Transition-state dynamics in complex quantum systems

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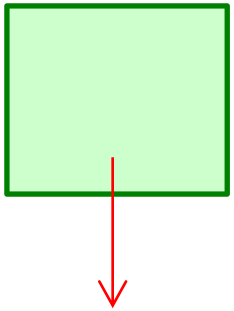
G.F. Bertsch and K. Hagino, arXiv:2105.12073 [quant-ph]
K. Hagino and G.F. Bertsch, arXiv:2106.152501 [quant-ph]

Workshop on Modern Developments in Quantum Chaos, Sep. 20-24, 2021

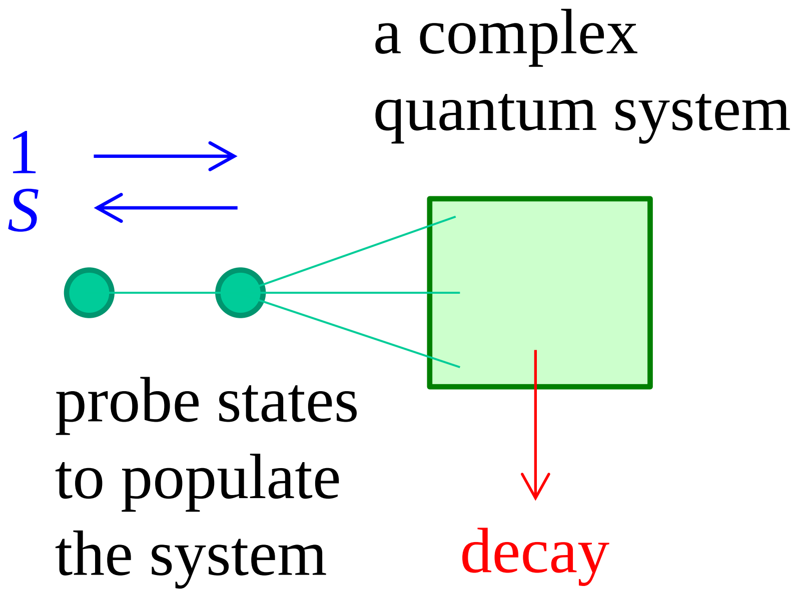
Introduction: decay of a complex quantum system

a complex
quantum system



decay

Introduction: decay of a complex quantum system



$$T(E) = 1 - |S(E)|^2$$

Fyodorov's talk on Wednesday

$$S(E) \to S(E + i\eta)$$

also:
Y.V. Fyodorov, D.V. Savin, and H.-J. Sommers, J. Phys. A38, 10731 (2005)

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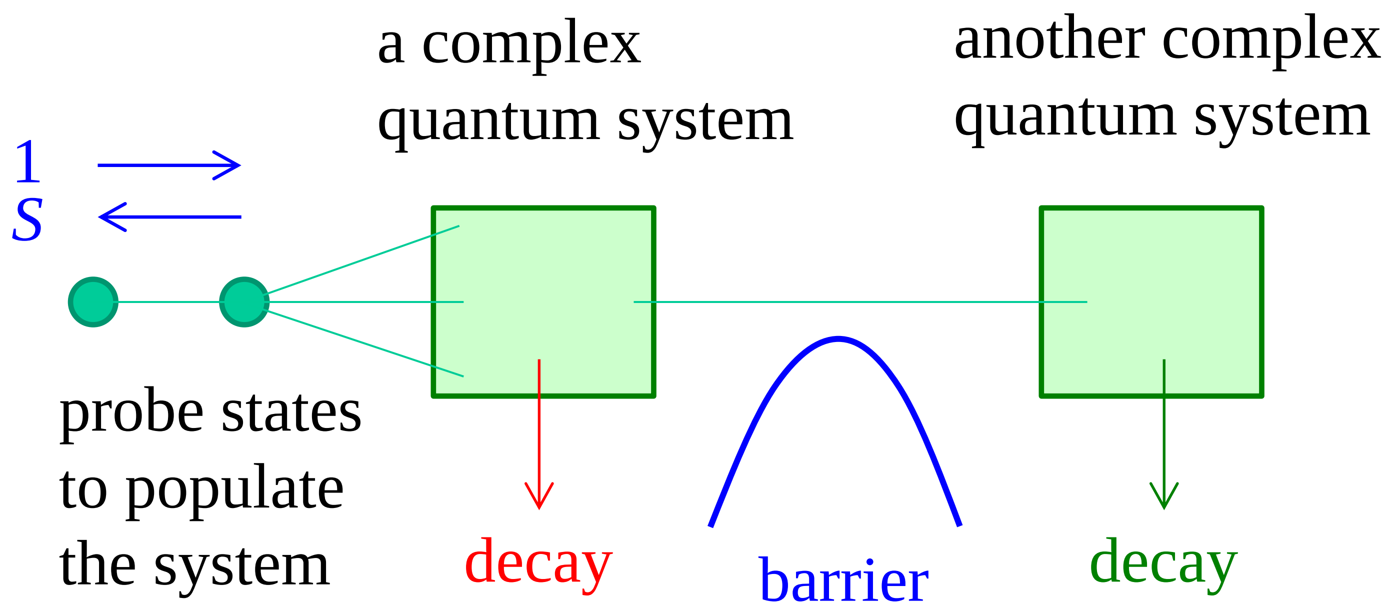

