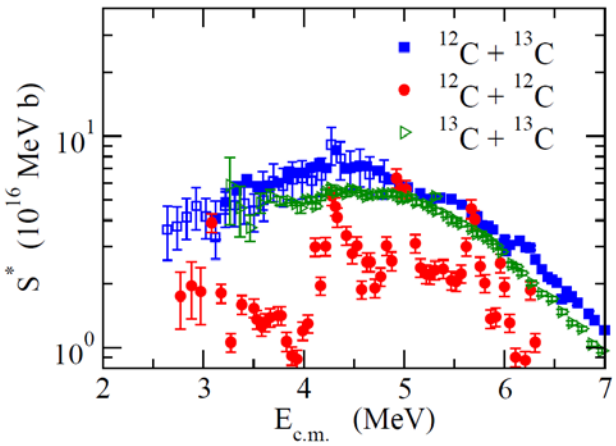
Subbarrier fusion of carbon isotopes
~ from resonance structure to fusion oscillations ~



Kouichi Hagino, *Tohoku University*
Neil Rowley, *IPN Orsay*

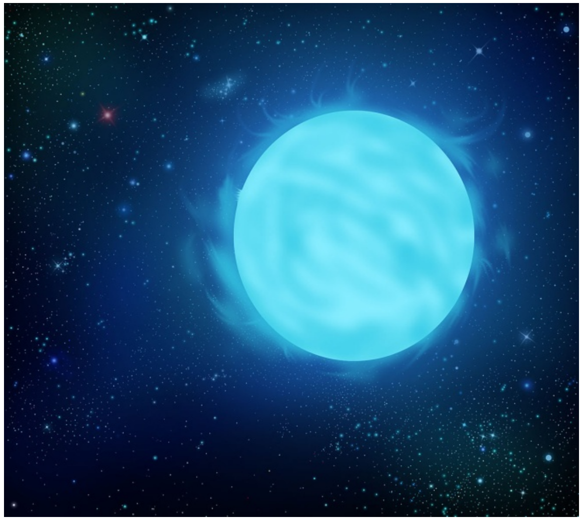
TOHOKU UNIVERSITY

1. *Introduction: $^{12}C + {}^{12}C$ fusion*
2. *Molecular resonances at subbarrier energies*
3. *Fusion oscillations at above barrier energies*
4. *Summary*

Introduction: ^{12}C + ^{12}C fusion

$^{12}C+^{12}C$ fusion : a key reaction in nuclear astrophysics

Carbon burning in massive stars



$^{12}C+^{12}C \rightarrow \alpha + ^{20}Ne$

$^{12}C+^{12}C \rightarrow p + ^{23}Na$

stellar evolution

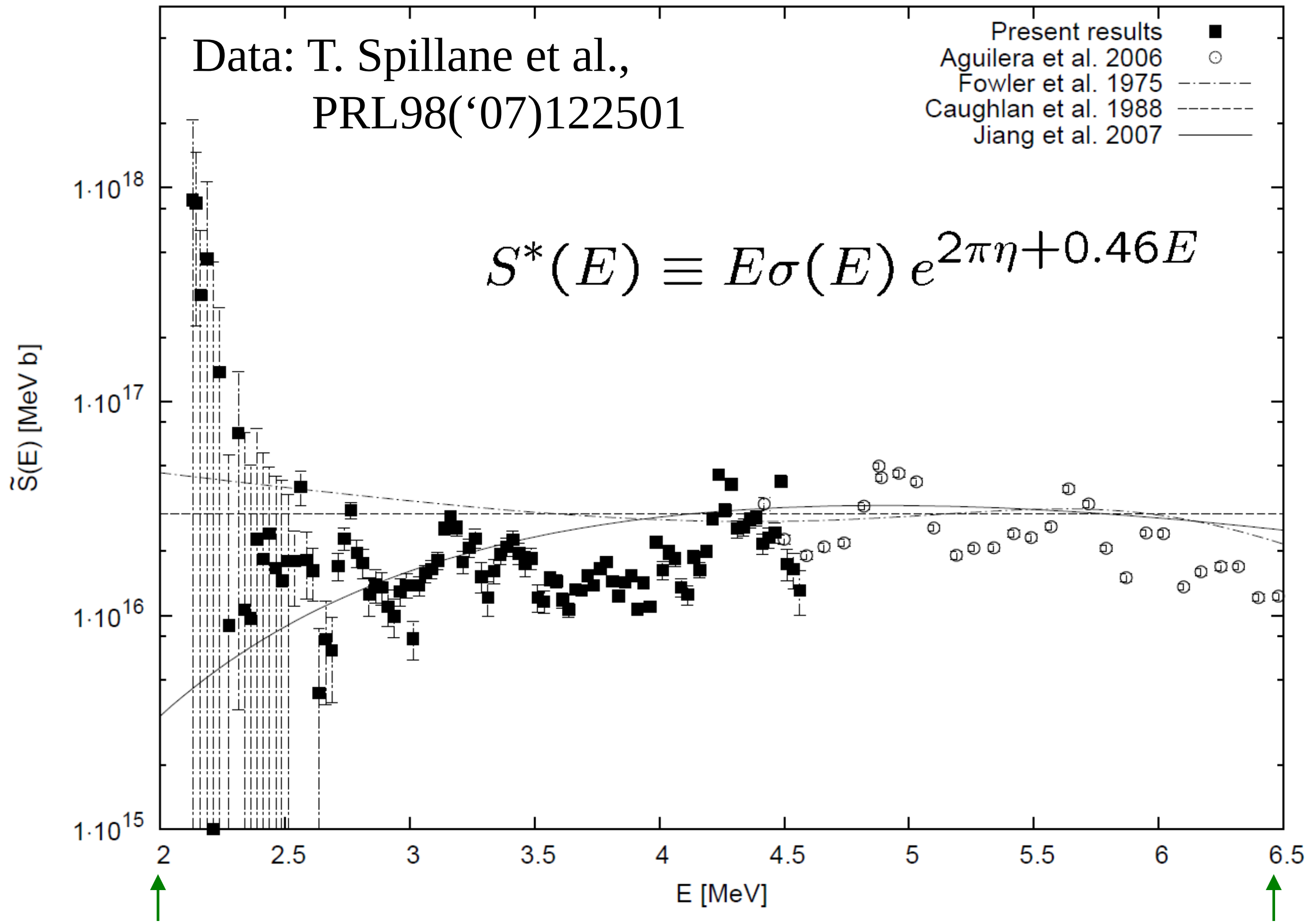
Type Ia supernovae

X-ray superburst

deep layer of the outer crust in accreting neutron stars

important to understand $^{12}C+^{12}C$ fusion at deep subbarrier energies

Experimental data at low energies



T. Spillane et al., PoS (NIC X) 016

$E_G \sim 2$ MeV (T=0.85 GK) $\sim V_b$

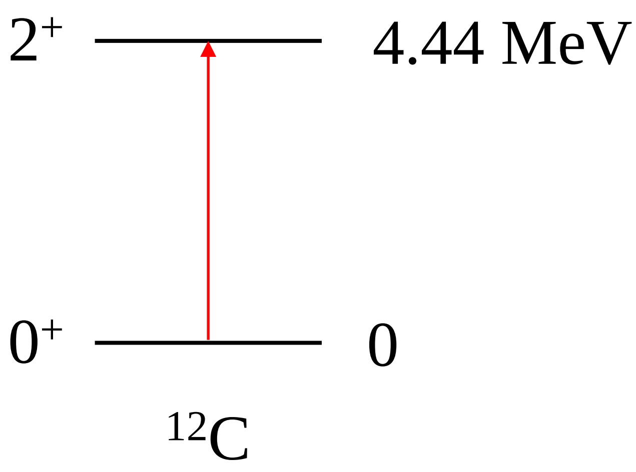
✓ pronounced resonance structures
"molecular resonances"

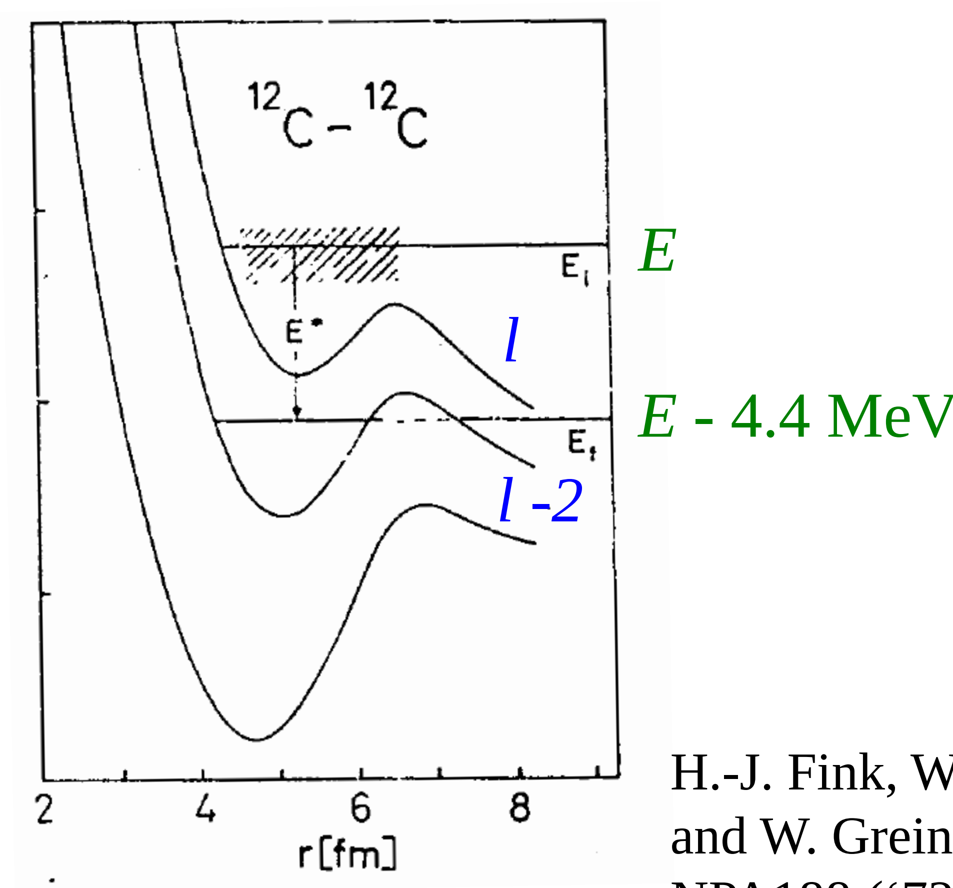
✓ difficult to extrapolate down to E_G

Theoretical calculations

- Nogami-Imanishi model (B. Imanishi, PL 27B (‘68) 267, NPA125 (‘69) 33)
- Band-crossing model (Y. Kondo, T. Matsuse, Y. Abe, PTP59 (‘78)465)
- Double resonance model (W. Scheid, W. Greiner, R. Lemmer, PRL25 (‘70) 176)

