# Heavy-Ion Fusion Reactions around the Coulomb Barrier

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TOHOKU UNIVERSITY

cf. Experimental aspects of H.I. Fusion reactions:
lectures by Prof. Mahananda Dasgupta (ANU)


3.11 earthquake

after 1 month

# Heavy-Ion Fusion Reactions around the Coulomb Barrier

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✧Fusion reactions and quantum tunneling

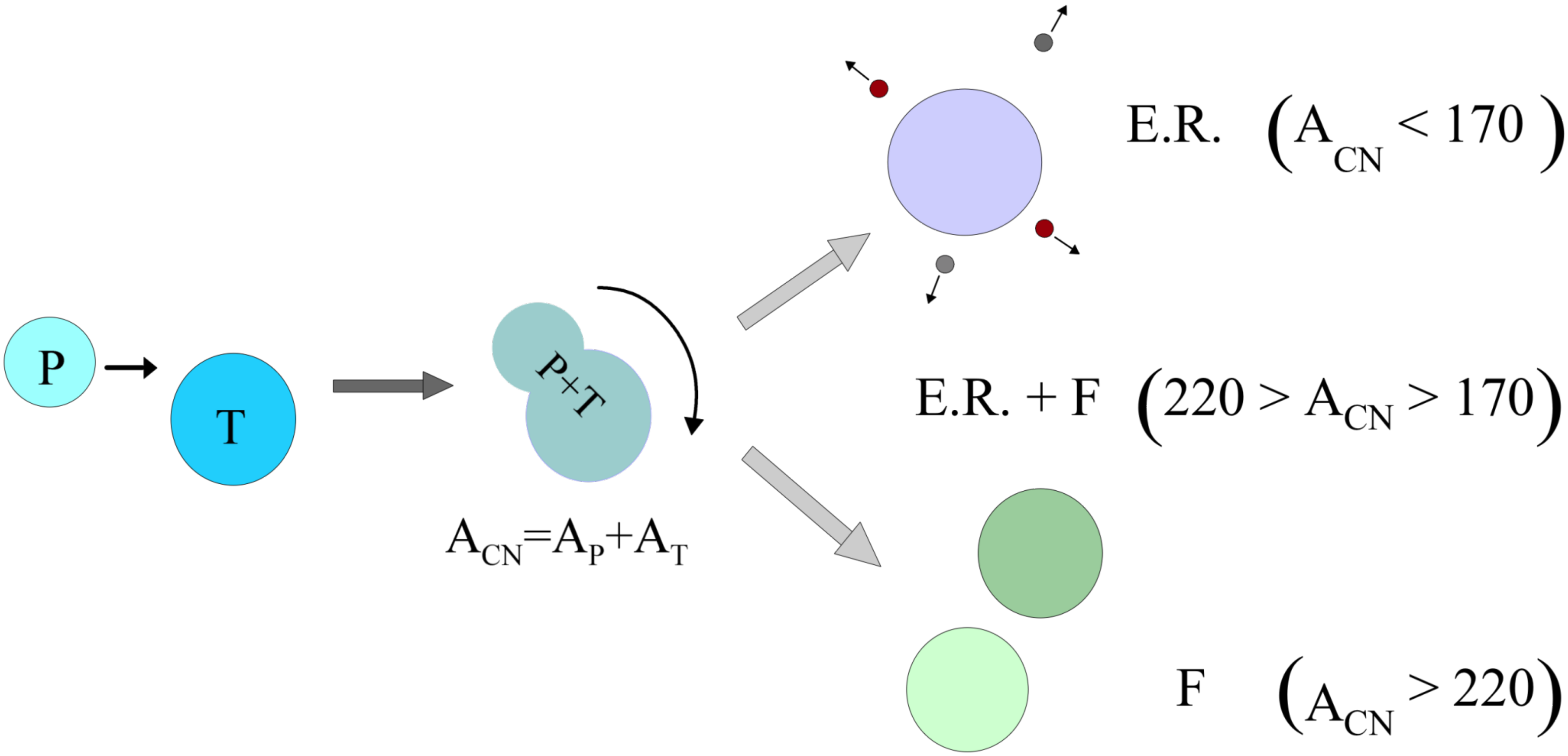
✧**Basics of the Coupled-channels method**

✧Concept of Fusion barrier distribution

✧Quasi-elastic scattering and quantum reflection

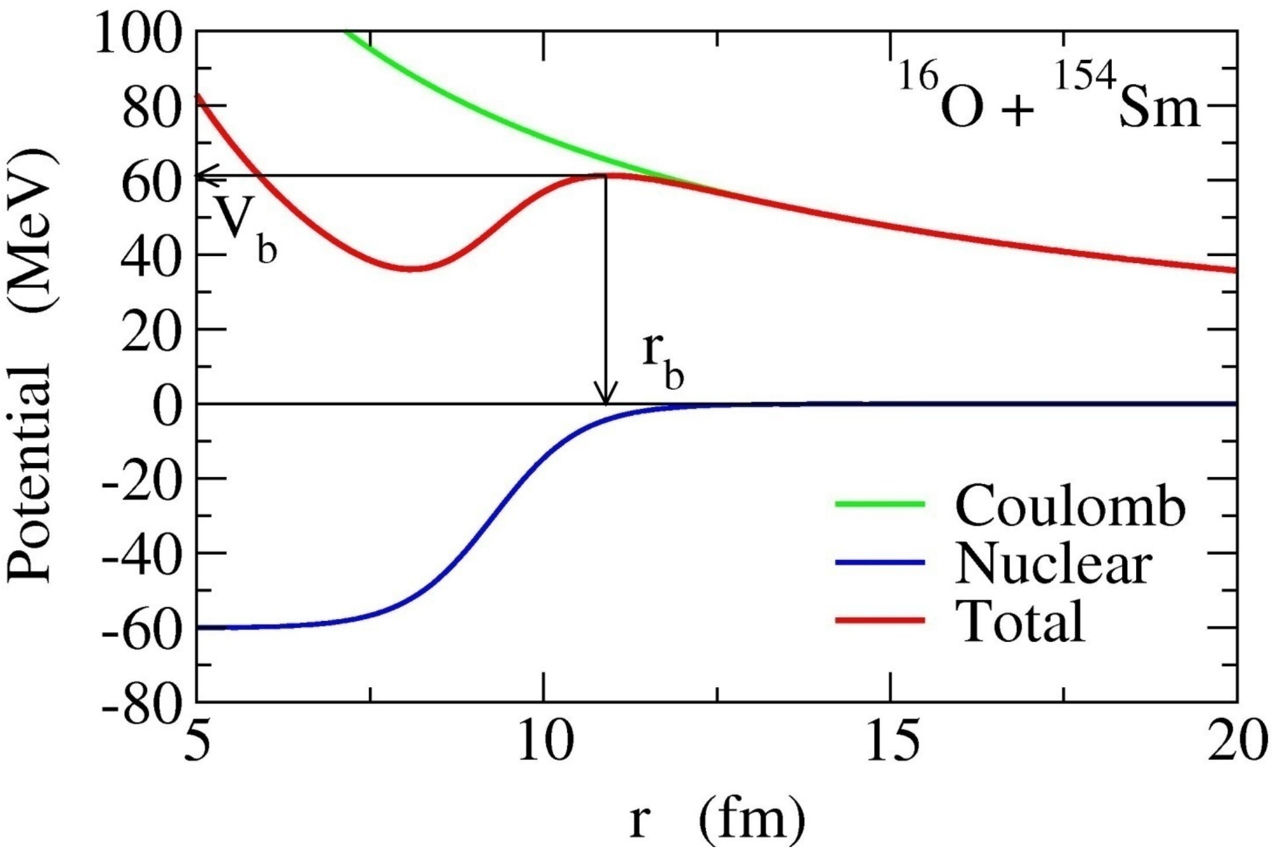
cf. Experimental aspects of H.I. Fusion reactions: lectures by Prof. Mahananda Dasgupta (ANU)

# Fusion: compound nucleus formation




courtesy: Felipe Canto

## Inter-nucleus potential




Two forces:

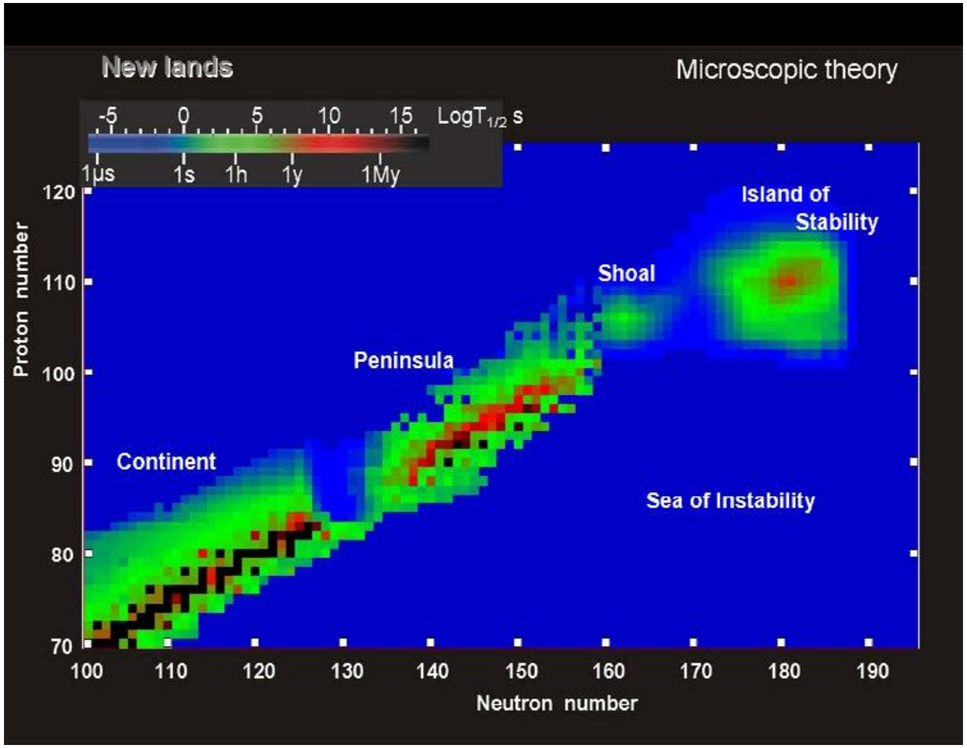
1. Coulomb force
   Long range, repulsive
2. Nuclear force
   Short range, attractive

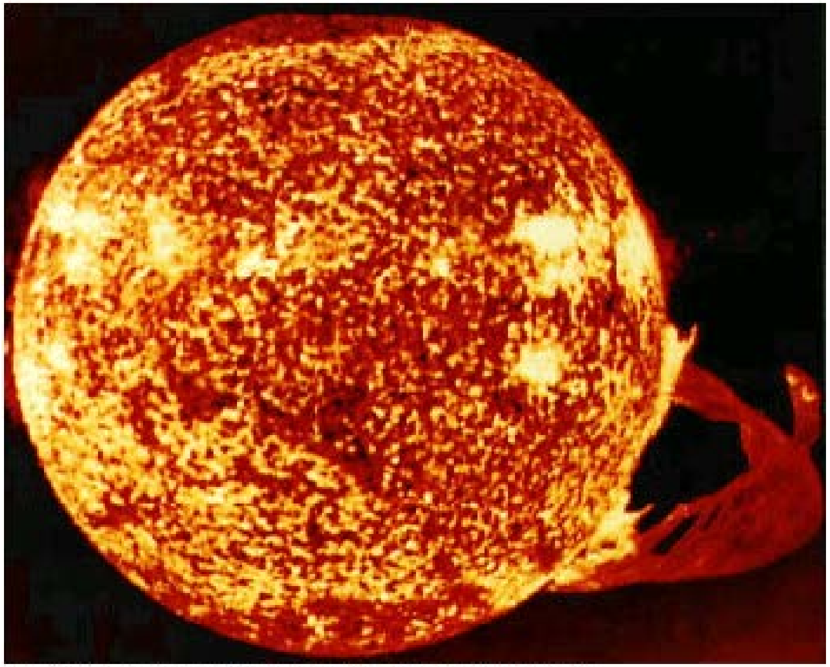
Potential barrier due to the compensation between the two (Coulomb barrier)

- above barrier
- sub-barrier
- deep subbarrier

# Why subbarrier fusion?

Two obvious reasons:



NASA, Skylab space station December 19, 1973, solar flare reaching 588 000 km off solar surface

discovering new elements
(SHE by cold fusion reactions)

nuclear astrophysics
(fusion in stars)

# Why subbarrier fusion?

Two obvious reasons:

✓discovering new elements (SHE)
✓nuclear astrophysics (fusion in stars)

Other reasons:

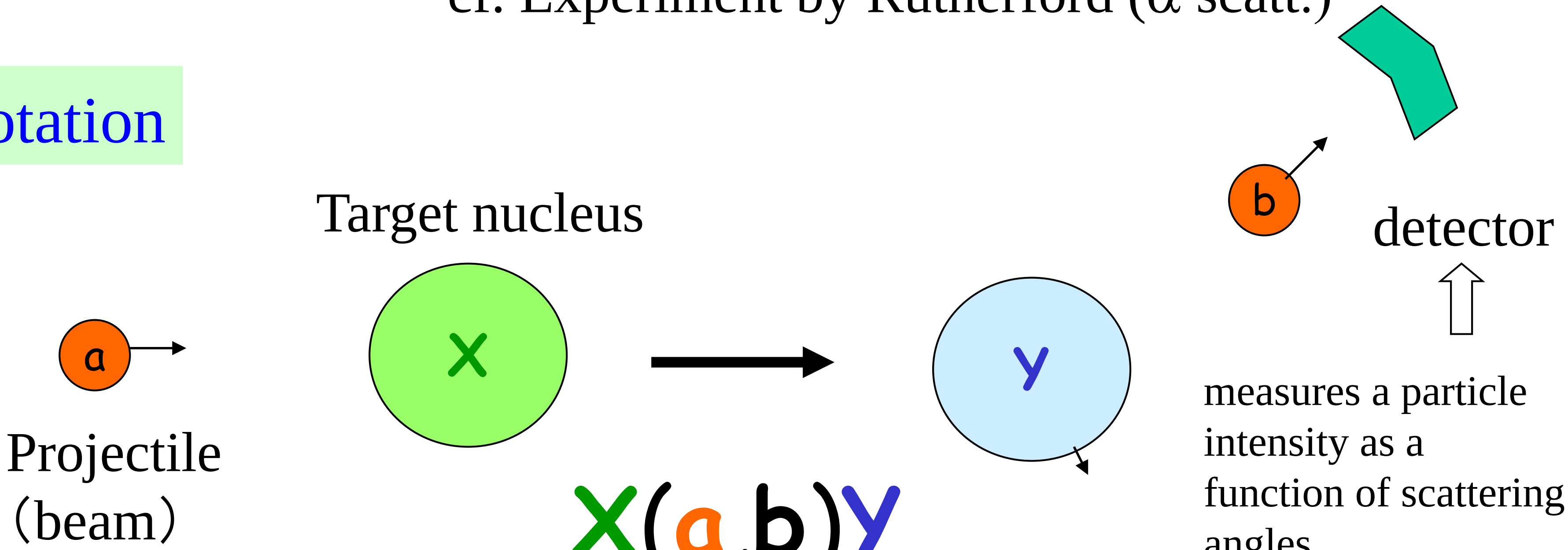
✓reaction mechamism
**strong interplay between reaction and structure**
(channel coupling effects)
cf. high *E* reactions: much simpler reaction mechanism
✓many-particle tunneling
cf. alpha decay: fixed energy
tunneling in atomic collision: less variety of intrinsic motions

# Basic of nuclear reactions

Shape, interaction, and excitation structures of nuclei ⟸ scattering expt.

cf. Experiment by Rutherford ($\alpha$ scatt.)

## Notation




measures a particle intensity as a function of scattering angles

X(a,b)Y

$^{208}$Pb($^{16}$O,$^{16}$O)$^{208}$Pb : $^{16}$O+$^{208}$Pb elastic scattering

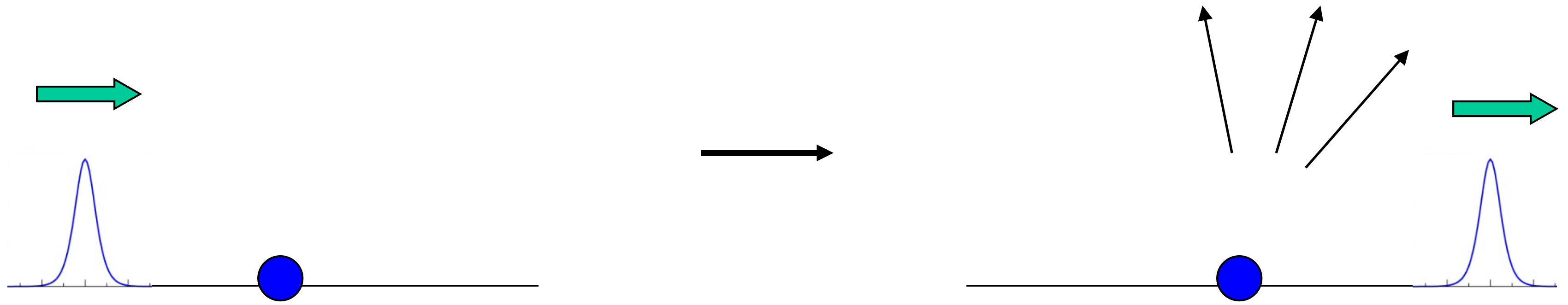
$^{208}$Pb($^{16}$O,$^{16}$O')$^{208}$Pb : $^{16}$O+$^{208}$Pb inelastic scattering

$^{208}$Pb($^{17}$O,$^{16}$O)$^{209}$Pb : 1 neutron transfer reaction

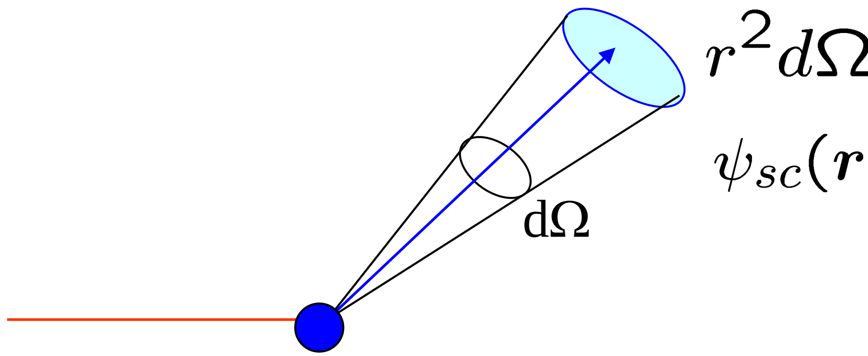
# Scattering Amplitude

$$\psi(\boldsymbol{r}) \;\; \to e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + f(\theta)\frac{e^{ikr}}{r}$$

= (incident wave) + (scattering wave)



# Differential cross section




$$\psi_{sc}(\boldsymbol{r}) \sim f(\theta)\frac{e^{ikr}}{r}$$

The number of scattered particle through the solid angle of dΩ per unit time:

$$N_{\mathsf{scatt}} = \boldsymbol{j}_{sc} \cdot \boldsymbol{e}_r \, r^2 d\Omega$$

$$\boldsymbol{j}_{sc} = \frac{\hbar}{2im}\left[\psi_{sc}^{*}\boldsymbol{\nabla}\psi_{sc} - c.c.\right] \sim \frac{k\hbar}{m}\frac{|f(\theta)|^2}{r^2}\boldsymbol{e}_r$$

(flux for the scatt. wave)

$$\boxed{\frac{d\sigma}{d\Omega} = |f(\theta)|^2}$$

# Scattering Amplitude

partial wave decomposition

Motion of Free particle: $-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi = \frac{\boldsymbol{k}^2\hbar^2}{2m}\psi$

$$\psi(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = \sum_{l=0}^{\infty}(2l+1)\, i^l j_l(kr) P_l(\cos\theta)$$

$$\rightarrow \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)i^l\left[e^{-i(kr-l\pi/2)} - e^{i(kr-l\pi/2)}\right]P_l(\cos\theta)$$

In the presence of a potential: $\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - E\right]\psi = 0$

Asymptotic form of wave function

$$\psi(\boldsymbol{r}) \rightarrow \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)i^l\left[e^{-i(kr-l\pi/2)} - S_l\, e^{i(kr-l\pi/2)}\right]P_l(\cos\theta)$$

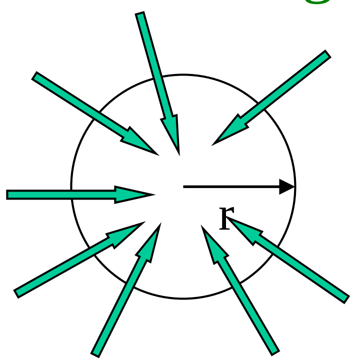
$$= e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \underbrace{\left[\sum_l (2l+1)\frac{S_l - 1}{2ik}P_l(\cos\theta)\right]}_{f(\theta)}\frac{e^{ikr}}{r}$$

$f(\theta)$ (scattering amplitude)

(note)

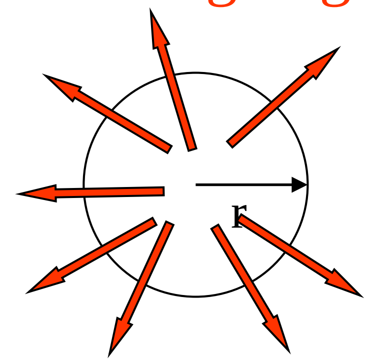
$$\psi(\boldsymbol{r}) \to \frac{i}{2k}\sum_l (2l+1)\, i^l\, \frac{1}{r}\left[\underbrace{e^{-i(kr-l\pi/2)}}_{\psi_{\text{in}}} - \underbrace{S_l e^{i(kr-l\pi/2)}}_{\psi_{\text{out}}}\right] P_l(\cos\theta)$$

Total incoming flux




$$j_{\text{in}}^{\text{net}} = \frac{k\hbar}{m}\cdot\frac{\pi}{k^2}\sum_l (2l+1)$$

Total outgoing flux




$$j_{\text{out}}^{\text{net}} = \frac{k\hbar}{m}\cdot\frac{\pi}{k^2}\sum_l (2l+1)|S_l|^2$$

➡ If only elastic scattering: $|S_l| = 1$ (flux conservation)

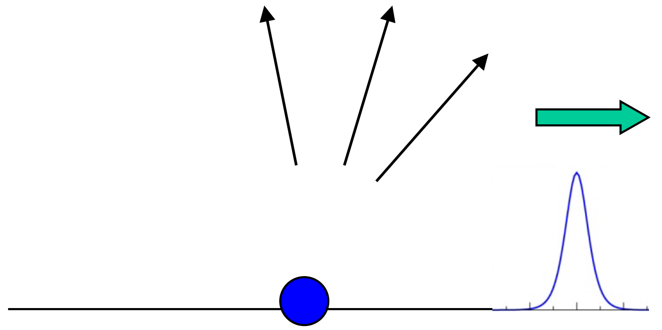
$$S_l = e^{2i\delta_l}$$

$\delta_l$ : phase shift

# Optical potential and Absorption cross section

## Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

## Optical potential

$$\boxed{V_{\mathrm{opt}}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)}$$

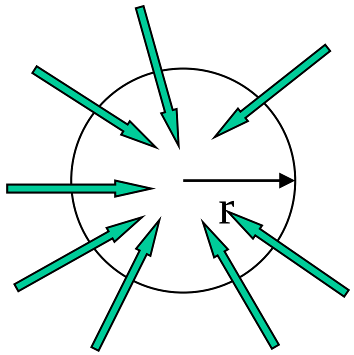
$$\Longrightarrow \quad \boldsymbol{\nabla}\cdot\boldsymbol{j} = \cdots = -\frac{2}{\hbar}W|\psi|^2$$

(note) Gauss's law

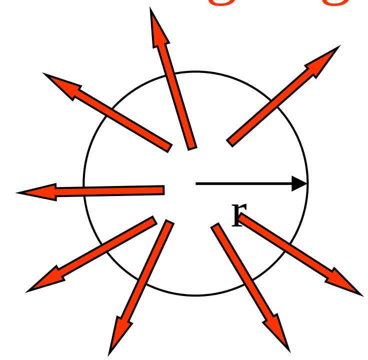
$$\int_S \boldsymbol{j}\cdot\boldsymbol{n}\,dS = \int_V \boldsymbol{\nabla}\cdot\boldsymbol{j}\,dV$$

$$\psi(\boldsymbol{r}) \to \frac{i}{2k}\sum_l (2l+1)\, i^l \,\frac{1}{r}\left[\underbrace{e^{-i(kr-l\pi/2)}}_{\psi_{\text{in}}} - \underbrace{S_l e^{i(kr-l\pi/2)}}_{\psi_{\text{out}}}\right] P_l(\cos\theta)$$

Total incoming flux




Total outgoing flux




$$j_{\text{in}}^{\text{net}} = \frac{k\hbar}{m}\cdot\frac{\pi}{k^2}\sum_l (2l+1)$$

$$j_{\text{out}}^{\text{net}} = \frac{k\hbar}{m}\cdot\frac{\pi}{k^2}\sum_l (2l+1)|S_l|^2$$

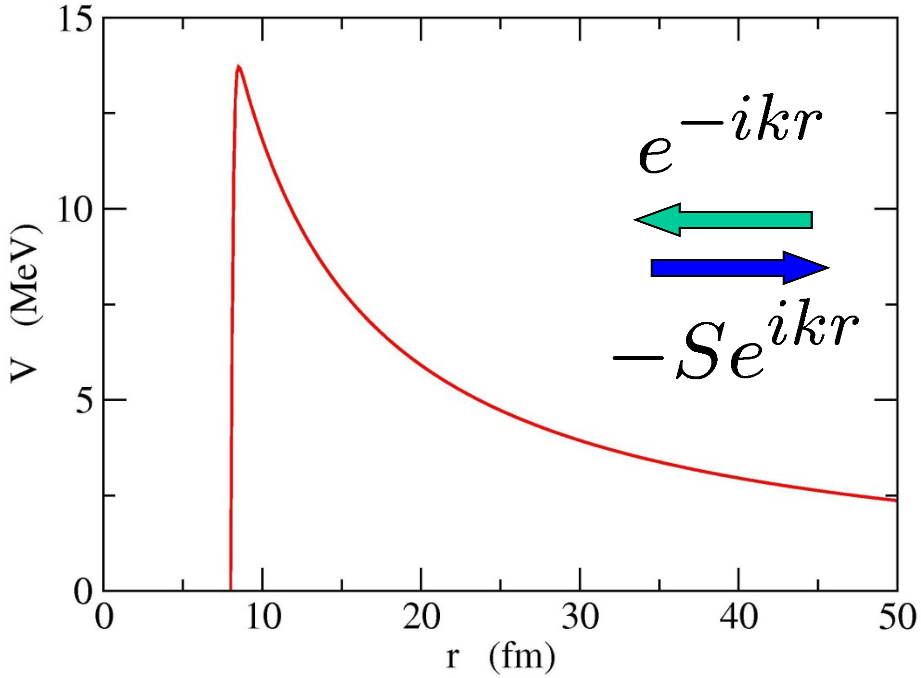
Net flux loss: $$j_{\text{in}}^{\text{net}} - j_{\text{out}}^{\text{net}} = \frac{k\hbar}{m}\cdot\frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$

Absorption cross section:

$$\boxed{\sigma_{\text{abs}} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)}$$

In the case of three-dimensional spherical potential:

$$\psi(\boldsymbol{r}) \to \frac{i}{2k}\sum_l (2l+1)\, i^l\, \frac{1}{r}\left[e^{-i(kr-l\pi/2)} - S_l e^{i(kr-l\pi/2)}\right] P_l(\cos\theta)$$



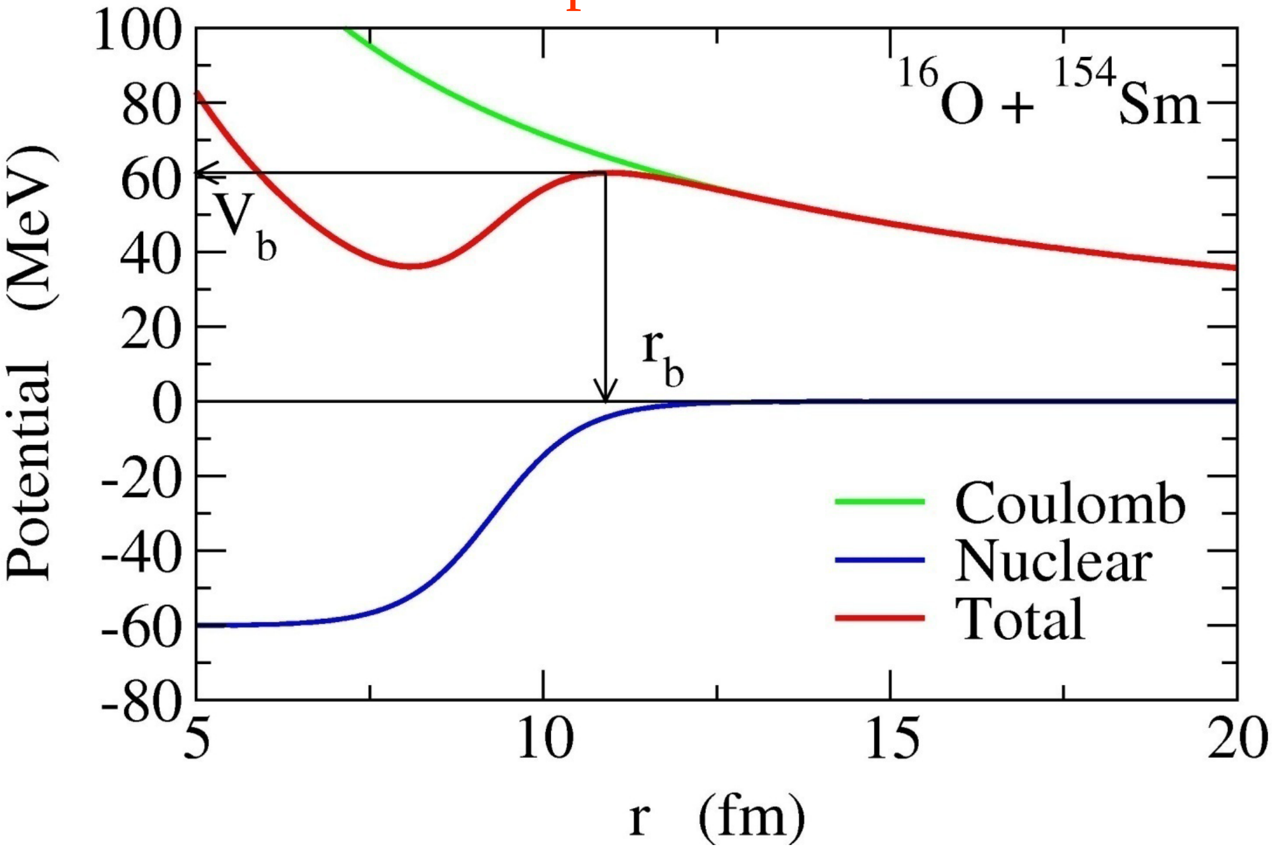

$-S_l \sim R$ (reflection coeff.) $\Longrightarrow$ $P = |T|^2 = 1 - |S_l|^2$

$$\boxed{\sigma_{\rm abs} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2) = \frac{\pi}{k^2}\sum_l (2l+1)P_l}$$

# Overview of heavy-ion reactions

Heavy-ion: Nuclei heavier than $^{4}$He

Inter-nucleus potential

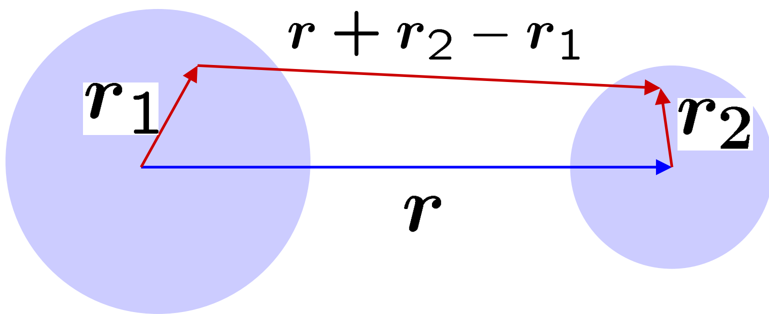



Two forces:

1. Coulomb force
   Long range, repulsive
2. Nuclear force
   Short range, attractive

Potential barrier due to the compensation between these two (Coulomb barrier)

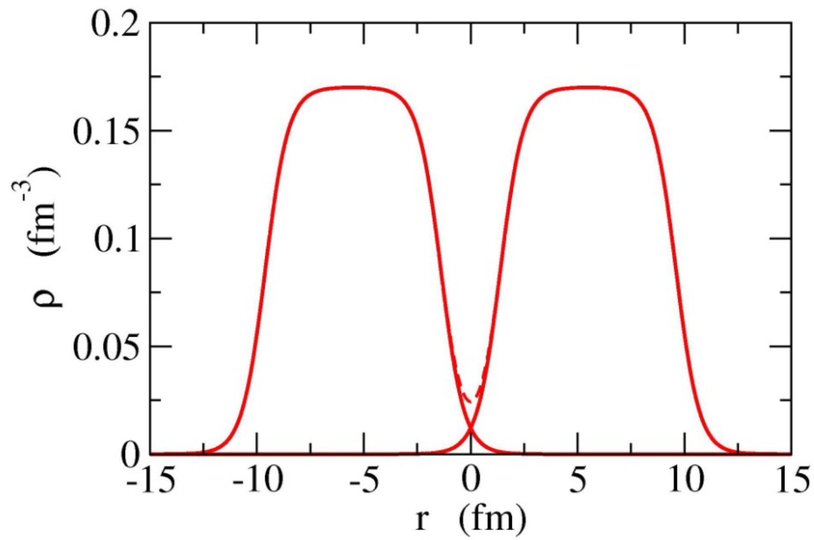
- Double Folding Potential




$$V_{DF}(\boldsymbol{r}) = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2\, \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2) \times v_{nn}(\boldsymbol{r}+\boldsymbol{r}_2-\boldsymbol{r}_1)$$

cf. Michigan 3 range Yukawa (M3Y) interaction

$$v_{nn}(r) = 7999\frac{e^{-4r}}{4r} - 2134\frac{e^{-2.5r}}{2.5r} - 276\,\delta(\boldsymbol{r}) \quad \text{(MeV)}$$




$$\rho(r) \sim \frac{\rho_0}{1+\exp[(r-R_d)/a_d]}$$

$a_d \sim 0.54$ (fm)

- Phenomenological potential

$$\boxed{V_{WS}(\boldsymbol{r}) = -\frac{V_0}{1+\exp[(r-R_0)/a]}}$$

$a \sim 0.63$ (fm)

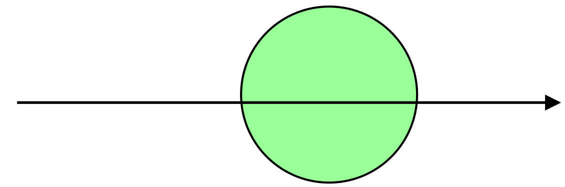
# Three important features of heavy-ion reactions

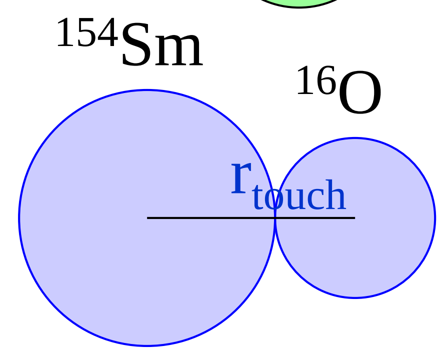
1. Coulomb interaction: important
2. Reduced mass: large → (semi-) classical picture

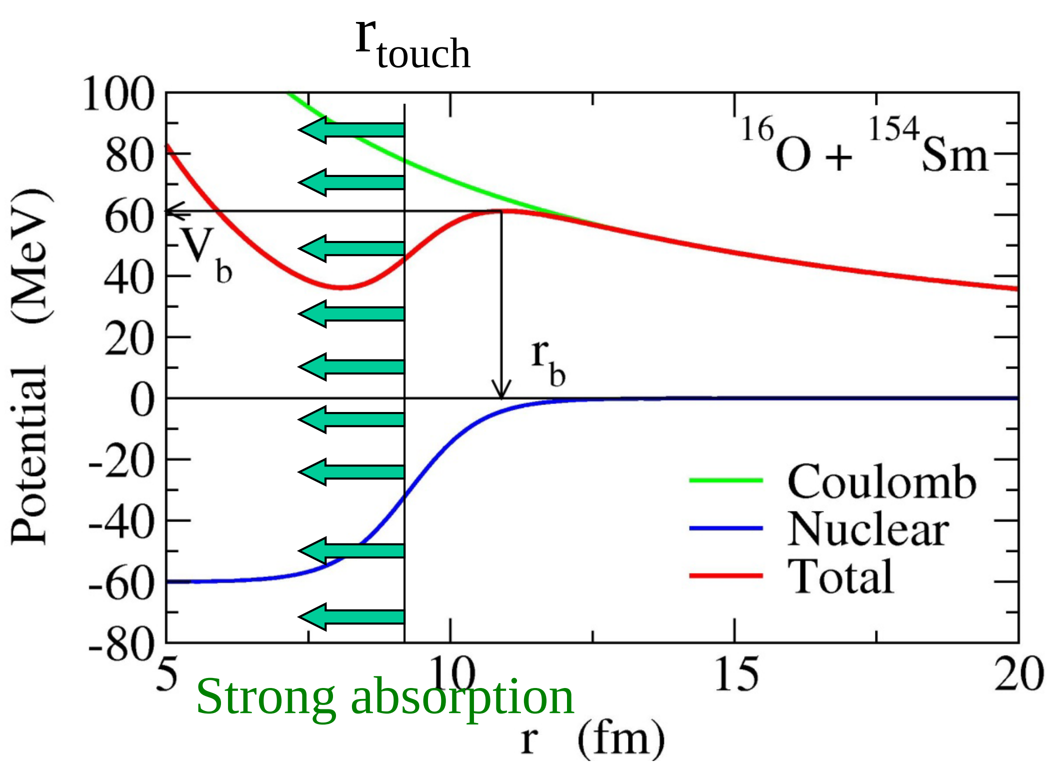
$$\mu = m_T m_P/(m_T + m_P)$$

concept of trajectory

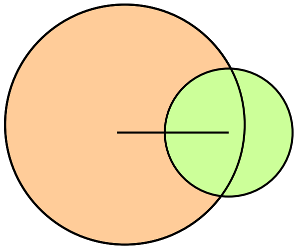
3. Strong absorption inside the Coul. barrier






*Automatic* compound nucleus formation once touched (assumption of strong absorption) the region of large overlap

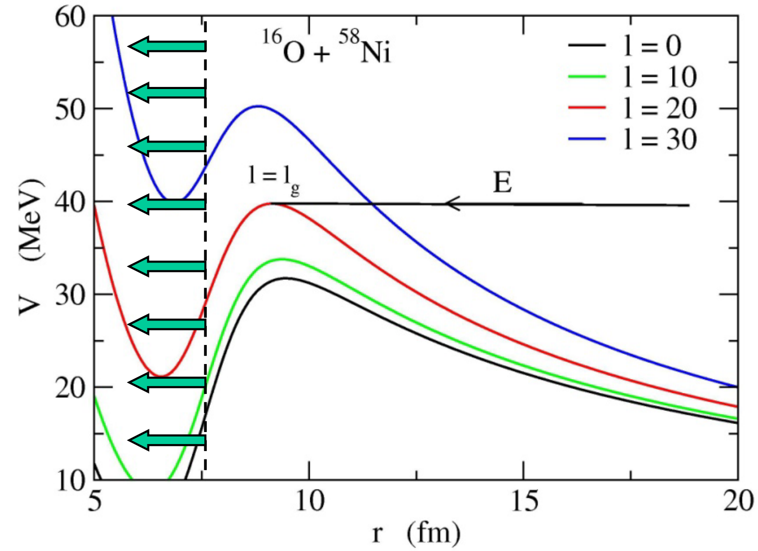


- High level density (CN)
- Huge number of d.o.f.

Relative energy is quickly lost
and converted to internal energy

Formation of hot CN (fusion reaction)

Strong absorption




$l < l_g$ : can access to the strong absorption

$l \geq l_g$ : cannot access cassically

# Partial decomposition of reaction cross section

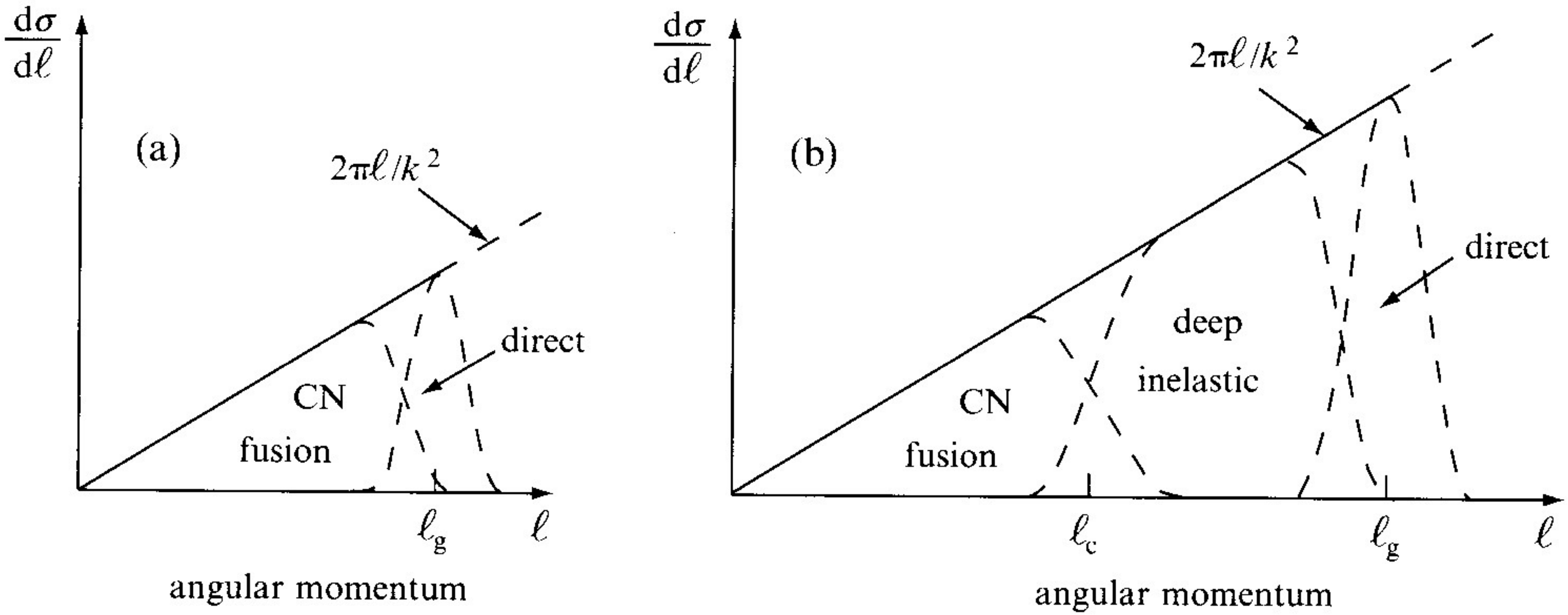
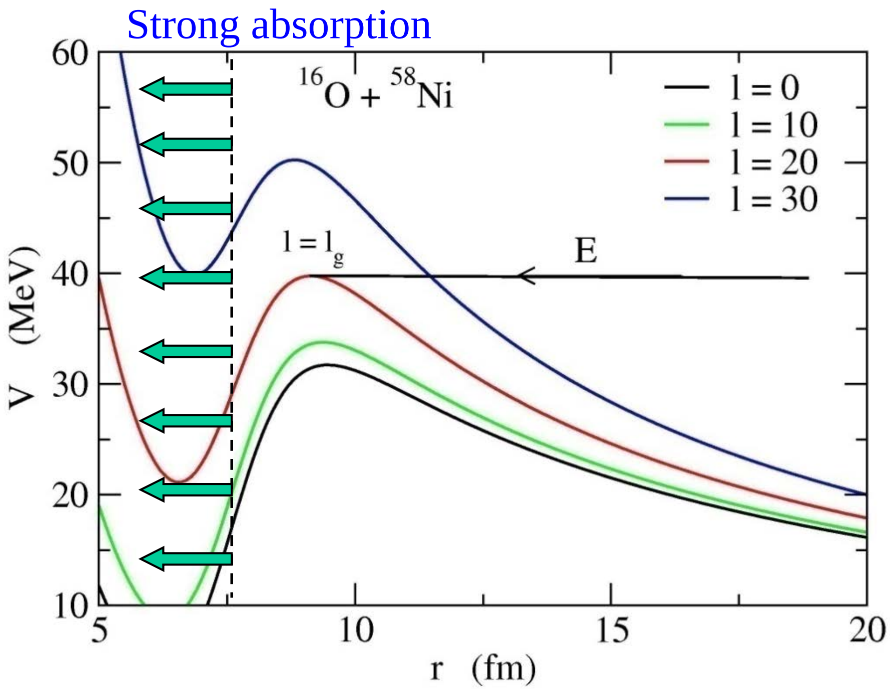
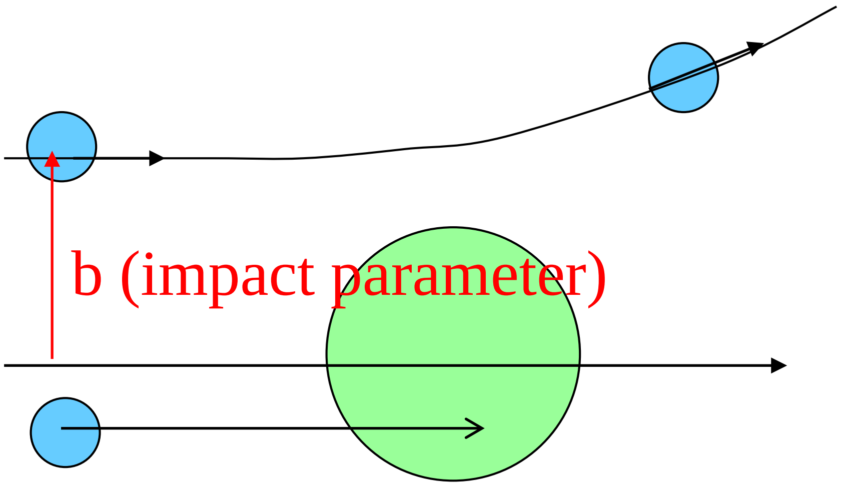



**Figure 4.18** Schematic decomposition of the total heavy-ion reaction cross section into contributions from different partial waves when (a) the grazing angular momentum (quantum number $\ell_g$) is below the critical angular momentum (quantum number $\ell_c$) that can be carried by the compound nucleus, and (b) when $\ell_g$ exceeds $\ell_c$. In both (a) and (b) the straight line is obtained from Equation (4.3) and the dashed areas indicate regions in which different types of heavy-ion nuclear reaction mechanisms predominate.