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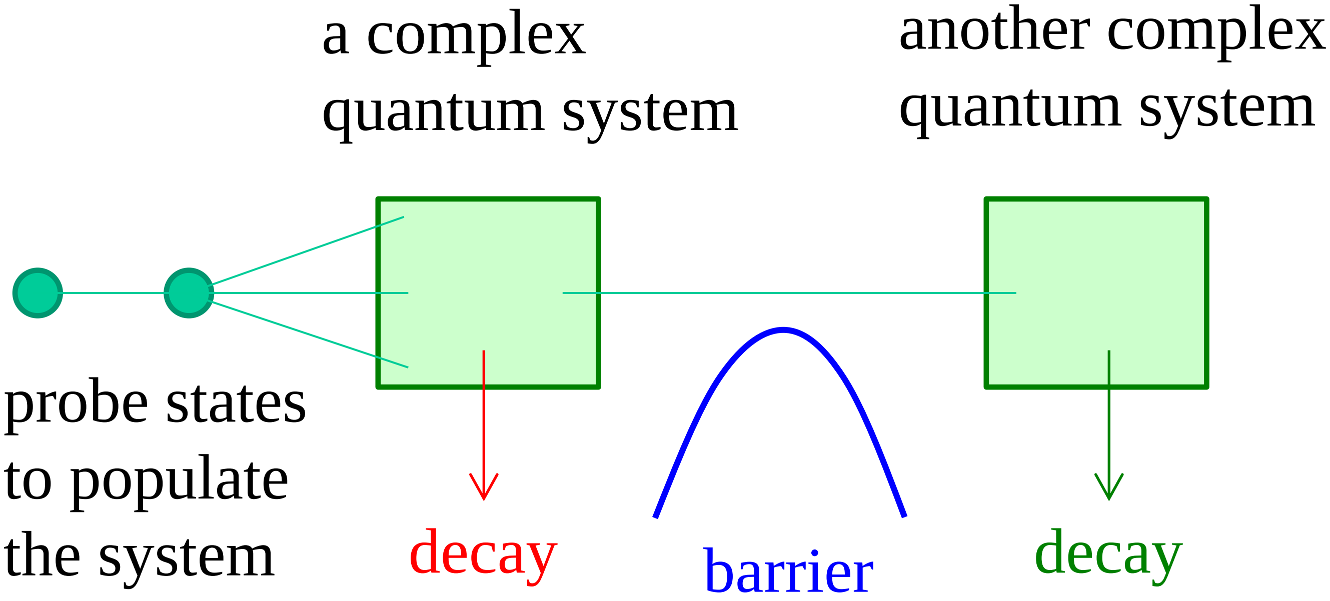
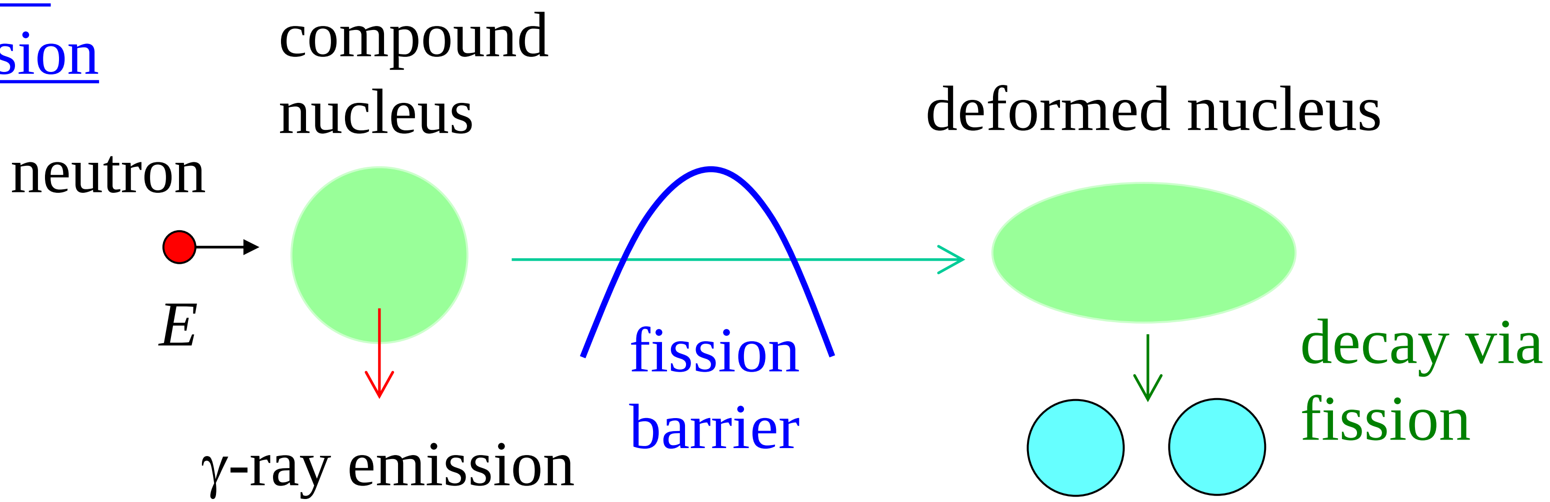
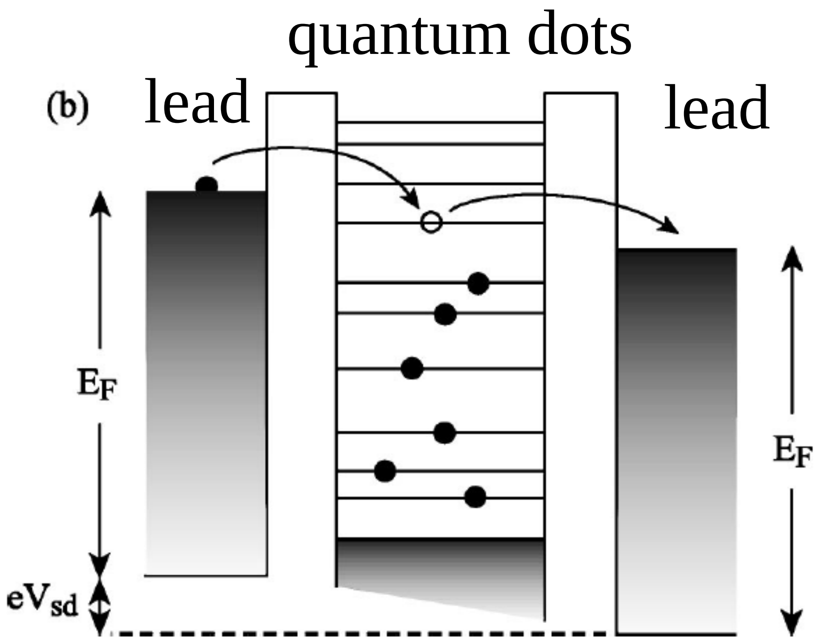


an example: nuclear fission



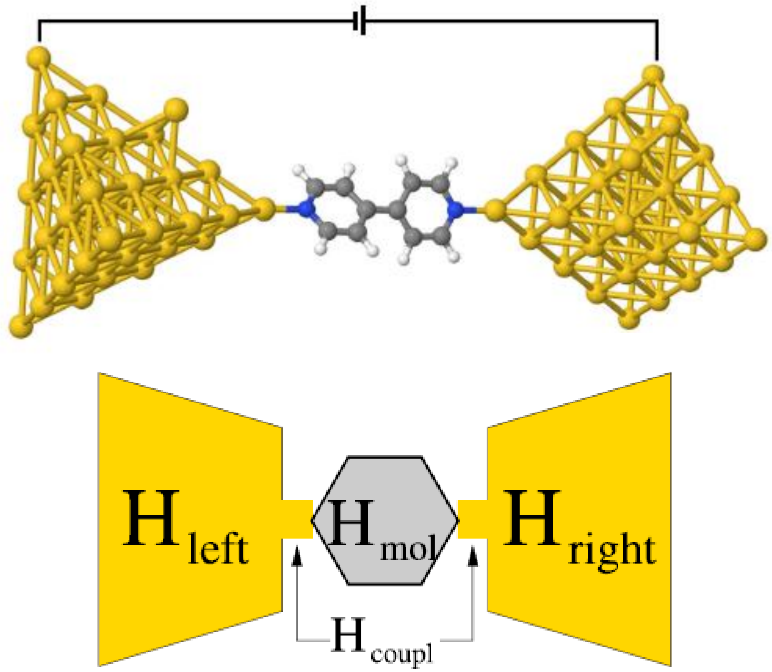
other examples: the problem of electron transport