* the basic concept is all same





H.-J. Fink, W. Scheid, and W. Greiner, NPA188 (‘72) 259

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- Nogami-Imanishi model (B. Imanishi, PL 27B (‘68) 267, NPA125 (‘69) 33)
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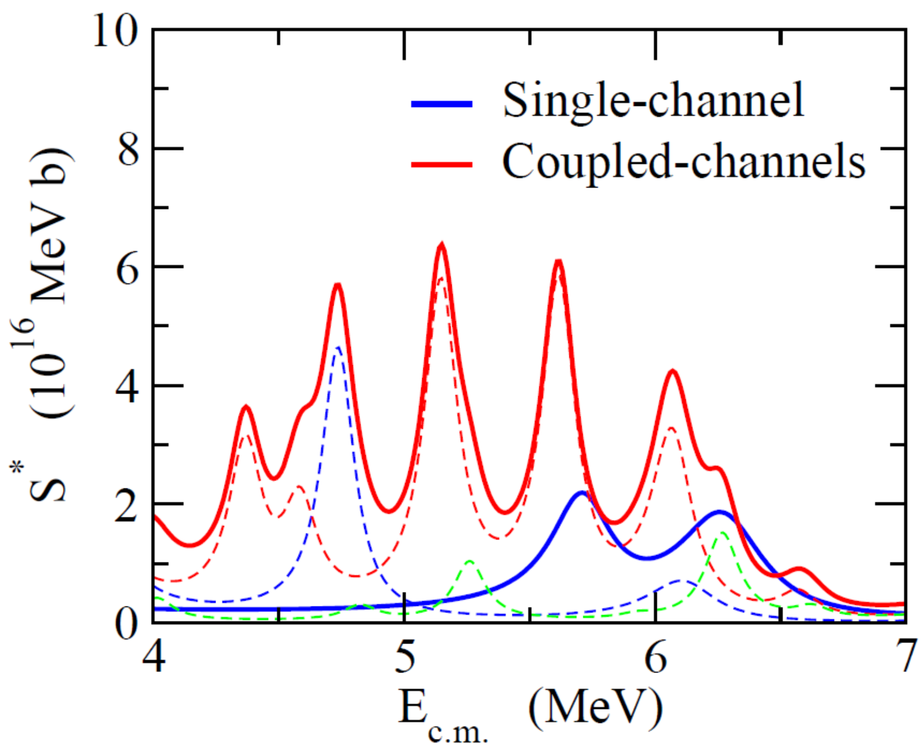
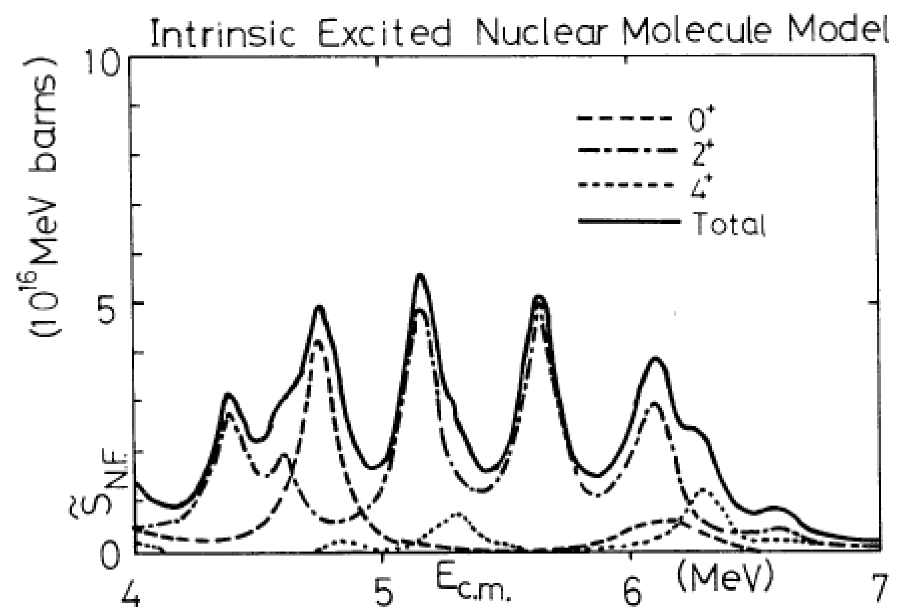
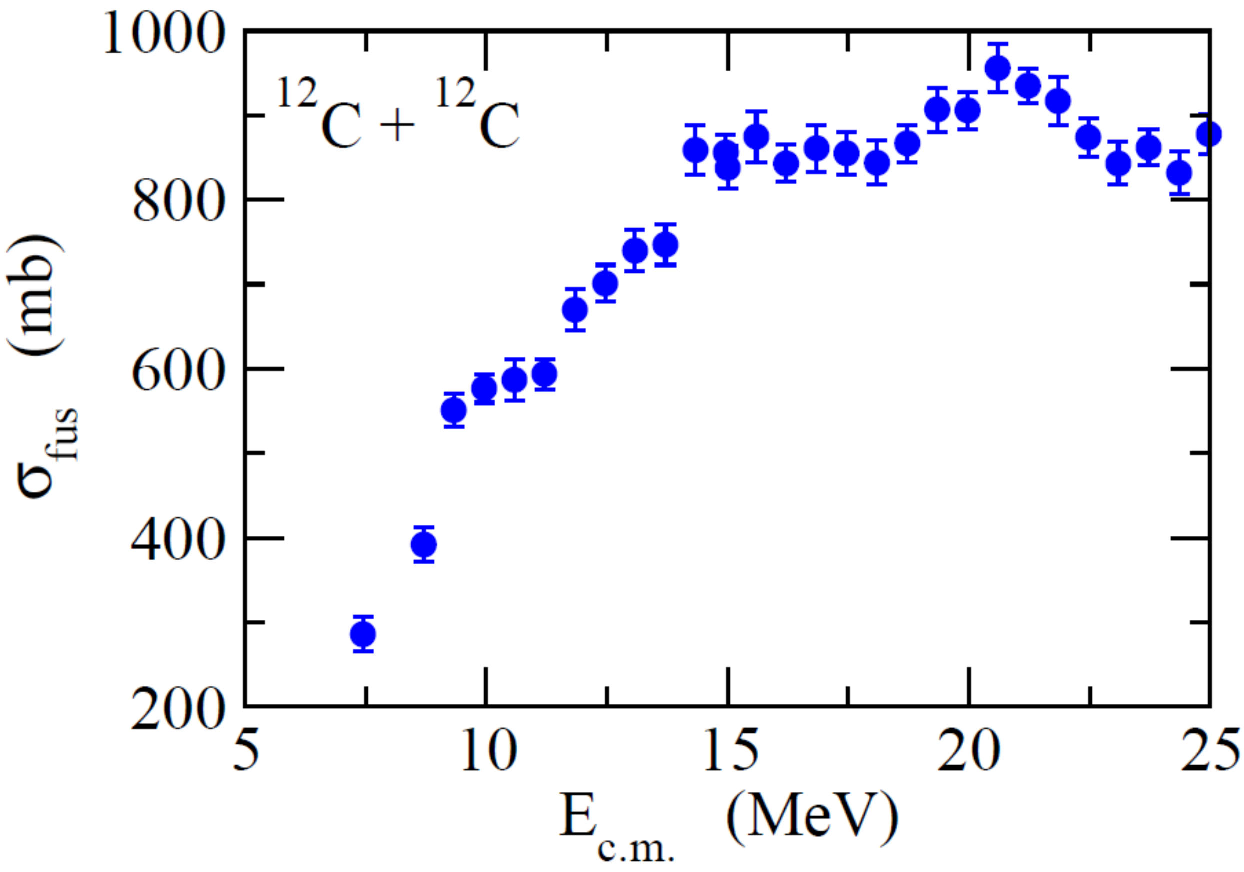




Y. Kondo, T. Matsuse, and Y. Abe, PTP 59 (‘78) 465

Experimental data at above barrier energies



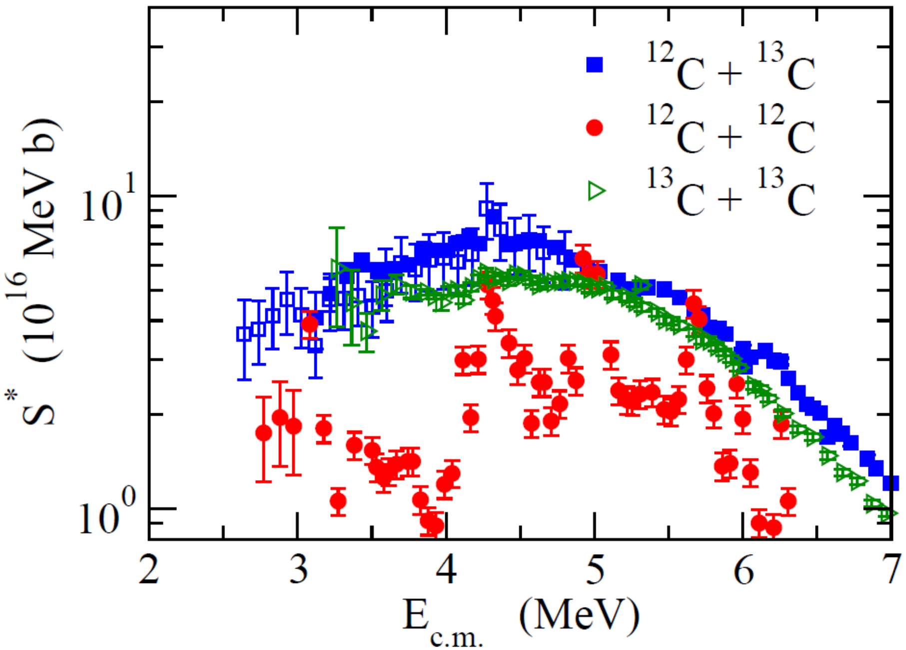
Data: D.G. Kovar et al., PRC20 (‘79) 1305

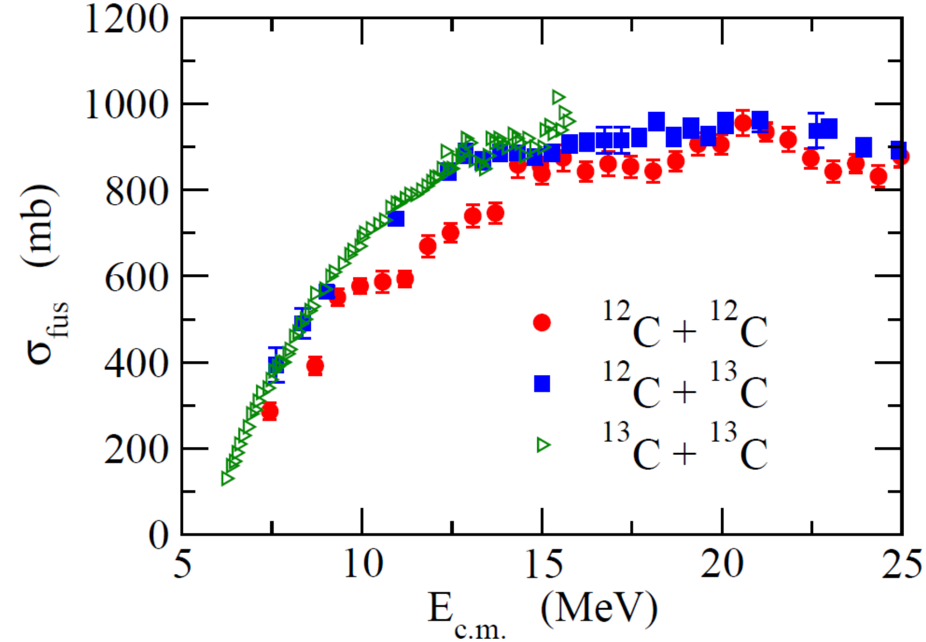
✓ fusion oscillations

⟵ successive contributions of individual partial waves

(N. Poffe, N. Rowley, and R. Lindsay, NPA410 (‘83) 498)

Comparison with other C+C systems



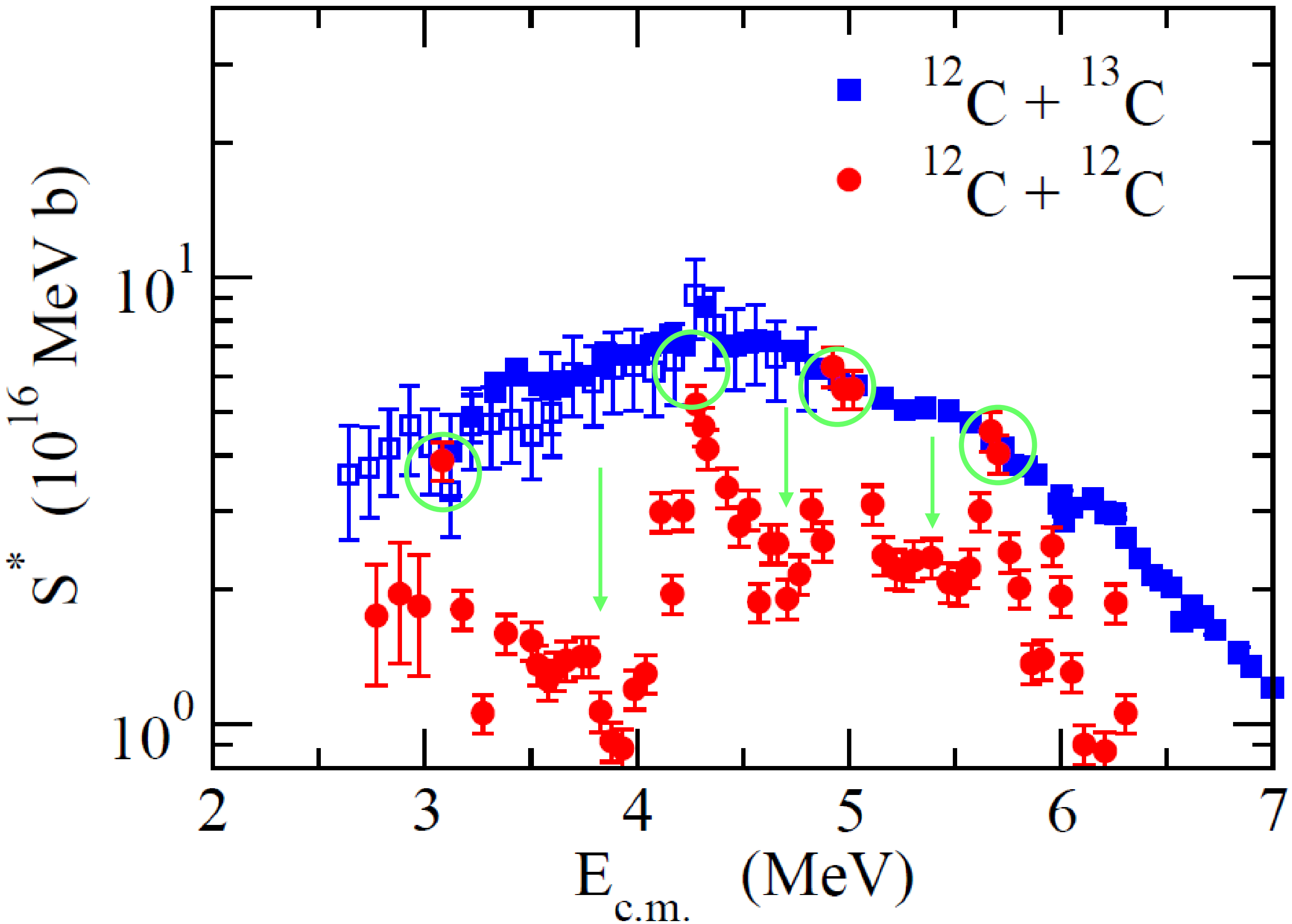


fusion cross sections for $^{12}C+^{13}C$, $^{13}C+^{13}C$: much less structured

How can one understand the systematics?
- from $^{12}C+^{12}C$ to $^{12}C+^{13}C$, $^{13}C+^{13}C$
  origins for the resonances/oscillations?
- from low to high energies