Taken from J.S. Lilley, "Nuclear Physics"

# Classical Model for heavy-ion fusion reactions




$$l_{cl} = kb \qquad k = \sqrt{2\mu E/\hbar^2}$$

$l < l_g$ : can access to the strong absorption region classically

$$\Rightarrow b_g = l_g/k$$

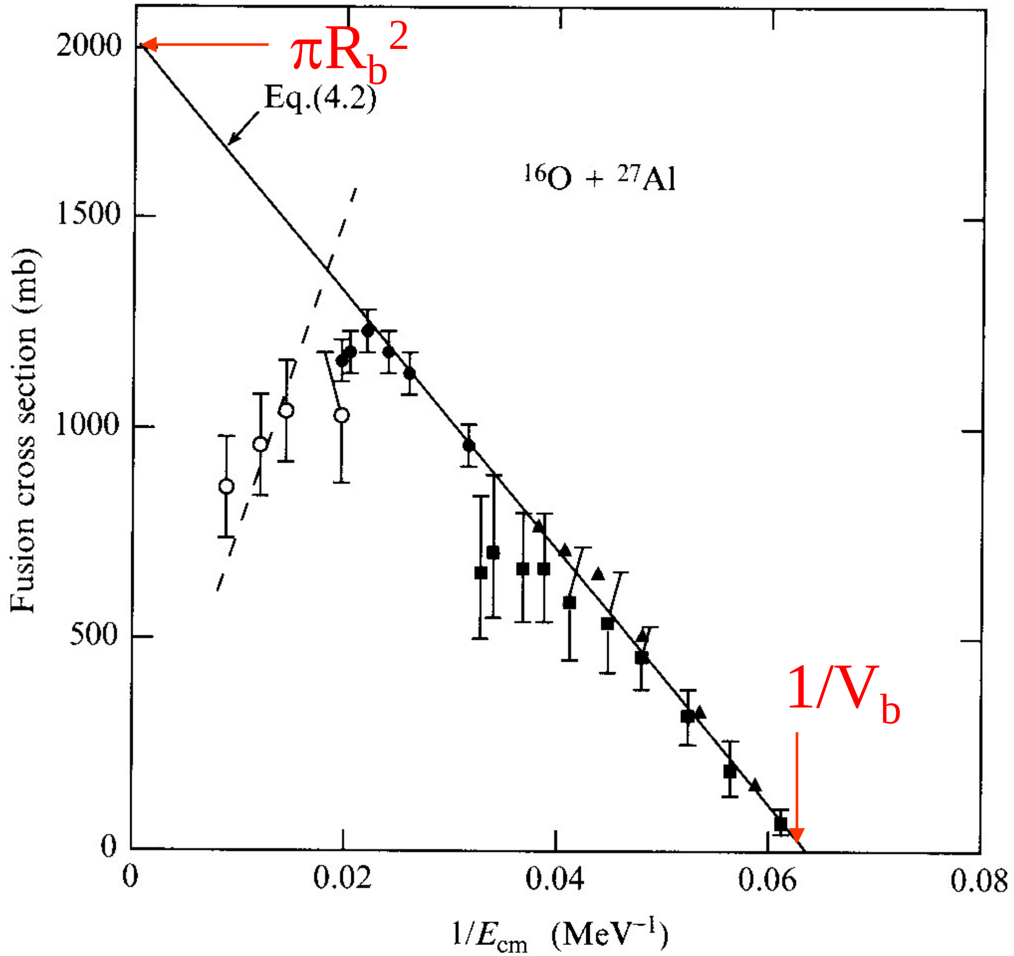
$$\boxed{\sigma^{cl} = 2\pi \int_0^{b_g} b\,db = \pi\, b_g^2}$$

$$V_b + \frac{(kb_g)^2\hbar^2}{2\mu R_b^2} = E$$

$$\boxed{\sigma_{\rm fus}^{cl}(E) = \pi R_b^2 \left(1 - \frac{V_b}{E}\right)}$$

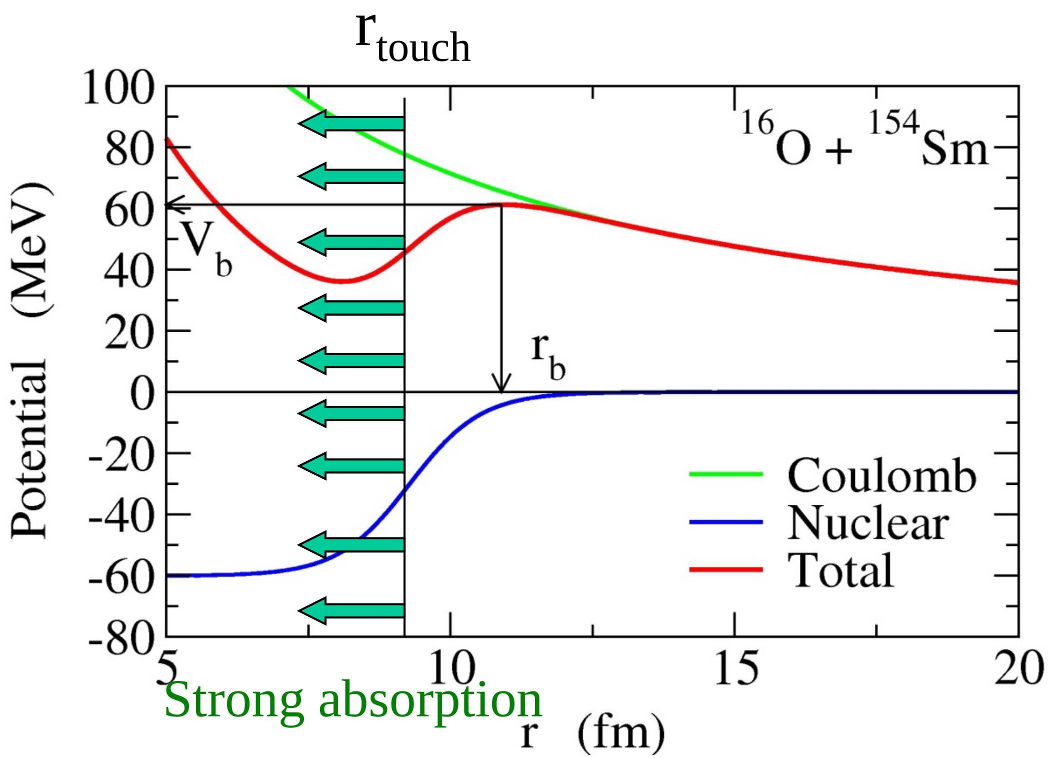
$$\sigma^{cl}_{\mathrm{fus}}(E) = \pi R_b^2 \left(1 - \frac{V_b}{E}\right)$$

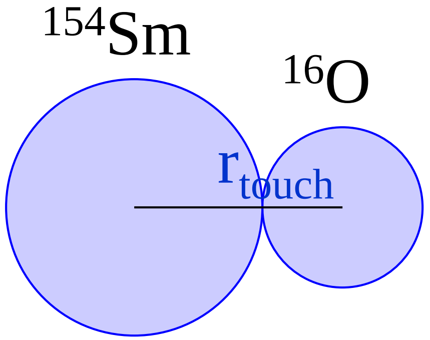
Classical fusion cross section is proportional to $1/E$




Taken from J.S. Lilley, "Nuclear Physics"

# Fusion reaction and Quantum Tunneling




Automatic CN formation once touched (assumption of strong absorption)

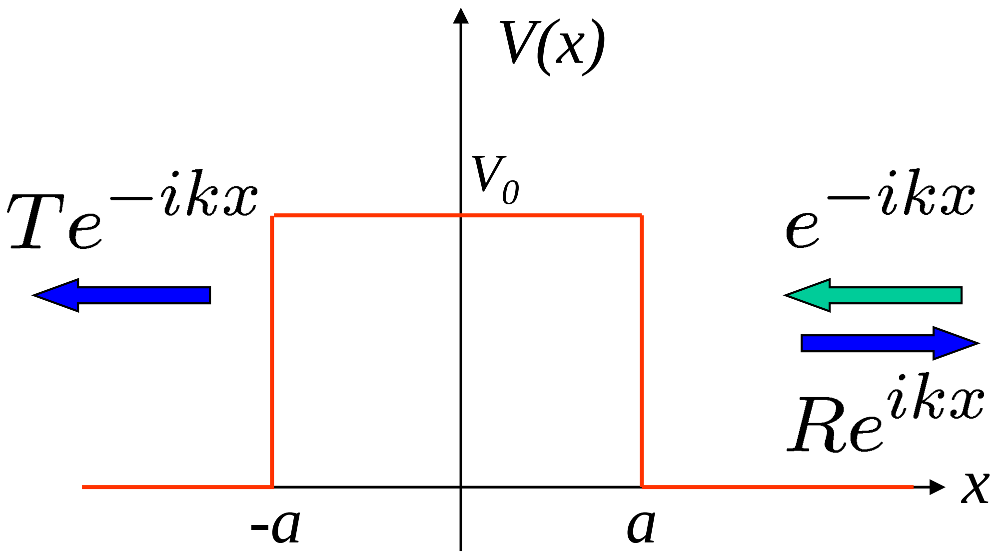
Probability of fusion
= prob. to access to $r_{touch}$

Penetrability of barrier

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

Fusion takes place by quantum tunneling at low energies!

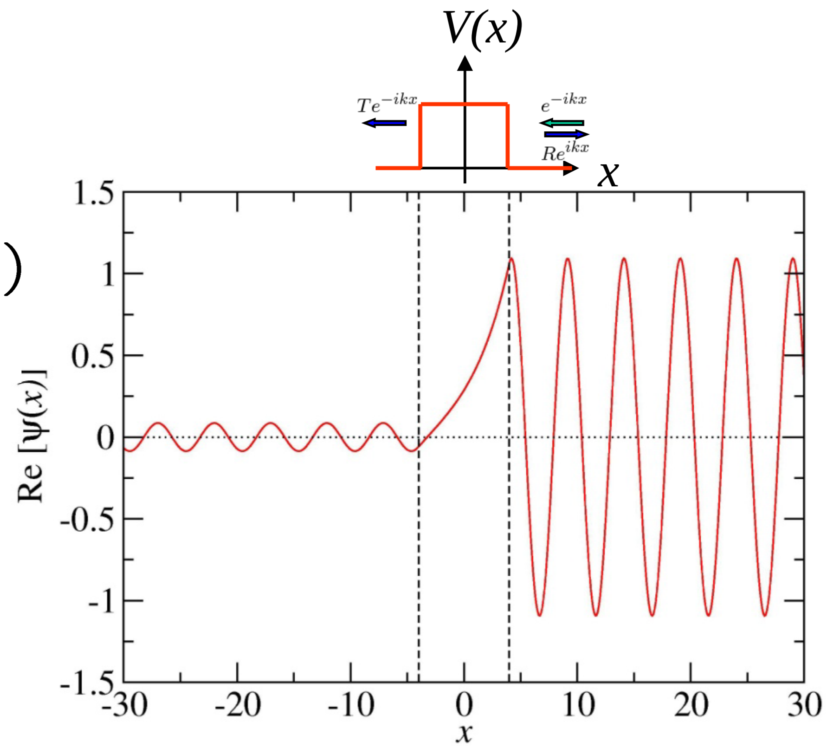
# Quantum Tunneling Phenomena




$$
\begin{aligned}
\psi(x) &= T\,e^{-ikx} && (x \leq -a) \\
&= A\,e^{-\kappa x} + B\,e^{\kappa x} && (-a < x < a) \\
&= e^{-ikx} + R\,e^{ikx} && (x \geq a)
\end{aligned}
$$

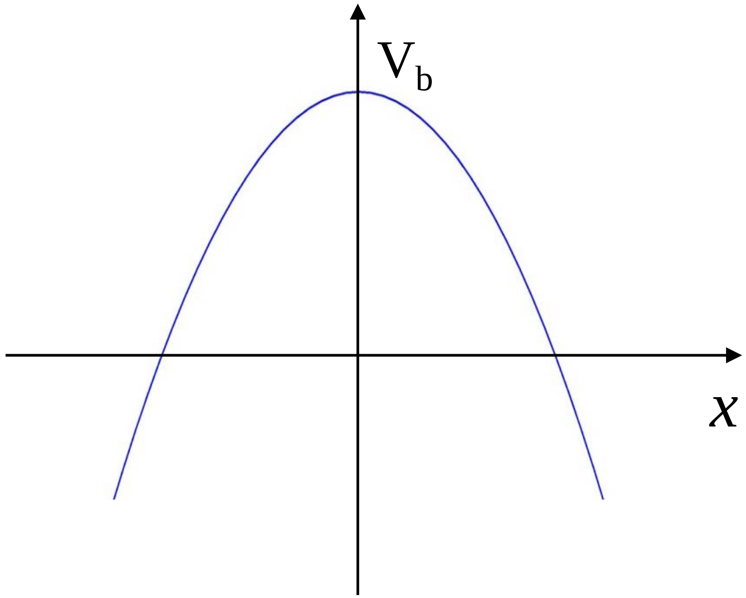
$$
\begin{aligned}
k &= \sqrt{2mE/\hbar^2} \\
\kappa &= \sqrt{2m(V_0 - E)/\hbar^2}
\end{aligned}
$$

Tunnel probability: $P(E) = |T|^2$

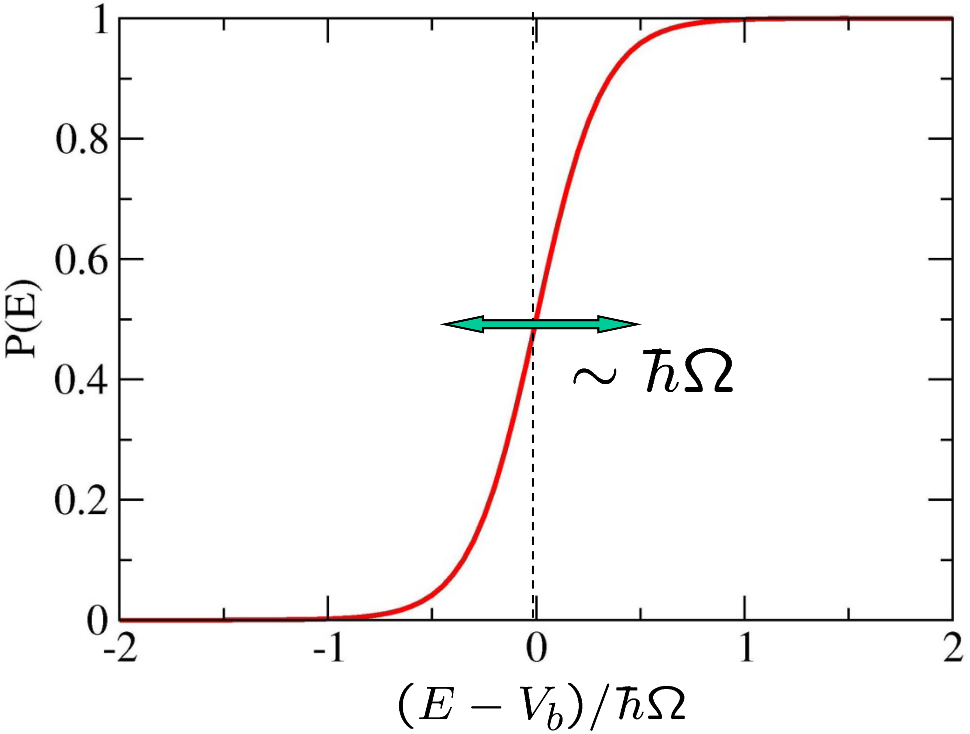


# For a parabolic barrier……

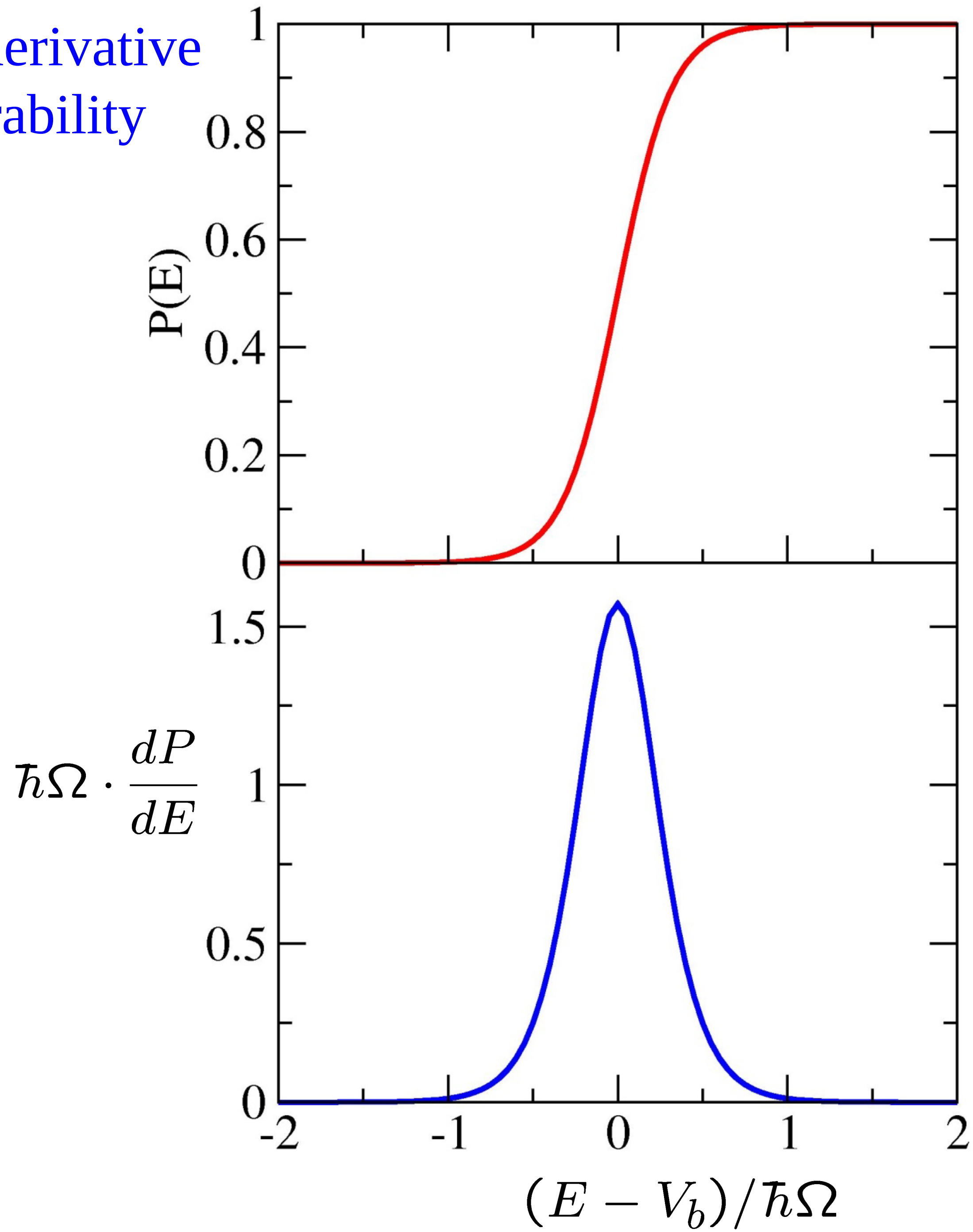
$$V(x) = V_b - \frac{1}{2} m \Omega^2 x^2$$




$$\boxed{P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}}$$



# Energy derivative of penetrability




(note) Classical limit

$$\begin{aligned} P(E) &= \theta(E - V_b) \\ dP/dE &= \delta(E - V_b) \end{aligned}$$

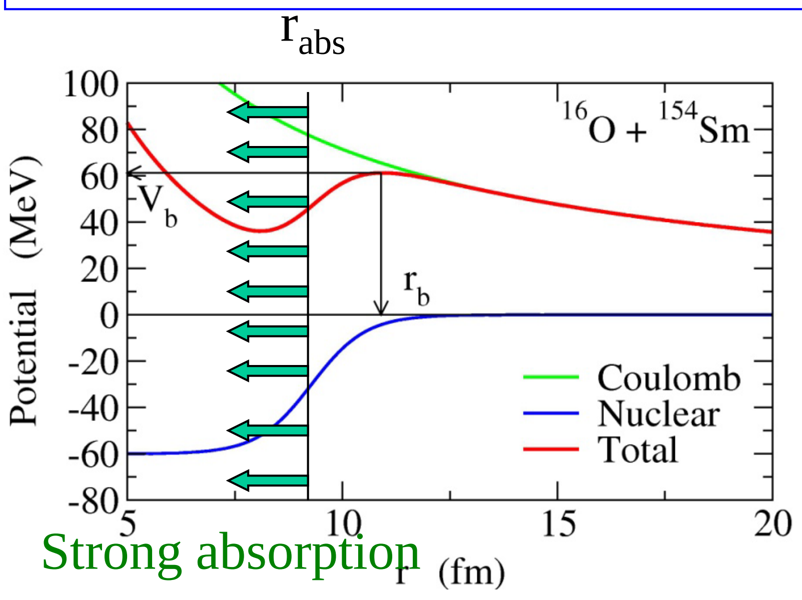
# Potential Model: its success and failure

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+V(r)+\frac{l(l+1)^2}{2\mu r^2}-E\right]u_l(r)=0$$

Asymptotic boundary condition: $u_l(r)\rightarrow H_l^{(-)}(kr)-S_l\,H_l^{(+)}(kr)$

Fusion cross section: $\sigma_{\mathrm{fus}} = \frac{\pi}{k^2}\sum_l (2l+1)P_l$

Mean angular mom. of CN: $\langle l\rangle = \frac{\sum_l l(2l+1)P_l}{\sum_l (2l+1)P_l}$

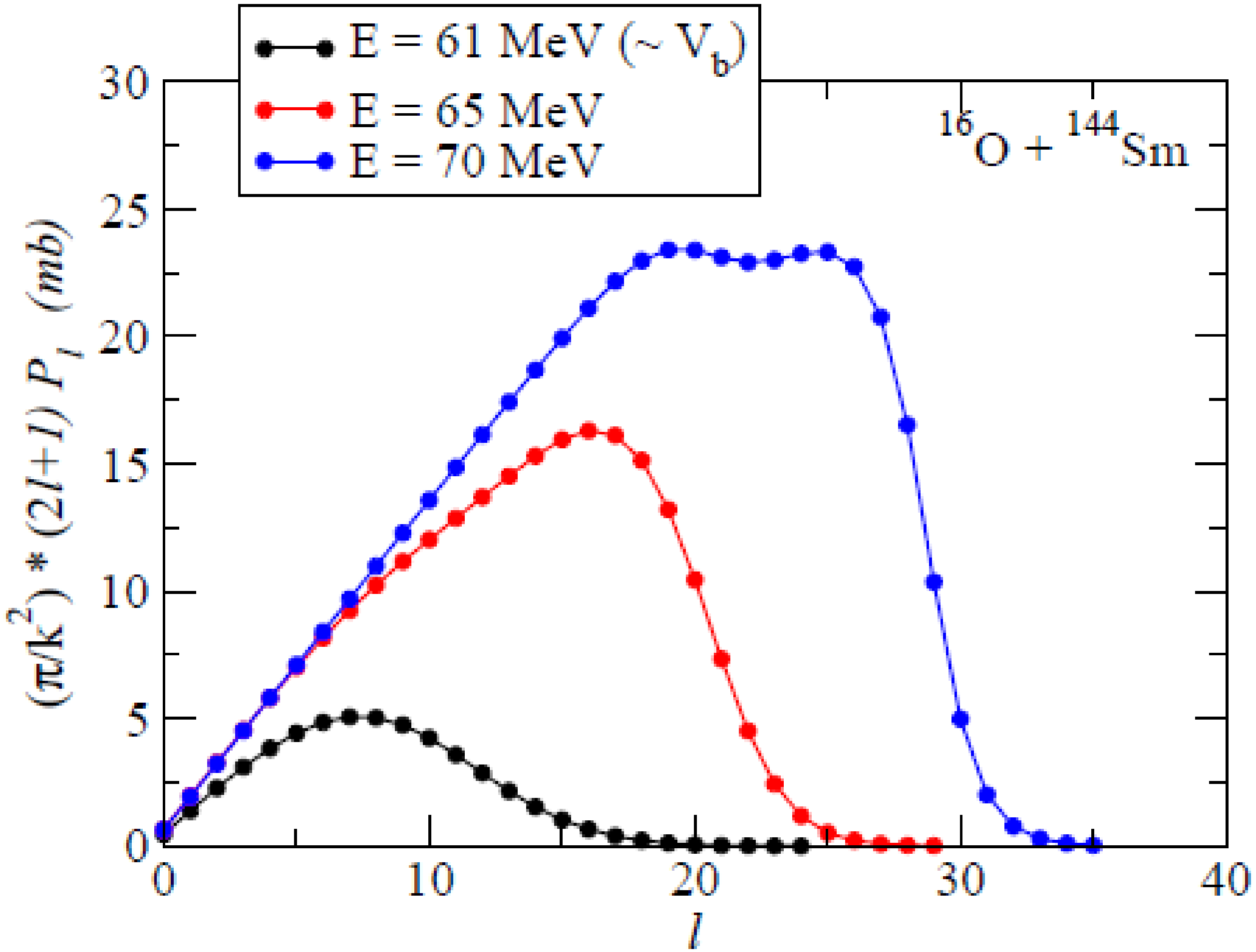



Strong absorption

$$P_l = 1-|S_l|^2$$

Fusion cross section: $$\sigma_{\text{fus}} = \frac{\pi}{k^2}\sum_l (2l+1)P_l$$

Mean angular mom. of CN: $$\langle l \rangle = \frac{\sum_l l(2l+1)P_l}{\sum_l (2l+1)P_l}$$



# Wong's formula

C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

i) Approximate the Coul. barrier by a parabola: $V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$

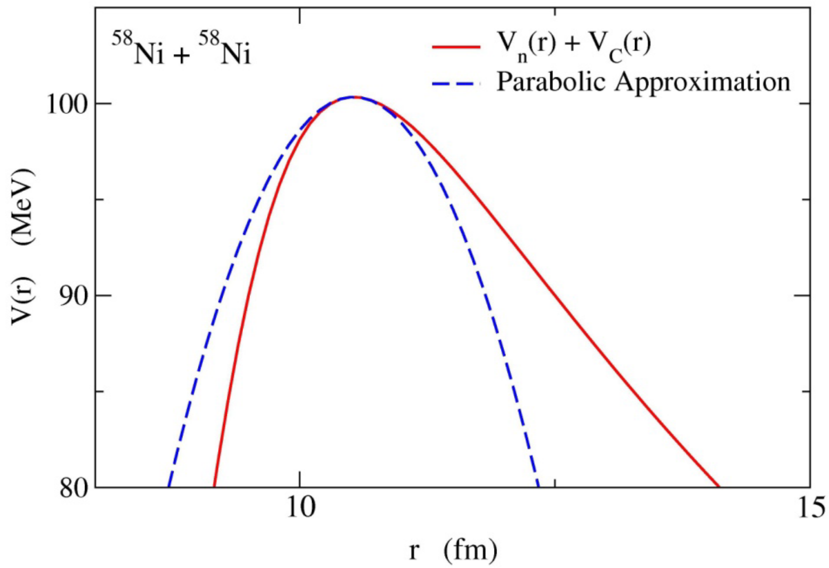
$$\Rightarrow \quad P_0(E) = 1\Big/\left(1+\exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]\right)$$

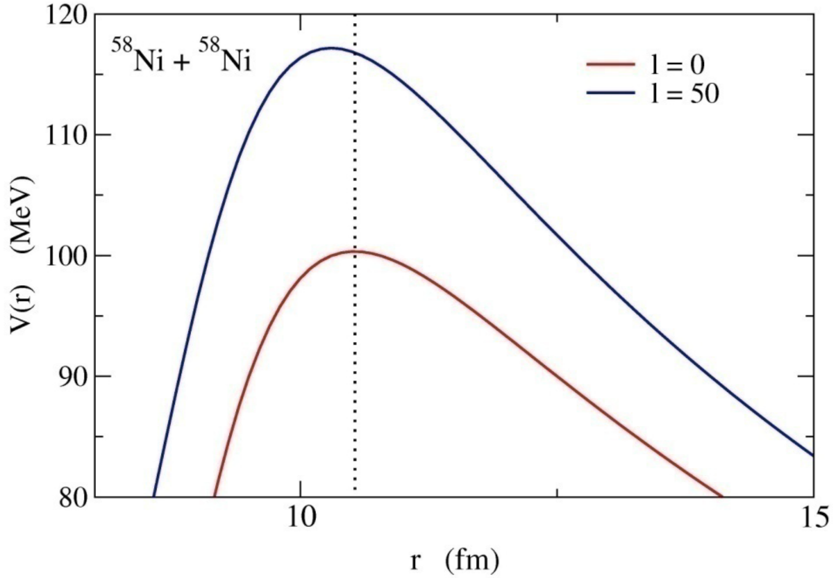
ii) Approximate $P_l$ by $P_0$:

$$P_l(E) \sim P_0\left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}\right)$$

(assume *l*-independent Rb and curvature)

iii) Replace the sum of *l* with an integral



$$\boxed{\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \log\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]}$$

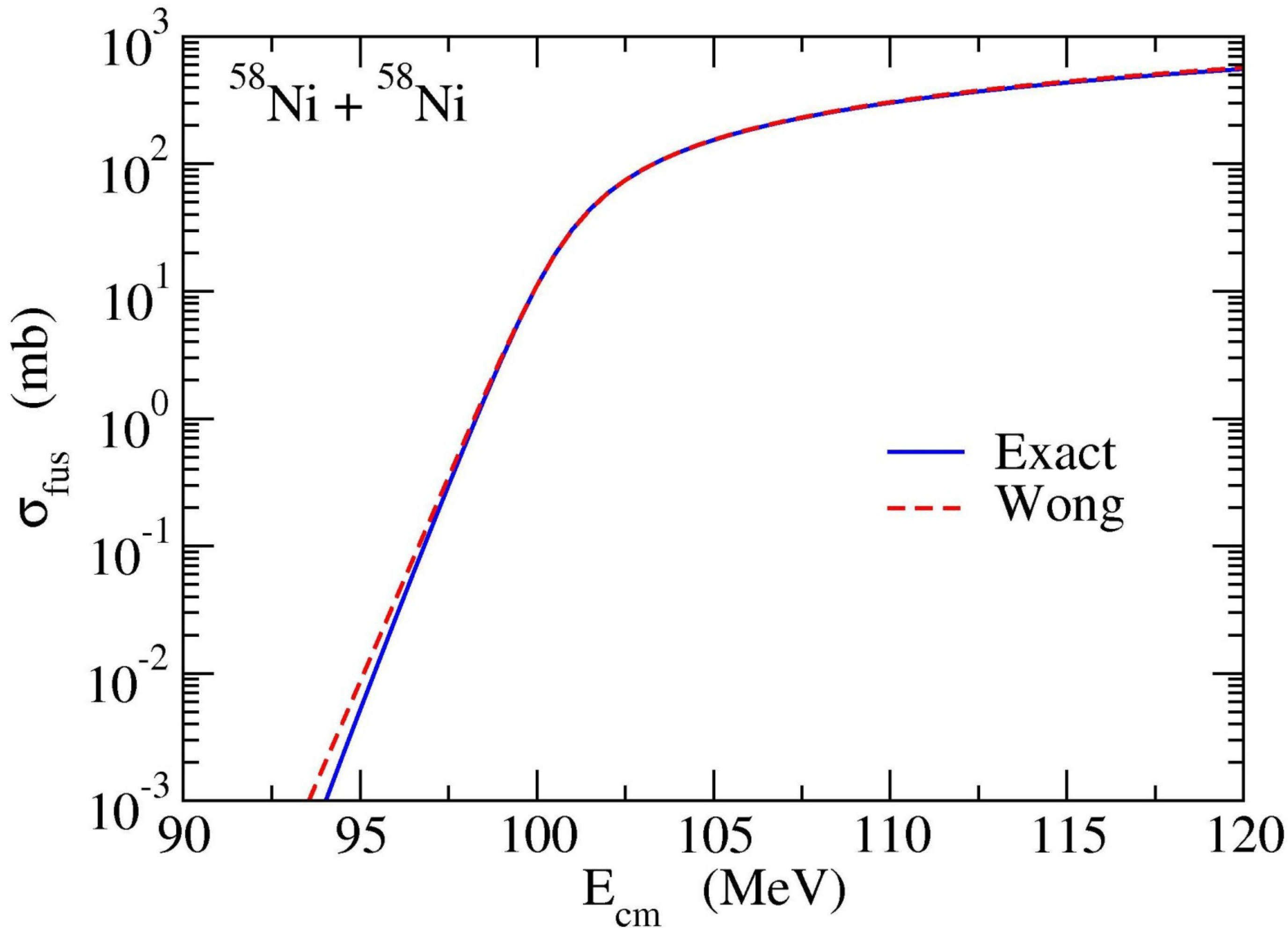
(note) For $E \gg V_b$ $\quad 1 \ll \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)$

$$\Longrightarrow \sigma_{\text{fus}}(E) \sim \pi R_b^2 \left(1 - \frac{V_b}{E}\right) = \sigma_{\text{fus}}^{cl}(E)$$

(note)

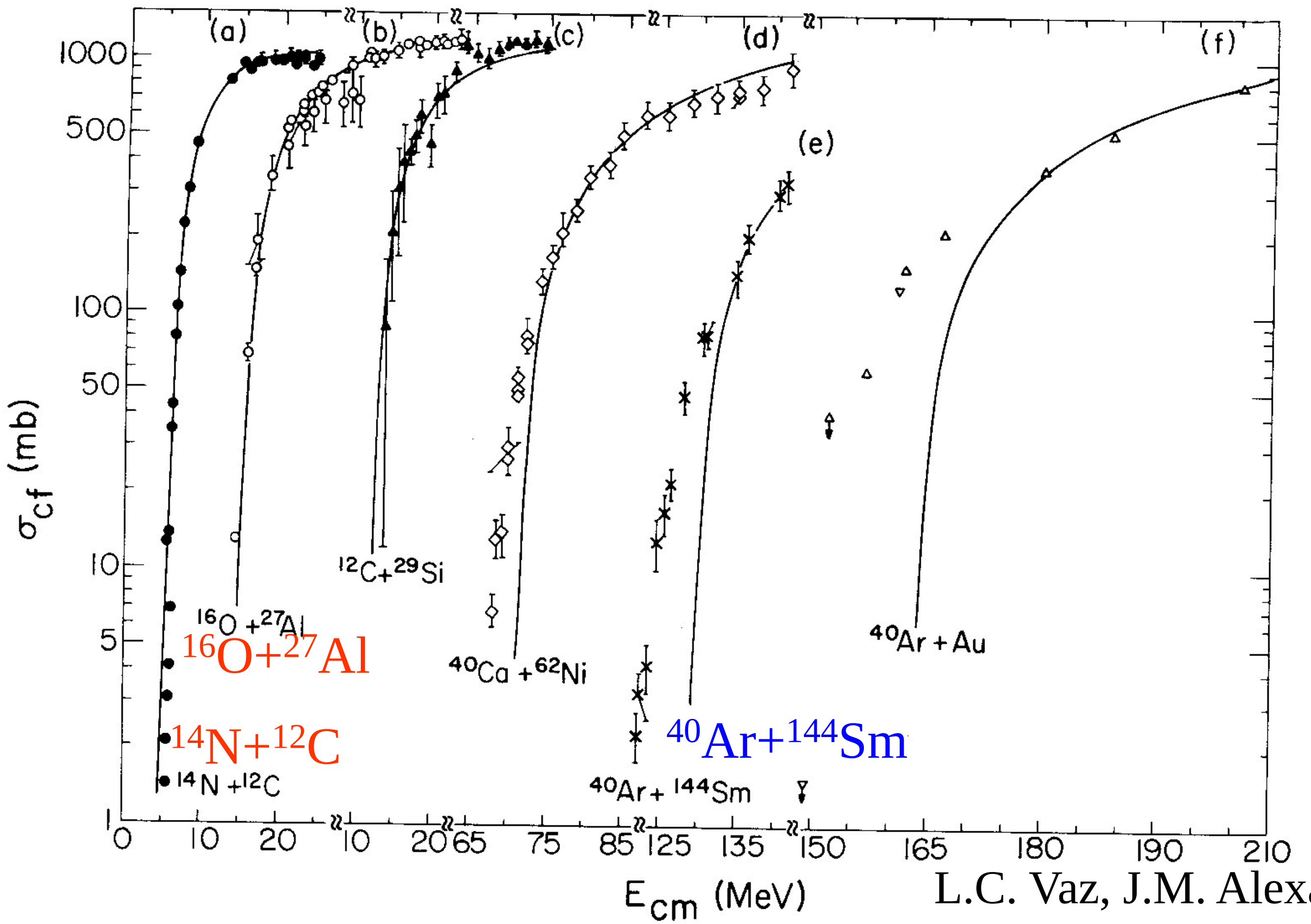
$$\boxed{\frac{d(E\sigma_{\text{fus}})}{dE} = \frac{\pi R_b^2}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]} = \pi R_b^2 \cdot P_{l=0}(E)}$$

$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E}\, R_b^2\, \log\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$



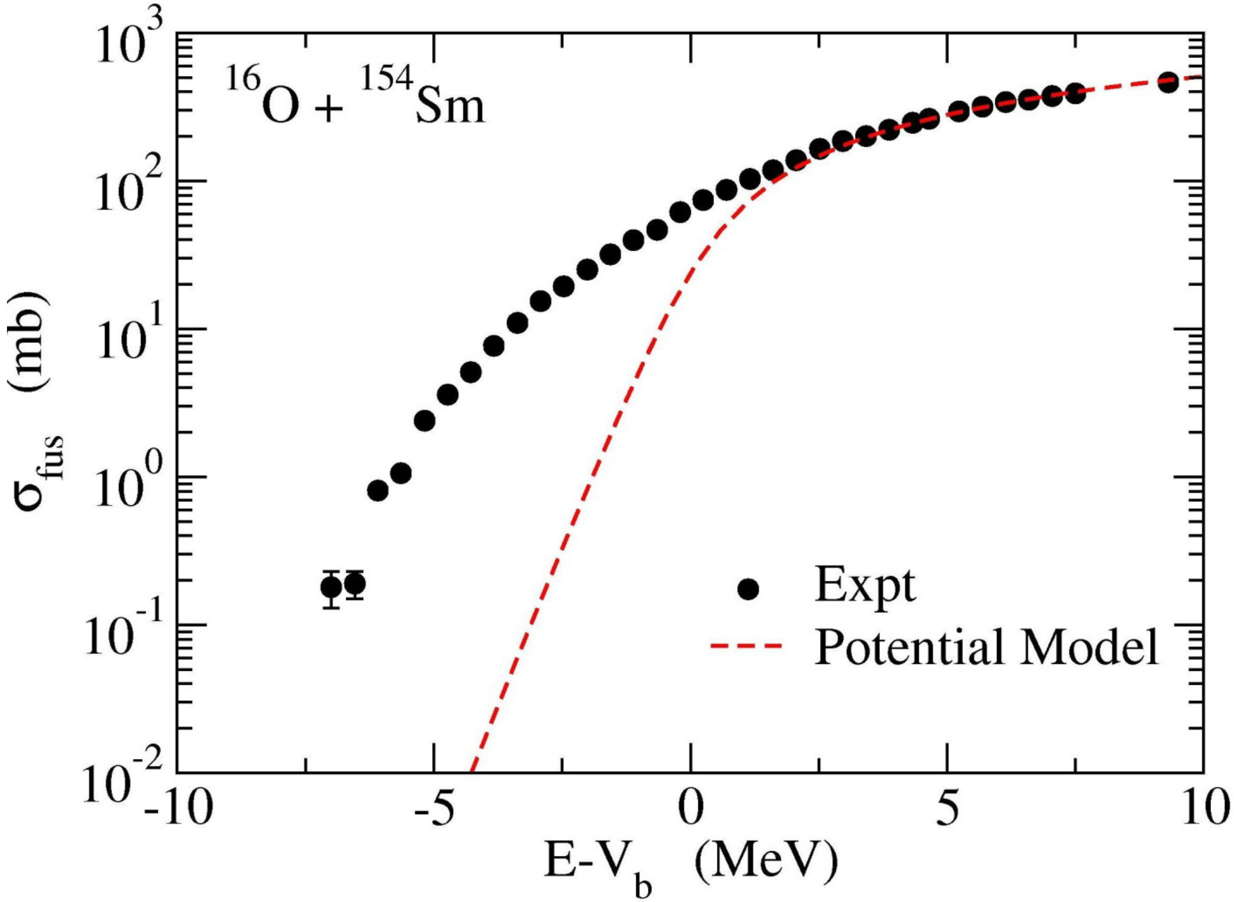
# Comparison between prediction of pot. model with expt. data

Fusion cross sections calculated with a static energy independent potential




L.C. Vaz, J.M. Alexander, and G.R. Satchler, Phys. Rep. 69('81)373

- ➢Works well for relatively light systems
- ➢Underpredicts $\sigma_{fus}$ for heavy systems at low energies




Potential model:
Reproduces the data reasonably well for
$E > V_b$

Underpredicts $\sigma_{fus}$ for
$E < V_b$

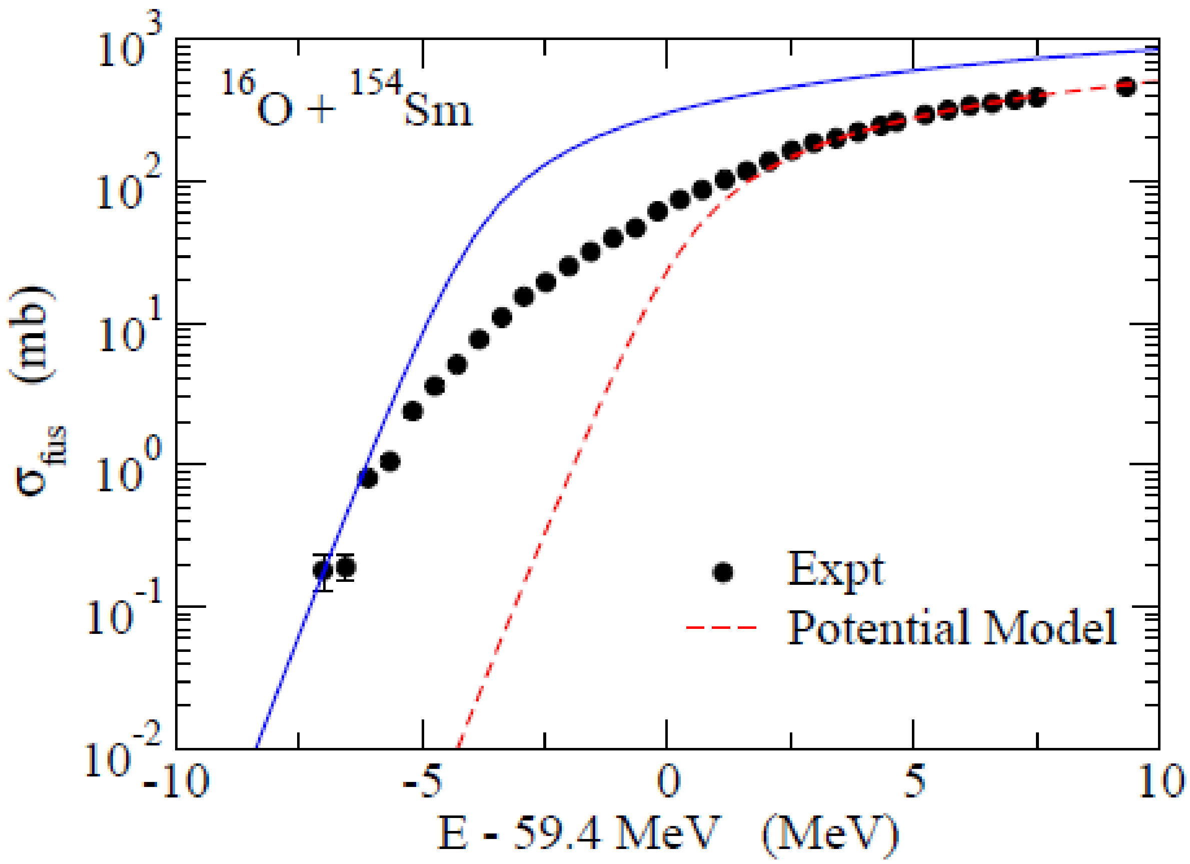
What is the origin?

Data: J.R. Leigh et al., PRC52('95)3151

cf. seminal work:
R.G. Stokstad et al., PRL41('78)465
PRC21('80)2427

Inter-nuclear Potential is poorly parametrized?
Other origins?

With a deeper nuclear potential (but still within a potential model)…..



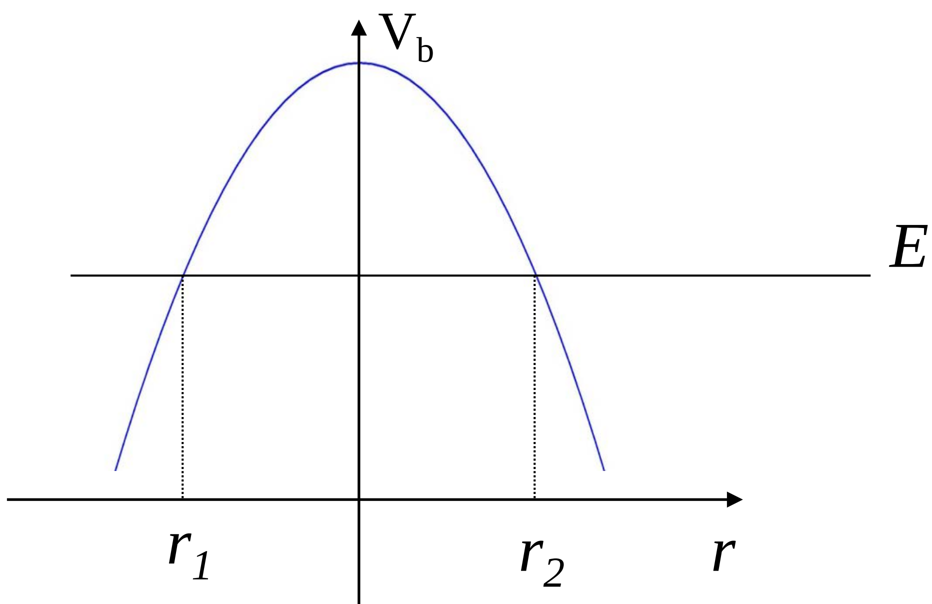
# Potential Inversion

$$P_0(E) = \frac{1}{\pi R_b^2}\frac{d(E\sigma_{\rm fus})}{dE}$$

(note)

$$P_0(E) = 1/[1 + S_0(E)], \quad S_0(E) = \int_{r_1}^{r_2} dr \sqrt{\frac{2\mu}{\hbar^2}(V(r) - E)}$$

$$\boxed{t(E) \equiv r_2 - r_1 = -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2\mu}}\int_E^{V_b}\frac{\frac{dS_0(E')}{dE'}}{\sqrt{E'-E}}\,dE'}$$




$$\sigma_{\rm fus} \longrightarrow V(r)$$

Semi-classical app.

