Mapping from quasi-elastic scattering to fusion reactions

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   : fusion and quasi-elastic barrier distributions
2. *Recent results on quasi-elastic barrier distribution: role of non-collective excitations*
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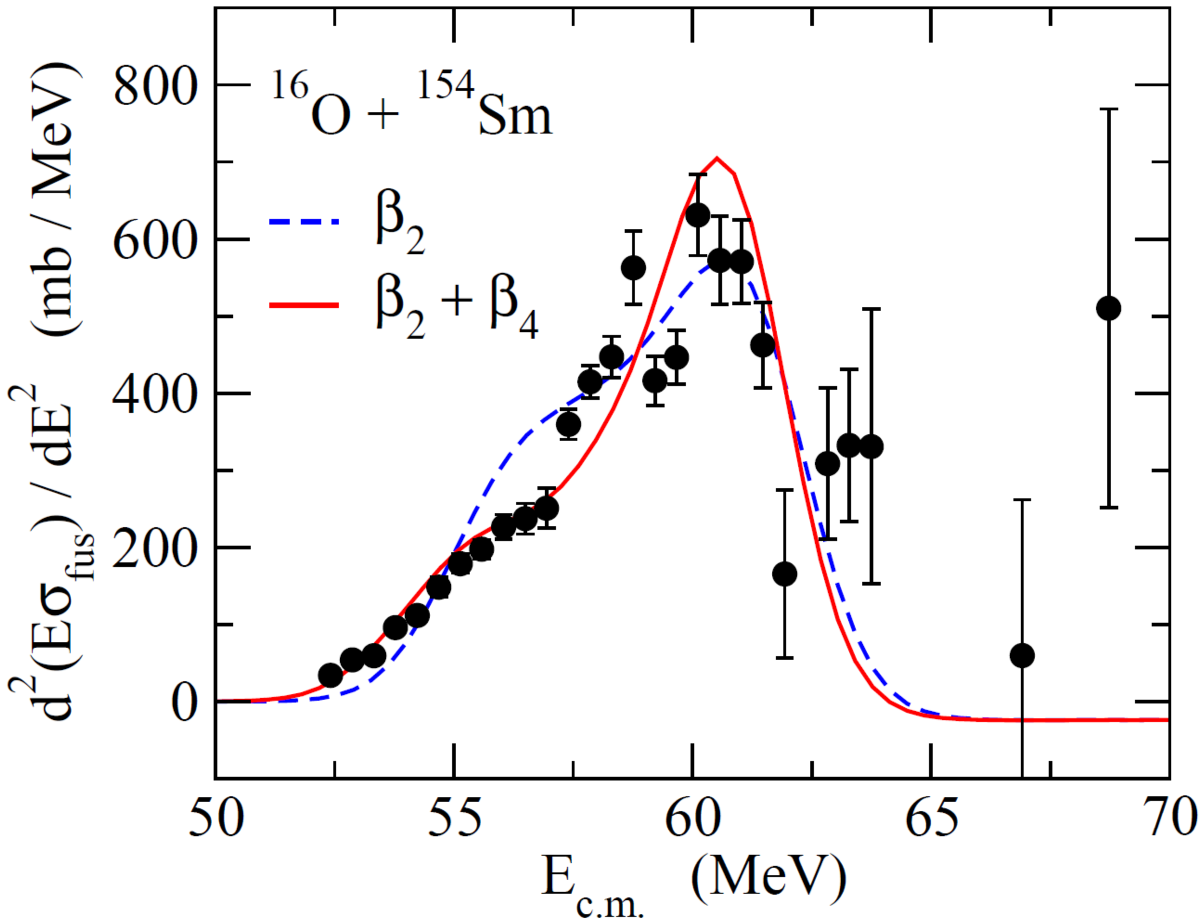
Introduction

Fusion barrier distribution

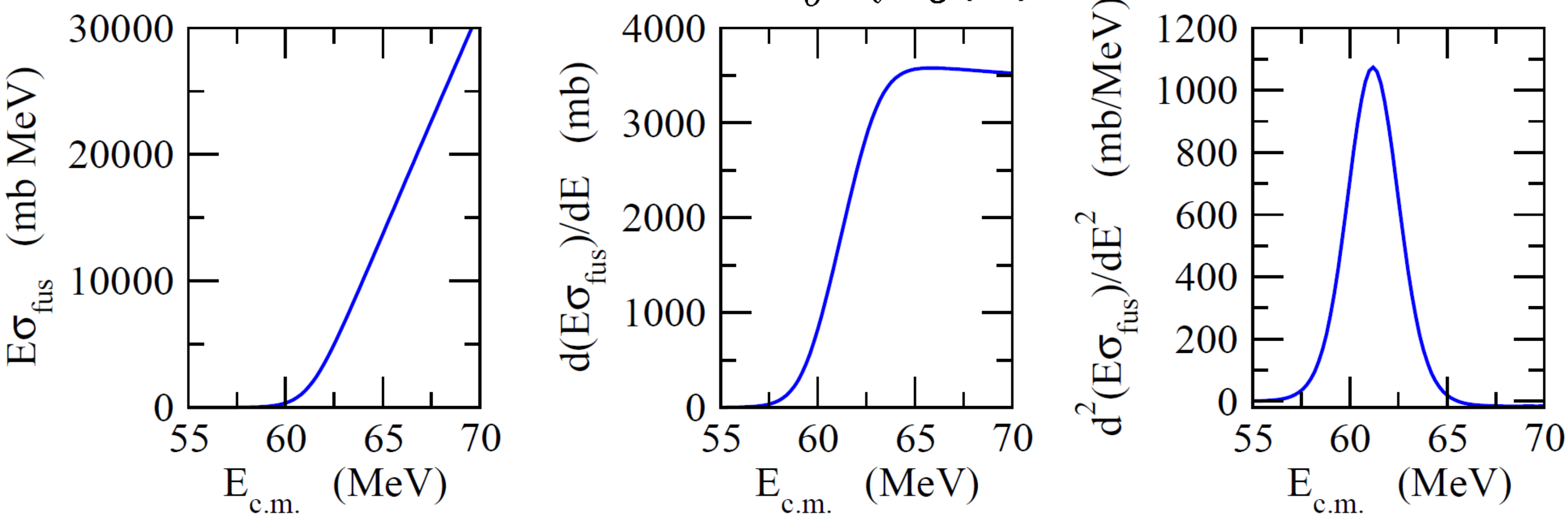
$$D_{\rm fus}(E) = \frac{d^2(E\sigma)}{dE^2}$$

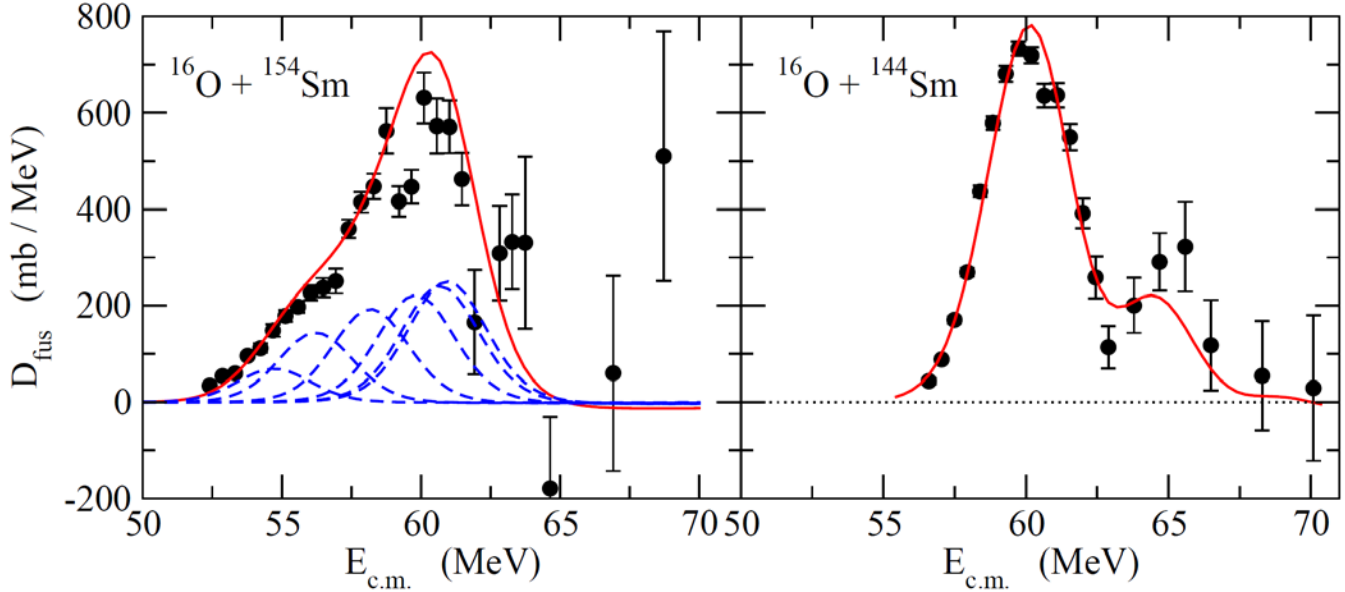
N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254('91) 25
J.X. Wei, J.R. Leigh et al., PRL67('91) 3368