cf. most of the previous studies: $^{12}C+^{12}C$ only

Molecular resonances at subbarrier energies



M. Notani, X.D. Tang et al., PRC85('12)014607

off-resonance: fusion inhibition

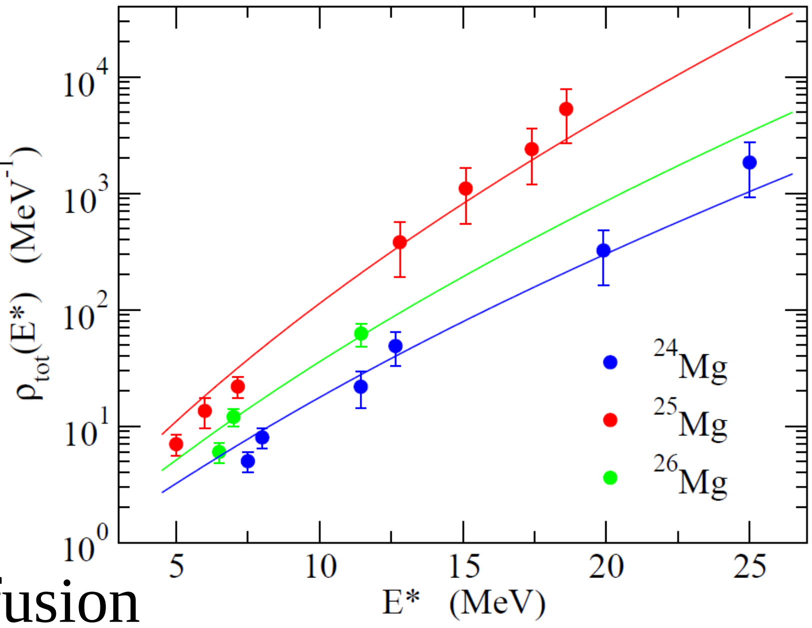
on-resonance: match with $^{12}C+^{13}C$

Jiang's conjecture: C.L. Jiang et al., PRL110('13)072701

properties of compound nucleus (^{24}Mg)?

^{12}C+^{12}C reaction:

- ✓ level density of ^{24}Mg : small (e-e)
- ✓ small fusion Q-value



Q = +13.9 MeV (^{12}C+^{12}C)
+16.3 MeV (^{12}C+^{13}C)
+22.5 MeV (^{13}C+^{13}C)

→ small E^* for ^{24}Mg in ^{12}C+^{12}C fusion

$$\sigma \sim \sum_J \sigma_{\rm cap}^J \left[1 - e^{-2\pi\Gamma_J/D_J}\right]$$

large hindrance factor

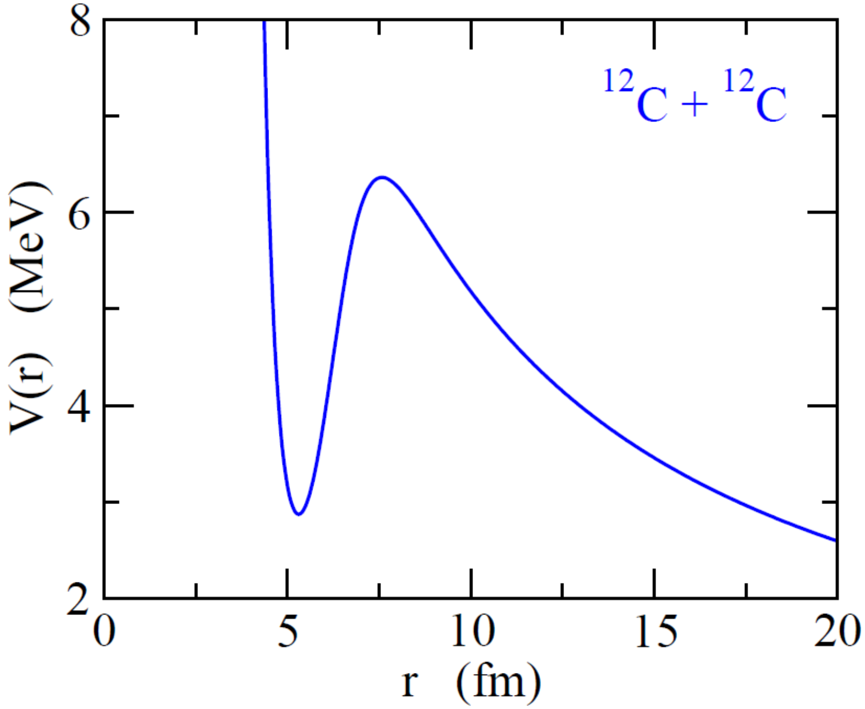
$D_J = 1/\rho_J$
Γ_J : CN width

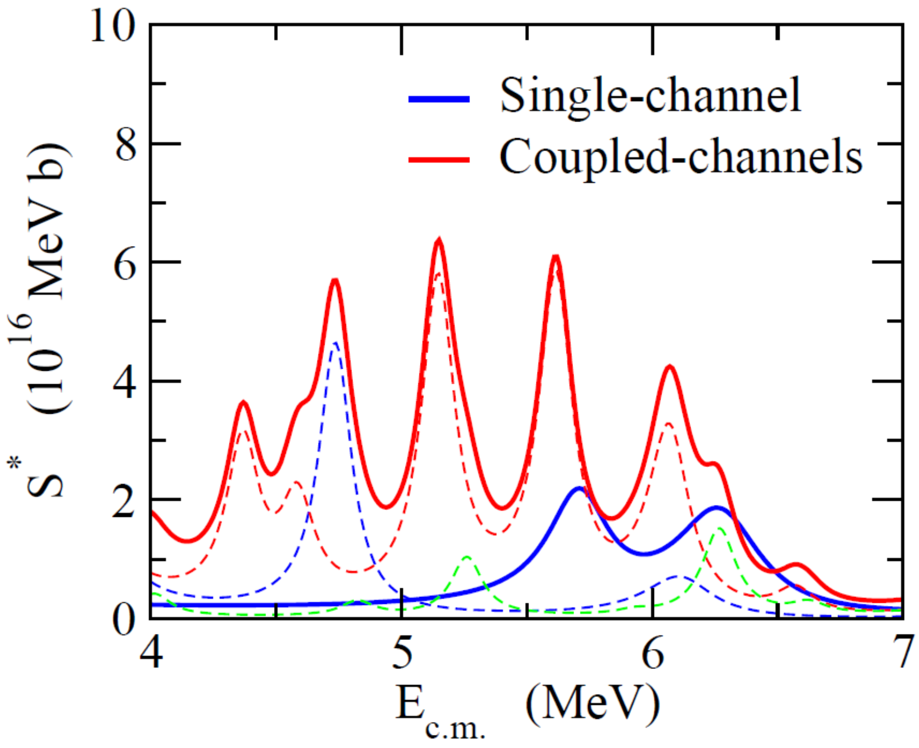
incorporate this idea in the coupled-channels calculations?

C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP('78))

- ✓ two-range Woods-Saxon + Coulomb for the real part
- ✓ a Woods-Saxon for the imaginary part





C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP('78))

- ✓ two-range Woods-Saxon + Coulomb for the real part
- ✓ a Woods-Saxon for the imaginary part

$$W(r) = -W_0 \cdot f_{\rm WS}(r) \to -w_0\, \rho_J(E^*) \cdot f_{\rm WS}(r)$$

G. Helling, W. Scheid, W. Greiner, PL 36B ('71) 64
H.-J. Fink, W. Scheid, W. Greiner, NPA188 ('72) 259
J.M. Quesada, M. Lozano, G. Madurga, PLB125 ('83) 14
M.V. Andres, Quesada, Lozano, Madurga, NPA443 ('85) 380

- ✓ *E* and *J* dependent imaginary potential
- ✓ system dependence through ρ(*E*)

cf. Fermi's golden rule

$$\frac{dw}{dt} = \frac{2\pi}{\hbar}\,|\langle\psi_{\rm CN}|V_{\rm int}|\psi_{\rm elastic}\rangle|^2\,\rho_J(E^*)$$

C.C. calculations with level-density-dependent imaginary potential

^{12}C-^{12}C potential (Kondo, Matsuse, Abe, PTP('78))

- ✓ two-range Woods-Saxon + Coulomb for the real part
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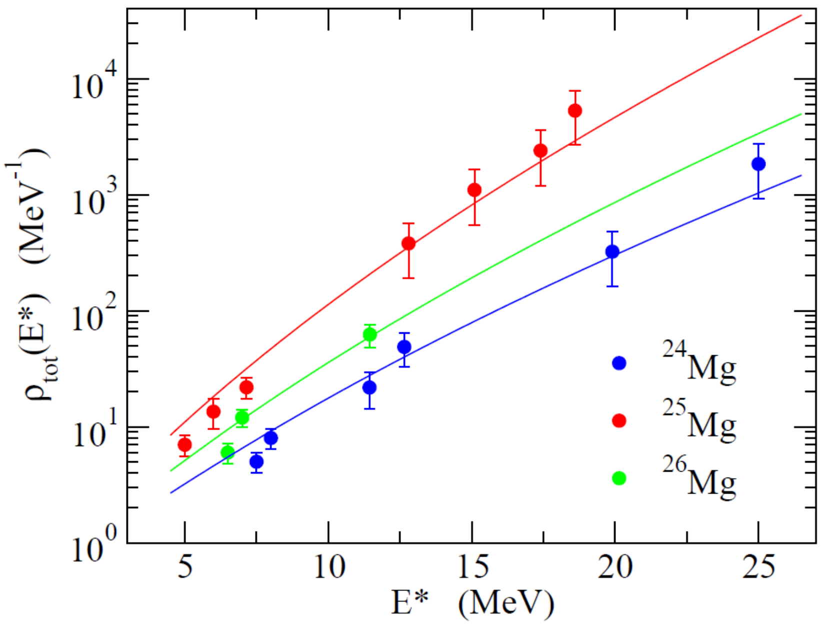
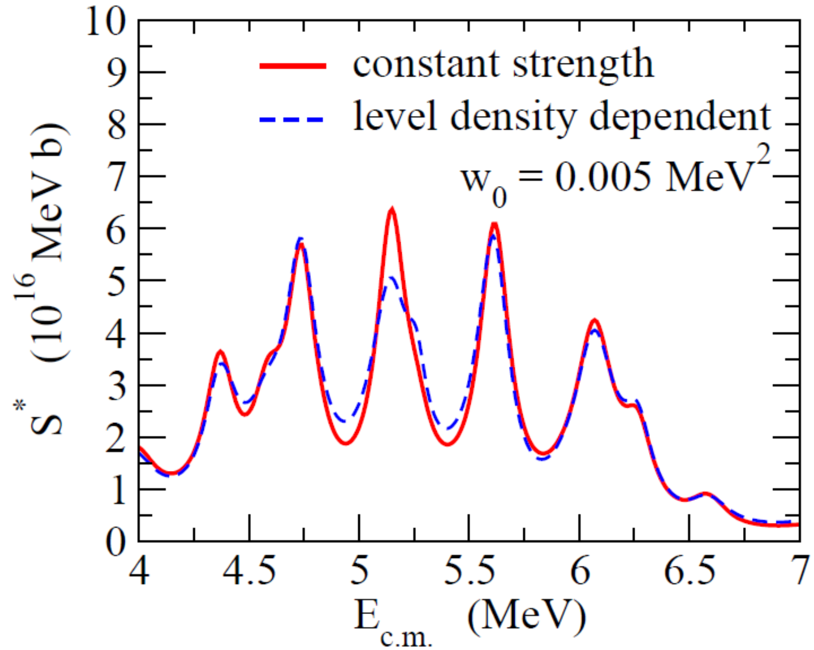
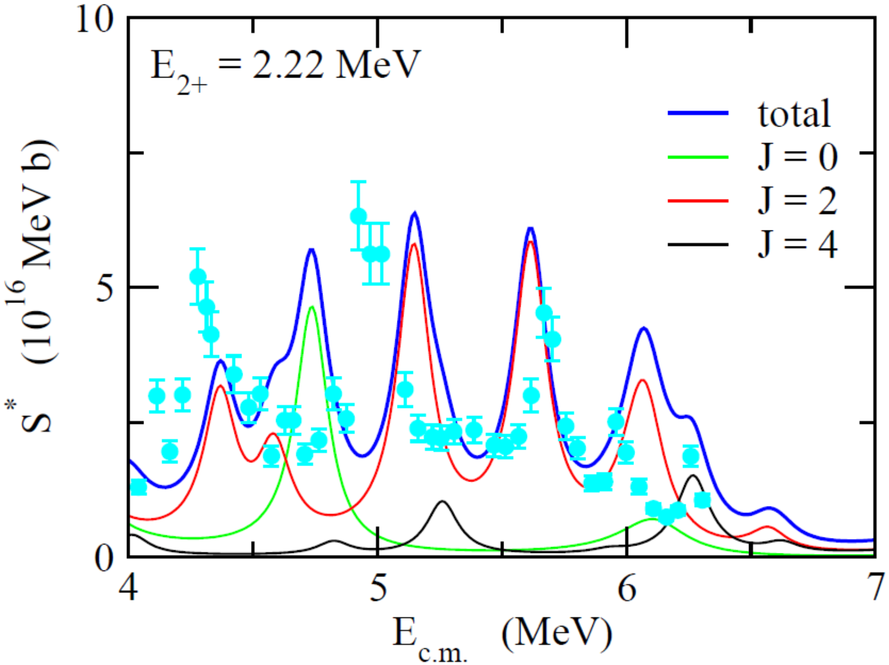
$$W(r) = -W_0 \cdot f_{\rm WS}(r) \rightarrow -w_0\, \rho_J(E^*) \cdot f_{\rm WS}(r)$$

