KPZ equation, its renormalization and invariant measures

Tadahisa Funaki

University of Tokyo

The KPZ equation is a nonlinear stochastic partial differential equation, being ill-posed because of inconsistency between the nonlinearity and the roughness of the noise. We consider the KPZ equation on a whole line and introduce a renormalization procedure appropriate for finding invariant measures. The Cole-Hopf transform applied to this approximating equation leads to a linear stochastic heat equation (SHE) with a smeared noise having an extra complex nonlinear term. Under the time average, this complex term can be replaced by a simple linear term with a specific constant $\frac{1}{24}$, at least under the situation that the corresponding tilt process is stationary. As a result, it is shown that the distribution of a two-sided geometric Brownian motion with a height shift given by Lebesgue measure is invariant under the evolution determined by the SHE. We also discuss the multi-component case.

This is a joint work with Jeremy Quastel.