M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48('98)401

$$\sim \pi R_b^2 \, P_{l=0}(E)$$





K.H. and N. Takigawa, PTP128 ('12) 1061

Quasi-elastic barrier distribution

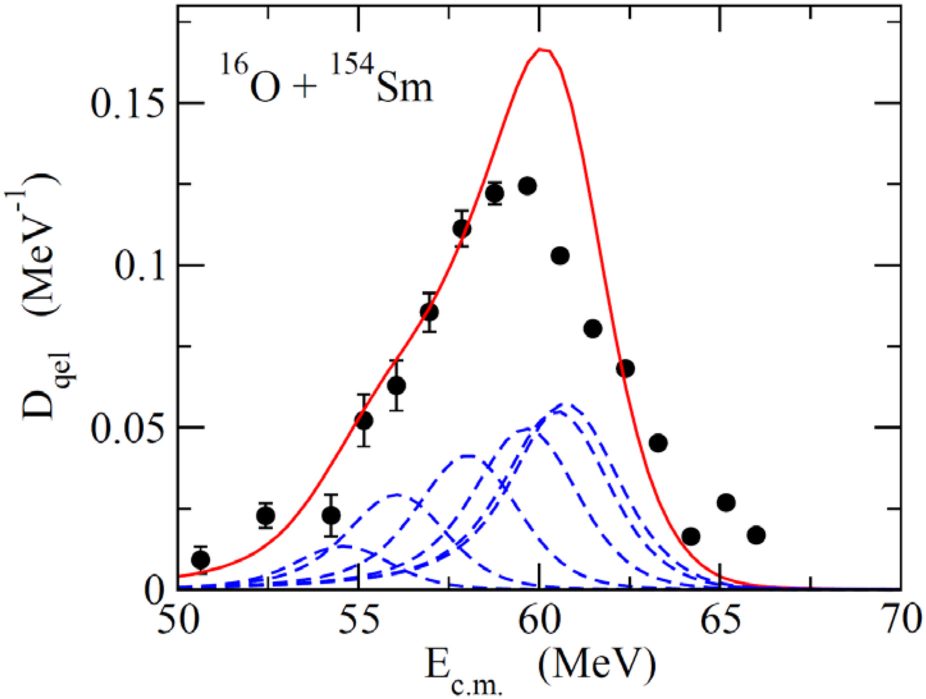
$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}\right)$$

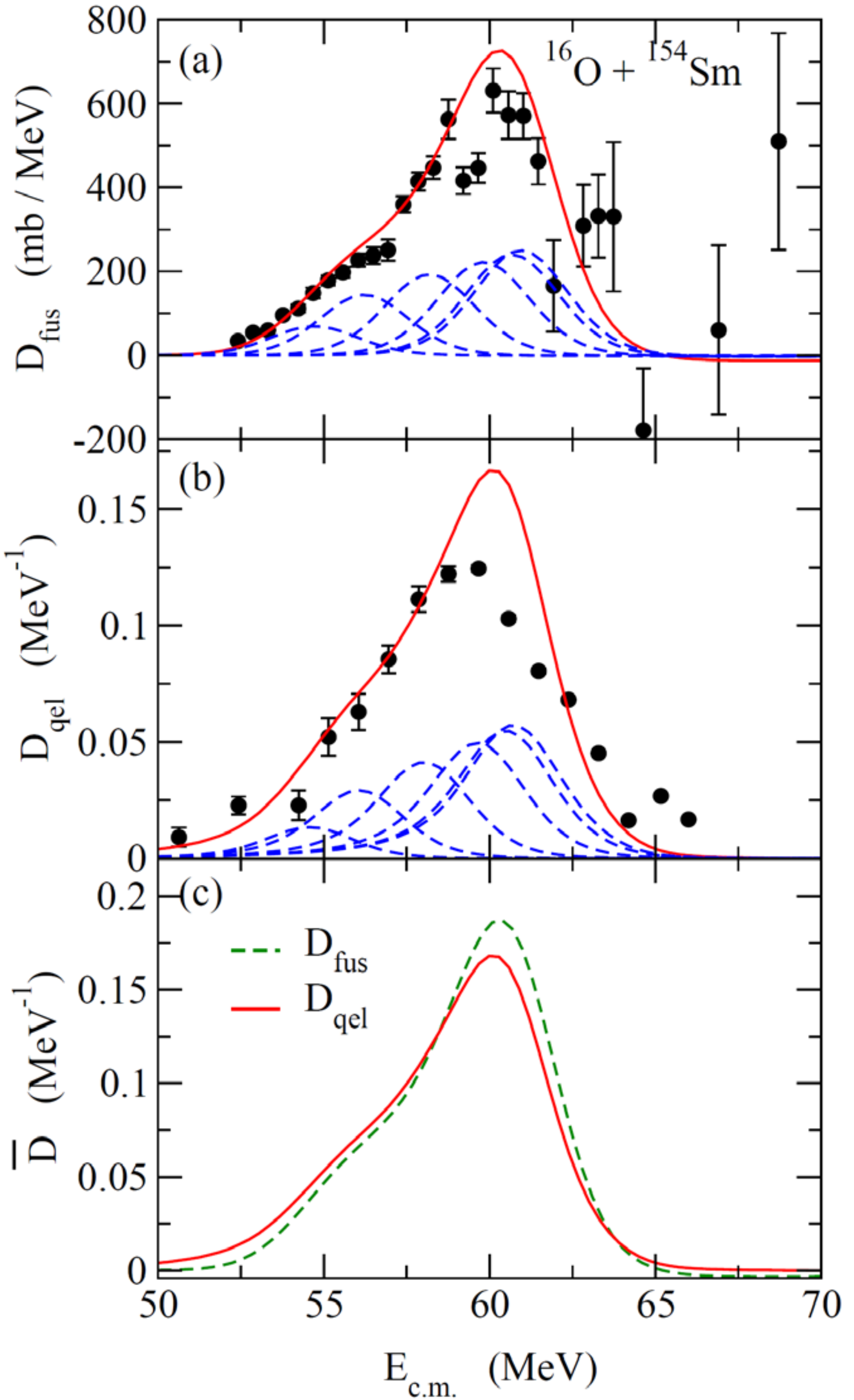
H. Timmers et al., NPA584('95)190

Quasi-elastic scattering:

A sum of all the reaction processes other than fusion
(elastic + inelastic + transfer + ……)

$$P_{l=0}(E) = 1 - R_{l=0}(E) \sim 1 - \frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}$$

