KPZ-class interfaces in turbulent liquid crystal

beyond a “mere” confirmation

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Acknowledgment

Masaki Sano, Tomohiro Sasamoto,
Herbert Spohn, Michael Prähofer, Takuma Akimoto, …

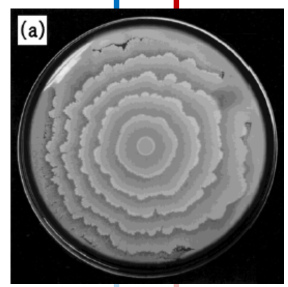
Growing interfaces

Important in both industry (e.g. solid-state device) and basic science

Non-local growth

Metal dendrite

snowflake

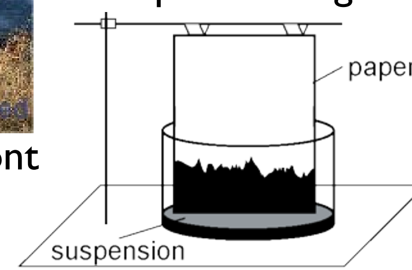


Bacterial colony

Local growth

Burning front

Paper wetting



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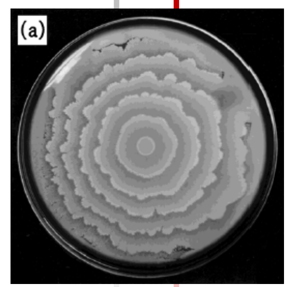

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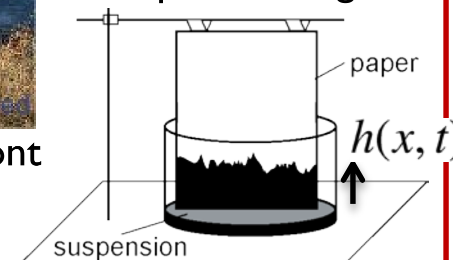


KPZ universality class

- KPZ equation: $\frac{\partial}{\partial t}h(x,t) = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \sqrt{D}\,\eta(x,t)$

  $\delta h \sim t^{\beta}$ or L^{α}, $\xi \sim t^{1/z}$

  $\alpha = \frac{1}{2}, \beta = \frac{1}{3}, z = \frac{3}{2}$ in 1d

- In 1d, many properties are exactly solvable, despite being out of equilibrium. KPZ is central in the studies of universality out of equilibrium.

- Deep connection to random matrix theory / combinatorics / integrable systems. Interests are far beyond understanding interfaces!

3 Important "Sub-classes"

(recall yesterday's review talks)

Circular (curved) interfaces

- Init. cond. : point or curved line
- Asymptotics : GUE Tracy-Widom distribution, Airy_2 process
- Shown for : TASEP [Johansson CMP 2000], PNG, PASEP [Tracy & Widom CMP 2009], KPZ eq. [Sasamoto & Spohn 2010, Amir *et al.* 2011, etc.] … (list is not complete)

Flat interfaces

- Init. cond. : straight line
- Asymptotics : GOE Tracy-Widom distribution, Airy_1 process
- Shown for : PNG [Prähofer-Spohn PRL 2000], TASEP, KPZ eq. [Calabrese-Le Doussal PRL 2011] ..

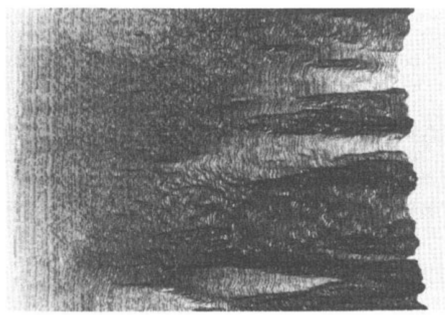
Stationary interfaces

- Init. cond. : stationary interface (1d-Brownian motion)
- Asymptotics : Baik-Rains F_0 distribution, $\text{Airy}_{\text{stat}}$ process
- Shown for : PNG [Baik-Rains JSP 2000], TASEP, KPZ eq. [Imamura-Sasamoto 2012, Borodin *et al.* 2014] ..

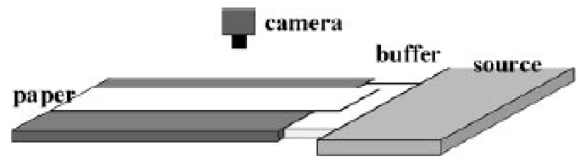
NB1) Scaling exponents are the same. NB2) Other subclasses exist.

Situation in Experiments

Rough interfaces are ubiquitous, but not so universal ?

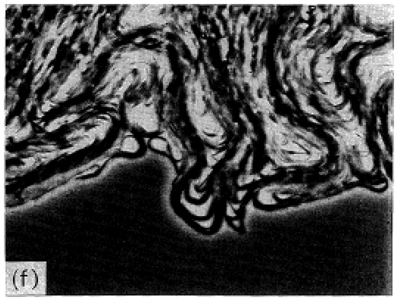


- flow in porous media $\alpha = 0.81$ $\beta = 0.65$
  [Horváth *et al.*, 1991]



- paper wetting $\alpha = 0.73$ $\beta = 0.60$
  [Kobayashi *et al.*, 2005]

- growth of plant callus
  [Galeano *et al.*, 2003]
  $\alpha = 0.86, \beta = 0.17$



- bacteria colony $\alpha = 0.78$
  [Wakita *et al.*, 1997]

cf. $\alpha_{\mathrm{KPZ}} = 1/2, \quad \beta_{\mathrm{KPZ}} = 1/3$

Small, but growing # of experiments showing 1d-KPZ exponents

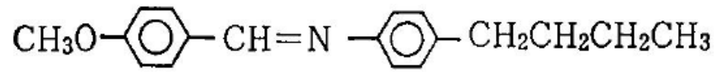
- Slow combustion of paper [Myllys's talk; 1997-]
- Colony of mutant bacteria [Wakita *et al.*, 1997]
- Turbulent liquid crystal [Takeuchi & Sano, 2010-]
- Tumor-like & tumor cells [Albano's talk; 2010-]
- Particle deposition on coffee ring [Yunker's talk; 2013]
- Chemical waves in disordered media [Atis' talk; 2014]

Advantages
- simple growth mechanism
- precise control
- many experimental runs

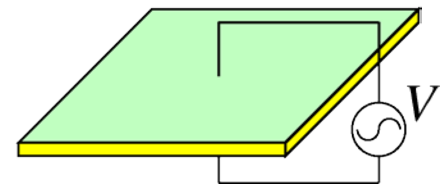
⇨ high statistical accuracy

Electroconvection

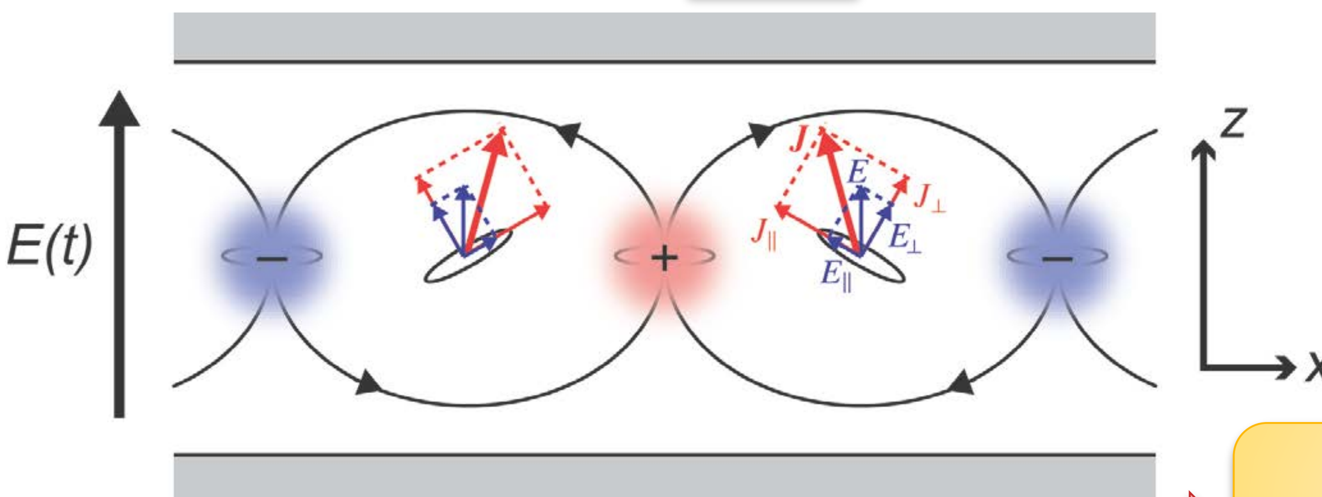
Nematic liquid crystal

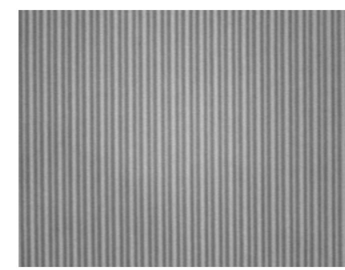
- Rod-like molecule (e.g., MBBA )
- Anisotropic material properties $\varepsilon_{\parallel} \neq \varepsilon_{\perp}, \sigma_{\parallel} \neq \sigma_{\perp}, \cdots$ ($\parallel$: along long axis, $\perp$: along short axis)