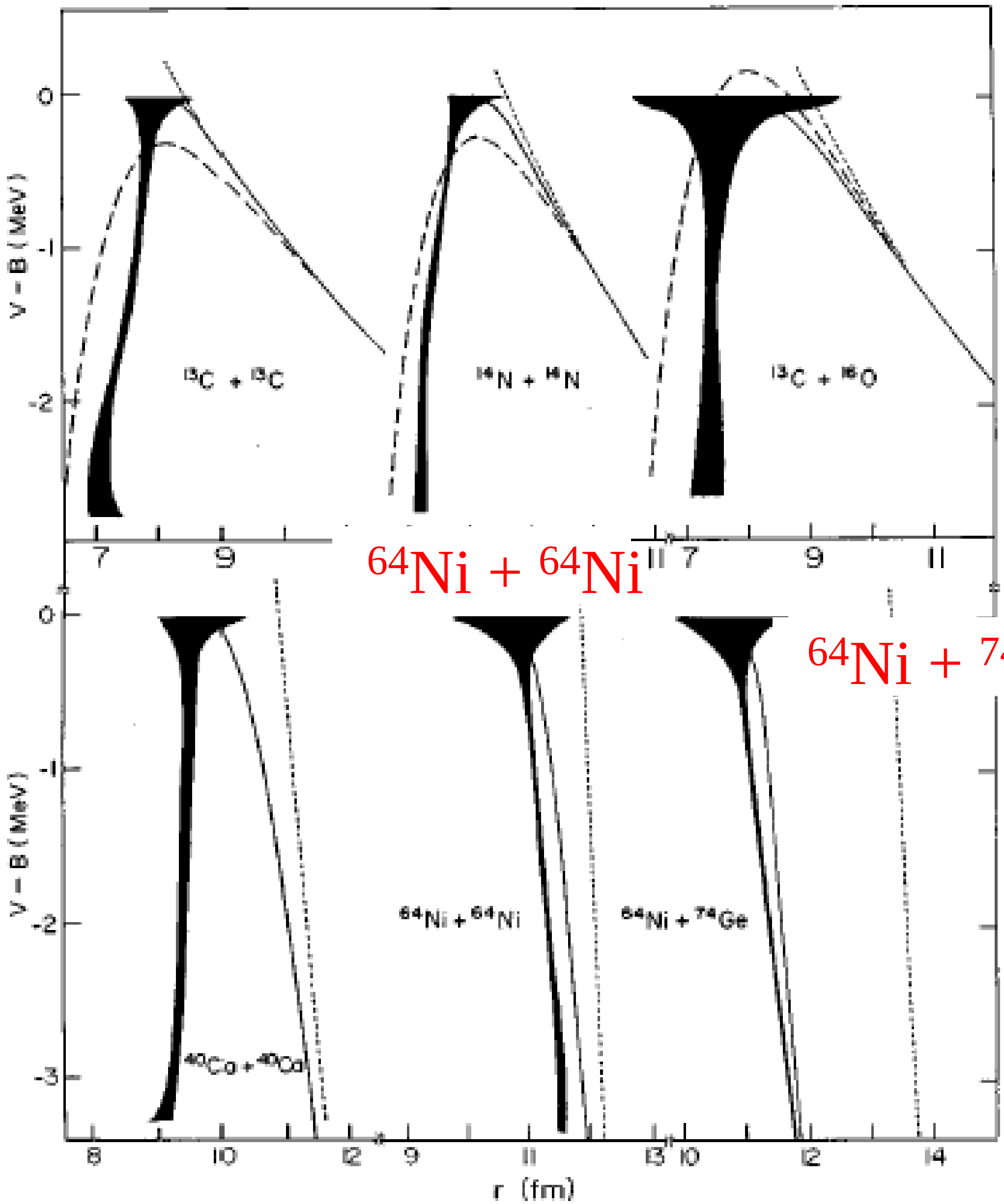
- Energy independent
- local
- single-ch.

- Potential inversion

$\sigma_{fus} \xrightarrow{\quad} V(r)$

Semi-classical app.

- Energy independent
- local
- single-ch.



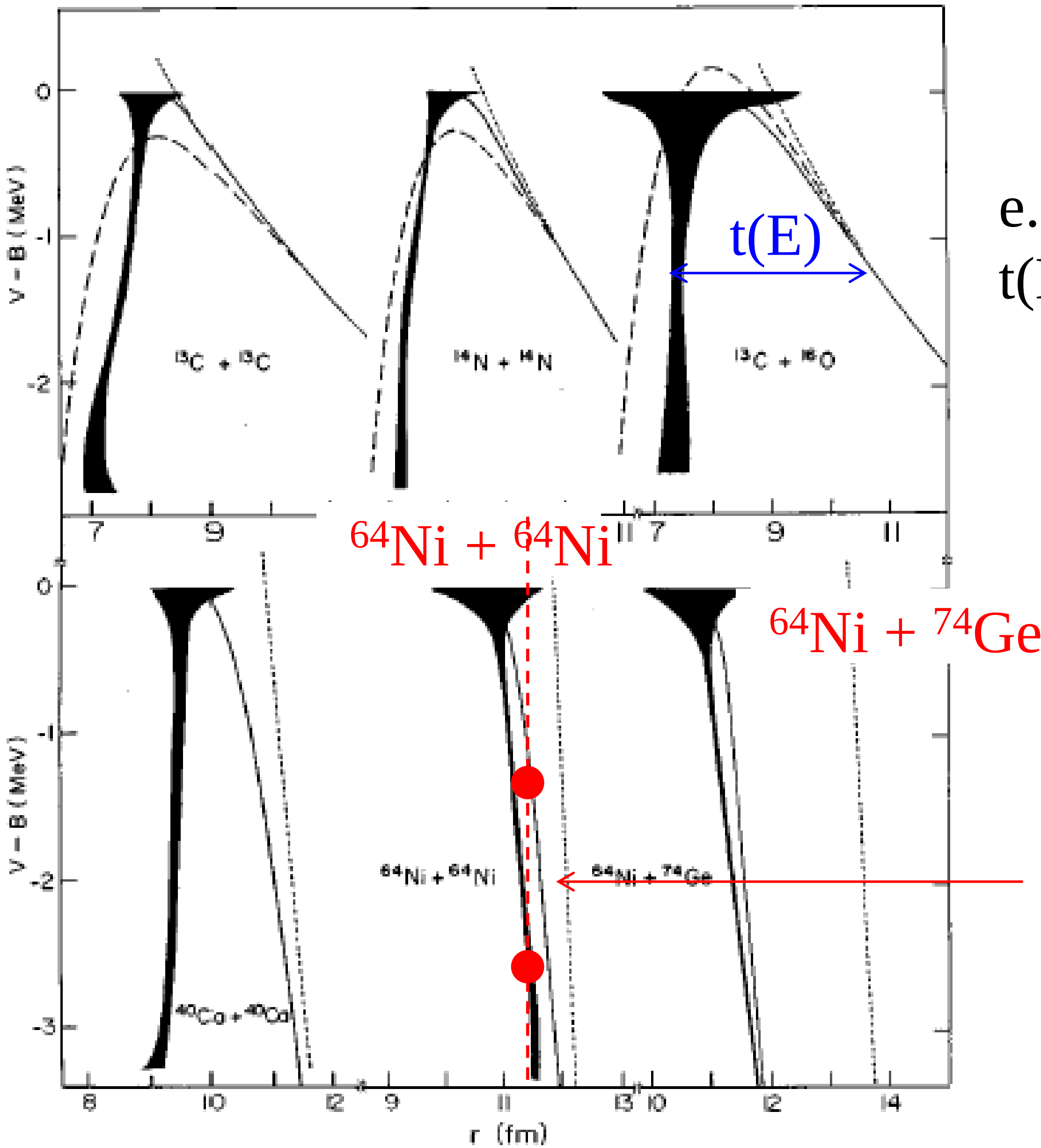

$^{64}$Ni + $^{74}$Ge

Unphysical potentials

Beautiful demonstration of C.C. effects

A.B. Balantekin, S.E. Koonin, and J.W. Negele, PRC28('83)1565

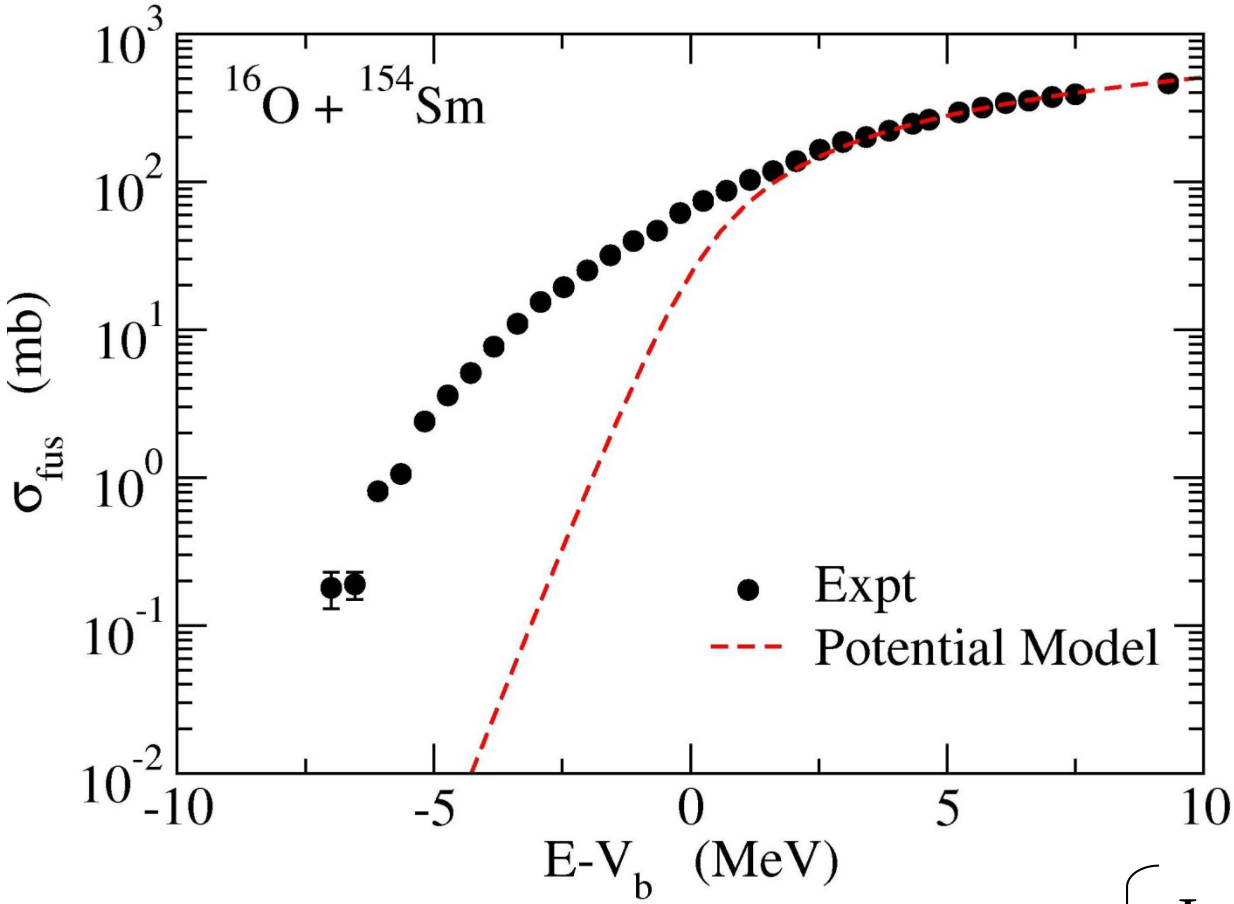
- Potential inversion




e.g.,
t(E) = 3 +/- 0.2 fm

double valued potential
(unphysical !!)

# Fusion cross sections calculated with a static energy independent potential



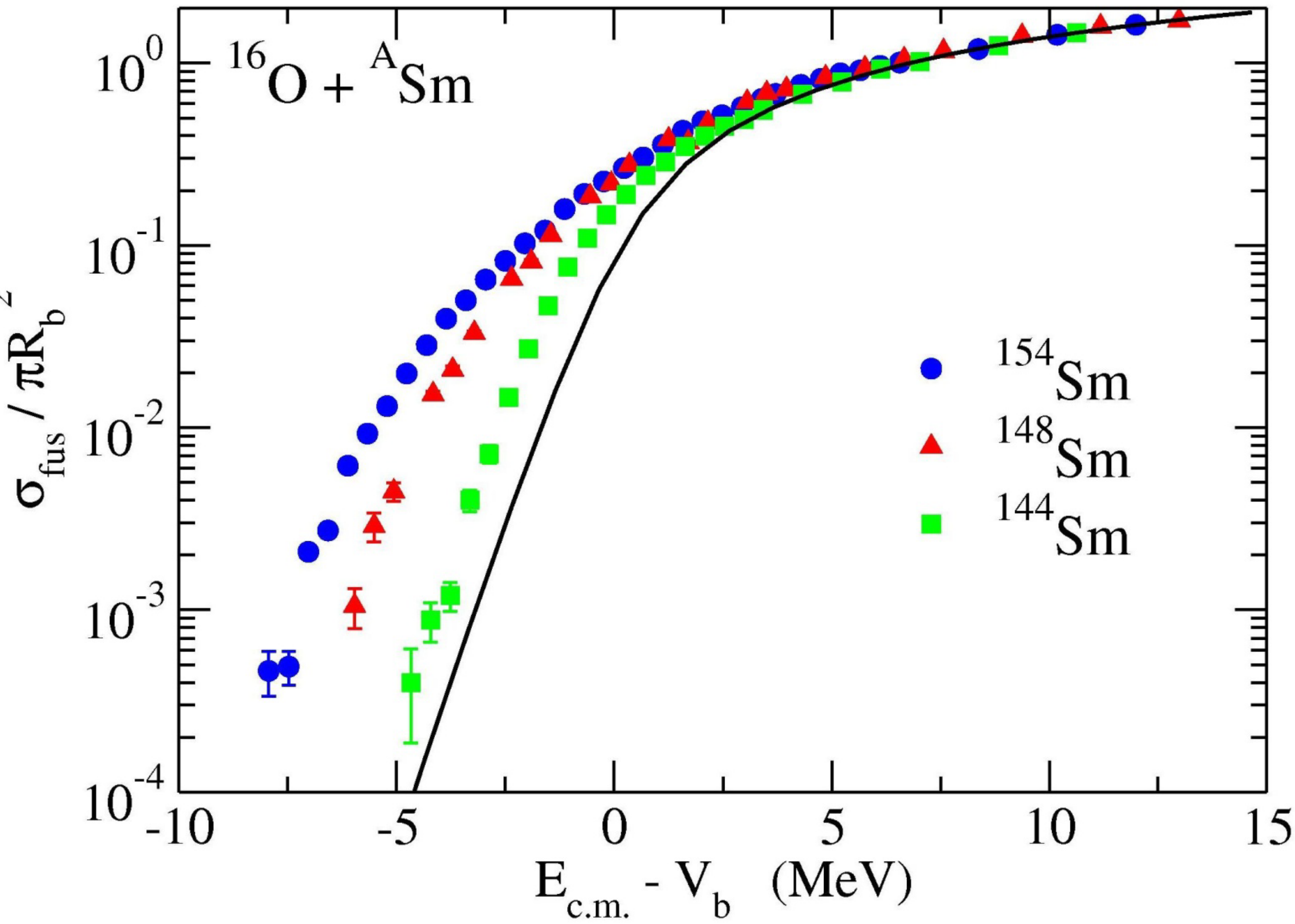

Potential model:
Reproduces the data reasonably well for
$E > V_b$

Underpredicts $\sigma_{fus}$ for
$E < V_b$

What is the origin?

~~Inter-nuclear Potential is poorly parametrized?~~
Other origins?

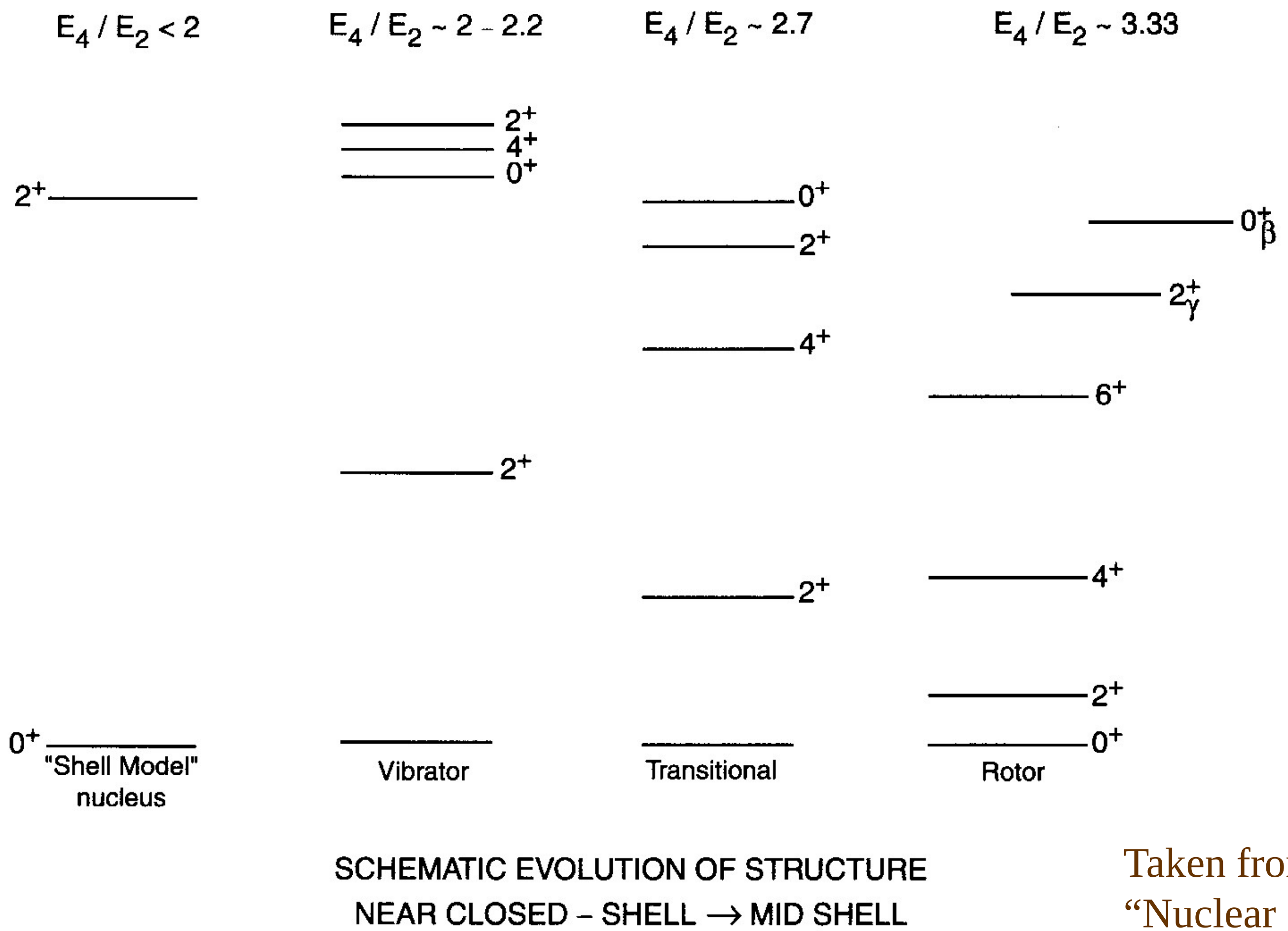
# Target dependence of fusion cross section




Strong target dependence at $E < V_b$

# Low-lying collective excitations in atomic nuclei

Low-lying excited states in even-even nuclei are collective excitations, and strongly reflect the pairing correlation and shell strucuture




Taken from R.F. Casten, “Nuclear Structure from a Simple Perspective”

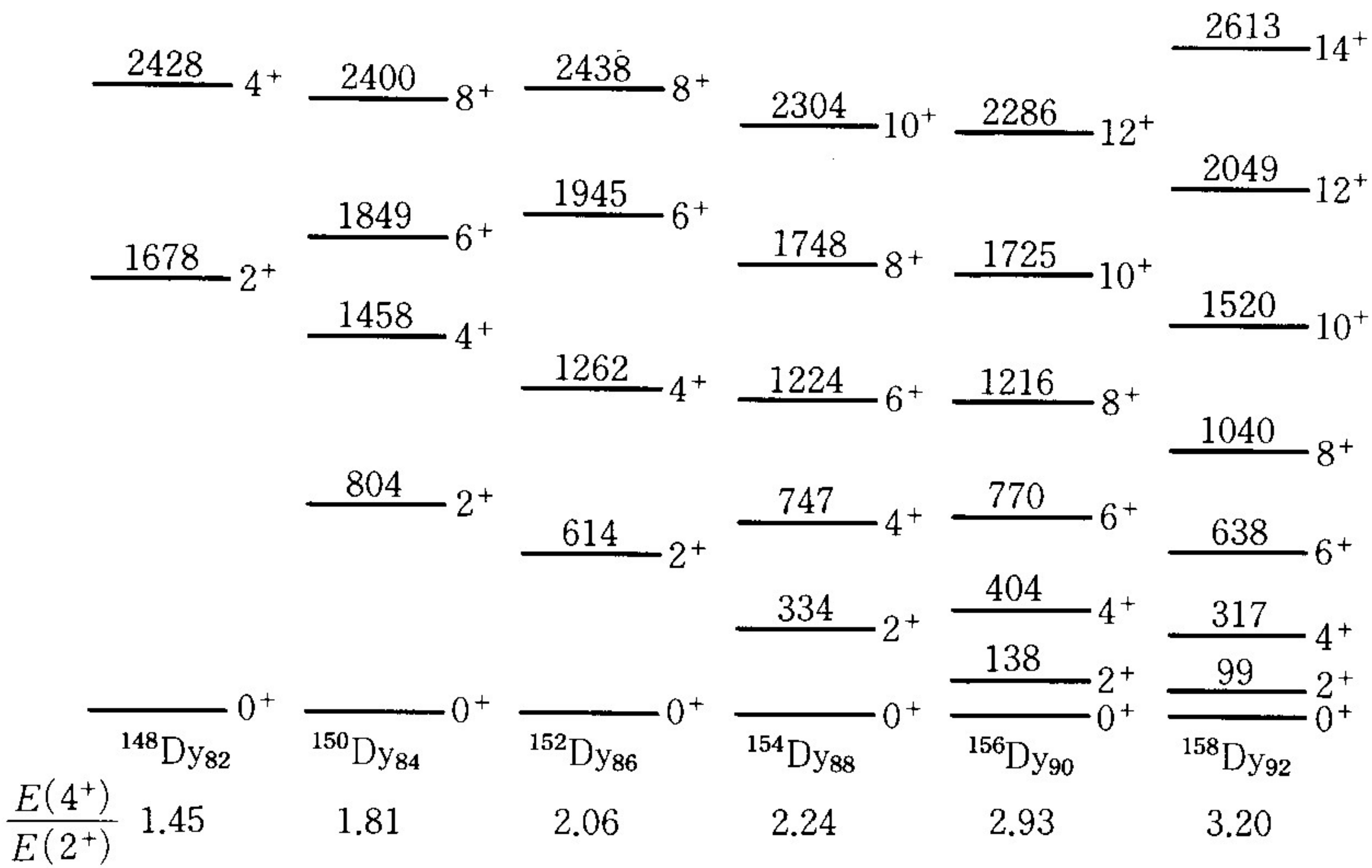



**図 3-4　Dy** アイソトープの低励起スペクトル．励起エネルギーの単位は keV.

# Effect of collective excitation on $\sigma_{fus}$: rotational case

Excitation spectra of $^{154}$Sm

| (MeV) | |
|---|---|
| 0.903 | $8^+$ |
| 0.544 | $6^+$ |
| 0.267 | $4^+$ |
| 0.082 | $2^+$ |
| 0 | $0^+$ |

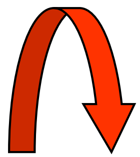
$^{154}$Sm

$$\boxed{E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}}$$

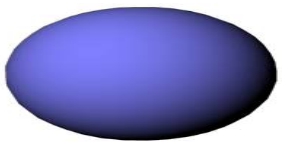
cf. Rotational energy of a rigid body
(Classical mechanics)

$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

$$(I = \mathcal{J}\omega, \;\; \omega = \dot{\theta})$$



$^{154}$Sm is deformed



# Effect of collective excitation on $\sigma_{fus}$: rotational case

Comparison of energy scales

$$V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$$

Tunneling motion: $E_{\text{tun}} \sim \hbar\Omega \sim$ 3.5 MeV (barrier curvature)

Rotational motion: $E_{\text{rot}} \sim E_{2+} \sim 0.08\text{MeV}$

$$E_{\text{tun}} \gg E_{\text{rot}} = I(I+1)\hbar^2/2\mathcal{J} \to 0$$

$$\longleftrightarrow \mathcal{J} \to \infty$$

The orientation angle of $^{154}$Sm does not change much during fusion

(note)

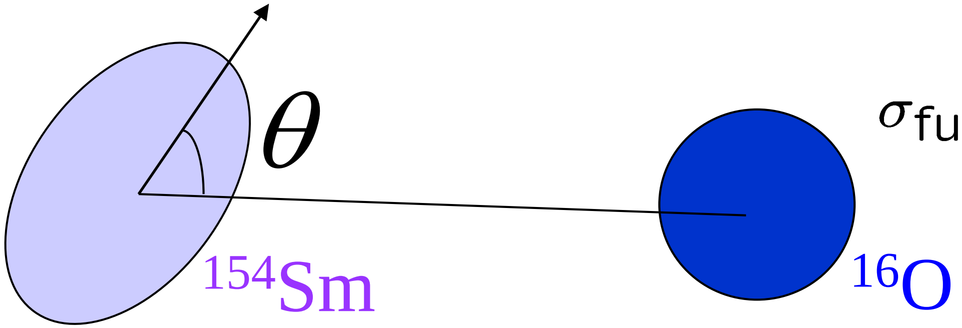
Ground state ($0^+$ state) when reaction starts


Mixing of all orientations with an equal weight

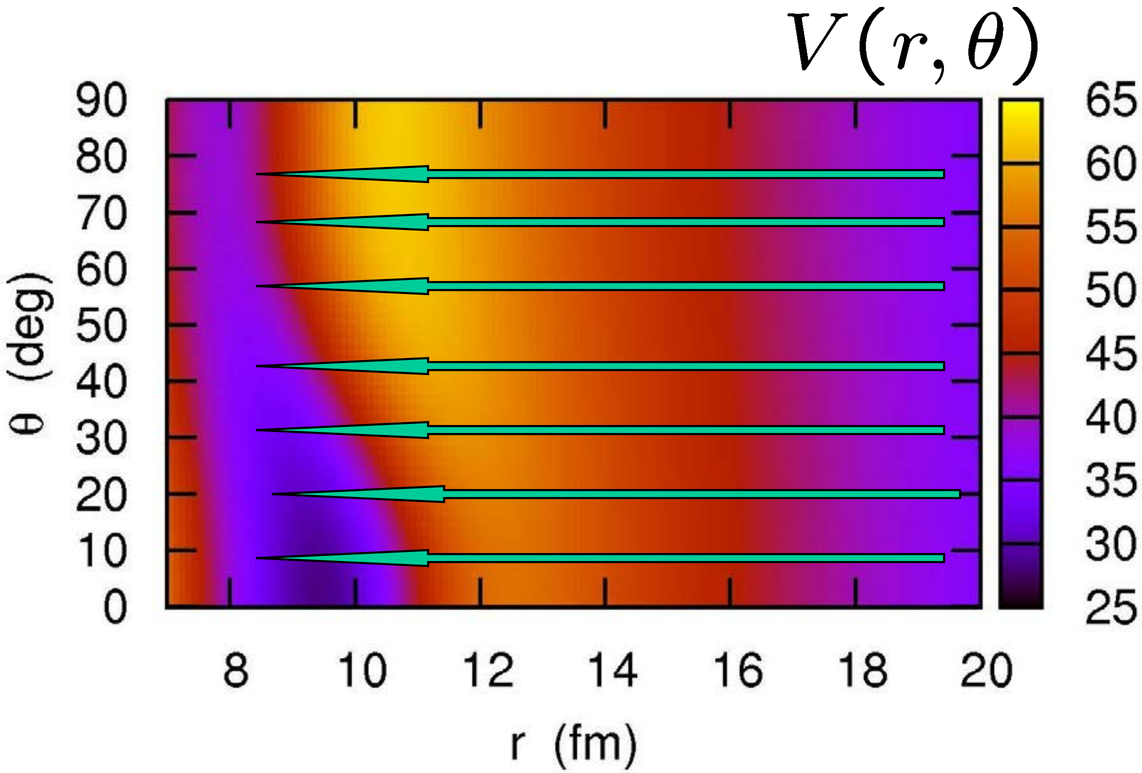
$$\boxed{\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\text{fus}}(E;\theta)}$$

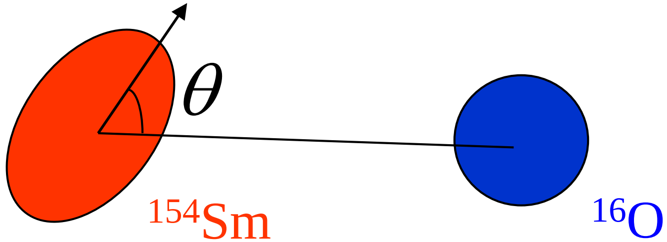
# Effect of collective excitation on $\sigma_{fus}$: rotational case

The orientation angle of $^{154}$Sm does not change much during fusion

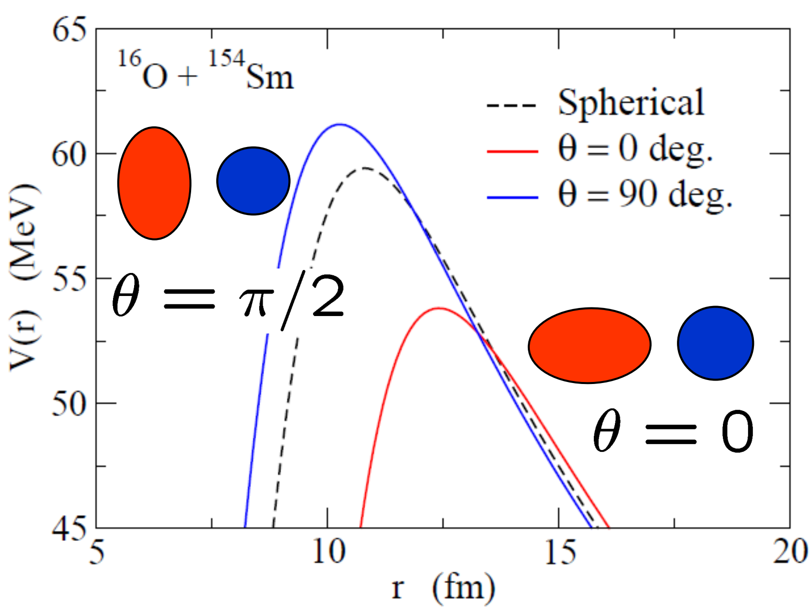


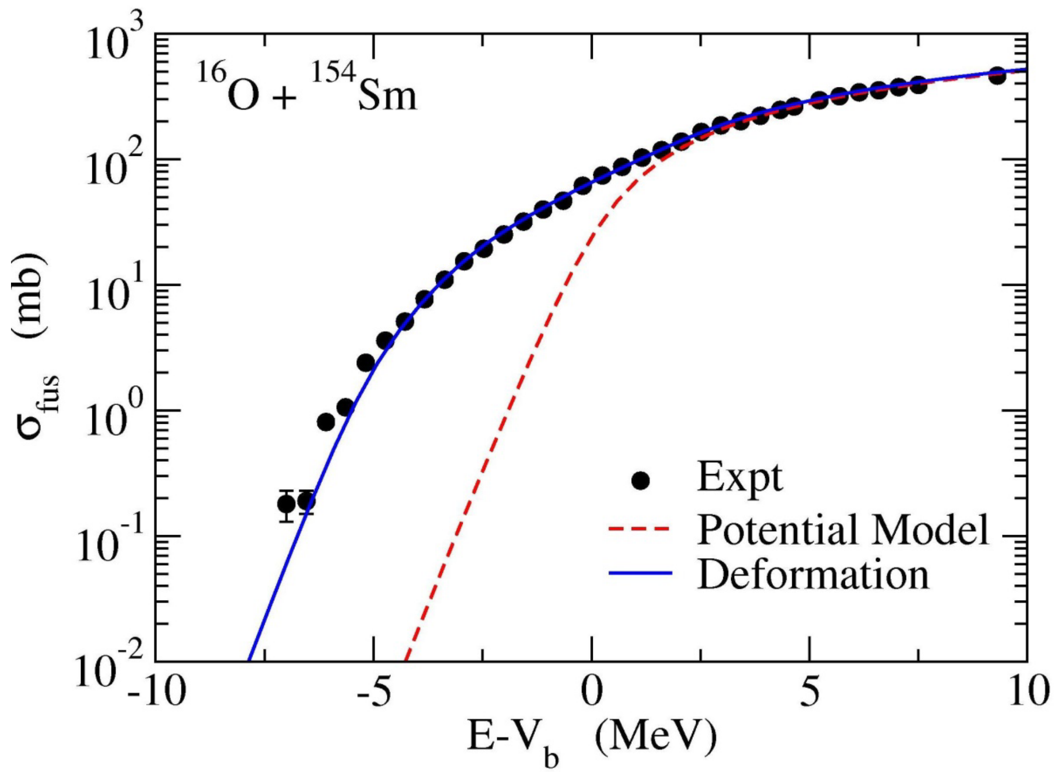

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$






$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\rm fus}(E;\theta)$$




The barrier is lowered for θ=0
because an attraction works from large distances.

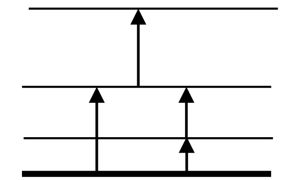
The barrier increases for θ=π/2.
because the rel. distance has
to get small for the attraction to work

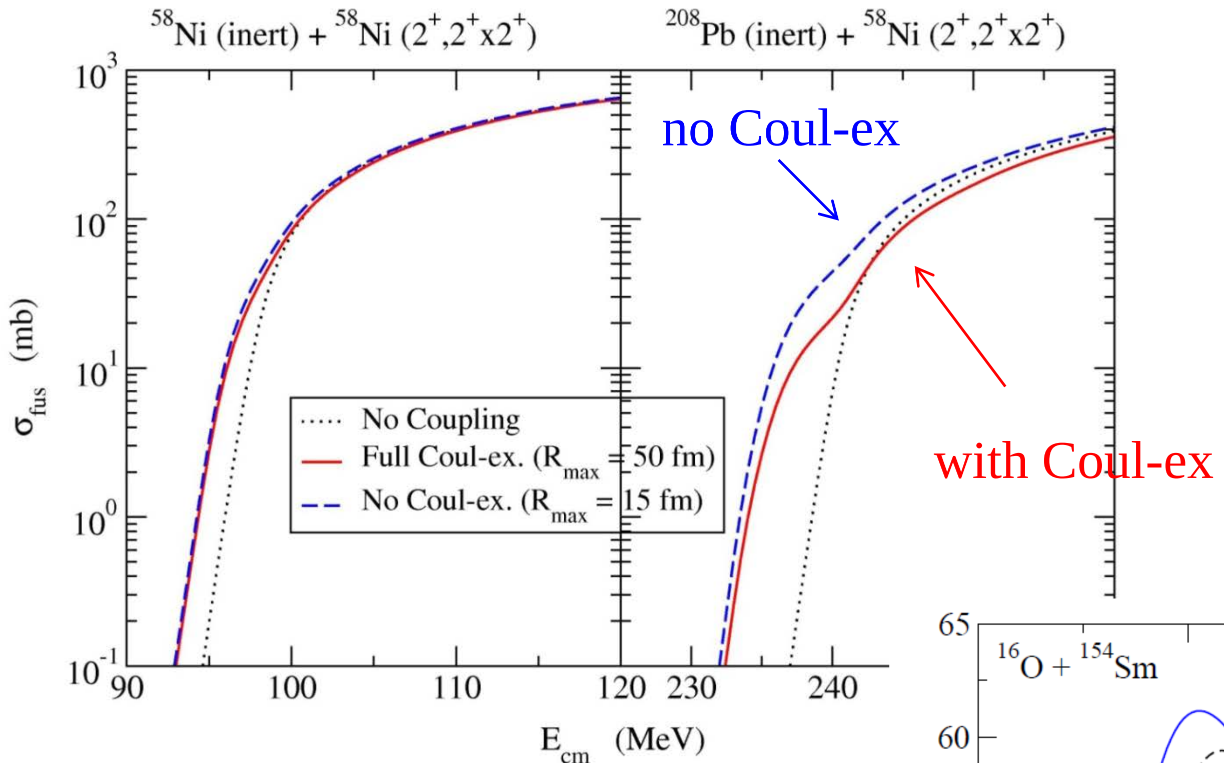
Def. Effect: enhances $\sigma_{\rm fus}$ by a factor of 10 ~ 100

Fusion: interesting probe for nuclear structure

# Two effects of channel couplings

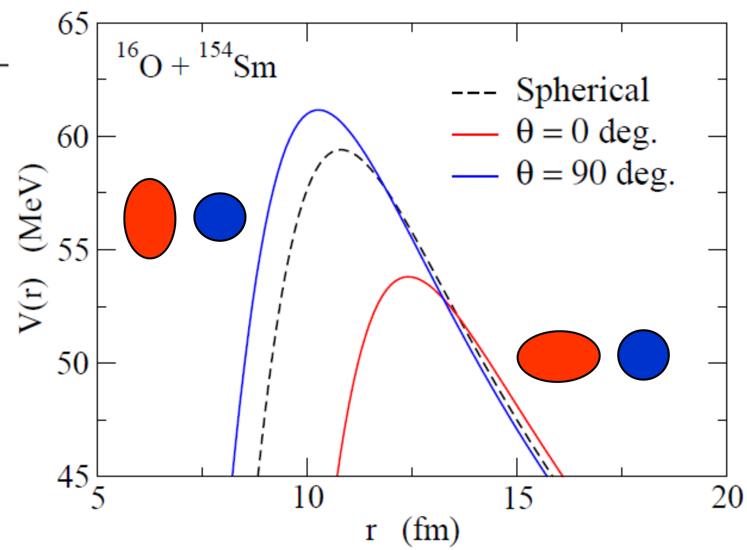
✓energy loss due to inelastic excitations





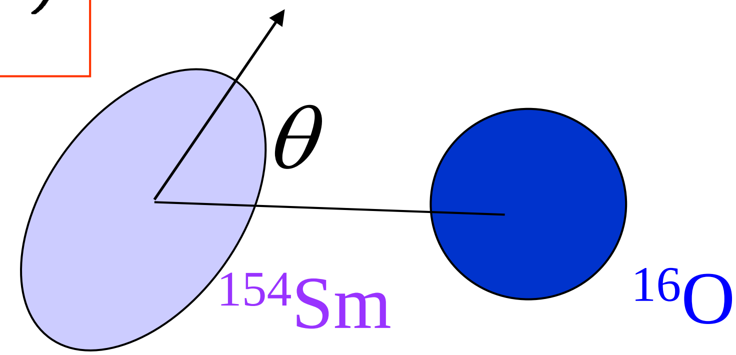

✓dynamical modification of the Coulomb barrier

→ large enhancement of fusion cross sections



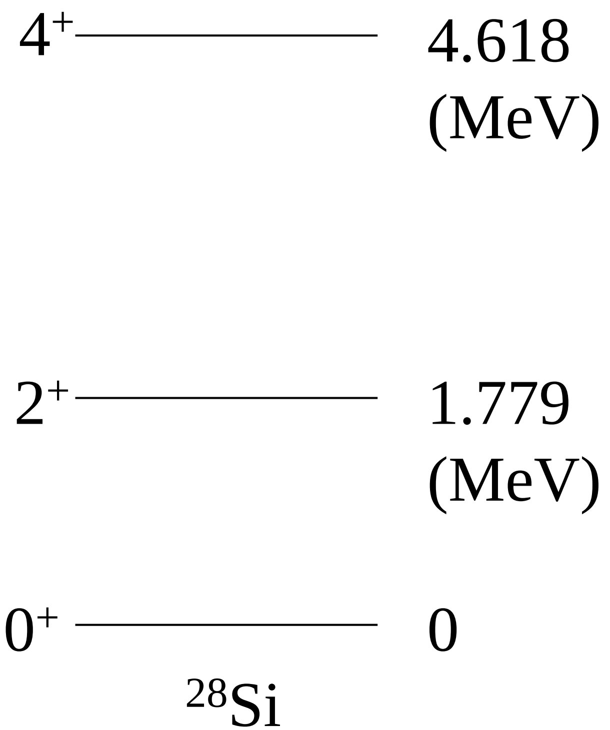

cf. 2-level model: Dasso, Landowne, and Winther, NPA405('83)381

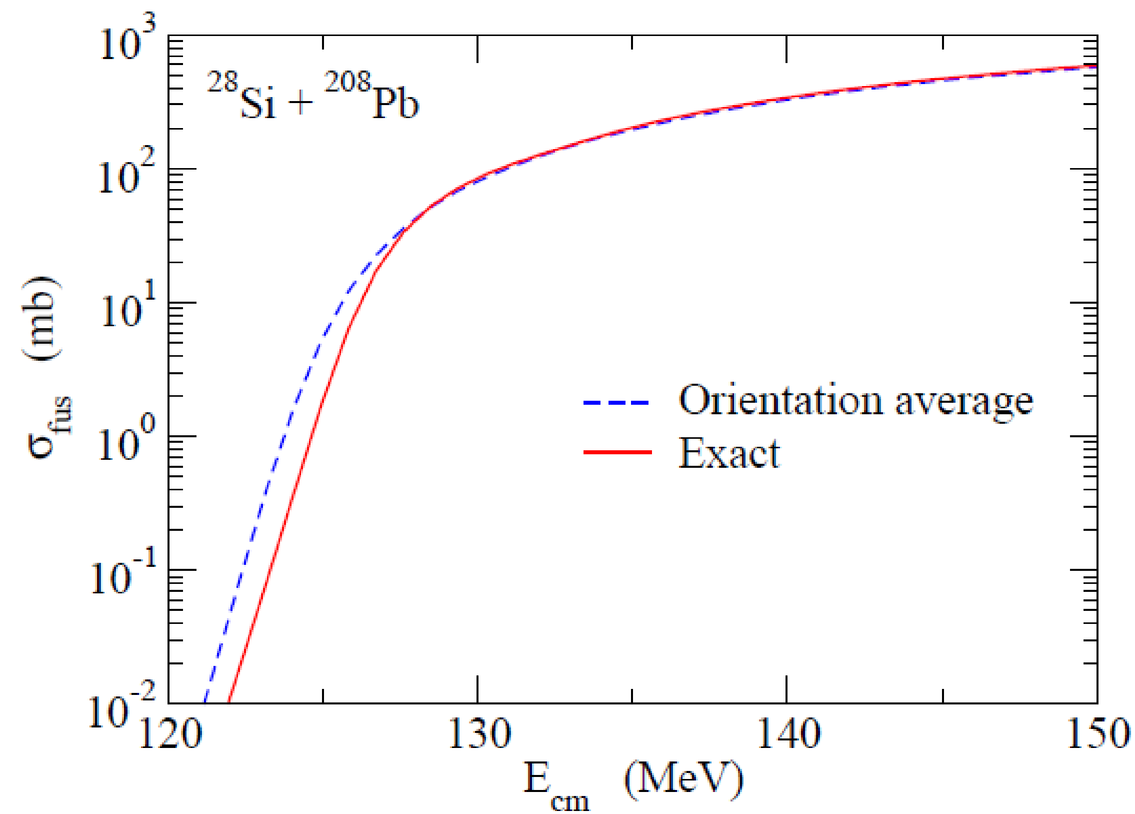
$$\boxed{\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\rm fus}(E;\theta)}$$




One warning:

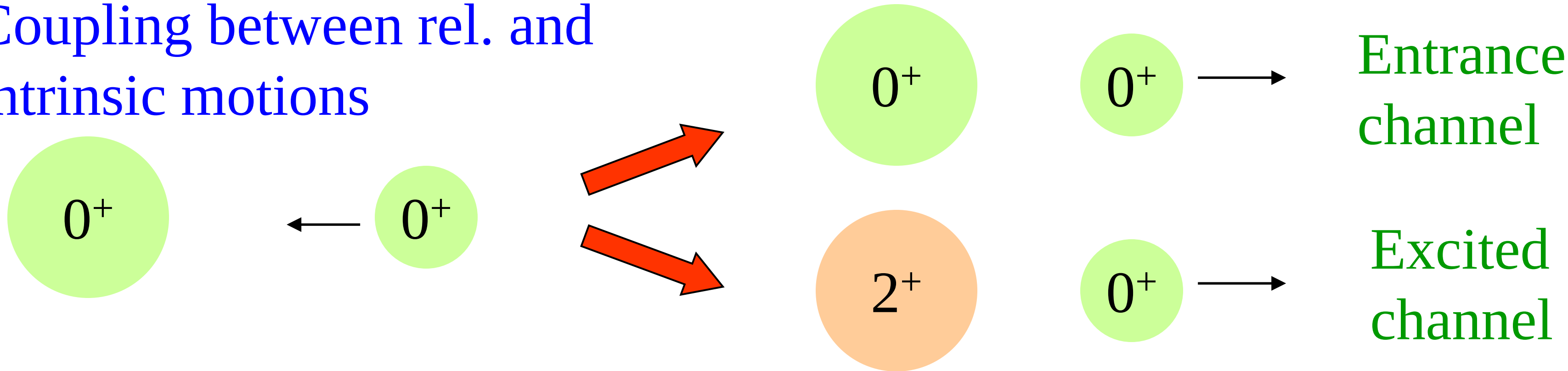
Don't use this formula for light deformed nuclei, e.g., $^{28}$Si



