Collective excitations of Lambda hypernuclei

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4. Summary

Introduction

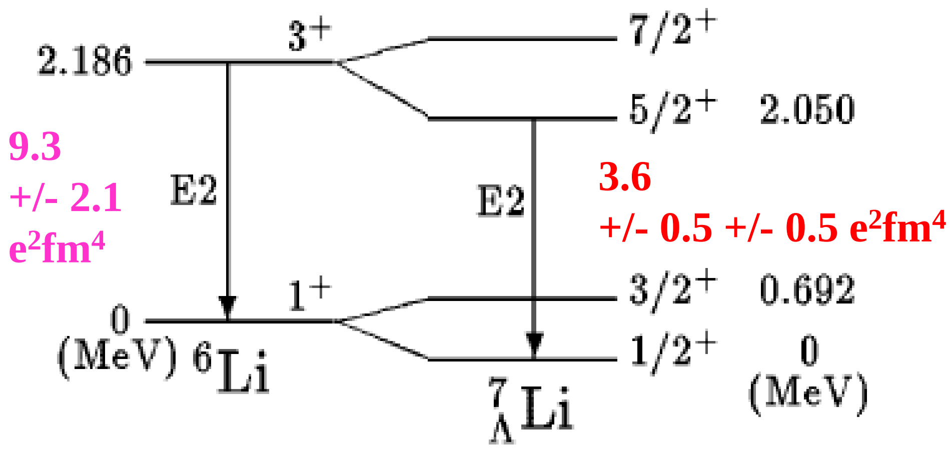
Impurity effects: one of the main interests of hypernuclear physics

how does Λ affect several properties of atomic nuclei?

- size, shape, density distribution, single-particle energy, shell effect, fission barrier……

the most prominent example:

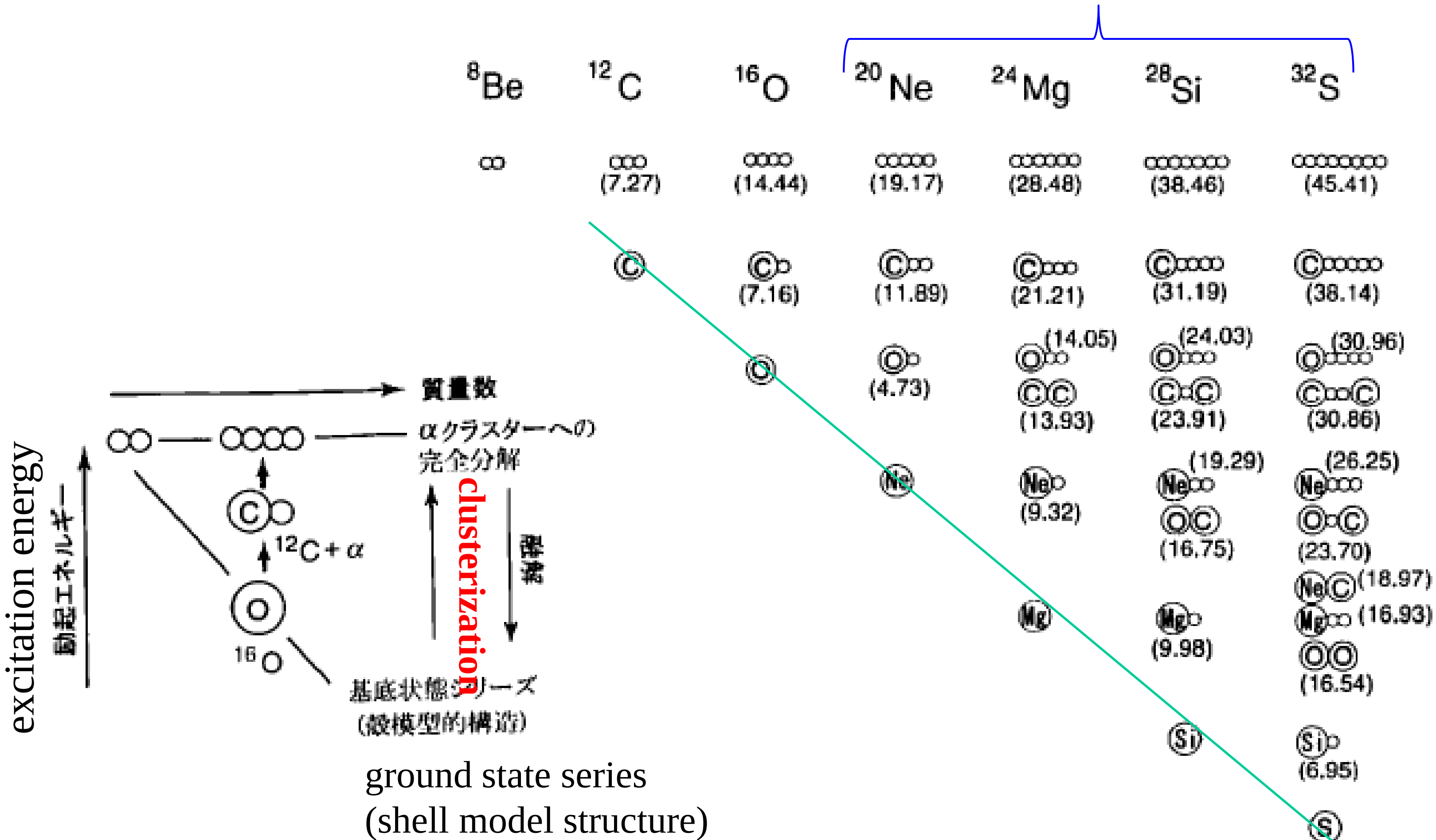
the reduction of B(E2) in $^{7}_{\Lambda}$Li



about 19% reduction of nuclear size
(shrinkage effect)

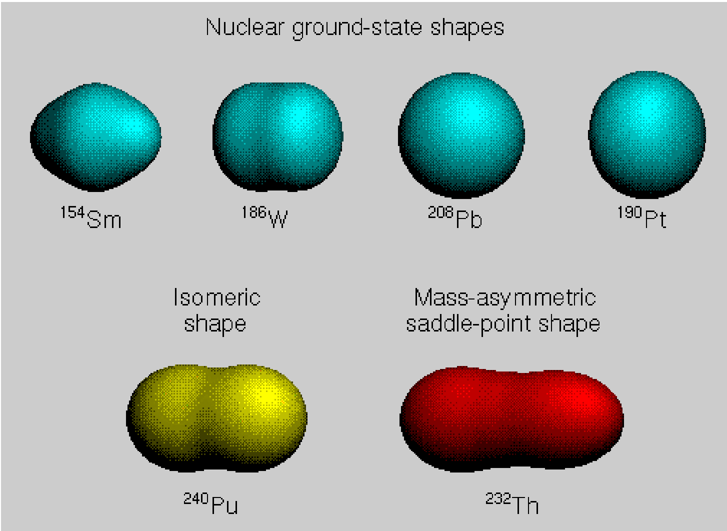
K. Tanida et al., PRL86('01)1982

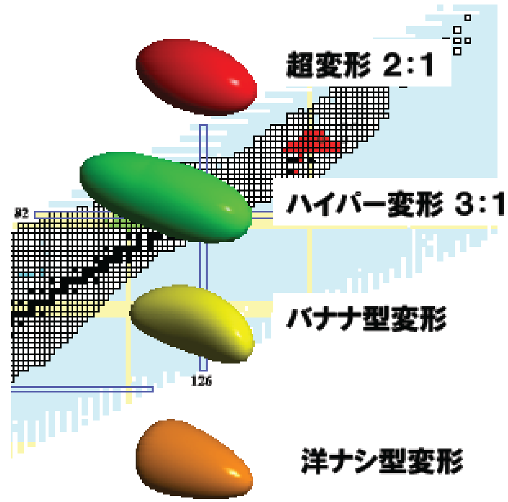
how about heavier nuclei?



the g.s. has a shell model-like structure for nuclei heavier than Be
(cluster-like structure appears in the excited states：threshold rule)

Shell model (mean-field) structure and nuclear deformation





http://t2.lanl.gov/tour/sch001.html

- many open-shell nuclei are deformed in the ground state
  - ✓characterstic feature of finite many-body systems
  - ✓spontaneous symmetry breaking of (rotational) symmetry
- B(E2) for deformed nuclei

$$B(E2:2^+\to 0^+)=\frac{1}{16\pi}\cdot Q_0^2$$

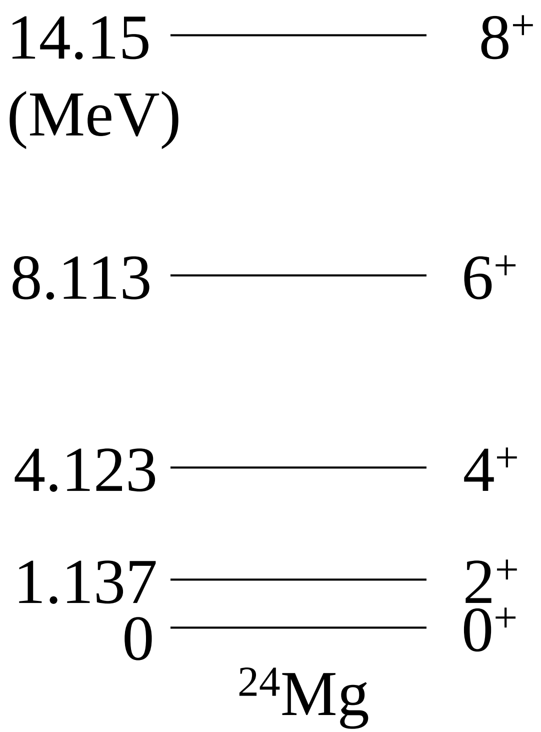
$$Q_0\sim\sqrt{\frac{16\pi}{5}}\frac{3}{4\pi}Ze\,R_0^2\,\beta$$