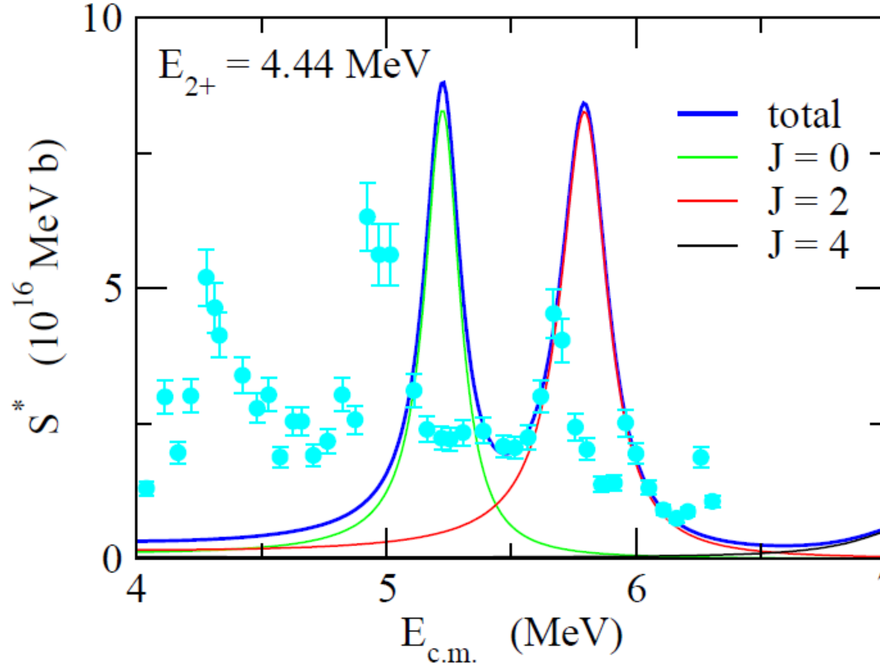


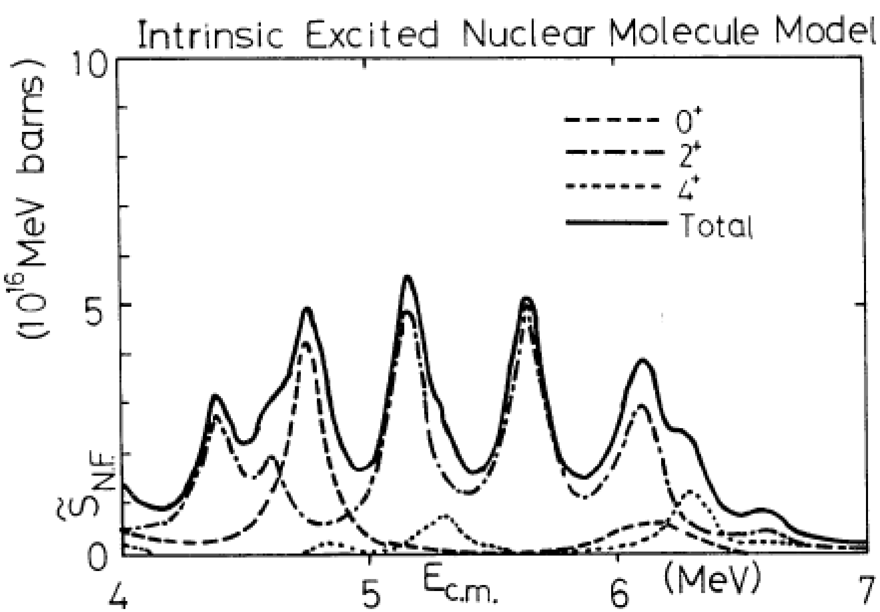


$$\rho_J(E^*) = \frac{(2J+1)e^{-(J+1/2)^2/2\sigma^2}}{4\sigma^3\sqrt{2\pi}} \frac{\sqrt{\pi}}{12} \frac{e^{2\sqrt{aE^*}}}{a^{1/4}(E^*)^{5/4}} \qquad \left(\sigma^2 = 0.088\, aA^{2/3}\sqrt{\frac{E^*}{a}}\right)$$

<u>some mystery?</u>

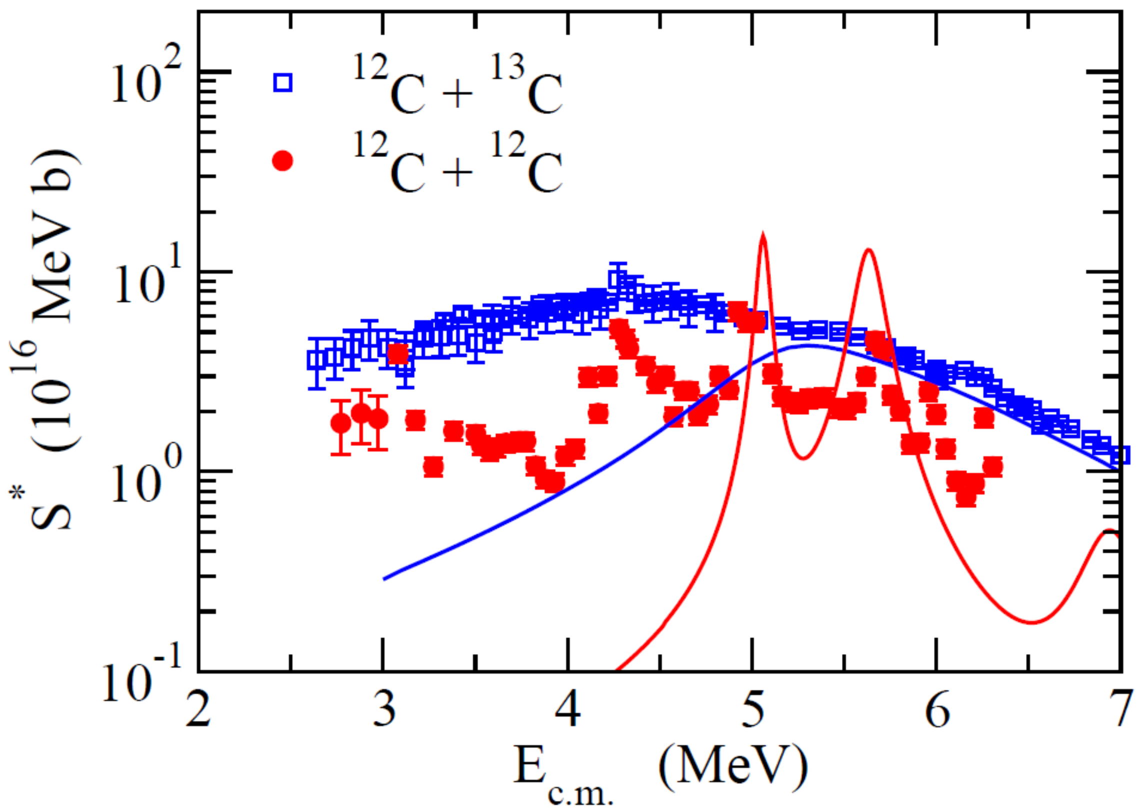






use slightly different input parameters from Kondo-Matsuse-Abe, but keep the physical value of E_{2+} (= 4.44 MeV)

Results of coupled-channels calculations



^{12}C (0^{+}, 2^{+}: 4.44)
^{13}C ($1/2^{-}$, $3/2^{-}$:3.68)
+ mutual excitations

✓ structured $^{12}C+^{12}C$
✓ smooth $^{12}C+^{13}C$

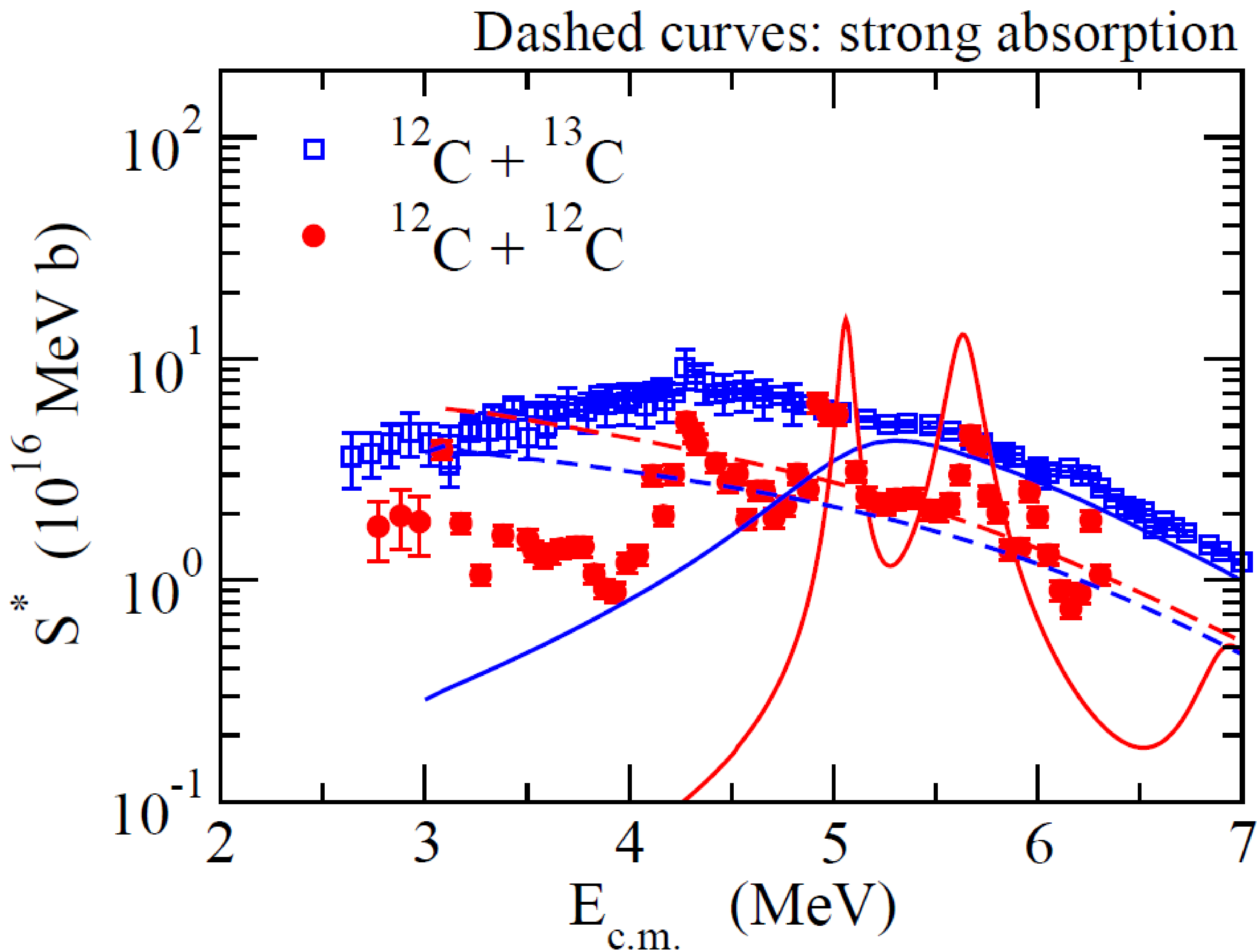
system dependence: qualitatively reproduced

underestimate of fusion cross sections at deep subbarrier energies:

→ couplings to 3^{-} and 0_{2}^{+} (Hoyle state)?

cf. role of Hoyle state in $^{12}C+^{12}C$:

M. Assuncao and P. Descouvemont, PLB723 ('13) 355



Dashed curves: strong absorption
$^{12}C + ^{13}C$
$^{12}C + ^{12}C$
S^* (10^{16} MeV b)
10^2
10^1
10^0
10^{-1}
2
3
4
5
6
7
$E_{c.m.}$ (MeV)