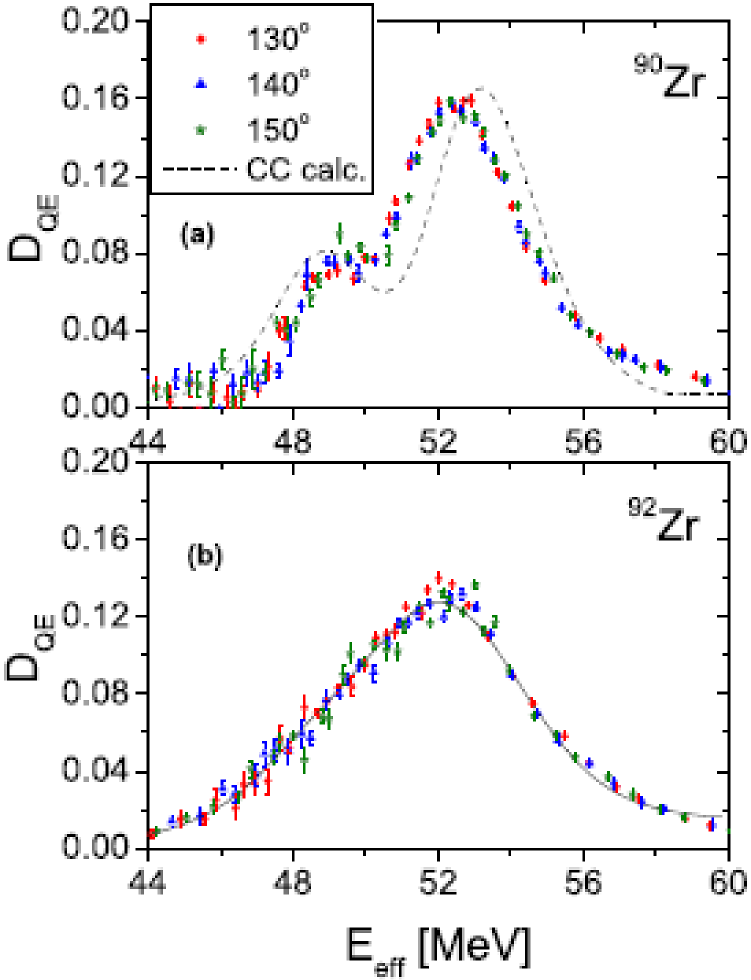




D_{fus} and D_{qel}: behave similarly to each other

K.H. and N. Rowley, PRC69('04)054610

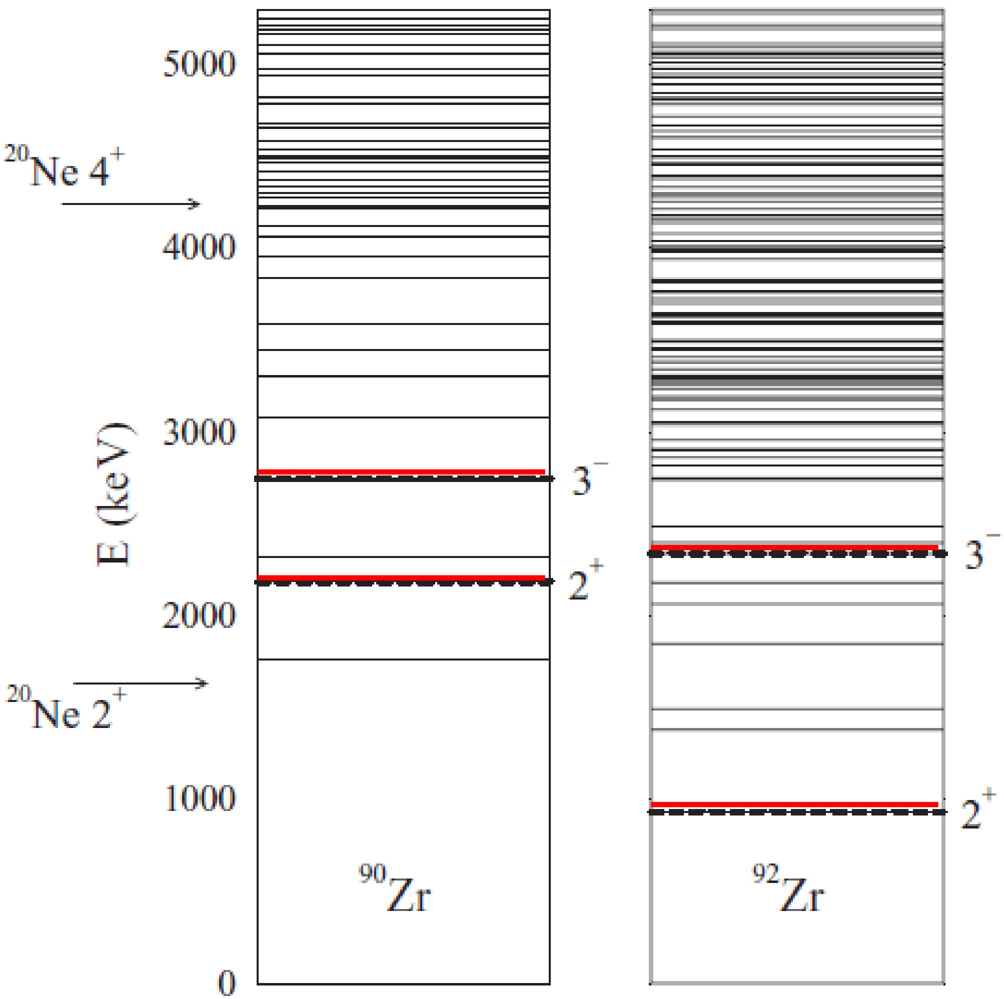
Quasi-elastic barrier distributions for $^{20}Ne + ^{90,92}Zr$



E. Piasecki et al., PRC80('09)054613

- C.C. results are almost the same between the two systems
- Yet, quite different barrier distribution and Q-value distribution

non-collective excitations?



red: collective levels

a typical model space for conventional C.C. calculations

non-collective levels

35 levels (^{90}Zr), 87 levels (^{92}Zr) up to 5 MeV

^{90}Zr (Z=40 sub-shell closure, N=50 shell closure)

^{92}Zr = ^{90}Zr + 2n

effects of the difference in the level densities?

a problem: the nature of non-collective states is poorly known (the energy, spin, parity only) i.e., no information on the coupling strengths

Random matrix model

Coupled-channels equations:

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \epsilon_k - E\right]\psi_k(\boldsymbol{r}) + \sum_{k'}\langle\phi_k|V_{\rm coup}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r}) = 0$$

$|\phi_k\rangle$: complicated non-collective states

random numbers

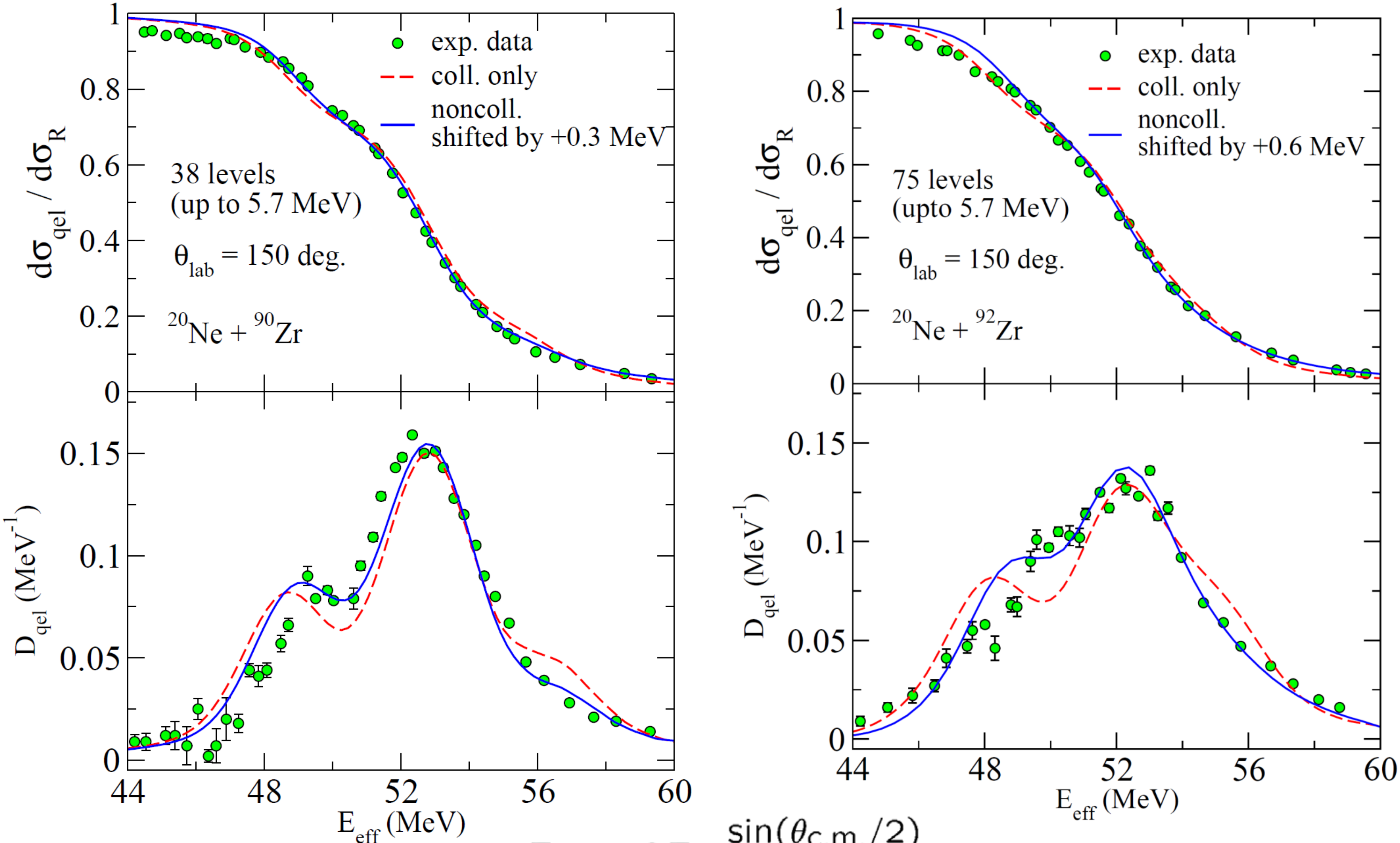
level density

$$\overline{V_{ij}(r)} = 0,$$

$$\overline{V_{ij}(r)V_{kl}(r')} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\frac{w_0}{\sqrt{\rho(\epsilon_i)\rho(\epsilon_j)}} \times e^{-\frac{(\epsilon_i-\epsilon_j)^2}{2\Delta^2}} \cdot e^{-\frac{(r-r')^2}{2\sigma^2}} \cdot h(r)h(r')$$