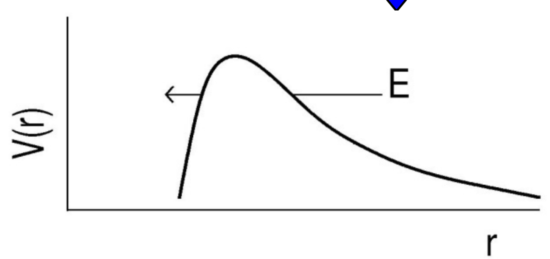



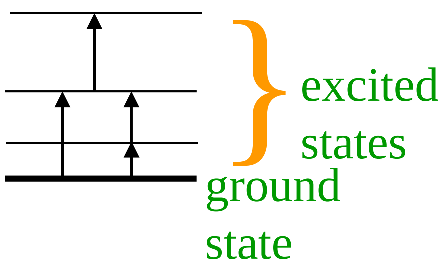
# More quantal treatment: Coupled-Channels method

Coupling between rel. and intrinsic motions




$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\mathrm{coup}}(\boldsymbol{r},\xi)$$




$$H_0(\xi)\phi_k(\xi) = \epsilon_k\,\phi_k(\xi)$$

$$\boxed{\Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi)}$$

$$\begin{cases} H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi) \\ \Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi) \end{cases}$$

$$H_0(\xi)\phi_k(\xi) = \epsilon_k\,\phi_k(\xi)$$

Schroedinger equation: $(H-E)\Psi(\boldsymbol{r},\xi) = 0$

$\langle\phi_k|$

$$\langle\phi_k|H-E|\Psi\rangle = 0$$

or

$$\boxed{\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \epsilon_k - E\right]\psi_k(\boldsymbol{r}) + \sum_{k'}\langle\phi_k|V_{\rm coup}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r}) = 0}$$

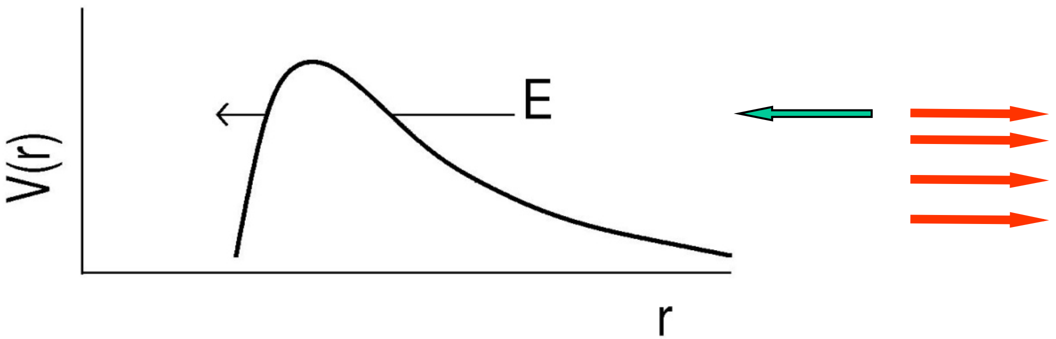
Coupled-channels equations

# Coupled-channels equations

$$\underbrace{\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \epsilon_k - E\right]\psi_k(\boldsymbol{r})}_{\text{equation for } \psi_k} + \sum_{k'}\underbrace{\langle\phi_k|V_{\mathrm{coup}}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r})}_{\text{transition from } \phi_\kappa \text{ to } \phi_{k'}} = 0$$

equation for $\psi_k$ transition from $\phi_\kappa$ to $\phi_{k'}$

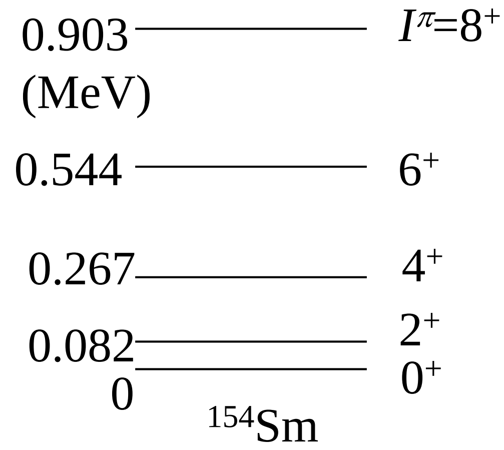
## boundary condition:




$$\boxed{\begin{array}{lll} \psi_n(r) \;\to & e^{-ik_0r} - S_0\,e^{ik_0r} & (n=0) \\ & -S_n\,e^{ik_nr} & (n\neq 0) \end{array}}$$

$$P(E) = 1 - \sum_n |S_n|^2 \qquad k_n = \sqrt{2\mu(E-\epsilon_n)/\hbar^2}$$

## Angular momentum coupling




$$H_0(\xi)\phi_{nIm_I}(\xi) = \epsilon_{nI}\,\phi_{nIm_I}(\xi)$$

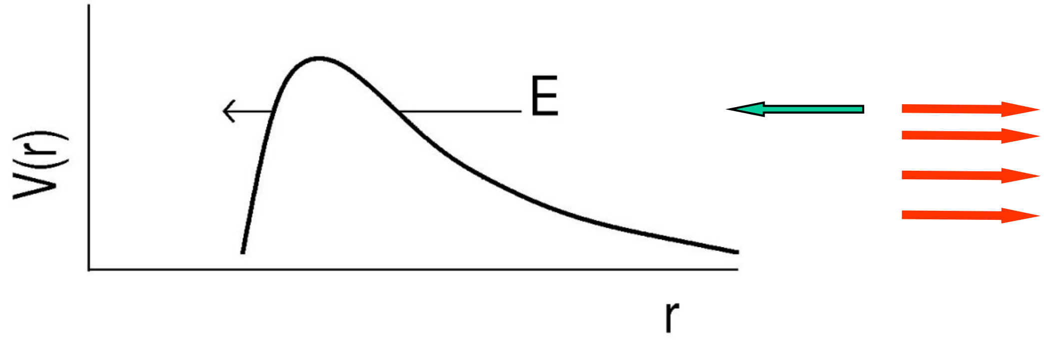
Total ang. mom.: $\boldsymbol{I} + \boldsymbol{l} = \boldsymbol{J}$

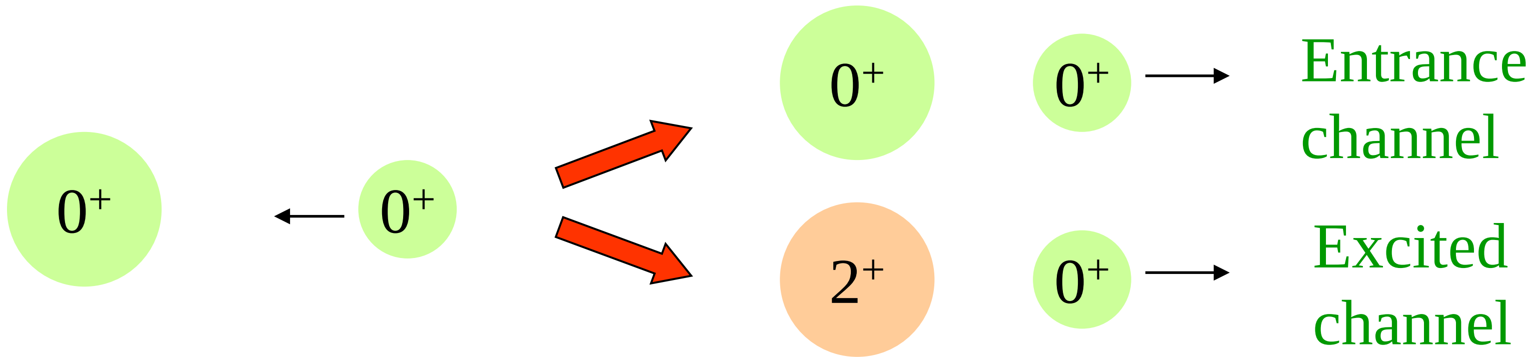
$$\Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi) = \sum_{n,l,I} \frac{u_{nlI}(r)}{r}[Y_l(\hat{\boldsymbol{r}})\phi_{nI}(\xi)]^{(JM)}$$

$$\langle [Y_l\phi_{nI}]^{(JM)}|H-E|\Psi\rangle = 0$$

$$\boxed{\begin{aligned}&\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_0(r) - E + \epsilon_{nI}\right]u_{nlI}(r)\\ &\quad + \sum_{n'l'I'}\langle [Y_l\phi_{nI}]^{(JM)}|V_{\rm coup}(r)|[Y_{l'}\phi_{n'I'}]^{(JM)}\rangle\, u_{n'l'I'}(r) = 0\end{aligned}}$$

## Boundary condition (with ang. mom. coupling)



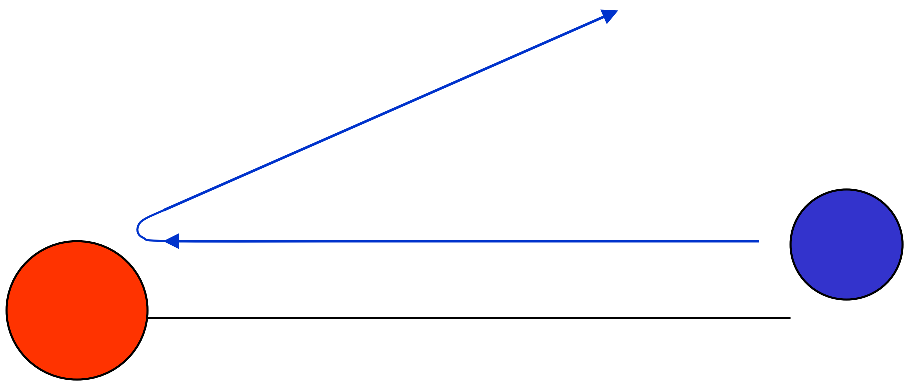




$$\Psi(\boldsymbol{r},\xi) = \sum_{n,l,I} \frac{u_{nlI}(r)}{r}[Y_l(\hat{\boldsymbol{r}})\phi_{nI}(\xi)]^{(JM)}$$

$$\boxed{u_{nlI}(r) \to H_l^{(-)}(k_{nI}r)\delta_{n,n_i}\delta_{l,l_i}\delta_{I,I_i} - \sqrt{\frac{k_0}{k_{nI}}}\, S_{nlI}\, H_l^{(+)}(k_{nI}r)}$$

$$P_l(E) = 1 - \sum_{nI} |S_{nlI}|^2 \qquad \sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

# Excitation structure of atomic nuclei



Excite the target nucleus via collision with the projectile nucleus

How does the targ. respond to the interaction with the proj.?

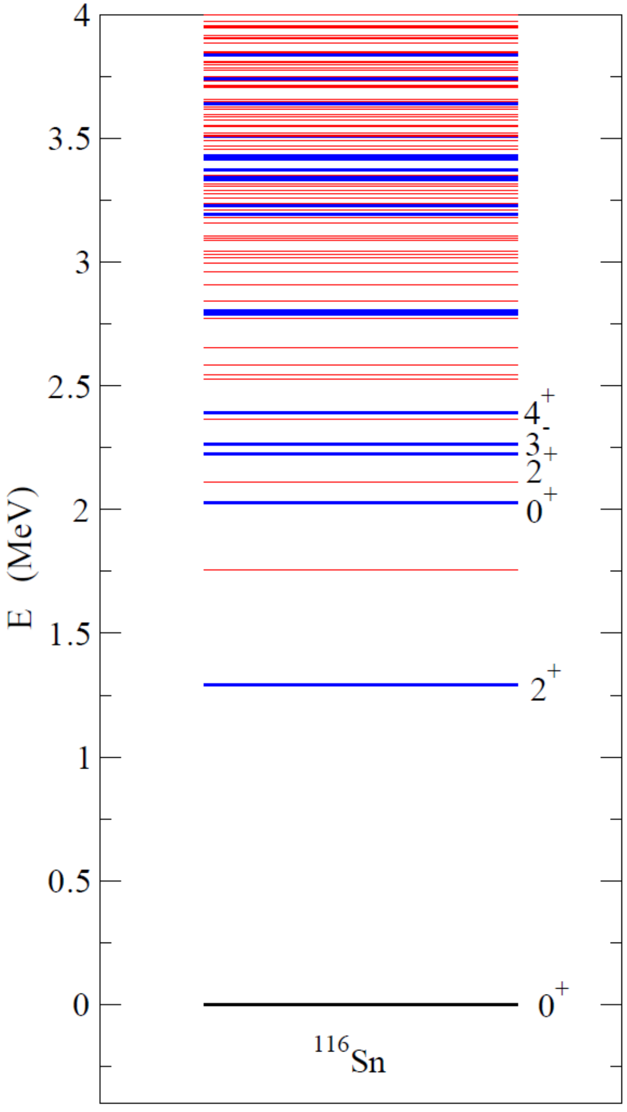
Standard approach: analysis with the coupled-channels method

- Inelastic cross sections
- Elastic cross sections
- Fusion cross sections

S-matrix $S_{nlI}$

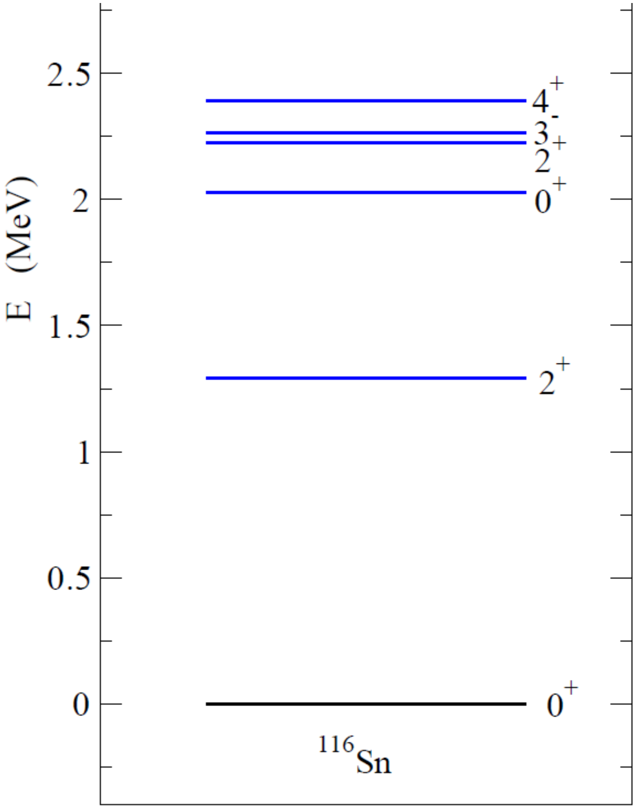
# How to perform coupled-channels calculations?

1. Modeling: selection of excited states to be included



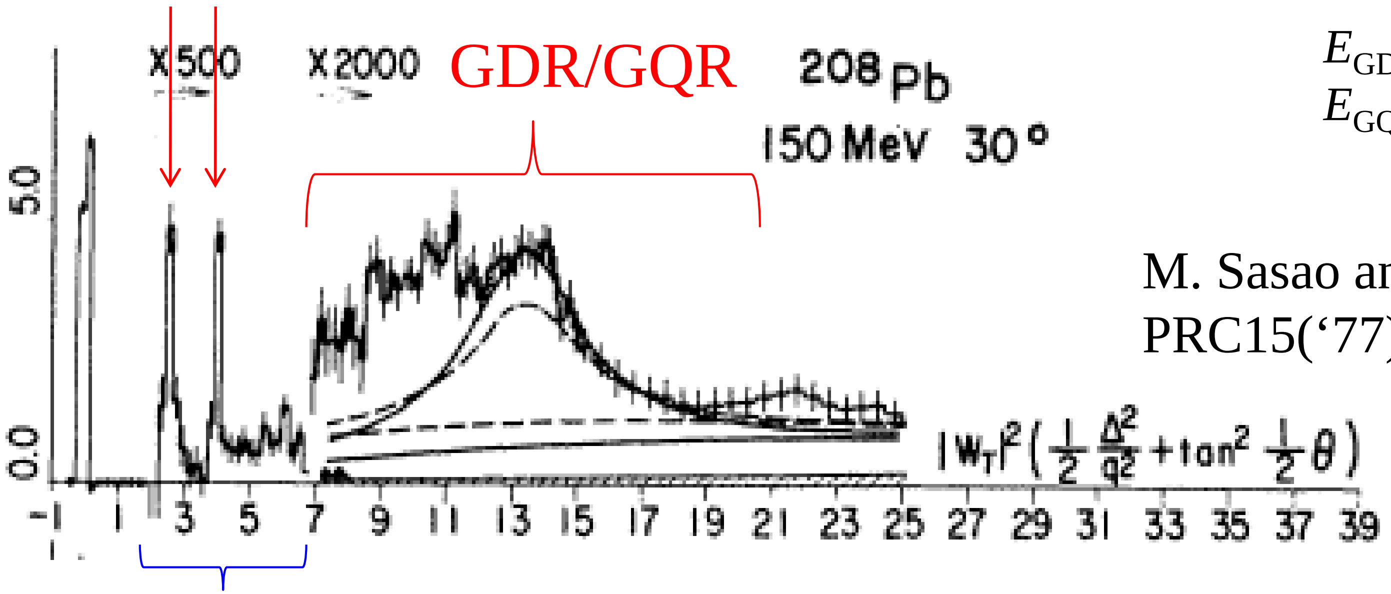

low-lying collective states only

S. Raman et al., PRC43('91)521



# typical excitation spectrum: electron scattering data

low-lying collective excitations




$E_x$ (MeV)

$E_{GDR} \sim 79A^{-1/3}$ MeV
$E_{GQR} \sim 65A^{-1/3}$ MeV

M. Sasao and Y. Torizuka, PRC15('77)217

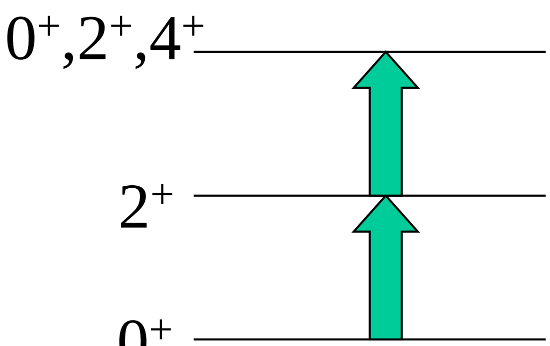
low-lying non-collective excitations

- Giant Resonances: high $E_x$, smooth mass number dependence
  $\longrightarrow$ adiabatic potential renormalization
- Low-lying collective excitations: barrier distributions, strong isotope dependence
- Non-collective excitations: either neglected completely or implicitly treated through an absorptive potential

## 2. Nature of collective states: vibration? or rotation?

### a) Vibrational coupling

excitation operator: $\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a + a^\dagger)$




$\frac{(a^\dagger)^2}{\sqrt{2}}|0\rangle$

$a^\dagger|0\rangle$

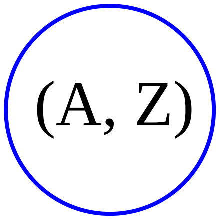
$|0\rangle$

$$|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$$
$$\epsilon_n = (n + 1/2)\hbar\omega$$

$$\langle n|O|n'\rangle = \frac{\beta}{\sqrt{4\pi}}\left(\sqrt{n'}\,\delta_{n,n'-1} + \sqrt{n'+1}\,\delta_{n,n'+1}\right)$$
$$= \begin{pmatrix} 0 & F & 0 \\ F & \epsilon & \sqrt{2}F \\ 0 & \sqrt{2}F & 2\epsilon \end{pmatrix}$$

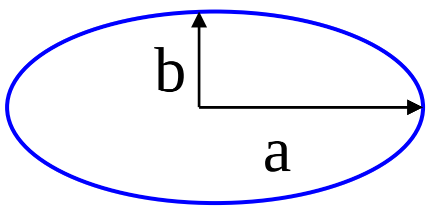
# Vibrational excitations

## Bethe-Weizacker formula: Mass formula based on Liquid-Drop Model

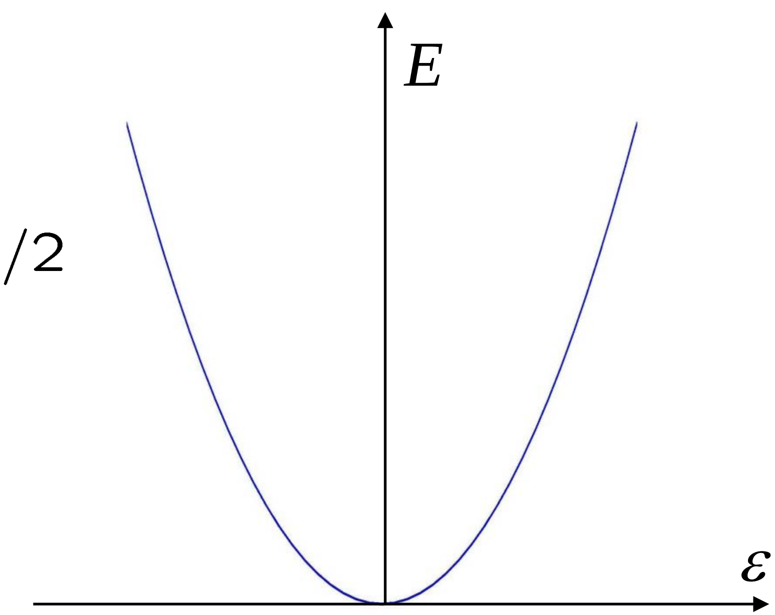



$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

For a deformed shape,




$$\begin{aligned} a &= R \cdot (1+\epsilon) \\ b &= R \cdot (1+\epsilon)^{-1/2} \end{aligned}$$

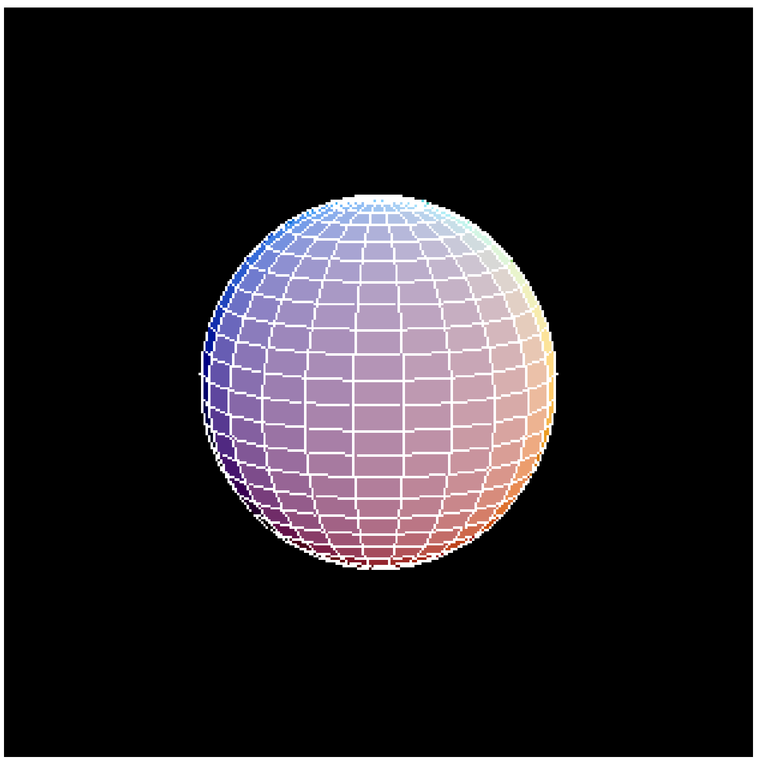


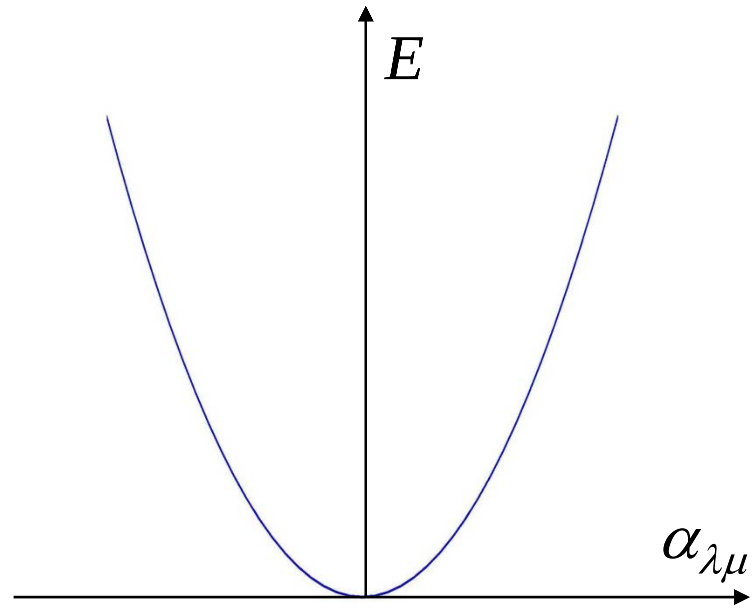

$$\begin{aligned} E_{\mathrm{surf}} &= E_{\mathrm{surf}}^{(0)} (1 + 2\epsilon^2/5 + \cdots) \\ E_C &= E_C^{(0)} (1 - \epsilon^2/5 + \cdots) \end{aligned}$$

In general $R(\theta,\phi) = R_0\left(1 + \sum_{\lambda,\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}\right)$

$$V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda\,|\alpha_{\lambda\mu}|^2$$

Harmonic oscillation





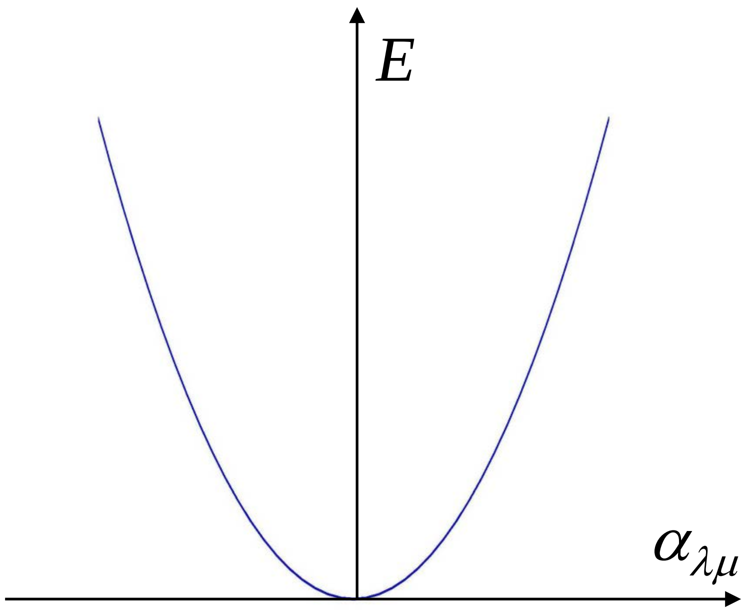

λ=2: Quadrupole vibration

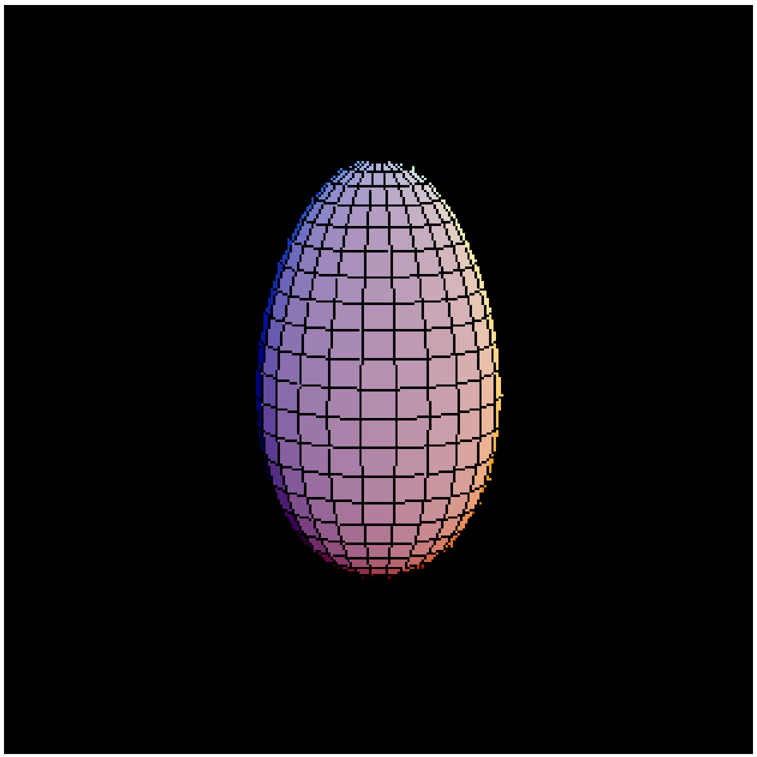
Movie: Dr. K. Arita (Nagoya Tech. U.)
http://www.phys.nitech.ac.jp/~arita/

In general $R(\theta,\phi) = R_0\left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right)$

$$V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$

Harmonic oscillation



λ=3: Octupole vibration

Movie: Dr. K. Arita (Nagoya Tech. U.)
http://www.phys.nitech.ac.jp/~arita/

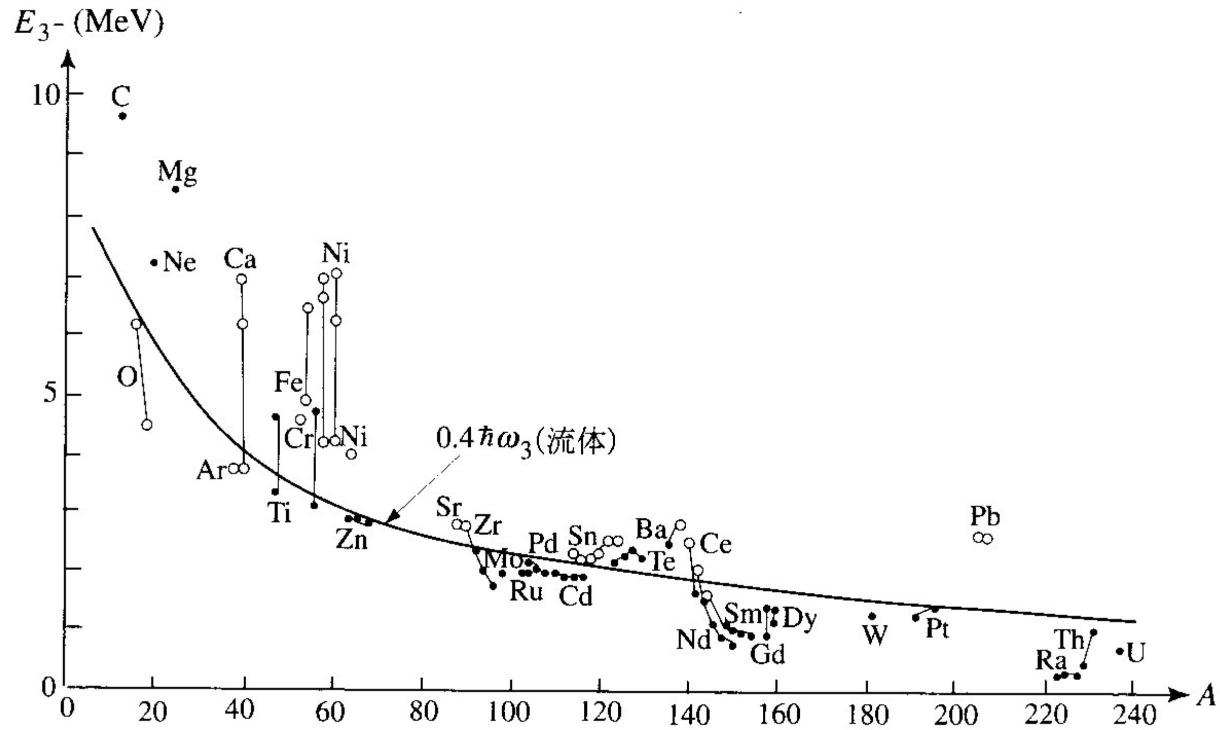



図 3.3　偶々核の第 1 励起 $3^-$ 状態の励起エネルギー

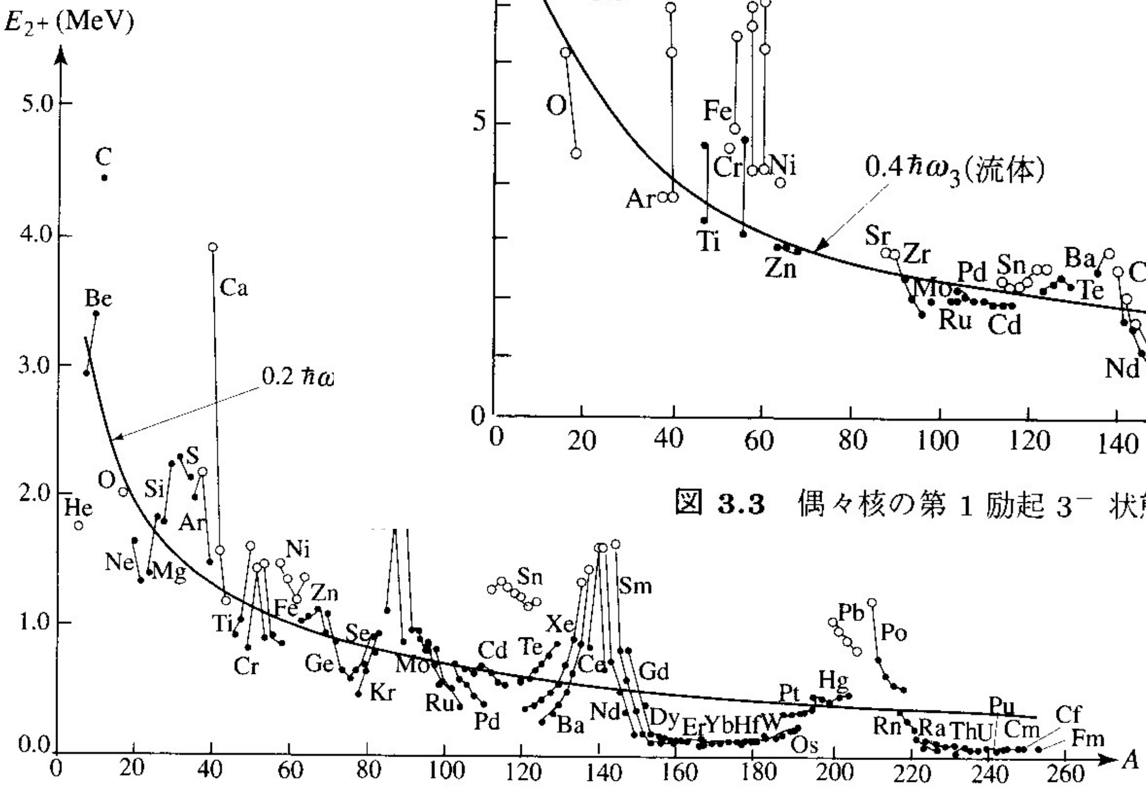
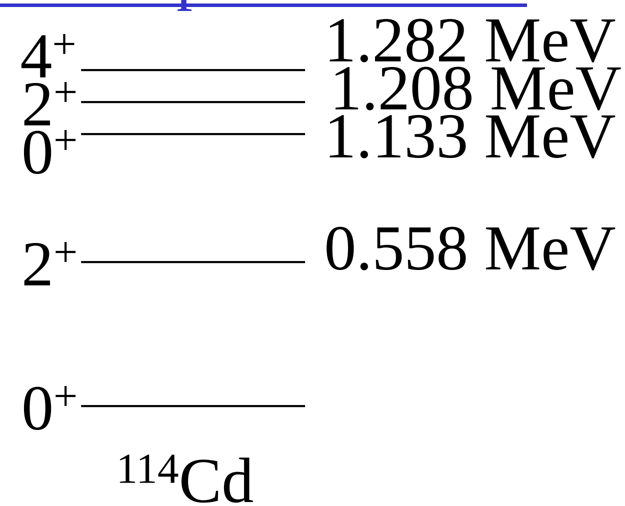



図 3.2　偶々核の第 1 励起 $2^+$ 状態の励起エネルギー

Double phonon states



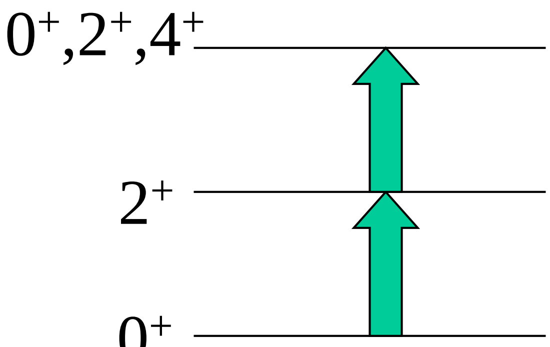

Microscopic description

⇨ Random phase approximation (RPA)

## 2. Nature of collective states: vibration? or rotation?

### a) Vibrational coupling

excitation operator: $\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a + a^\dagger)$




$$\frac{(a^\dagger)^2}{\sqrt{2}}|0\rangle$$

$$a^\dagger|0\rangle$$

$$|0\rangle$$

$$|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$$

$$\epsilon_n = (n + 1/2)\hbar\omega$$

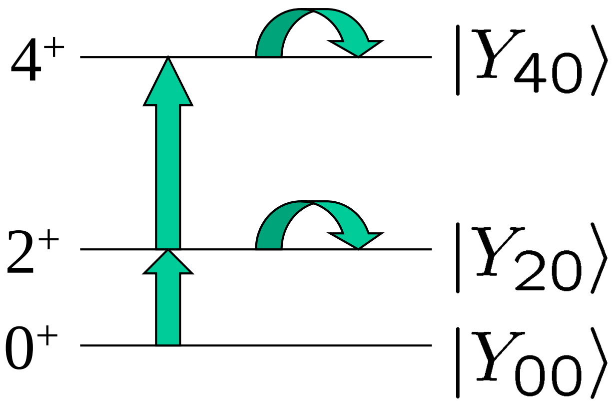
$$\langle n|O|n'\rangle = \frac{\beta}{\sqrt{4\pi}}\left(\sqrt{n'}\,\delta_{n,n'-1} + \sqrt{n'+1}\,\delta_{n,n'+1}\right)$$

$$= \begin{pmatrix} 0 & F & 0 \\ F & \epsilon & \sqrt{2}F \\ 0 & \sqrt{2}F & 2\epsilon \end{pmatrix}$$

2. Nature of collective states: vibration? or rotation?

## b) Rotational coupling

excitation operator: $\hat{O} = \beta Y_{20}(\theta)(+\beta_4 Y_{40}(\theta) + \cdots)$

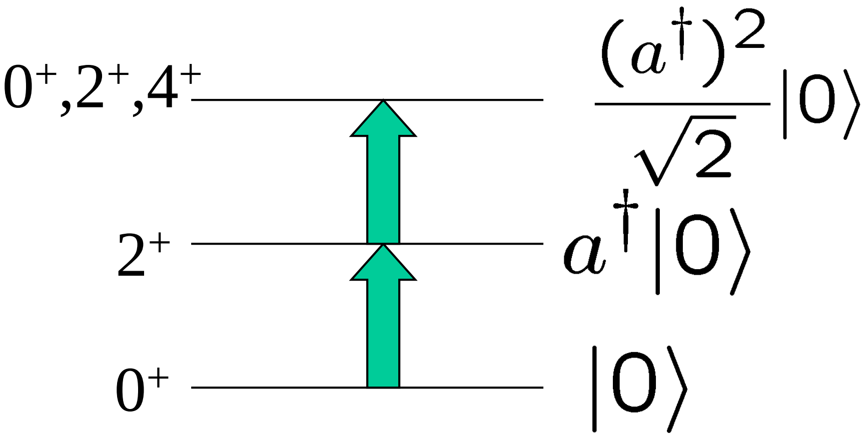



$$
\begin{aligned}
|I\rangle &= |Y_{I0}\rangle \\
\epsilon_I &= \frac{I(I+1)}{6} \cdot E_{I=2}
\end{aligned}
$$

$$
\begin{aligned}
\langle I|O|I'\rangle &= \sqrt{\frac{5\cdot(2I+1)(2I'+1)}{4\pi}} \begin{pmatrix} I & 2 & I' \\ 0 & 0 & 0 \end{pmatrix}^2 \\
&= \begin{pmatrix} 0 & F & 0 \\ F & \epsilon + \frac{2\sqrt{5}}{7}F & \frac{6}{7}F \\ 0 & \frac{6}{7}F & \frac{10\epsilon}{3} + \frac{20\sqrt{5}}{77}F \end{pmatrix}
\end{aligned}
$$

## Vibrational coupling

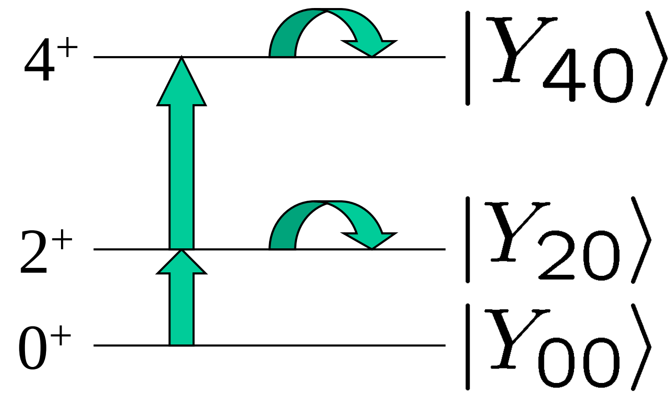
$$\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a + a^\dagger)$$




$$\begin{pmatrix} 0 & F & 0 \\ F & \epsilon & \sqrt{2}F \\ 0 & \sqrt{2}F & 2\epsilon \end{pmatrix}$$

## Rotational coupling

$$\hat{O} = \beta Y_{20}(\theta)$$




$$\begin{pmatrix} 0 & F & 0 \\ F & \epsilon + \frac{2\sqrt{5}}{7}F & \frac{6}{7}F \\ 0 & \frac{6}{7}F & \frac{10\epsilon}{3} + \frac{20\sqrt{5}}{77}F \end{pmatrix}$$

$$F = \frac{\beta}{\sqrt{4\pi}}$$

cf. reorientation term

# 3. Coupling constants and coupling potentials

## Deformed Woods-Saxon model:

$$V_{WS}(r) = -\frac{V_0}{1+\exp[(r-R_0)/a]}$$

$$= -\frac{V_0}{1+\exp[(r-R_P-R_T)/a]}$$

$$R_T \to R_T\left(1+\sum_\mu \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)\right)$$

excitation operator

$$\boxed{V_{WS}(\boldsymbol{r}) = -\frac{V_0}{1+\exp[(r-R_0-R_T\,\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}}))/a]}}$$

# Coupling Potential: Collective Model

$$R(\theta,\phi) = R_T \left(1 + \sum_\mu \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)\right)$$

- Vibrational case

$$\alpha_{\lambda\mu} = \frac{\beta_\lambda}{\sqrt{2\lambda+1}} (a^\dagger_{\lambda\mu} + (-)^\mu a_{\lambda\mu})$$

- Rotational case

Coordinate transformation to the body-fixed rame

$$\alpha_{\lambda\mu} = \sqrt{\frac{4\pi}{2\lambda+1}} \beta_\lambda Y_{\lambda\mu}(\theta_d,\phi_d) \qquad \text{(for axial symmetry)}$$

In both cases $\quad \beta_\lambda = \dfrac{4\pi}{3Z_T R_T^\lambda}\sqrt{\dfrac{B(E\lambda)\uparrow}{e^2}}$

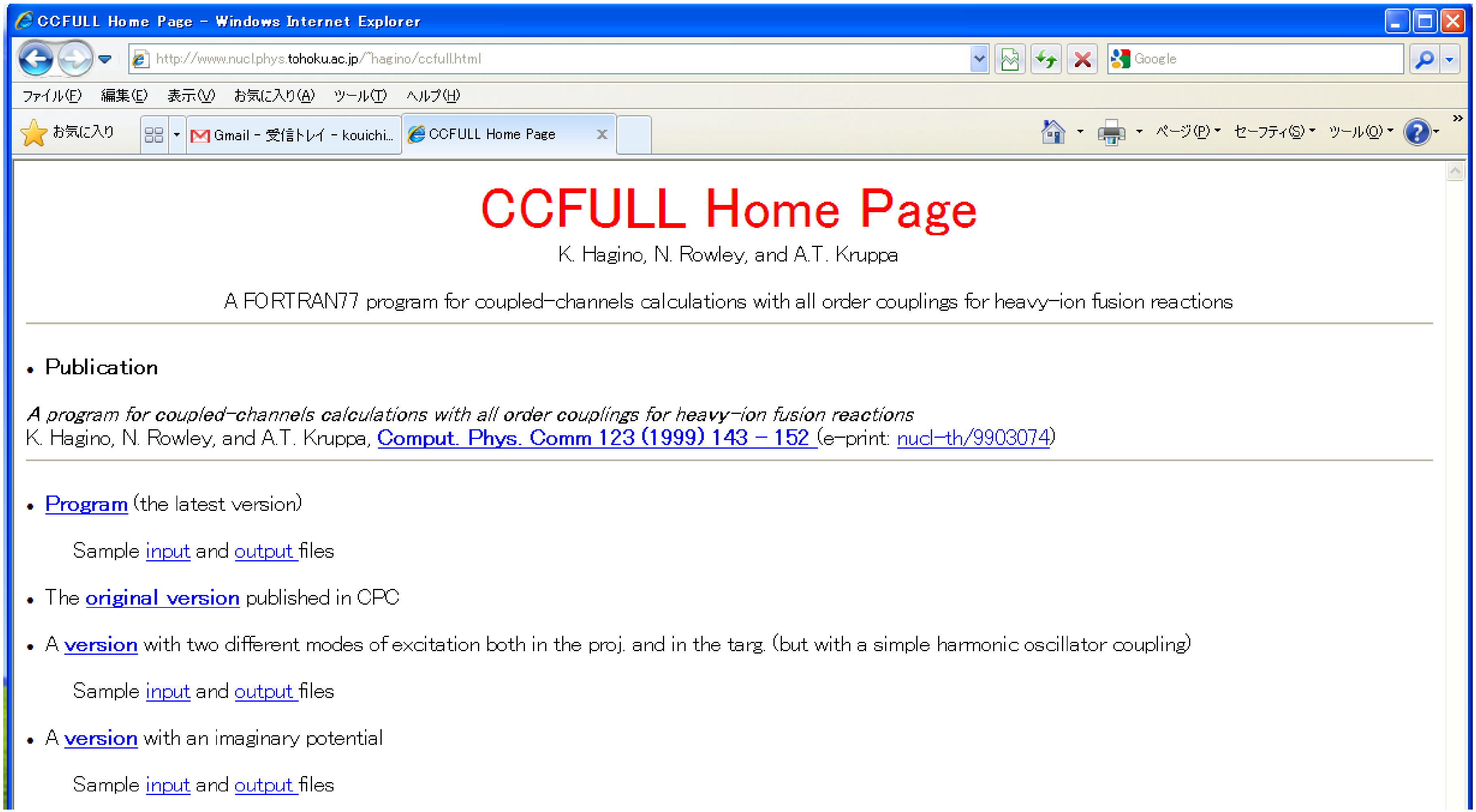
(note) coordinate transformation to the rotating frame ( $\hat{\boldsymbol{r}} = 0$ )

$$\sum_\mu \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi) \to \sqrt{\frac{2\lambda+1}{4\pi}}\alpha_{\lambda 0}$$