Fusion oscillations at above barrier energies

high-E : high level density of CN $\longrightarrow$ overlapping resonances

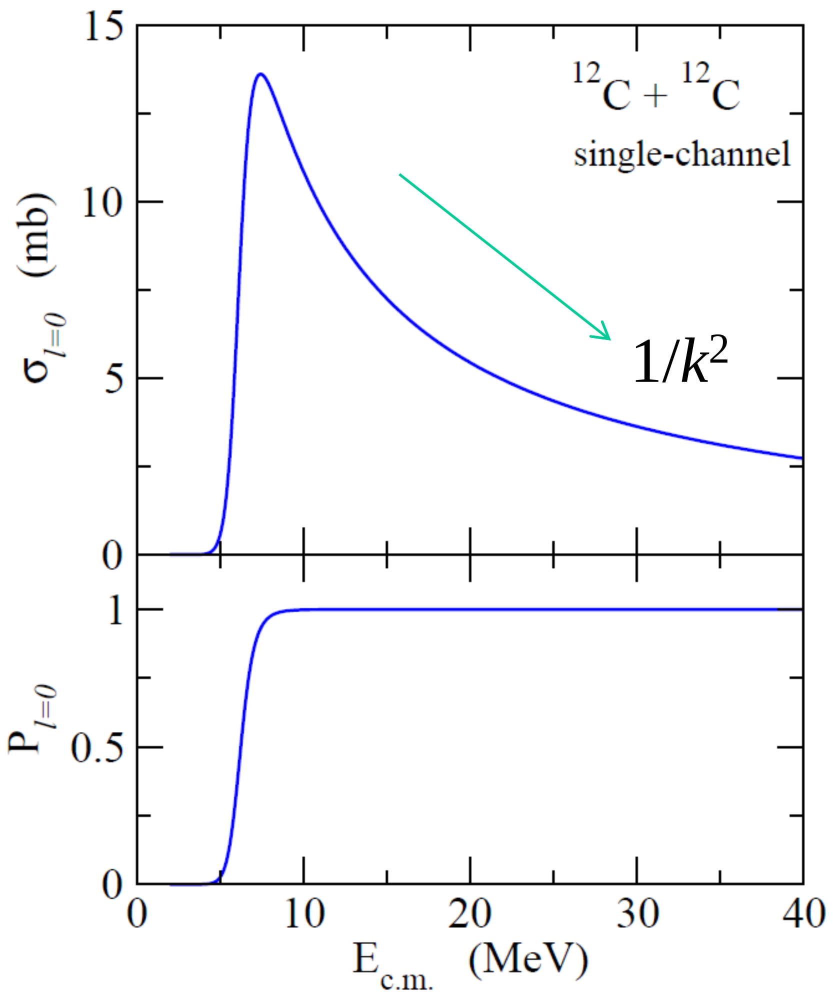
$\longrightarrow$ strong absorption

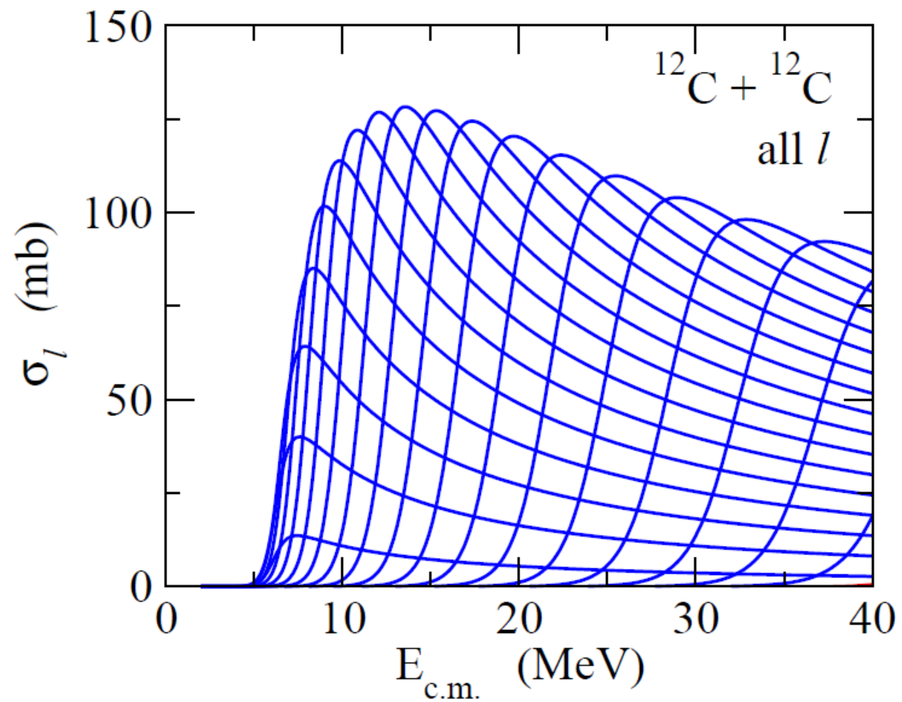
$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

$P_l(E)$: barrier penetrability

Fusion oscillations at above barrier energies

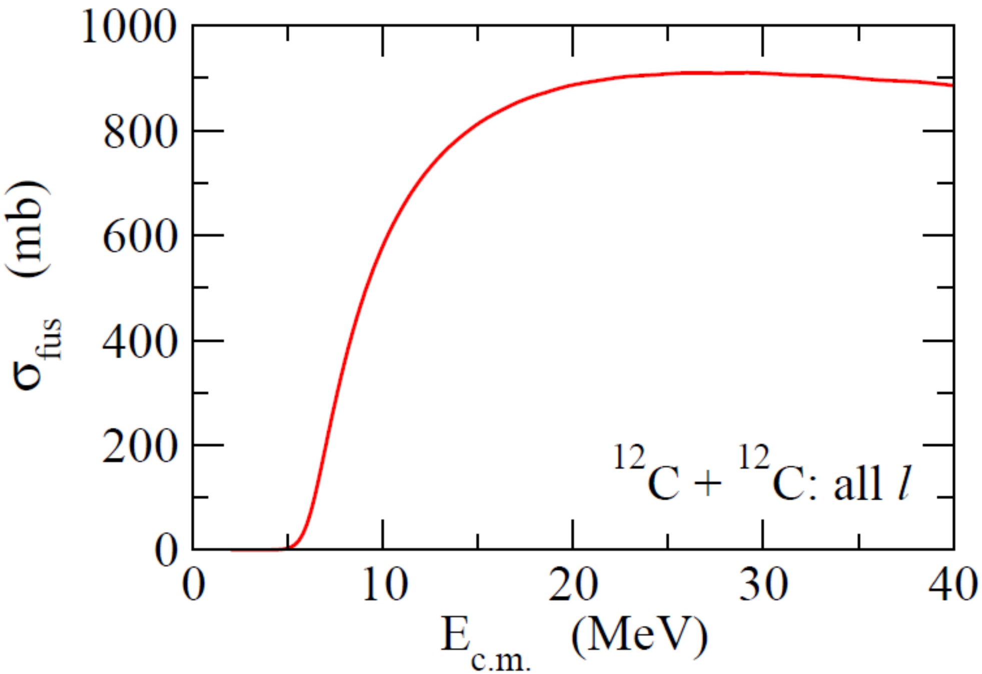
$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

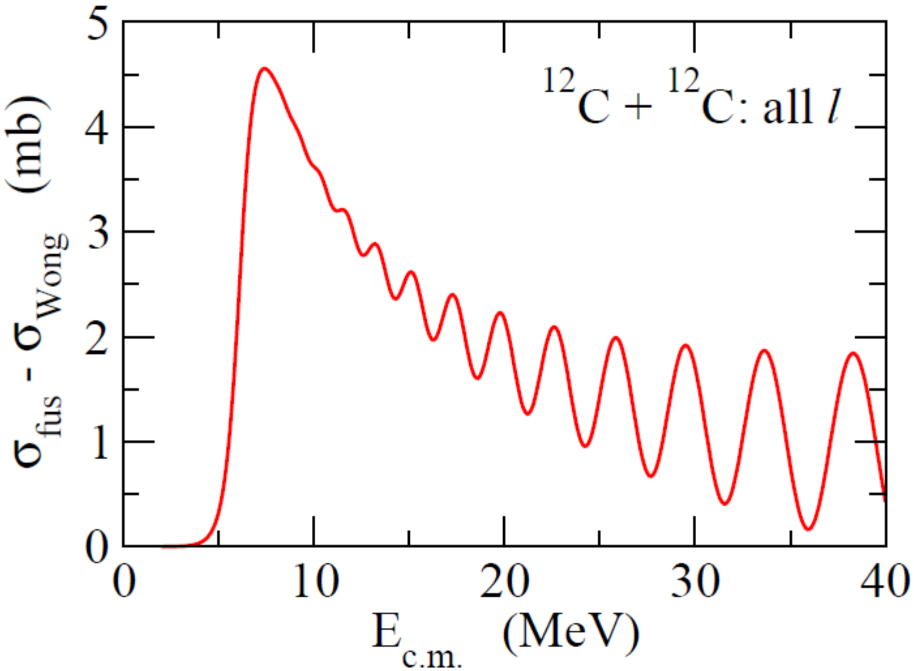




discrete l-sum

→ (oscillatory) structure in $\sigma_{\rm fus}$



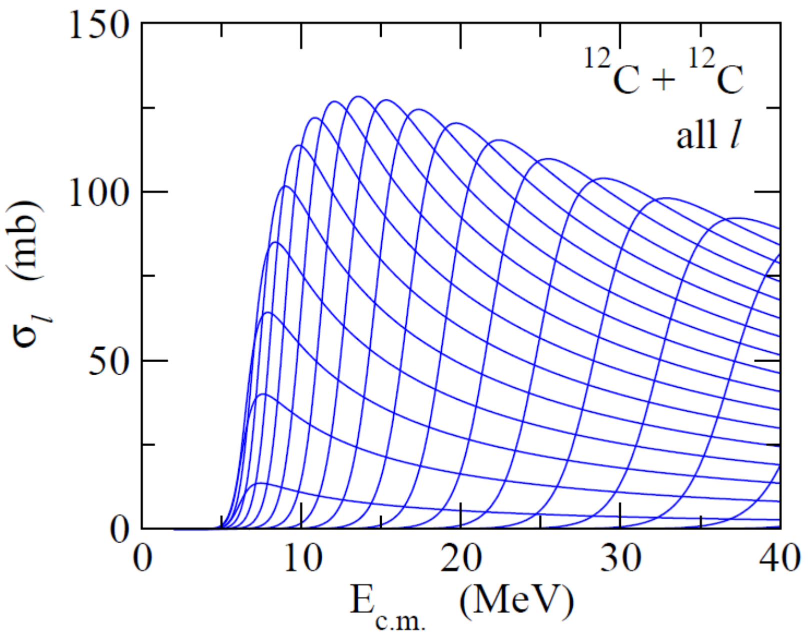


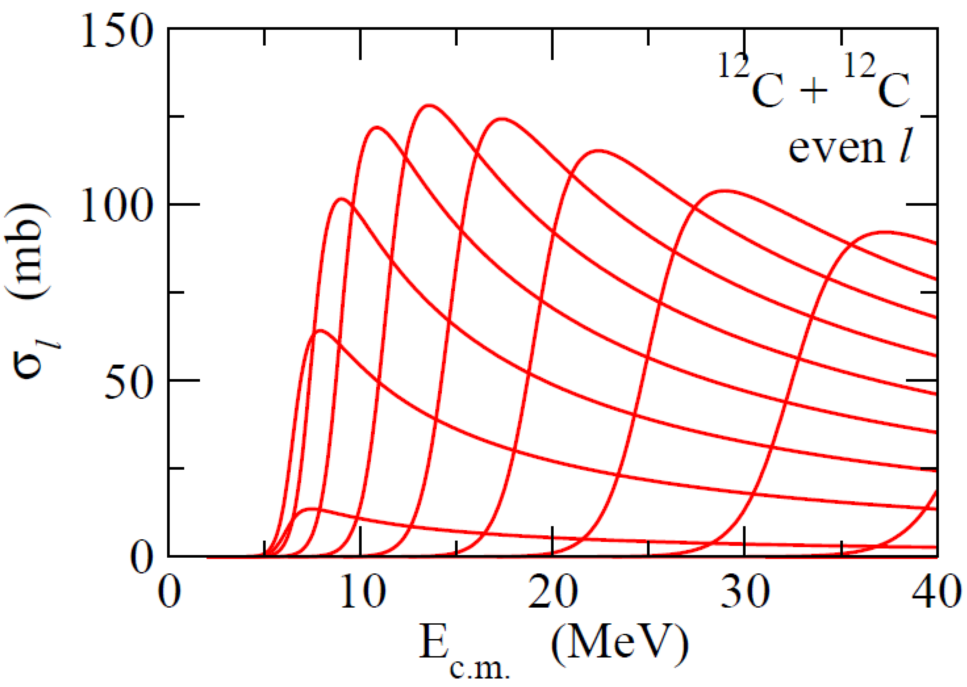
* practically, the oscillations are invisible ($\Delta\sigma \sim 1$ mb) if all-*l* are summed