tunnel junction



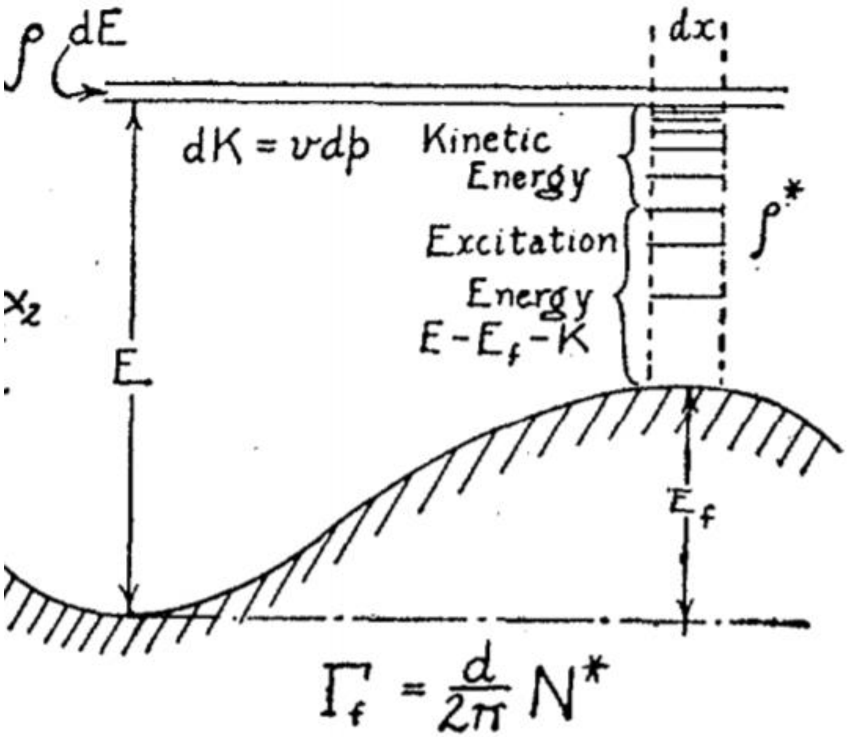
Y. Alhassid,
Rev. Mod. Phys. 72, 895 (2000)

molecular bridge



M.Thoss and F. Evers,
JCP148, 030901 (2018)

the transition state theory



N. Bohr and J.A. Wheeler,
Phys. Rev. 56, 426 (1939)

cf. RKKM theory
in chemistry

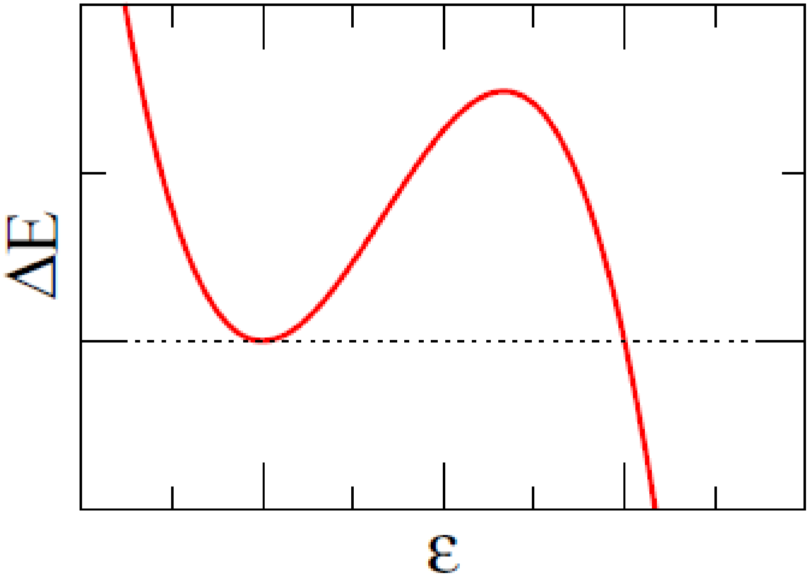
tunneling probability

cf. D.M. Brink and U. Smilansky,
Nucl. Phys. A405 (‘83) 301

$$\Gamma_f = \frac{1}{2\pi\rho_{\rm gs}(E^*)}\int_0^{E^*-B_f}\rho_{\rm sd}(E^*-B_f-K)dK \to \frac{1}{2\pi\rho_{\rm gs}(E^*)}\sum_c T_c$$

✓ decay dynamics: entirely determined at the saddle
✓ does not depend on what will happen after the barrier

the transition state theory



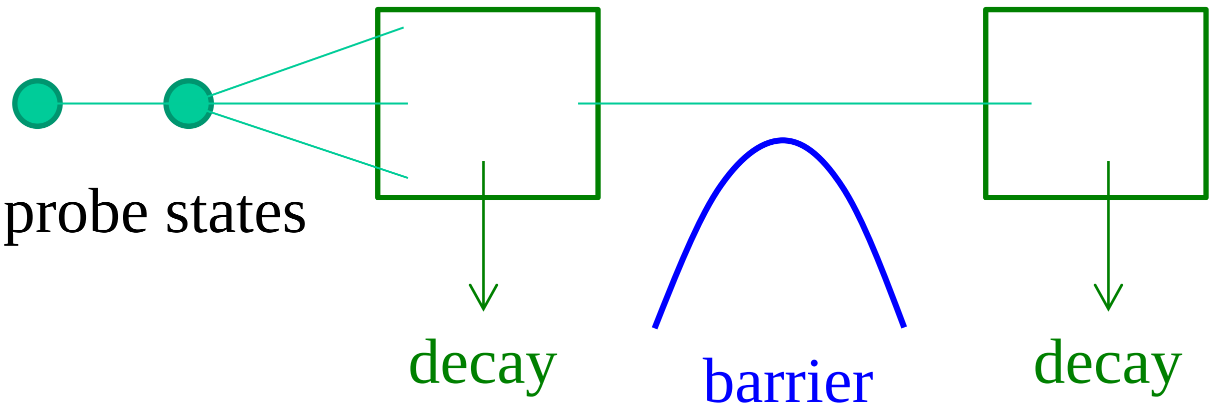
$$\Gamma_f = \frac{1}{2\pi\rho_{\mathrm{gs}}(E^*)}\sum_c T_c$$