D. Agassi, C.M. Ko, and H.A. Weidenmuller, Ann. of Phys. 107('77)140
cf. Deep Inelastic Collisions

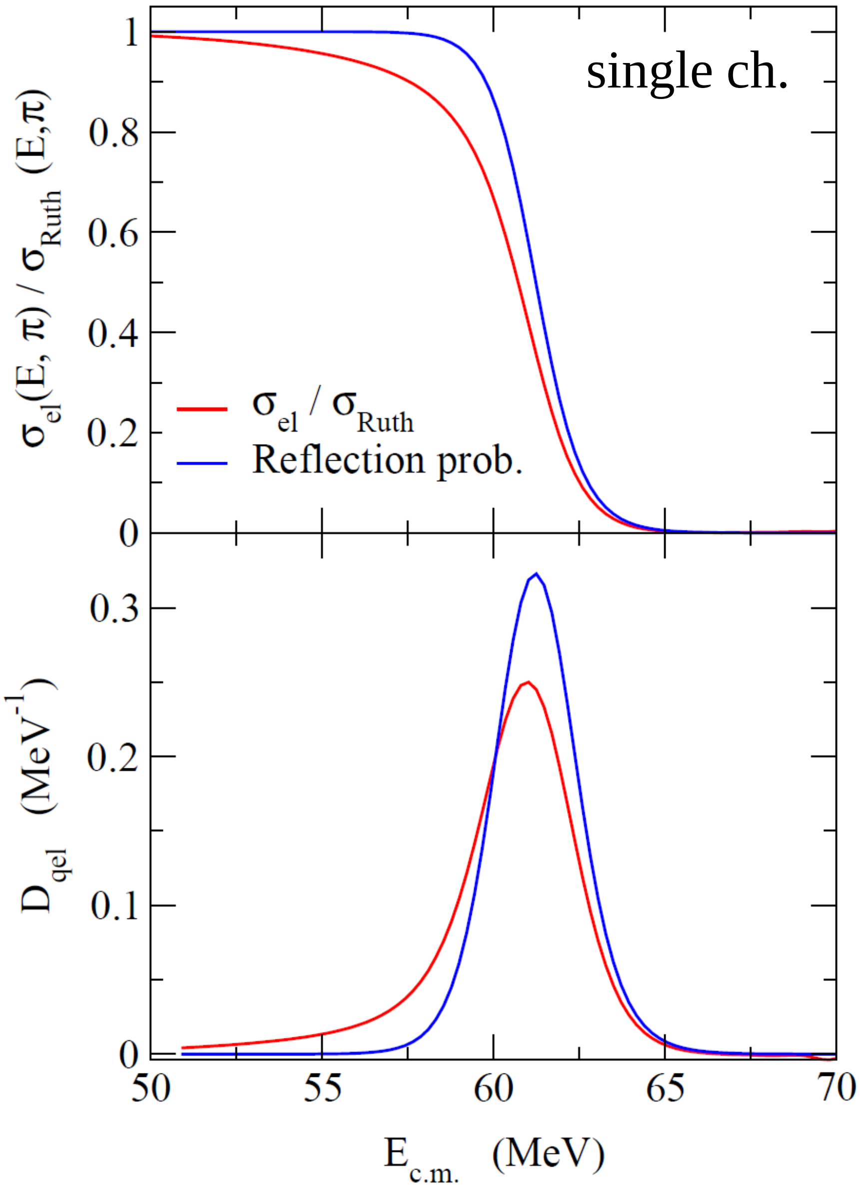
Results



$$E_{\text{eff}} = 2E\frac{\sin(\theta_{\text{c.m.}}/2)}{1+\sin(\theta_{\text{c.m.}}/2)}$$

S. Yusa, K.H., and N. Rowley, PRC88('13)054621

Problems with quasi-elastic barrier distributions

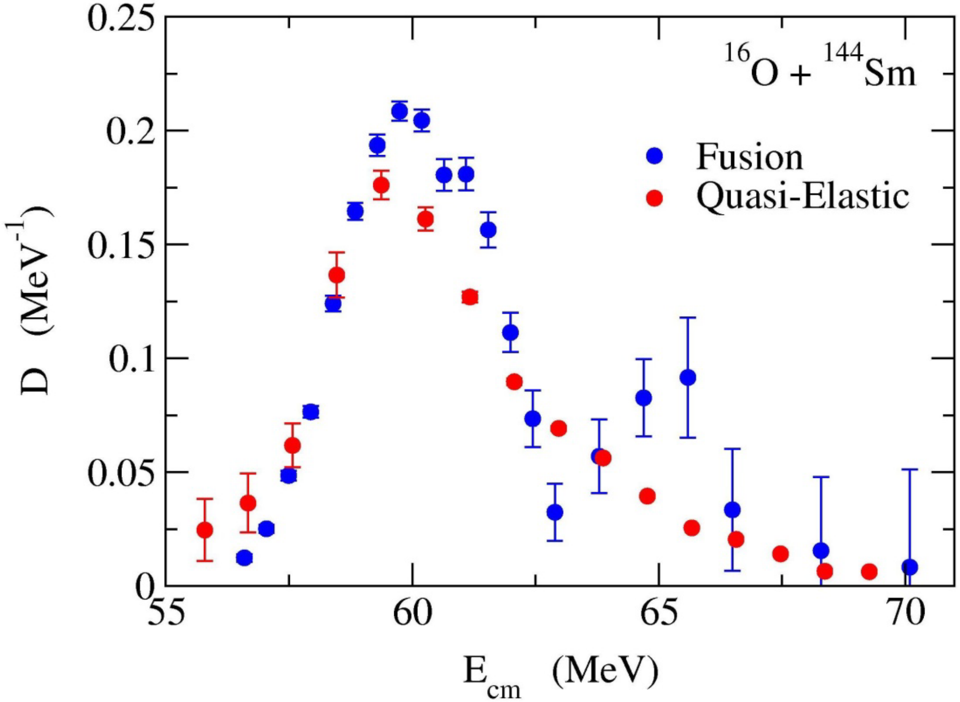


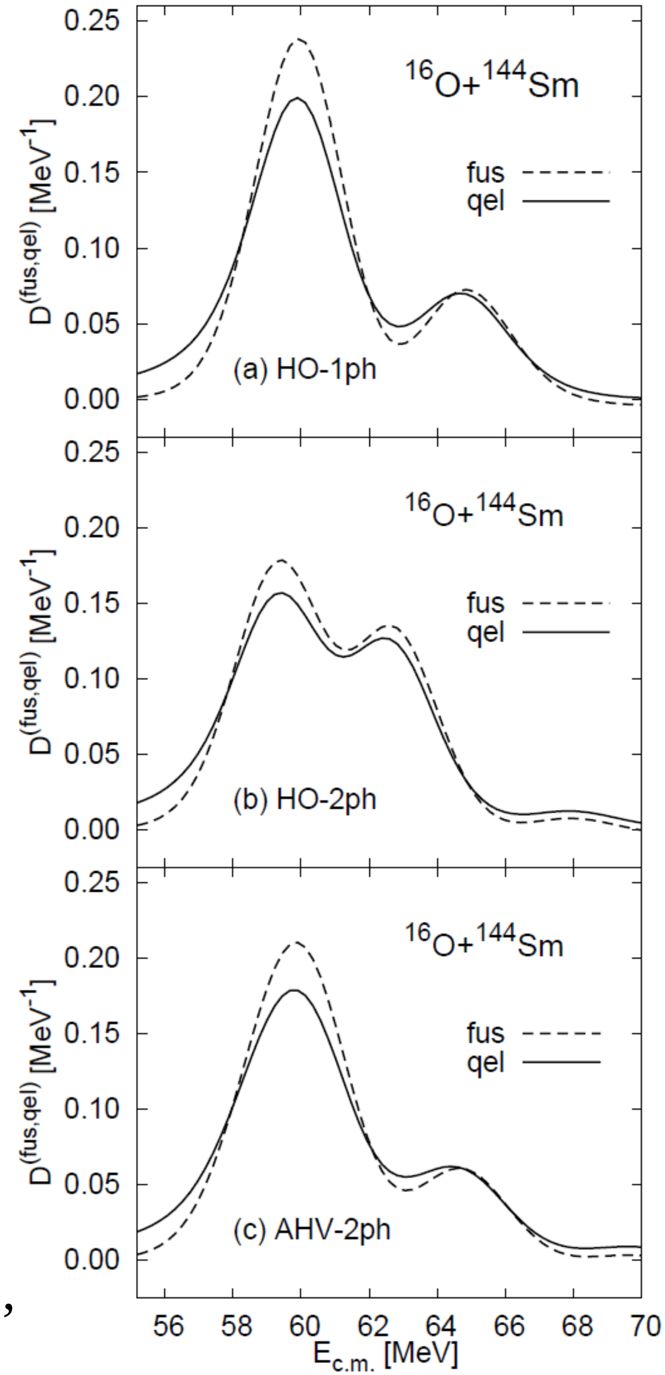
$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}\right)$$

