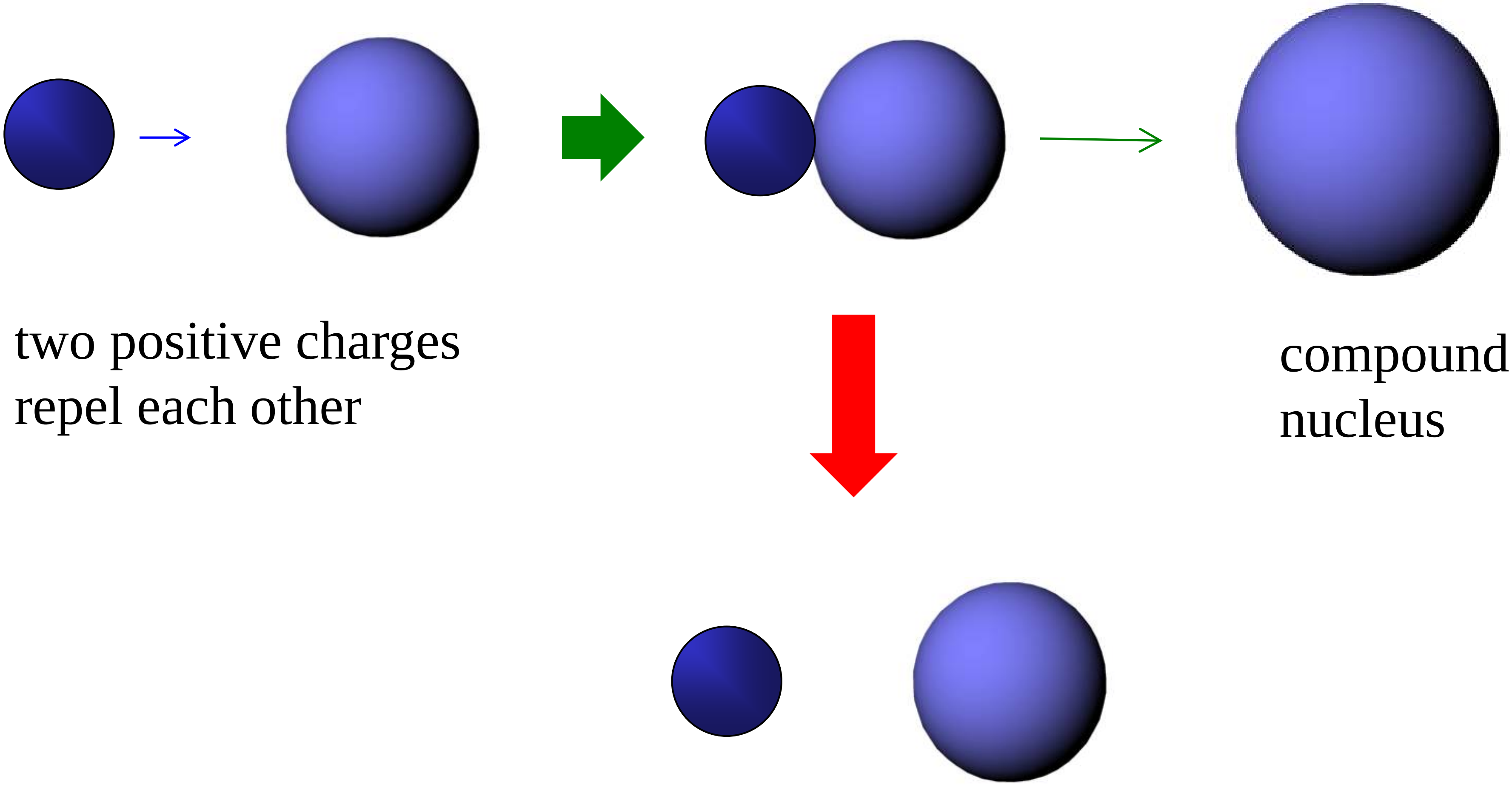


How to synthesize SHE?

Nuclear fusion reactions

e.g.,

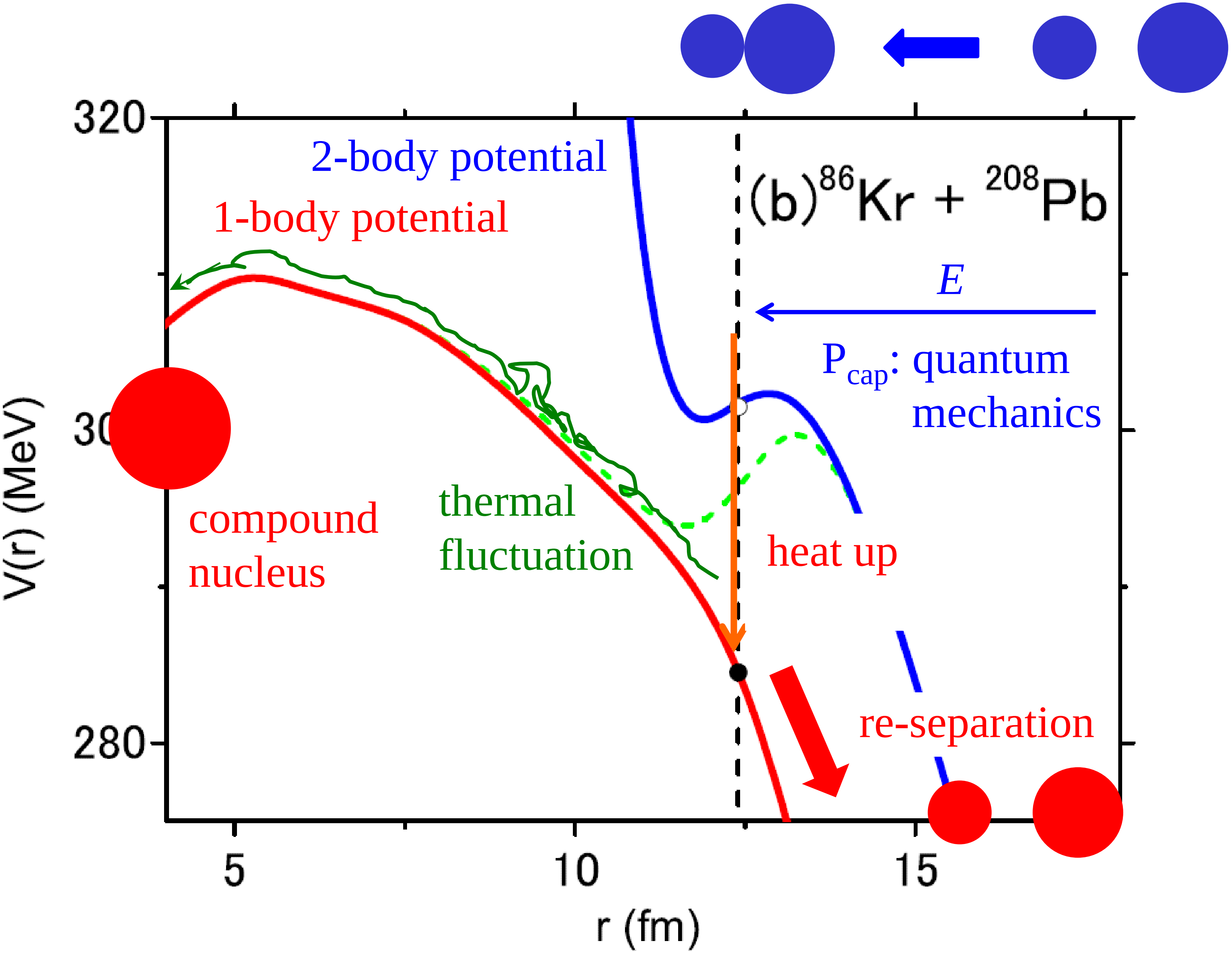
$^{70}\text{Zn} + {}^{209}\text{Bi} \longrightarrow {}^{279}113$



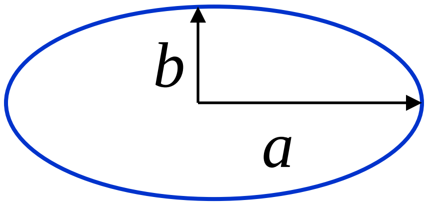
two positive charges repel each other

compound nucleus

re-separation


2-body potential
1-body potential
(b)^{86}Kr + ^{208}Pb
E
P_{cap}: quantum mechanics
compound nucleus
thermal fluctuation
heat up
re-separation
320
300
280
5
10
15
V(r) (MeV)
r (fm)