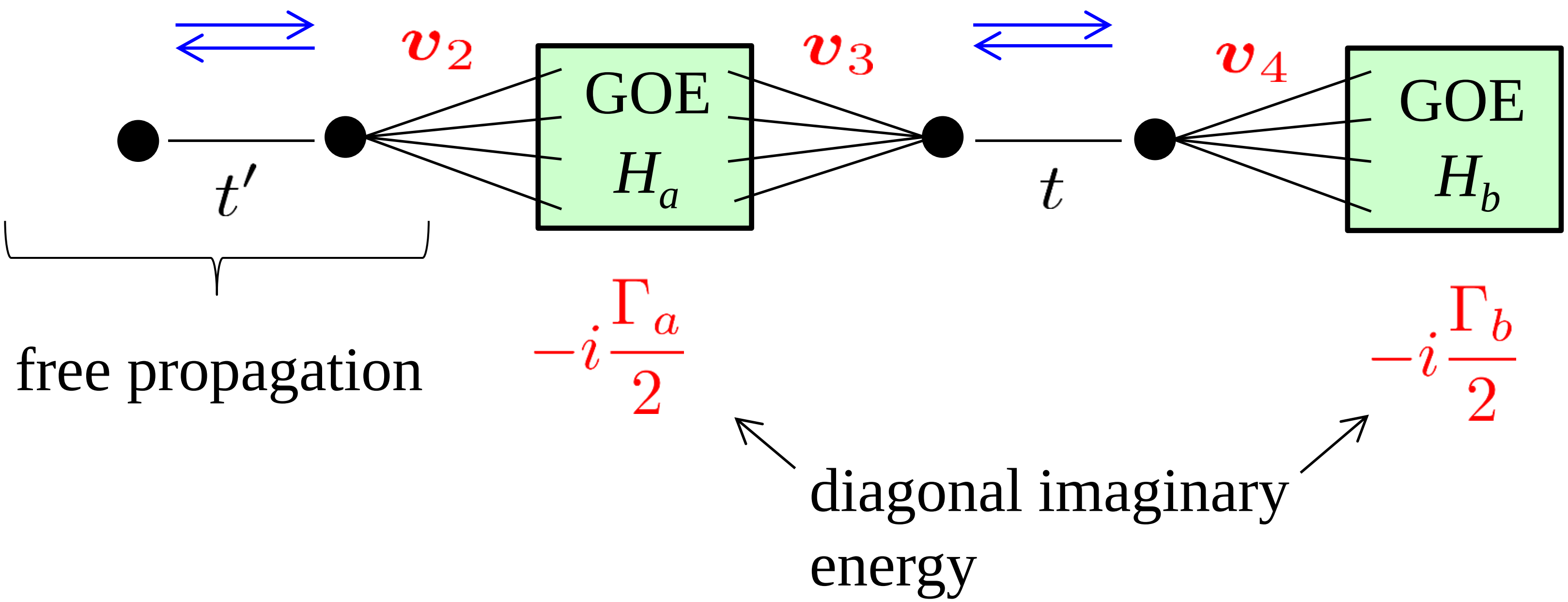
✓ decay dynamics: the saddle only
✓ the insenstitivity property

a question which we ask:

Can one derive the properties of the transition state theory based on a *microscopic* many-body Hamiltonian?

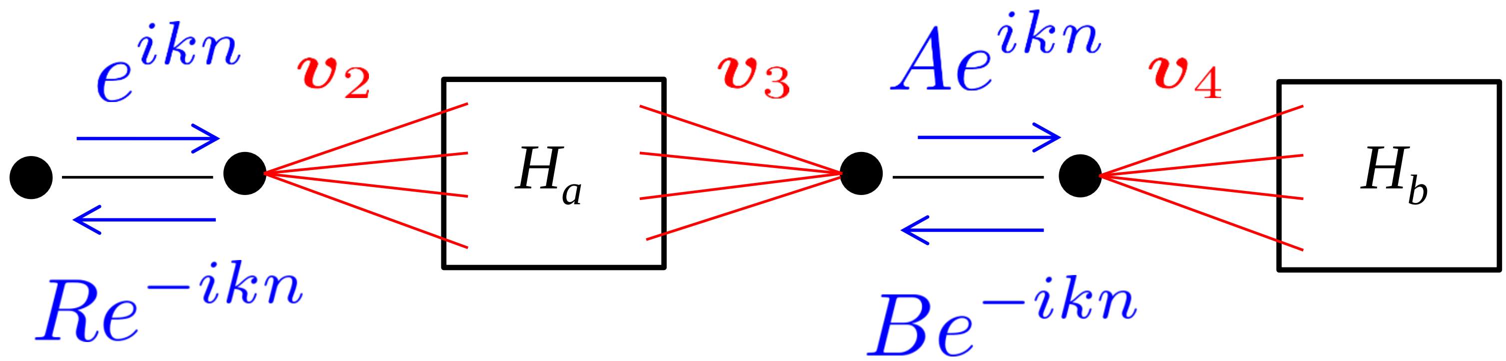
2 GOE Model





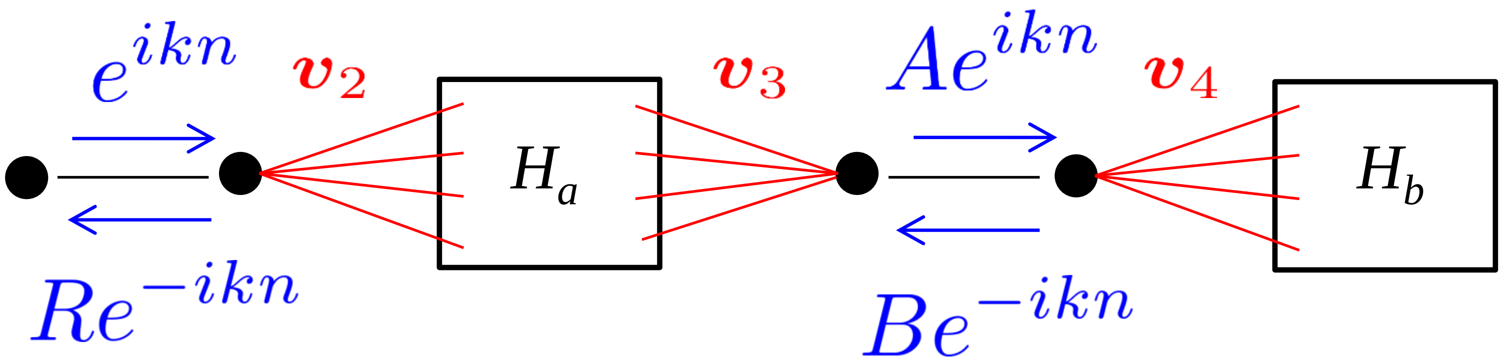
v_k (k = 2,3, 4): random interactions

G.F. Bertsch and K. Hagino, arXiv:2105.12073 [quant-ph]



$$T(E) = T_a(E) + T_b(E) = 1 - |R(E)|^2$$

$$\begin{aligned} T_a(E) &= 1 - |R(E)|^2 - |A(E)|^2 + |B(E)|^2 \\ T_b(E) &= |A(E)|^2 - |B(E)|^2 \end{aligned}$$