D_{qel} and D_{fus}: behave similarly, but not identically

the effect of nuclear distortion of the classical trajectory

K.H. and N. Rowley, PRC69('04)054610



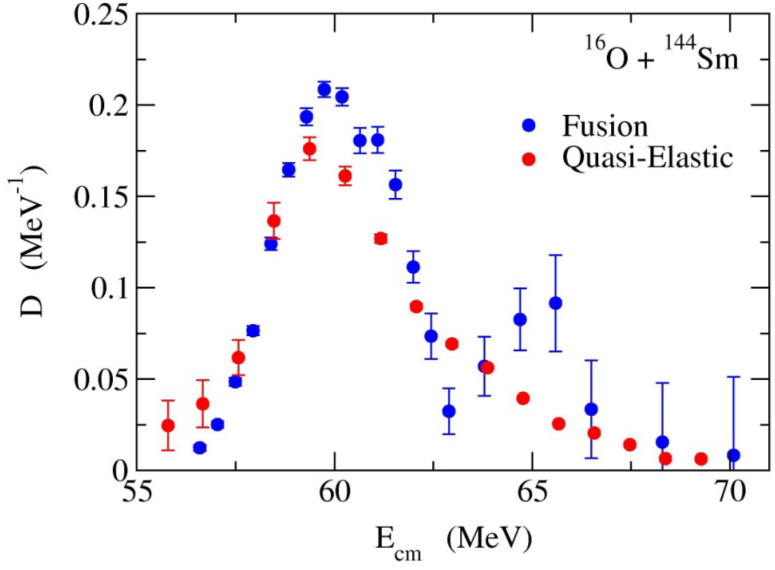


M. Zamrun F. and K.H.,
PRC77(‘08)014606

Problems with quasi-elastic barrier distributions

$$D_{\mathrm{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\mathrm{qel}}(E,\pi)}{\sigma_{\mathrm{Ruth}}(E,\pi)}\right)$$

- D_{qel} and D_{fus}: behave similarly, but not identically



- D_{qel} : not applicable to symmetric systems

$$\sigma(\theta) = |f(\theta) \pm f(\pi - \theta)|^2$$

⟶ diverges at $\theta = \pi$

Sum-of-differences (SOD) method

J.T. Holdeman and R.M. Thaler, PRL14('65)81, PR139('65)B1186
C. Marty, Z. Phys. A309('83)261, A322('85)499

$$\sigma_R \sim 2\pi \int_{\theta_{\rm min}}^{\pi} \sin\theta\, d\theta\, (\sigma_{\rm Ruth}(\theta) - \sigma_{\rm el}(\theta))$$

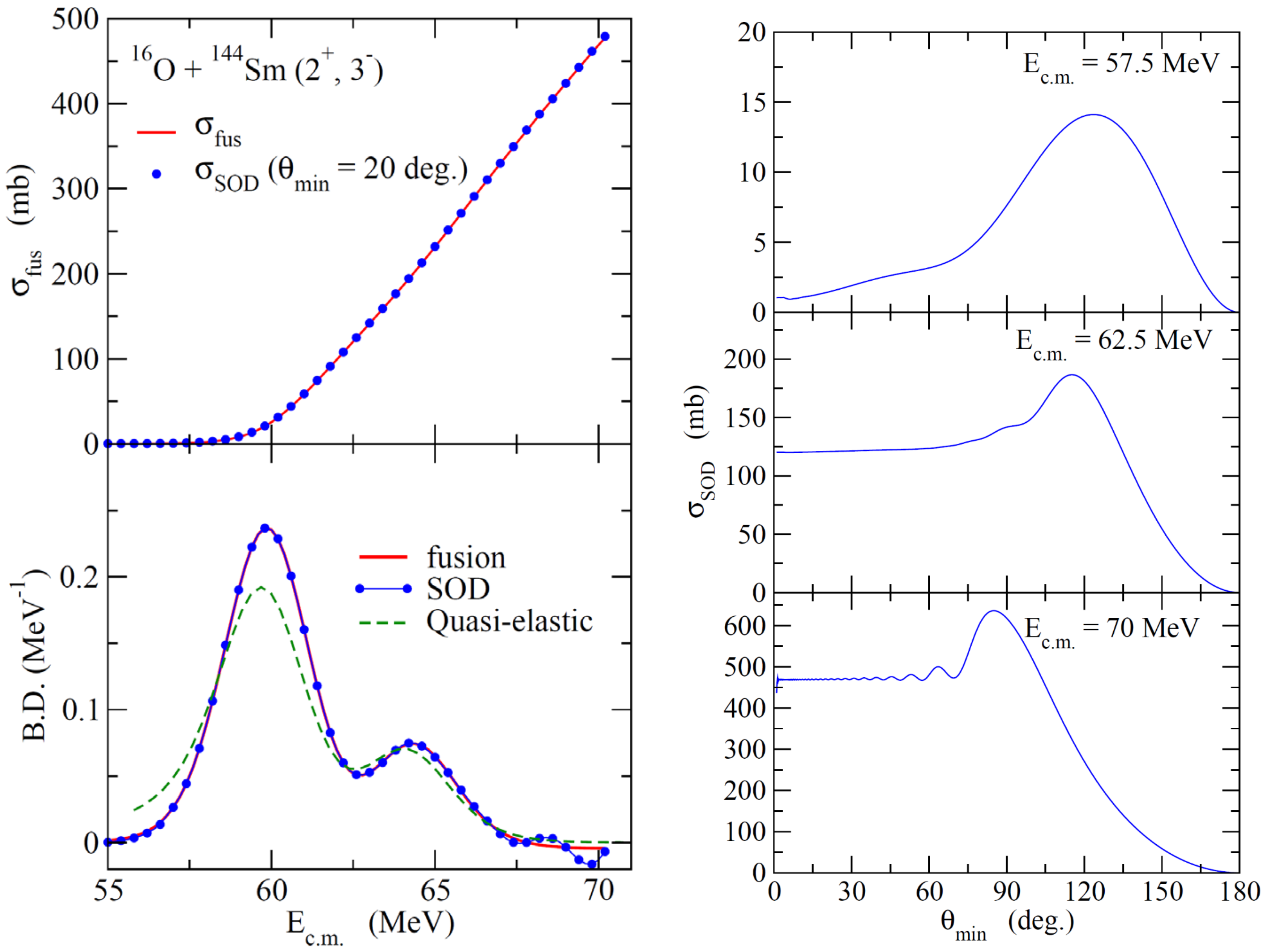
expt.: H. Wojciechowski et al., PRC16('77)1767
H. Oeschler et al., NPA325('79)463
T. Yamaya et al., PLB417('98)7 etc.

generalization (K.H. and N. Rowley, in preparation)

$$\sigma_R = \sigma_{\rm fus} + \sigma_{\rm inel} + \sigma_{\rm tr}$$

$$\sigma_{\rm fus} \sim 2\pi \int_{\theta_{\rm min}}^{\pi} \sin\theta\, d\theta\, (\sigma_{\rm Ruth}(\theta) - \sigma_{\rm qel}(\theta))$$

$\longrightarrow$ D_{fus} from σ_{qel}?