⇒ A change in B(E2) can be interpreted as a change in β

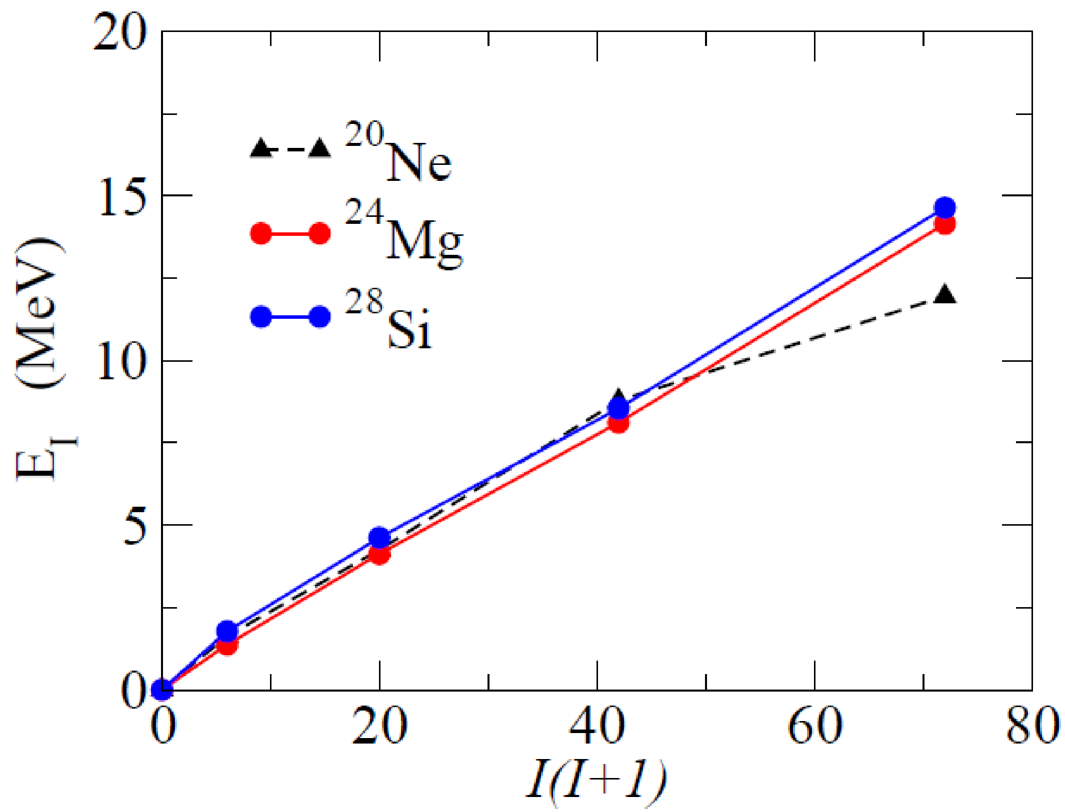
sd-shell nuclei : prominent nuclear deformation

an evidence for deformation

rotational spectrum

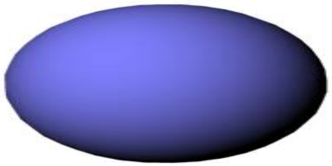


$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$



How is the deformation altered due to an addition of Λ particle?

Self-consistent mean-field method:



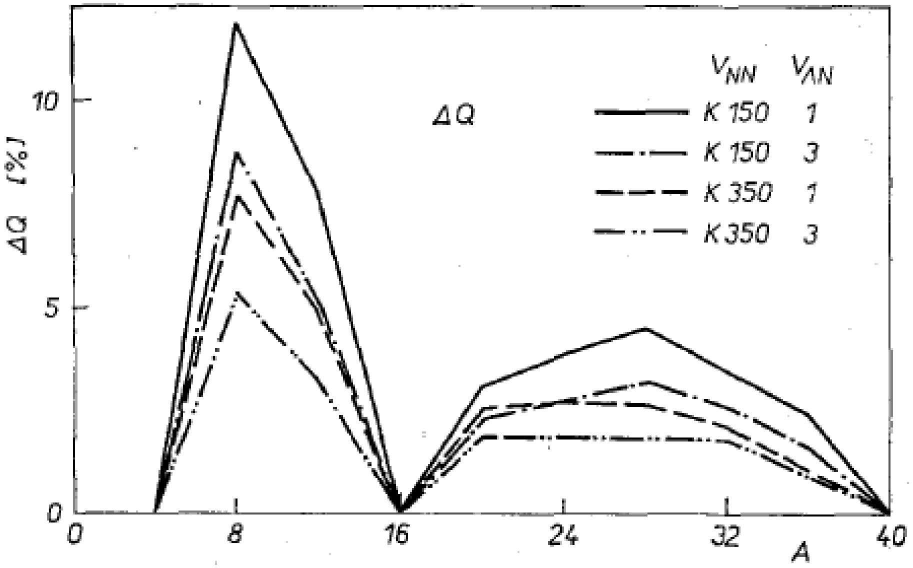
optimized shape can be automatically determined

= suitable for discussion of shape of hypernuclei

➢ First application to deformed hypernucleus

J. Zofka, Czech. J. Phys. B30('80)95

Hartree-Fock calculations with V_{NN}: 3 range Gauss
$V_{\Lambda N}$: 2 range Gauss

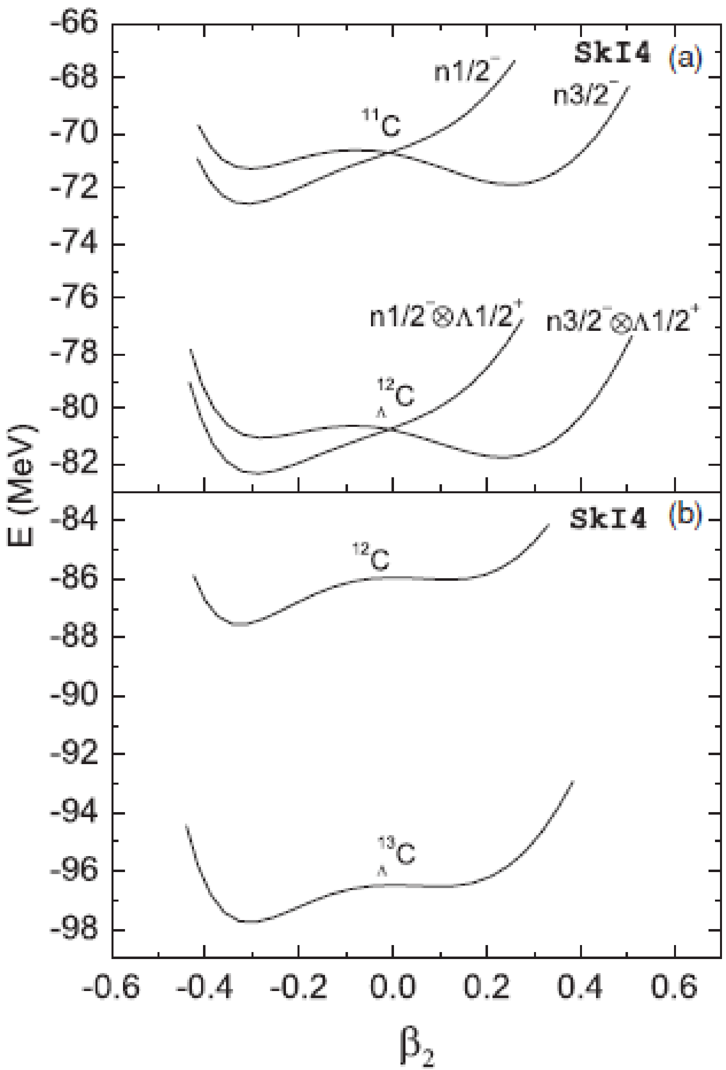


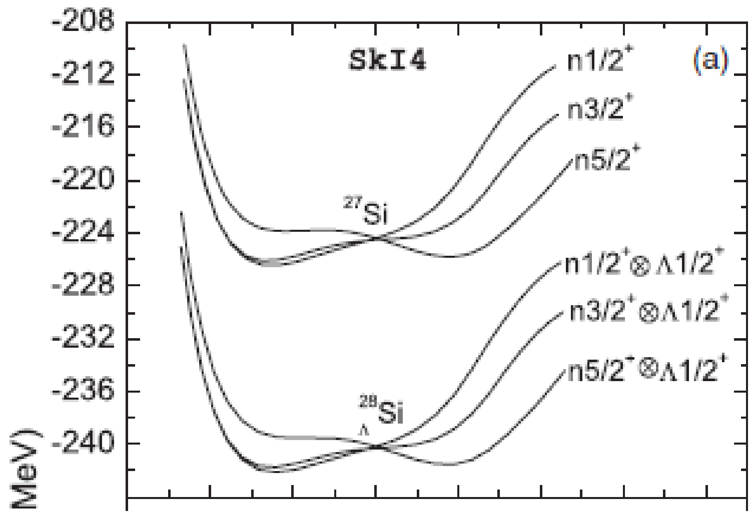
Λ changes the Q-moment (deformation) at most by 5%

e.g., $\beta = 0.5 \longrightarrow \beta = 0.475$

Shape of hypernuclei

Recent Skyrme-Hartree-Fock +BCS calculation by Zhou *et al.*
(with assumption of axial symmetry for simplicity)