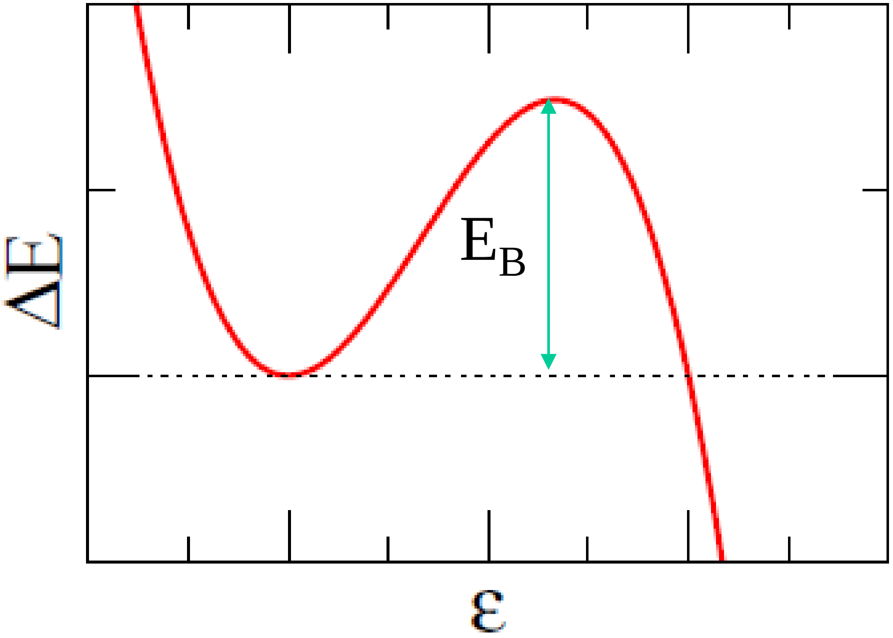
(note) fission barrier in the liquid drop model



$$a = R\cdot(1+\epsilon)$$
$$b = R\cdot(1+\epsilon)^{-1/2}$$

$ab^2 = R^3 =$ constant

$$\begin{aligned}\Delta E &= \Delta E_{\rm surf} + \Delta E_{\rm coul} \\ &= E_S^{(0)}\left\{\frac{2}{5}(1-x)\epsilon^2 - \frac{4}{105}(1+2x)\epsilon^3 + \cdots\right\}\end{aligned}$$

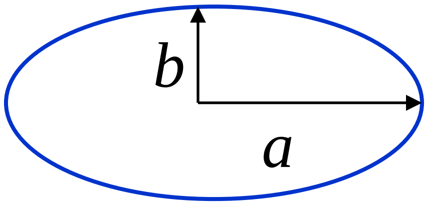


$$E_S^{(0)} = +a_S A^{2/3}$$

$$x \equiv \frac{E_C^{(0)}}{2E_S^{(0)}} = \frac{a_C}{2a_S}\cdot\frac{Z^2}{A} \sim \frac{1}{53.3}\cdot\frac{Z^2}{A}$$

$$E_C^{(0)} = a_C Z(Z-1)/A^{1/3}$$

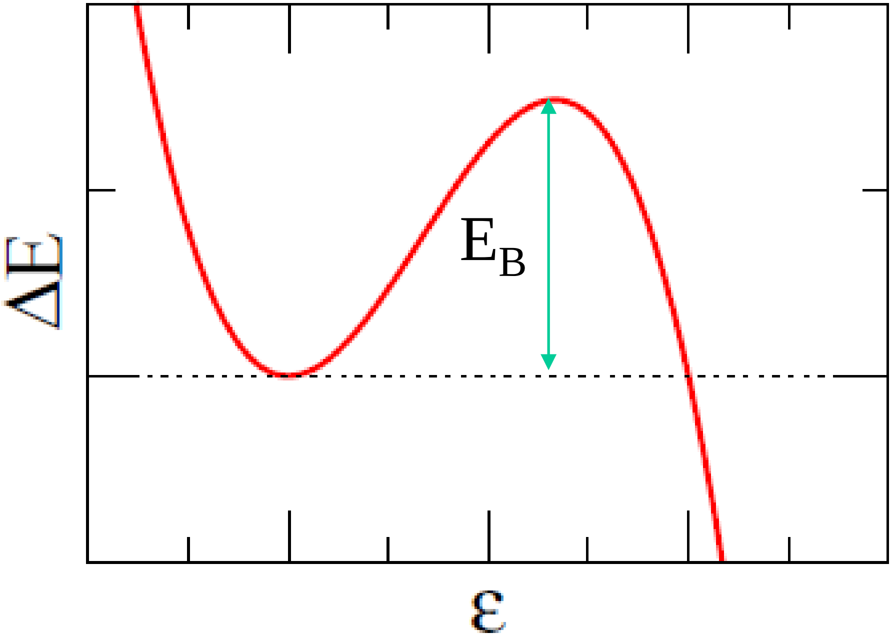
(note) fission barrier in the liquid drop model



$$a = R\cdot(1+\epsilon)$$
$$b = R\cdot(1+\epsilon)^{-1/2}$$

$$ab^2 = R^3 = \text{constant}$$

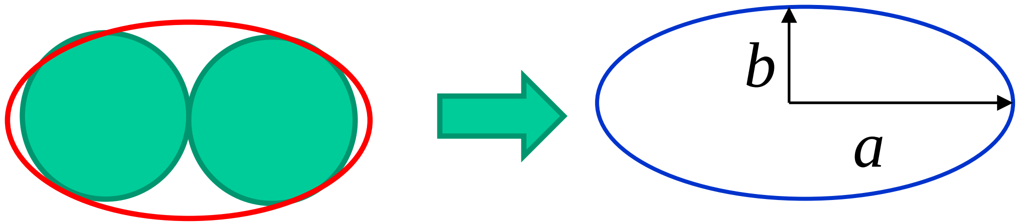
$$\begin{aligned}\Delta E &= \Delta E_{\text{surf}} + \Delta E_{\text{coul}}\\ &= E_S^{(0)}\left\{\frac{2}{5}(1-x)\epsilon^2 - \frac{4}{105}(1+2x)\epsilon^3 + \cdots\right\}\end{aligned}$$



fission barrier:

$$\epsilon_B = \frac{21(1-x)}{3(1+2x)}$$
$$E_B = \frac{98}{15}\cdot\frac{(1-x)^3}{(1+2x)^2}\cdot E_S^{(0)}$$

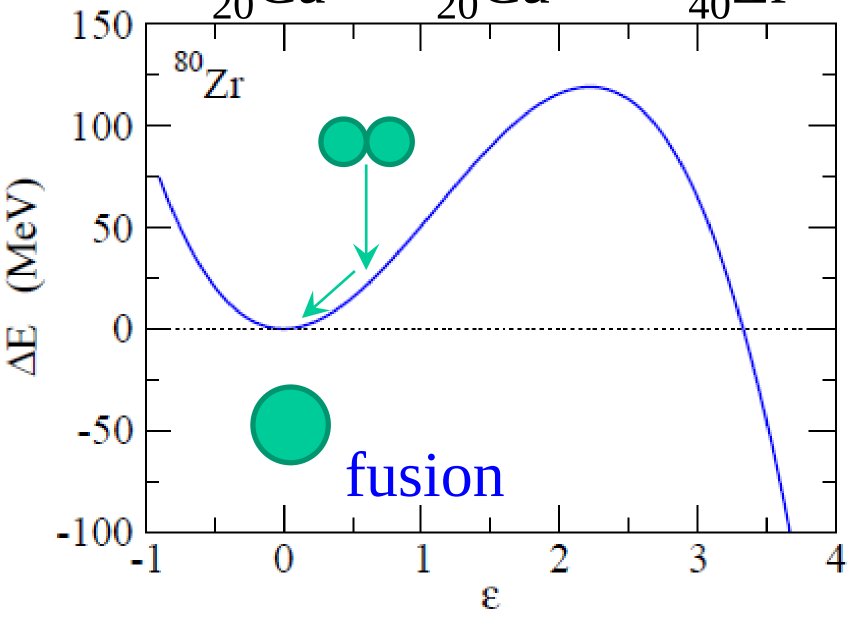
if two identical nuclei contact :



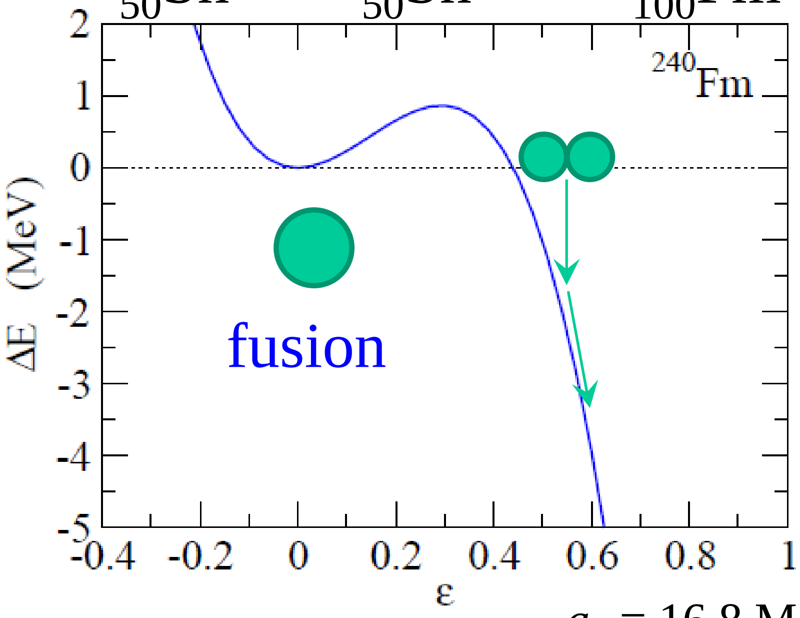
$$a = R_0 \cdot (1+\epsilon)$$
$$b = R_0 \cdot (1+\epsilon)^{-1/2}$$

$$\frac{a}{b} \sim \frac{2R}{R} = 2 \rightarrow \epsilon \sim 0.587$$

$^{40}_{20}\mathrm{Ca} + ^{40}_{20}\mathrm{Ca} \longrightarrow ^{80}_{40}\mathrm{Zr}$