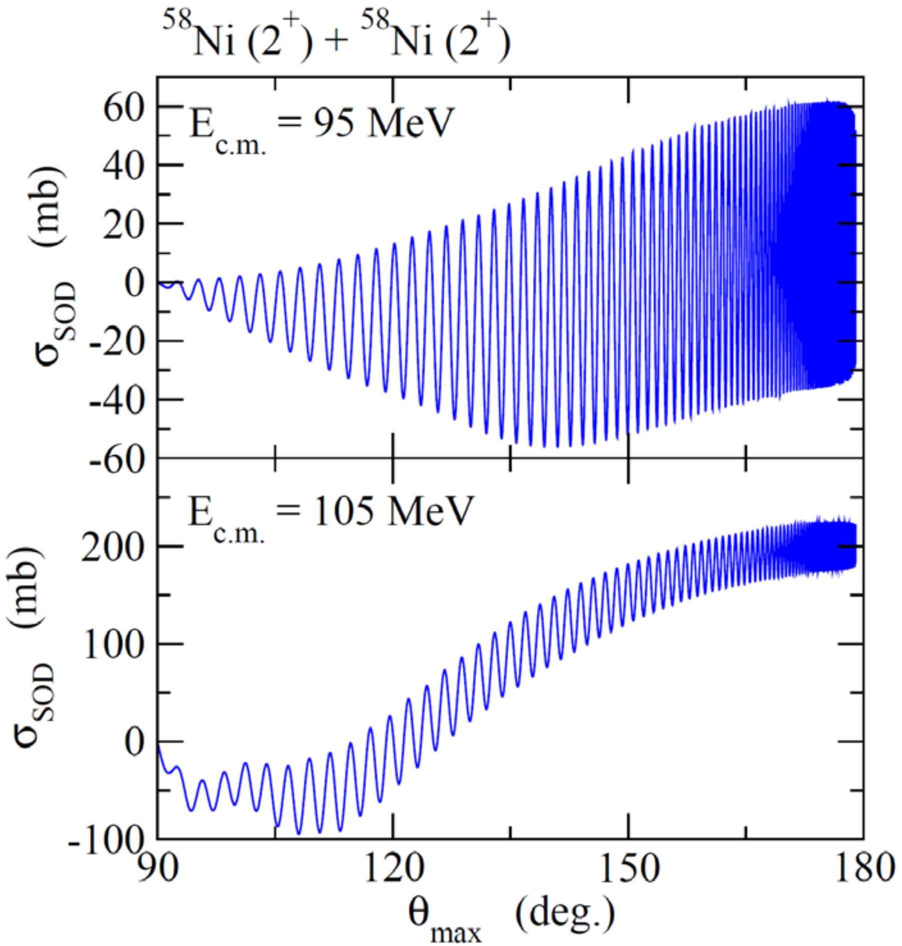


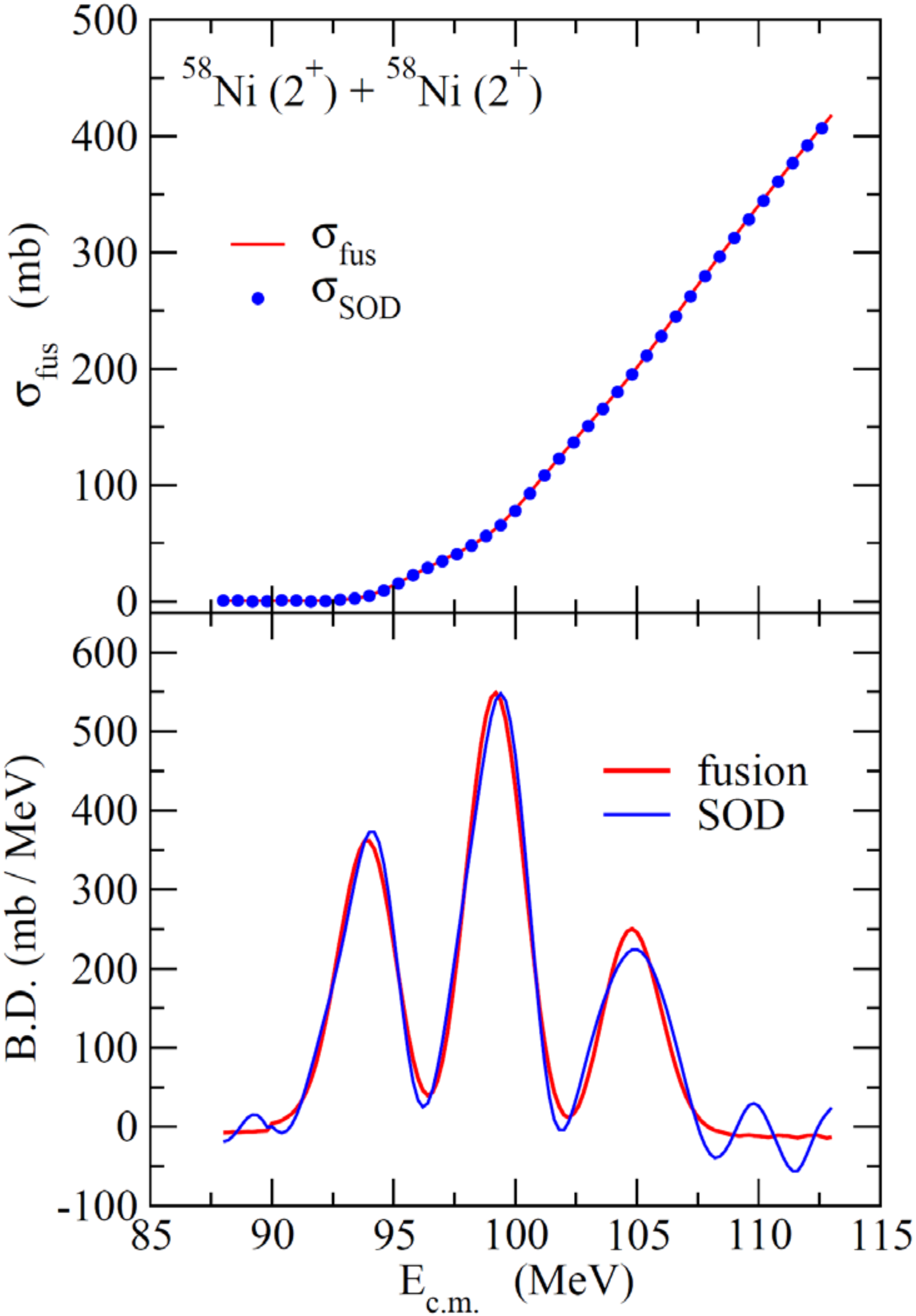
^{16}O + ^{144}Sm (2$^+$, 3$^-$)
σ_{fus}
σ_{SOD} (θ_{min} = 20 deg.)
σ_{fus} (mb)
B.D. (MeV^{-1})
fusion
SOD
Quasi-elastic
$E_{c.m.}$ (MeV)
$E_{c.m.}$ = 57.5 MeV
$E_{c.m.}$ = 62.5 MeV
$E_{c.m.}$ = 70 MeV
σ_{SOD} (mb)
θ_{min} (deg.)

Symmetric sysmtes

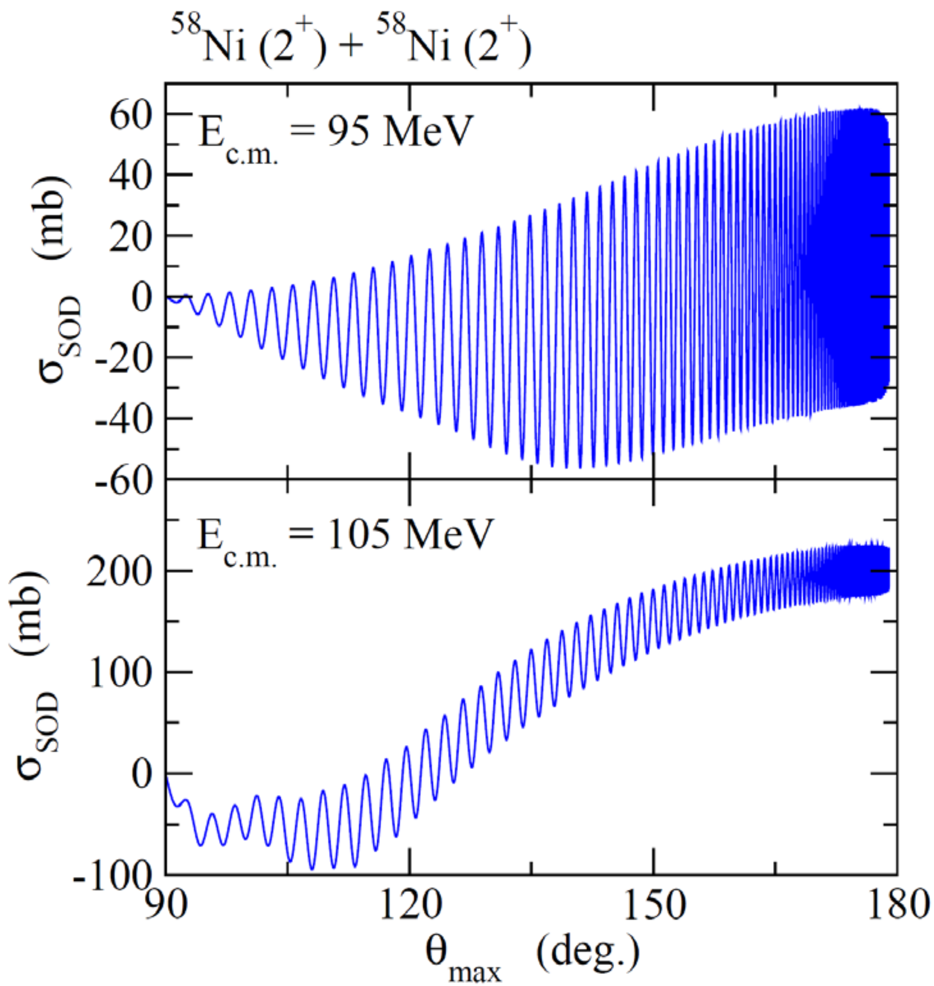
$$\sigma_{\text{fus}} \sim 2\pi \int_{\theta_{\text{min}}}^{\pi} \sin\theta \, d\theta \, (\sigma_{\text{Ruth}}(\theta) - \sigma_{\text{qel}}(\theta))$$

$$\longrightarrow \quad \boxed{\sigma_{\text{fus}} \sim 2\pi \int_{\pi/2}^{\theta_{\text{max}}} \sin\theta \, d\theta \, (\sigma_{\text{Mott}}(\theta) - \sigma_{\text{qel}}(\theta))}$$