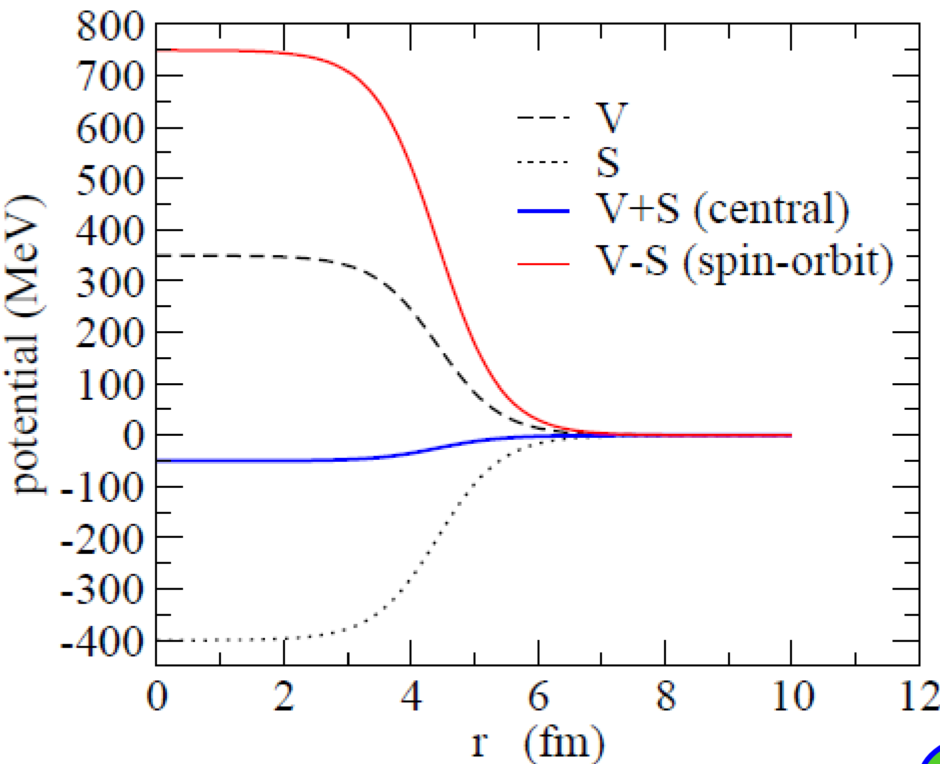
- similar deformation between the hypernuclei and the core nuclei
- hypernuclei: slightly smaller deformation than the core

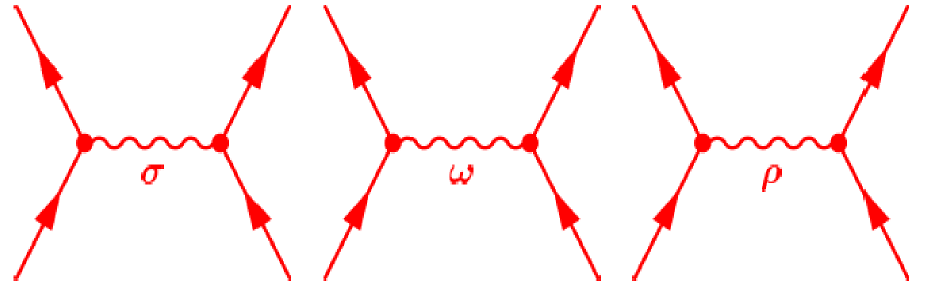
X.-R. Zhou *et al.*, PRC76(‘07) 034312

Deformation of Λ hypernuclei

Recent Skyrme-Hartree-Fock calculations by Zhou *et al.*

→ How about Relativistic Mean-Field (RMF) approach?





non-relativistic reduction

$$V_{\text{cent}} = V + S$$

(strong cancellation between *V* and *S*)

$$V_{\text{ls}} = \frac{m}{m - (V-S)/2}(V-S)$$

changes in *V* and *S* due to a Λ particle are emphasized (only in RMF)

cf. D. Vretenar et al., PRC57('98)R1060

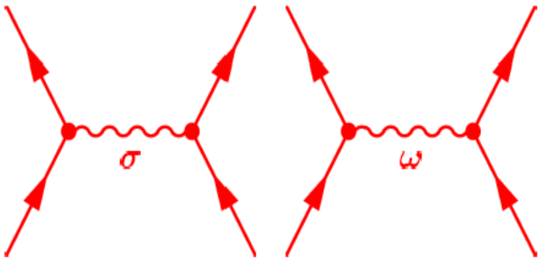
RMF for deformed hypernuclei

$$\mathcal{L} = \mathcal{L}_N + \bar{\psi}_\Lambda \left[\gamma_\mu \left(i\partial^\mu - g_{\omega\Lambda}\omega^\mu\right) - m_\Lambda - g_{\sigma\Lambda}\sigma\right] \psi_\Lambda$$

$$g_{\omega\Lambda} = \frac{2}{3} g_{\omega N} \longleftarrow \text{quark model}$$

$$g_{\sigma\Lambda} = 0.621 g_{\sigma N} \longleftarrow {}^{17}{}_{\Lambda}\mathrm{O}$$

cf. D. Vretenar et al., PRC57('98)R1060



Λσ and Λω couplings

variational principle

$$\left\{ \begin{array}{l} \left[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta\left(m_\Lambda + g_{\sigma\Lambda}\sigma(\boldsymbol{r})\right) + g_{\omega\Lambda}\omega^0(\boldsymbol{r})\right]\psi_\Lambda = \epsilon_\Lambda\psi_\Lambda \\ \left[-\boldsymbol{\nabla}^2 + m_\omega^2\right]\omega^0(\boldsymbol{r}) = g_\omega\rho_v(\boldsymbol{r}) + g_{\omega\Lambda}\psi_\Lambda^\dagger(\boldsymbol{r})\psi_\Lambda(\boldsymbol{r}) \end{array} \right.$$

etc.

→ self-consistent solution (iteration)

RMF for deformed hypernuclei

self-consistent solution (iteration)

→ (intrinsic) Quadrupole moment

$$Q = \sqrt{\frac{16\pi}{5}} \int d\boldsymbol{r}\, [\rho_v(\boldsymbol{r}) + \psi_\Lambda^\dagger(\boldsymbol{r})\psi_\Lambda(\boldsymbol{r})]\, r^2 Y_{20}(\hat{\boldsymbol{r}})$$

Application to hypernuclei

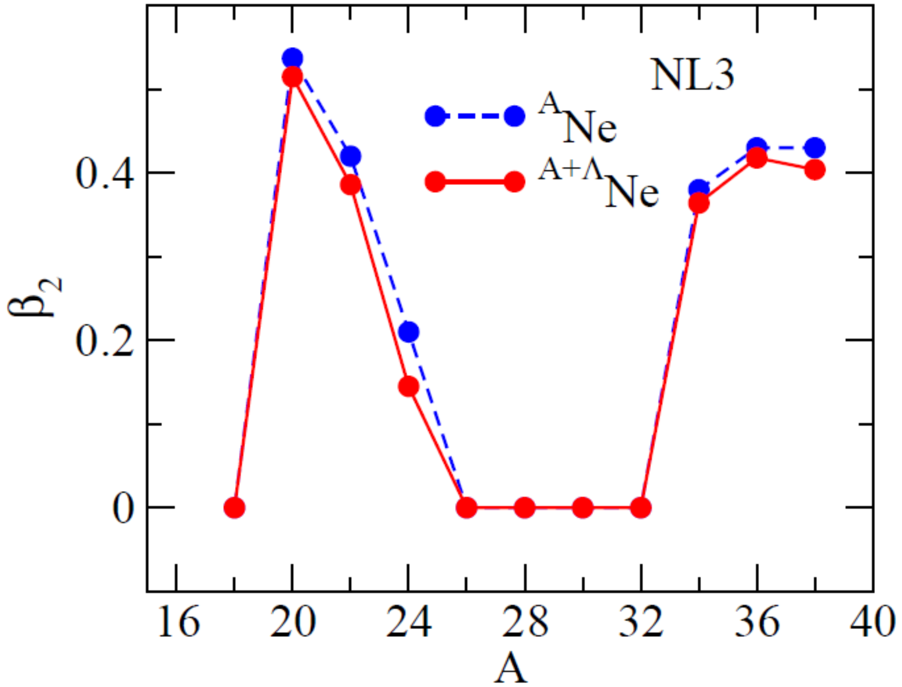
- parameter sets: NL3 and NLSH
- Axial symmetry
- pairing among nucleons: Const. gap approach

  $\Delta_n = 4.8/N^{1/3} \quad \Delta_p = 4.8/Z^{1/3}$ (MeV)
- Λ particle: the lowest s.p. level ($K^\pi = 1/2^+$)
- Basis expansion with deformed H.O. wf
- Deformation parameter:

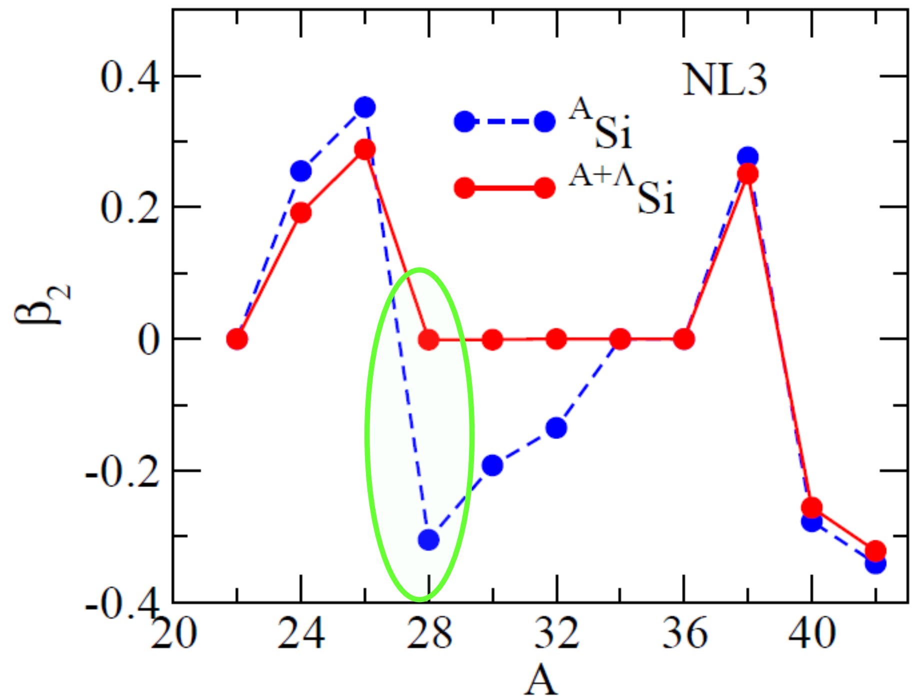
$$Q = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} (A_c + 1) R_0^2 \beta$$

$$R_0 = 1.2\, A_c^{1/3} \text{ (fm)}$$

Ne isotopes



Si isotopes



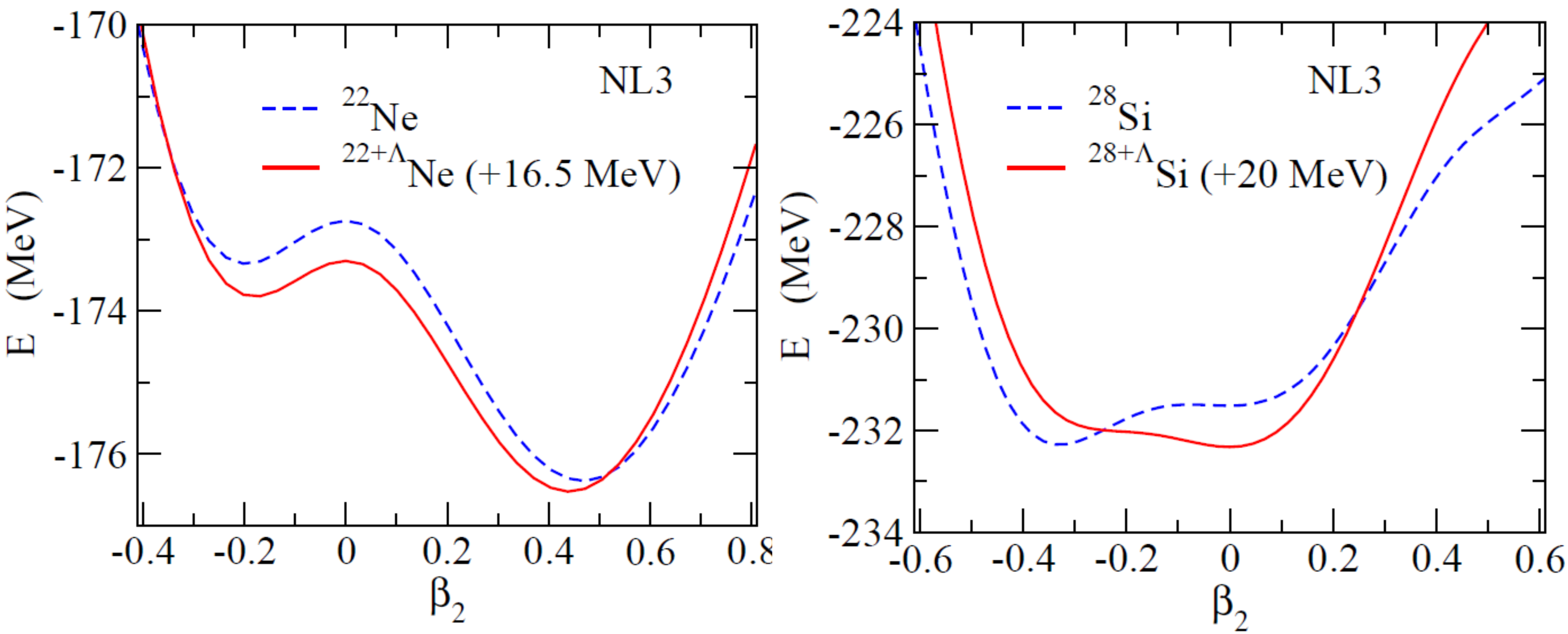
- in most cases, similar deformation between the core and the hypernuclei
- hypernuclei: slightly smaller deformation than the core

⟶ conclusions similar to Skyrme-Hartree-Fock (Zhou *et al.*)

Exception: $^{29}_{\Lambda}$Si oblate (^{28}Si) $\xrightarrow{\Lambda}$ spherical ($^{29}_{\Lambda}$Si)

Myaing Thi Win and K.H., PRC78('08)054311

Potential energy surface (constraint Hartree-Fock)

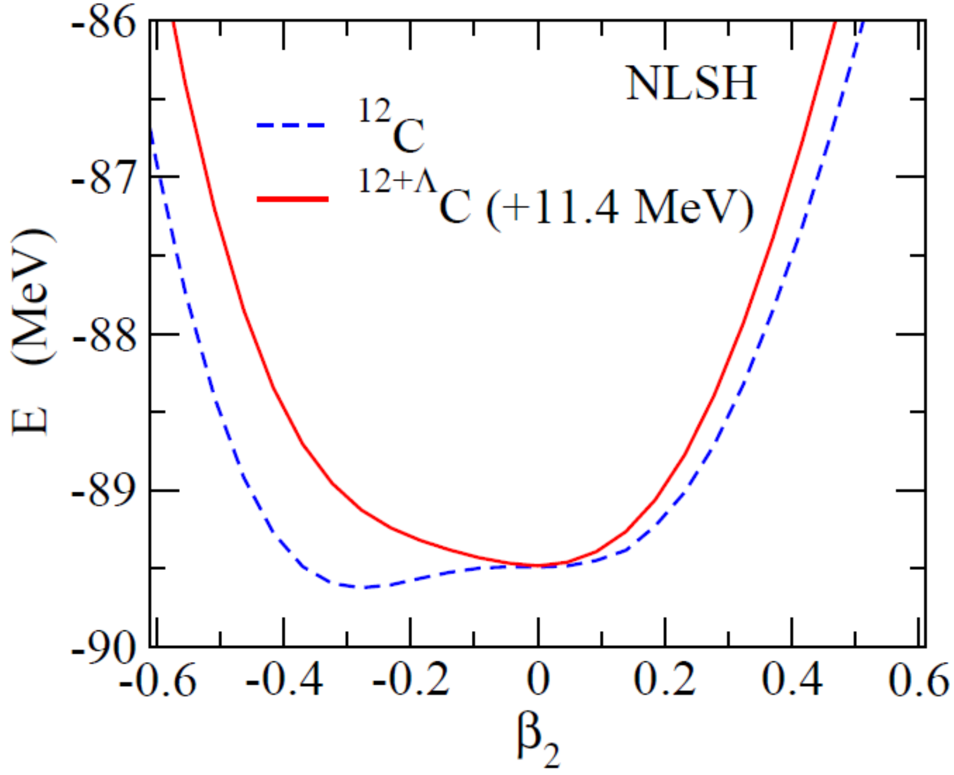


If the energy curve is relatively flat, a large change in nuclear deformation can occur due to an addition of Λ particle

the same conclusion also with NLSH and/or with another treatment of pairing correlation (constant G approach)