  ⇨ Convection driven by electric field
  (Carr-Helfrich instability)



- Interesting case: $\varepsilon_{\parallel} < \varepsilon_{\perp}, \ \sigma_{\parallel} > \sigma_{\perp}$ (true of MBBA)





observed by transmitted light

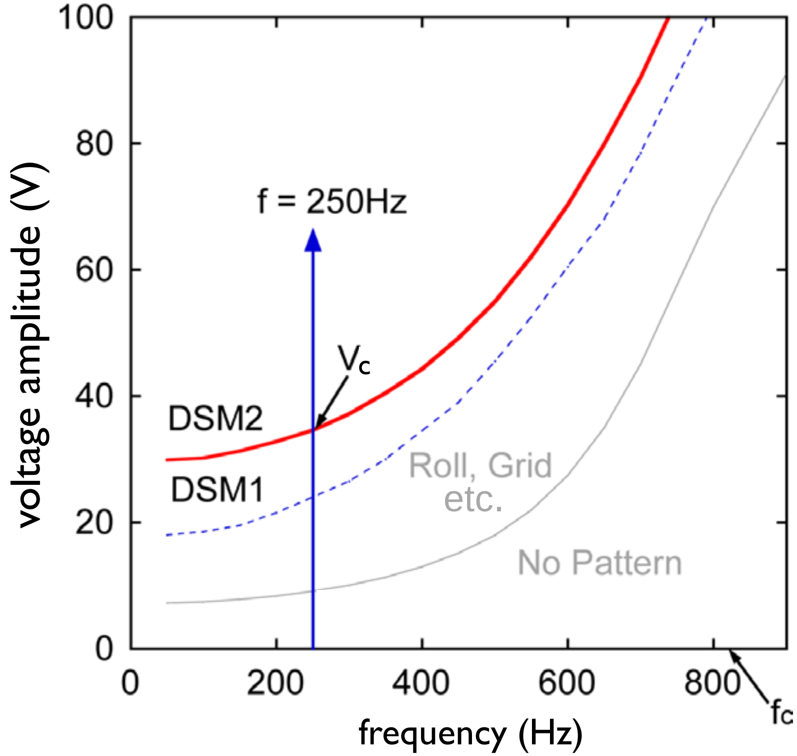
recall Ohm's law $\boldsymbol{J} = \sigma \boldsymbol{E}$

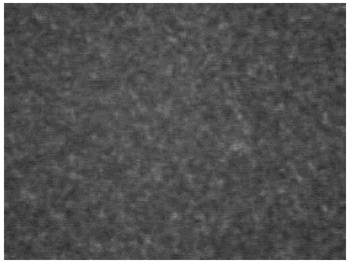
⇨ formation of static convection rolls (Williams domain)

Phase Diagram

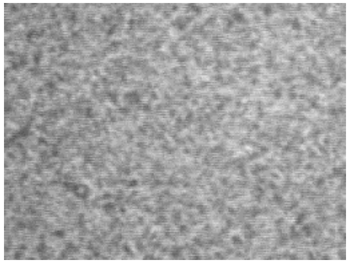
As applied voltage is increased,
convection undergoes a series of transitions toward turbulent (chaotic) states

phase diagram (MBBA; planar alignment)

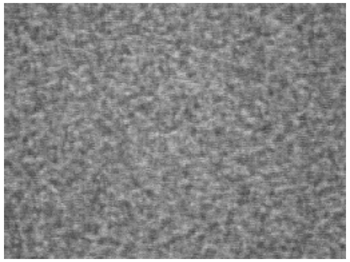




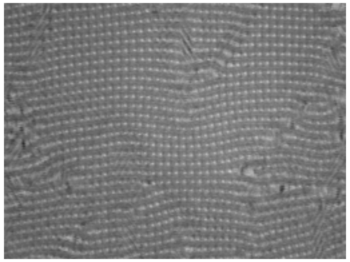
Dynamic Scattering Mode 2 (DSM2)



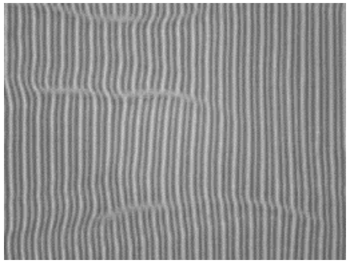
DSM2 nucleation ($V \gg V_c$)



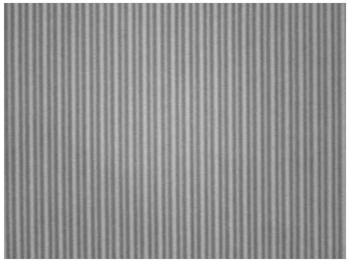
Dynamic Scattering Mode 1 (DSM1)



grid pattern



Fluctuating Williams domain

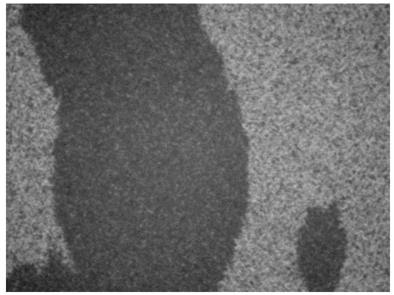


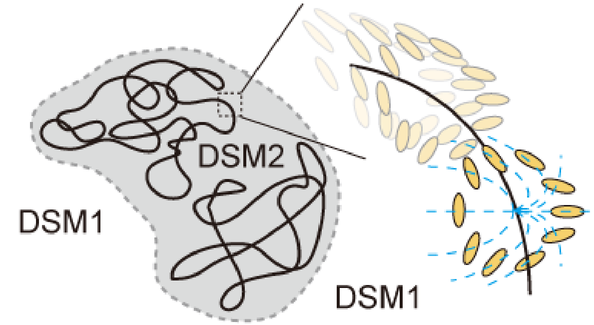
Williams domain

— 100 μm

Two Turbulent States : DSM1 & DSM2

nucleation if $V \gg V_c$





DSM2 = topological-defect turbulence
(analogy with "quantum turbulence"?)

0V → 72V → 0V ($V_c \approx 30$ V, speed x3)

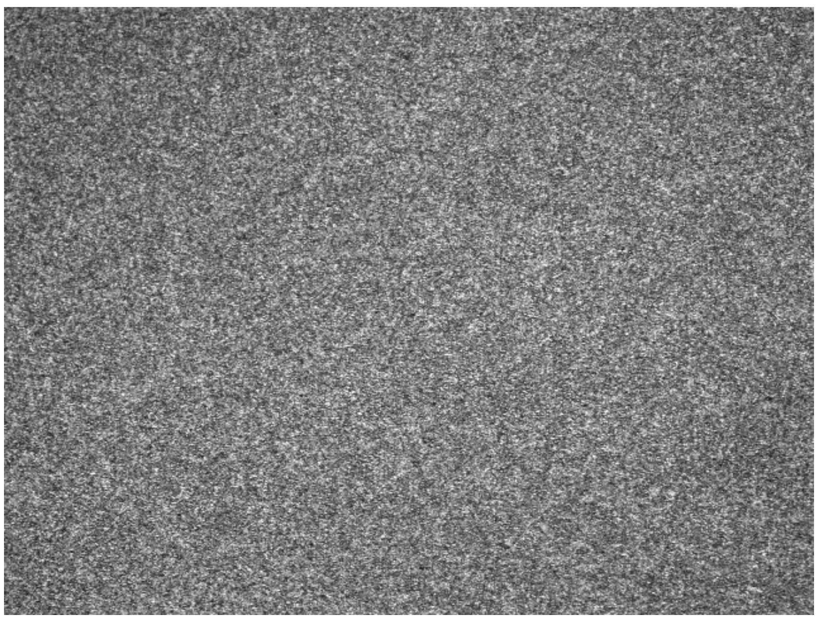
Under applied voltage,
defects are driven by local chaotic flow

- effectively short-range interactions
- no effect of quenched disorder
- (by switching voltage on & off) many runs with a single sample

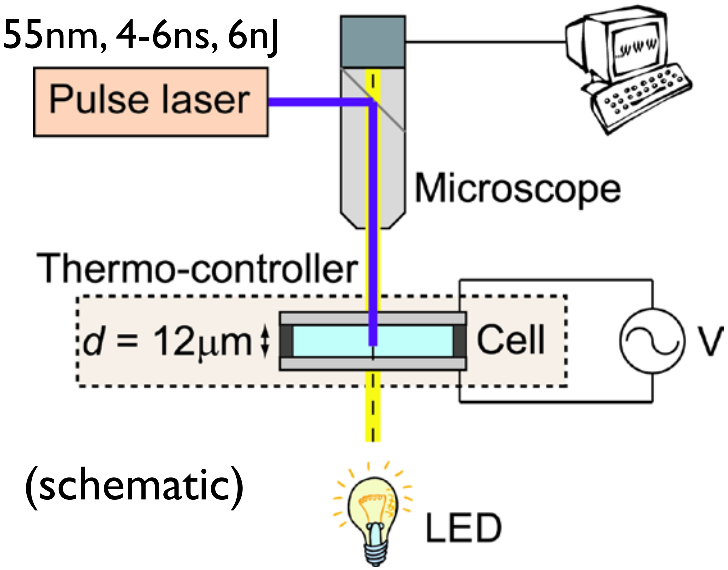
We focus on growing DSM2 interfaces and study their fluctuations