effect of symmetrization: fusion oscillations in light symmetric systems

fusion of identical spin-zero bosons: wf has to be symmetric

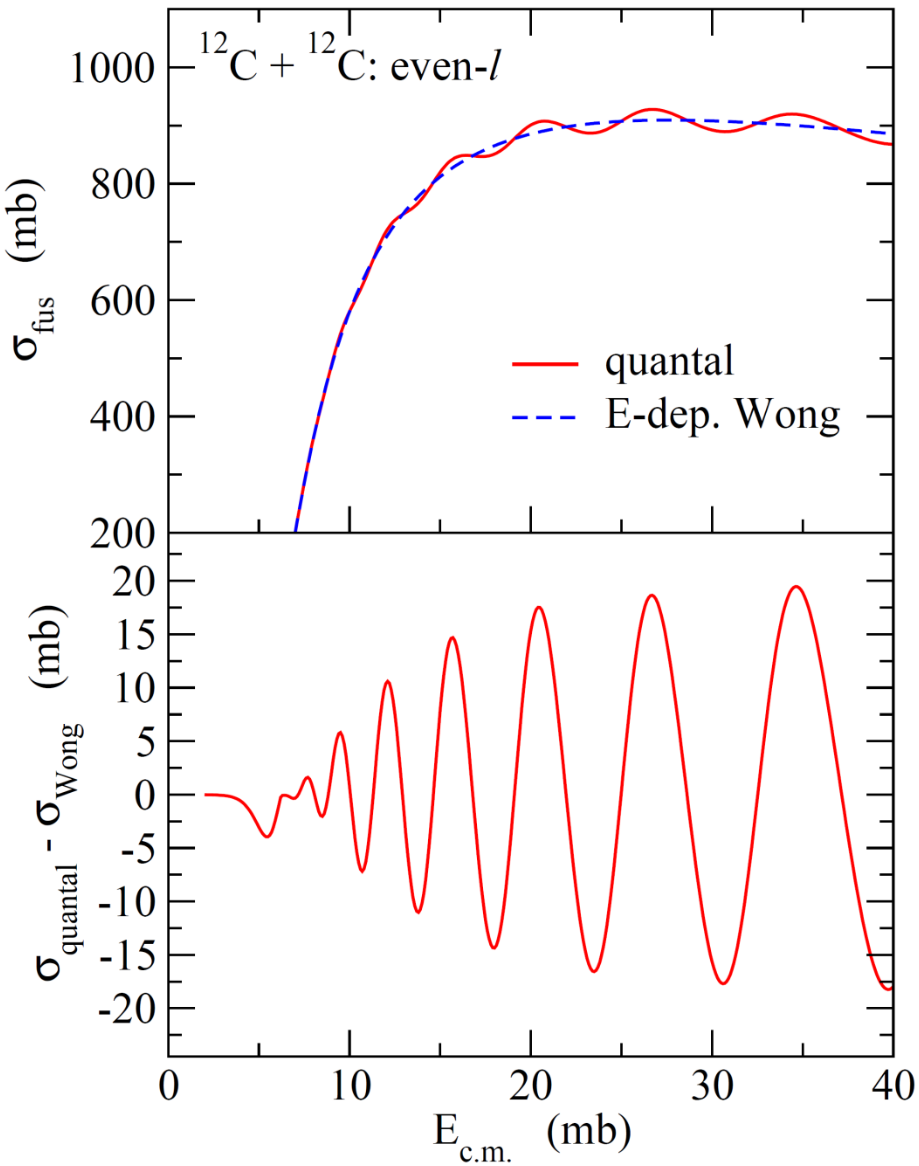
$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \to \frac{\pi}{k^2}\sum_l (1+(-)^l)(2l+1)P_l(E)$$



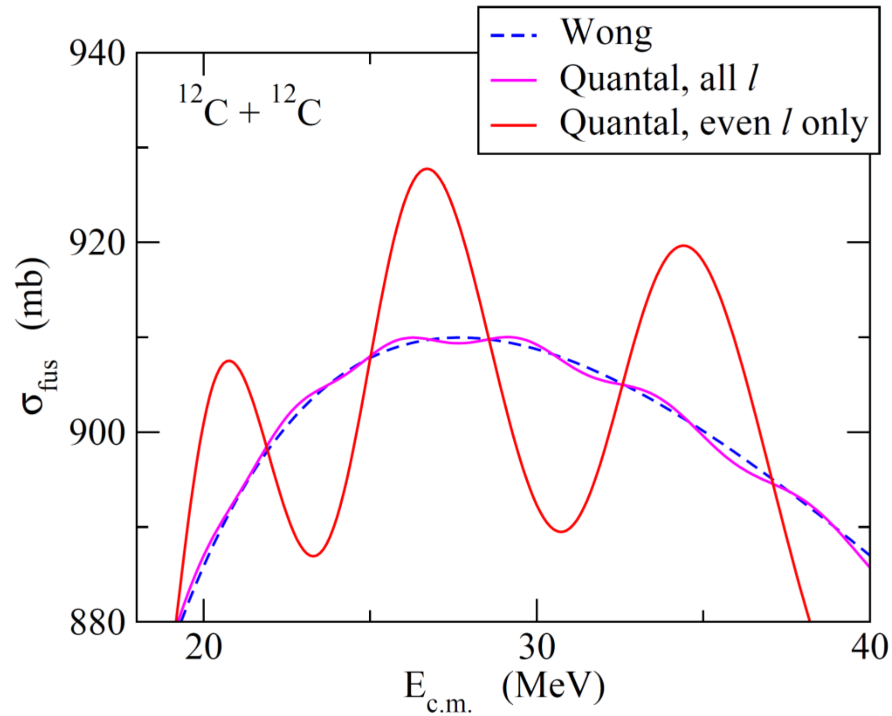


✓the angular mom. is quantized in units of 2-hbar

✓a larger amplitude of fusion oscillations



$^{12}C + ^{12}C$: even-l
quantal
E-dep. Wong
σ_{fus} (mb)
$\sigma_{quantal} - \sigma_{Wong}$ (mb)
$E_{c.m.}$ (mb)
1000
800
600
400
200
20
15
10
5
0
-5
-10
-15
-20
0
10
20
30
40



Wong
Quantal, all l
Quantal, even l only
$^{12}C + ^{12}C$
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
940
920
900
880
20
30
40

Analytic formula for fusion oscillations

N. Poffe, N. Rowley, and R. Lindsay, Nucl. Phys. A410 ('83) 498
N. Rowley and K. Hagino, in preparation

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (1\pm(-)^l)(2l+1)P_l(E)$$

$$\sim \sigma_{\rm E-Wong} \pm 2\pi R_E^2\,\frac{\hbar\Omega_E}{E}\,e^{-\xi}\,\sin(\pi l_g)$$

⟵ Poisson sum formula

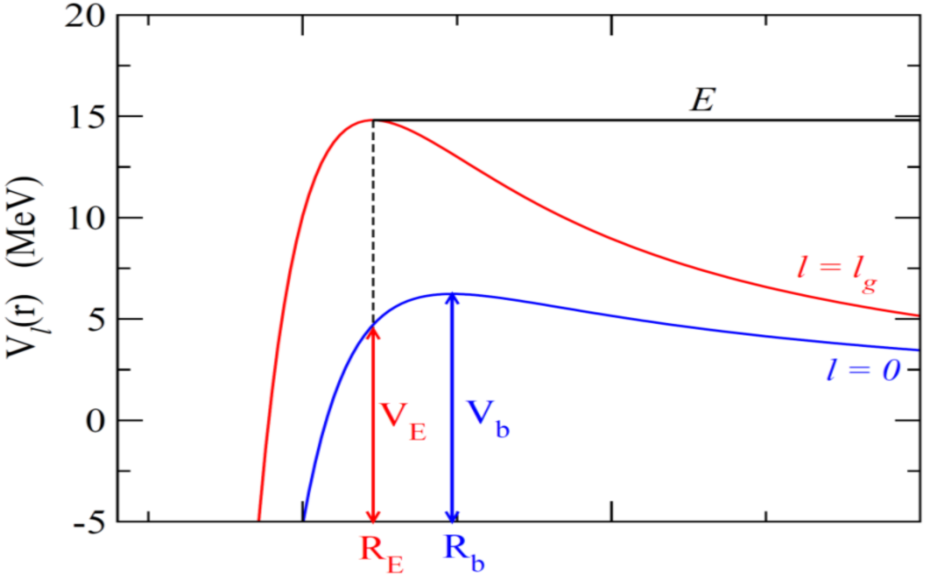
$$\xi = \pi\cdot\frac{\hbar\Omega_E}{2l_g+1}\cdot\frac{\mu R_E^2}{\hbar^2}$$

(note)

$$2l_g + 1 \gg \pi\hbar\Omega_E\cdot\frac{\mu R_E^2}{\hbar^2}$$

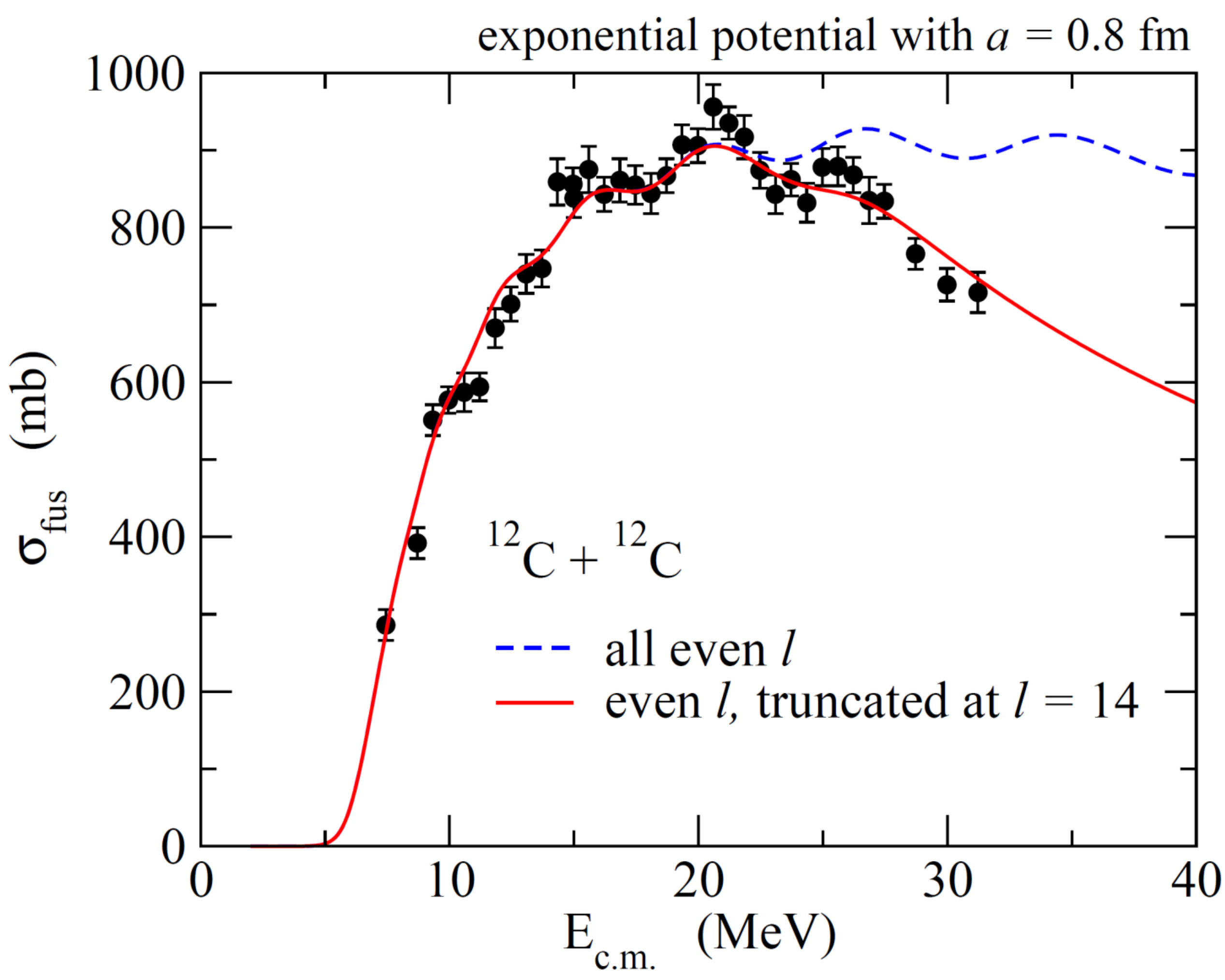
in order for the osc. to be visible

⟶ light symmetric systems



i) Comparison with the experimental data: $^{12}C + ^{12}C$

$^{12}C_{g.s.}: 0^+ \longrightarrow$ the relative w.f. has to be spatially symmetric