Myaing Thi Win and K.H., PRC78('08)054311

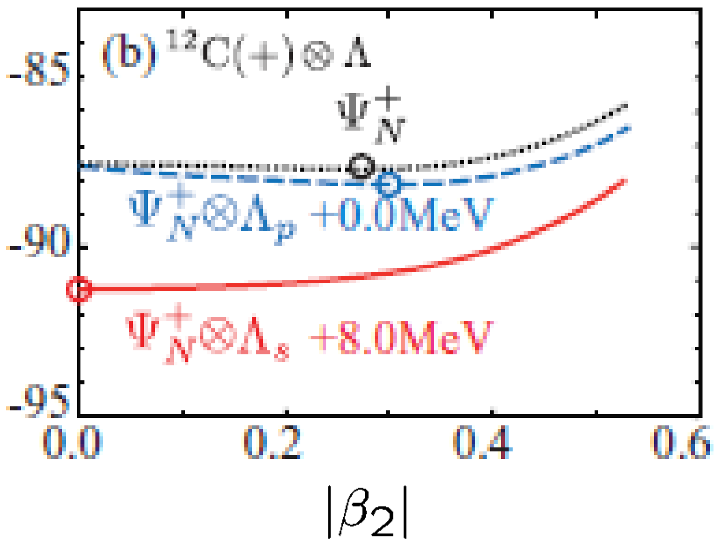
Another example: $^{13}{}_{\Lambda}C$



oblate ⟶ spherical

Myaing Thi Win and K.H., PRC78('08)054311

cf. recent AMD calculations



M. Isaka, K. Kimura, A. Dote, and A. Ohnishi, PRC83('11)044323

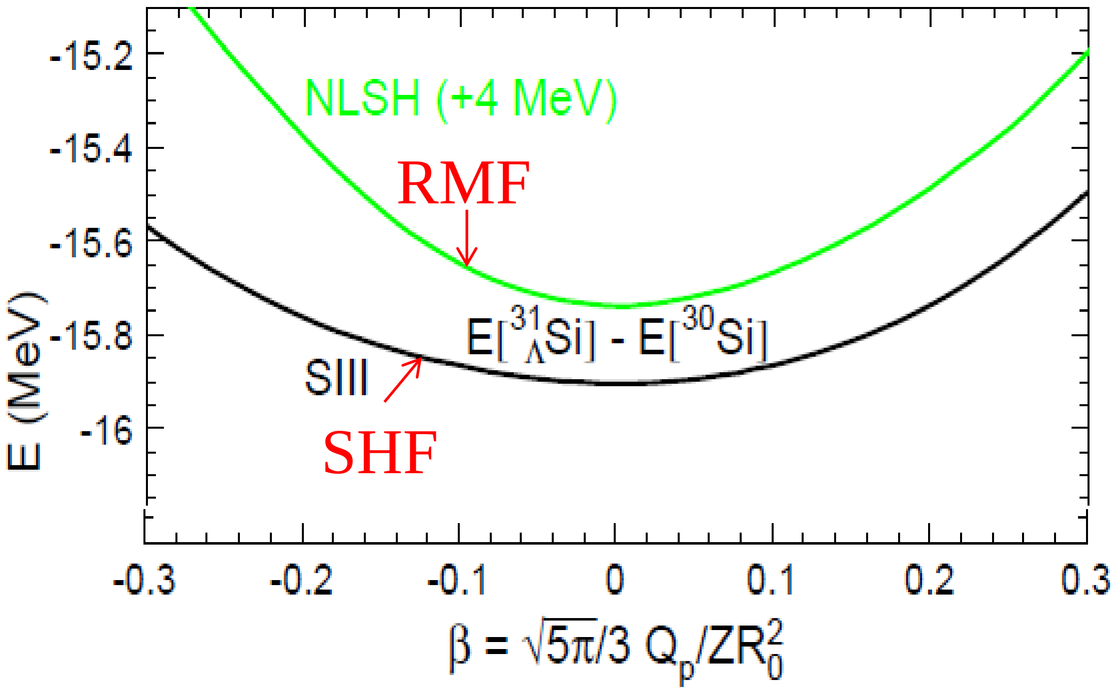
Comparison between RMF and SHF

- Gain of binding energy= $E_{^{30+\Lambda}Si} - E_{^{30}Si}$
  - in spherical configuration

$$\Delta E = E(\beta = -0.2) - E(\beta = 0) \approx 0.14\text{MeV} \quad \text{(SHF)}$$

$$\Delta E = E(\beta = -0.2) - E(\beta = 0) \approx 0.3\text{MeV} \quad \text{(RMF)}$$

- Larger effect of NΛ force in RMF



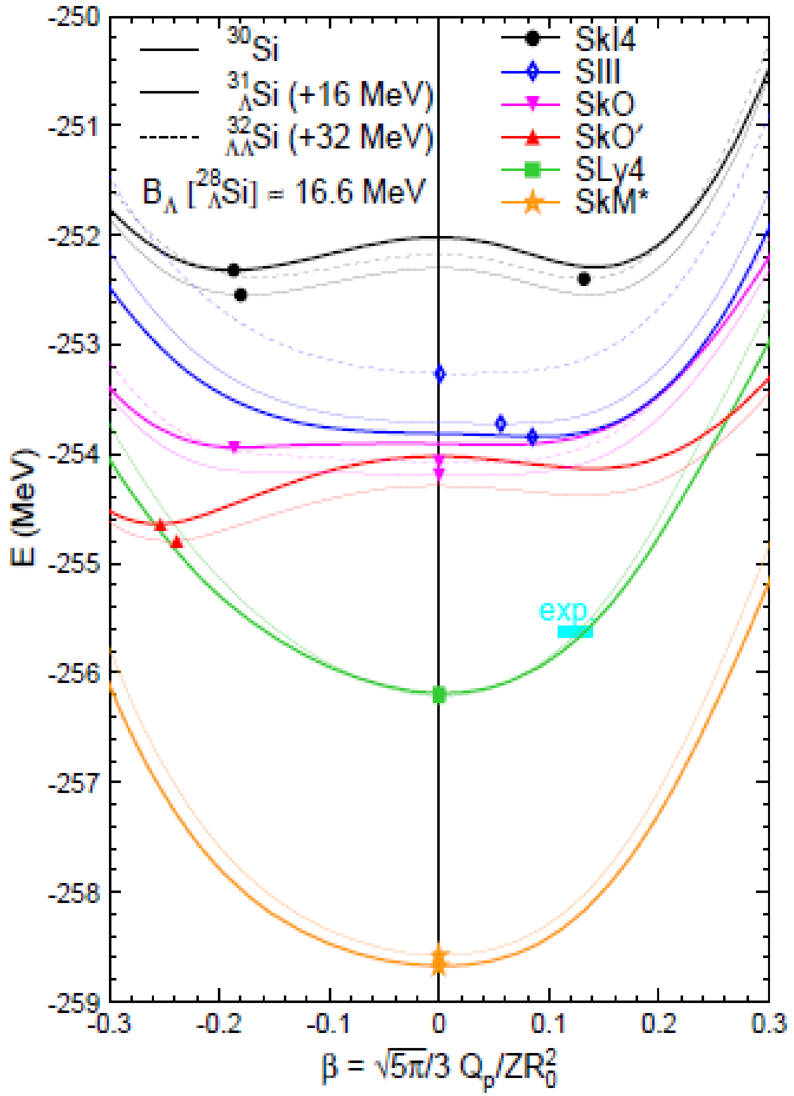
H.-J.Schulze, M.Thi Win, K.Hagino,H.Sagawa Prog.Theo.Phys123('10) 569

Systematic comparison with
Skyrme-Hartree-Fock method:

- ➢Stronger influence of Λ in RMF than in SHF
- ➢Disappearance of deformation can happen also with SHF if the energy curve is very flat

H.-J. Schulze, Myaing Thi Win, K.H., H. Sagawa, PTP123(‘10)569

A key point is a flatness of potential energy curve

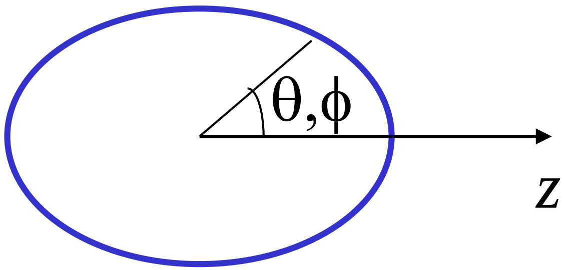


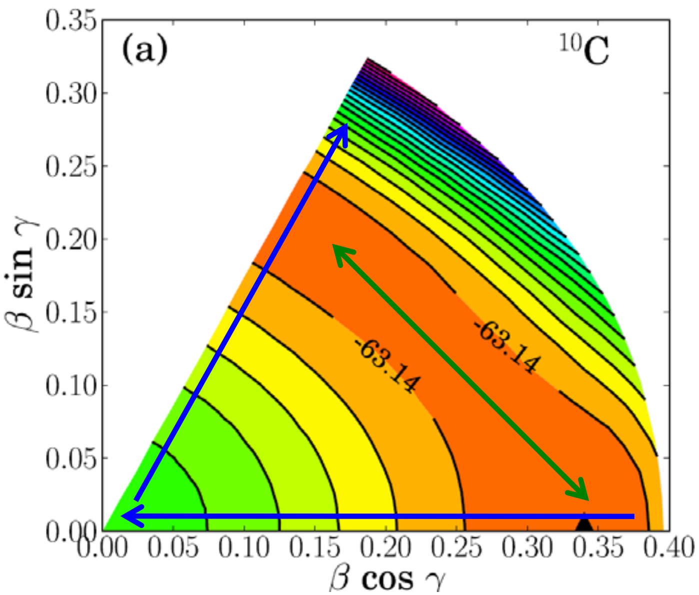
3D Hartree-Fock calculation for hypernuclei

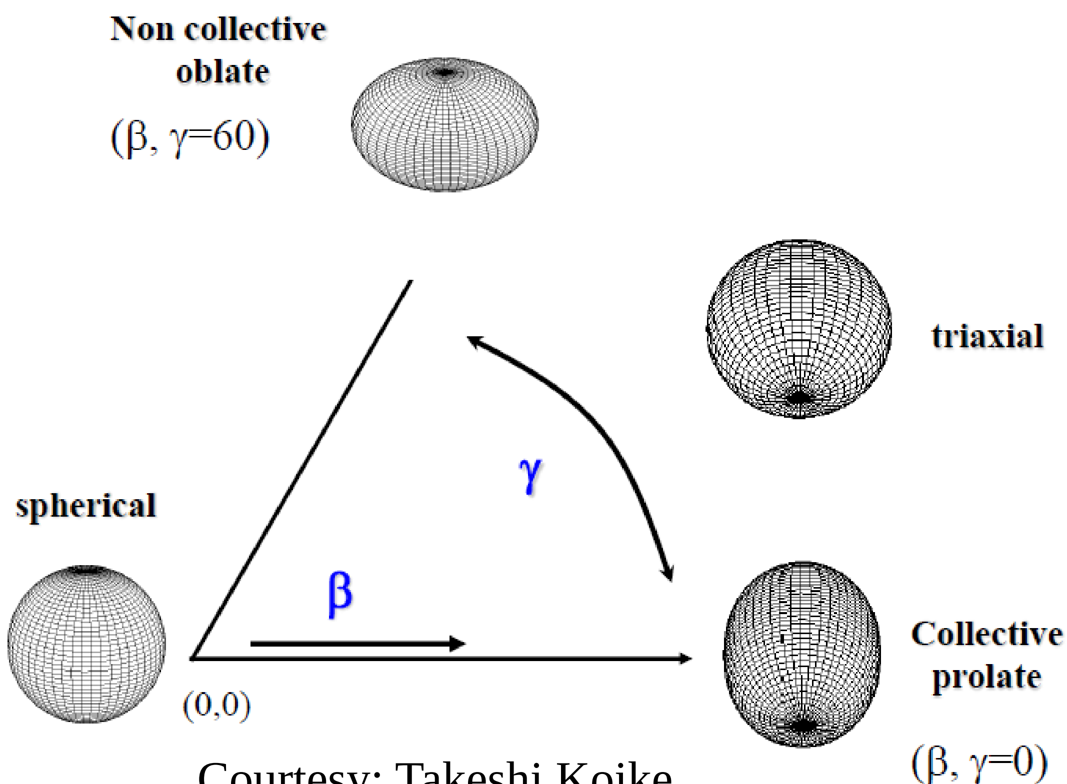
So far, axial symmetric shape has been assumed for simplicity