Experimental Setup

- Quasi-2d cell: $16\,\mathrm{mm} \times 16\,\mathrm{mm} \times 12\,\mu\mathrm{m}$
- Nematic liquid crystal: MBBA
- Homeotropic alignment (to work with isotropic growth)
- Temperature control: $T = 25\,^\circ\mathrm{C}$
- Nucleation of DSM2 by UV pulse laser



$26\mathrm{V}, 250\mathrm{Hz}$ Speed x2, — $200\mu\mathrm{m}$

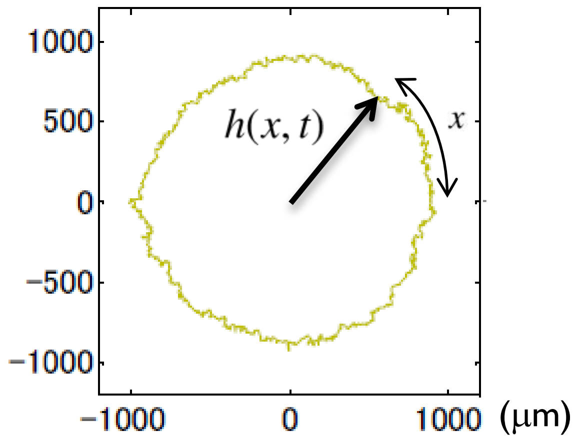


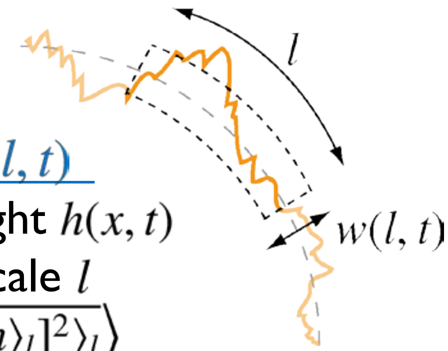
(schematic)

Rough interface appears

Scaling Exponents

interfaces at $t = 2, 7, 12, \cdots, 27$sec





interface width $w(l,t)$

= std. of local height $h(x,t)$ over length scale l

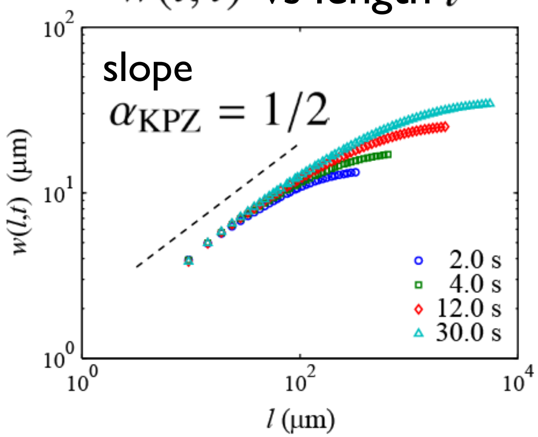
$$= \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_l]^2 \rangle_l} \right\rangle$$

Family-Vicsek scaling

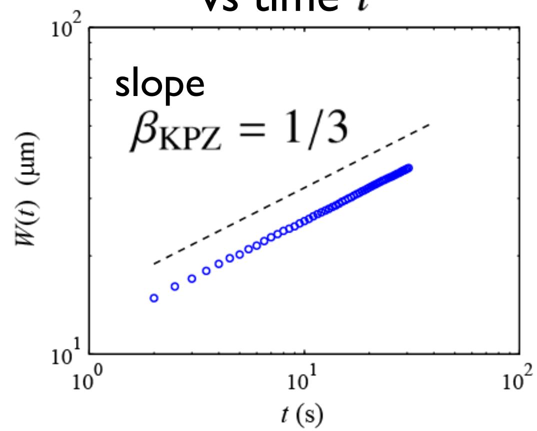
$$w(l,t) \sim t^\beta F(lt^{-1/z}) \sim \begin{cases} l^\alpha & (l \ll l_*) \\ t^\beta & (l \gg l_*) \end{cases}$$

$$l_* \sim t^{1/z}, z = \alpha/\beta$$

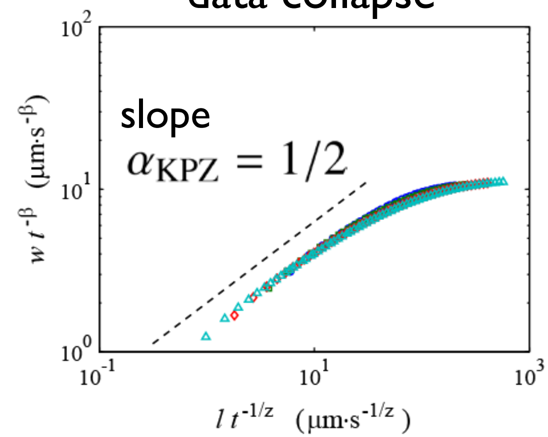
$w(l,t)$ vs length l



vs time t



data collapse



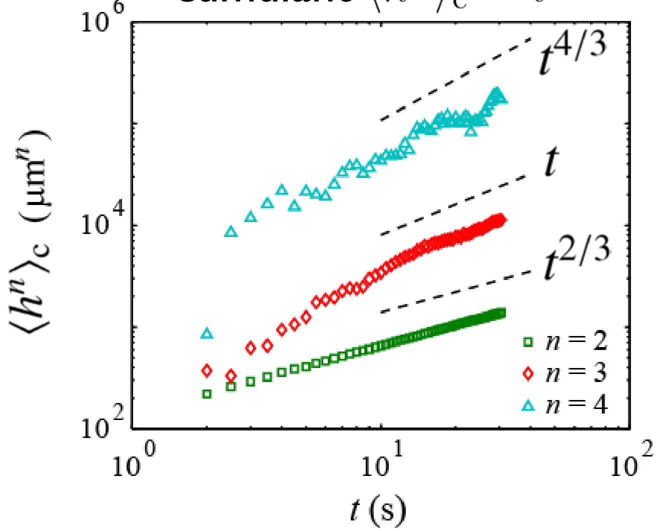
Both exponents α and β agree with the KPZ class

Toward Distribution

Key quantity: nth-order cumulant $\langle h^n \rangle_c$

$$\langle h^2 \rangle_c \equiv \langle \delta h^2 \rangle \qquad (\delta h \equiv h(x,t) - \langle h \rangle)$$
$$\langle h^3 \rangle_c \equiv \langle \delta h^3 \rangle$$
$$\langle h^4 \rangle_c \equiv \langle \delta h^4 \rangle - 3\langle \delta h^2 \rangle^2$$

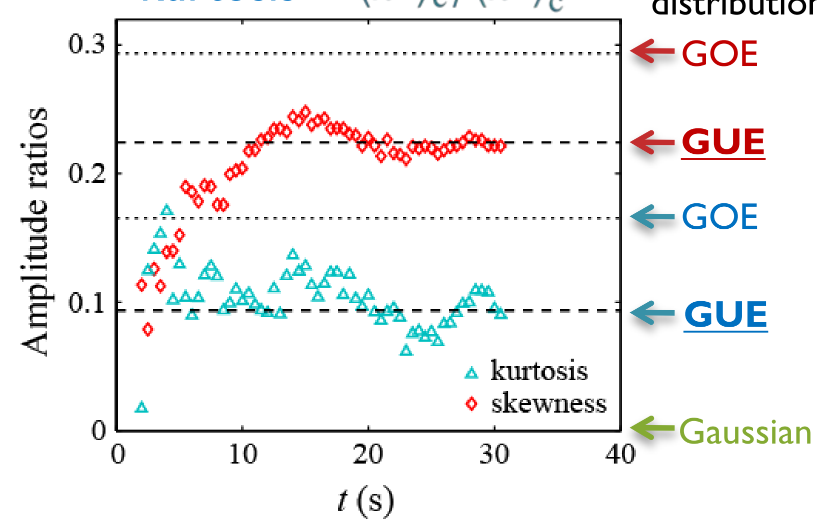
cumulant $\langle h^n \rangle_c \sim t^{n/3}$



skewness $\langle h^3 \rangle_c / \langle h^2 \rangle_c^{3/2}$

kurtosis $\langle h^4 \rangle_c / \langle h^2 \rangle_c^2$

Tracy-Widom distribution for...