# Deformed Woods-Saxon model (collective model)

## CCFULL

K.H., N. Rowley, and A.T. Kruppa,
Comp. Phys. Comm. 123('99)143




# CCFULL Home Page

K. Hagino, N. Rowley, and A.T. Kruppa

A FORTRAN77 program for coupled-channels calculations with all order couplings for heavy-ion fusion reactions

- Publication

*A program for coupled-channels calculations with all order couplings for heavy-ion fusion reactions*
K. Hagino, N. Rowley, and A.T. Kruppa, Comput. Phys. Comm 123 (1999) 143 - 152 (e-print: nucl-th/9903074)

- Program (the latest version)

  Sample input and output files

- The original version published in CPC
- A version with two different modes of excitation both in the proj. and in the targ. (but with a simple harmonic oscillator coupling)

  Sample input and output files

- A version with an imaginary potential

  Sample input and output files

http://www.nucl.phys.tohoku.ac.jp/~hagino/ccfull.html

# CCFULL

K.H., N. Rowley, and A.T. Kruppa,
Comp. Phys. Comm. 123('99)143

## i) all order couplings

$$V_{\rm coup}(r,\hat{O}) = V^{(N)}_{\rm coup}(r,\hat{O}) + V^{(C)}_{\rm coup}(r,\hat{O})$$

## Nuclear coupling:

$$V^{(N)}_{\rm coup}(r,\hat{O}) = -\frac{V_0}{1+\exp[(r-R_0-R_T\hat{O})/a]}$$

## Coulomb coupling:

$$V^{(C)}_{\rm coup}(r,\hat{O}) = \frac{3}{2\lambda+1}\,Z_PZ_Te^2\,\frac{R_T^\lambda}{r^{\lambda+1}}\,\hat{O}$$

CCFULL

K.H., N. Rowley, and A.T. Kruppa, Comp. Phys. Comm. 123('99)143

i) all order couplings

$$V_{\rm coup}^{(N)}(r,\hat{O}) = -\frac{V_0}{1+\exp[(r-R_0-R_T\hat{O})/a]}$$

$$\sim V_N(r) - R_T\hat{O}\,\frac{dV_N(r)}{dr}$$

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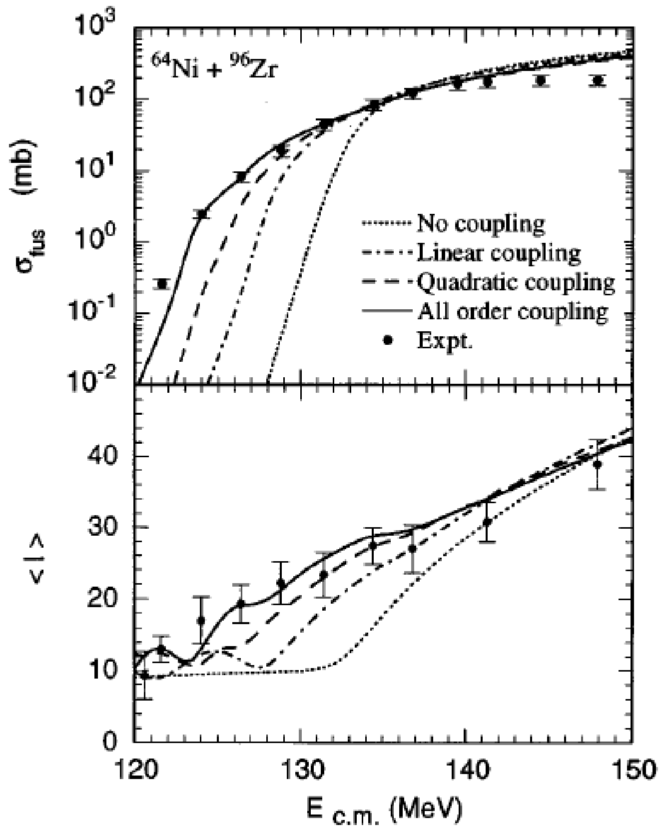
K.H., N. Rowley, and A.T. Kruppa,
Comp. Phys. Comm. 123('99)143

## i) all order couplings

$$V^{(N)}_{\rm coup}(r,\hat{O}) = -\frac{V_0}{1+\exp[(r-R_0-R_T\hat{O})/a]}$$

$$\sim \xcancel{V_N(r) - R_T\hat{O}\,\frac{dV_N(r)}{dr}}$$

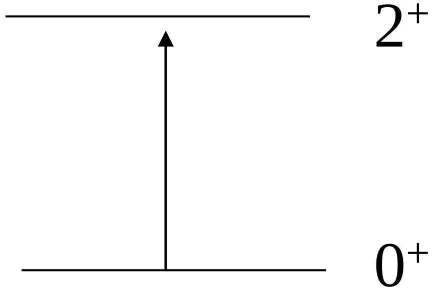



K.H., N. Takigawa, M. Dasgupta,
D.J. Hinde, and J.R. Leigh, PRC55('97)276

CCFULL

K.H., N. Rowley, and A.T. Kruppa, Comp. Phys. Comm. 123('99)143

## ii) isocentrifugal approximation

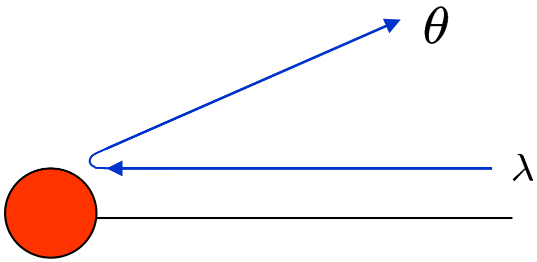


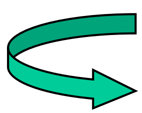

***l=J-2, J, J+2***

***l=J***

***J=l+I***

| Truncation | Dimension |
|---|---|
| $2^+$ | 4 $\longrightarrow$ 2 |
| $4^+$ | 9 $\longrightarrow$ 3 |
| $6^+$ | 16 $\longrightarrow$ 4 |
| $8^+$ | 25 $\longrightarrow$ 5 |



Iso-centrifugal approximation:

λ: independent of excitations

$$\boxed{\frac{l(l+1)\hbar^2}{2\mu r^2} \rightarrow \frac{J(J+1)\hbar^2}{2\mu r^2}}$$

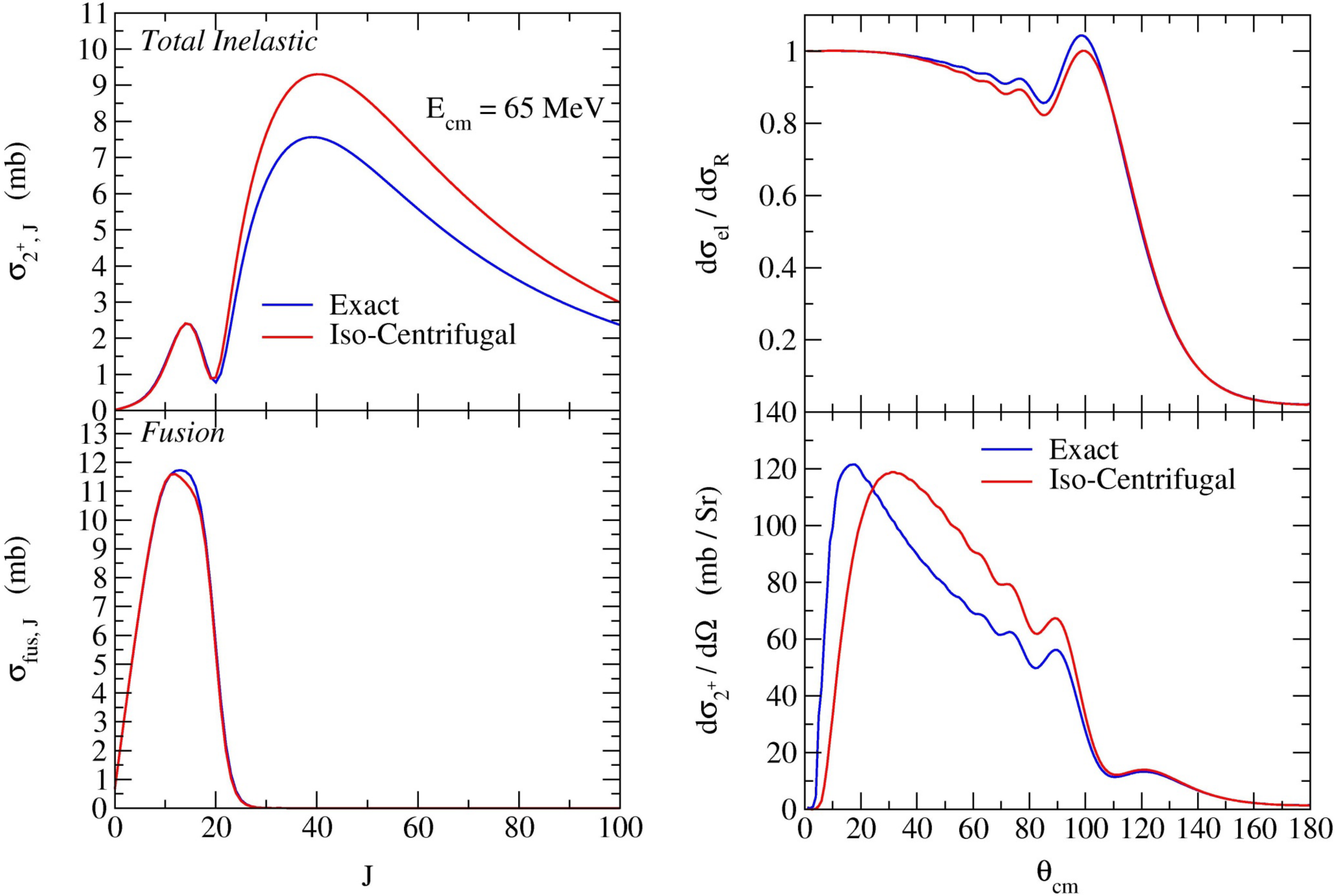
$$V_{\rm coup}(\boldsymbol{r},\xi) = f(r)Y_\lambda(\hat{\boldsymbol{r}})\cdot T_\lambda(\xi)$$

transform to the rotating frame

$$\rightarrow \sqrt{\frac{2\lambda+1}{4\pi}}f(r)T_{\lambda 0}(\xi)$$

"Spin-less system"

# $^{16}O + ^{144}Sm\ (2^+)$




K.H. and N. Rowley, PRC69('04)054610

CCFULL

K.H., N. Rowley, and A.T. Kruppa, Comp. Phys. Comm. 123('99)143

## iii) incoming wave boundary condition (IWBC)

$$\boxed{\sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)P_l \qquad (P_l = 1-|S_l|^2)}$$

### (1) Complex potential

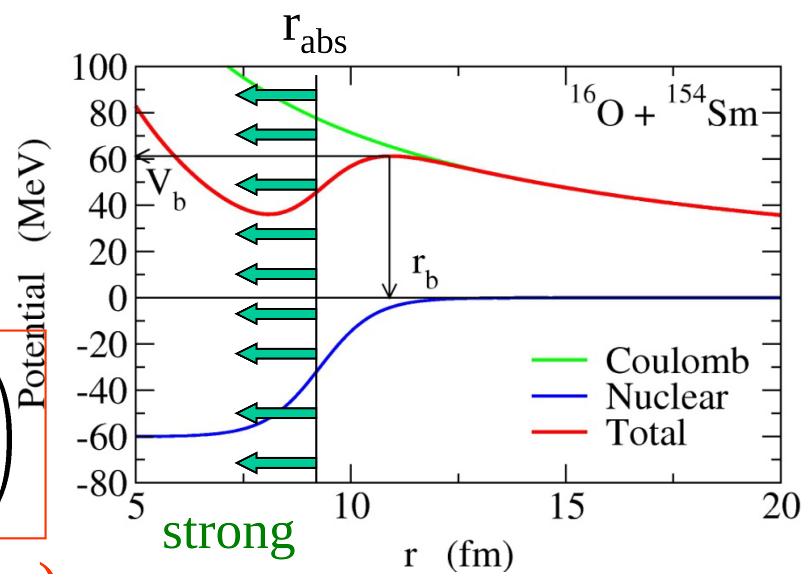
$$V(r) = V_R(r) - iW(r)$$

### (2) IWBC

*limit of large W (strong absorption)*

$$\boxed{u_l(r) = T_l \exp\left(-i\int_{r_{abs}}^{r} k_l(r')dr'\right)}$$

(Incoming Wave Boundary Condition)




strong absorption

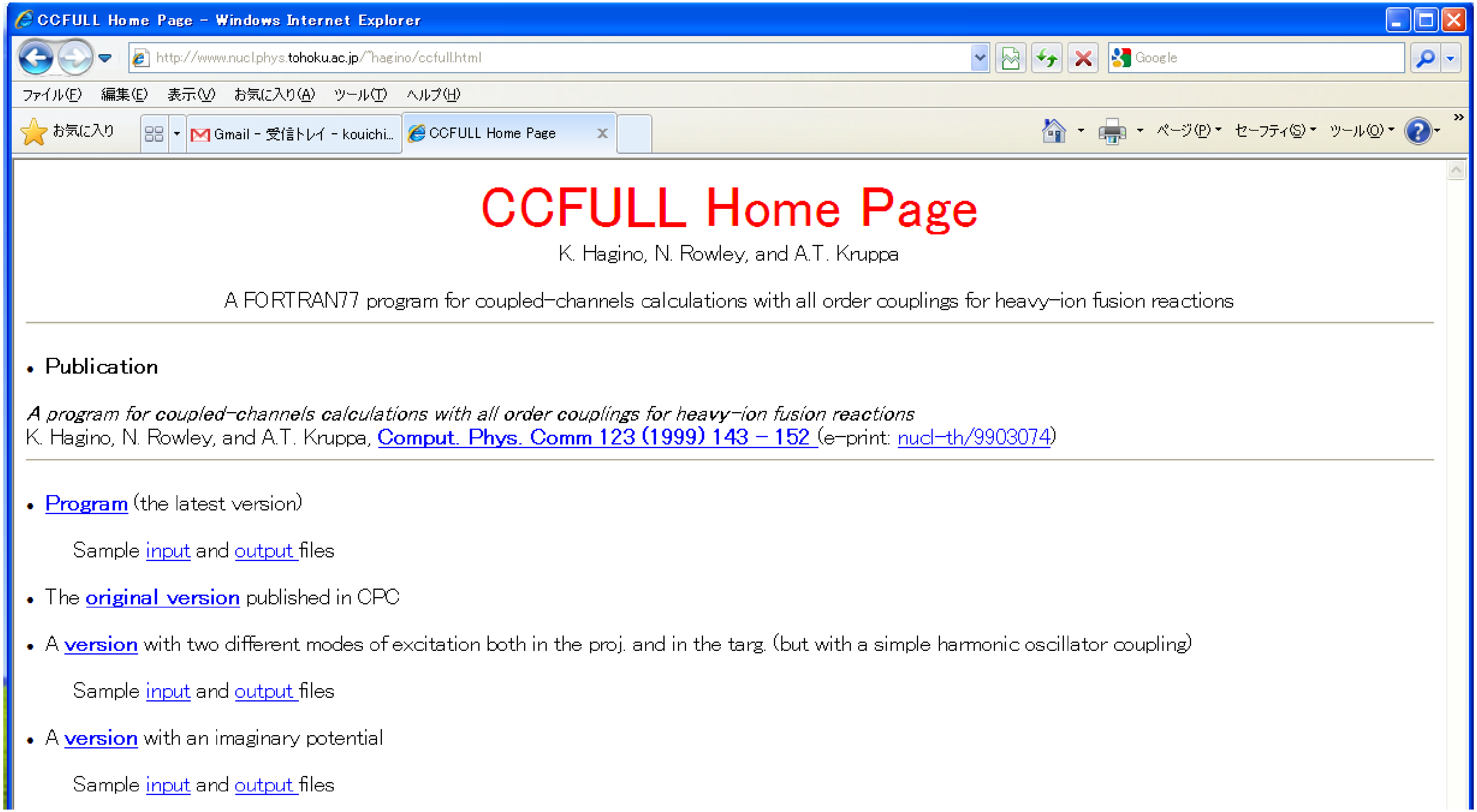
$$k_l(r) = \sqrt{2\mu/\hbar^2[E - V_R(r) - l(l+1)\hbar^2/2\mu r^2]}$$

- Only Real part of Potential
- More efficient at low energies $\quad P_l = |T_l|^2$

cf. $|S_l| \sim 1$ at low $E$

# CCFULL

K.H., N. Rowley, and A.T. Kruppa,
Comp. Phys. Comm. 123('99)143




http://www.nucl.phys.tohoku.ac.jp/~hagino/ccfull.html

ccfull.inp

```
16.,8.,144.,62.
1.2,-1,1.06,0
1.81,0.205,3,1
1.66,0.11,2,0
6.13,0.733,3,1
0,0.,0.3
105.1,1.1,0.75
55.,70.,1.
30,0.05
```

reaction system
($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62)

$r_p$, $Ivibrot_p$, $r_t$, $Ivibrot_t$
(inert projectile, and vib. for targ.)

ccfull.inp

```
16.,8.,144.,62.

1.2,-1,1.06,0

1.81,0.205,3,1

1.66,0.11,2,0

6.13,0.733,3,1

0,0.,0.3

105.1,1.1,0.75

55.,70.,1.

30,0.05
```

← reaction system ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62)

← $r_p$, Ivibrot$_p$, $r_t$, Ivibrot$_t$ (inert projectile, and vib. for targ.)

$$V^{(N)}_{\text{coup}}(r,\hat{O}) = -\frac{V_0}{1+\exp[(r-R_0-R_T\hat{O})/a]}$$

$$R_T = r_t A_t^{1/3} \quad \text{(fm)}$$

If Ivibrot$_t$ =0: O = $O_{vib}$
Ivibrot$_t$ =1: O = $O_{rot}$
Ivibrot$_t$ = -1: O = 0 (inert)

similar for the projectile

ccfull.inp

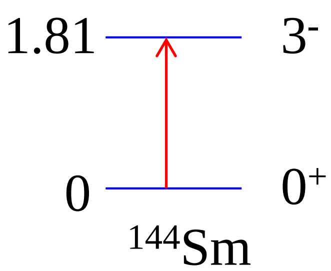
```
16.,8.,144.,62.
1.2,-1,1.06,0
1.81,0.205,3,1
1.66,0.11,2,0
6.13,0.733,3,1
0,0.,0.3
105.1,1.1,0.75
55.,70.,1.
30,0.05
```

($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62)

(inert projectile, and vib. for targ.)

properties of the targ. excitation

$E_{1st}$ = 1.81 MeV
$\beta$ = 0.205
$\lambda$ = 3
$N_{phonon}$ = 1




coupling to $3^-$ vibrational state in the target with def. parameter $\beta$ = 0.205

$$\alpha_{\lambda\mu} = \frac{\beta_\lambda}{\sqrt{2\lambda+1}}(a^\dagger_{\lambda\mu} + (-)^\mu a_{\lambda\mu})$$

$$\beta_\lambda = \frac{4\pi}{3Z_T R_T^\lambda}\sqrt{\frac{B(E\lambda)\uparrow}{e^2}}$$

ccfull.inp

```
16.,8.,144.,62.
1.2,-1,1.06,0
1.81,0.205,3,1
1.66,0.11,2,0
6.13,0.733,3,1
0,0.,0.3
105.1,1.1,0.75
55.,70.,1.
30,0.05
```

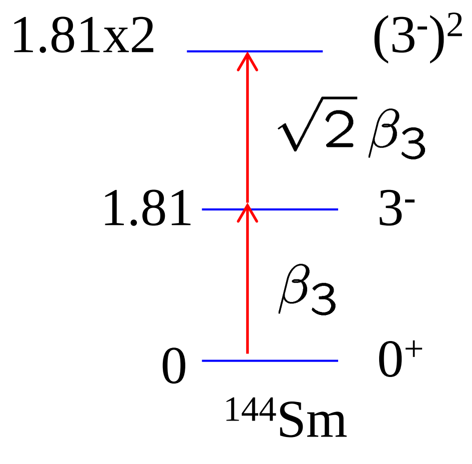
← ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62)

← (inert projectile, and vib. for targ.)

← properties of the targ. excitation

(*note*) if $N_{phonon}$ = 2: double phonon excitation

$E_{1st}$ = 1.81 MeV
$\beta$ = 0.205
$\lambda$ = 3
$N_{phonon}$ = 2



ccfull.inp

```
16.,8.,144.,62.
1.2,-1,1.06,0
1.81,0.205,3,1
1.66,0.11,2,0
6.13,0.733,3,1
0,0.,0.3
105.1,1.1,0.75
55.,70.,1.
30,0.05
```

16.,8.,144.,62. ← ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62)

1.2,-1,1.06,0 ← (inert projectile, and vib. for targ.)

1.81,0.205,3,1 ← properties of the targ. excitation

(*note*) if Ivibrott = 1 (rot. coup.)

the input line would look like:

0.08,0.306,0.05,3 instead of 1.81,0.205,3,1

$E_{2+}$ $\beta_2$ $\beta_4$ $N_{rot}$

3 excitated states ($N_{rot}$=3) + g.s.

| | |
|---|---|
| 6x7x0.08/6 | $6^+$ |
| 4x5x0.08/6 | $4^+$ |
| 0.08 | $2^+$ |
| 0 | $0^+$ |

ccfull.inp

```
16.,8.,144.,62.

1.2,-1,1.06,0

1.81,0.205,3,1

1.66,0.11,2,0

6.13,0.733,3,1

0,0.,0.3

105.1,1.1,0.75

55.,70.,1.

30,0.05
```

16.,8.,144.,62. ← ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62)

1.2,-1,1.06,0 ← (inert projectile, and vib. for targ.)

1.81,0.205,3,1 ← properties of the targ. excitation

1.66,0.11,2,0 ← same as the previous line, but the second mode of excitation in the target nucleus (vibrational coupling only)

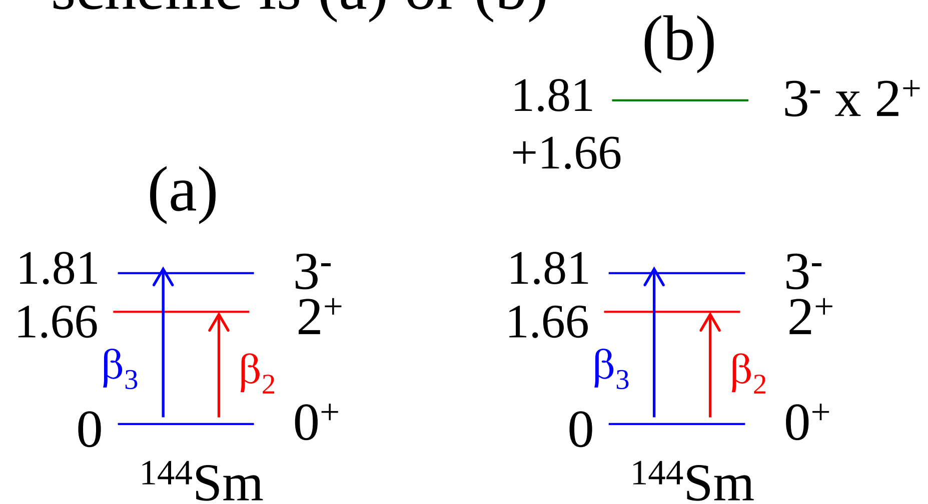
$N_{phonon} = 0$ ➔ no second mode

ccfull.inp

| | |
|---|---|
| 16.,8.,144.,62. | ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62) |
| 1.2,-1,1.06,0 | (inert projectile, and vib. for targ.) |
| 1.81,0.205,3,1 | properties of the targ. excitation |
| 1.66,0.11,2,*1* | second mode in the targ. |
| 6.13,0.733,3,1 | |
| 0,0.,0.3 | |
| 105.1,1.1,0.75 | |
| 55.,70.,1. | |
| 30,0.05 | |

(note) if $N_{phonon}$ = 1:

the code will ask you while you run it whether your coupling scheme is (a) or (b)



ccfull.inp

| | |
|---|---|
| 16.,8.,144.,62. | ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62) |
| 1.2,-1,1.06,0 | (inert projectile, and vib. for targ.) |
| 1.81,0.205,3,1 | properties of the targ. excitation |
| 1.66,0.11,2,1 | second mode in the targ. |
| 6.13,0.733,3,1 | properties of the proj. excitation (similar as the third line) |
| 0,0.,0.3 | (will be skipped for an inert projectile) |
| 105.1,1.1,0.75 | |
| 55.,70.,1. | |
| 30,0.05 | |

ccfull.inp

```
16.,8.,144.,62.
1.2,-1,1.06,0
1.81,0.205,3,1
1.66,0.11,2,1
6.13,0.733,3,1
0,0.,0.3
105.1,1.1,0.75
55.,70.,1.
30,0.05
```

16.,8.,144.,62. ← ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62)

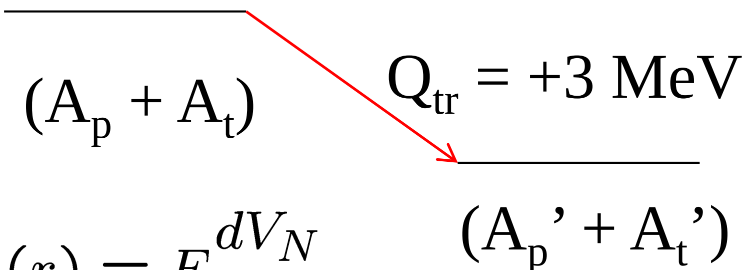
1.2,-1,1.06,0 ← (inert projectile, and vib. for targ.)

1.81,0.205,3,1 ← properties of the targ. excitation

1.66,0.11,2,1 ← second mode in the targ.

6.13,0.733,3,1 ← properties of the proj. excitation (similar as the third line)

0,0.,0.3 ← transfer coupling (g.s. to g.s.)




$$F_{tr}(r) = F\frac{dV_N}{dr}$$

* no transfer coup. for $F$ = 0

ccfull.inp

| | |
|---|---|
| 16.,8.,144.,62. | ← ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62) |
| 1.2,-1,1.06,0 | ← (inert projectile, and vib. for targ.) |
| 1.81,0.205,3,1 | ← properties of the targ. excitation |
| 1.66,0.11,2,1 | ← second mode in the targ. |
| 6.13,0.733,3,1 | ← properties of the proj. excitation (similar as the third line) |
| 0,0.,0.3 | ← transfer coupling (g.s. to g.s.) |
| 105.1,1.1,0.75 | ← potential parameters |
| 55.,70.,1. | |
| 30,0.05 | |

$$V_N(r) = -\frac{V_0}{1+\exp[(r-R_0)/a]}$$

$V_0$ = 105.1 MeV, $a$ = 0.75 fm

$R_0 = 1.1 * (A_p^{1/3} + A_t^{1/3})$ fm

ccfull.inp

| | |
|---|---|
| 16.,8.,144.,62. | ($A_p$=16, $Z_p$=8, $A_t$=144, $Z_t$=62) |
| 1.2,-1,1.06,0 | (inert projectile, and vib. for targ.) |
| 1.81,0.205,3,1 | properties of the targ. excitation |
| 1.66,0.11,2,1 | second mode in the targ. |
| 6.13,0.733,3,1 | properties of the proj. excitation<br>(similar as the third line) |
| 0,0.,0.3 | transfer coupling (g.s. to g.s.) |
| 105.1,1.1,0.75 | potential parameters |
| 55.,70.,1. | $E_{\mathrm{min}}$, $E_{\mathrm{max}}$, $\Delta E$ (c.m. energies) |
| 30,0.05 | $R_{\mathrm{max}}$, $\Delta r$ |

ccfull.inp

```
16.,8.,144.,62.
1.2,-1,1.06,0
1.81,0.205,3,1
1.66,0.11,2,1
6.13,0.733,3,1
0,0.,0.3
105.1,1.1,0.75
55.,70.,1.
30,0.05
```

OUTPUT

```
16O + 144Sm Fusion reaction
-------------------------------------------------
Phonon Excitation in the targ.: beta_N= 0.205, beta_C= 0.205,
              r0= 1.06(fm), omega= 1.81(MeV), Lambda= 3, Nph= 1
-------------------------------------------------
Potential parameters: V0= 105.10(MeV), r0= 1.10(fm),
                          a= 0.75(fm),power= 1.00
Uncoupled barrier: Rb=10.82(fm), Vb= 61.25(MeV),
                        Curv=4.25(MeV)
-------------------------------------------------
Ecm (MeV)   sigma (mb)      <l>
-------------------------------------
55.00000   0.97449E-02   5.87031
56.00000       0.05489   5.94333
57.00000       0.28583   6.05134
58.00000       1.36500   6.19272
59.00000       5.84375   6.40451
....................

69.00000     427.60179  17.16365
70.00000     472.46037  18.08247
```

In addition, “cross.dat” : fusion cross sections only

# Coupled-channels equations and barrier distribution

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{J(J+1)\hbar^2}{2\mu r^2}+V_0(r)-E+\epsilon_n\right]u_n(r)$$
$$+\sum_{n'}\langle\phi_n|V_{\rm coup}(r,\xi)|\phi_{n'}]\rangle\,u_{n'}(r)=0$$

$$u_n(r)\to H_J^{(-)}(k_nr)\delta_{n,n_i}-\sqrt{\frac{k_0}{k_n}}\,S_n\,H_J^{(+)}(k_nr)$$

$$P_J(E)=1-\sum_n|S_n|^2 \qquad \sigma_{\rm fus}(E)=\frac{\pi}{k^2}\sum_J(2J+1)P_J(E)$$

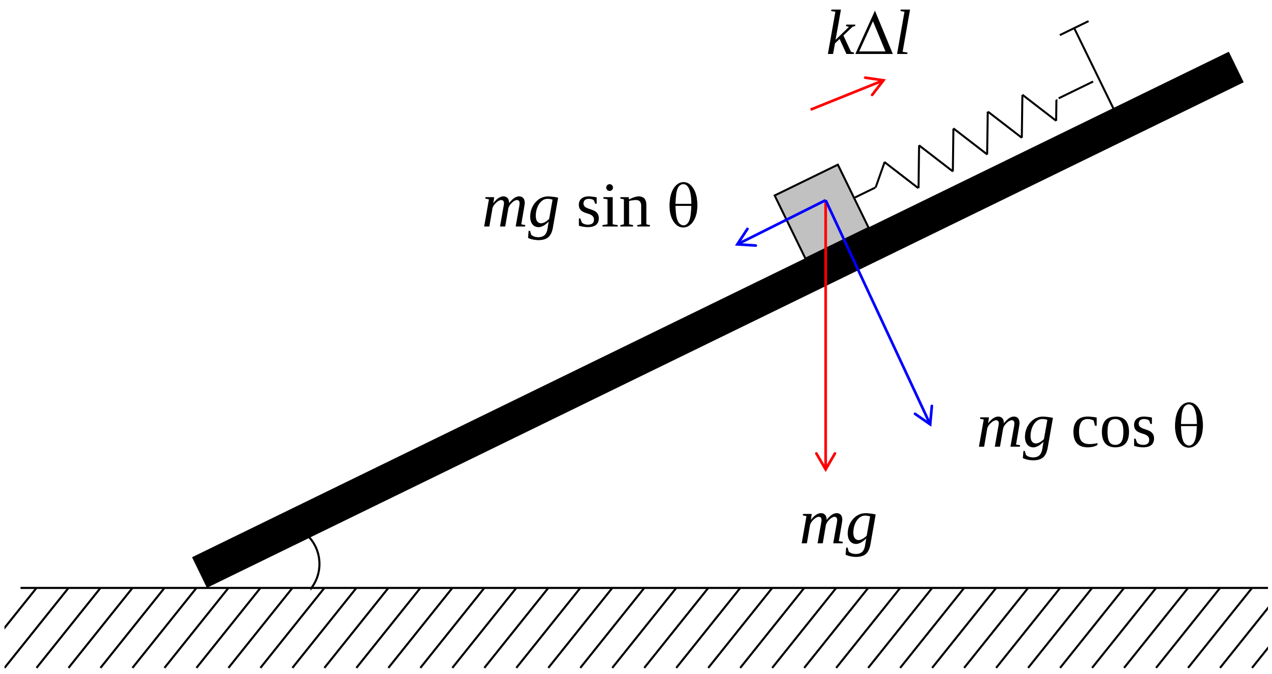
Calculate $\sigma_{fus}$ by numerically solving the coupled-channels equations

Let us consider a limiting case in order to understand (interpret) the numerical results

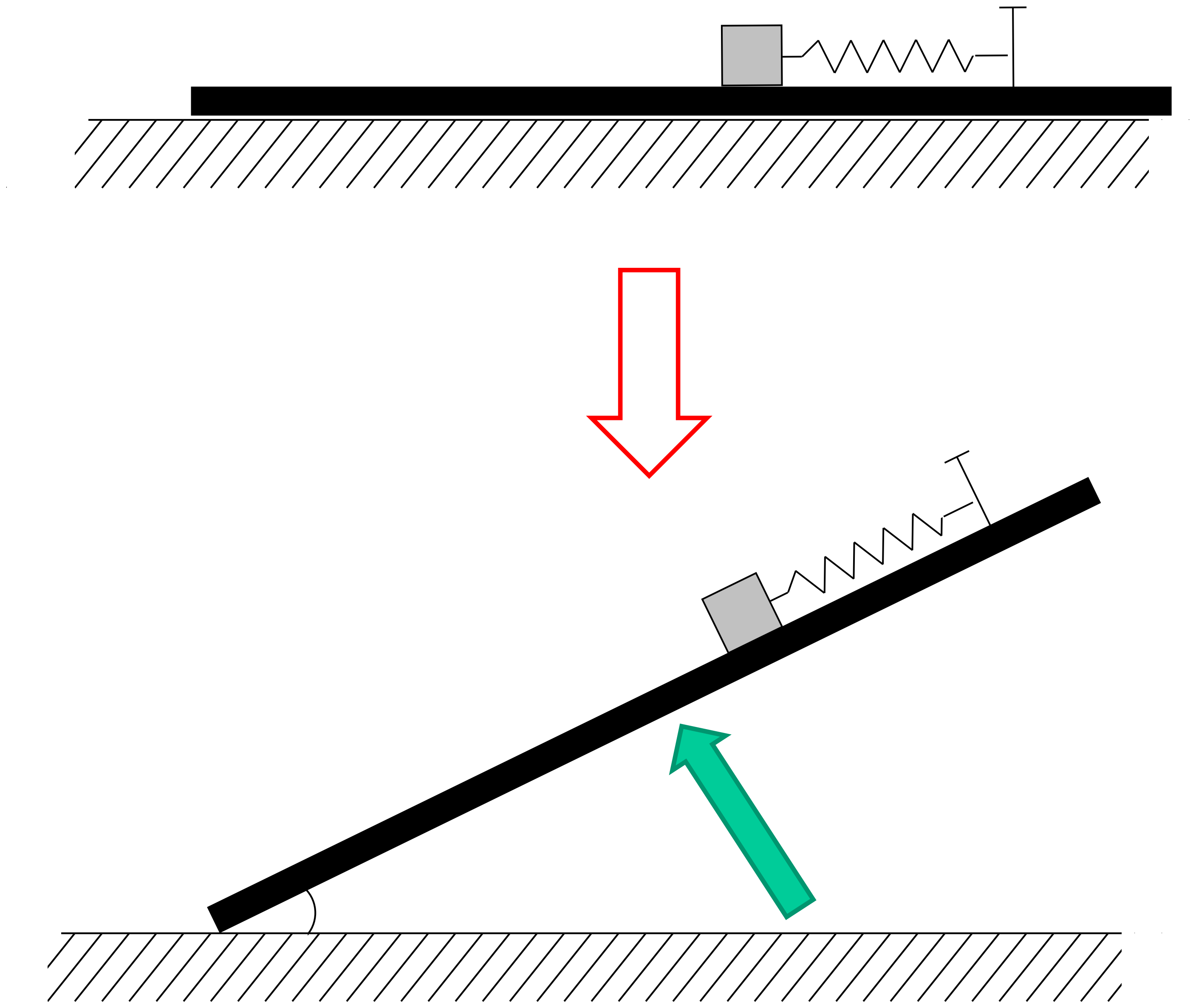
- $\varepsilon_{nI}$: very large *Adiabatic limit*
- $\varepsilon_{nI} = 0$ *Sudden limit*

# Comparison of two time scales

spring on a board



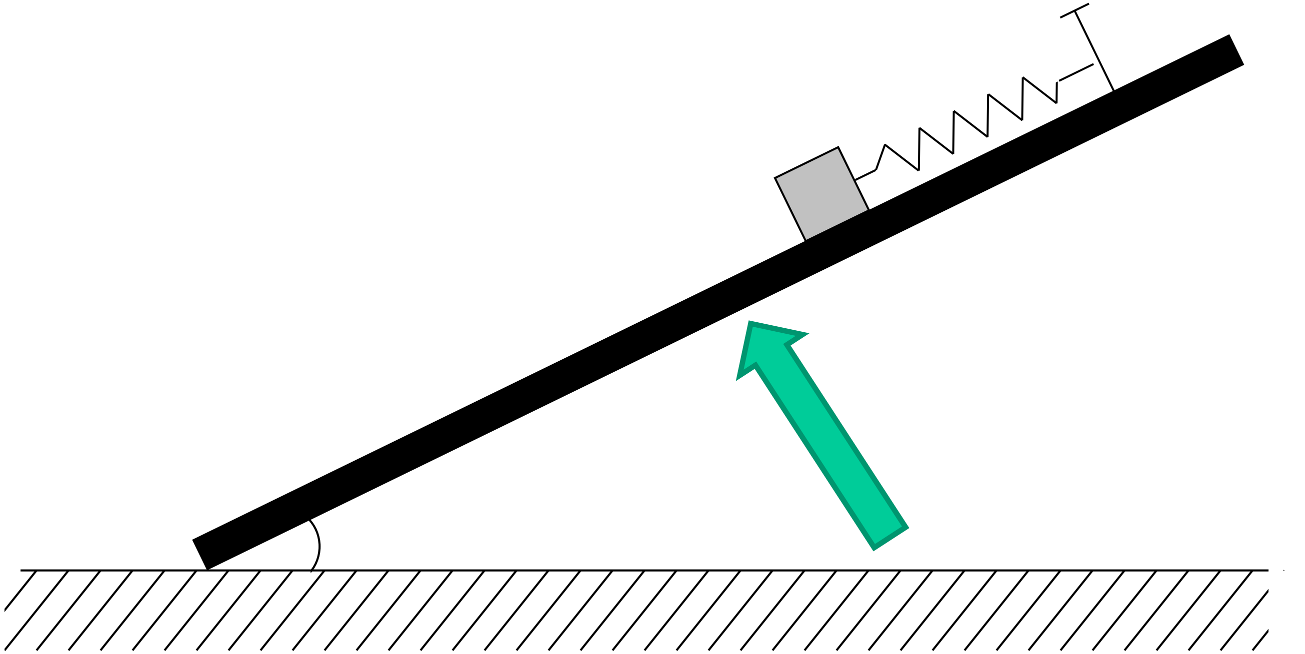

static case: $mg \sin\theta = k\Delta l \rightarrow \Delta l = mg \sin\theta / k$



move very slowly? or move instantaneously?

# Comparison of two time scales

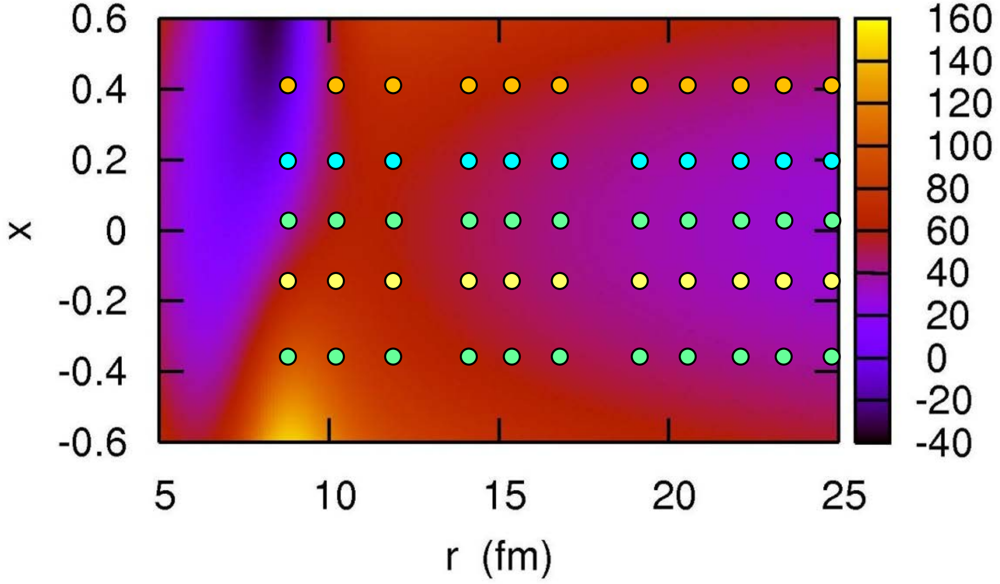
similar related example: spring on a moving board



move very slowly? or move instantaneously?