→ Effect of Λ particle on triaxial deformation?

$$R(\theta,\phi) = R_0\left[1+\beta\cos\gamma Y_{20}(\theta)+\frac{1}{\sqrt{2}}\beta\sin\gamma(Y_{22}(\theta,\phi)+Y_{2-2}(\theta,\phi))\right]$$







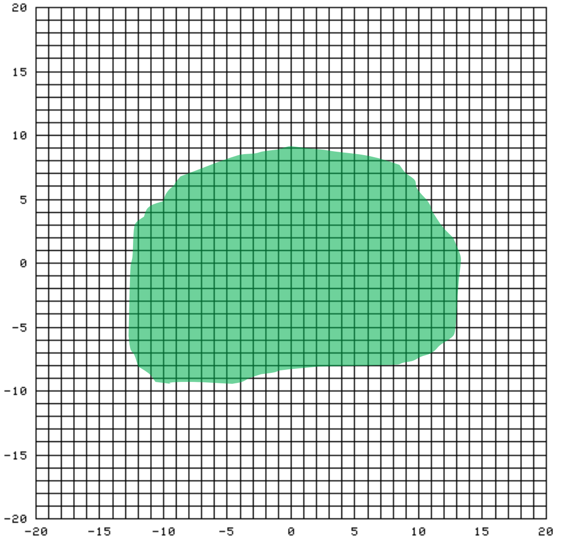
Courtesy: Takeshi Koike

Skyrme-Hartree-Fock calculations for hypernuclei

3D calcaulations with non-relativistic Skyrme-Hartree-Fock:
the most convenient and the easiest way

- 3D mesh calculation ("lattice Hartree-Fock")
- Imaginary time evolution of single-particle wave functions
- computer code "ev8" available
  P. Bonche, H. Flocard, and P.-H. Heenen,
  NPA467('87)115, CPC171('05)49

$$\phi_k(x,y,z) \sim \phi_k(n_x\Delta x, n_y\Delta y, n_z\Delta z)$$
$$\phi_k(x,y,z) = \lim_{\tau\to\infty} e^{-\hat{h}\tau}\phi_k^{(0)}(x,y,z)$$



(note)
$$\begin{aligned} e^{-\hat{h}\tau}\phi^{(0)} &= e^{-\hat{h}\tau}\sum_k C_k\phi_k \\ &= \sum_k e^{-e_k\tau}C_k\phi_k \\ &\to e^{-e_0\tau}C_0\phi_0 \qquad (\tau\to\infty) \end{aligned}$$

Skyrme-Hartree-Fock calculations for hypernuclei

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  P. Bonche, H. Flocard, and P.-H. Heenen,
  NPA467('87)115, CPC171('05)49

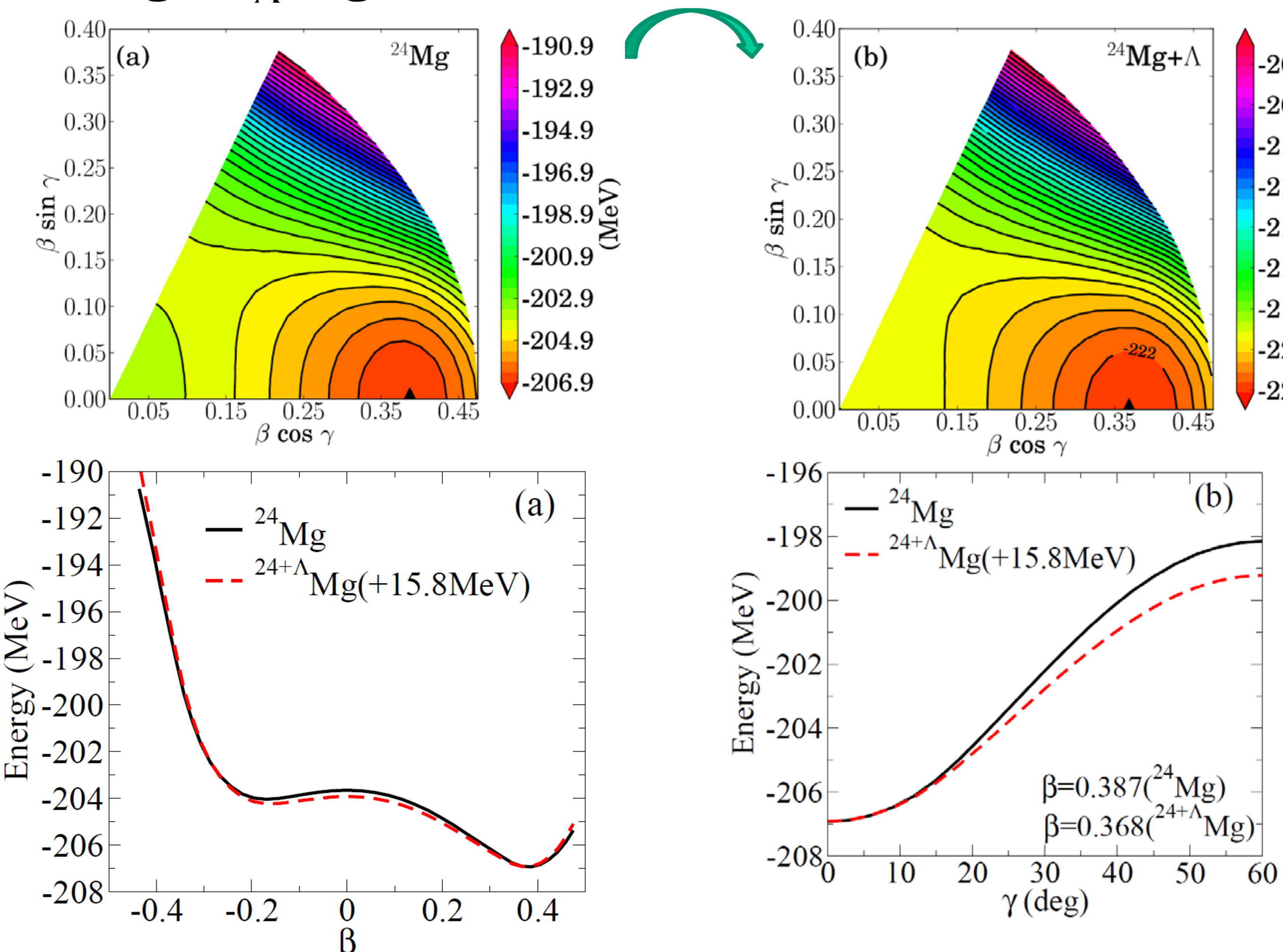
→ extension to hypernuclei

$$v_{\Lambda N}(\boldsymbol{r}_\Lambda, \boldsymbol{r}_N) = t_0(1 + x_0 P_\sigma)\delta(\boldsymbol{r}_\Lambda - \boldsymbol{r}_N) + \cdots$$
$$v_{\Lambda NN}(\boldsymbol{r}_\Lambda, \boldsymbol{r}_1, \boldsymbol{r}_2) = t_3\delta(\boldsymbol{r}_\Lambda - \boldsymbol{r}_1)\delta(\boldsymbol{r}_\Lambda - \boldsymbol{r}_2)$$

M. Rayet, NPA367('81)381

∗ Interaction No.1 of Yamamoto *et al.* + SGII (NN)
(Y. Yamamoto, H. Bando, and J. Zofka, PTP80('88)757)
∗ Pairing among nucleons: BCS approximation with d.d. contact force
∗ Λ particle: the lowest energy state