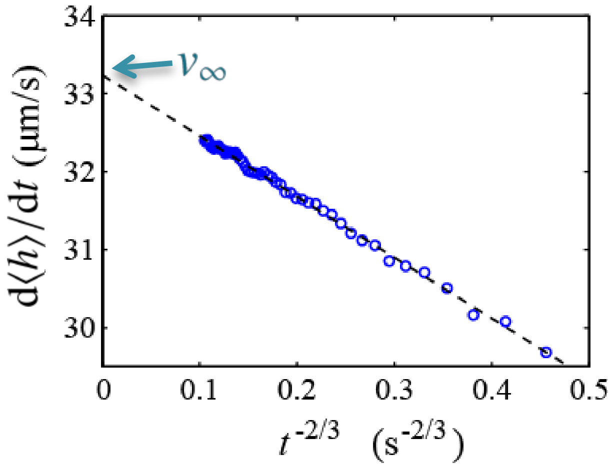
This suggests $h(t) \simeq v_\infty t + (\Gamma t)^{1/3} \chi$ (χ : GUE Tracy-Widom distribution)

with some constant parameters v_∞, Γ

Parameter Estimation

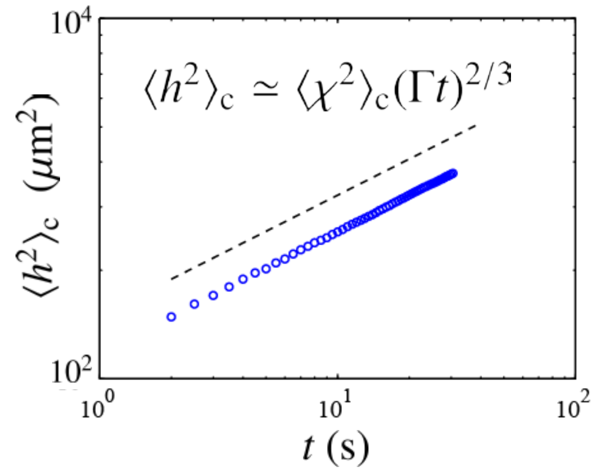
$$h(t) \simeq v_\infty t + (\Gamma t)^{1/3}\chi$$

- Linear growth rate v_∞ … measured from $\dfrac{\mathrm{d}\langle h\rangle}{\mathrm{d}t} \simeq v_\infty + at^{-2/3}$



⇨ $v_\infty = 33.24(4)\ \mu\mathrm{m/s}$

- Amplitude Γ of $t^{1/3}$-fluctuations … from 2nd-order cumulant



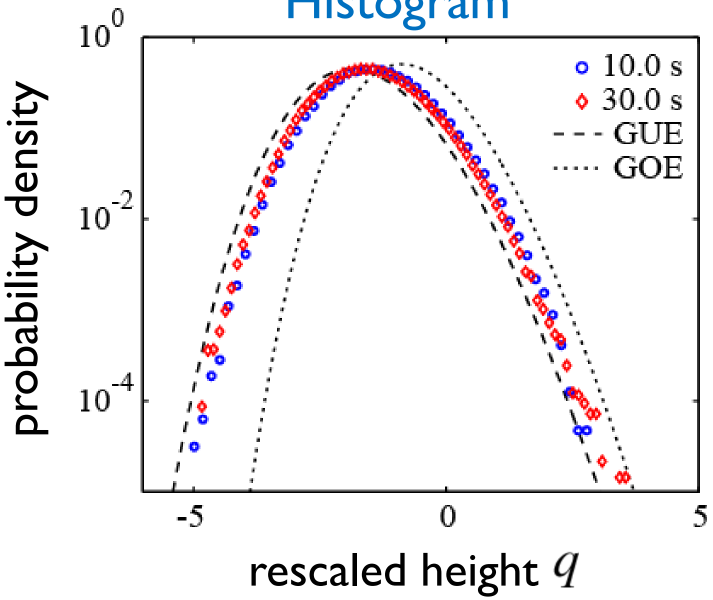
⇨ $\Gamma = 2.29(3) \times 10^3\ \mu\mathrm{m}^3/\mathrm{s}$

Parameter values determined

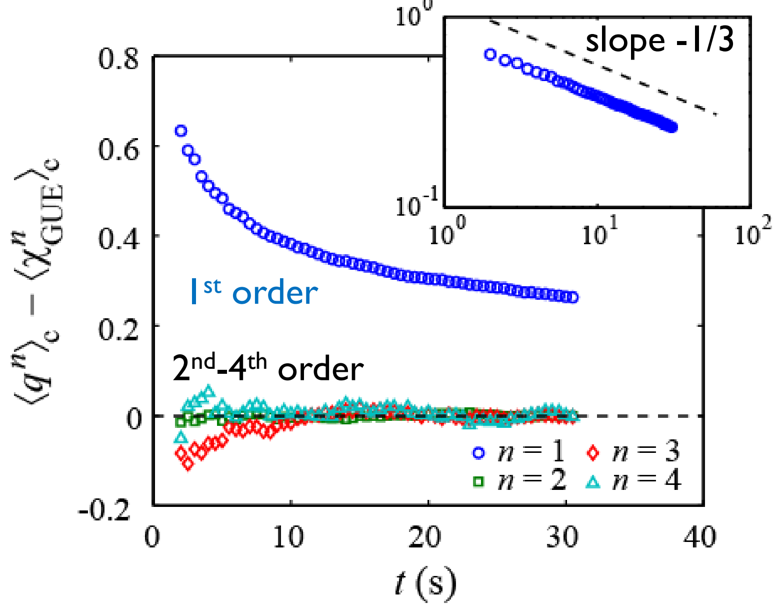
One-Point Distribution

With the rescaled height $q \equiv (h - v_\infty t)/(\Gamma t)^{1/3} \simeq \chi$

Histogram



Difference from GUE-TW distribution

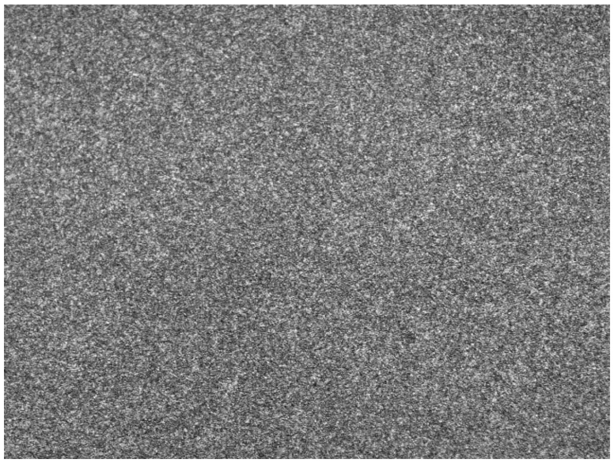


Interface fluctuations precisely agree with the GUE-TW distribution up to the 4th order cumulant. Finite-time effect $\sim t^{-1/3}$ for the mean.

GUE-TW statistics for the circular interfaces is confirmed experimentally

Geometry Dependence

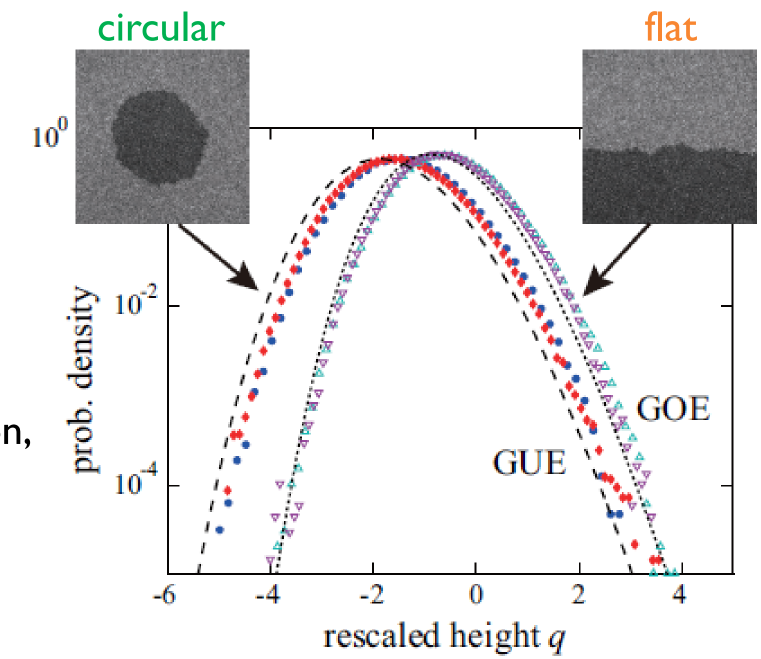
Flat interfaces can also be created by shooting line-shaped laser pulses



26V, 250Hz Speed x5, — 200μm

Same KPZ exponents are found.

measuring the distribution,



Same exponents,

but different distributions!!

