A few caveats and exact solutions for the 1D KPZ equation

Tomohiro Sasamoto

Tokyo Institute of Technology

In this talk we explain a few points one has to be careful when dealing with the KPZ equation and its universality.

In one dimension, the KPZ equation reads

$$\partial_t h(x,t) = \tfrac{1}{2}\lambda(\partial_x h(x,t))^2 + \nu\partial_x^2 h(x,t) + \sqrt{D}\eta(x,t)$$

where η is the Gaussian noise with mean zero and covariance $\langle \eta(x,t)\eta(x',t')\rangle = \delta(x-x')\delta(t-t')$. Naively, this may look innocent but it is known that this equation is not well-defined as it is. A well-defined notion of the KPZ equation (or the solution of it) is given by using the Cole-Hopf transformation $Z(x,t) = e^{h(x,t)}$ [1].

The exact solution for the one-point height distribution for the 1D KPZ equation was discovered for the narrow wedge situation ($Z(x,0) = \delta(x)$) in [2, 3], by combining the connection of the KPZ equation to ASEP and a previous analysis of ASEP in [4]. It was also shown that one can arrive at the same formula by using replica analysis [5, 6].

In the $t \to \infty$ limit, the above height distribution for the KPZ equation tends to the GUE Tracy-Widom distribution from random matrix theory. This shows that the KPZ equation is in the KPZ universality class. A subtle point here is the definition of the KPZ universality. One also has to be careful about the difference between the KPZ universality and the universality of the KPZ equation.

References

[1] L. Bertini and G. Giacomin, Stochastic Burgers and KPZ equations from particle systems, Commun. Math. Phys., 183: 571–607, 1997.

[2] T. Sasamoto and H. Spohn, One-Dimensional Kardar-Parisi-Zhang Equation: An Exact Solution and its Universality, *Phys. Rev. Lett.*, 104: 230602, 2010.

[3] G. Amir, I. Corwin and J. Quastel, Probability distribution of the free energy of the continuum directed random polymer in 1 + 1 dimensions, *Com. Pure. Appl. Math.*, 64: 466-537, 2011.

[4] C. A. Tracy and H. Widom, Asymptotics in ASEP with step initial condition, *Comnum. Math. Phys.*, 290: 129–154, 2009.

[5] V. Dotsenko, Bethe ansatz derivation of the Tracy-Widom distribution for one-dimensional directed polymers, *EPL*, 90: 20003, 2010.

[6] P. Calabrese, P. Le Doussal and A. Rosso, Free-energy distribution of the directed polymer at high temperature, *EPL*, 90: 20002, 2010.