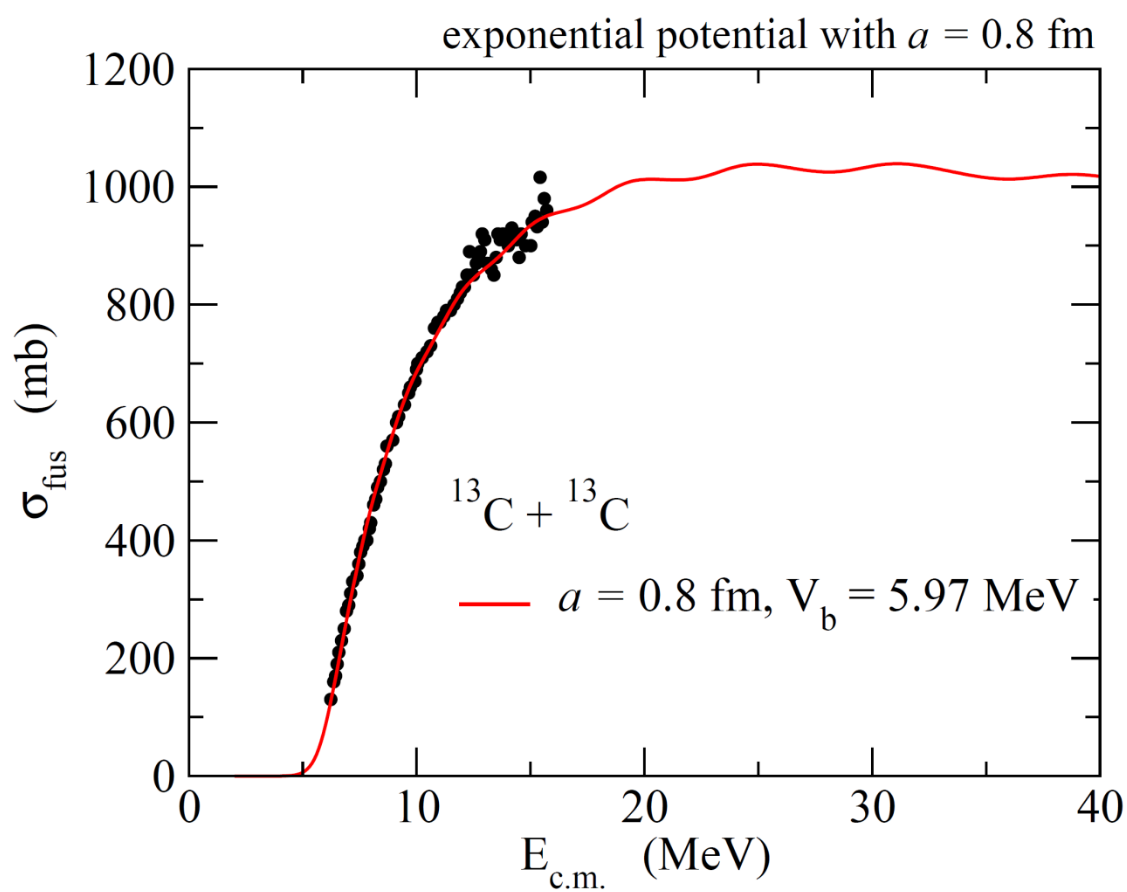


ii) $^{13}C + ^{13}C$

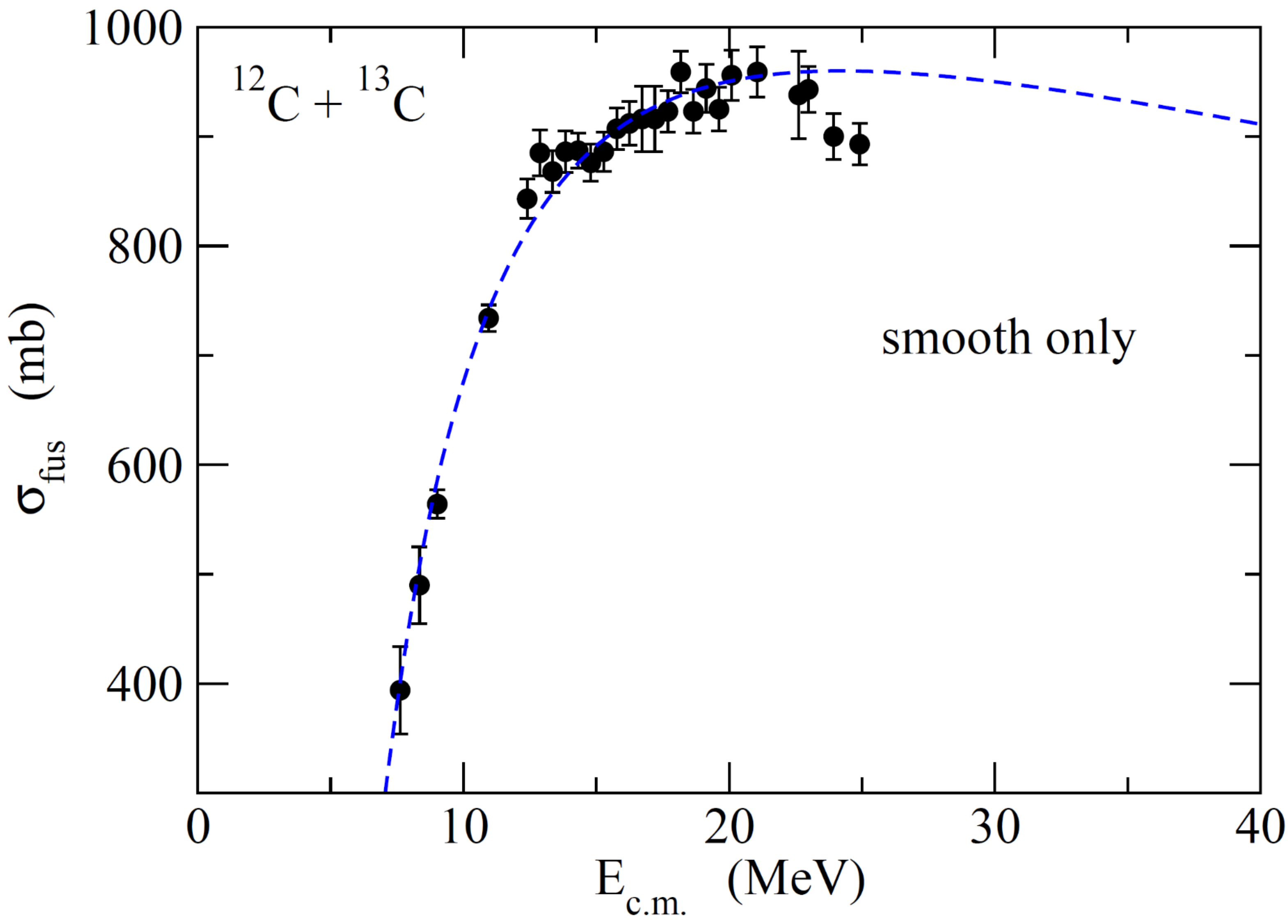
$^{13}C_{g.s.}: 1/2^-$ $\longrightarrow$ the relative w.f. has to be spatially symmetric for $S = 0$
spatially anti-symmetric for $S = 1$

$$\sum_l \to \frac{1}{4}\sum_l \left(1+(-1)^l\right) + \frac{3}{4}\sum_l \left(1-(-1)^l\right)$$

$$\sigma_{\rm osc} = \frac{1}{2}\sigma_{\rm osc}({\rm odd}-l)$$

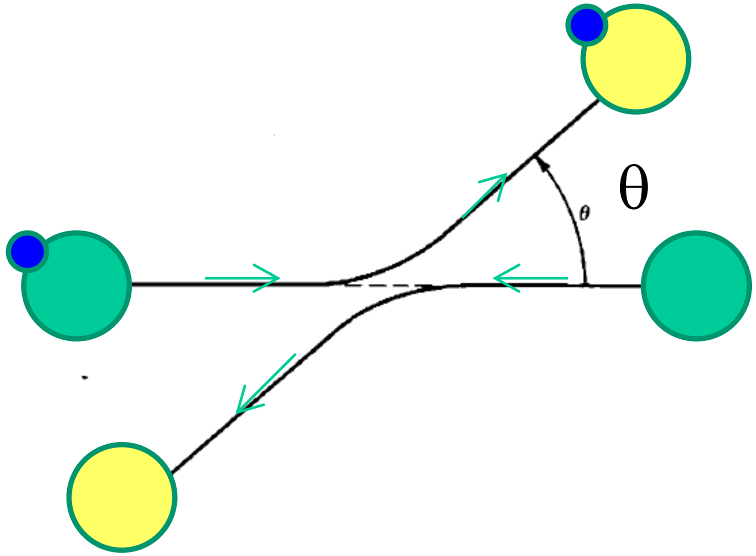


iii) $^{12}C + ^{13}C$



$^{12}C + ^{13}C$
smooth only
1000
800
600
400
0
10
20
30
40
σ_{fus} (mb)
$E_{c.m.}$ (MeV)

role of elastic transfer



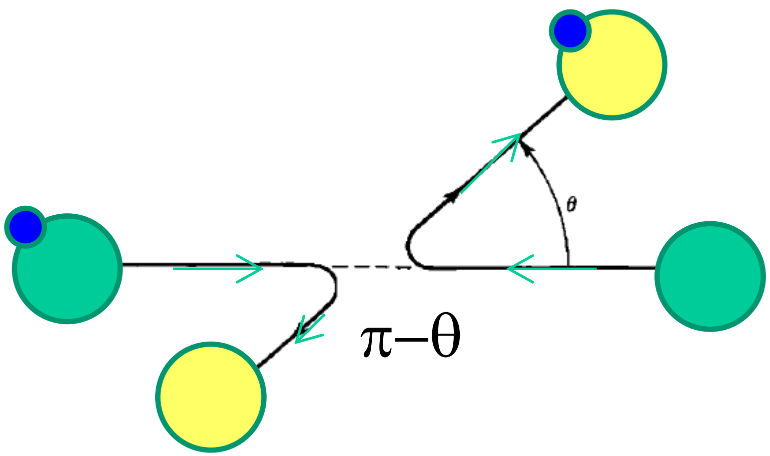
elastic scattering

$f_{\mathrm{el}}(\theta)$

indistinguishable

transfer

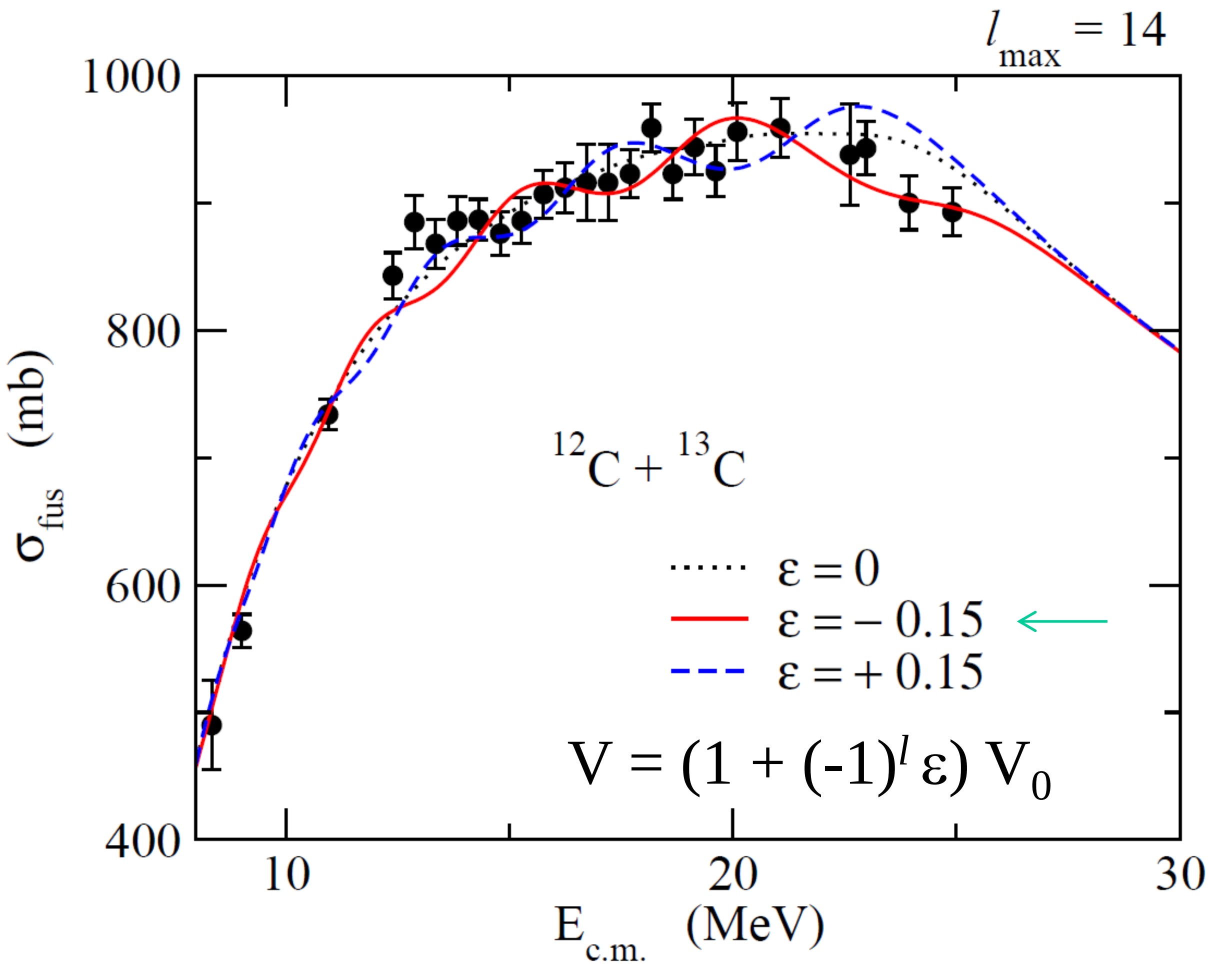
$f_{\mathrm{trans}}(\pi - \theta)$



$$f(\theta) \rightarrow f_{\mathrm{el}}(\theta) + f_{\mathrm{trans}}(\pi - \theta)$$

parity-dependent potential

✓W. von Oertzen and H.G. Bohlen, Phys. Rep. 19C(‘75) 1
✓A. Vitturi and C.H. Dasso, Nucl. Phys. A458 (‘86) 157
✓A. Kabir, M.W. Kermode and N. Rowley, Nucl. Phys. A481(‘88) 94