circular : $h(t) \simeq v_\infty t + (\Gamma t)^{1/3} \chi_{\mathrm{GUE}}$

flat : $h(t) \simeq v_\infty t + (\Gamma t)^{1/3} \chi_{\mathrm{GOE}}$

NB) $\chi_{\mathrm{GOE}} = 2^{-2/3} \chi_{\mathrm{GOE}}^{\mathrm{standard}}$

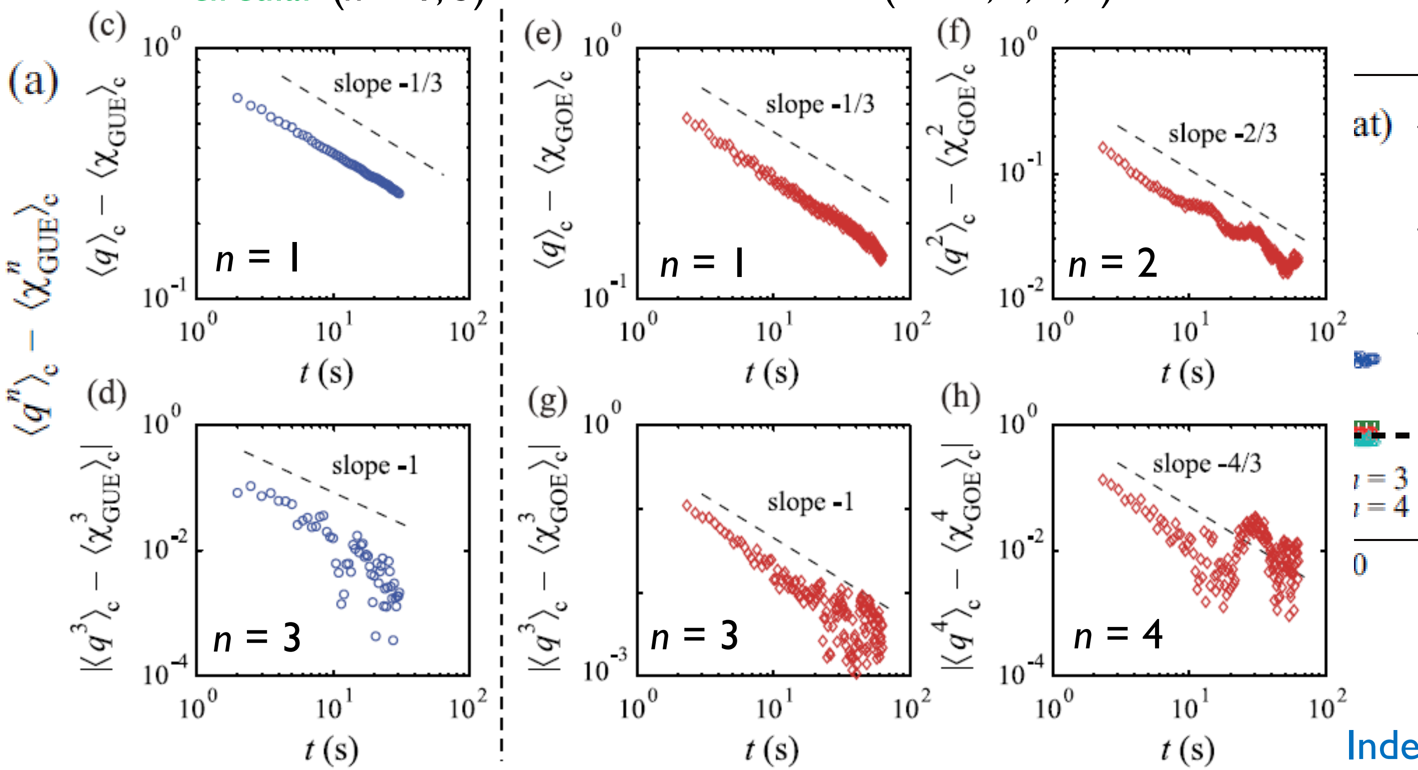
GUE & GOE Tracy-Widom distributions are directly seen.

"Curved KPZ subclass" & "flat KPZ subclass" are confirmed.

Finite-Time Corrections

Differences in nth-order cumulants

circular (n = 1, 3) flat (n = 1, 2, 3, 4)



Finite-t corrections in nth-order cumulants $\sim \mathcal{O}(t^{-n/3})$ $(n \le 4)$

$$h \simeq v_\infty t + (\Gamma t)^{1/3}\chi + \Gamma' \chi'$$

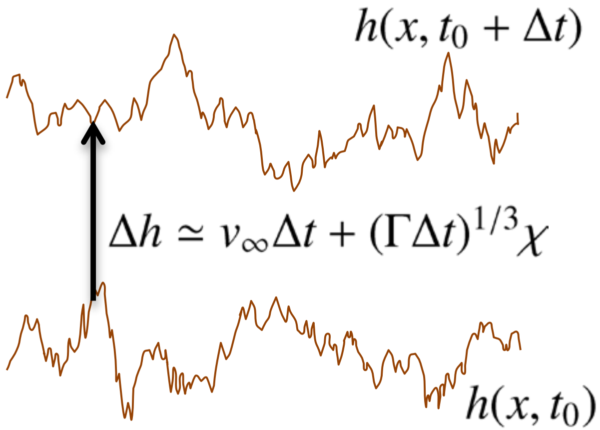
KPZ eq.: $\mathcal{O}(t^{-n/3})$ for $n \le 4$, but $\mathcal{O}(t^{-4/3})$ for $n \ge 5$. [Sasamoto-Spohn PRL 2010]

PNG, TASEP, PASEP: $\mathcal{O}(t^{-1/3})$ for n = 1, $\mathcal{O}(t^{-2/3})$ in moments for $n \ge 2$. [Ferrari-Frings JSP 2011; Baik-Jenkins Ann. Probab. 2013 (PNG)]

Flat-Stationary Crossover

[Takeuchi, PRL 110, 210604 (2013)]

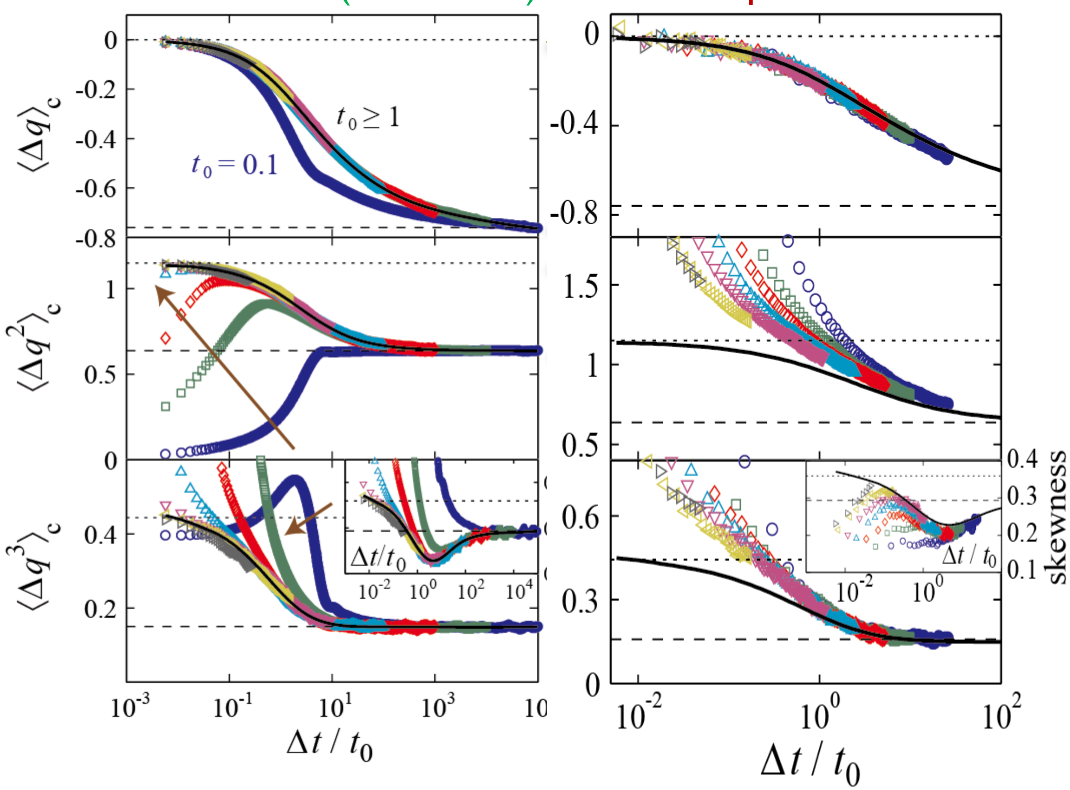
Stationary subclass is theoretically established, but it is never reached in practice.
However, approach, or crossover to the stationary subclass can be studied.



rescaled height difference

$$\Delta q \equiv \frac{\Delta h - v_\infty \Delta t}{(\Gamma \Delta t)^{1/3}} \simeq \chi$$

PNG model (simulation) experiment



- Scaling functions $\langle \Delta q^n \rangle_c \simeq G_n(\Delta t / t_0)$ describing flat-stationary crossover are found.
- Experiment seems to indicate the same scaling functions, so universal!