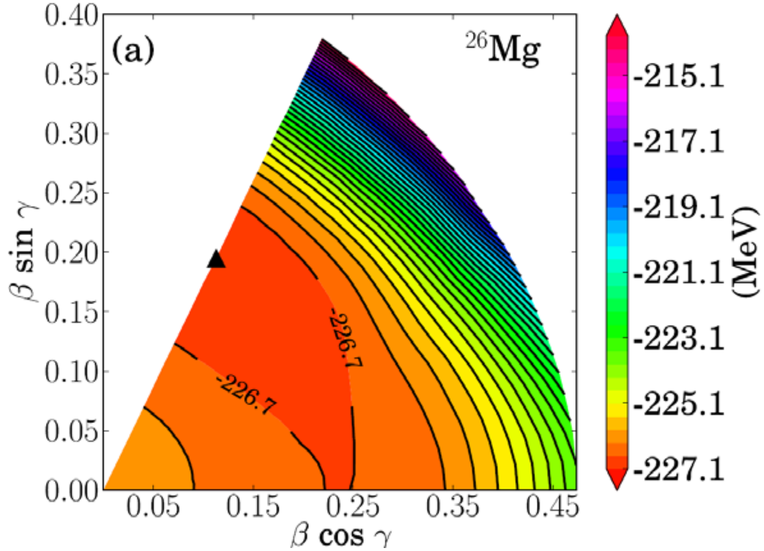
^{24}Mg, $^{25}{}_{\Lambda}$Mg

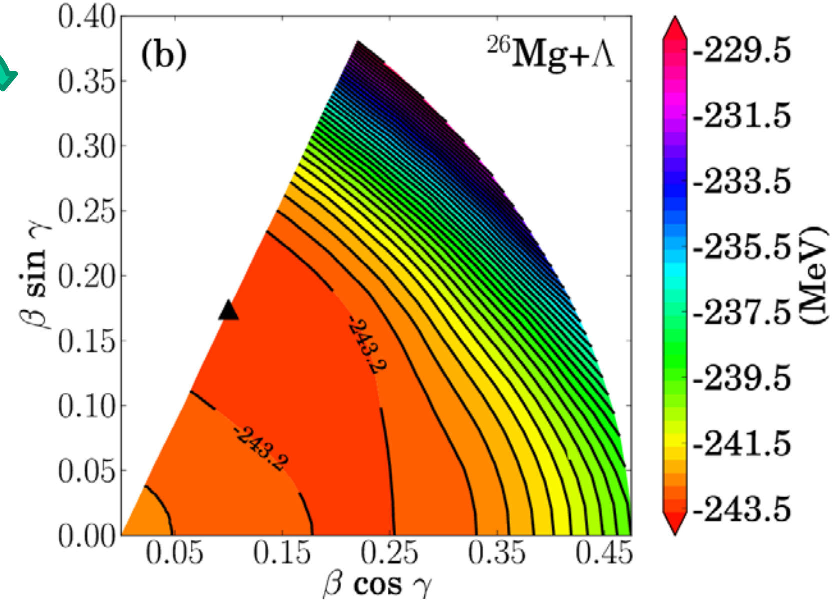


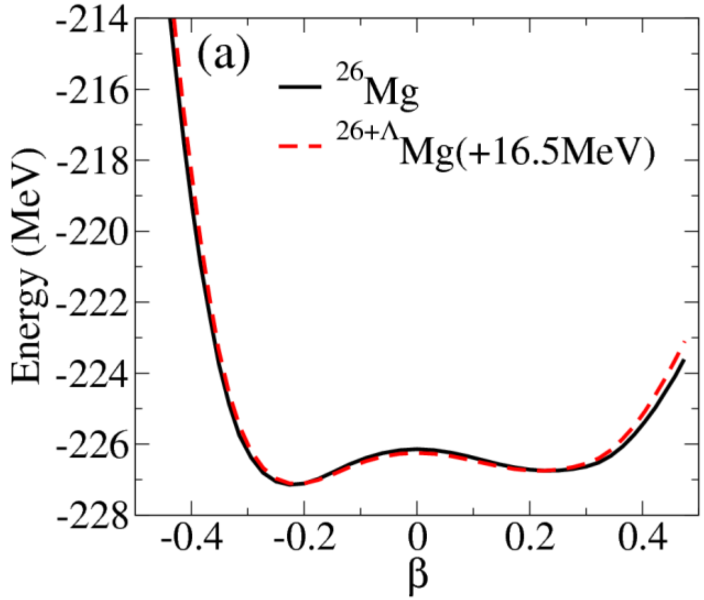
Myaing Thi Win, K.H., T. Koike, Phys. Rev. C83(‘11)014301

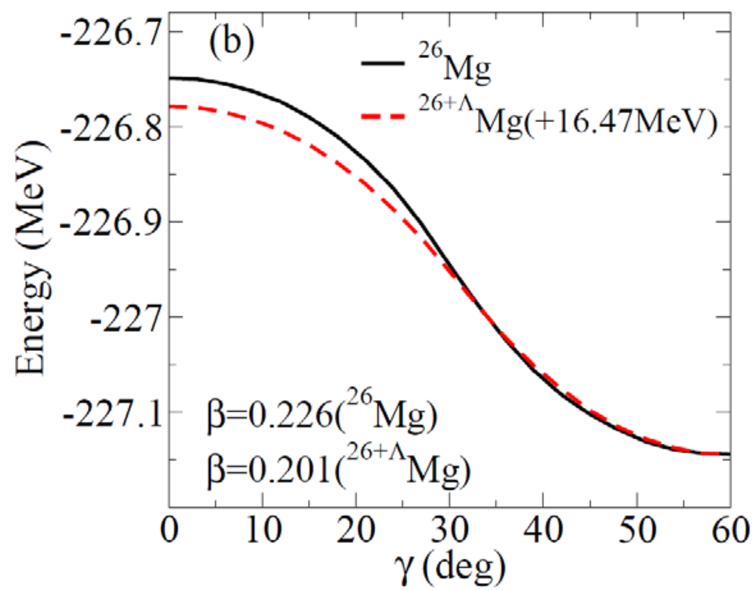
^{26}Mg, $^{27}{}_{\Lambda}$Mg

+Λ





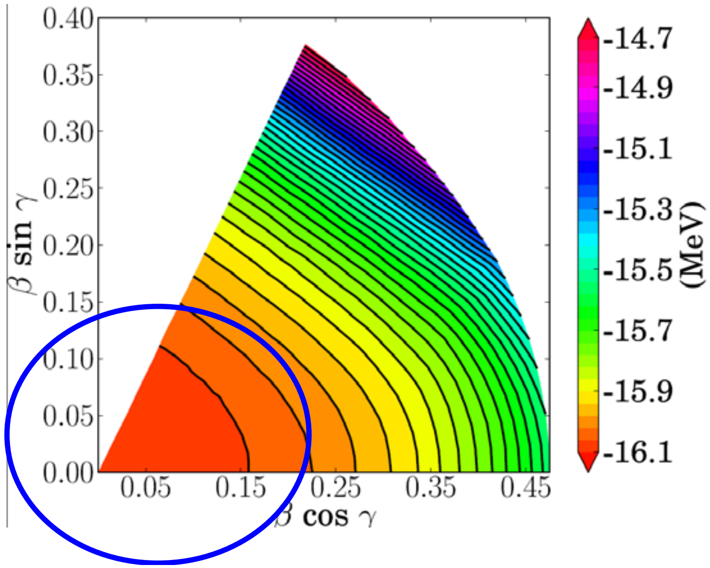




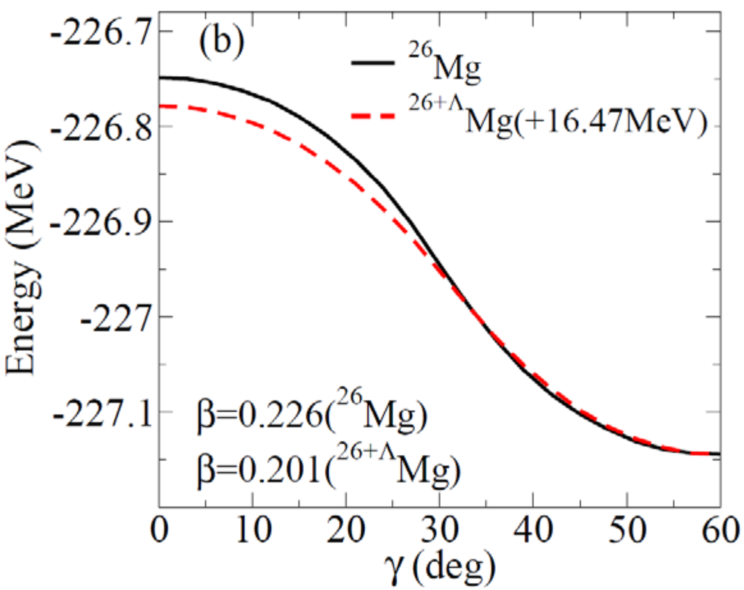
Myaing Thi Win, K.H., T. Koike, Phys. Rev. C83('11)014301

Discussions

$$E_{^{25}_{\Lambda}Mg}(\beta,\gamma) - E_{^{24}Mg}(\beta,\gamma)$$



- ➢Deformation is driven to speherical when Λ is in the lowest state
- ➢Prolate configuration is prefered for the same value of β



All of ^{24}Mg, ^{26}Mg, ^{26}Si, ^{28}Si show that Λ makes the curvature along the γ direction somewhat smaller

⇓

Experiment? (the energy of 2_2^+ state)

quantitative estimat：RPA or GCM

or Bohr Hamiltonian

Rotational Excitation of hypernuclei

Collective spectrum of a hypernucleus: half-integer spin

"Bohr Hamiltonian" for the *core* part:

$$\mathcal{H}_{\text{coll}} = T_{\text{vib}} + \frac{1}{2}\sum_{k=1}^{3}\frac{\hat{I}_k^2}{2\mathcal{J}_k} + V_{\text{coll}}(\beta,\gamma)$$