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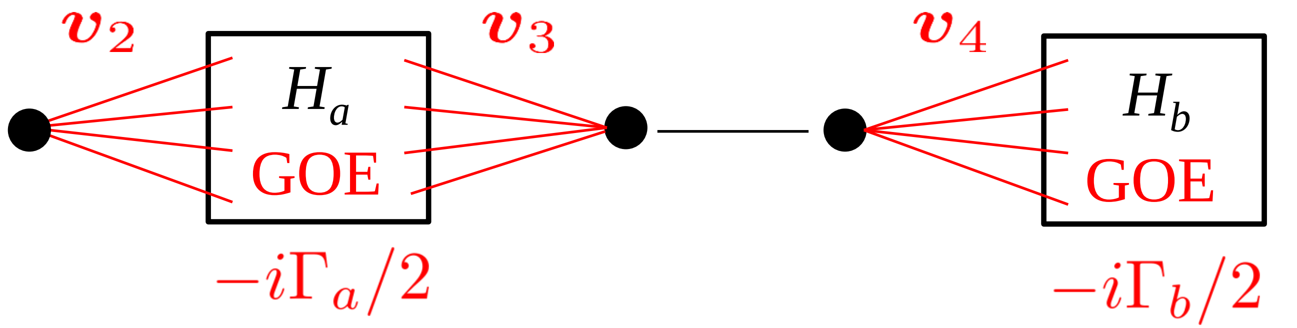
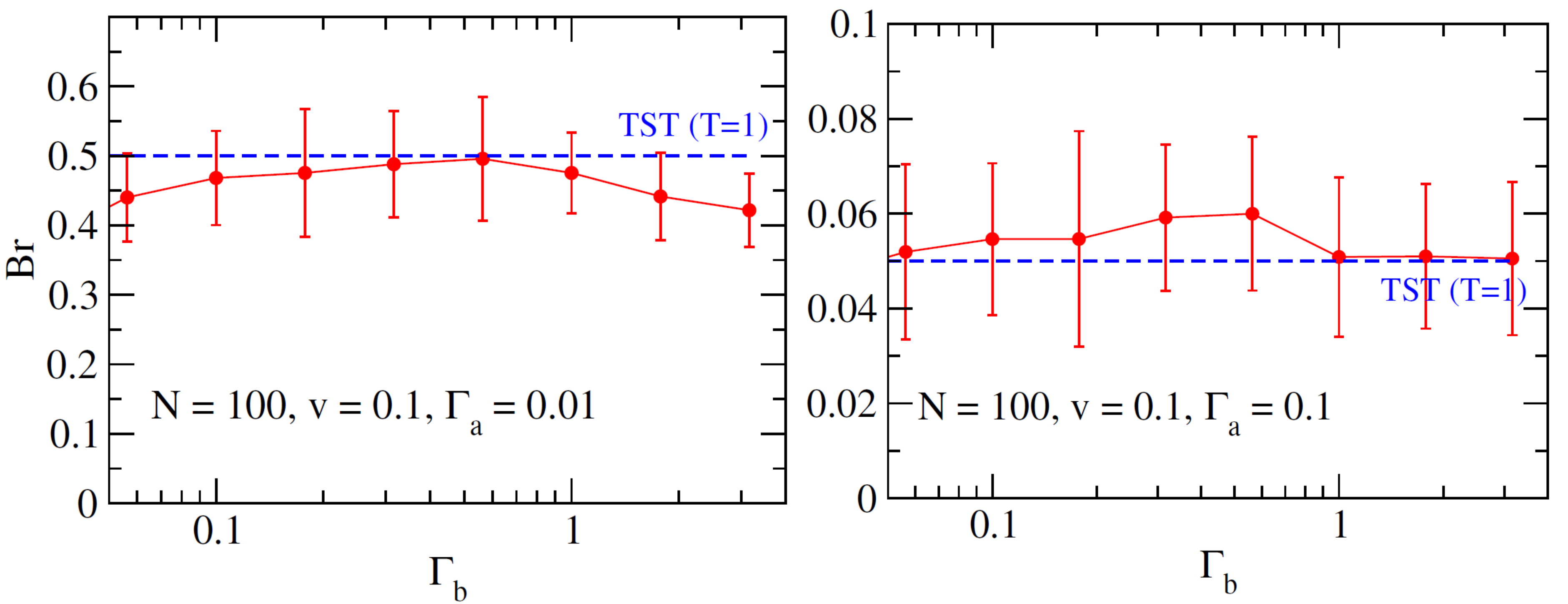
branching ratio: $$Br = \frac{\int dE\, T_b(E)}{\int dE\, T_a(E)}$$

the relative decay probability from b: $$P_b(E) = \frac{T_b(E)}{T(E)}$$

branching ratios



the average and the variance with 20 ensembles



Branching ratios: **<u>insensitive</u>** to Γ_b ← the main assumption of the TST

*also with analytical study with Stieltjes transform

G.F. Bertsch and K.H., arXiv:2105.12073

fluctuations of decay rates

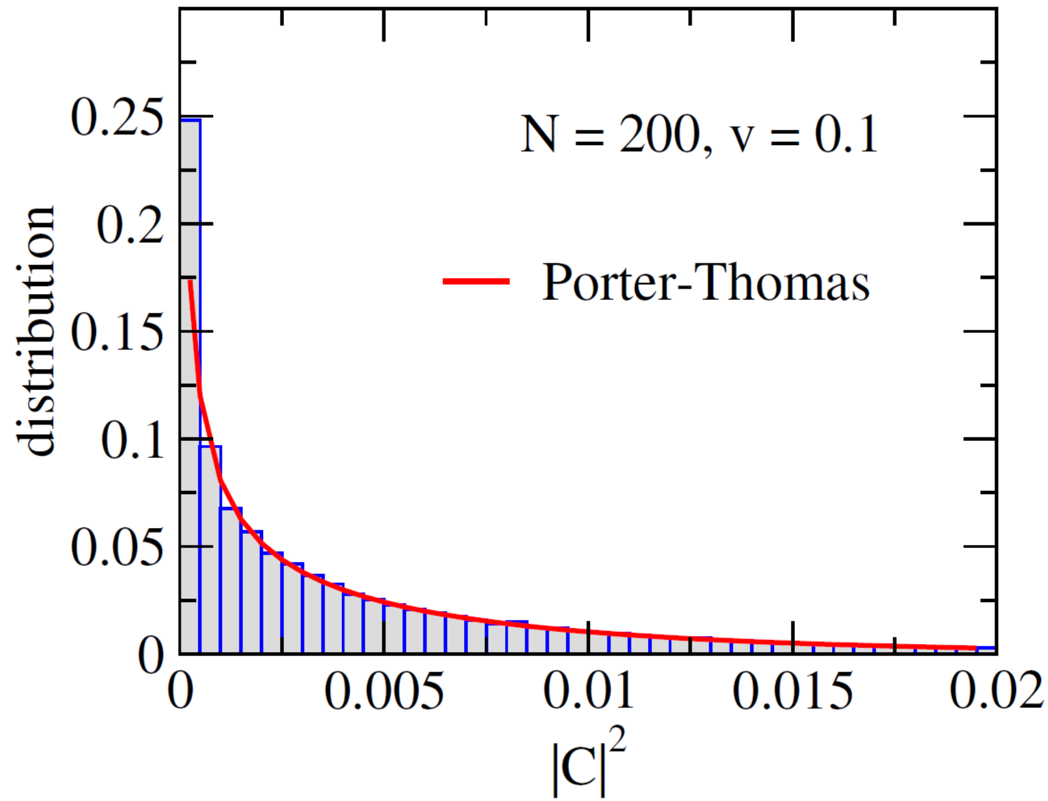
a common *belief*: a fluctuation of decay rates follows the Porter-Thomas distribution