Spatial Correlation Function

Predictions for solvable models:

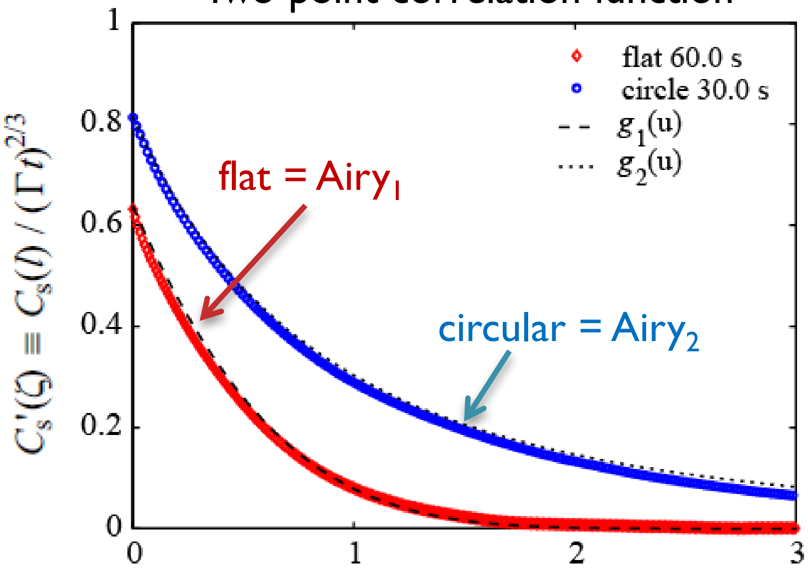
$$C_s(l,t) \equiv \langle h(x+l,t)h(x,t)\rangle - \langle h\rangle^2 \simeq (\Gamma t)^{2/3} g_i(\zeta)$$

with i = 1 (flat), i = 2 (circular),

$$\zeta \equiv l\sqrt{\Gamma/2v_\infty}(\Gamma t)^{-2/3} \quad g_i(\zeta) \equiv \langle \mathcal{A}_i(t'+\zeta)\mathcal{A}_i(t')\rangle - \langle \mathcal{A}_i(t')\rangle^2$$

$\mathcal{A}_i(t')$: Airy$_i$ process (cf. Airy$_2$ = largest-eigenvalue dynamics in Dyson's Brownian motion of GUE matrices)

Two-point correlation function



rescaled length ζ

Correlation of flat / circular interfaces is governed by the Airy$_1$ / Airy$_2$ process

Qualitatively different decay

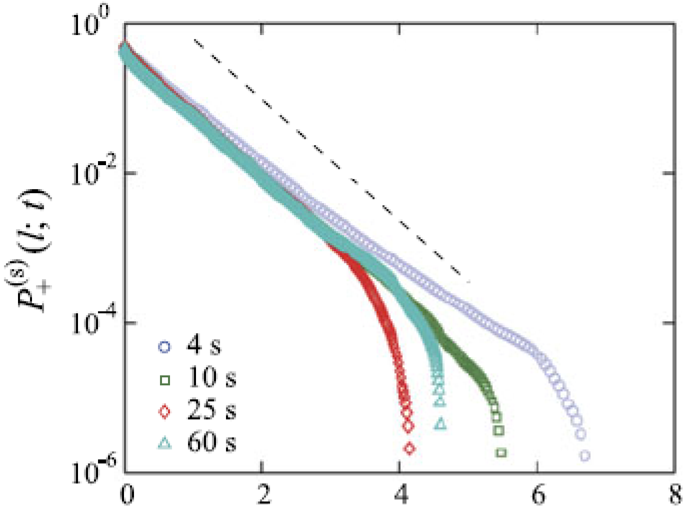
- $g_2(\zeta) \sim \zeta^{-2}$ (circular)
- $g_1(\zeta)$: faster than exponential (flat)

Spatial Persistence

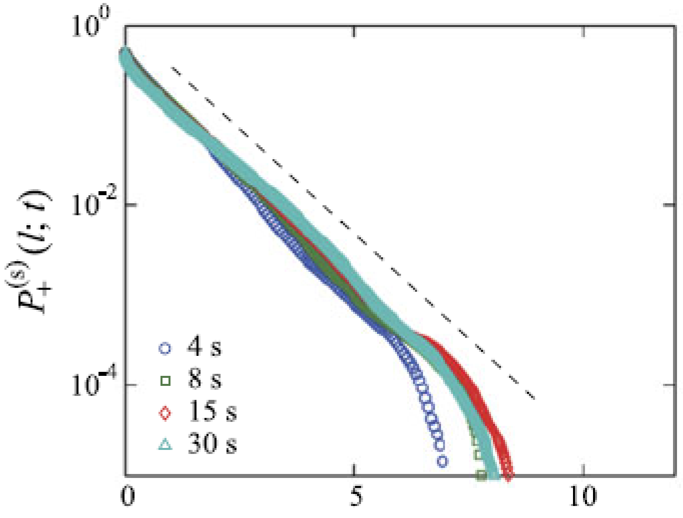
Spatial Persistence probability $P_{\pm}^{(s)}(l;t)$

= probability that $\delta h \equiv h(x,t) - \langle h \rangle$ keeps the same sign over length l in space at fixed time t

flat interfaces



circular interfaces

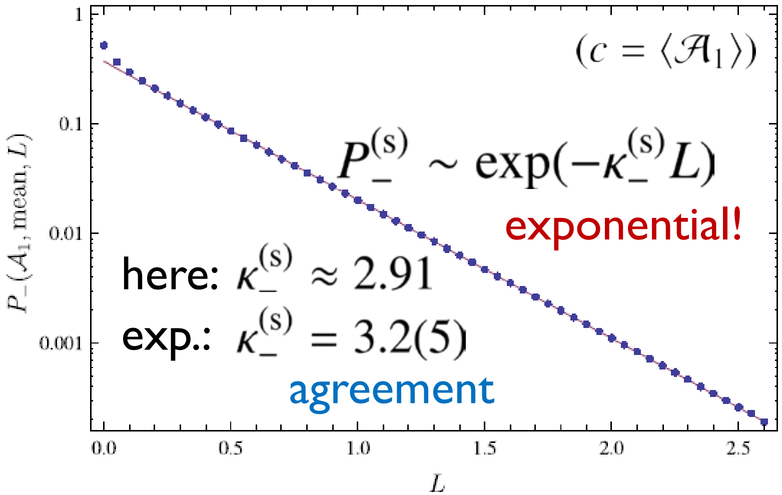


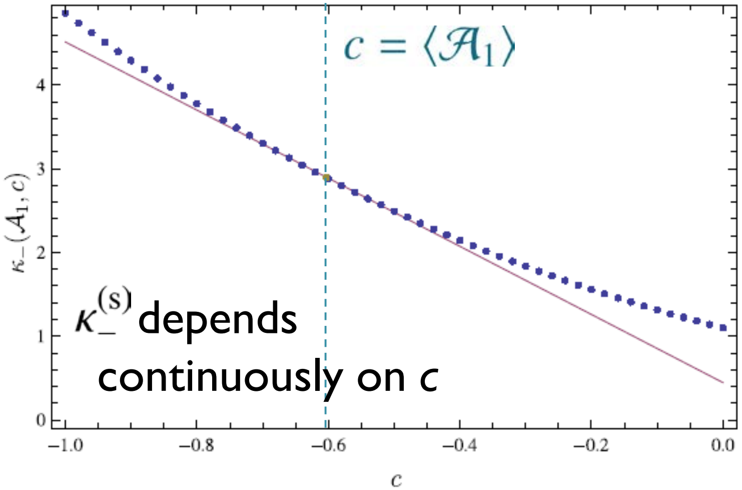
- Exponential decay $P_{\pm}^{(s)} \sim \exp(-\kappa_{\pm}^{(s)}\zeta)$ with symmetric coefficients $\kappa_{+}^{(s)} = \kappa_{-}^{(s)}$
- This should indicate the persistence property of the $Airy_1$/$Airy_2$ processes.
- $\kappa_{\pm}^{(s)}$ expected to be universal $\kappa_{\pm}^{(s)} \approx 2.0$ (flat) $\kappa_{\pm}^{(s)} \approx 0.9$ (circular)

Spatial Persistence

Ferrari and Frings [J. Stat. Mech. 2013, P02001] derived determinantal expressions for the persistence probability of the Airy$_1$/Airy$_2$ processes (probability that $(\mathcal{A}_i(t') - c)$ is negative/positive for $0 \le t' \le L$) for *negative* fluctuations.

- Numerical evaluation (Airy$_1$)





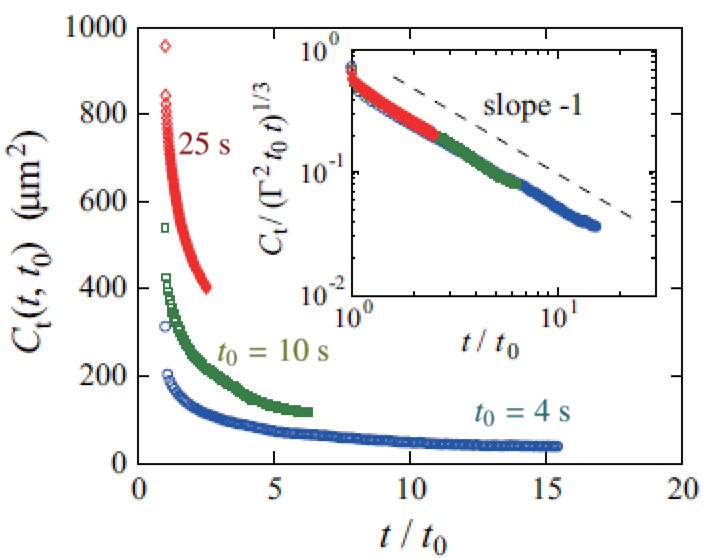
- New open problems
  - Can one solve the persistence probability for positive fluctuations?
  - Experiment showed $\kappa_+^{(s)} \approx \kappa_-^{(s)}$ at $c = \langle \mathcal{A}_i \rangle$, while $\kappa_\pm^{(s)}(c)$ depends on c. Can this equality be shown, or it's just a coincidence?