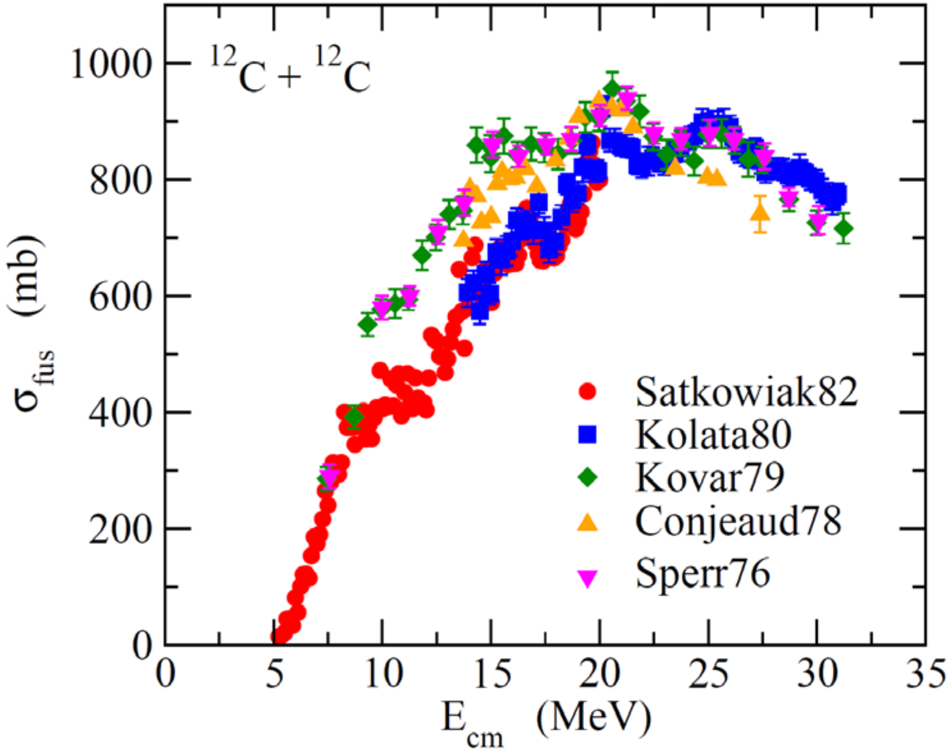
exponential potential with a = 0.9 fm

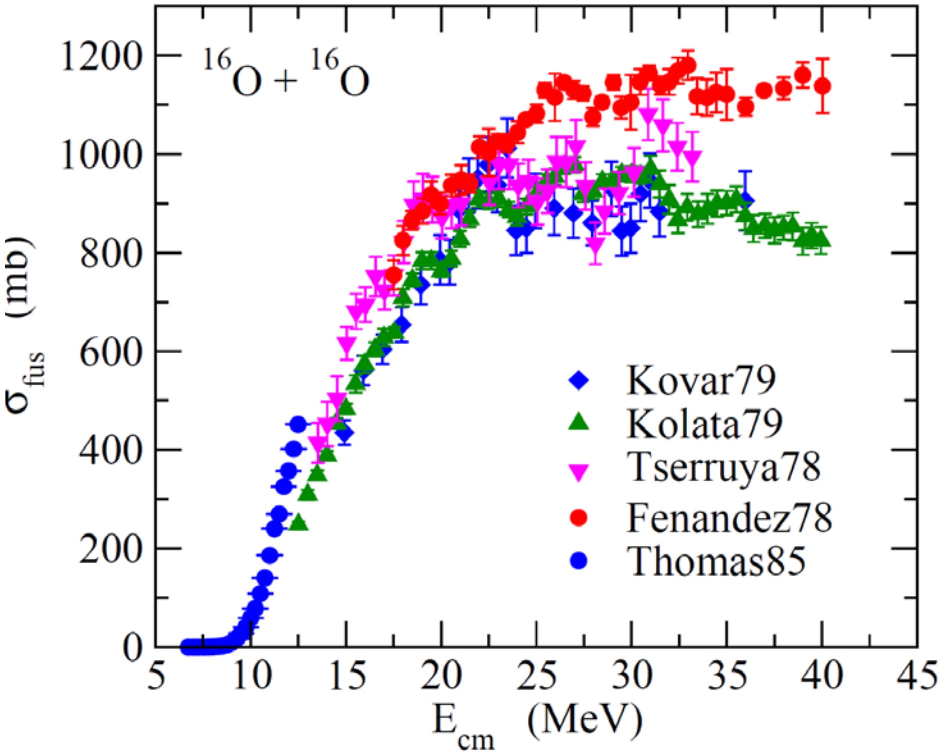
the sign of ε:
consistent with Baye's simple rule

D. Baye, J. Deenen, and Y. Salmon,
Nucl. Phys. A289(‘77) 511
D. Baye,
Nucl. Phys. A460 (‘86)581

RGM with two-center HO shell model

Fusion oscillations with SOD method





The expt. data: rather scattered

- ✓systematic errors
- ✓missing evaporation channels

⟶ σ_{fus} from Sum-of-Differences (SOD) method?

Sum-of-differences (SOD) method

J.T. Holdeman and R.M. Thaler, PRL14('65)81, PR139('65)B1186
C. Marty, Z. Phys. A309('83)261, A322('85)499

$$\sigma_R \sim 2\pi \int_{\theta_{\rm min}}^{\pi} \sin\theta\, d\theta\, (\sigma_{\rm Ruth}(\theta) - \sigma_{\rm el}(\theta))$$

expt.: H. Wojciechowski et al., PRC16('77)1767
H. Oeschler et al., NPA325('79)463
T. Yamaya et al., PLB417('98)7 etc.

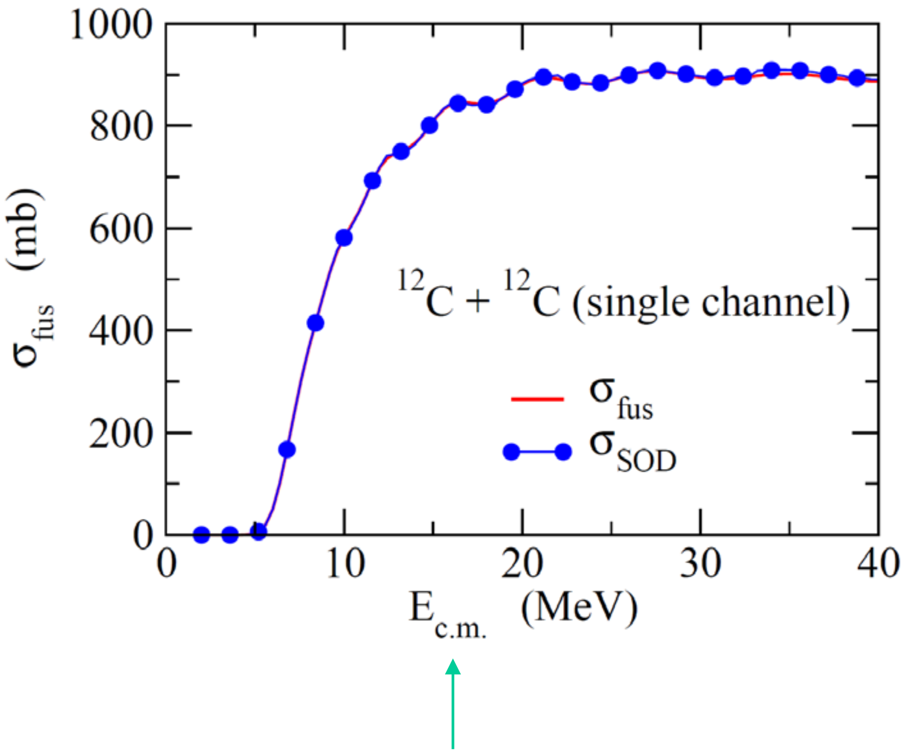
generalization (K.H. and N. Rowley, in preparation)

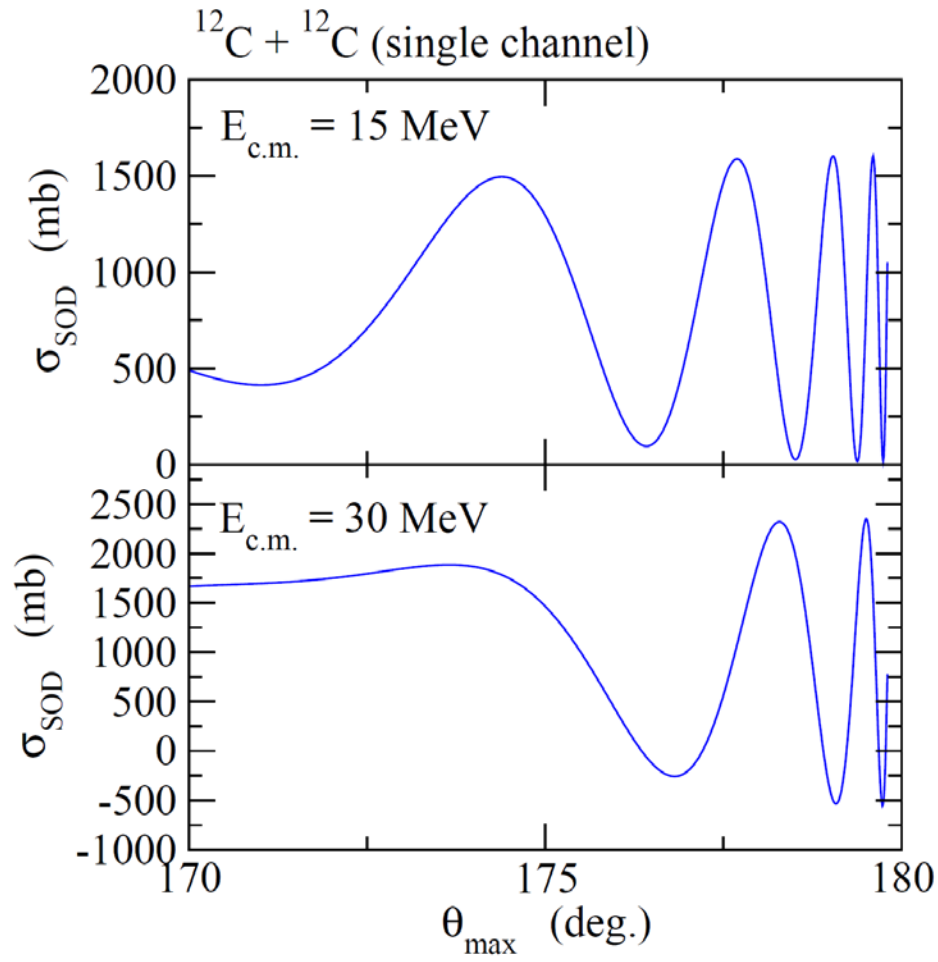
$$\sigma_R = \sigma_{\rm fus} + \sigma_{\rm inel} + \sigma_{\rm tr}$$

$$\sigma_{\rm fus} \sim 2\pi \int_{\theta_{\rm min}}^{\pi} \sin\theta\, d\theta\, (\sigma_{\rm Ruth}(\theta) - \sigma_{\rm qel}(\theta))$$

for symm. systems: $\sigma_{\rm fus} \sim 2\pi \int_{\pi/2}^{\theta_{\rm max}} \sin\theta\, d\theta\, (\sigma_{\rm Mott}(\theta) - \sigma_{\rm qel}(\theta))$

$$\sigma_{\rm SOD} = 2\pi \int_{\pi/2}^{\theta_{\rm max}} \sin\theta\, d\theta\, (\sigma_{\rm Mott}(\theta) - \sigma_{\rm qel}(\theta))$$





average of a maximum and a minimum in $\sigma_{\rm SOD}$

Summary

sub-barrier fusion of C+C systems

- Molecular resonances at subbarrier energies

  $^{12}C + ^{12}C$: well pronounced resonance structure

  $^{13}C + ^{13}C$, $^{12}C + ^{13}C$: rather smooth

  ← CN ^{24}Mg: low level density (low Q-value, e-e nucleus)

  cf. Jiang's conjecture

- Fusion oscillations: successive contribution of discrete centrifugal bararriers

  $^{12}C(0^+) + ^{12}C(0^+)$
  $^{13}C(1/2^-) + ^{13}C(1/2^-)$ } symmetrization of relative wave function

  $^{12}C + ^{13}C$ --- elastic transfer

- Sum-of-differences (SOD) method
  - an alternative way to obtain sfus
  - application to fusion oscillations?