$$\psi_i = \sum_k C_k^{(i)} |k\rangle$$

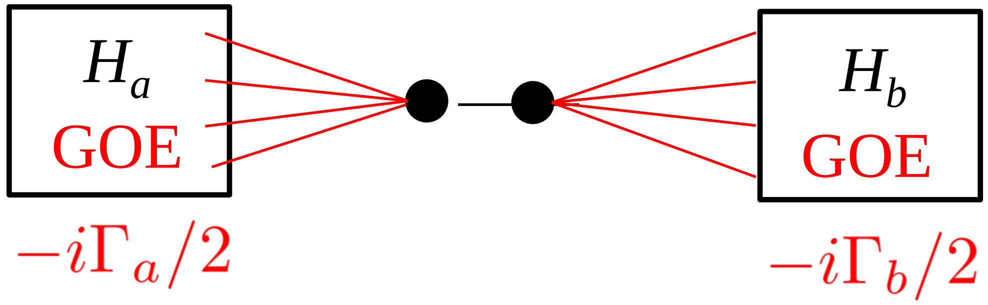
$$P(y) = \frac{1}{\sqrt{2\pi y}} e^{-y/2}$$

$$y = |C_k^{(i)}|^2 / \langle |C_k^{(i)}|^2 \rangle$$

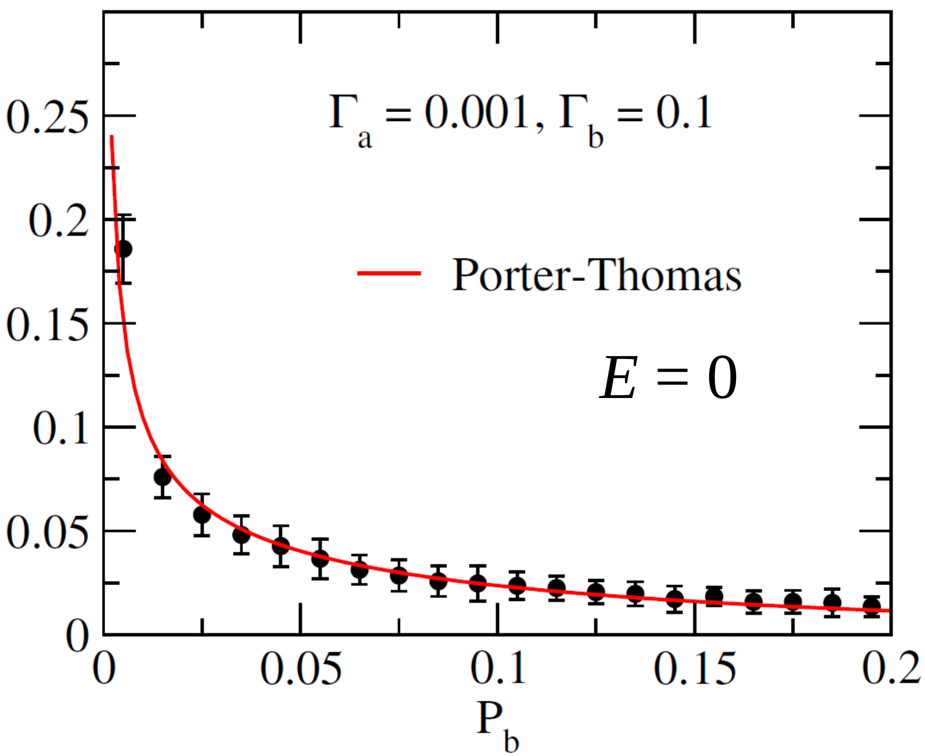
a distribution of wf amplitudes in GOE

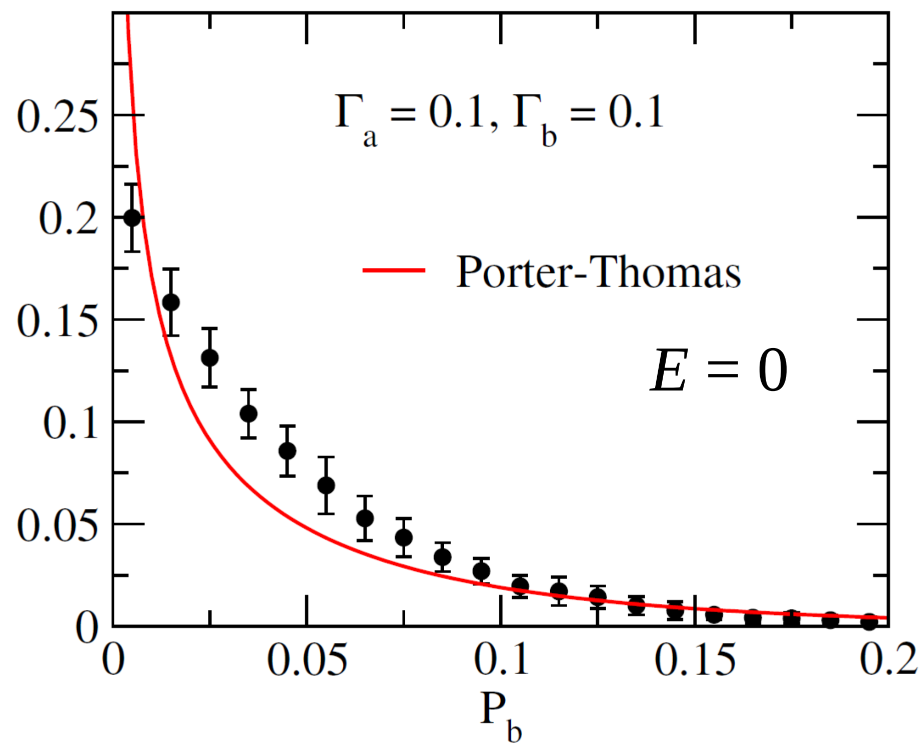


fluctuations of decay rates