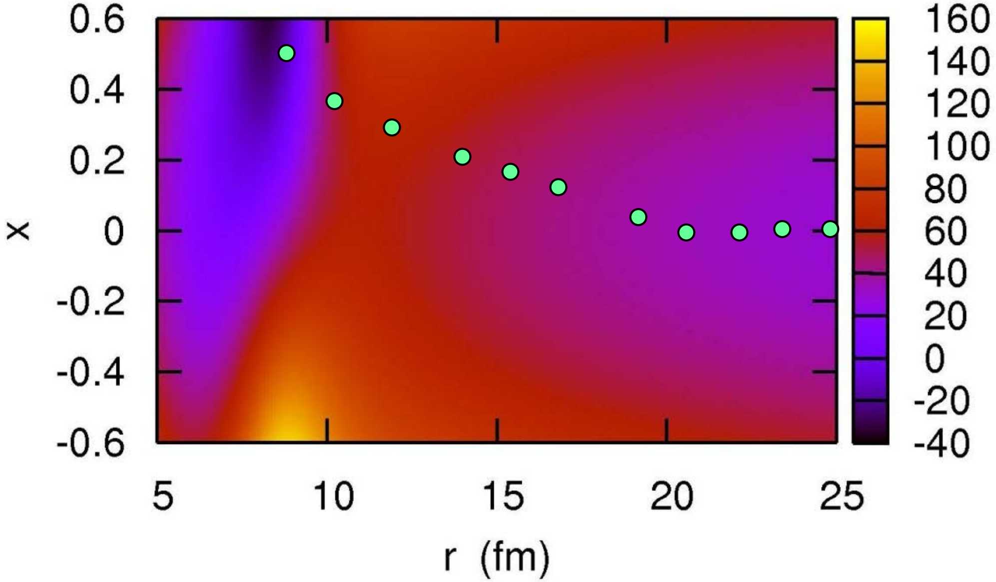
keep the original length ($\Delta l$ =0) "sudden limit"

always at the equilibrium length ($\Delta l = mg \sin \theta / k$ ) "adiabatic limit" fast reaction




large fluctuation

slow reaction




relative distance

+ small fluctuation
around the adiabatic path

# Two limiting cases: (i) adiabatic limit

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)$$

much slower rel. motion than the intrinsic motion

$\Updownarrow$

much larger energy scale for intrinsic motion than the typical energy scale for the rel. motion

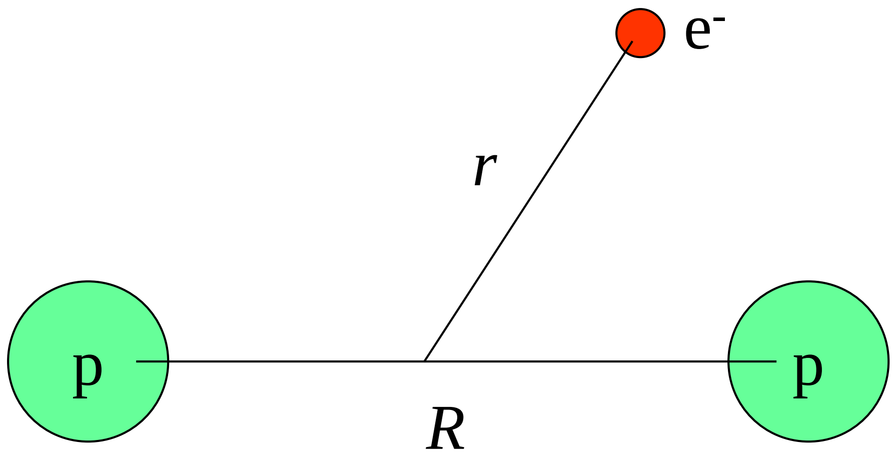
$$\hbar\Omega \ll \epsilon$$

(Barrier curvature v.s. Intrinsic excitation energy)

$$[H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)]\varphi_0(\xi;\boldsymbol{r}) = \epsilon_0(r)\,\varphi_0(\xi;\boldsymbol{r})$$

$\Rightarrow$ $$\boxed{H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi) \to \epsilon_0(r)}$$

# c.f. Born-Oppenheimer approximation for hydrogen molecule




$$[T_R + T_r + V(r,R)]\Psi(r,R) = E\Psi(r,R)$$

1. Consider first the electron motion for a fixed *R*

$$[T_r + V(r,R)]u_n(r;R) = \epsilon_n(R)u_n(r;R)$$

2. Minimize $\varepsilon_n(R)$ with respect to *R*

Or 2'. Consider the proton motion in a potential $\varepsilon_n(R)$

$$[T_R + \epsilon_n(R)]\phi_n(R) = E\phi_n(R)$$

# Adiabatic Potential Renormalization

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)$$

When ε is large,

$$\boxed{H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi) \to \epsilon_0(r)}$$

where

$$[H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)]\varphi_0(\xi;\boldsymbol{r}) = \epsilon_0(r)\,\varphi_0(\xi;\boldsymbol{r})$$

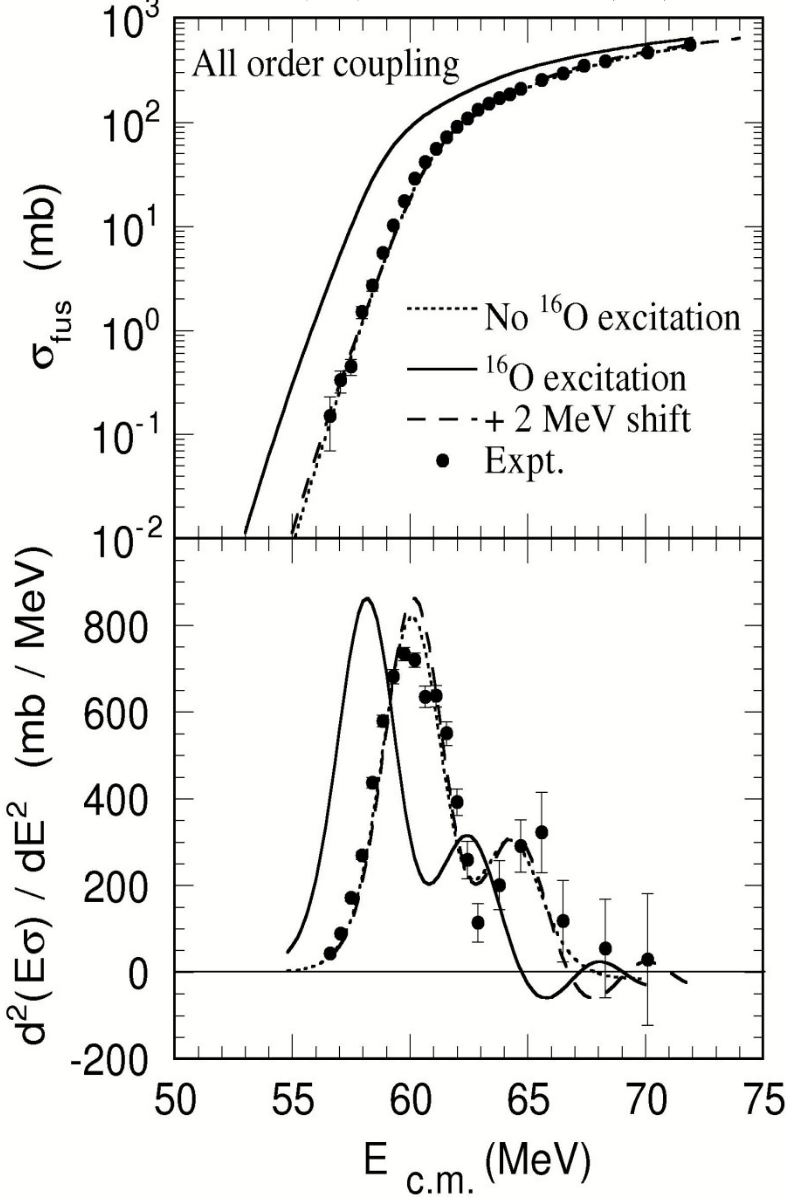
Fast intrinsic motion

→ Adiabatic potential renormalization

$$V_{\rm ad}(r) = V_0(r) + \epsilon_0(r)$$

Giant Resonances, $^{16}O(3^-)$ [6.31 MeV]

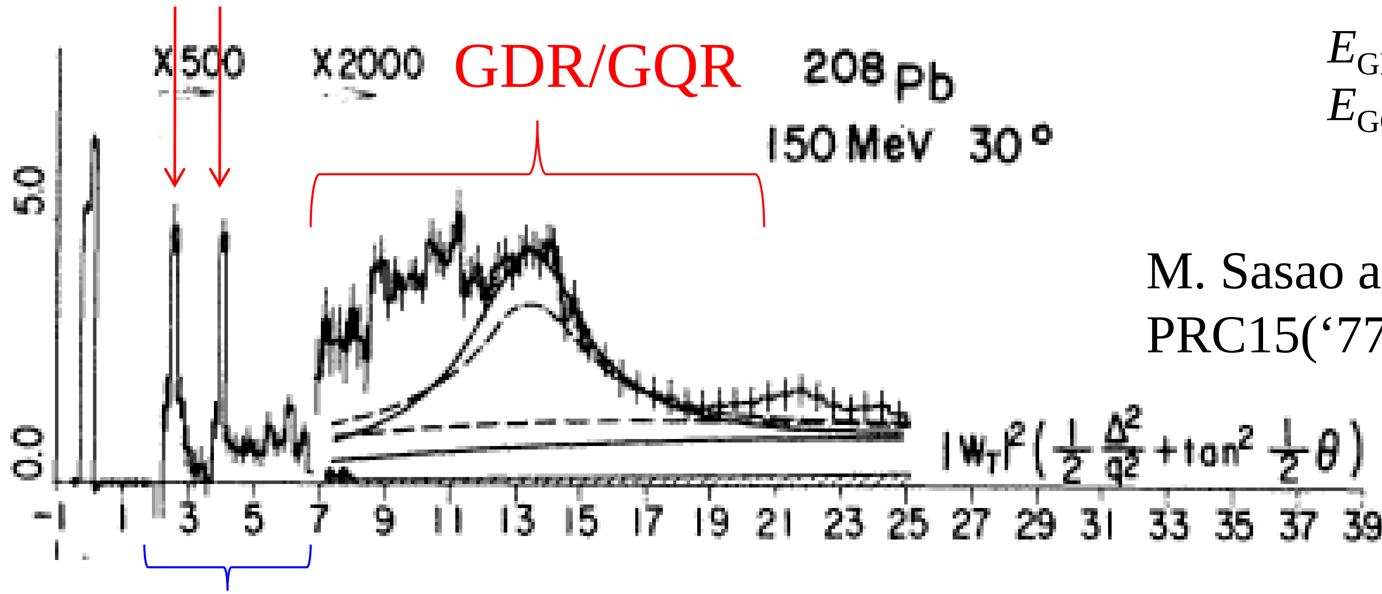
$^{16}O(3^-) + {}^{144}Sm(3^-)$




K.H., N. Takigawa, M. Dasgupta, D.J. Hinde, J.R. Leigh, PRL79('99)2014

## typical excitation spectrum: electron scattering data

low-lying collective excitations




$E_x$ **(MeV)**

$E_{\rm GDR} \sim 79A^{-1/3}$ MeV
$E_{\rm GQR} \sim 65A^{-1/3}$ MeV

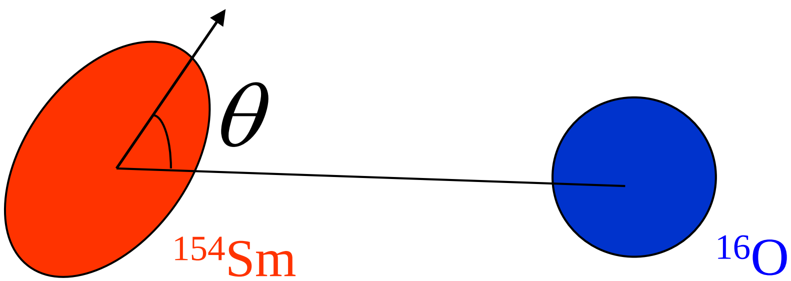
M. Sasao and Y. Torizuka, PRC15('77)217

low-lying non-collective excitations

- Giant Resonances: high $E_x$, smooth mass number dependence
  $\longrightarrow$ adiabatic potential renormalization
- Low-lying collective excitations: barrier distributions,
  strong isotope dependence
- Non-collective excitations: either neglected completely or
  implicitly treated through an absorptive potential

# Two limiting cases: (ii) sudden limit

$$\epsilon \to 0$$




$$\epsilon_I = I(I+1)\hbar^2/2\mathcal{J}$$

$$\mathcal{J} \to \infty$$

$$\sigma_{\mathrm{fus}}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\mathrm{fus}}(E;\theta)$$

Coupled-channels:

$$\begin{pmatrix} 0 & f(r) & 0 \\ f(r) & \frac{2\sqrt{5}}{7}f(r) & \frac{6}{7}f(r) \\ 0 & \frac{6}{7}f(r) & \frac{20\sqrt{5}}{77}f(r) \end{pmatrix} \xrightarrow{\text{diagonalize}} \begin{pmatrix} \lambda_1(r) & 0 & 0 \\ 0 & \lambda_2(r) & 0 \\ 0 & 0 & \lambda_3(r) \end{pmatrix}$$

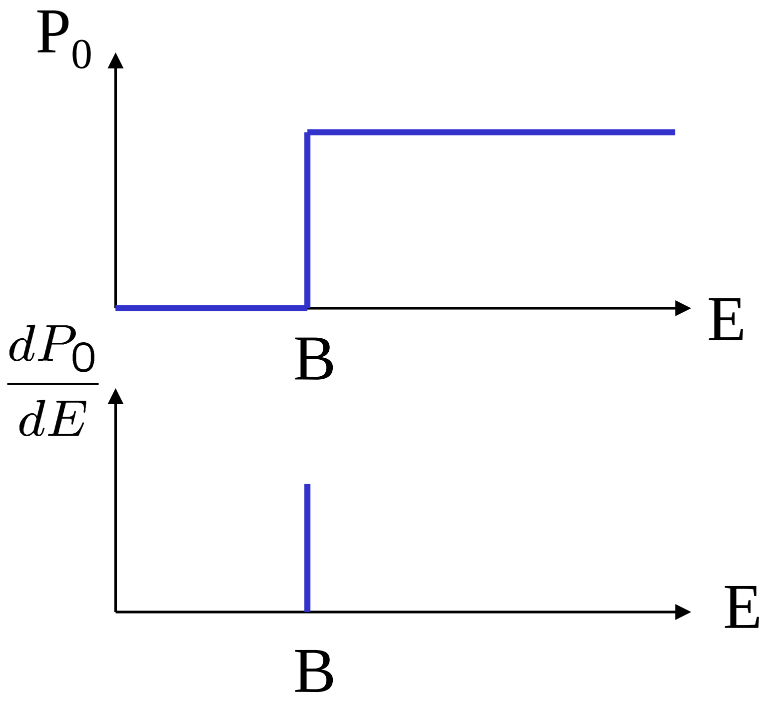
$$\Rightarrow \quad P(E) = \sum_i w_i\, P(E; V_0(r) + \lambda_i(r))$$

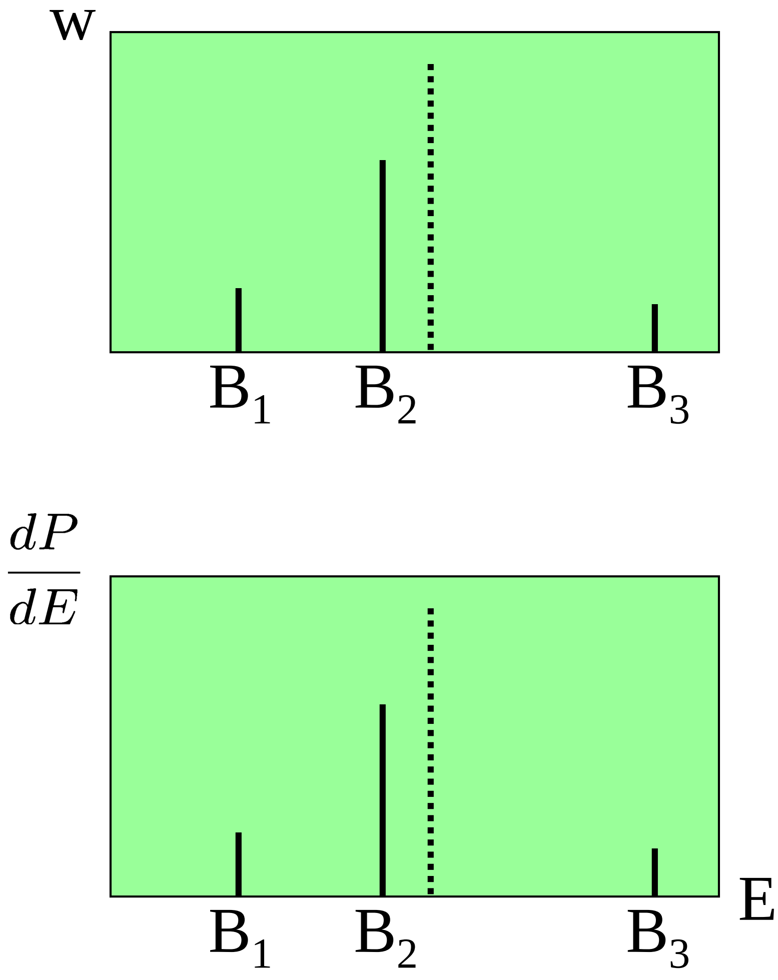
Slow intrinsic motion

→ Barrier Distribution

# Barrier distribution

$$P(E) = \sum_i w_i \, P(E; V_0(r) + \lambda_i(r))$$



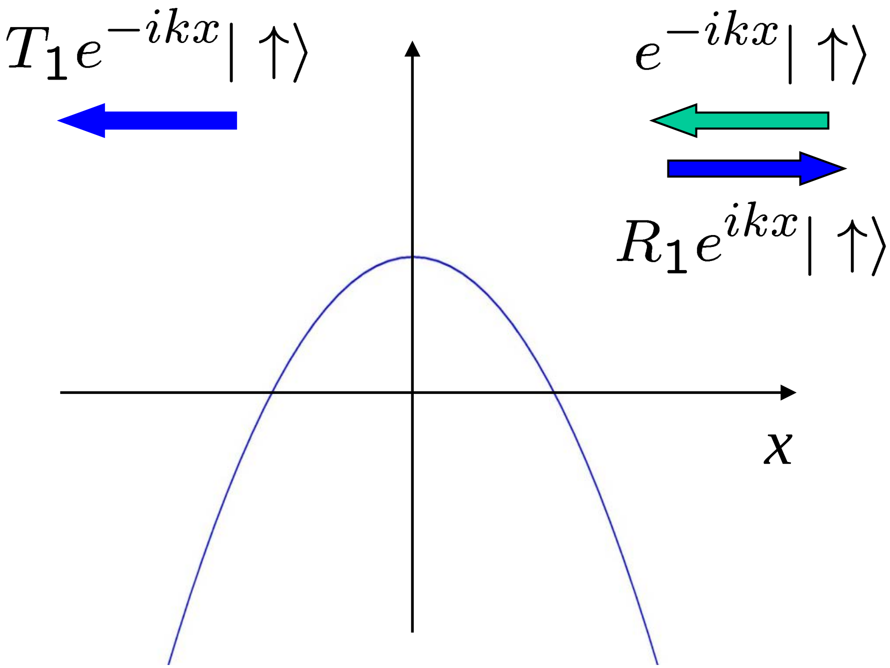



# Barrier distribution: understand the concept using a spin Hamiltonian

Hamiltonian (example 1): $$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) + \hat{\sigma}_z \cdot V_s(x)$$

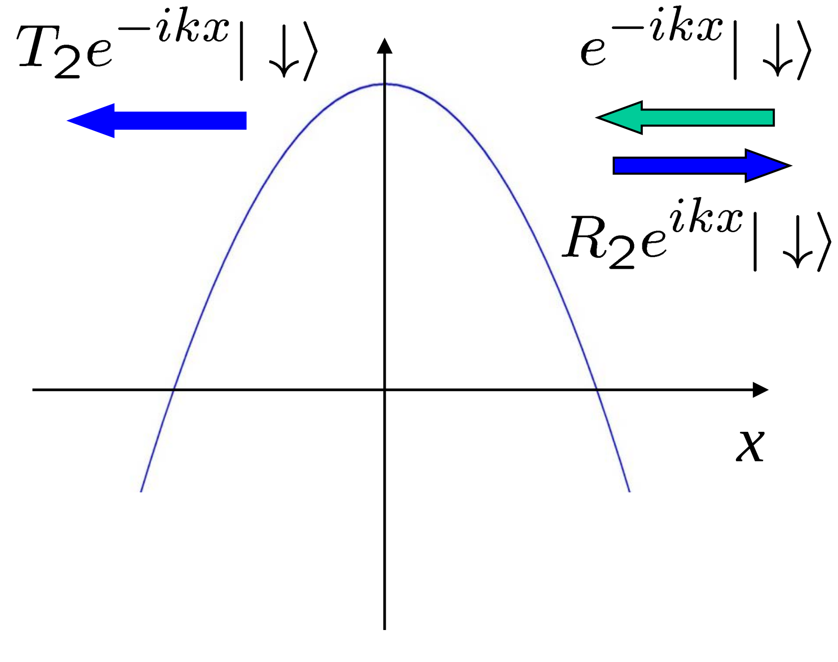
$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For Spin-up




$$V_1(x) = V_0(x) + V_s(x)$$

For Spin-down




$$V_2(x) = V_0(x) - V_s(x)$$

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) + \hat{\sigma}_z \cdot V_1(x)$$

Wave function
(general form)

$$\begin{aligned} \Psi(x) &= \psi_1(x)\,|\uparrow\rangle + \psi_2(x)\,|\downarrow\rangle \\ &= \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} \end{aligned}$$

The spin direction does not change during tunneling:

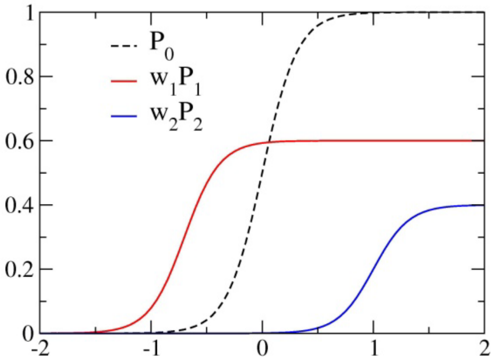
$$\boxed{P(E) = w_\uparrow P_1(E) + w_\downarrow P_2(E)}$$

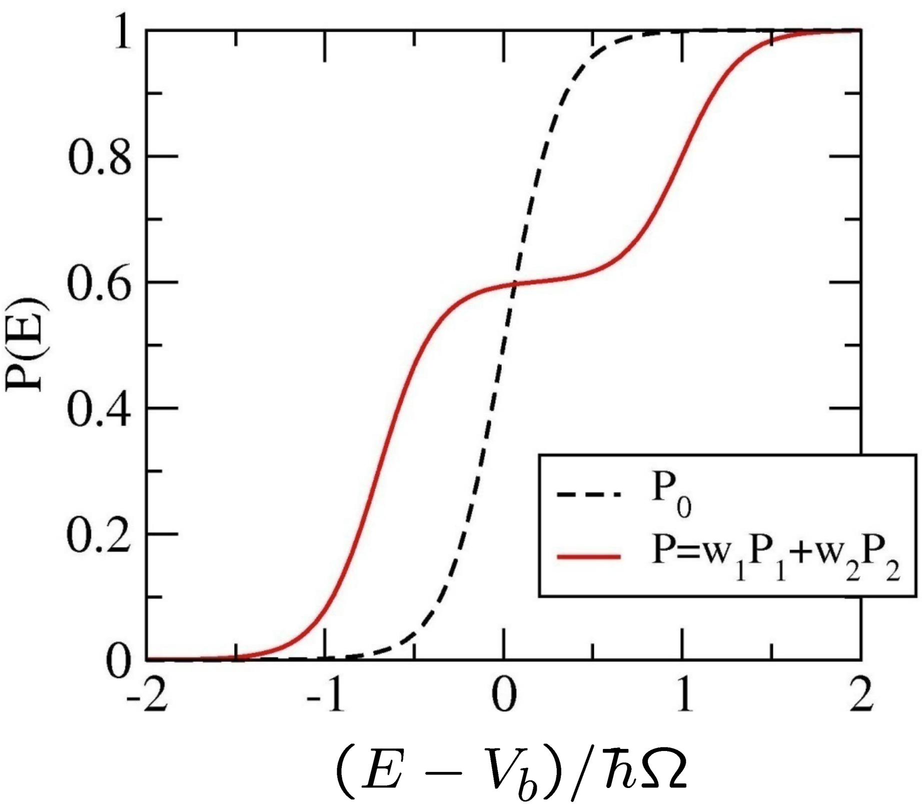
$$w_\uparrow + w_\downarrow = 1$$

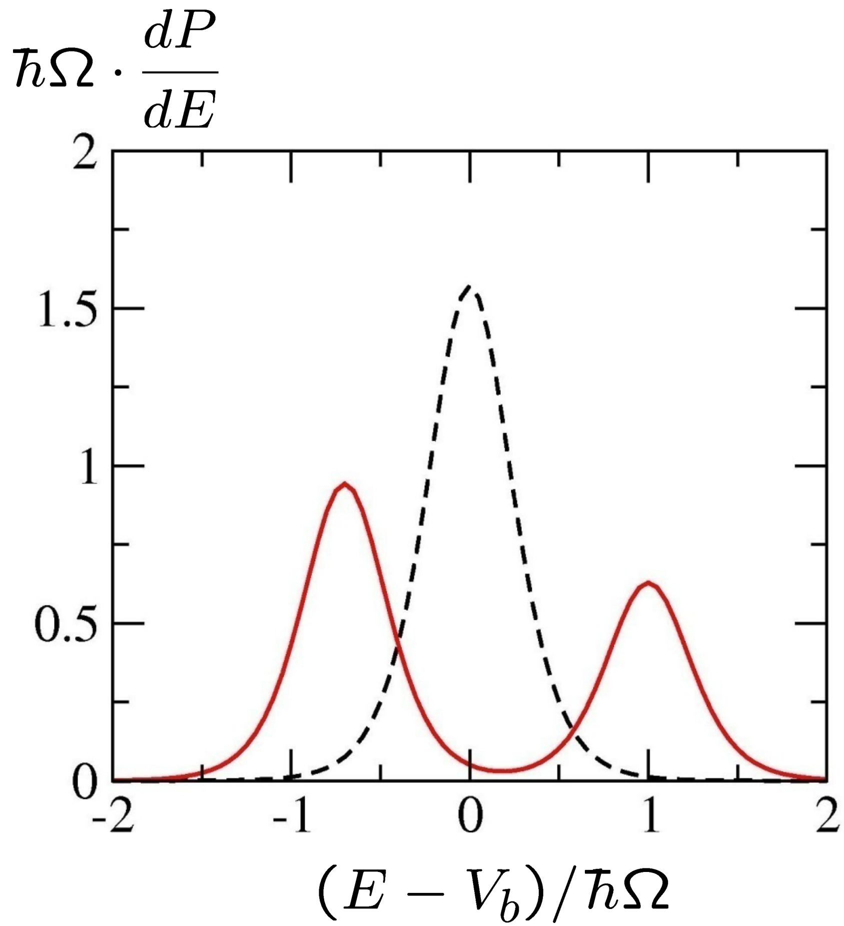
$$P(E) = w_{\uparrow} P_1(E) + w_{\downarrow} P_2(E)$$

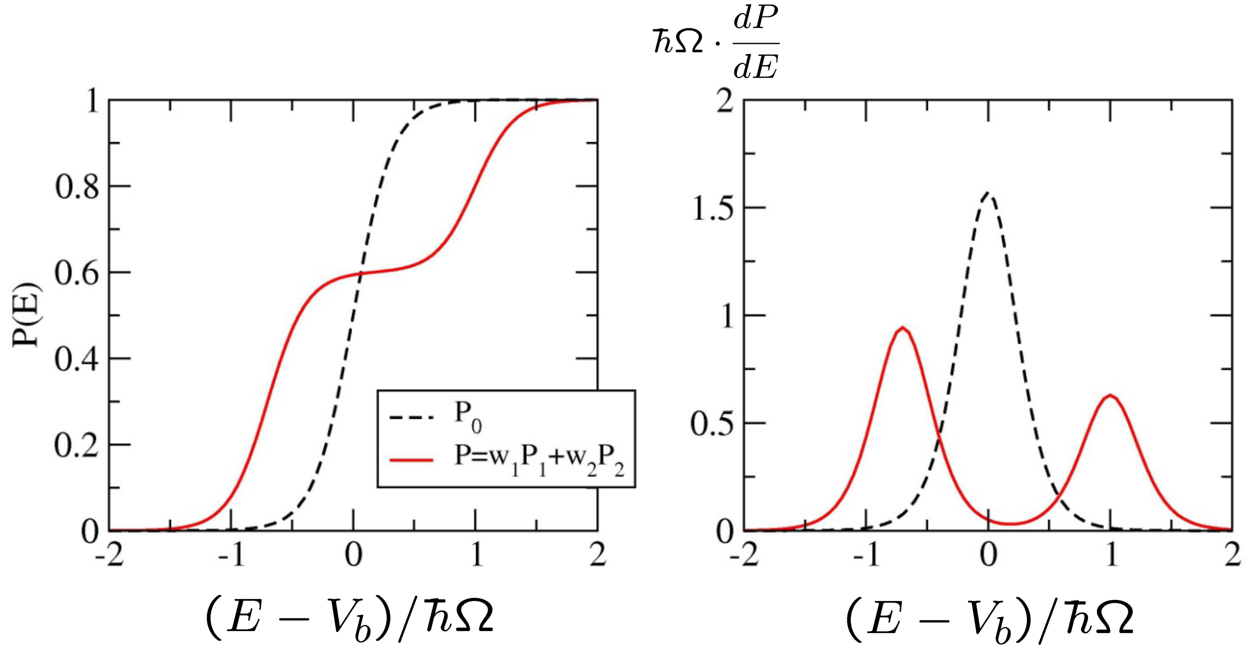
$\Rightarrow$ Tunneling prob. is a weighted sum of tunnel prob. for two barriers

$$\begin{cases} V_1(x) = V_0(x) + V_s(x) & \Longleftarrow \quad |\uparrow\rangle \\ V_2(x) = V_0(x) - V_s(x) & \Longleftarrow \quad |\downarrow\rangle \end{cases}$$






- Tunnel prob. is enhanced at $E < V_b$ and hindered $E > V_b$
- *dP/dE* splits to two peaks → "barrier distribution"
- The peak positions of *dP/dE* correspond to each barrier height
- The height of each peak is proportional to the weight factor

$$P(E) = w_\uparrow P_1(E) + w_\downarrow P_2(E)$$

$$\frac{dP}{dE} = w_\uparrow \frac{dP_1}{dE} + w_\downarrow \frac{dP_2}{dE}$$

simple 2-level model (Dasso, Landowne, and Winther, NPA405('83)381)

entrance channel

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+V_l(r)+\begin{pmatrix}0 & F\\ F & \epsilon\end{pmatrix}-E\right]\begin{pmatrix}u_0(r)\\ u_1(r)\end{pmatrix}=0$$

excited channel

$$\Longrightarrow\quad\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+V_l(r)+\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}-E\right]\begin{pmatrix}\phi_0(r)\\ \phi_1(r)\end{pmatrix}=0$$

$$\left\{\begin{array}{rcl}\phi_0(r) & = & \alpha\cdot u_0(r)+\beta\cdot u_1(r)\\ \phi_1(r) & = & \beta\cdot u_0(r)-\alpha\cdot u_1(r)\end{array}\right.$$

# simple 2-level model (Dasso, Landowne, and Winther, NPA405('83)381)

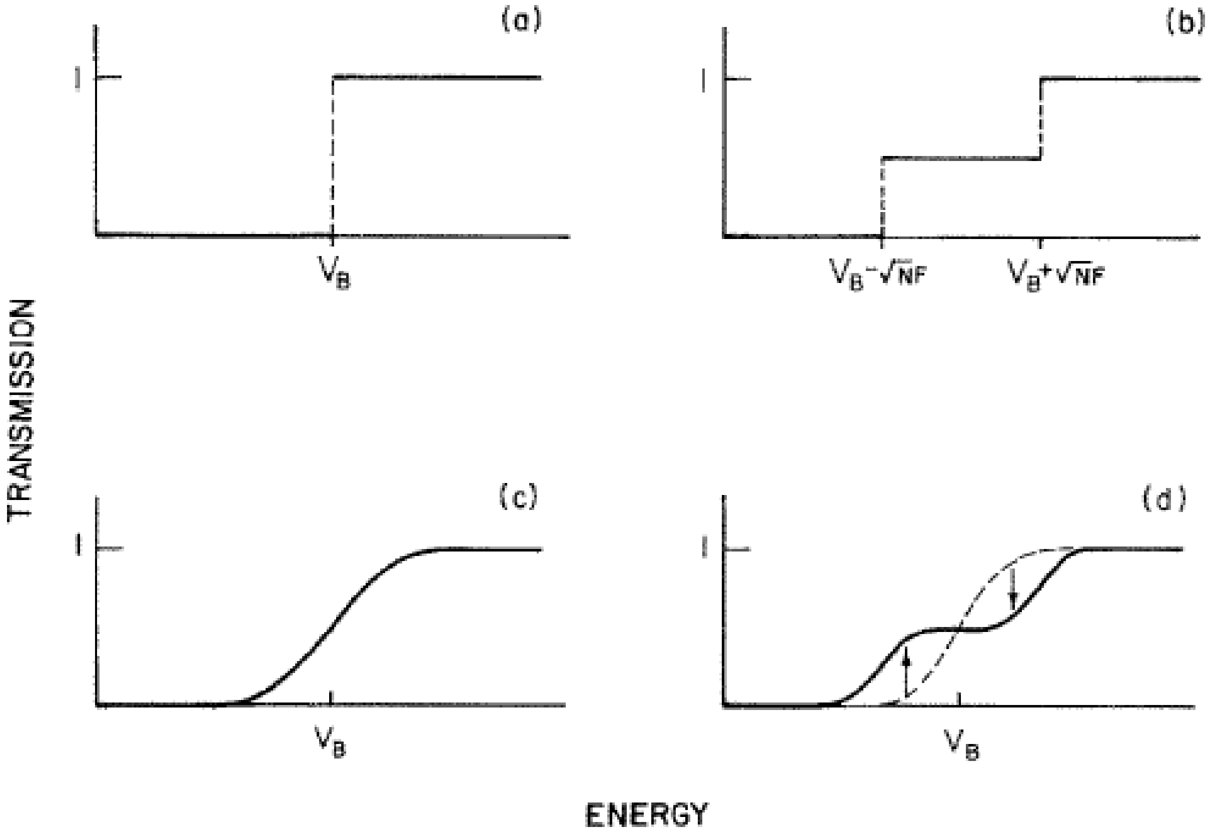



Fig. 1. Illustration of how channel coupling increases transmission at energies below the barrier and decreases it above. Parts (a) and (b) indicate the classical limits for no coupling and coupling, respectively, while parts (c) and (d) indicate how quantum mechanical effects modify the corresponding curves.

# Sub-barrier Fusion and Barrier distribution method

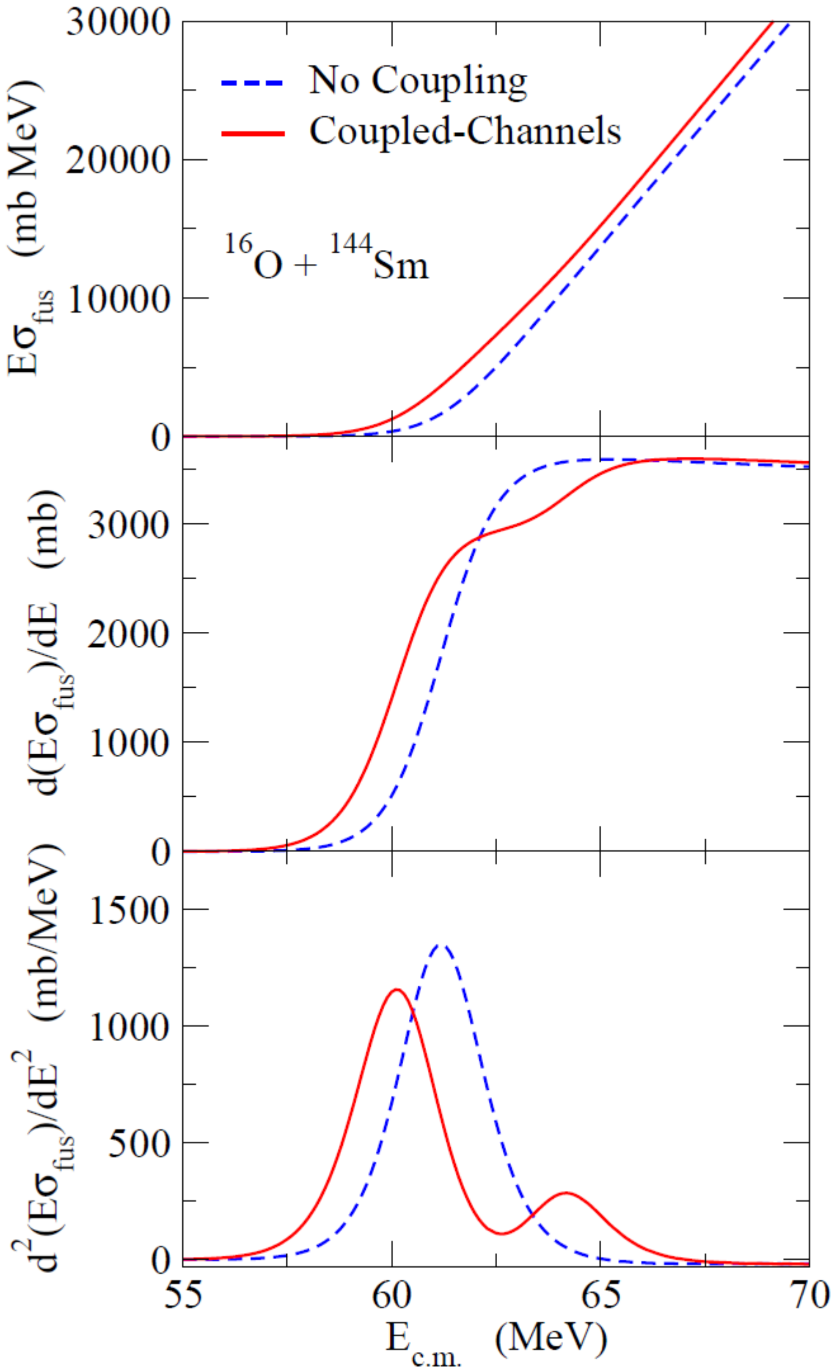
$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega}{2E} R_b^2 \log\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

$$\frac{d(E\sigma_{\rm fus})}{dE} = \frac{\pi R_b^2}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]} = \pi R_b^2 \cdot P_{l=0}(E)$$

$$D_{\rm fus}(E) \equiv \frac{d^2(E\sigma_{\rm fus})}{dE^2} \simeq \pi R_b^2 \frac{dP_{l=0}}{dE}$$

(Fusion barrier distribution)

N. Rowley, G.R. Satchler,
P.H. Stelson, PLB254('91)25




N. Rowley, G.R. Satchler, P.H. Stelson, PLB254('91)25

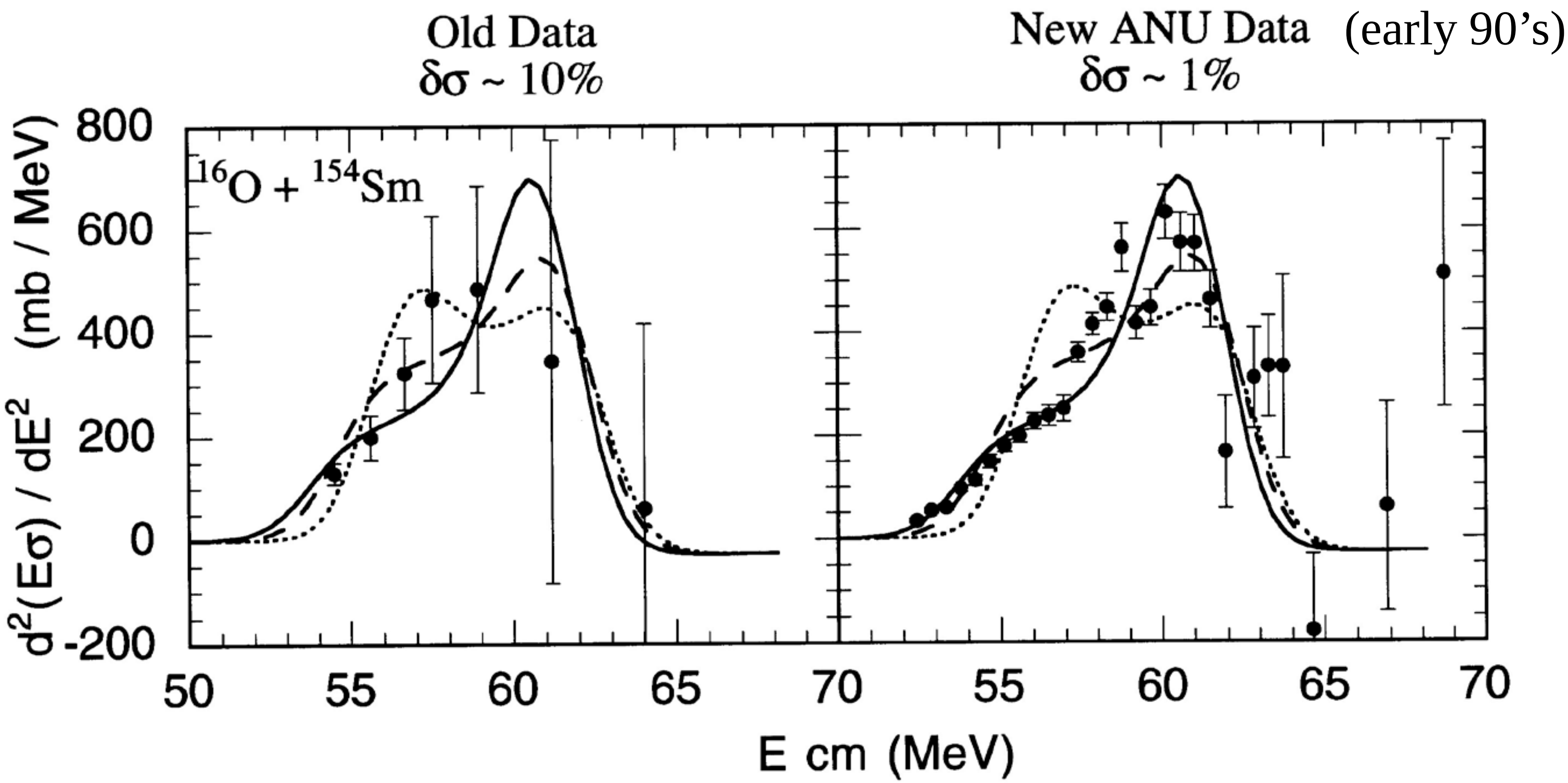
$$\frac{d}{dE}[E\sigma_{\rm fus}(E)] \propto P(E)$$
$$\frac{d^2}{dE^2}[E\sigma_{\rm fus}(E)] \propto \frac{dP}{dE}$$

centered on $E = V_b$

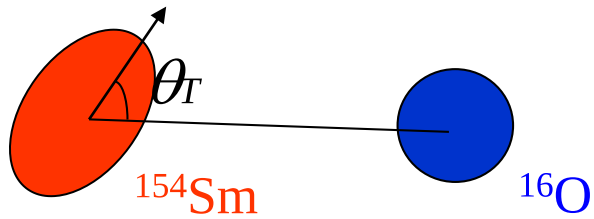
# Barrier distribution measurements

Fusion barrier distribution $$D_{\text{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}$$

Needs high precision data in order for the 2nd derivative to be meaningful

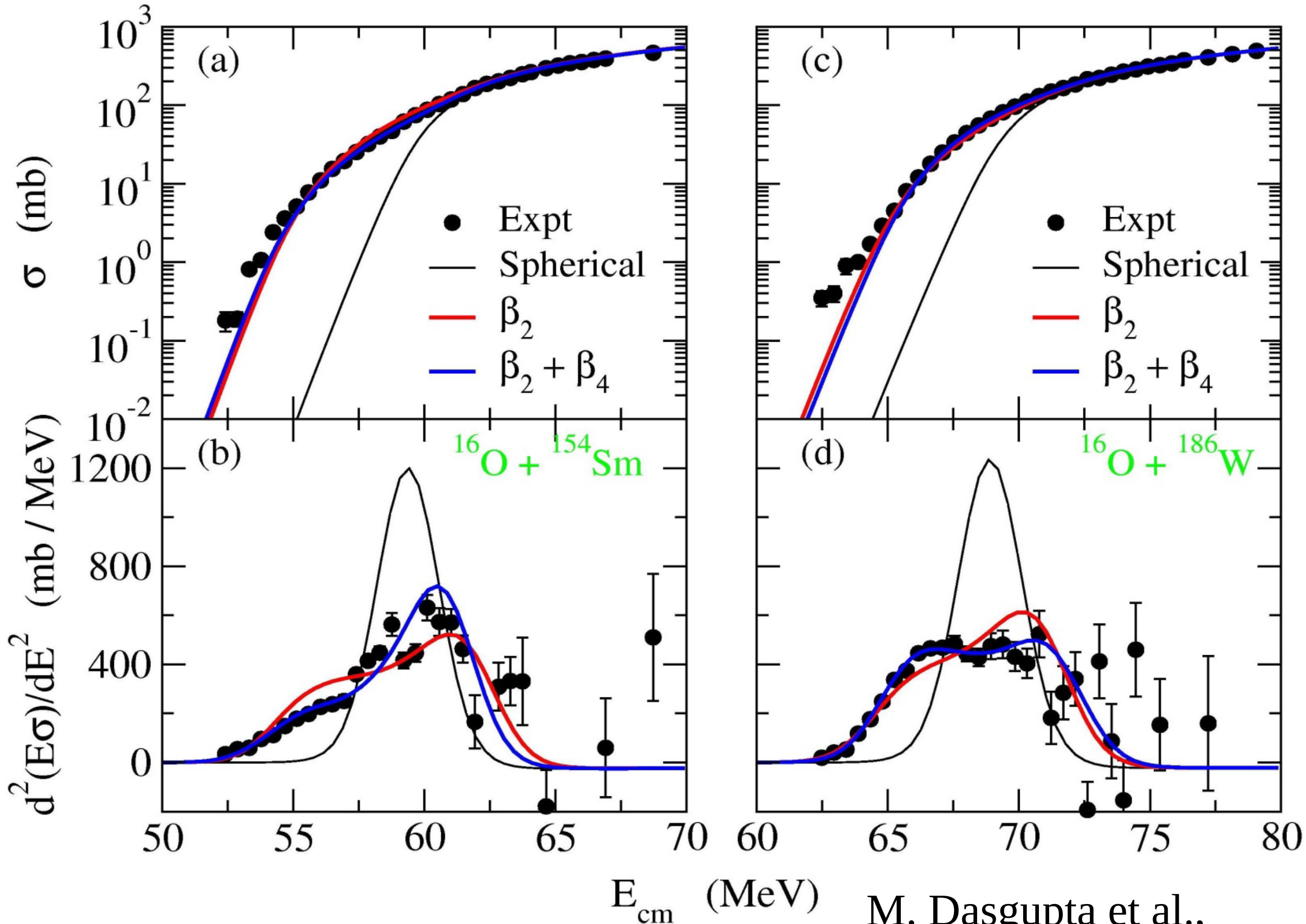


# Experimental Barrier Distribution




Requires high precision data

$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\text{fus}}(E;\theta_T)$$

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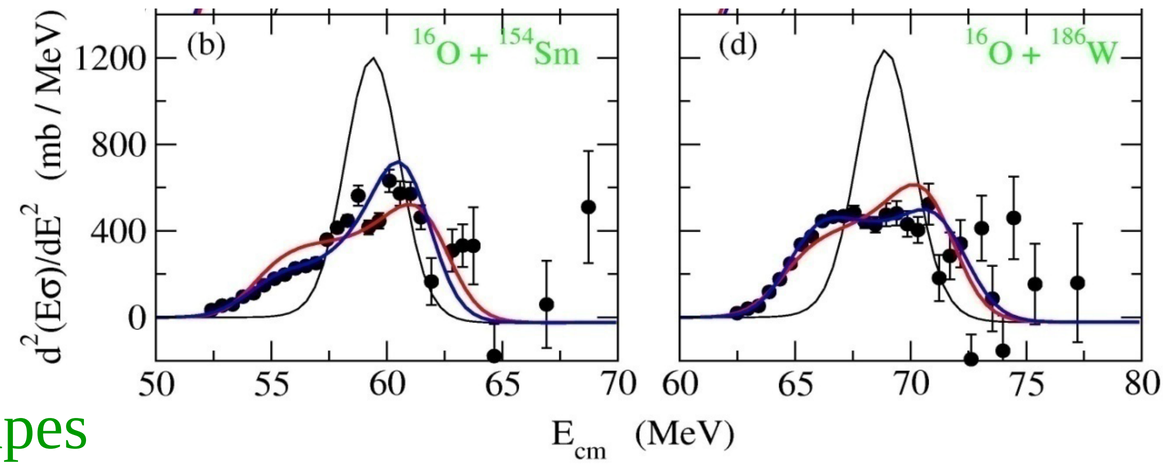
Annu. Rev. Nucl. Part. Sci. 48('98)401

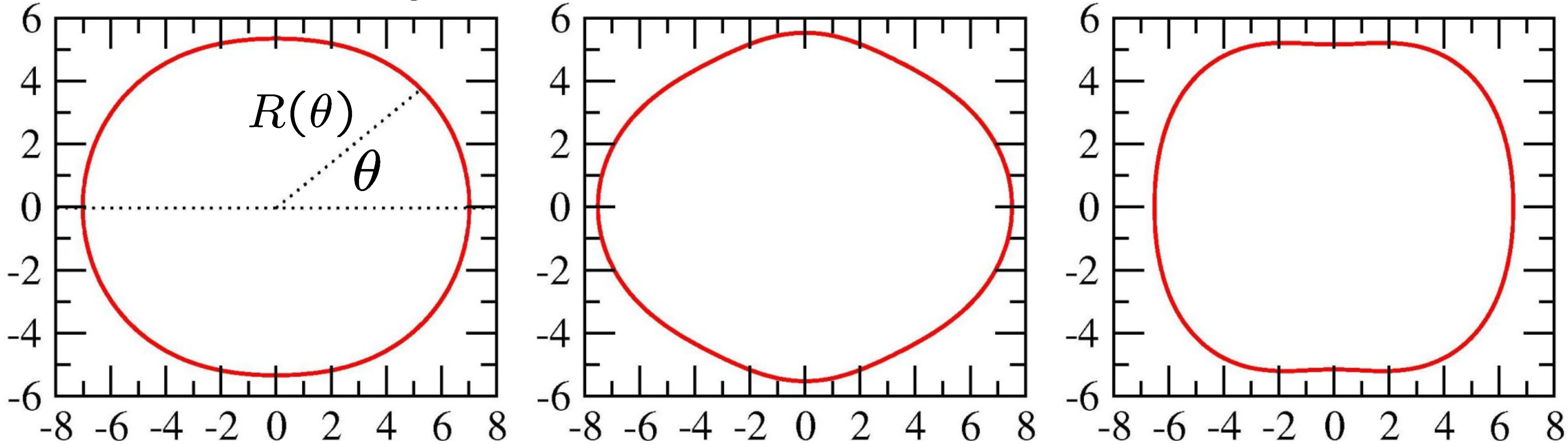
# Investigate nuclear shape through barrier distribution



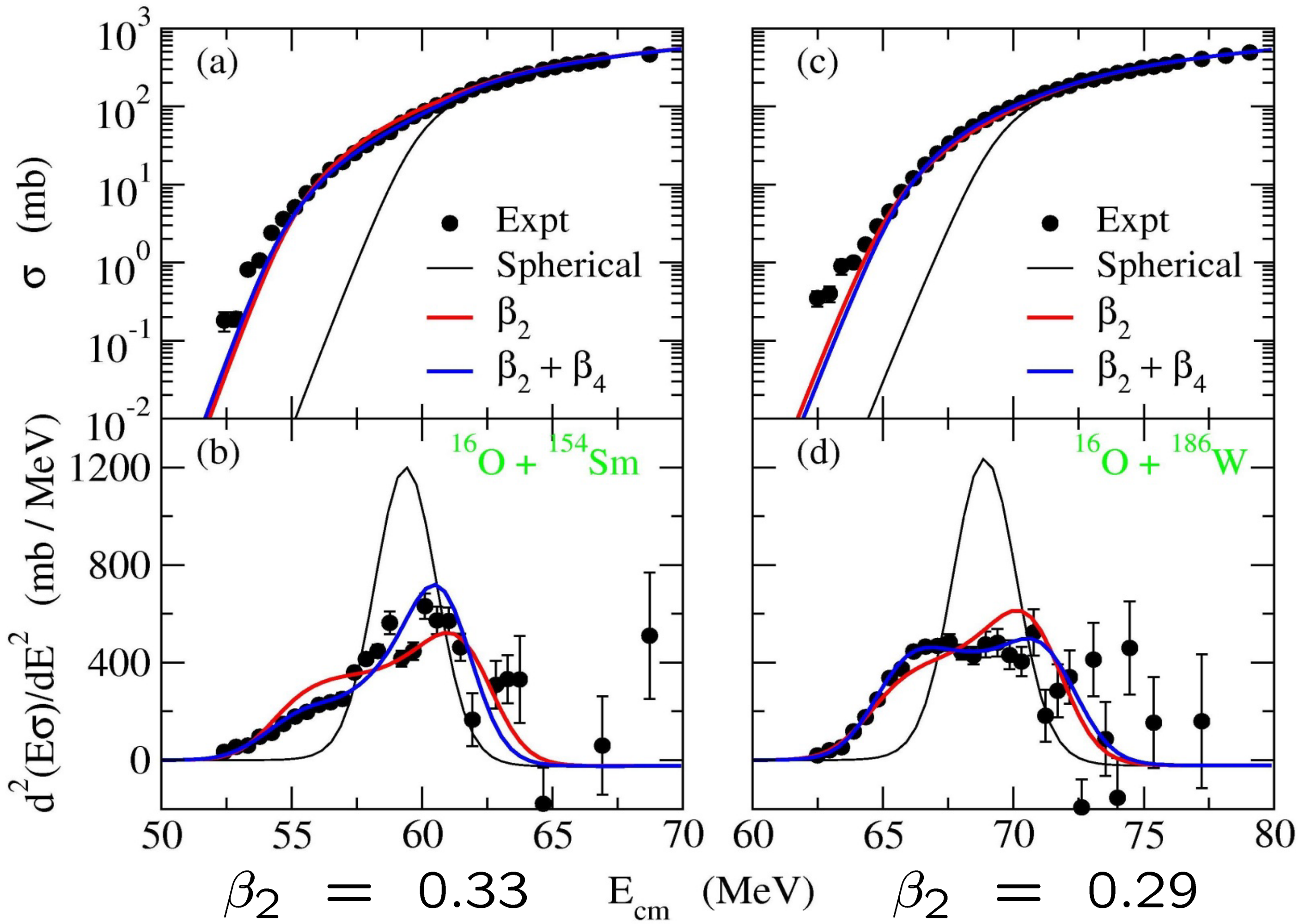

Nuclear shapes

$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

$$R_0 = 5.9 \text{ (fm)}, \quad \beta_2 = 0.3$$

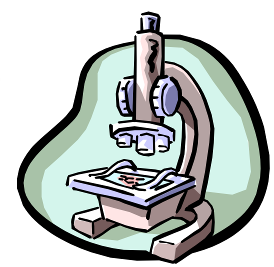



$\beta_4 = 0$ $\beta_4 = 0.1$ $\beta_4 = -0.1$




$\beta_2 = 0.33$ $\quad\quad$ $\beta_2 = 0.29$

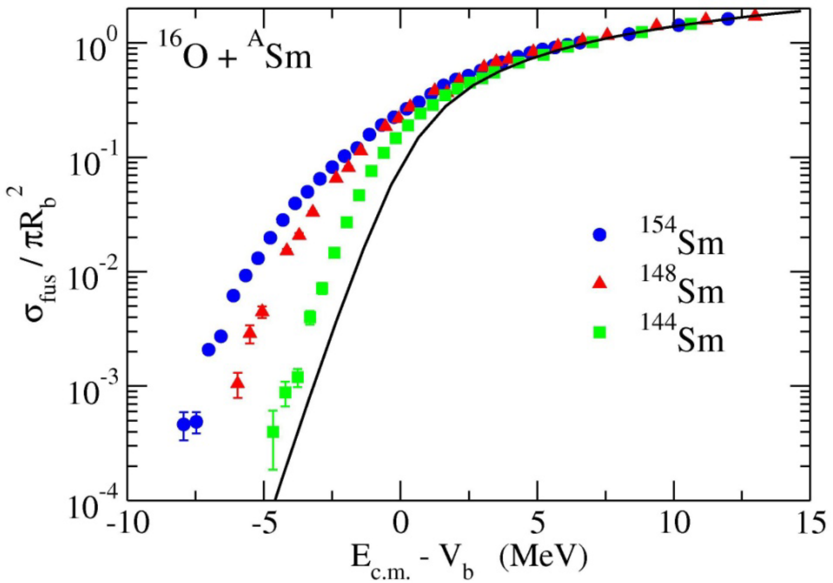
$\beta_4 = +0.05$ $\quad\quad$ $\beta_4 = -0.03$



By taking the barrier distribution, one can very clearly see the difference due to $\beta_4$!