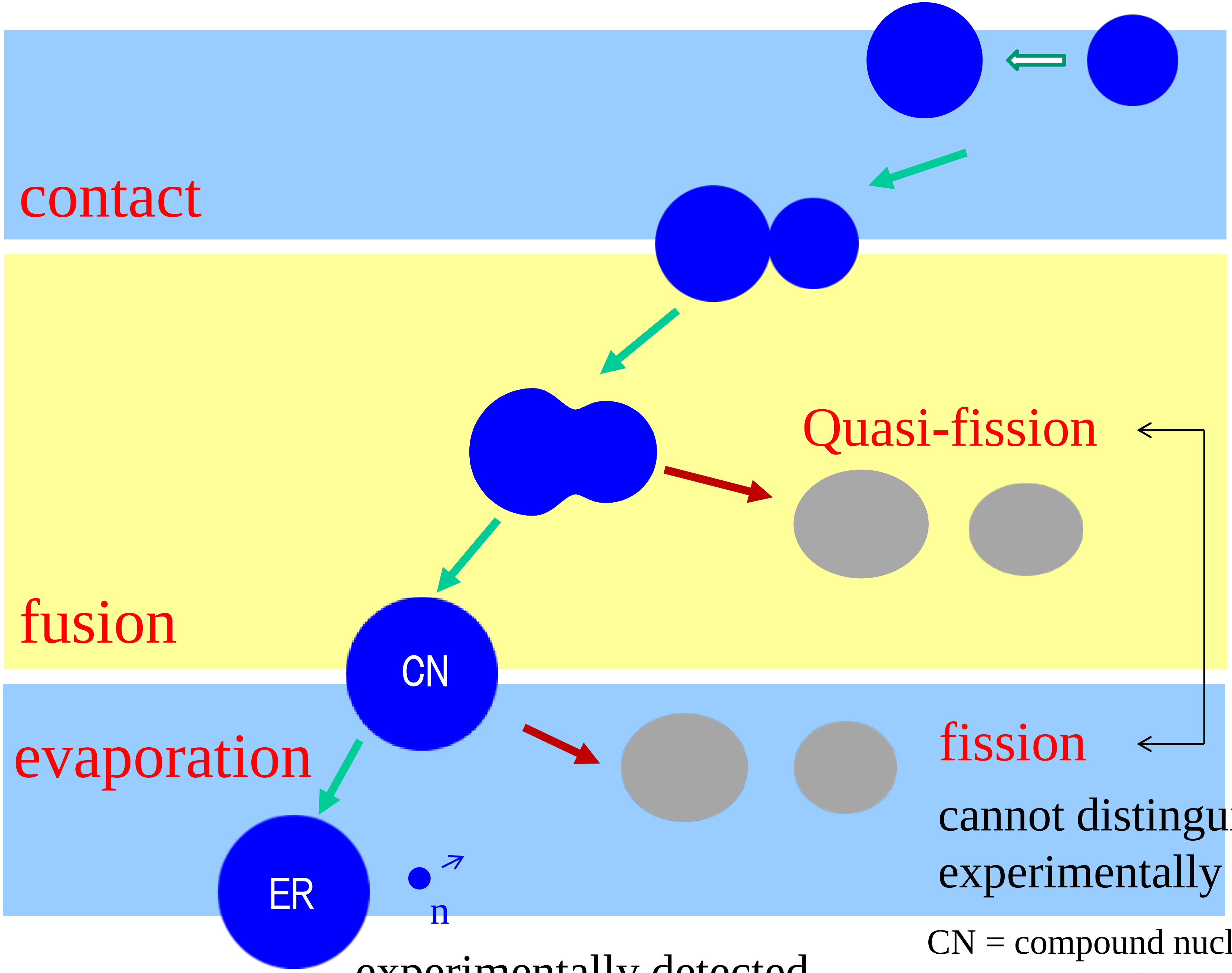
$^{120}_{50}\mathrm{Sn} + ^{120}_{50}\mathrm{Sn} \longrightarrow ^{240}_{100}\mathrm{Fm}$