$$P_b(E) = \frac{T_b(E)}{T(E)}$$





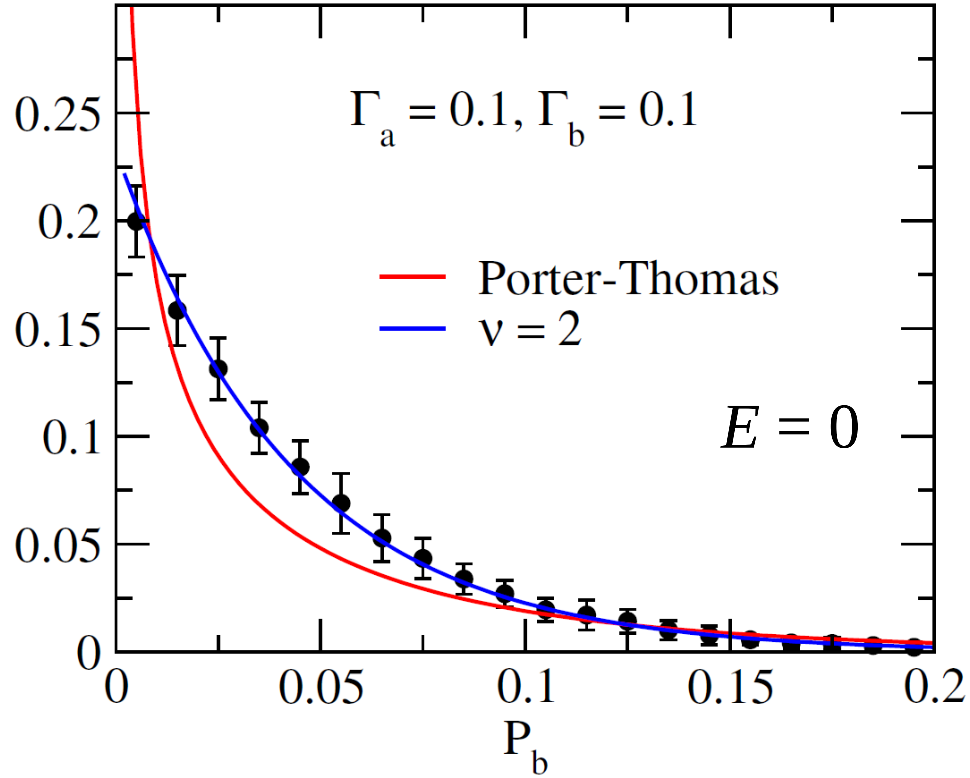
PT: reasonably good,
but with a significant deviation

fluctuations of decay rates

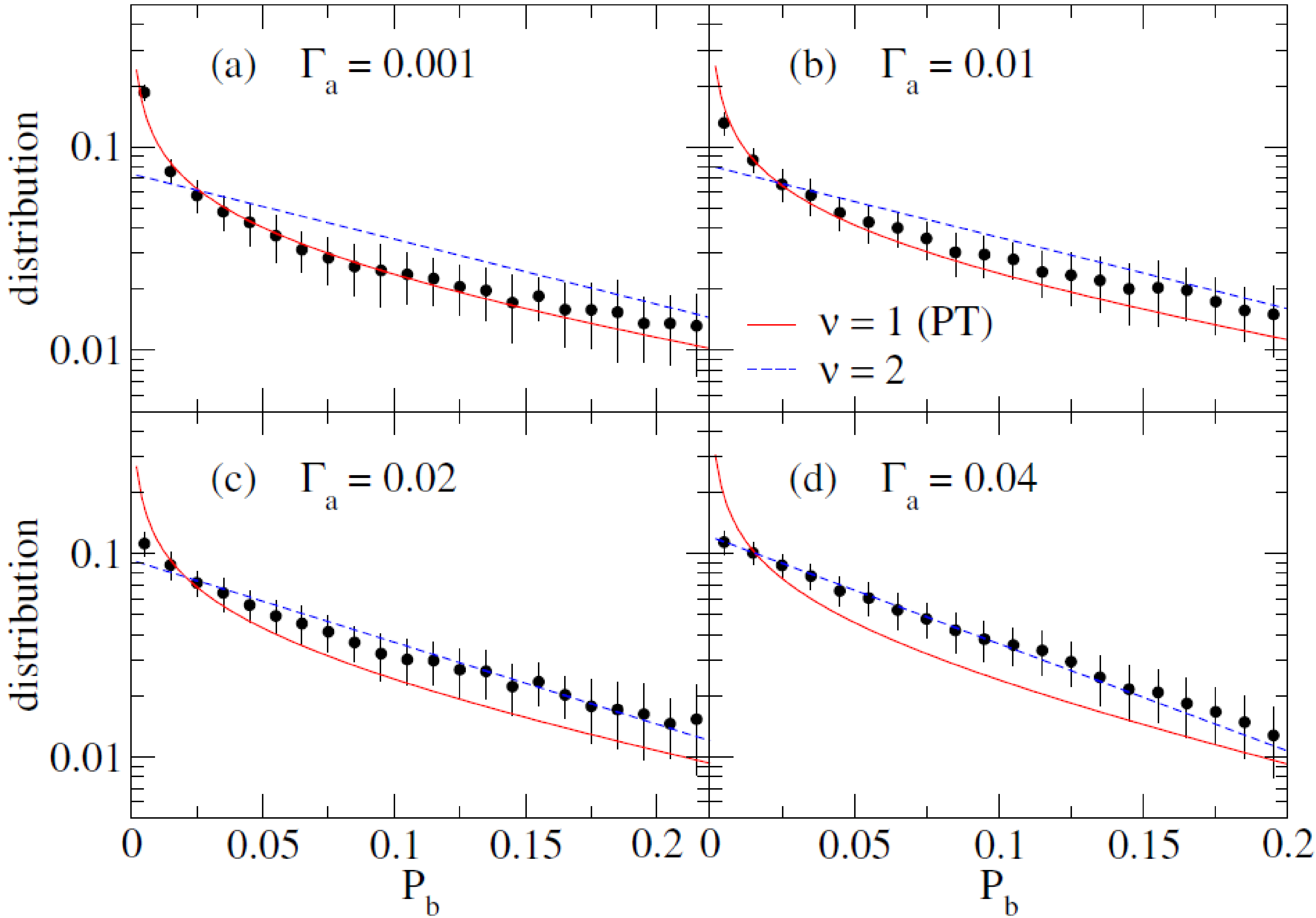
$$P_\nu(x) = \frac{\nu}{2x_0\Gamma(\nu/2)}\left(\frac{\nu x}{2x_0}\right)^{\nu/2-1} e^{-\nu x/2x_0}$$