Temporal Correlation Function

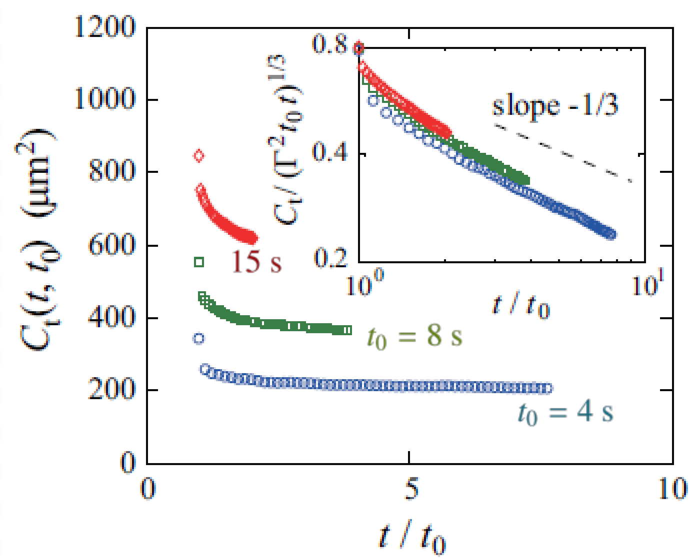
$$C_t(t, t_0) \equiv \langle h(x,t)h(x,t_0)\rangle - \langle h(x,t)\rangle\langle h(x,t_0)\rangle$$

analytically unsolved yet

flat



circular



- Natural scaling ansatz works

$$C_t(t, t_0) \simeq (\Gamma^2 t_0 t)^{1/3} F_t(t/t_0)$$

- Long-time asymptotics

$$F_t(t/t_0) \sim (t/t_0)^{-\bar{\lambda}} \quad \text{with} \quad \bar{\lambda} = 1$$

cf. Kallabis-Krug conjecture $\bar{\lambda} = \beta + d/z = 1$

- Stronger finite-time correction to the scaling ansatz.
- Singha's approximative theory [JSM 2005] works after modification. In particular,

$$C_t(t, t_0) > 0 \quad (t \to \infty)$$ (!)

local ergodicity breaking?

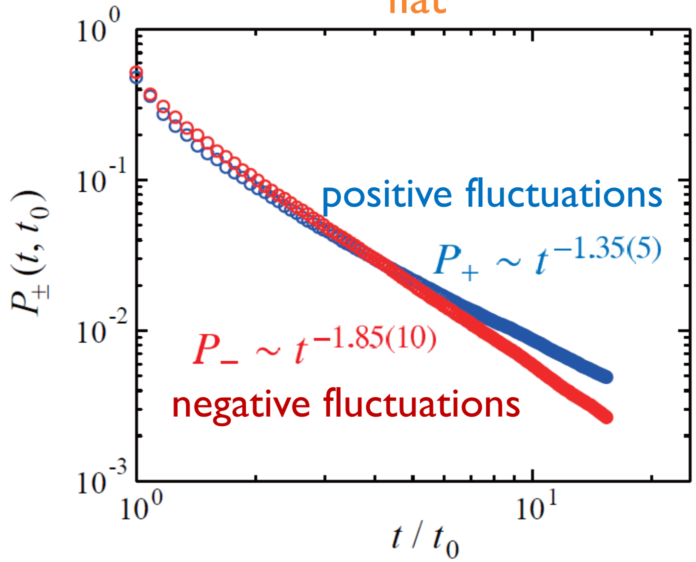
Temporal Persistence

typically, $P_{\pm}(t, t_0) \sim t^{-\theta_{\pm}^{(\mathrm{p})}}$

Persistence probability $P_{\pm}(t, t_0)$

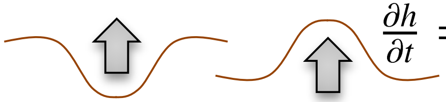
= joint probability that $\delta h \equiv h(x,t) - \langle h \rangle$ at a fixed position x
is positive (negative) at time t_0 and keeps the same sign until time t

flat



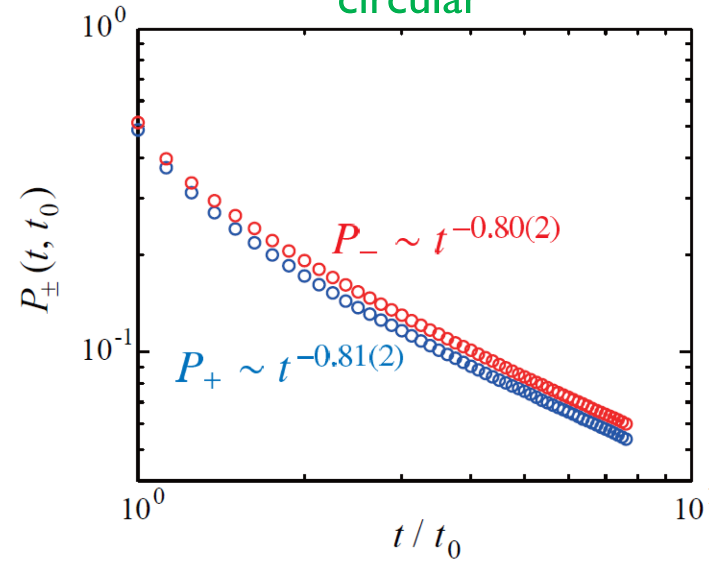
$$\theta_+^{(\mathrm{p})} = 1.35(5), \quad \theta_-^{(\mathrm{p})} = 1.85(10)$$

- $\theta_+^{(\mathrm{p})} < \theta_-^{(\mathrm{p})}$ due to the KPZ nonlinearity



$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$$

circular



$$\theta_+^{(\mathrm{p})} = 0.80(2), \quad \theta_-^{(\mathrm{p})} = 0.81(2)$$

- Asymmetry cancelled! $\theta_+^{(\mathrm{p})} = \theta_-^{(\mathrm{p})}$
- $\theta_{\pm}^{(\mathrm{p})} < 1$ consistent with local ergodicity breaking ($\because \int P_{\pm} \mathrm{d}t = \infty$)

Ergodicity Breaking?

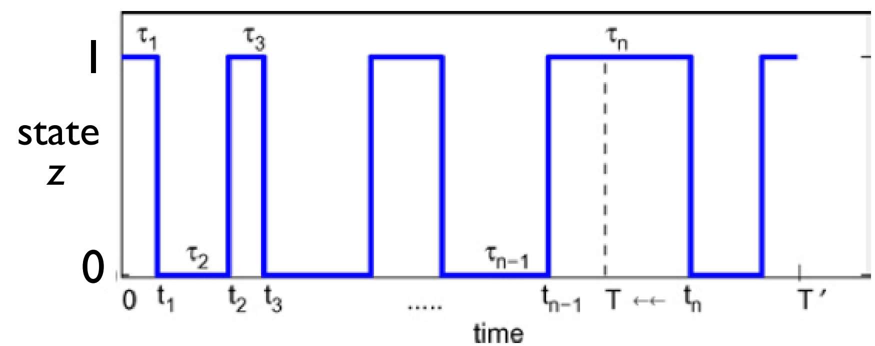
- "Weak ergodicity breaking" [J.-P. Bouchaud 1992]
  time average ≠ ensemble average, because of long & random trapping of trajectory

- "Dichotomous process" : simplest model

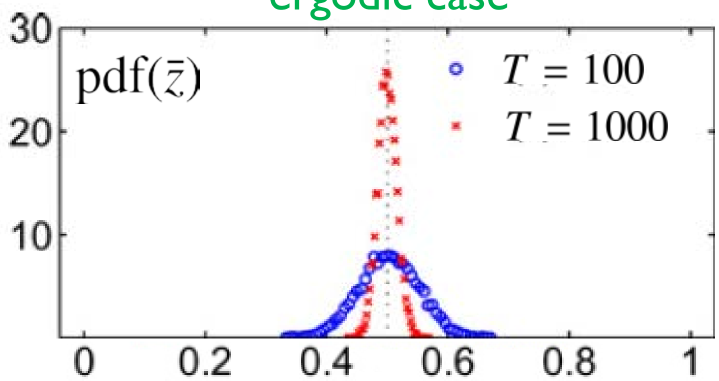
$$\mathrm{Prob}[\text{ duration of each state } > \tau] \sim \tau^{-\theta}$$