→ Fusion as a quantum tunneling microscope for nuclei

# Advantage of fusion barrier distribution




Fusion Cross sections

Very strong exponential energy dependence

Difficult to see differences due to details of nuclear structure

Plot cross sections in a different way: Fusion barrier distribution

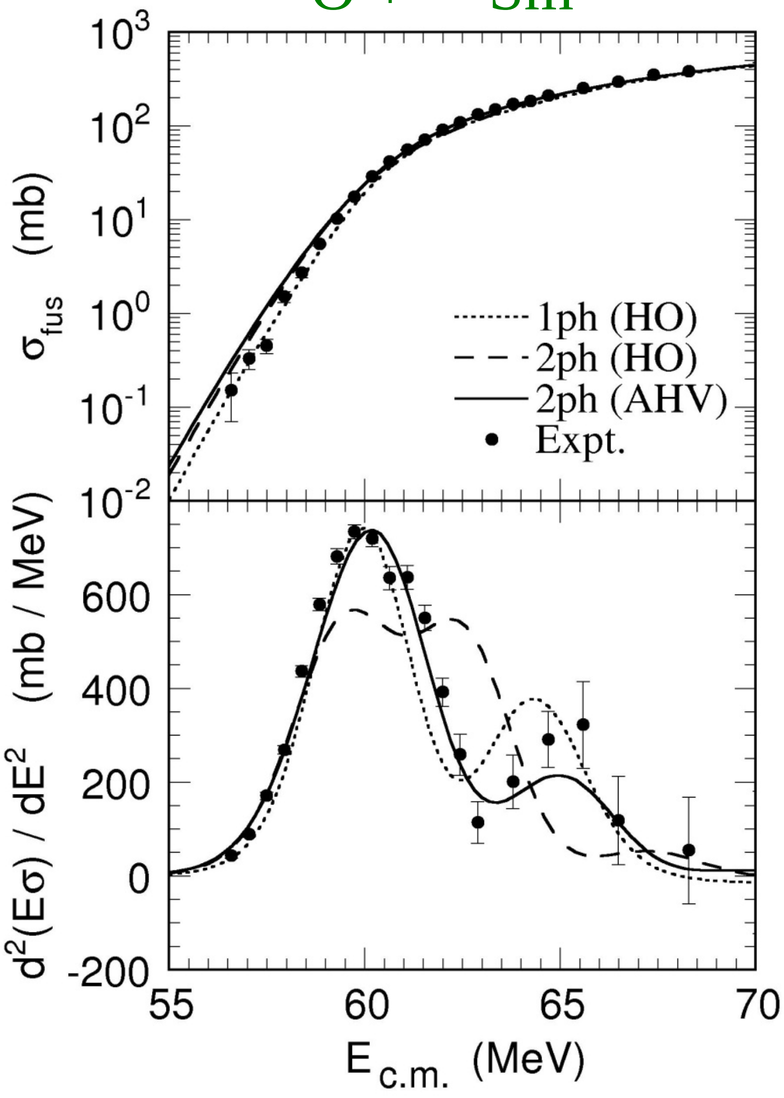
$$\boxed{D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}}$$

N. Rowley, G.R. Satchler, P.H. Stelson, PLB254('91)25

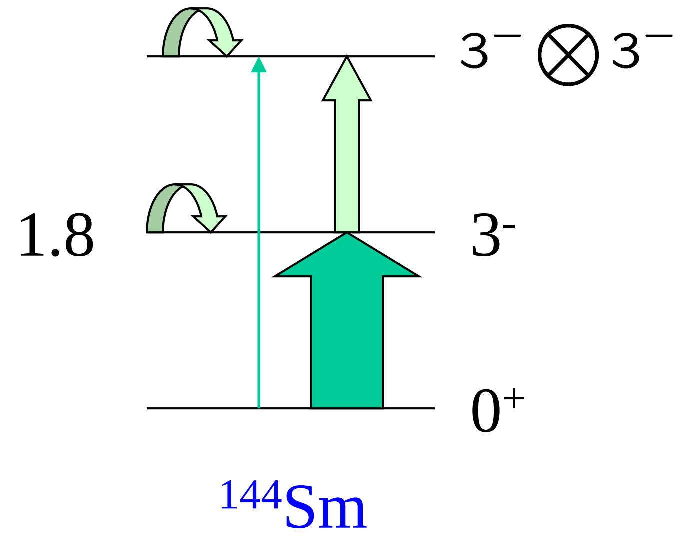
$\longrightarrow$ Function which is sensitive to details of nuclear structure

# Example for spherical vibrational system

$^{16}O + ^{144}Sm$




Anharmonicity of octupole vibration



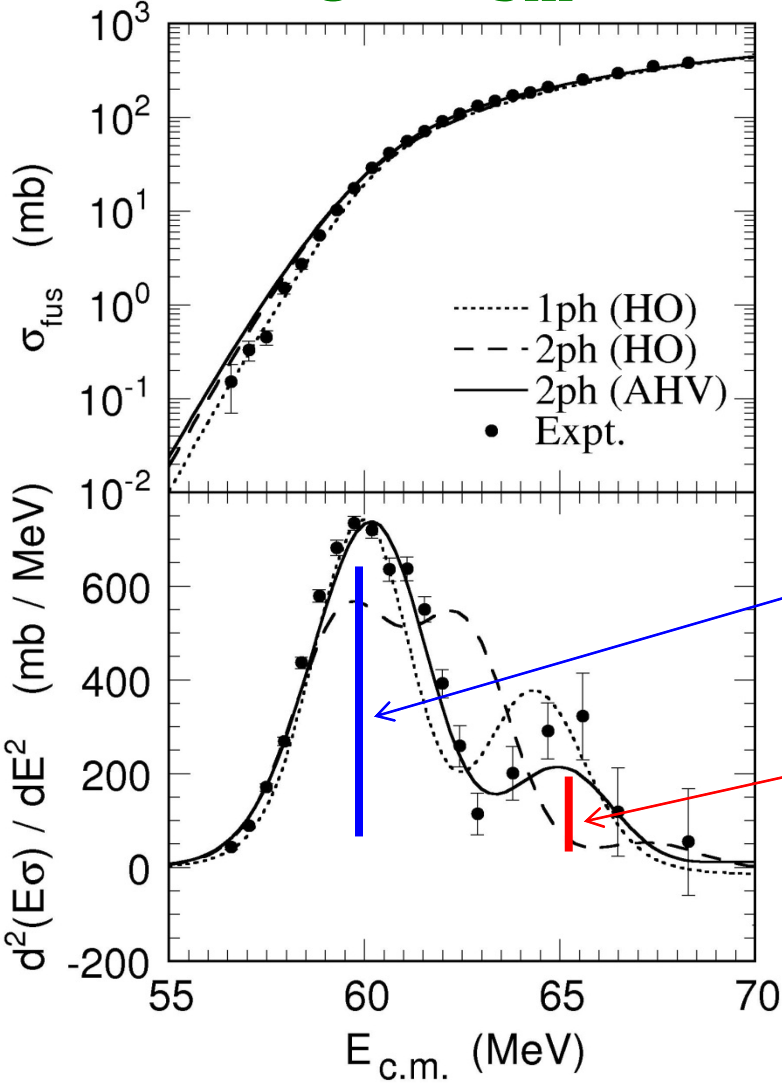

Quadrupole moment:

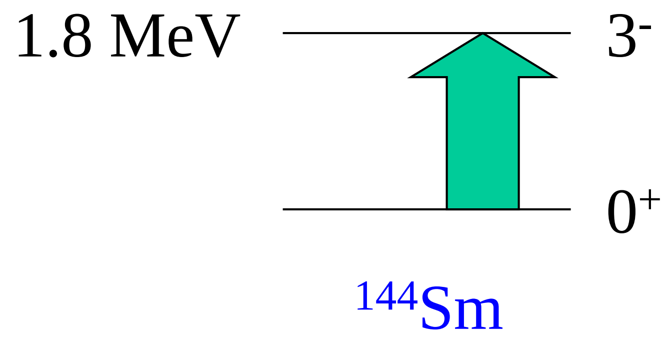
$$Q(3^-) = -0.70 \pm 0.02\mathrm{b}$$

K.Hagino, N. Takigawa, and S. Kuyucak, PRL79('97)2943

# Barrier distribution

$^{16}O + {}^{144}Sm$




$$\alpha|0^+\rangle + \beta|3^-\rangle$$

$$\beta|0^+\rangle - \alpha|3^-\rangle$$

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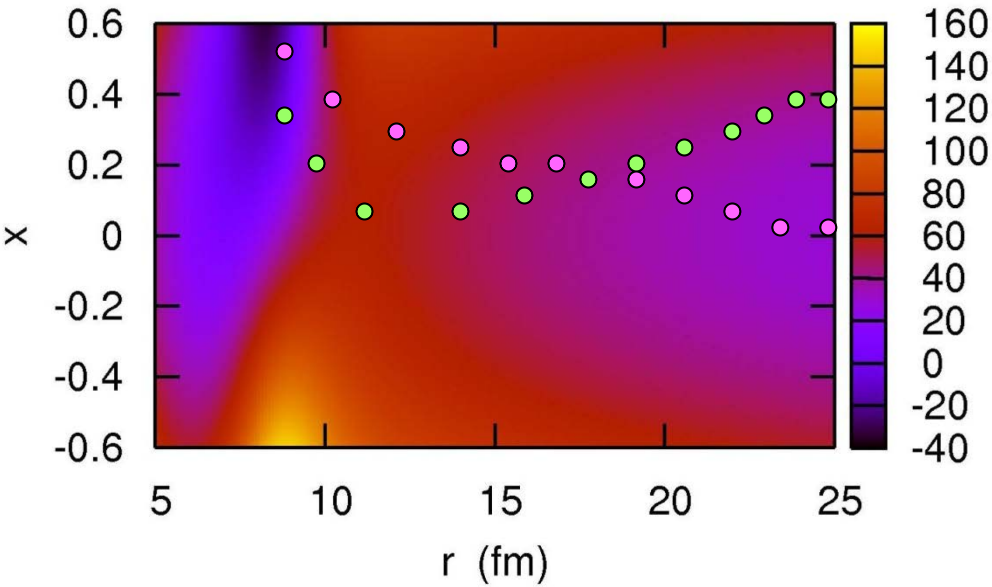
Coupling to excited states $\longrightarrow$ distribution of potential barrier

multi-dimensional potential surface

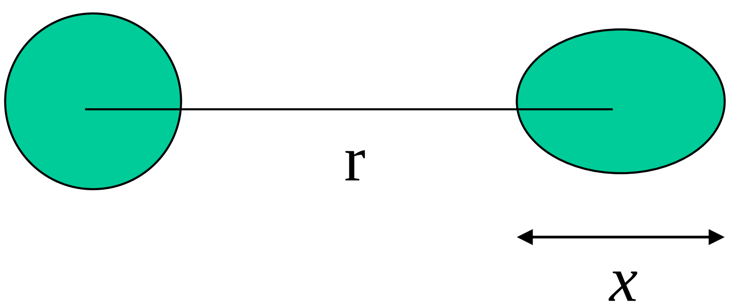



relative distance

single barrier

$\longrightarrow$ a collection of many barriers

$$P(E) = P[E, V(r)]$$
$$\to P(E) = \sum_{\alpha} w_{\alpha}\, P[E, V_{\alpha}(r)]$$



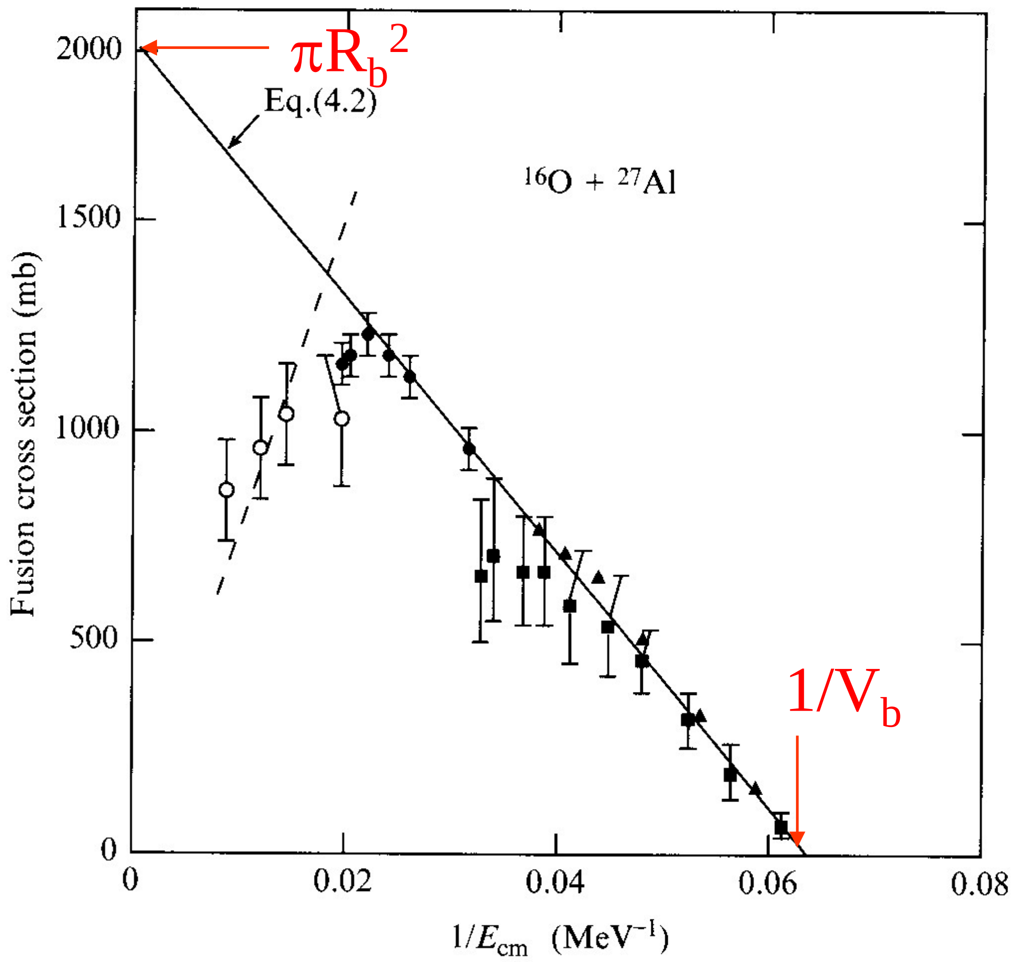

(intrinsic coordinate)

# Representations of fusion cross sections

## i) $\sigma_{fus}$ vs 1/E (~70's)

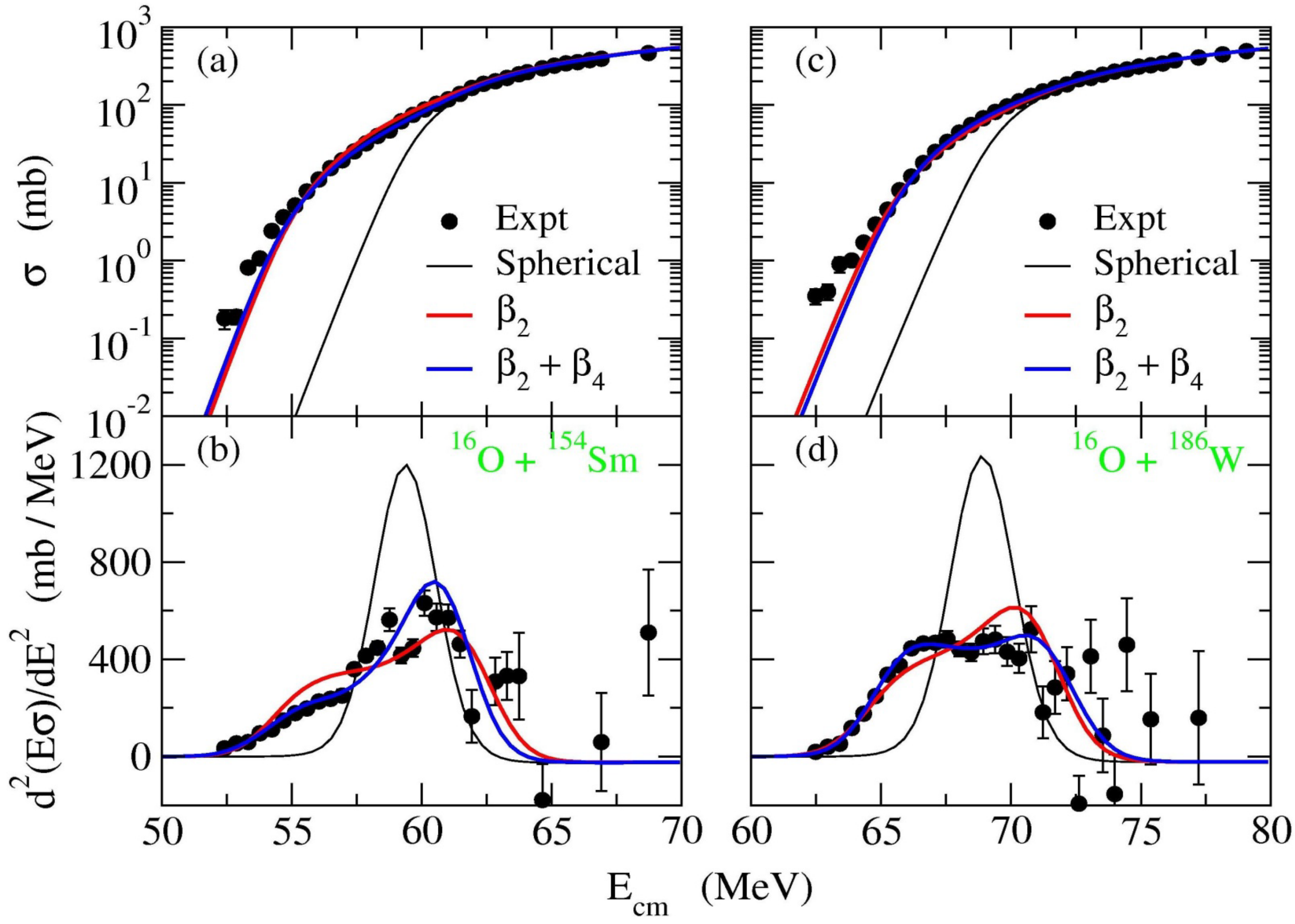
Classical fusion cross section is proportional to $1/E$ :

$$\sigma_{\text{fus}}^{cl}(E) = \pi R_b^2 \left(1 - \frac{V_b}{E}\right)$$




Taken from J.S. Lilley, "Nuclear Physics"

ii) barrier distribution (~90's)



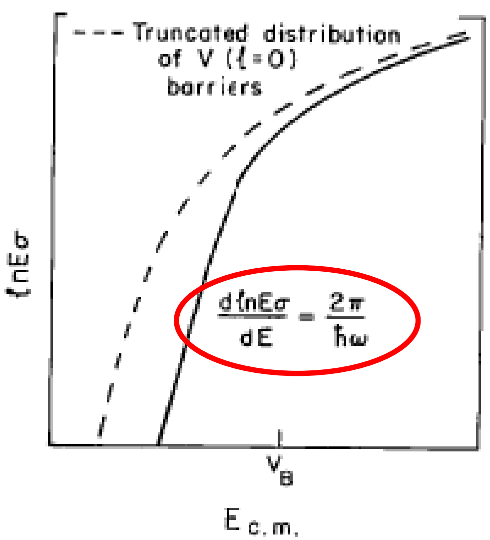

M. Dasgupta et al.,
Annu. Rev. Nucl. Part. Sci. 48('98)401

# iii) logarithmic derivative (~00's)

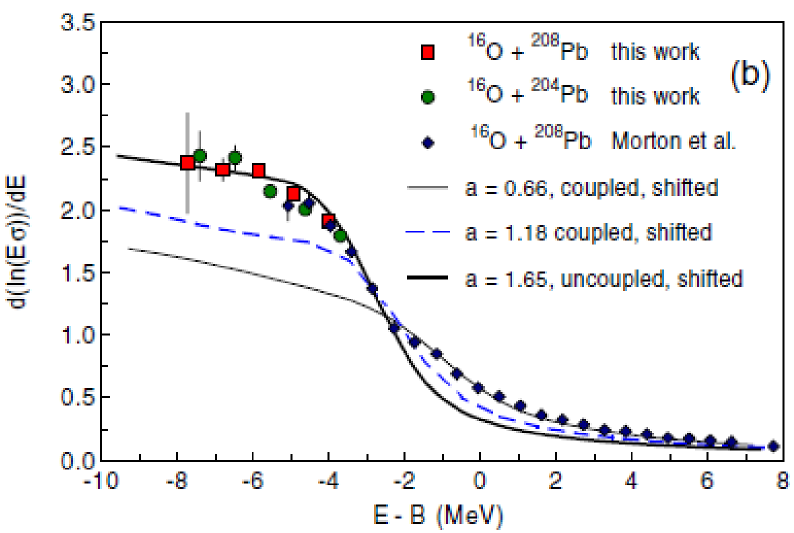
$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E}R_b^2 \log\left[1+\exp\left(\frac{2\pi}{\hbar\Omega}(E-V_b)\right)\right]$$
$$\sim \frac{\hbar\Omega}{2E}R_b^2 \exp\left(\frac{2\pi}{\hbar\Omega}(E-V_b)\right) \qquad (E \ll V_b)$$

$$\boxed{\frac{d}{dE}\log(E\sigma) = \frac{(E\sigma)'}{E\sigma} = \frac{2\pi}{\hbar\Omega}}$$

cf. $D_{\text{fus}} = (E\sigma)''$

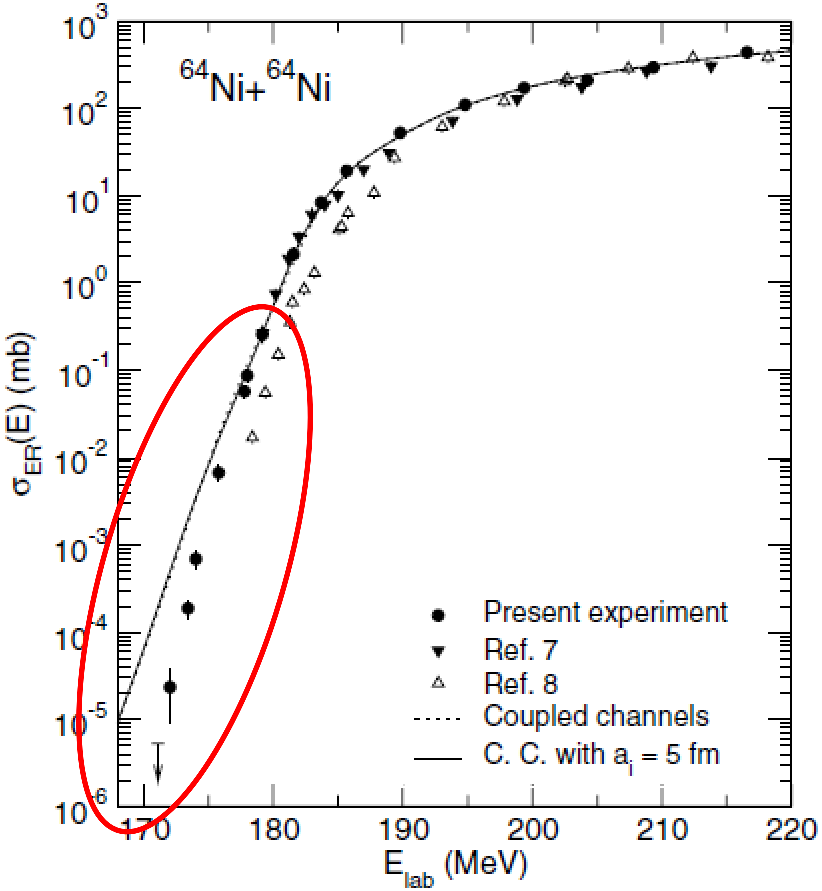


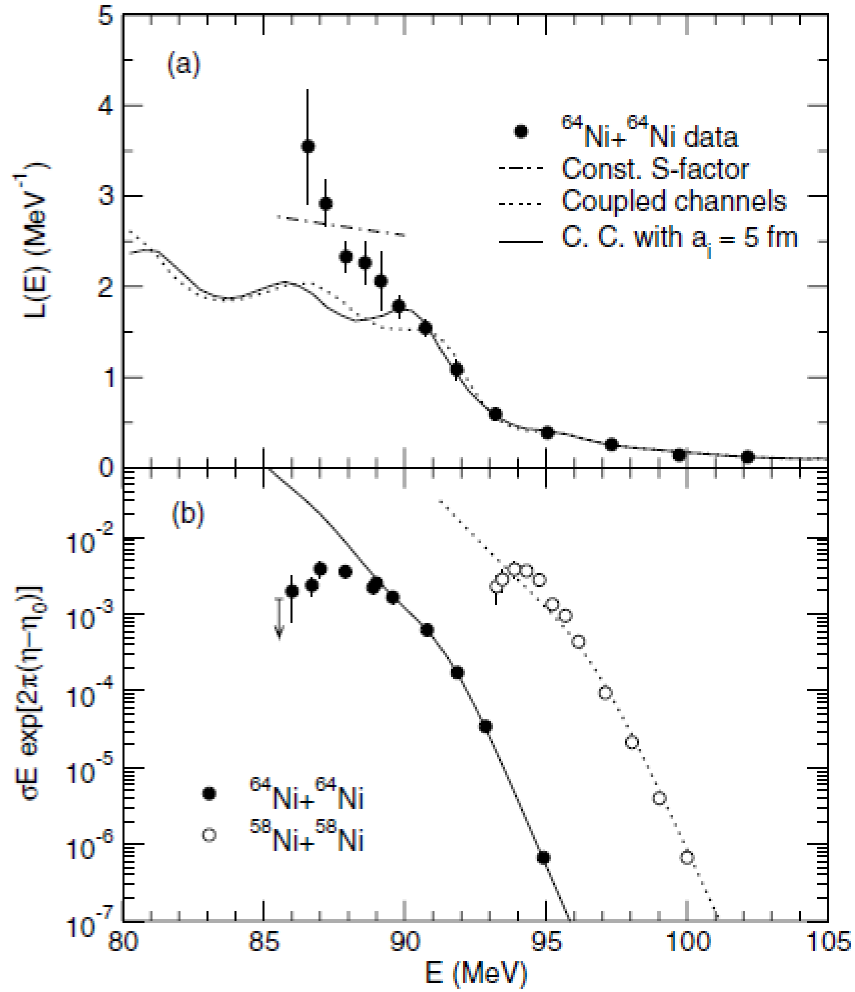

R. Vandenbosch,
Ann. Rev. Nucl. Part. Sci. 42('92)447




M. Dasgupta et al., PRL99('07) 192701

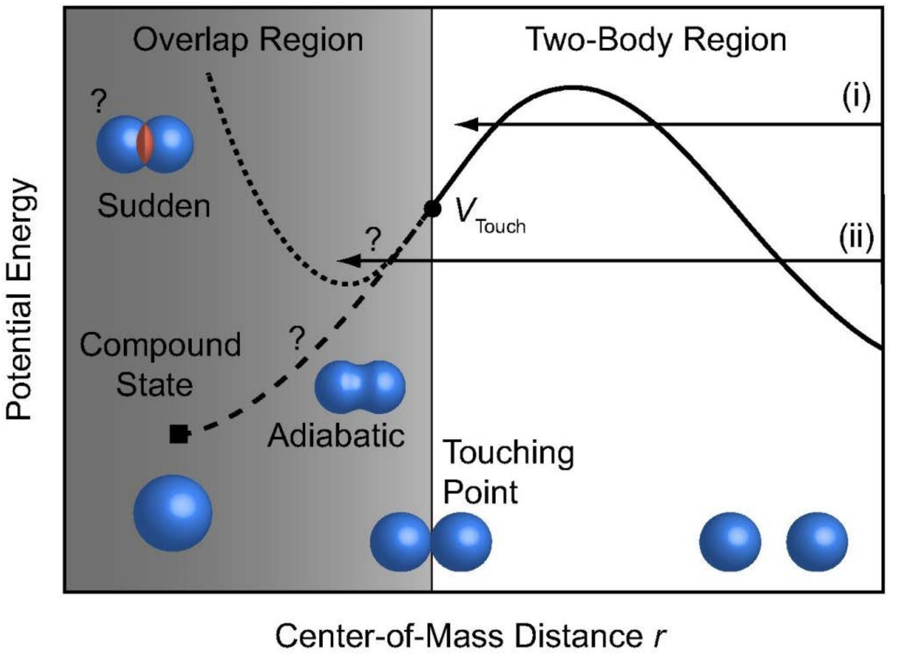
# deep subbarrier hindrance of fusion cross sections

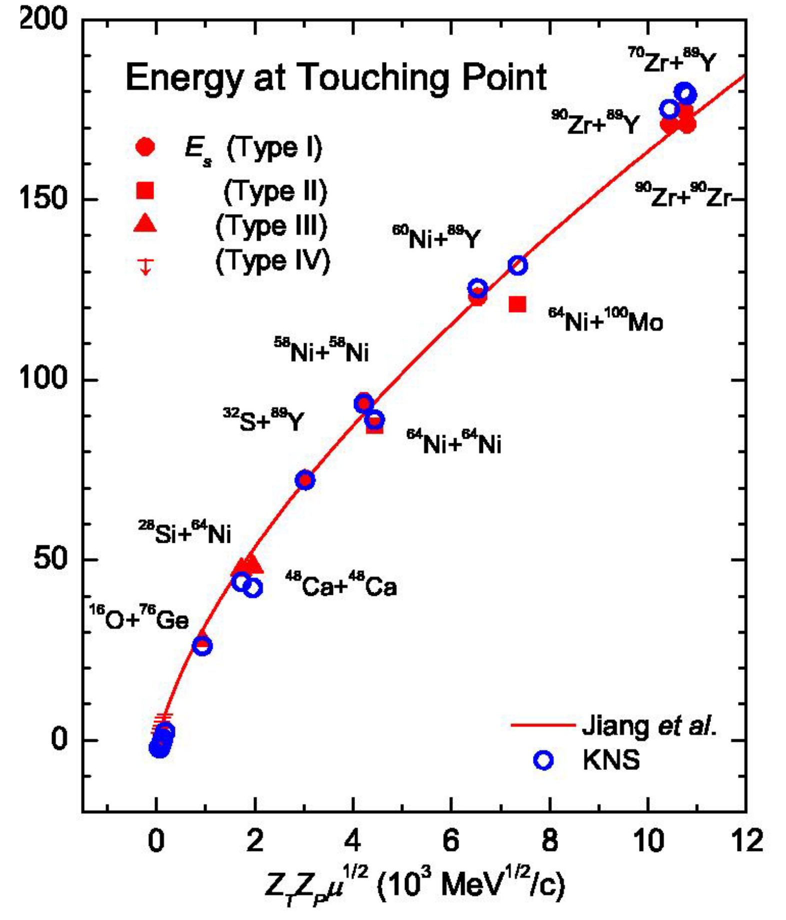



C.L. Jiang et al., PRL89(‘02)052701; PRL93(‘04)012701

# Systematics of the touching point energy and deep subbarrier hindrance



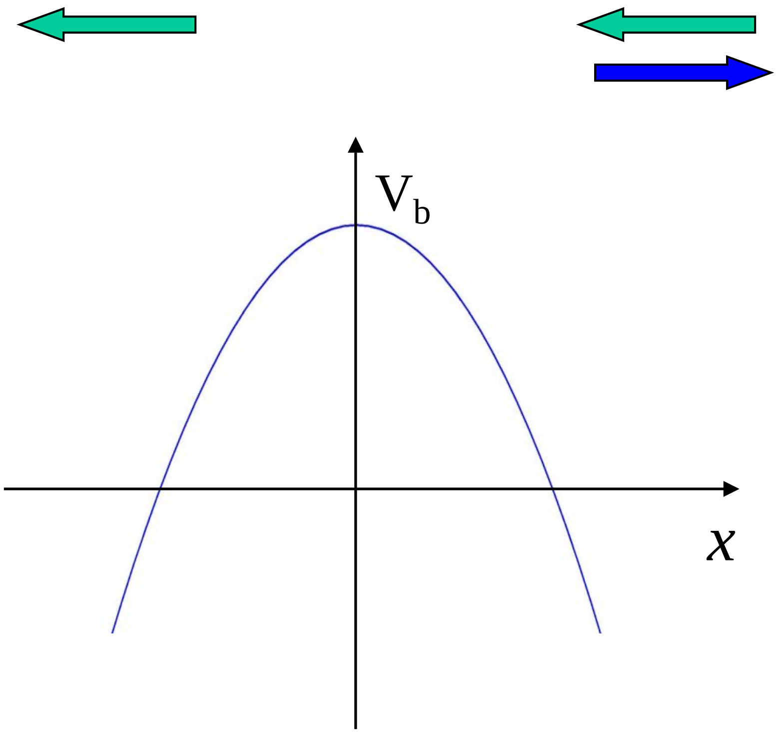



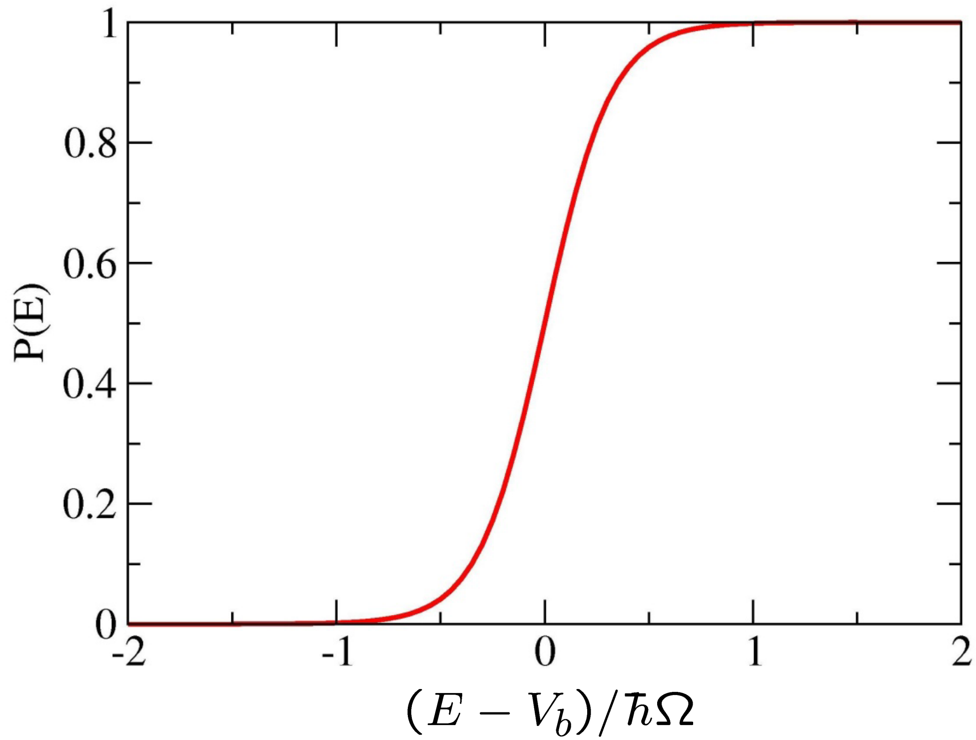

mechanism of deep subbarrier hindrance:
not yet been fully understood

how to model the dynamics after touching?

T. Ichikawa, K.H., A. Iwamoto, PRC75('07) 064612 & 057603

# Quantum reflection and quasi-elastic scattering



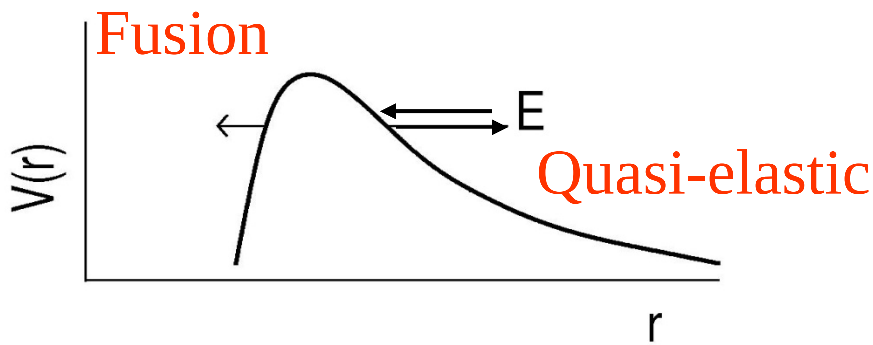

In quantum mechanics, reflection occurs even at $E > V_b$

Quantum Reflection

$$P(E) + R(E) = 1$$

Reflection prob. carries the same information as penetrability, and barrier distribution can be defined in terms of reflection prob.