threshold: $Z_1 * Z_2 = 1600 \sim 1800$

$a_S = 16.8$ MeV

$a_C = 0.72$ MeV



contact

fusion

Quasi-fission

evaporation

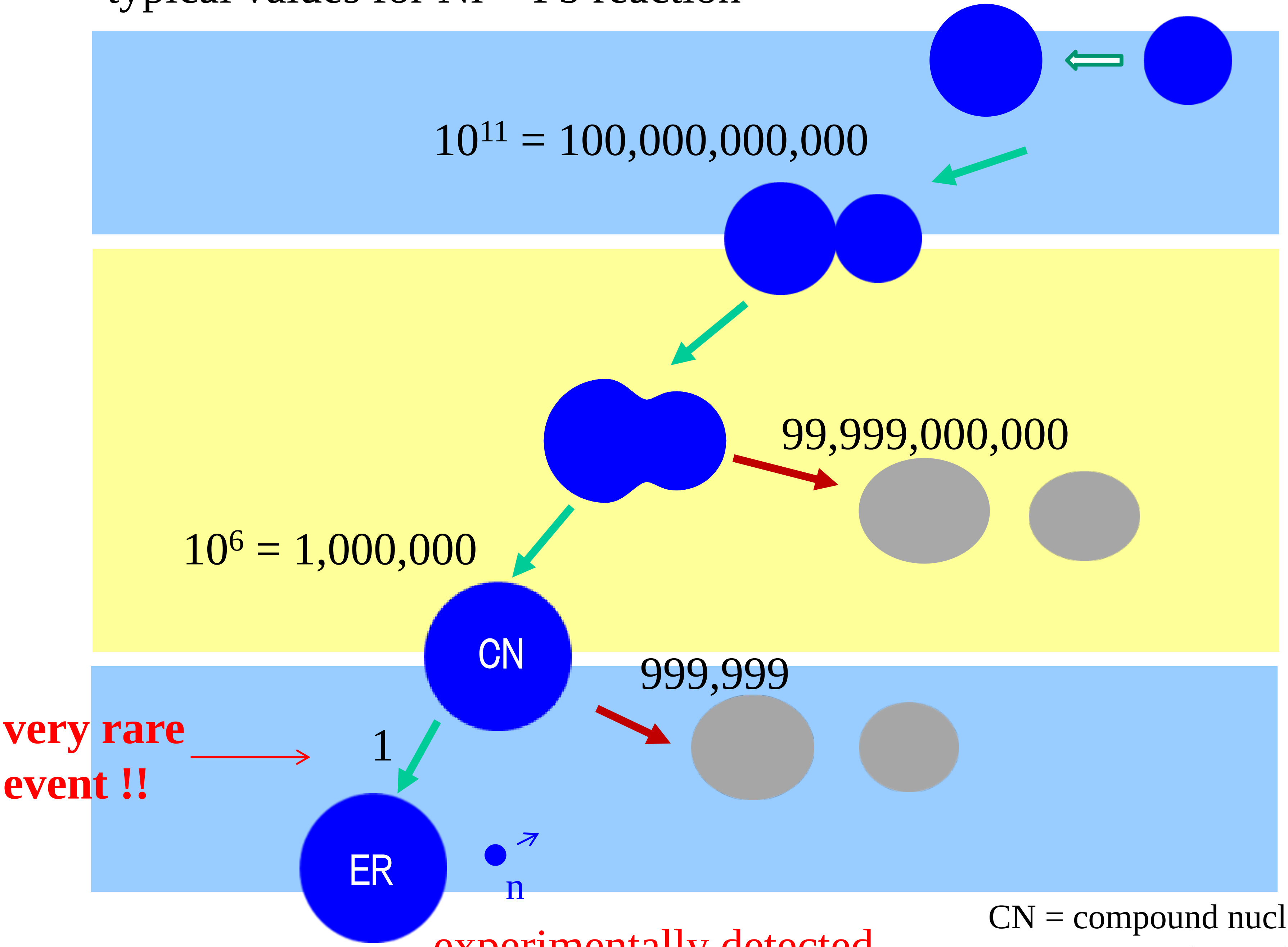
fission

cannot distinguish experimentally

experimentally detected

CN = compound nucleus
ER = evaporation residue

typical values for Ni + Pb reaction



10^{11} = 100,000,000,000

99,999,000,000

10^{6} = 1,000,000

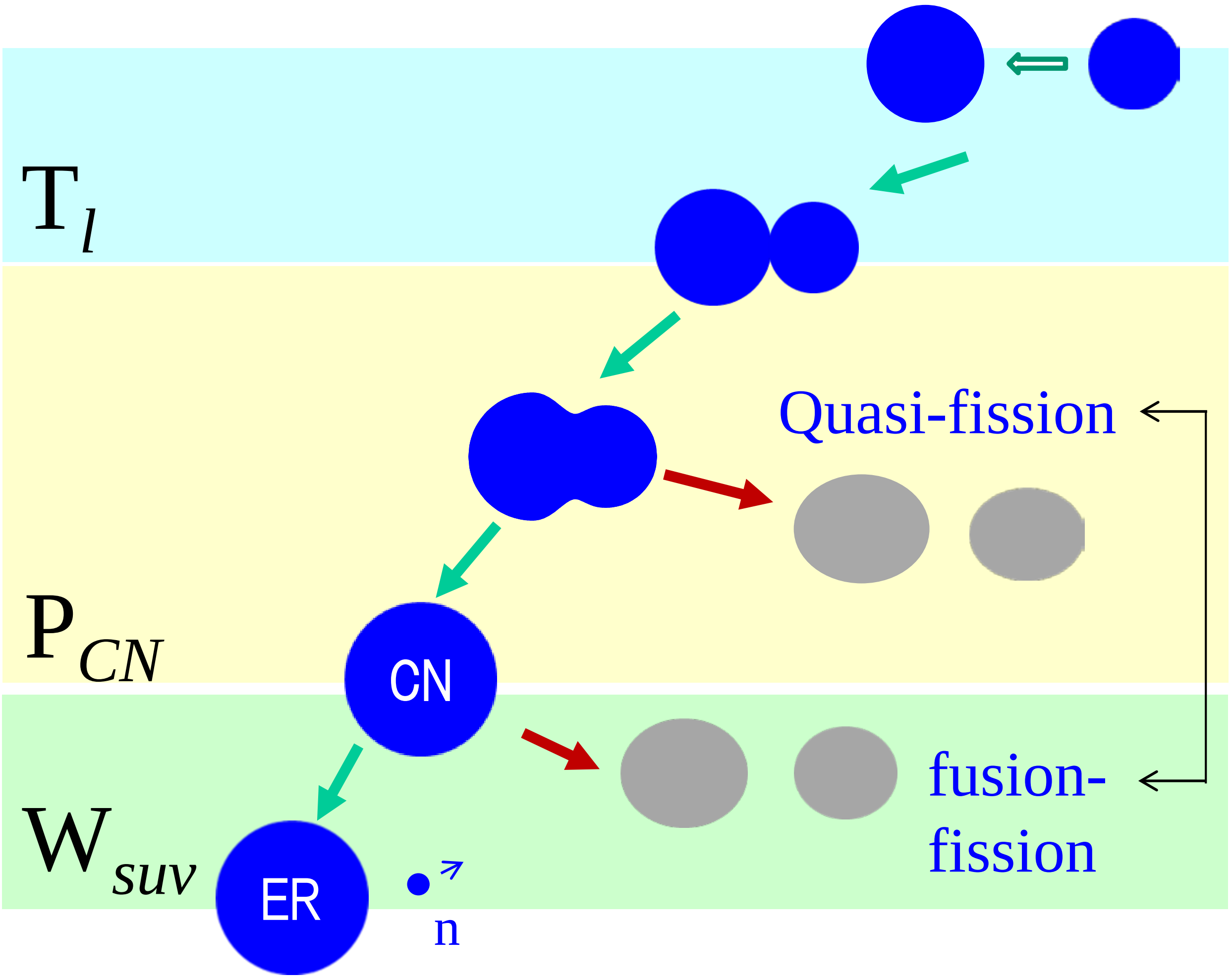
999,999

very rare event !!

1

experimentally detected

CN = compound nucleus
ER = evaporation residue



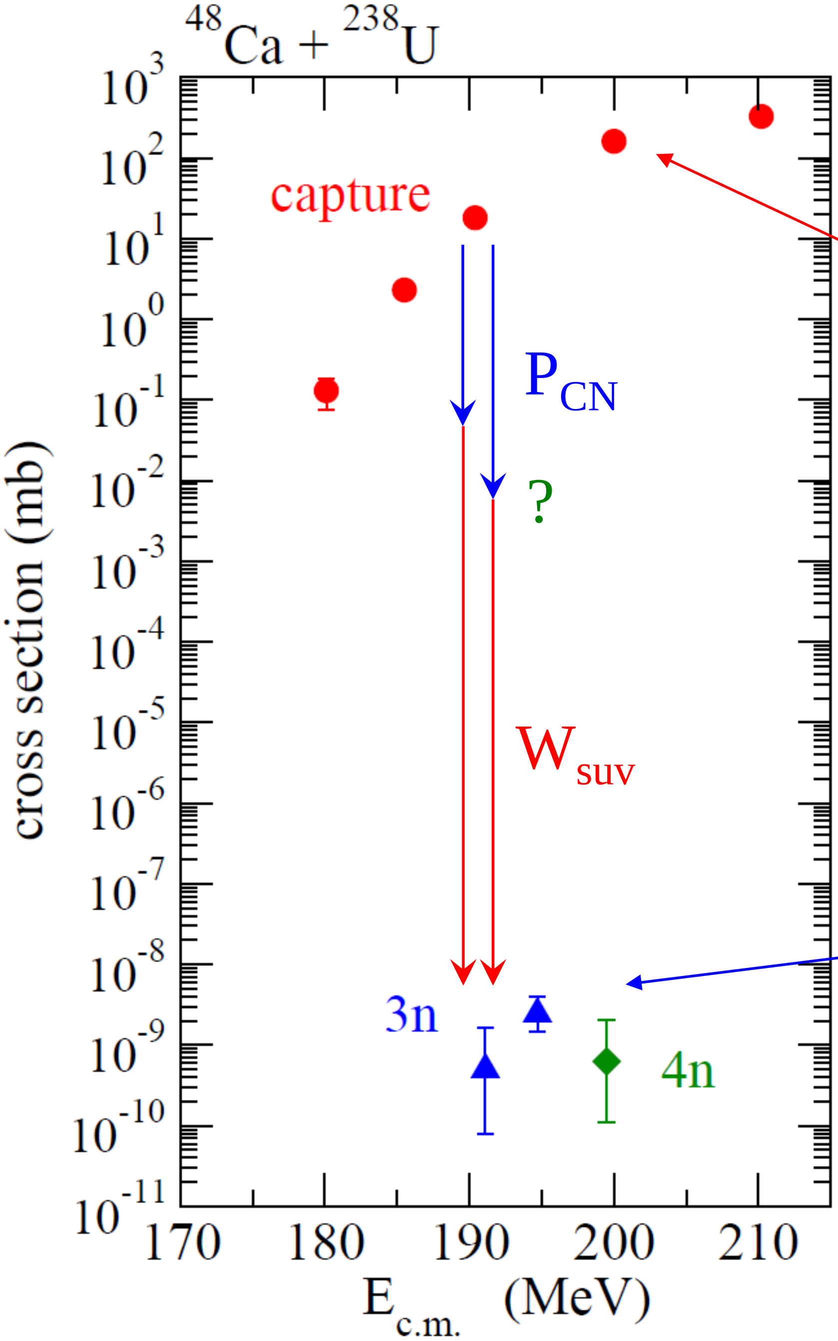
Coupled-channels

Langevin approach

statistical model

CN = compound nucleus
ER = evaporation residue

$$\boxed{\sigma_{\rm ER}(E) = \frac{\pi}{k^2}\sum_l (2l+1)T_l(E)\,P_{\rm CN}(E,l)\,W_{\rm suv}(E^*,l)}$$