$0 < \theta < 1$: weak ergodicity breaking

e.g.) time-avg. of z : $\bar{z}(T) = \frac{1}{T}\int z(t)\mathrm{d}t$

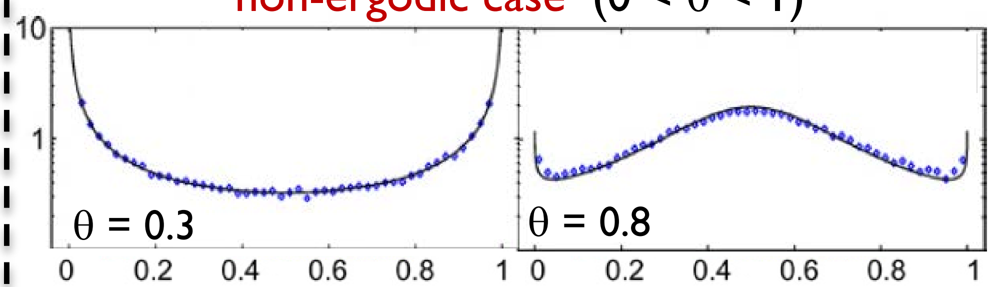


ergodic case



$$\mathrm{pdf}(\bar{z}) \to \delta(\bar{z} - \langle z \rangle)$$

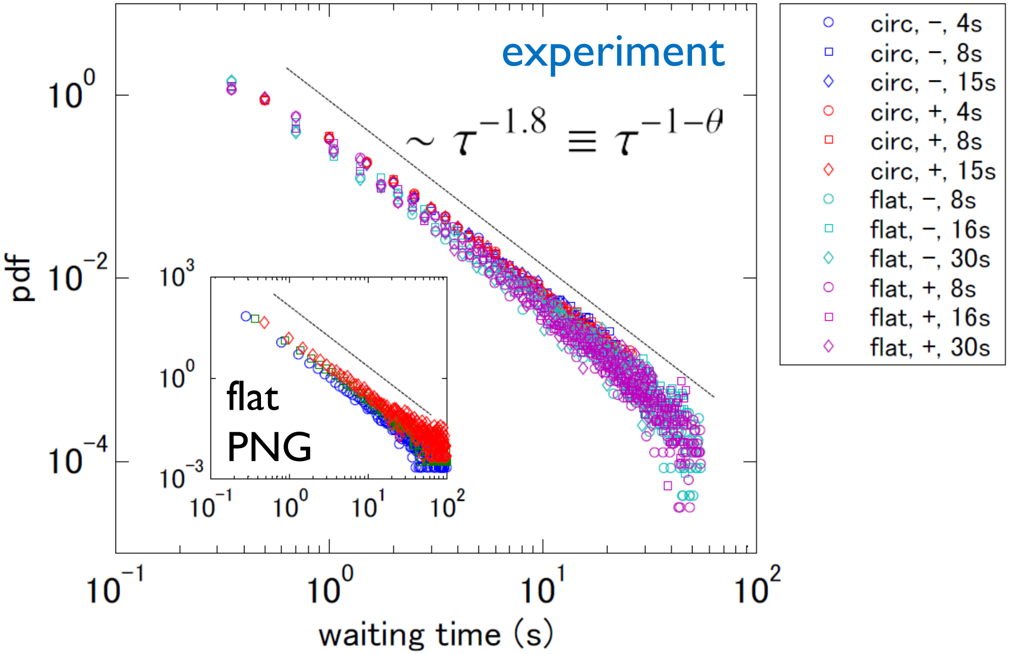
non-ergodic case (0 < θ < 1)



pdf converges to an asymptotic distribution
($\bar{z}$ remains stochastic).

KPZ Interfaces vs Dichotomous Process

- Dichotomous process is far too simple to describe KPZ.
  two-state model, no space, uncorrelated waiting times…
- But, let's naively compare... one can regard δh's sign as the state z.
  Waiting-time dist. is related to persistence prob.
  For dichotomous process, θ is the persistence exponent $\theta^{(p)}$. (for 0<θ<1)
- First, measure the waiting-time distribution.



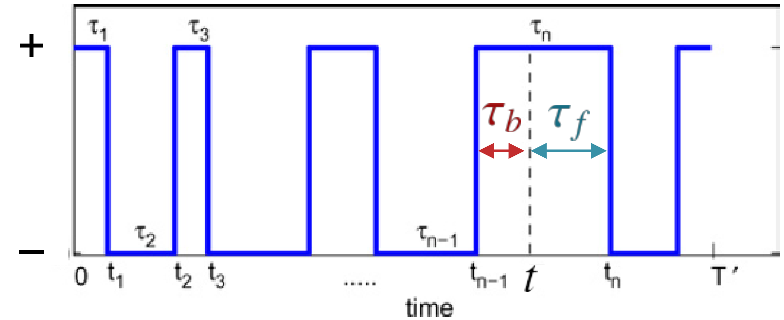
- Power-law waiting-time dist. with θ = 0.8, regardless of time, +/-, circular/flat.
- Maybe not too stupid to compare with dich. process?
- Dich. : $\theta = \theta^{(p)}$
  Circ : $\theta = \theta_{\pm}^{(p)} \approx 0.8$
  Flat : $\theta \neq \theta_{\pm}^{(p)} > 1$

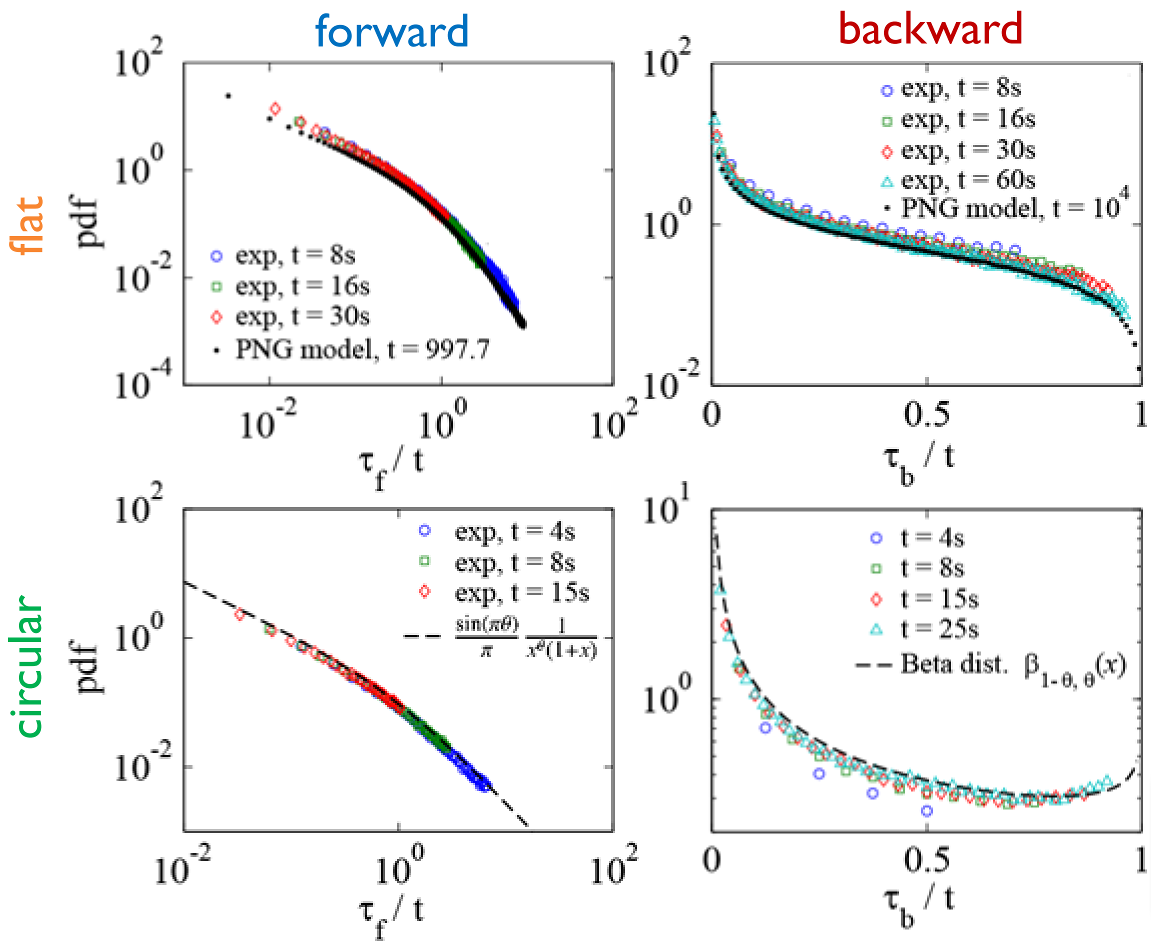
Recurrence Time Statistics

Forward recurrence time $\tau_f(t)$

Backward recurrence time $\tau_b(t)$

NB: persistence prob. $= \int_{\tau}^{\infty} \mathrm{pdf}(\tau_\mathrm{f}) \mathrm{d}\tau_\mathrm{f}$



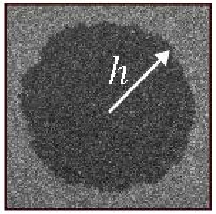


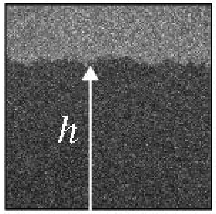
- All pdfs converge to different universal functions. (shown are data for positive fluctuations)
- Flat/circ. interfaces have same waiting-time dist., but different recur. time dist.
- Data for circ. interfaces nicely agree with exact results for dich. process!

w/ a few other results

Some set of time corr. props of circ. interfaces is reproduced by dich. process!

Summary





Turbulent liquid crystal shows us the beautiful physics (and even math!) of the (1+1)d KPZ class

Experimentally confirmed predictions (those based on exact solutions)

- Primary features of the flat/curved subclasses are evidenced. (KPZ exponents, GOE/GUE-TW dist., $\mathrm{Airy}_{1,2}$ processes)
- There are other confirmed predictions (e.g., max. height distribution).
- Universal crossover to the stationary subclass (Baik-Rains F_0 dist.) is detected.

Beyond a "mere" confirmation…

- What should be measured now is unsolved properties! (time correlation, spatial persistence, finite-time effects, ...)
- Time correlation looks particularly fascinating. (for circ. interfaces, "weak ergodicity breaking" may be a key concept)
- Physical understanding for universality? (subclass structure, bridging micro & macro)

Refs: KaT & Sano, J. Stat. Phys. 147, 853; KaT, PRL 110, 210604; KaT & Akimoto, in preparation