ν = the number of d.o.f.
(ν=1 for PT)

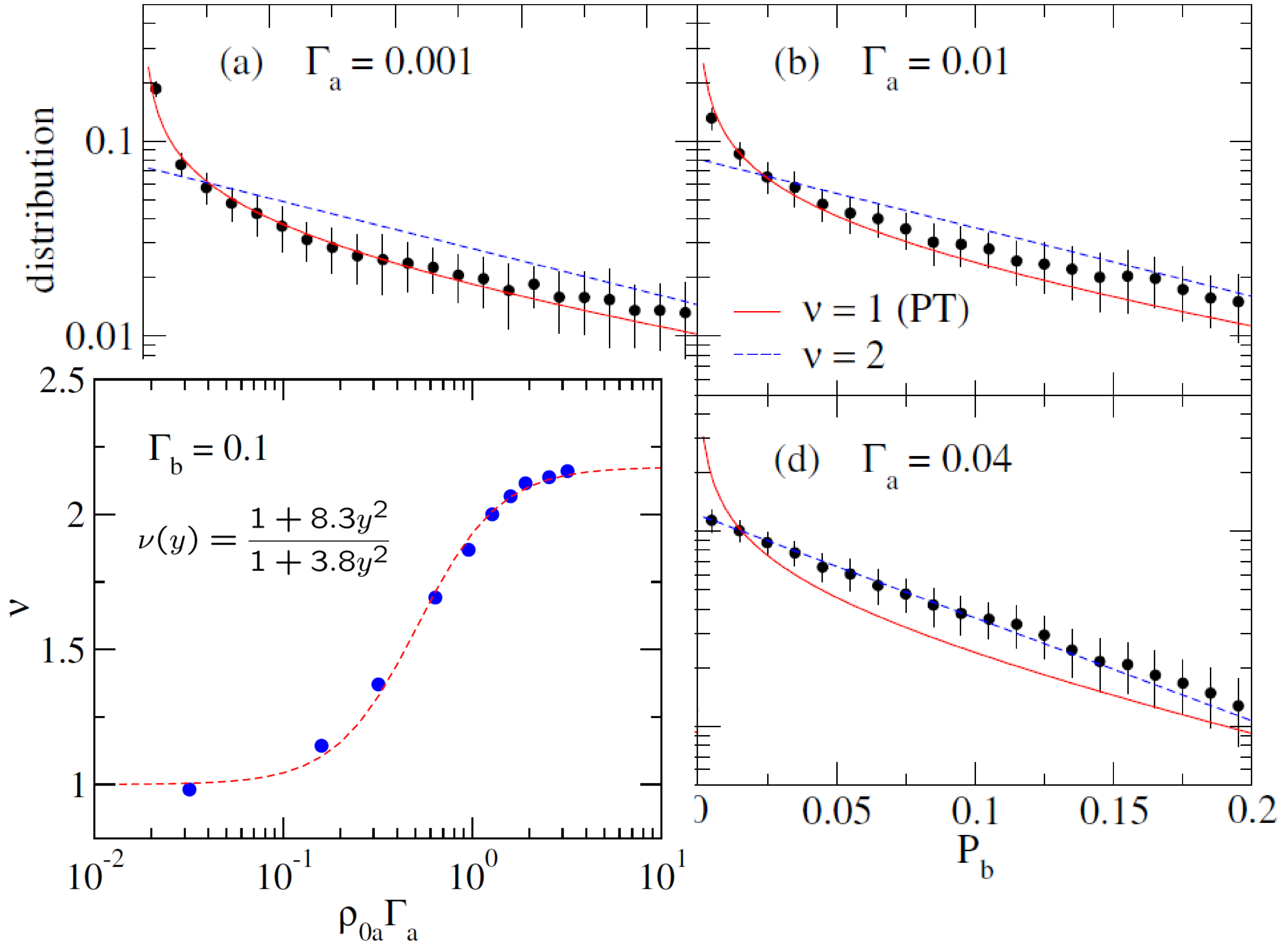
cf. GUE: ν=2



K. Hagino and G.F. Bertsch, arXiv:2106.152501 [quant-ph]



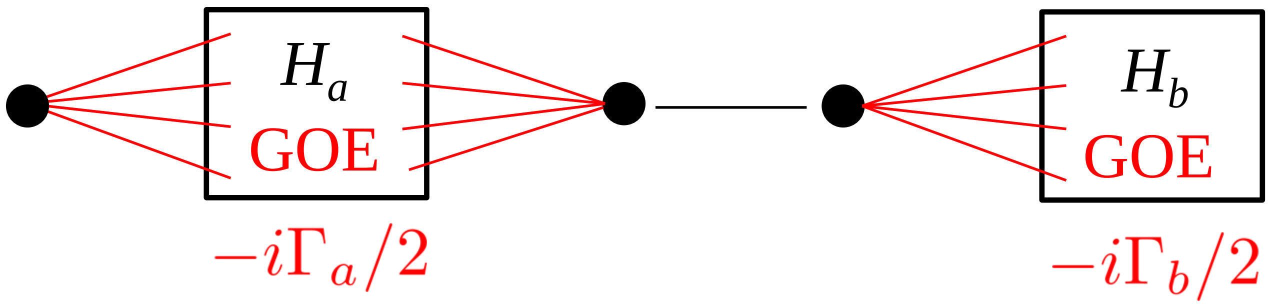
(a) $\Gamma_a = 0.001$
(b) $\Gamma_a = 0.01$
(c) $\Gamma_a = 0.02$
(d) $\Gamma_a = 0.04$
$\nu = 1$ (PT)
$\nu = 2$
distribution
0.1
0.01
0
0.05
0.1
0.15
0.2
P_b



(a) Γa = 0.001
(b) Γa = 0.01
(d) Γa = 0.04
distribution
0.1
0.01
ν = 1 (PT)
ν = 2
Γb = 0.1
ν(y) = (1 + 8.3y²)/(1 + 3.8y²)
ν
2.5
2
1.5
1
10⁻²
10⁻¹
10⁰
10¹
ρ0aΓa
0.05
0.1
0.15
0.2
Pb

Summary

2-GOE model for a decay of quantum complex systems



✓ decay rates: insensitive to Γ_b

→ the first microscopic realization of the transition-state theory

✓ fluctuations of the decay rates:

a transition from the Porter-Thomas to the ν=2 distribution

the boundary condition can make the distribution deviate from the PT

G.F. Bertsch and K. Hagino, arXiv:2105.12073, JPSJ in press.
K. Hagino and G.F. Bertsch, arXiv:2106.152501 [quant-ph]

➢ ongoing works

applications to nuclear fission: CI approach