<u>no experimental data for P_{CN}</u>

$$\sigma_{\mathrm{cap}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)T_l(E)$$

$$\xcancel{\sigma_{\mathrm{CN}}(E) \quad = \quad \frac{\pi}{k^2}\sum_l (2l+1)T_l(E) \times P_{\mathrm{CN}}}$$

not available

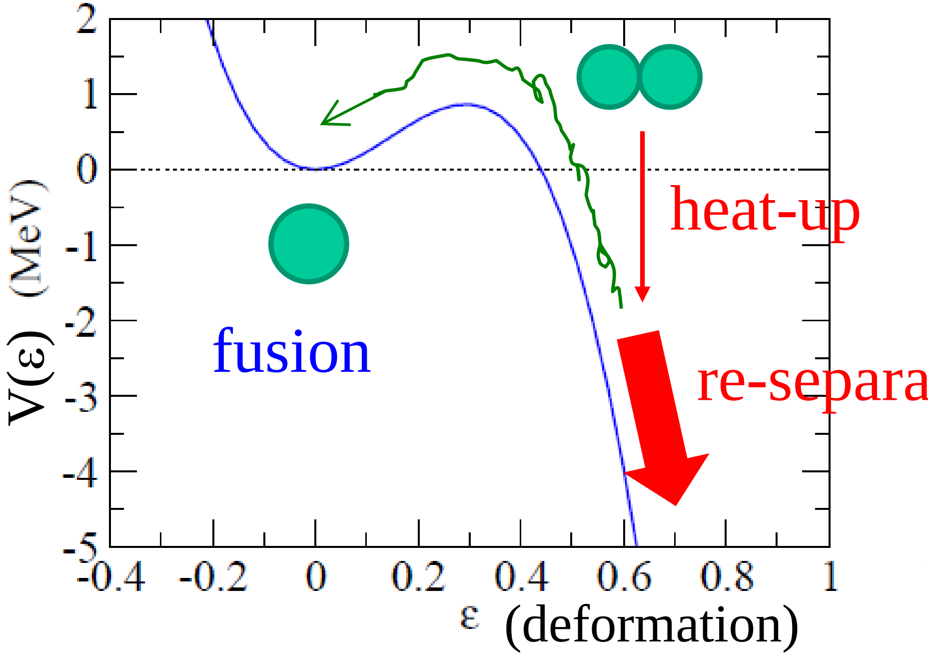
$$\sigma_{\mathrm{ER}}(E) \quad = \quad \frac{\pi}{k^2}\sum_l (2l+1)T_l(E) \times \boxed{P_{\mathrm{CN}} \cdot W_{\mathrm{suv}}}$$

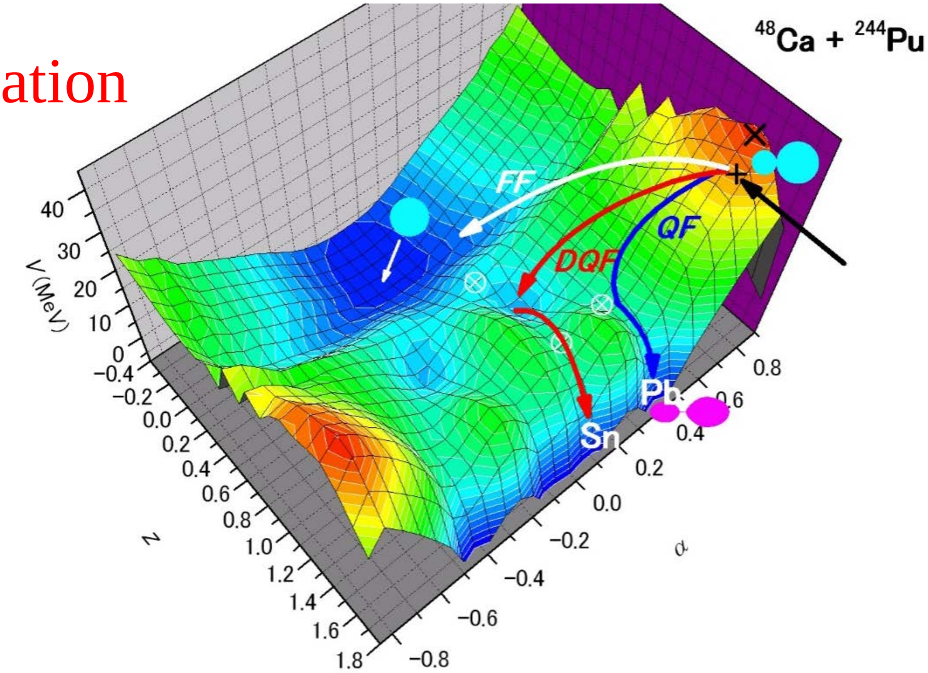
large uncertainties

Langevin approach

multi-dimensional extention

q:
- internuclear separation,
- deformation,
- asymmetry of the two fragments





thermal fluctuation

⟶ Langevin method
(Brownian method)

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$

γ: friction coefficient
R(*t*): random force

- ✓ Abe, Wada, Bao
- ✓ Aritomo, Ohta
- ✓ Zagrebaev

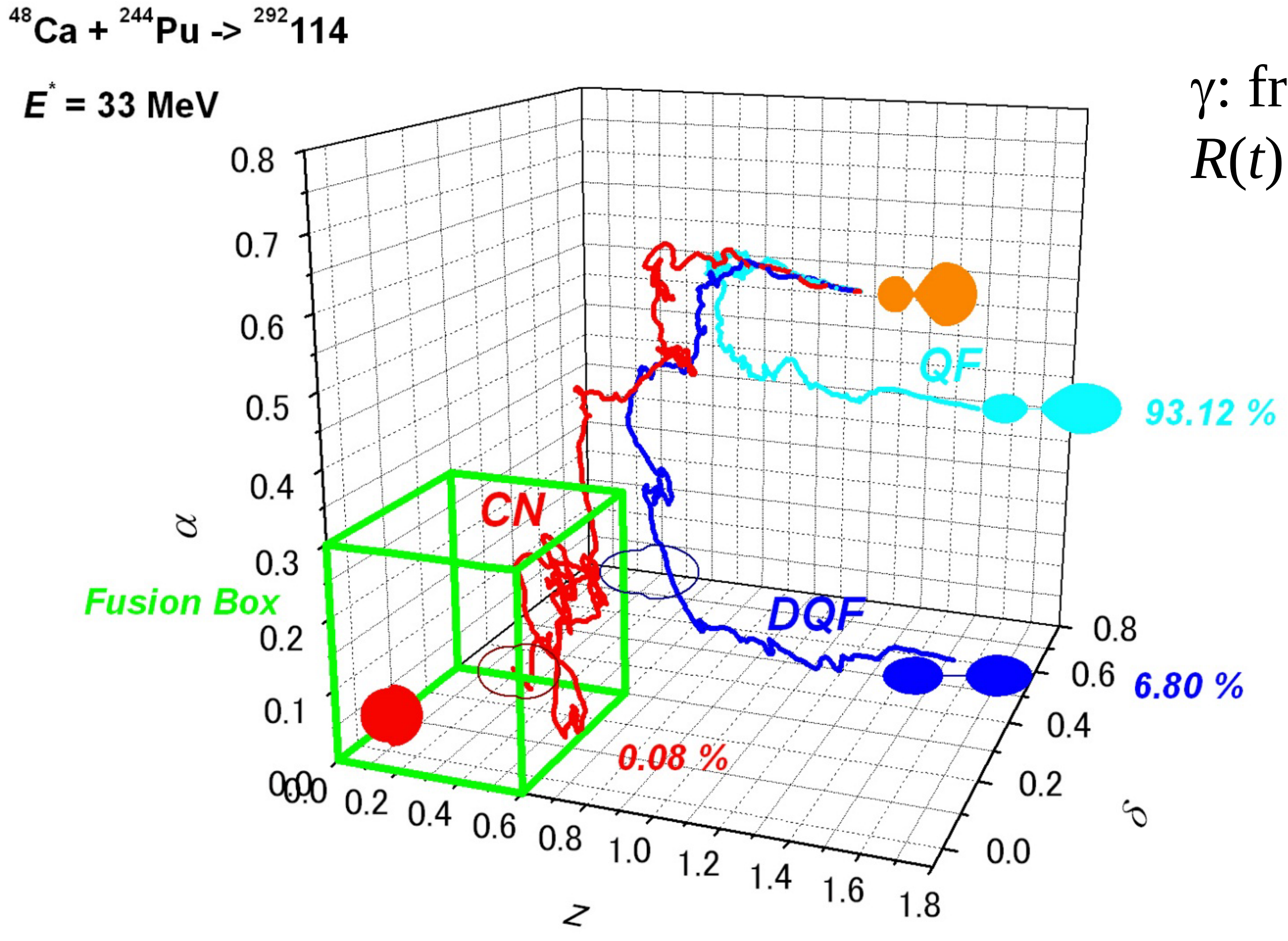
Theory: Lagenvin approach

multi-dimensional extension of:

$$m\frac{d^2q}{dt^2} = -\frac{dV(q)}{dq} - \gamma\frac{dq}{dt} + R(t)$$

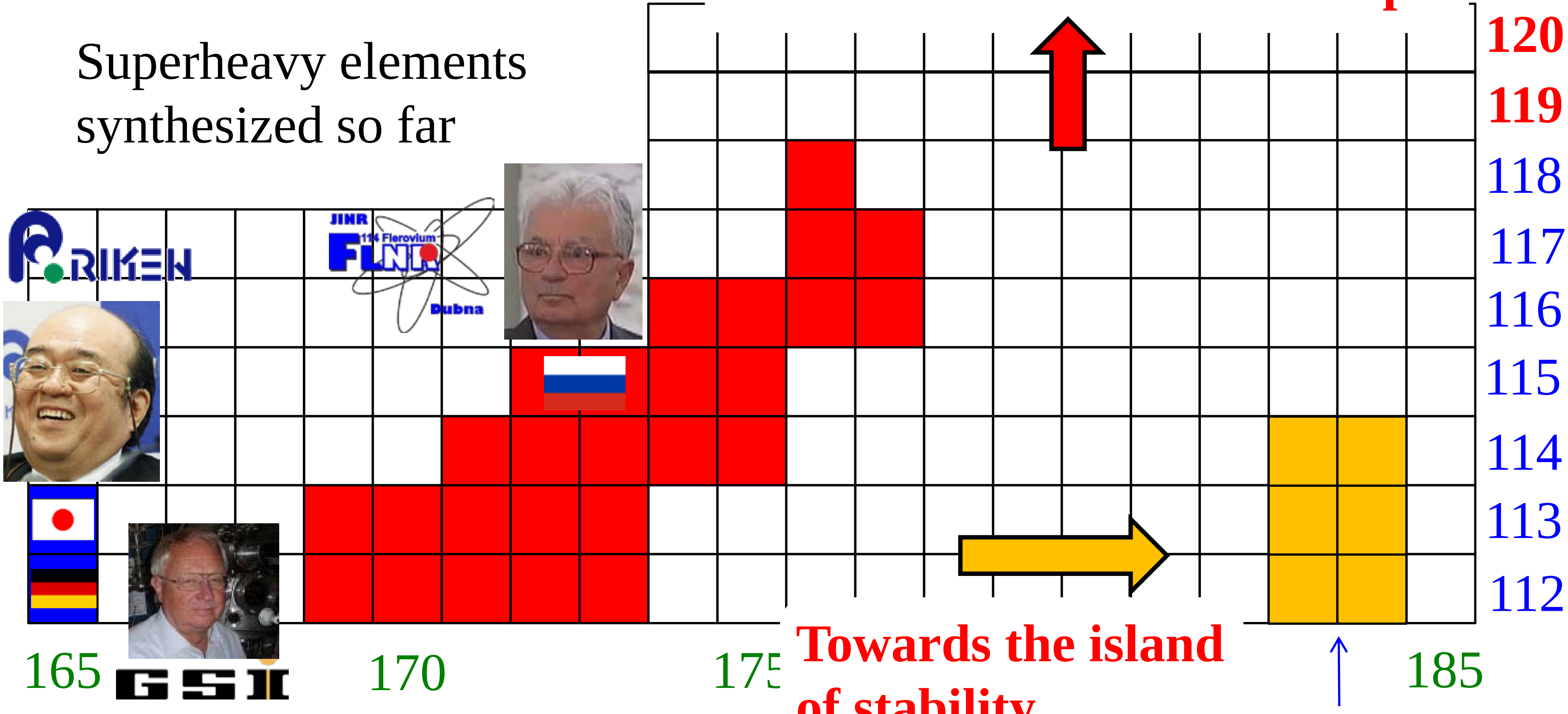
γ: friction coefficient

$R(t)$: random force



Future directions

Superheavy elements
synthesized so far



Towards Z=119 and 120 isotopes

Towards the island of stability

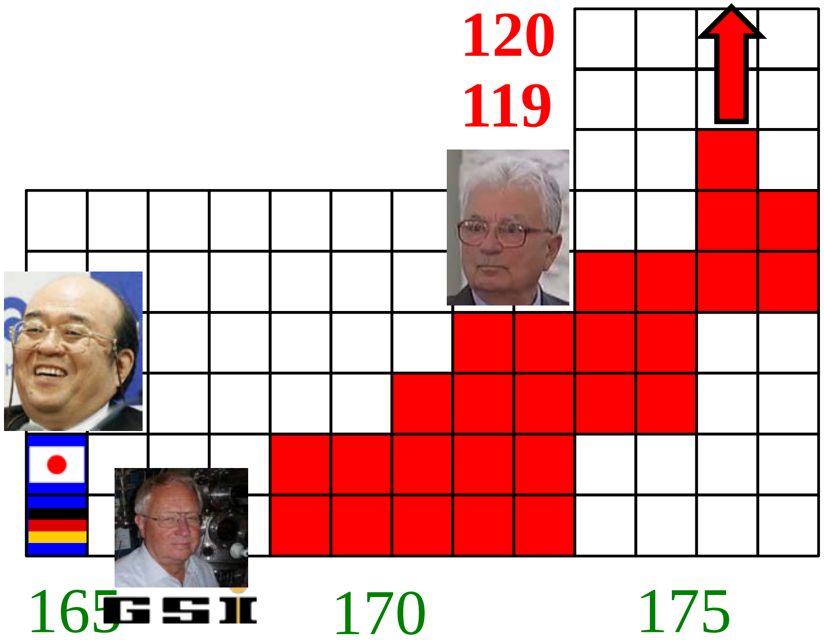
the island
of stability?

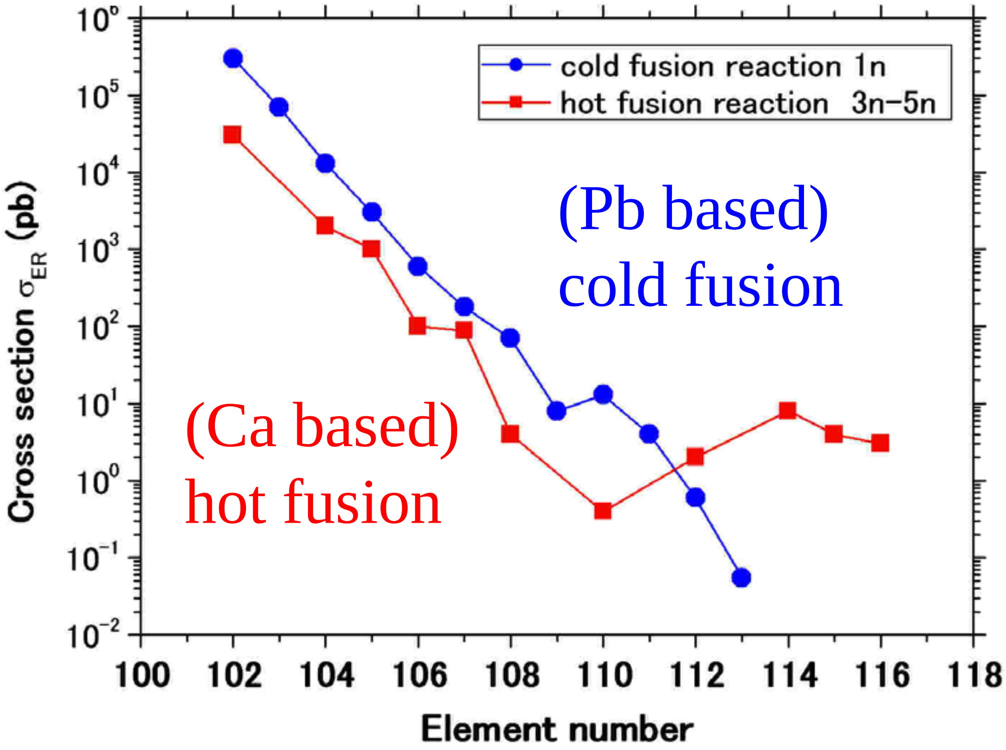
Theoretical issues:

- to understand the reaction dynamics
- to make a reliable theoretical prediction for fusion cross sections

Hot fusion for Z = 119 and 120

Towards Z=119 and 120 isotopes





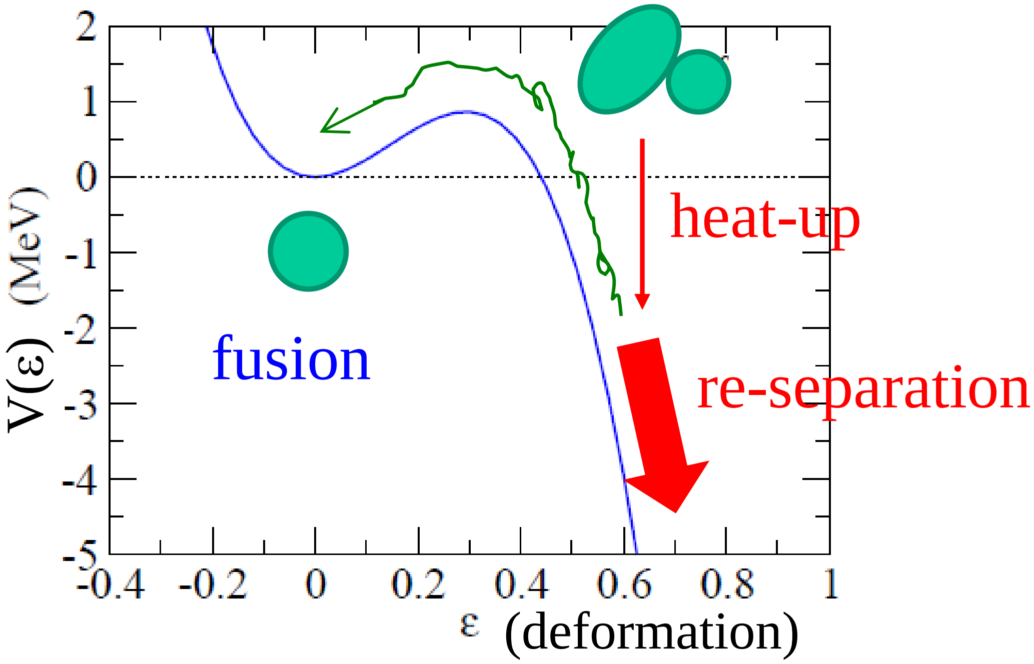
hot fusion: ^{48}Ca + actinide targets

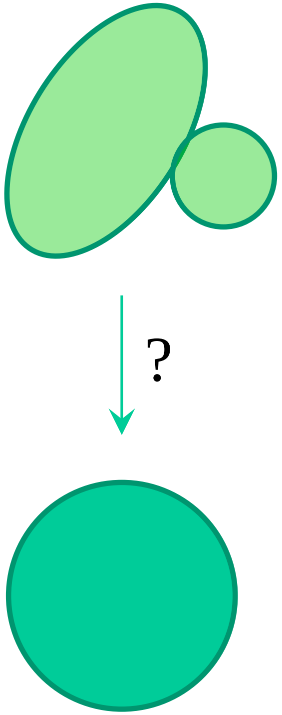
Dubna: ^{48}Ca + ^{249}Cf ($\beta_2 = 0.235$) → $^{297\text{-}x}$Og (Z=118) + xn