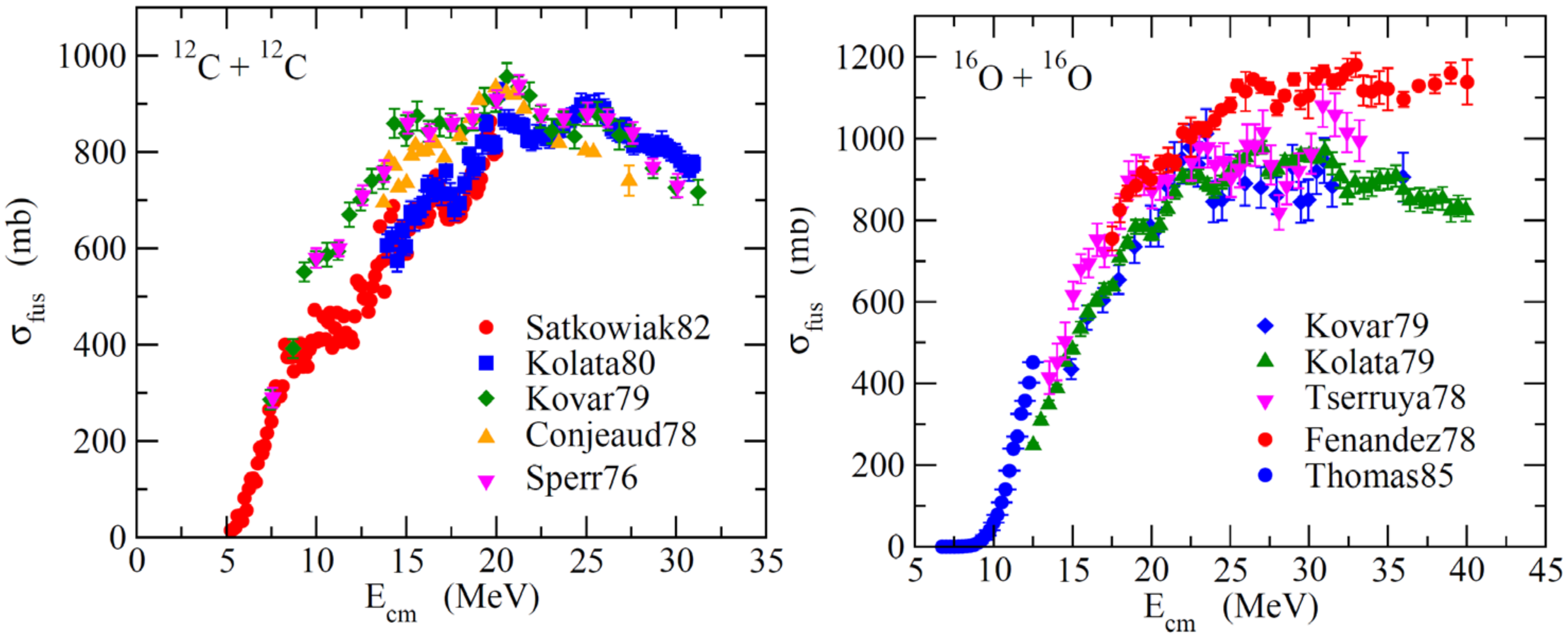




← average in the range of θ_{max} = (176.5, 179.5) deg.



Fusion of light symmetric systems: fusion oscillations

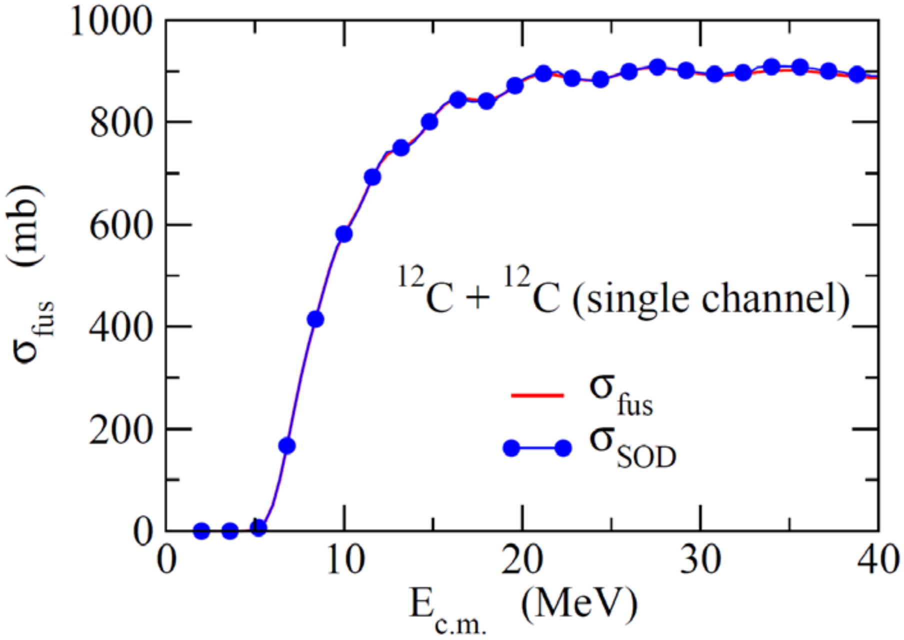


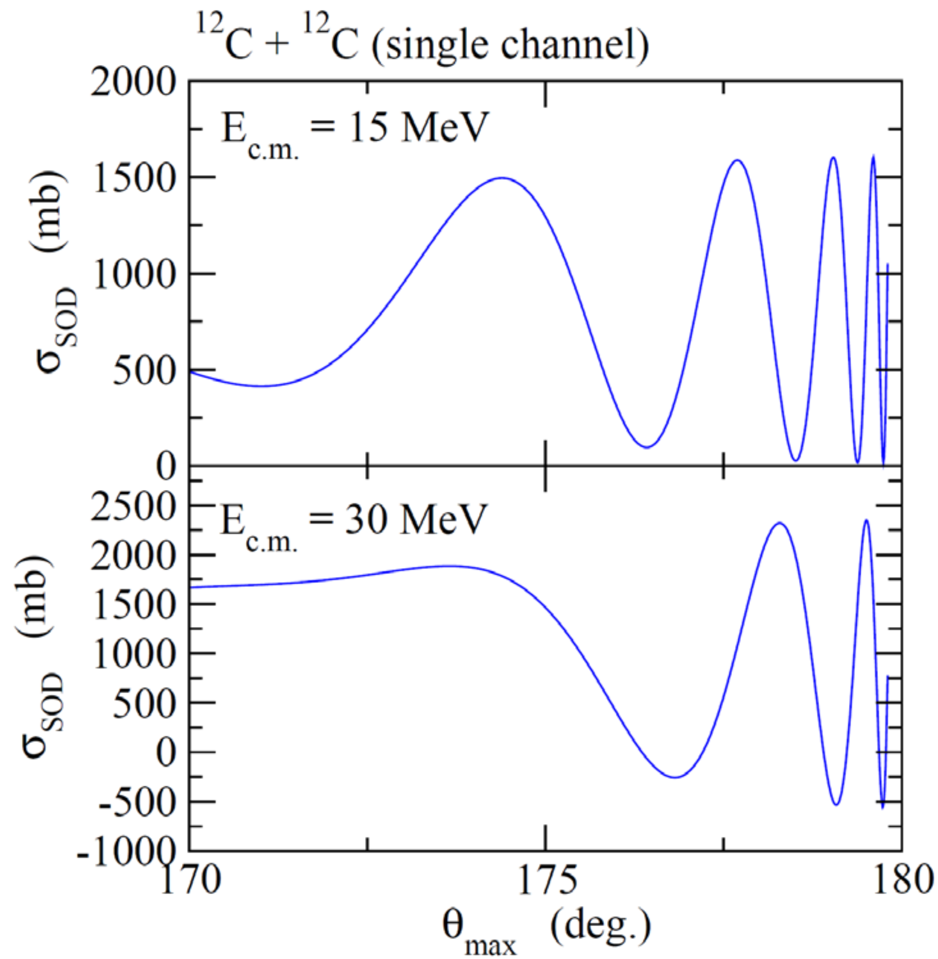
The expt. data: rather scattered

✓systematic errors

✓missing evaporation channels

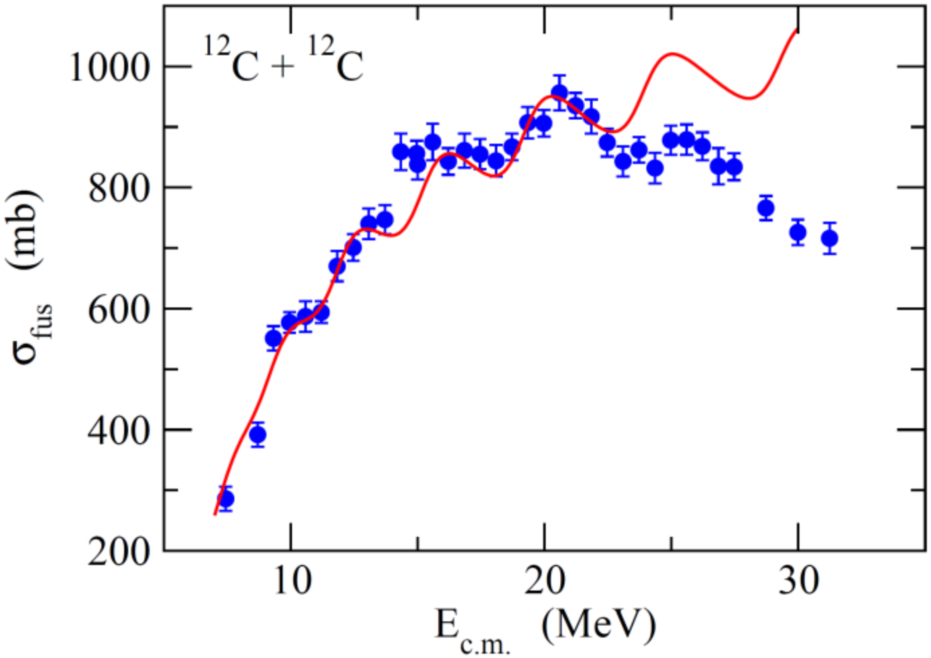
⟶ σ_{fus} from SOD?