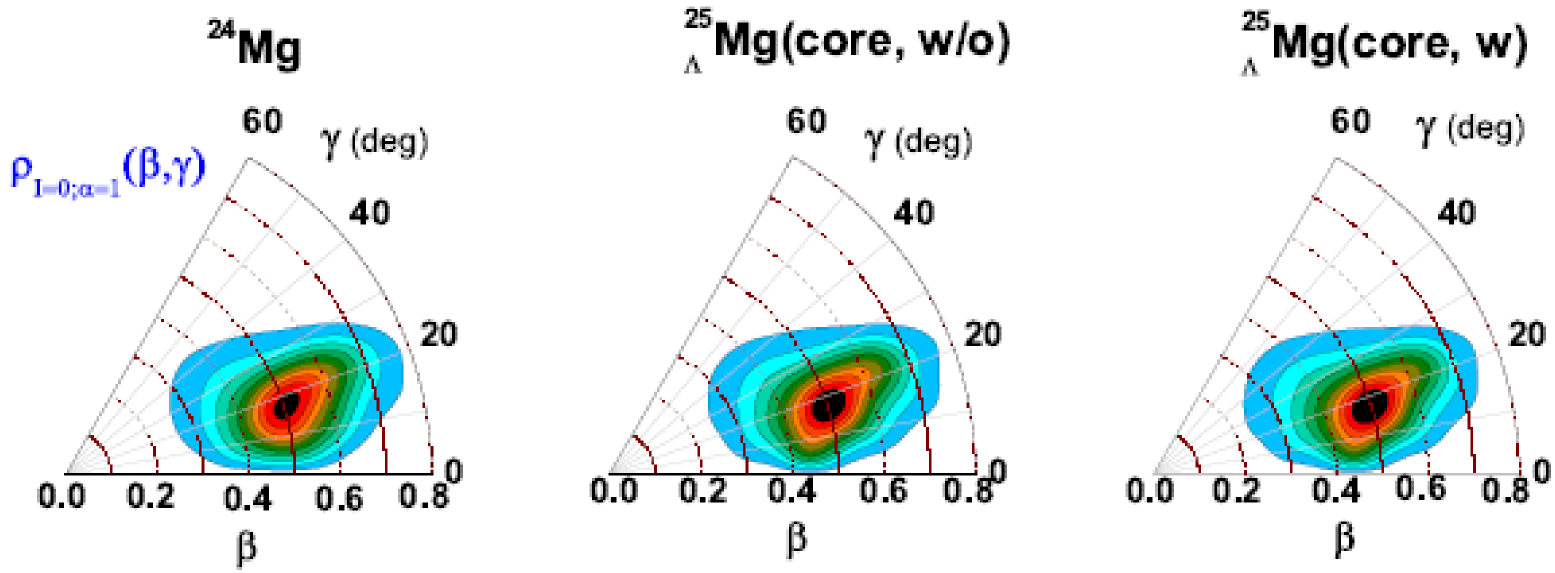
mass inertias: cranking approximation

(Inglis-Belyaev formula for the rotational inertia)

$$V_{\text{coll}}(\beta,\gamma) = E(\beta,\gamma) - \Delta V_{\text{vib}}(\beta,\gamma) - \Delta V_{\text{rot}}(\beta,\gamma)$$

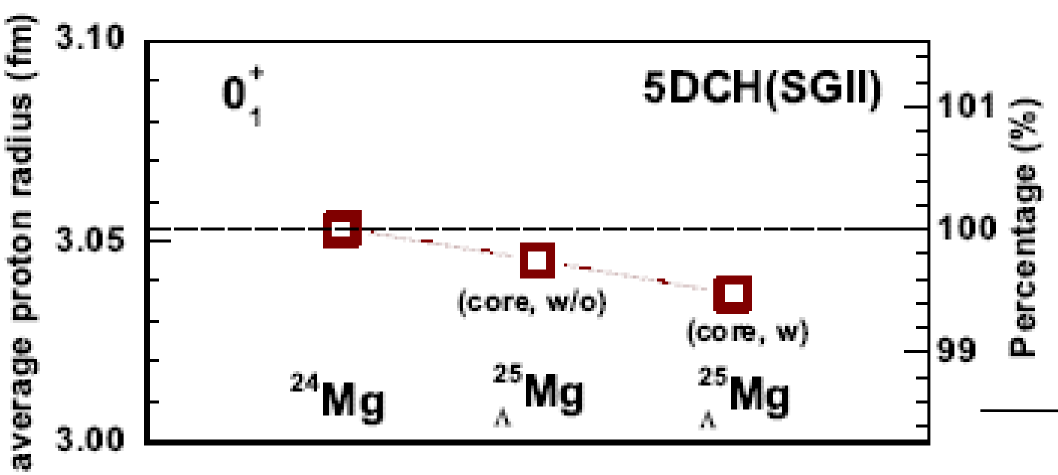
$$\begin{cases} (i)\ \ E(\beta,\gamma) = E_N(\beta,\gamma) \\ (ii)\ \ E(\beta,\gamma) = E_N(\beta,\gamma) + \int d\boldsymbol{r}\,\mathcal{E}_{N\Lambda}(\boldsymbol{r}) \end{cases}$$

Solution of Coll. H $\longrightarrow$ fluctuation of deformation parameters



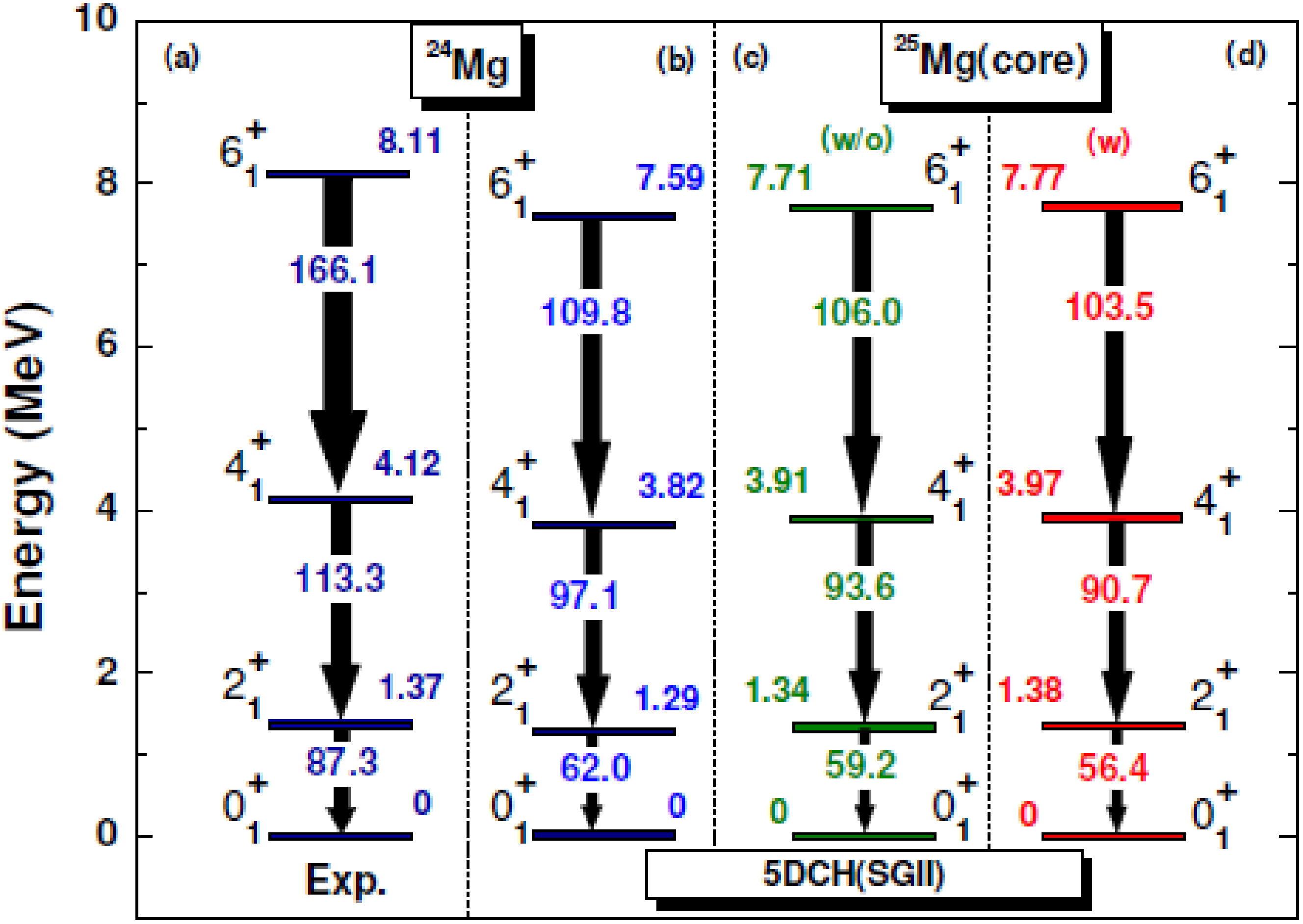
$$\langle\beta\rangle = 0.54$$
$$\langle\gamma\rangle = 20^\circ$$

$$\langle\beta\rangle = 0.52$$
$$\langle\gamma\rangle = 20.8^\circ$$



J.M. Yao, Z.P. Li, K.H. et al., arXiv: 1104.3200

$\longrightarrow$ much smaller change



J.M. Yao, Z.P. Li, K.H. et al., arXiv: 1104.3200

reduction of B(E2) from 2^+ to 0^+

^{24}Mg: B(E2) = 62.0 e^2fm^4

$^{25}{}_{\Lambda}$Mg: B(E2) = 56.4 e^2fm^4 (about 9% reduction)

Summary

Shape of Λ hypernuclei: from the view point of mean-field theory

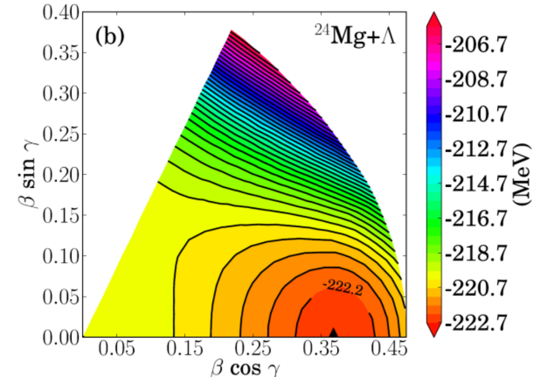
- ➢deformation: in important key word in the sd-shell region
- ➢RMF: stronger influence of Λ particle

⟶Shape of ^{28}Si : drastically changed due to Λ

- ➢SHF: weaker influence of Λ, but large def. change if PES is very flat

•3D calcaulations

•softening of γ-vibration?



Rotational excitations of Λ hypernuclei

- ➢about 9% reduction of B(E2) value

A challenging problem

- ➢full spectrum of a hypernucleus

odd mass, broken time reversal symmetry, half-integer spins

- spectrum of a double Λ hypernucleus?