role of deformation?

Hot fusion: ^{48}Ca + deformed actinide target

Effect of deformation





Open problems

- how is the shape evolved to a compound nucleus?
- Deformation: a quantum effect
  how does the deformation disappear during heat-up?

Quantum friction

Quantum friction

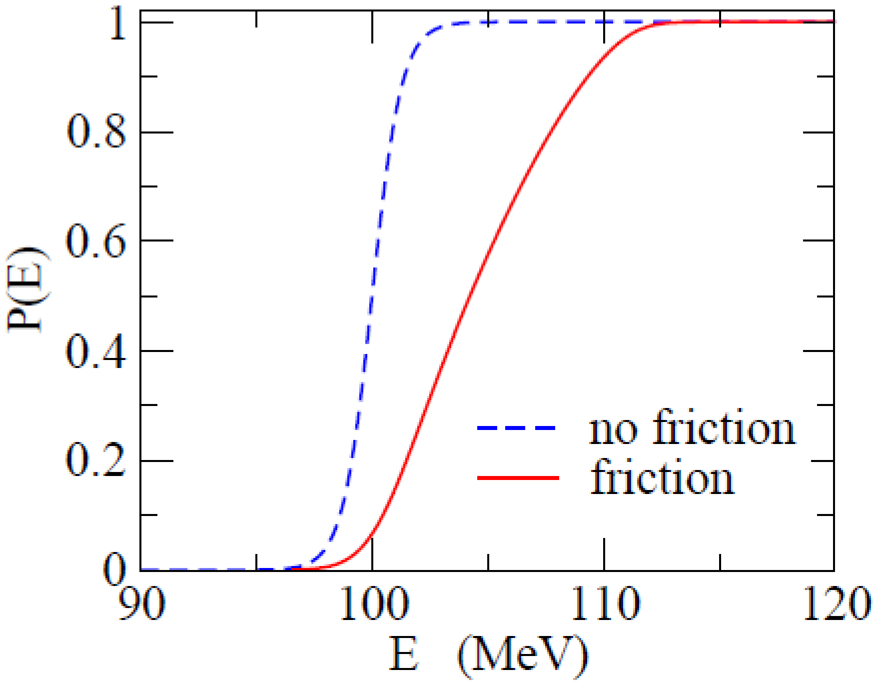
classical eq. of motion $\dot{p} = -V'(x) - \gamma p$

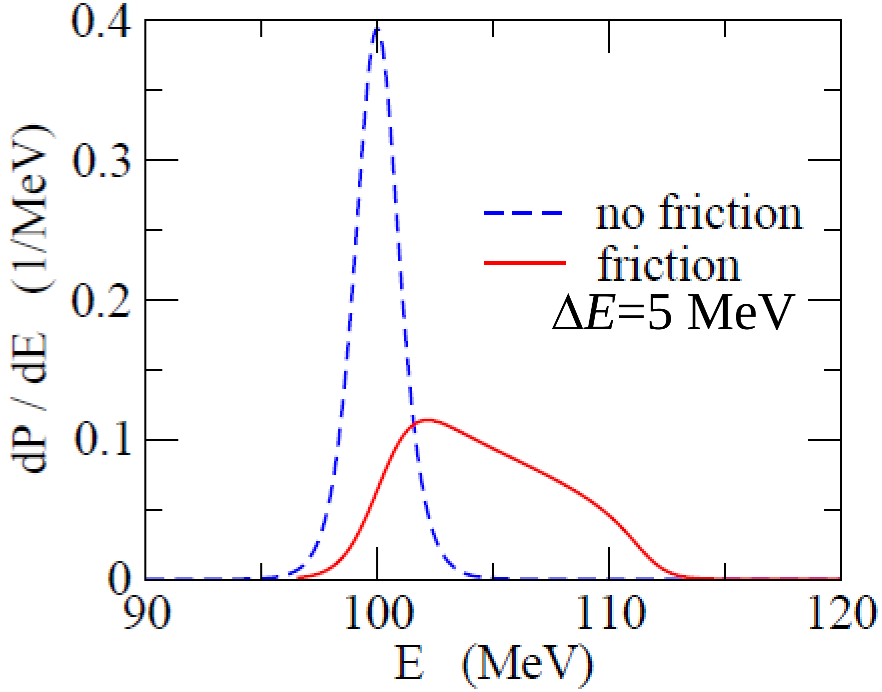
a quantization: Kanai model E. Kanai, PTP 3 (1948) 440)

$$\boxed{H = \frac{p^2}{2m} + V(x) \to \frac{\pi^2}{2m} e^{-\gamma t} + e^{\gamma t} V(x) \qquad (\pi = e^{\gamma t} p)}$$

(a quantal Hamiltonian which reproduces the classical eq. of motion)

time-dep. wave packet approach





M. Tokieda and K.H., PRC95 ('17) 054604

➢Towards Z=119 and 120 nuclei

Another issue

$^{48}_{20}\mathrm{Ca} + {}_{99}\mathrm{Es} \rightarrow 119$
$^{48}_{20}\mathrm{Ca} + {}_{100}\mathrm{Fm} \rightarrow 120$

the targets: not available with sufficient amounts

Dubna: $^{48}\mathrm{Ca} + {}^{249}_{98}\mathrm{Cf} \rightarrow {}^{297-x}\mathrm{Og}$ (Z=118) + xn

$^{249}_{98}\mathrm{Cf}$ (351 year)
$^{252}_{99}\mathrm{Es}$ (471.7 day)
$^{257}_{100}\mathrm{Fm}$ (100.5 day)

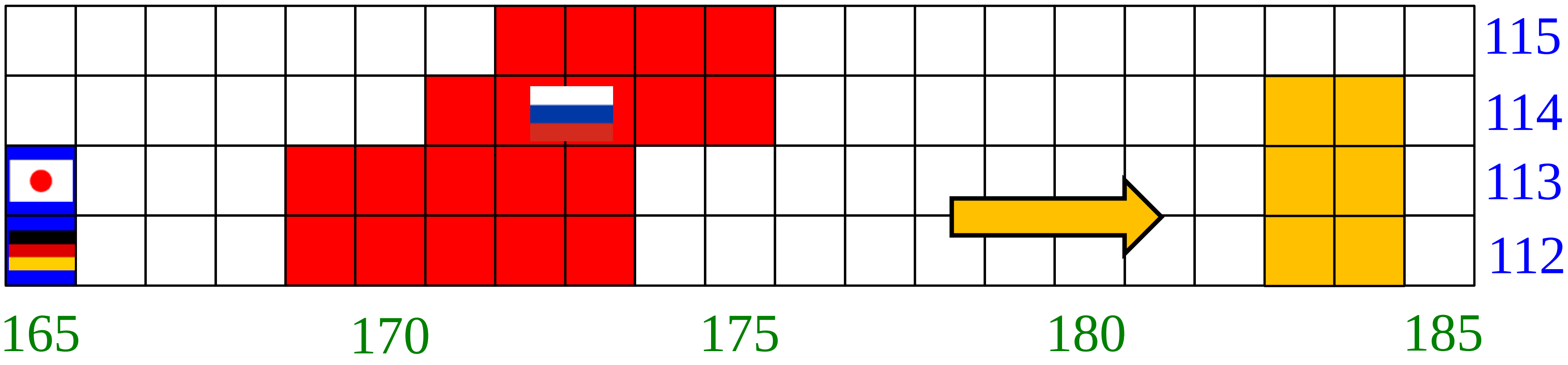
➡ $^{48}\mathrm{Ca} \rightarrow {}^{50}_{22}\mathrm{Ti}$, $^{51}_{23}\mathrm{V}$, $^{54}_{24}\mathrm{Cr}$ projectiles

cf. $^{46}_{21}\mathrm{Sc}_{25}$: relatively small neutron number

how much will fusion cross sections be reduced?