## Quasi-Elastic Scattering



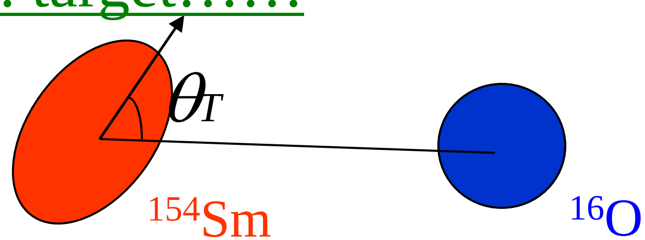

A sum of all the reaction processes other than fusion (elastic + inelastic + transfer + ……)

Detect all the particles which reflect at the barrier and hit the detector

Related to reflection

Complementary to fusion

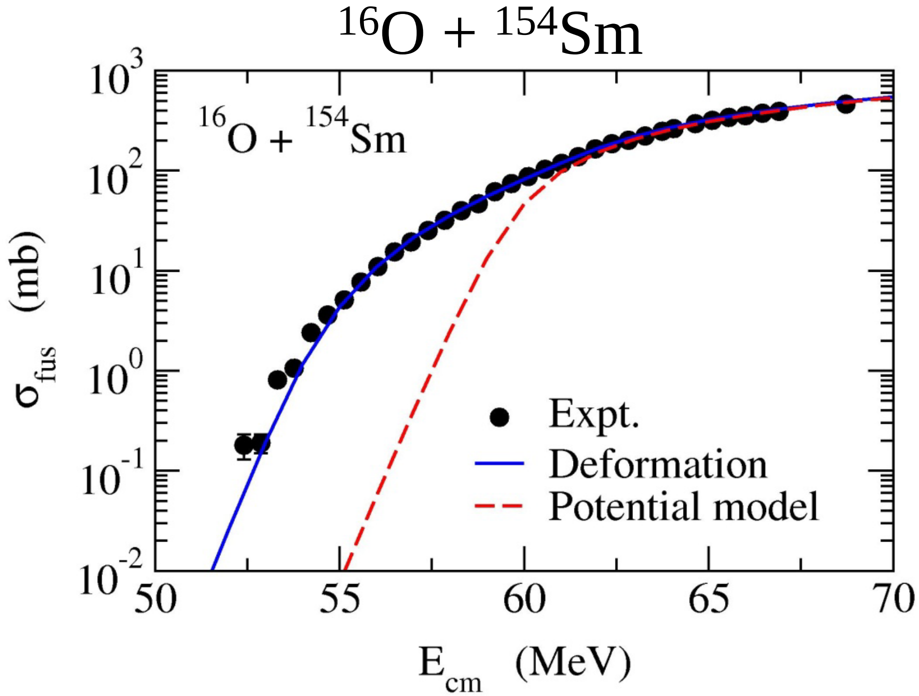
In case of a def. target……

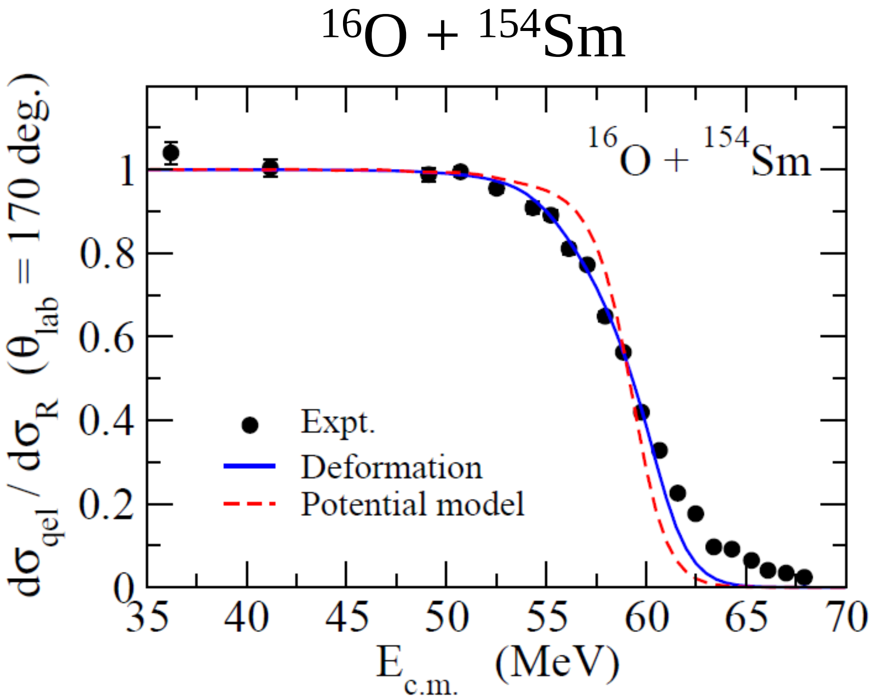


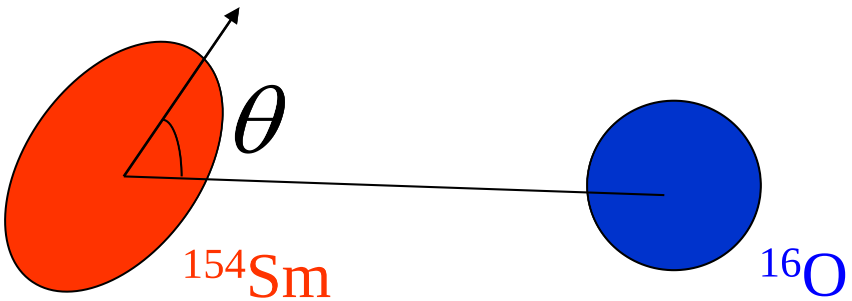

$$\begin{cases} \sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\text{fus}}(E;\theta_T) \\ \sigma_{\text{qel}}(E,\theta) = \sum_I \sigma(E,\theta) = \int_0^1 d(\cos\theta_T)\sigma_{\text{el}}(E,\theta;\theta_T) \end{cases}$$

Subbarrier enhancement of fusion cross sections

Quasi-elastic scattering (elastic + inelastic)



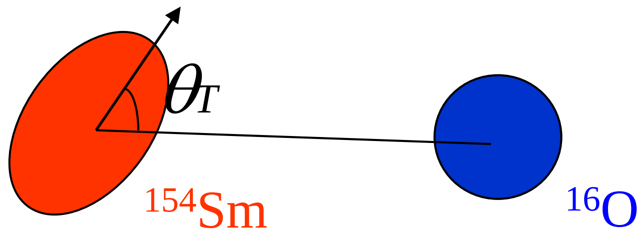




$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$

$$\sigma_{\text{qel}}(E,\theta) = \int_0^1 d(\cos\theta_T)\sigma_{\text{el}}(E,\theta;\theta_T)$$

# Quasi-elastic barrier distribution




$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\text{fus}}(E;\theta_T)$$

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

$$\sigma_{\text{qel}}(E,\theta) = \sum_I \sigma(E,\theta) = \int_0^1 d(\cos\theta_T)\sigma_{\text{el}}(E,\theta;\theta_T)$$

## Quasi-elastic barrier distribution:

$$\boxed{D_{\text{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\text{qel}}(E,\pi)}{\sigma_R(E,\pi)}\right)}$$

H. Timmers et al.,
NPA584('95)190

(note) Classical elastic cross section in the limit of strong Coulomb field:

$$\sigma_{\text{el}}^{cl}(E,\pi) = \sigma_R(E,\pi)\theta(V_b - E)$$

$$\frac{\sigma_{\text{el}}^{cl}(E,\pi)}{\sigma_R(E,\pi)} = \theta(V_b - E) = R(E)$$

# Quasi-elastic test function

Classical elastic cross section (in the limit of a strong Coulomb):

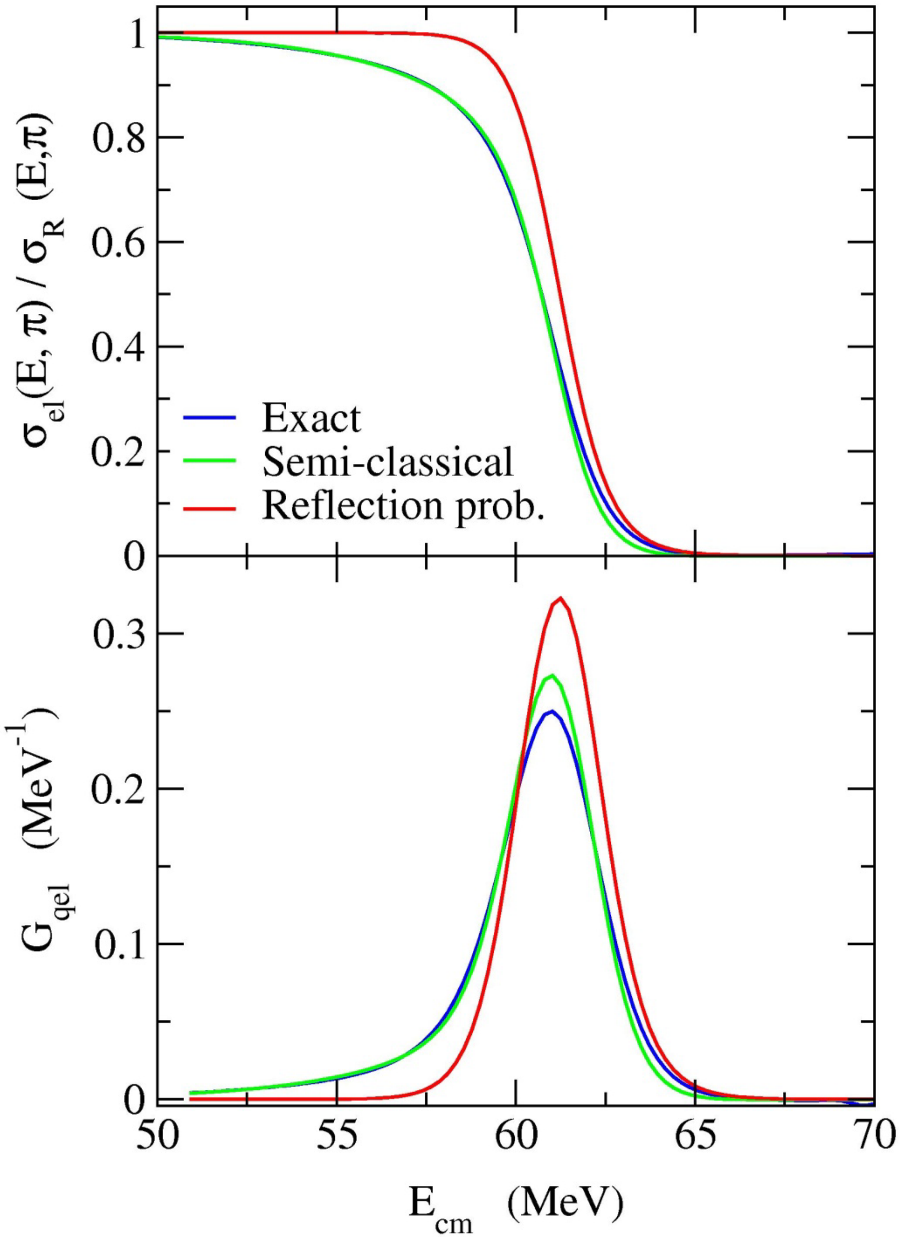
$$\sigma^{cl}_{\mathrm{el}}(E,\pi) = \sigma_R(E,\pi)\theta(V_b - E)$$

$$\boxed{\begin{aligned} \frac{\sigma^{cl}_{\mathrm{el}}(E,\pi)}{\sigma_R(E,\pi)} &= \theta(V_b - E) = R(E) \\ -\frac{d}{dE}\left(\frac{\sigma^{cl}_{\mathrm{el}}(E,\pi)}{\sigma_R(E,\pi)}\right) &= \delta(E - V_b) \end{aligned}}$$

Nuclear effects ⟵ Semi-classical perturbation theory

$$\boxed{\frac{\sigma_{\mathrm{el}}(E,\pi)}{\sigma_R(E,\pi)} \sim \left(1 + \frac{V_N(r_c)}{ka}\frac{\sqrt{2a\pi k\eta}}{E}\right)\cdot R(E)}$$

S. Landowne and H.H. Wolter, NPA351('81)171
K.H. and N. Rowley, PRC69('04)054610



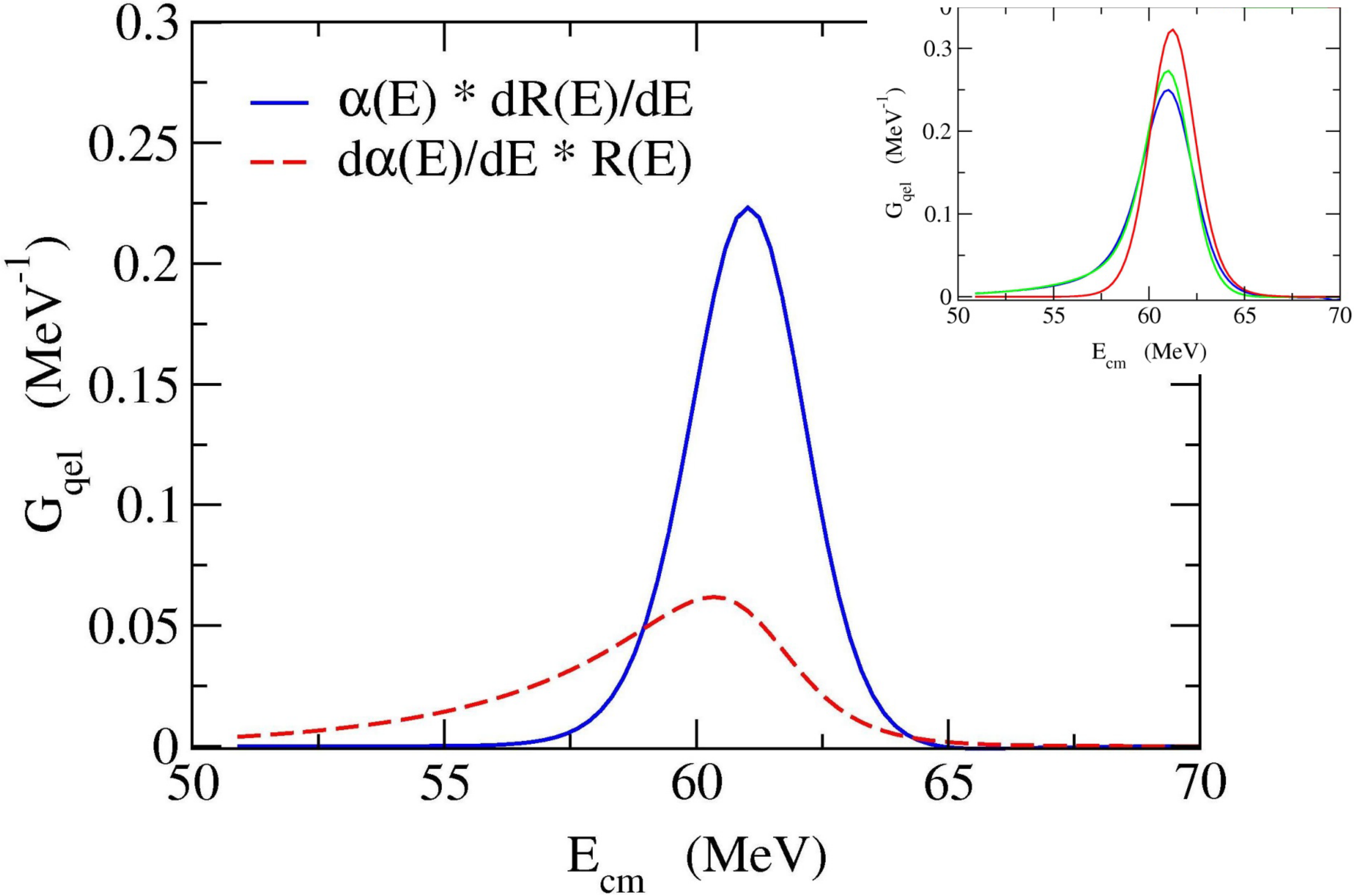

Quasi-elastic test function

$$\boxed{G_{\mathsf{qel}}(E) \equiv -\frac{d}{dE}\left(\frac{\sigma_{\mathsf{el}}(E,\pi)}{\sigma_R(E,\pi)}\right)}$$

- The peak position slightly deviates from $V_b$
- Low energy tail
- Integral over E: unity
- Relatively narrow width

$\Rightarrow$

Close analog to fusion b.d.

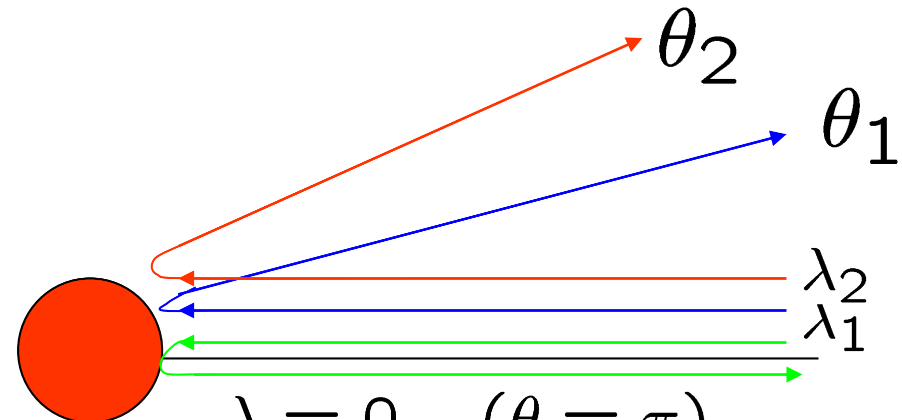


α(E) * dR(E)/dE
dα(E)/dE * R(E)
$G_{qel}$ (MeV$^{-1}$)
$E_{cm}$ (MeV)
0
0.05
0.1
0.15
0.2
0.25
0.3
50
55
60
65
70

# Scaling property

Expt.: impossible to perform at $\theta = \pi$

→ Relation among different $\theta$?

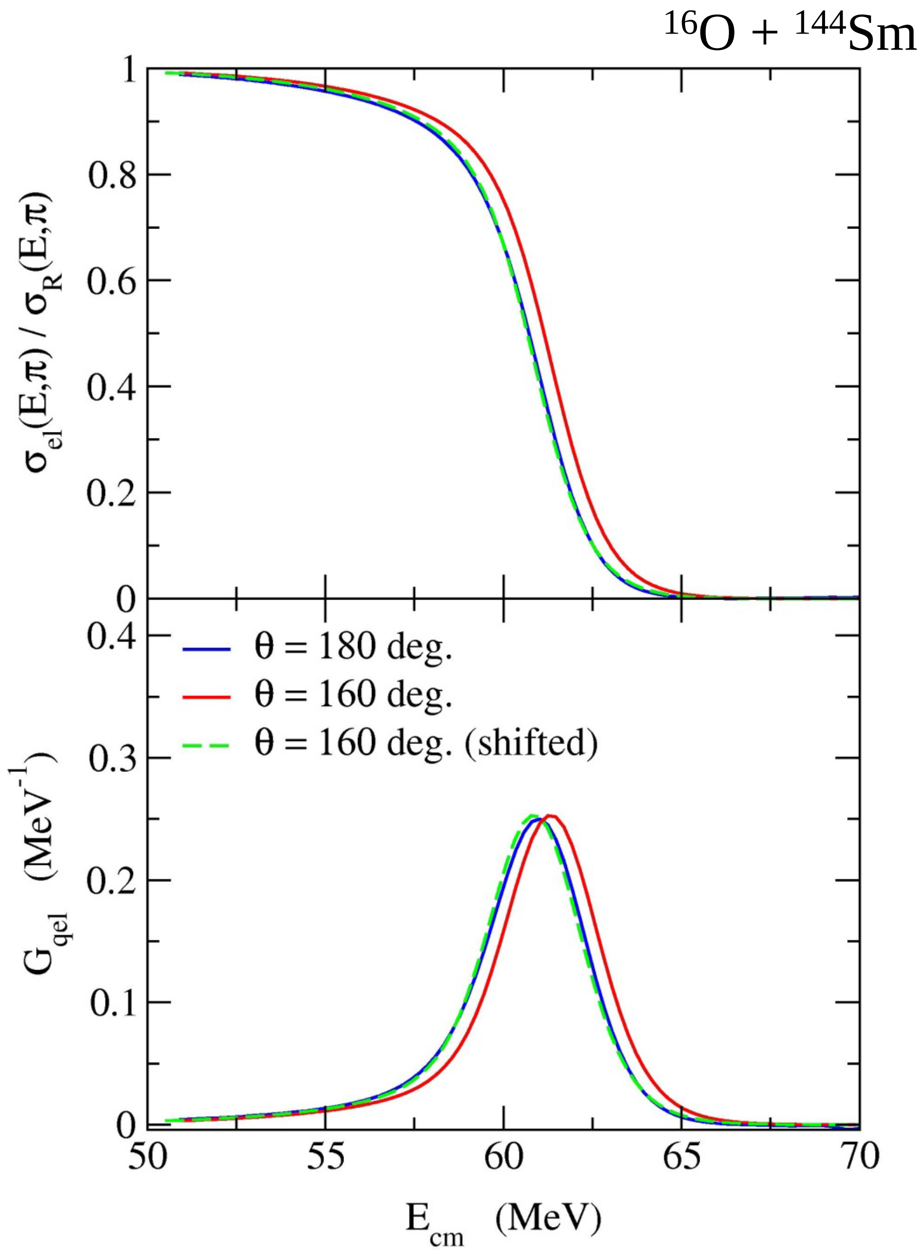


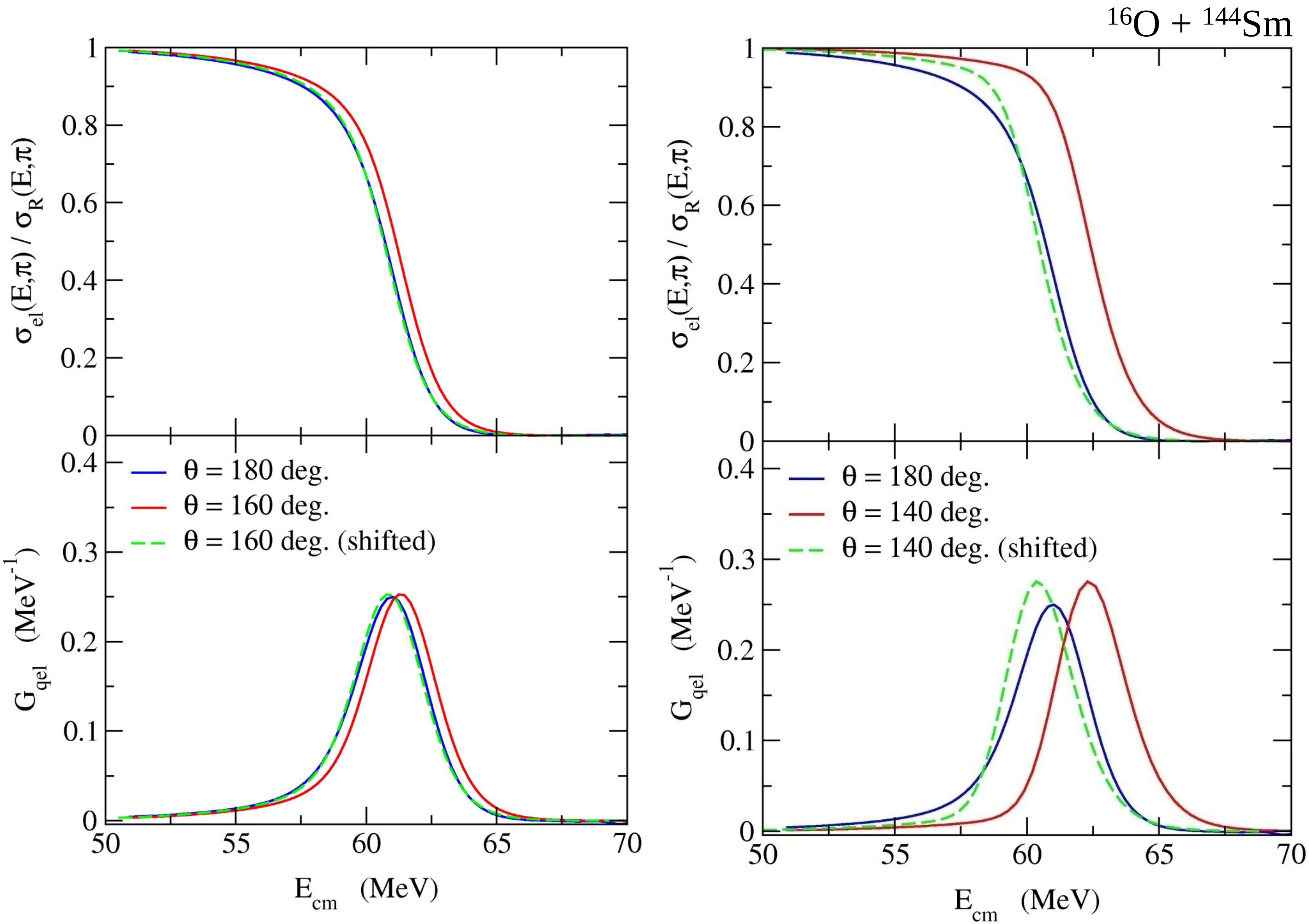

$\lambda = 0 \quad (\theta = \pi)$

## Effective energy:

$$\begin{aligned} E_{\text{eff}} &\sim E - \frac{\lambda_c^2 \hbar^2}{2\mu r_c^2} \\ &= 2E\,\frac{\sin(\theta/2)}{1+\sin(\theta/2)} \\ D_{\text{qel}}(E,\theta) &\sim D_{\text{qel}}(E_{\text{eff}},\pi) \end{aligned}$$

$$\lambda_c = \eta \cot(\theta/2)$$



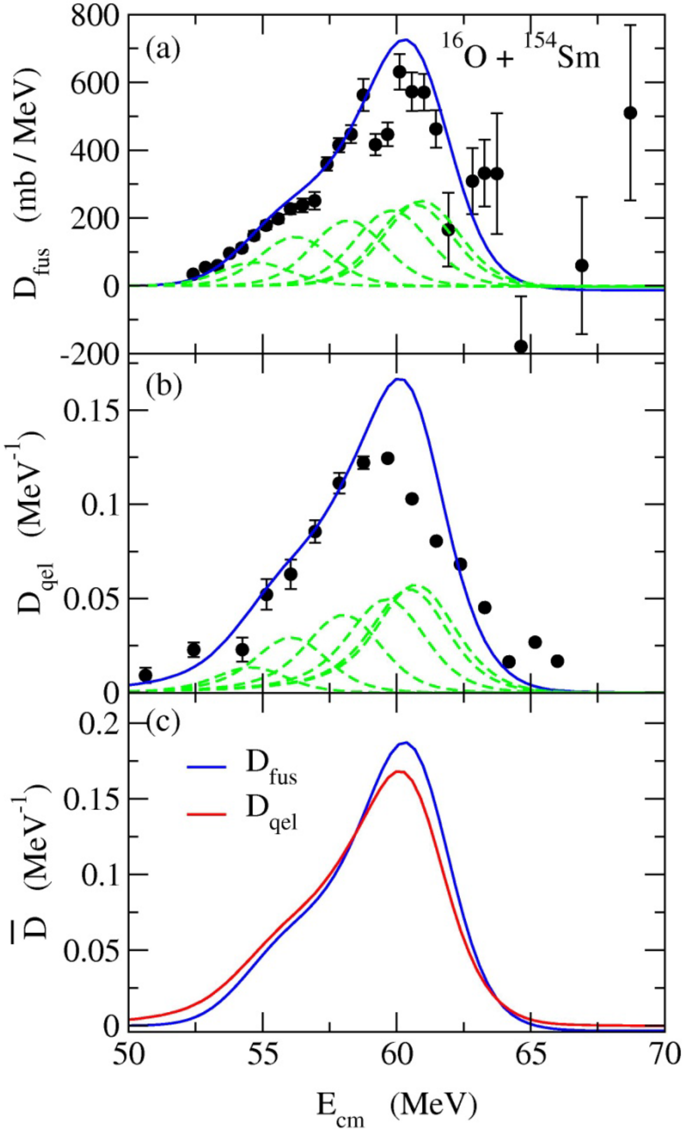


$^{16}$O + $^{144}$Sm
$\sigma_{el}(E,\pi) / \sigma_R(E,\pi)$
$G_{qel}$ (MeV$^{-1}$)
$E_{cm}$ (MeV)
θ = 180 deg.
θ = 160 deg.
θ = 160 deg. (shifted)
θ = 180 deg.
θ = 140 deg.
θ = 140 deg. (shifted)

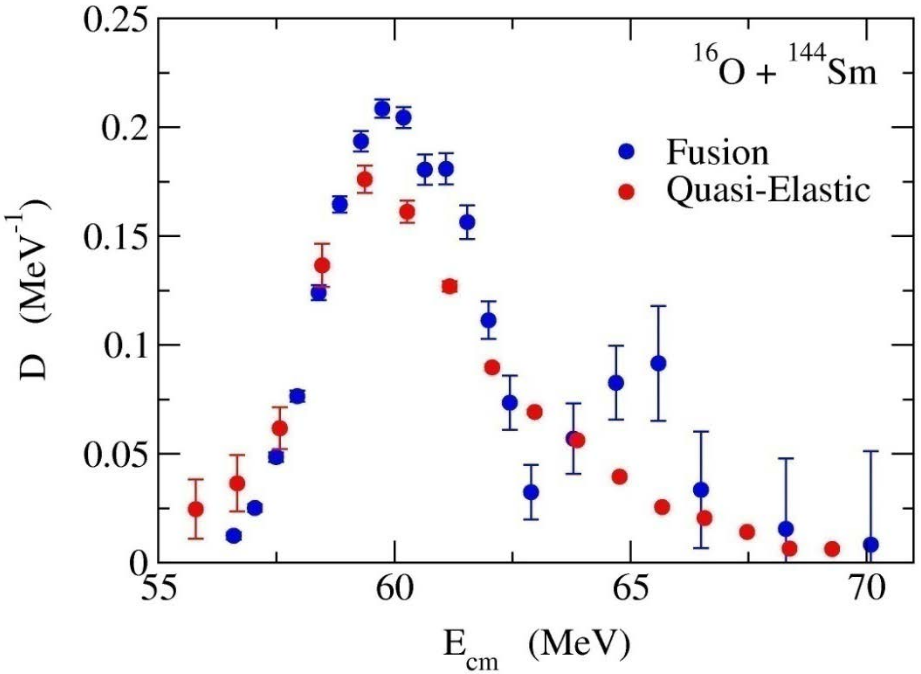
# Comparison of $D_{fus}$ with $D_{qel}$

Fusion $$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

Quasi-elastic $$D_{\text{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\text{qel}}(E,\pi)}{\sigma_R(E,\pi)}\right)$$




K.H. and N. Rowley, PRC69('04)054610




H. Timmers et al., NPA584('95)190

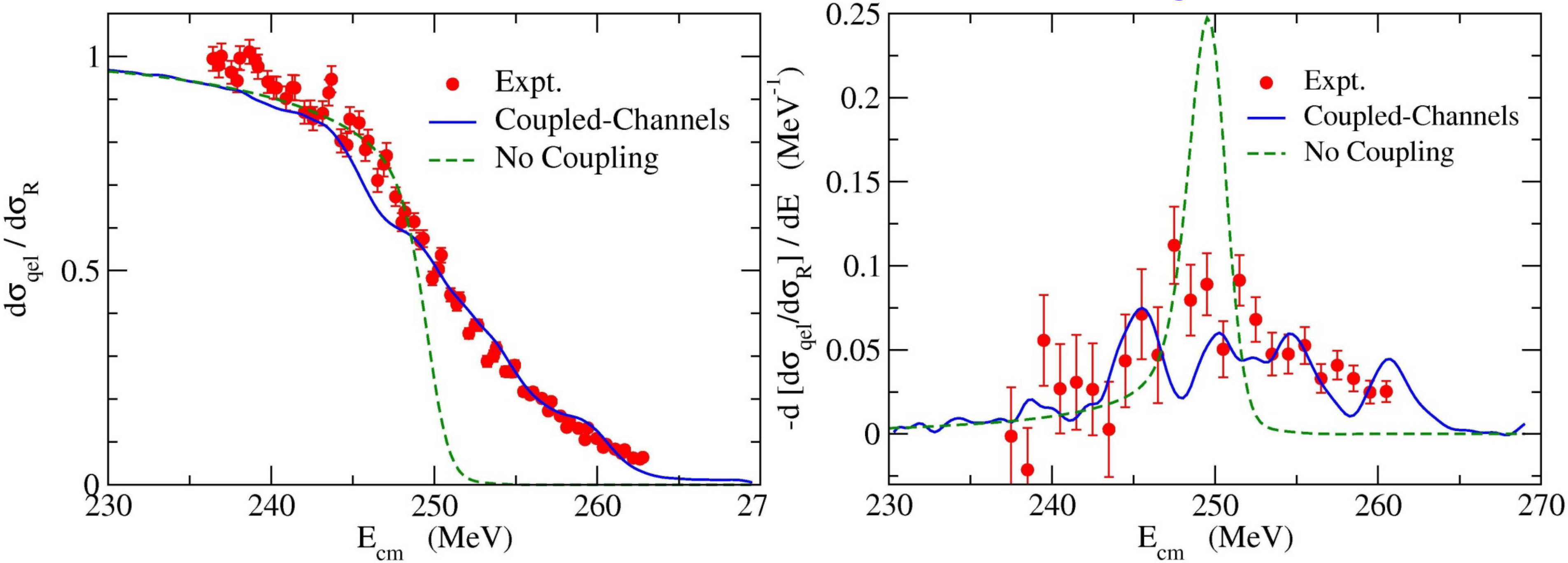
A gross feature is similar to each other

# Experimental barrier distribution with QEL scattering

$$D_{\mathrm{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\mathrm{qel}}(E,\pi)}{\sigma_R(E,\pi)}\right)$$

H. Timmers et al.,
NPA584('95)190

$^{70}$Zn + $^{208}$Pb




$^{70}$Zn : $E_2$ = 0.885 MeV, 2 phonon, $^{208}$Pb: $E_3$ = 2.614 MeV, 3 phonon

Muhammad Zamrun F., K. H., S. Mitsuoka, and H. Ikezoe, PRC77('08)034604.

Experimental Data: S. Mitsuoka et al., PRL99('07)182701

# Experimental advantages for $D_{qel}$

$$D_{\mathrm{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\mathrm{qel}}(E,\pi)}{\sigma_R(E,\pi)}\right) \qquad D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

- less accuracy is required in the data (1st vs. 2nd derivative)
- much easier to be measured

Qel : a sum of everything

$\longrightarrow$ a very simple charged-particle detector

Fusion : requires a specialized recoil separator

to separate ER from the incident beam

ER + fission for heavy systems

- several effective energies can be measured at a single-beam energy $\longleftrightarrow$ relation between a scattering angle and an impact parameter

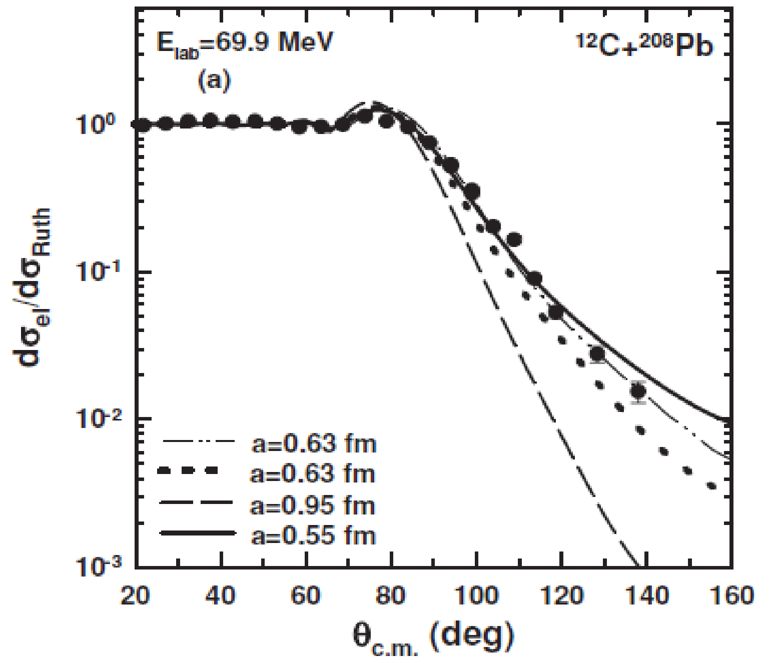
$$\boxed{E_{\mathrm{eff}} = 2E\,\sin(\theta/2)/[1+\sin(\theta/2)]}$$

# Deep subbarrier fusion and diffuseness anomaly

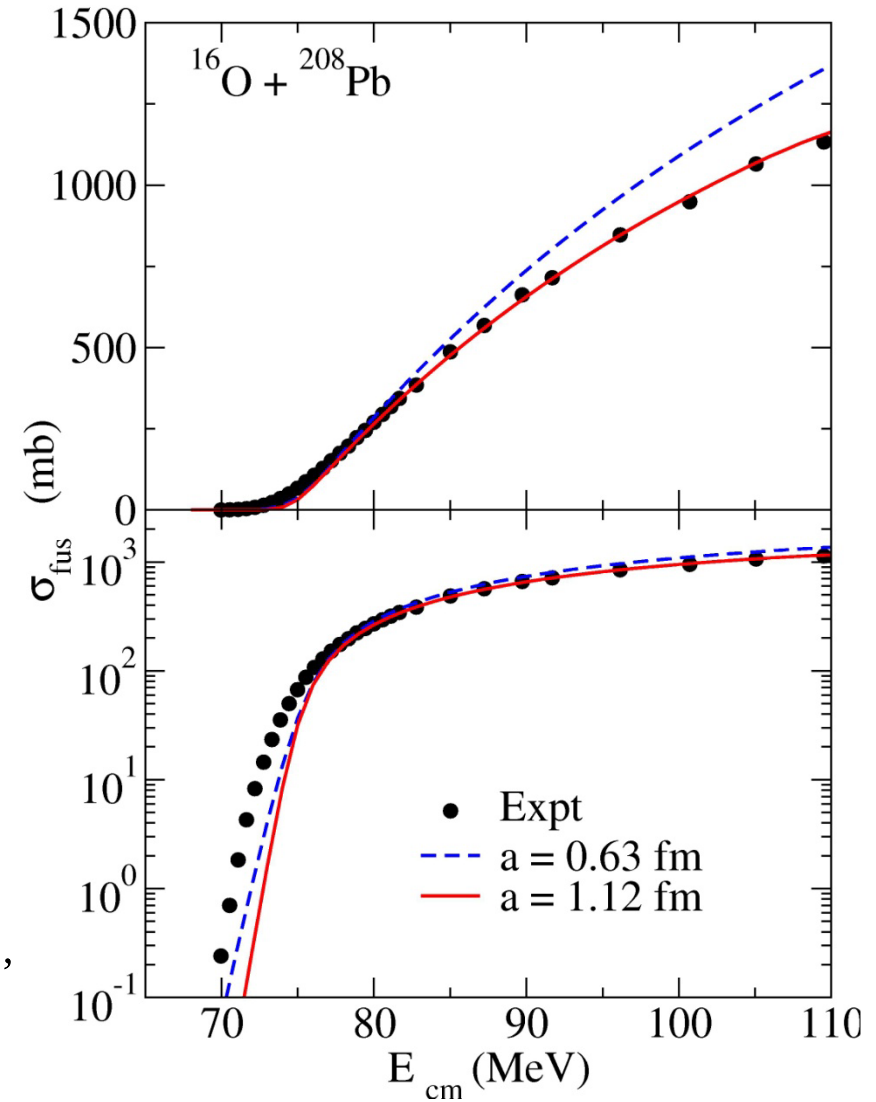
*Scattering processes:*

Double folding pot.
Woods-Saxon (a ~ 0.63 fm)

→ successful

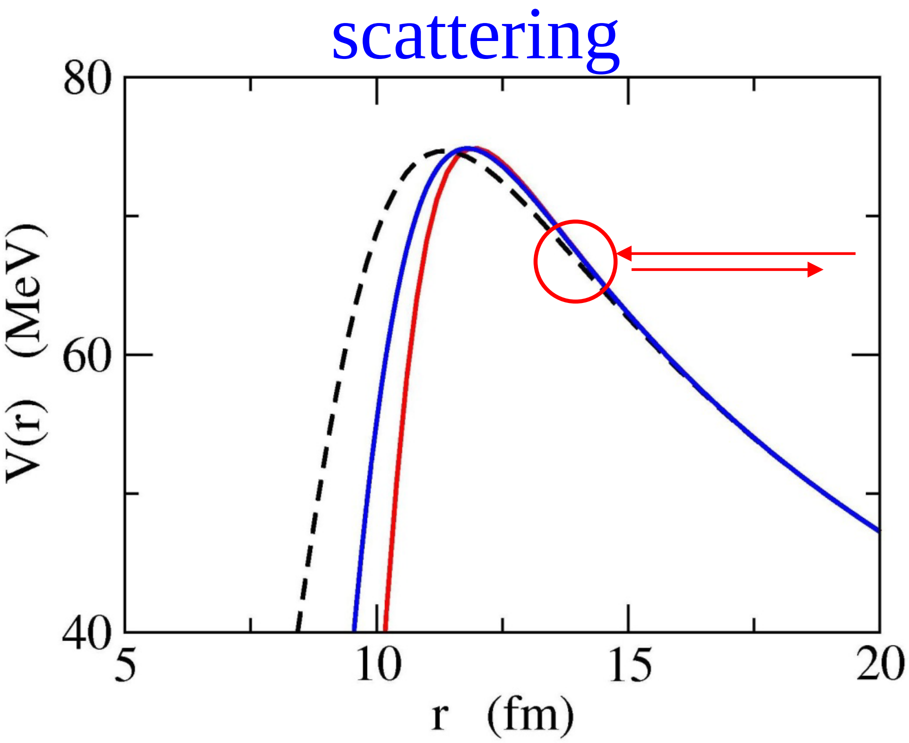


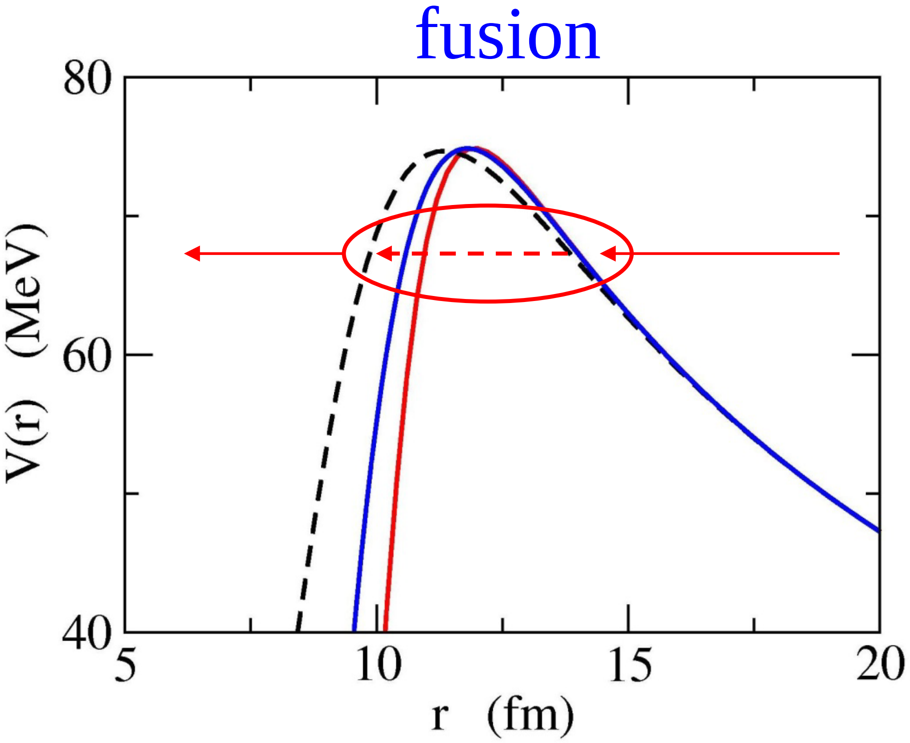

A. Mukherjee, D.J. Hinde, M. Dasgupta, K.H., et al.,
PRC75('07)044608




*Fusion process: not successful*

→ *a* ~ 1.0 fm required (if WS)

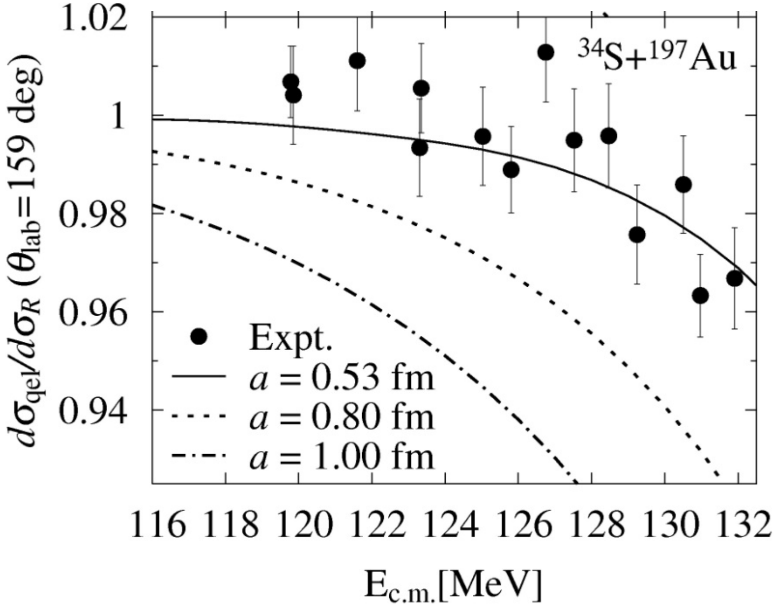


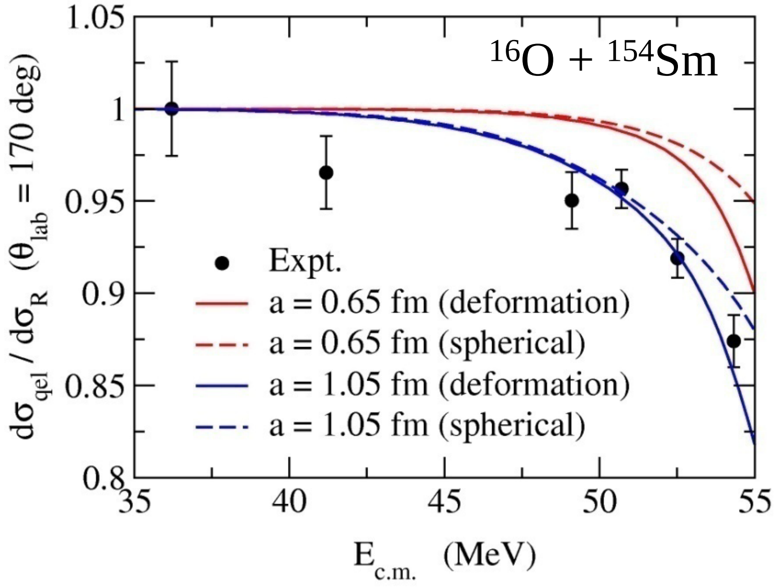




- How reliable is the DFM/WS?
- What is an optimum potential?

⇨ deduction of fusion barrier from exp. data?
(model independent analysis?)

# Quasi-elastic scattering at deep subbarrier energies?

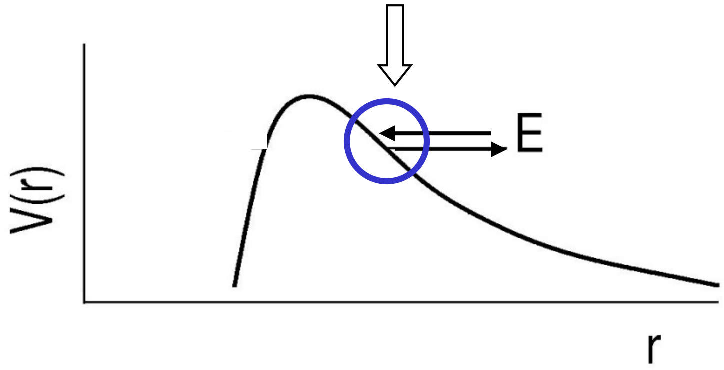



K.H., T. Takehi, A.B. Balantekin, and N. Takigawa, PRC71('05) 044612

K. Washiyama, K.H., M. Dasgupta, PRC73('06) 034607

QEL at deep subbarrier energies: sensitive only to the surface region




$$\frac{\sigma_{\rm el}(E,\pi)}{\sigma_R(E,\pi)} \sim \left(1+\frac{V_N(r_c)}{ka}\frac{\sqrt{2a\pi k\eta}}{E}\right)\cdot R(E)$$

$$\sim 1+\frac{V_N(r_c)}{ka}\frac{\sqrt{2a\pi k\eta}}{E}$$

## *Summary*

**Heavy-Ion Fusion Reactions around the Coulomb Barrier**

✧Fusion and quantum tunneling

Fusion takes place by tunneling

✧Basics of the Coupled-channels method

Collective excitations during fusion

✧Concept of Fusion barrier distribution

Sensitive to nuclear structure $$D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

✧Quasi-elastic scattering and quantum reflection

Complementary to fusion

Computer program: CCFULL

http://www.nucl.phys.tohoku.ac.jp/~hagino/ccfull.html

# References

## Nucleear Reaction in general


- G.R. Satchler, “*Direct Nuclear Reactions*”
- G.R. Satchler, “*Introduction to Nuclear Reaction*”
- R.A. Broglia and A. Winther, “*Heavy-Ion Reactions*”
- “*Treatise on Heavy-Ion Science*”, vol. 1-7
- D.M. Brink, “*Semi-classical method in nucleus-nucleus collisions*”
- P. Frobrich and R. Lipperheide, “*Theory of Nuclear Reactions*”


## Heavy-ion Fusion Reactions


- M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48(’98) 401
- A.B. Balantekin and N. Takigawa, Rev. Mod. Phys. 70(’98) 77
- Proc. of Fusion03, Prog. Theo. Phys. Suppl. 154(’04)
- Proc. of Fusion97, J. of Phys. G 23 (’97)
- Proc. of Fusion06, AIP, in press.

# Hamiltonian (example 3): more general cases

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) - \epsilon\sigma_z + \hat{\sigma}_x \cdot F(x)$$
$$= -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) + \begin{pmatrix} -\epsilon & F(x) \\ F(x) & \epsilon \end{pmatrix}$$

$$\Rightarrow \quad U(x)\begin{pmatrix} -\epsilon & F(x) \\ F(x) & \epsilon \end{pmatrix}U^\dagger(x) = \begin{pmatrix} \lambda_1(x) & 0 \\ 0 & \lambda_2(x) \end{pmatrix}$$

*x* dependent

$$\boxed{P(E) = \sum_i w_i(E)\, P(E; V_0(x) + \lambda_i(x))}$$

*E* dependent

K.H., N. Takigawa, A.B. Balantekin, PRC56('97)2104 $\quad w_i(E) \sim \text{constant}$

(note) Adiabatic limit: $\epsilon \to \infty \quad \Longrightarrow \quad w_i(E) = \delta_{i,0}$