average of a maximum and a minimum in σ_{SOD}

Fusion oscillations



N. Poffe, N. Rowley, R. Lindsay, NPA410('83) 498

Poisson sum rule
+ parabolic approximation

$$\sigma_{\text{fus}}(E) \sim \sigma_{\text{Wong}} + 2\pi R_b^2 \frac{\hbar\Omega}{E} e^{-\xi} \sin(\pi l_g)$$

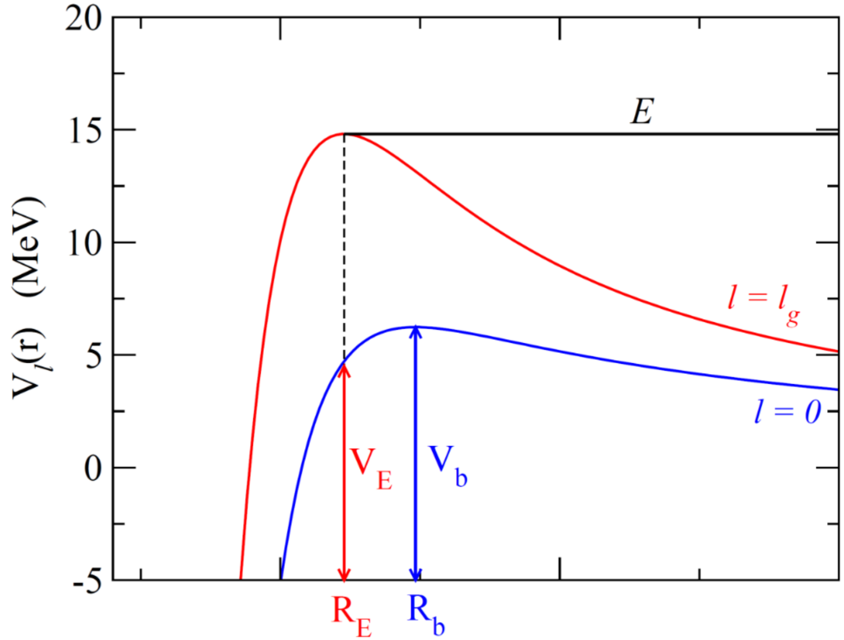
$$\xi = \pi \cdot \frac{\hbar\Omega}{2l_g + 1} \cdot \frac{\mu R_b^2}{\hbar^2}$$

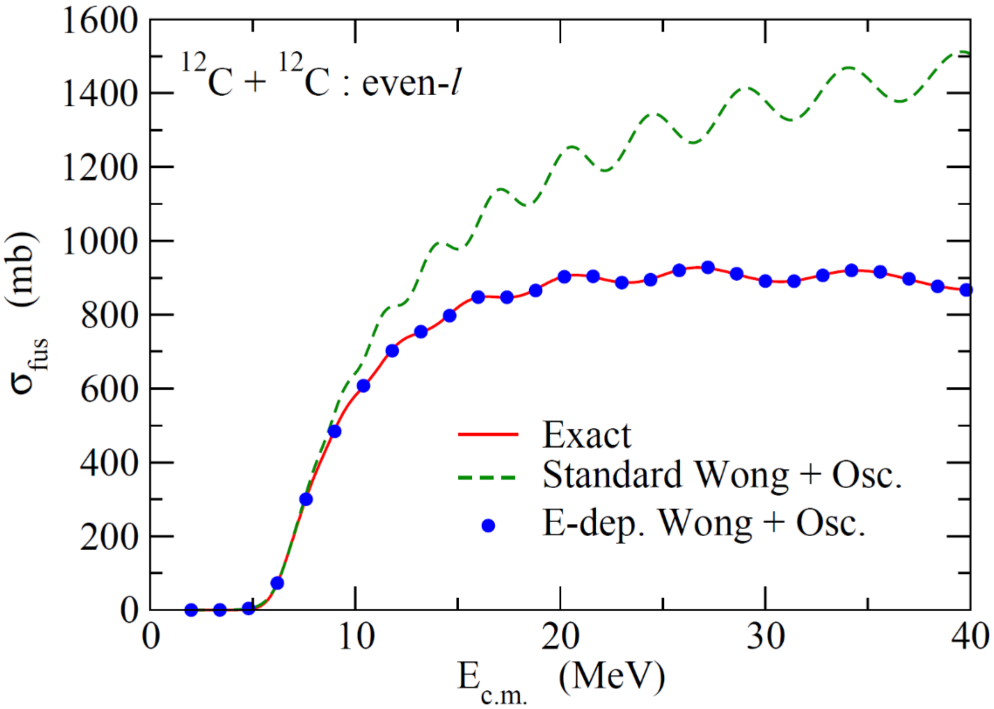
fusion oscillations:
successive contributions of the centrifugal barriers

cf. recent papers: H. Esbensen, PRC85('12) 064611
C.Y. Wong, PRC86('12) 064603
C. Simenel et al., PRC88 ('13) 024617

E-dependent Wong formula

N. Rowley, A. Kabir, and R. Lindsay, J. of Phys. G15('89)L269
N. Rowley and K. Hagino, in preparation





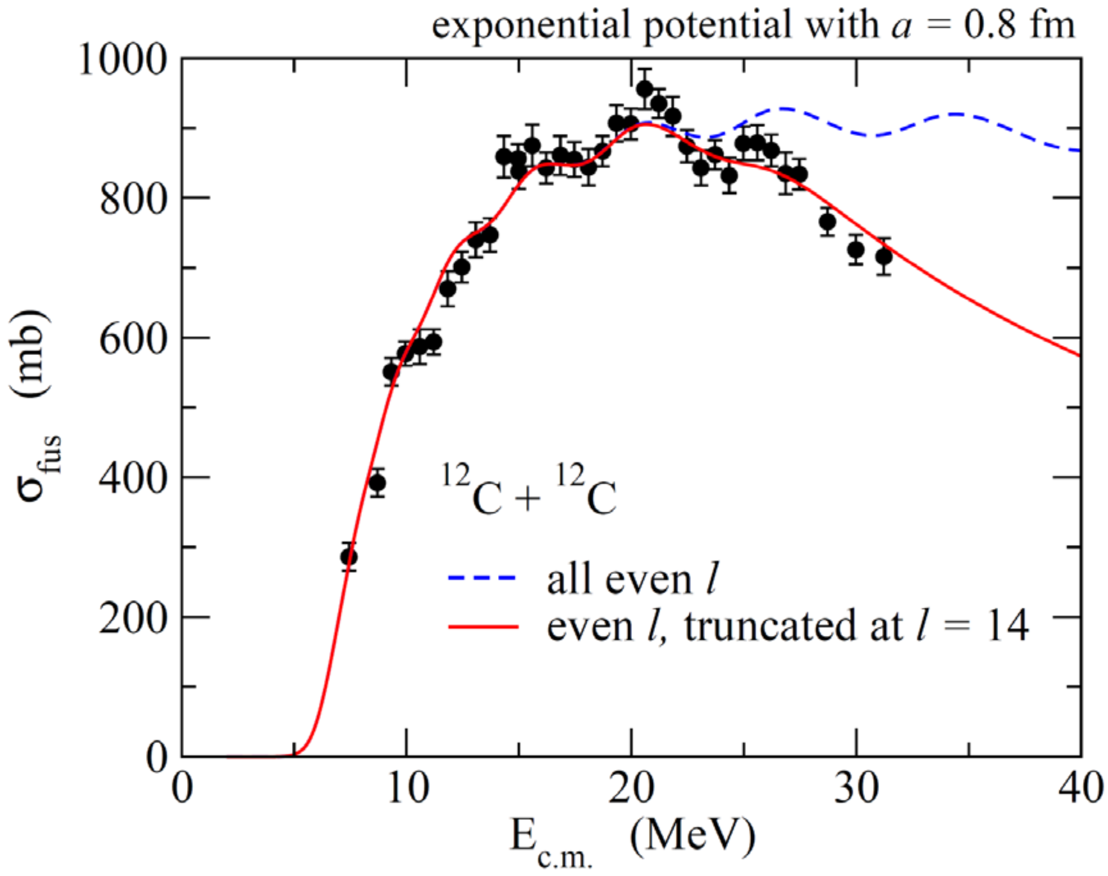
use V_b, R_b, and Ω for the grazing angular momentum, l_g

$\longrightarrow$ V_E, R_E, and Ω_E

comparison with the experimental data

$^{12}C_{g.s.} : 0^+ \longrightarrow$ the relative w.f. has to be spatially symmetric

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (1+(-)^l)(2l+1)P_l(E)$$



Summary

✓ Fusion barrier distribution $D_{\text{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}$

✓ Quasi-elastic barrier distribution $D_{\text{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\text{qel}}(E,\pi)}{\sigma_{\text{Ruth}}(E,\pi)}\right)$

✓ Sum-of-differenecs (SOD) method

$$\sigma_{\text{fus}} \sim 2\pi \int_{\theta_{\text{min}}}^{\pi} \sin\theta \, d\theta \, (\sigma_{\text{Ruth}}(\theta) - \sigma_{\text{qel}}(\theta))$$

D_{SOD}:

- closer correspondence to D_{fus} compared to D_{qel}
- applicable also to symmetric systems

➢ application to light symmetric systems? (fusion oscillations)