nobody still knows

Towards the island of stability



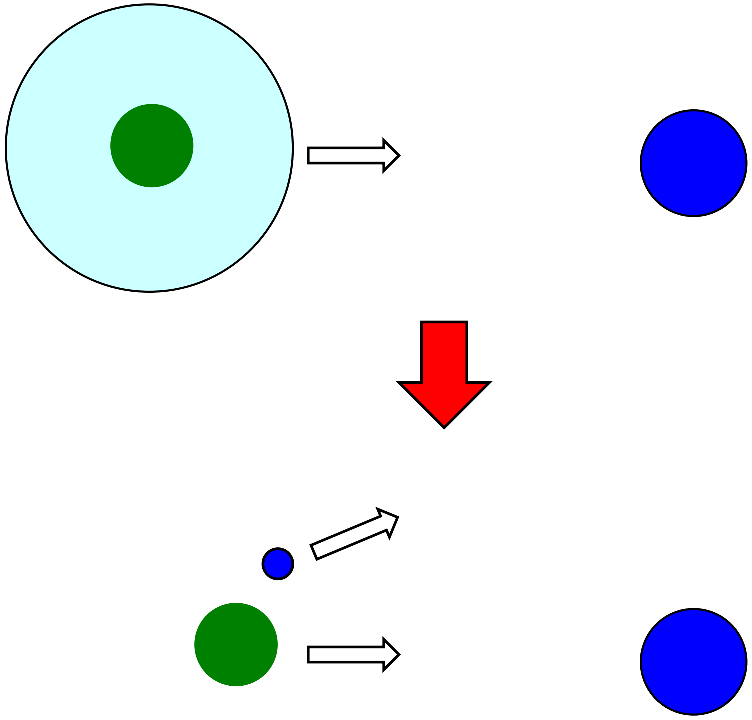
neutron-rich beams: indispensable

- ➢ how to deal with low beam intensity?
- ➢ reaction dynamics of neutron-rich beams?
  - ✓ capture: role of breakup and (multi-neutron) transfer?
  - ✓ diffusion: neutron emission during a shape evolution?
  - ✓ survival: validity of the statistical model?

structure of exotic nuclei

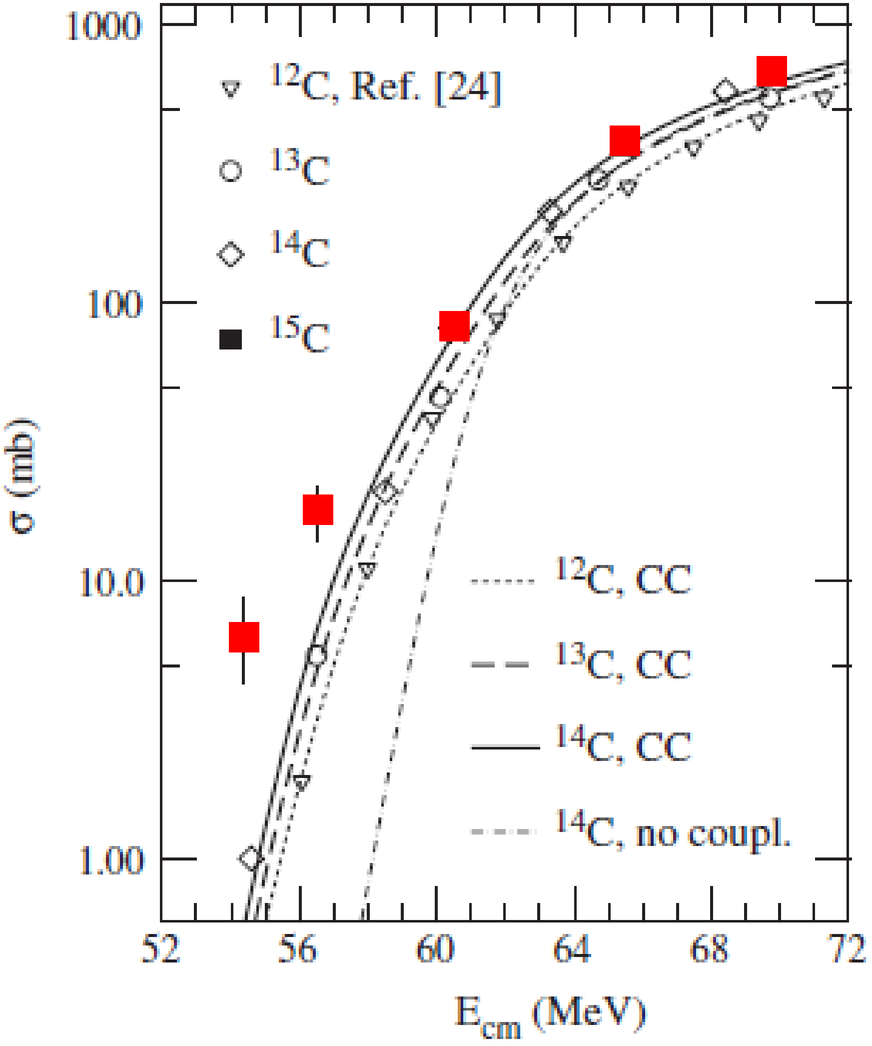
more studies are required

Fusion of halo nuclei



1. Lowering of potential barrier
   due to a halo structure
   ⟶ enhancement
2. effect of breakup
3. effect of transfer: equally important

$^{12,13,14,15}\mathrm{C} + {}^{232}\mathrm{Th}$

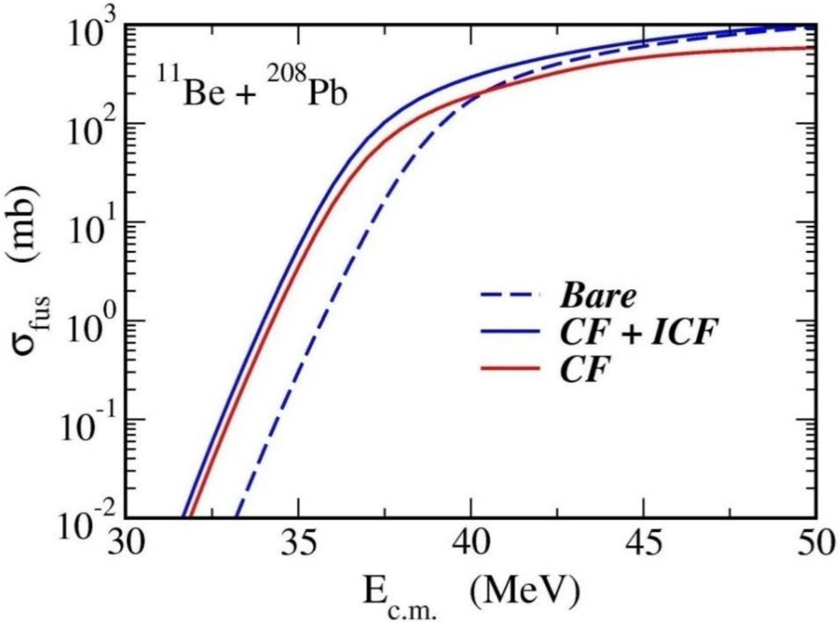


M. Alcorta et al.,
PRL106('11)172701

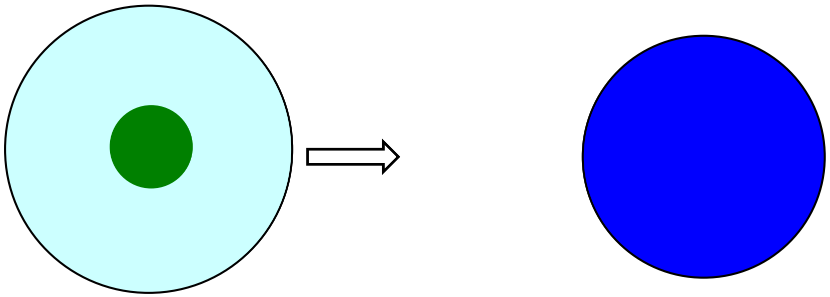
Future directions - 2

- Towards the island of stability

  neutron-rich beams: indispensable



K. Hagino, A. Vitturi, C.H. Dasso,
and S.M. Lenzi, Phys. Rev. C61 ('00) 037602



simultaneous treatment
of breakup and transfer

→ an important future problem

Summary

Heavy-ion fusion reactions around the Coulomb barrier

- ✓Strong interplay between nuclear structure and reaction
- ✓Quantum tunneling with various intrinsic degrees of freedom
- ✓coupled-channels approach

Remaining challenges

- ✓ microscopic understanding of heavy-ion fusion reactions

Future perspectives: superheavy elements

- ✓ how to reduce theoretical uncertainties?
- ✓ Towards heavier SHE (Z = 119, 120)
- ✓ Towards the island of stability

investigations of physics of SHE with neutron-rich nuclei as a keyword