



Dynamics of the KPZ interface in disordered media: theoretical and experimental studies

B. Kahng

Dept of Physics & Astronomy

Seoul National University, Korea

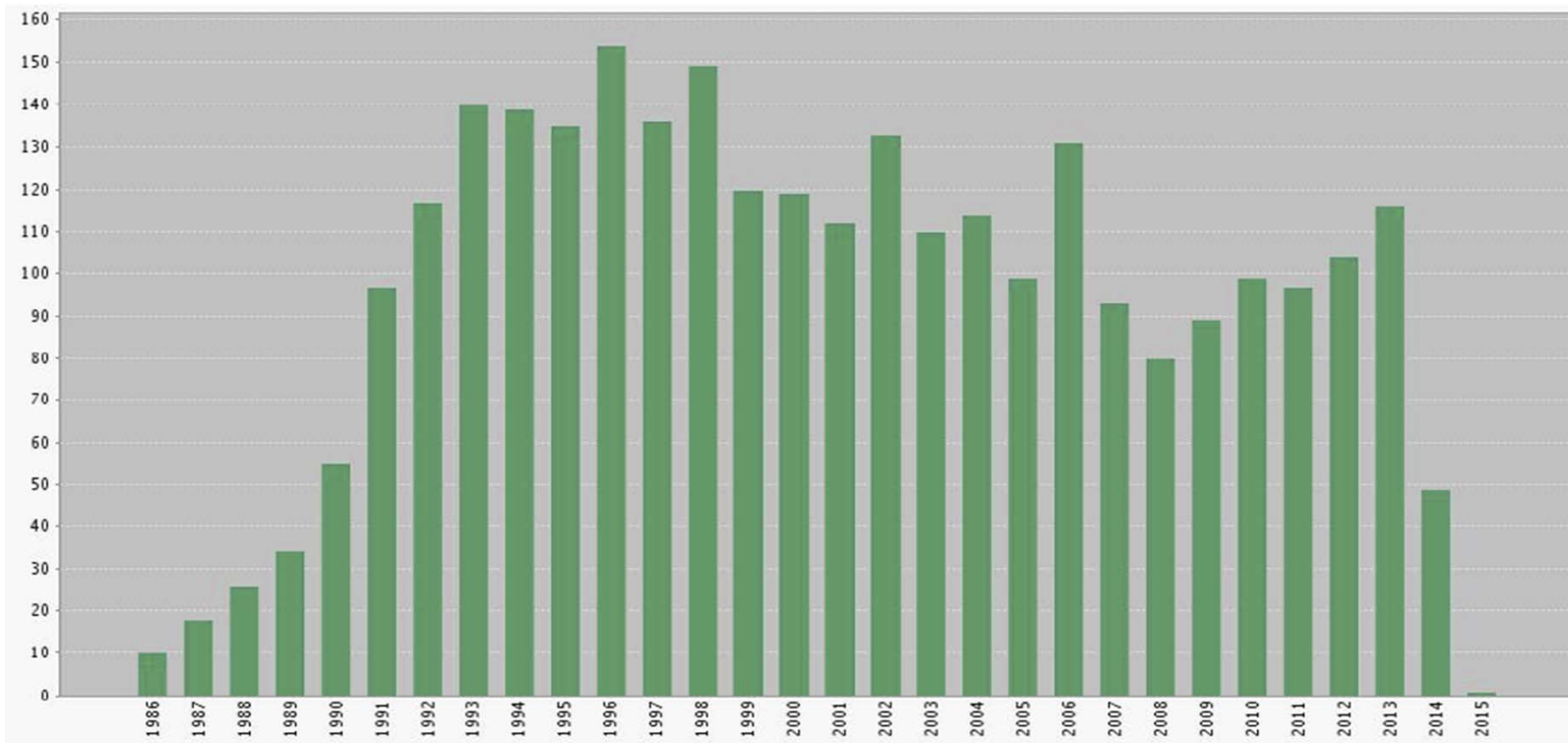
Collaborators:

Exp: Prof. Sug-Bong Choe (SNU)

Interface fluctuations and KPZ universality class,

At YITP, Kyoto Japan, Aug 20th-23rd, 2014

The number of citations for the KPZ paper since 1986 = 3012 (total)



Contents

Theory part

- Anisotropic KPZ equation with thermal noise in 2+1 dimensions: in weak coupling limit
- Anisotropic KPZ equation with quenched noise in 2+1 dimensions
- KPZ equation with negative nonlinear term in 1+1 dimensions

Experiment part

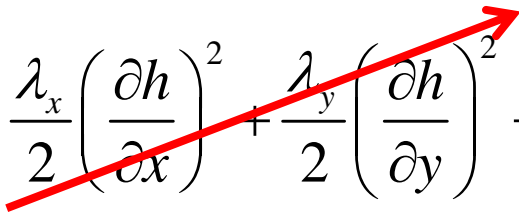
- Domain wall motion in disordered media driven by magnetic field
- Domain wall motion in disordered media driven by electric current

KPZ equation with thermal noise

$$\frac{\partial h(x,t)}{\partial t} = V + v \frac{\partial^2 h}{\partial x^2} + \frac{\lambda}{2} \left(\frac{\partial h}{\partial x} \right)^2 + \eta(x,t) \quad \langle (\Delta h_i)^2 \rangle^{1/2} \sim \begin{cases} t^\beta & t < L^z \\ L^\alpha & t > L^z \end{cases}$$

$\lambda \sim V$ α and β are invariant under $\lambda \leftrightarrow -\lambda$

Anisotropic KPZ equation

$$\frac{\partial h(x,y,t)}{\partial t} = v_x \frac{\partial^2 h}{\partial x^2} + v_y \frac{\partial^2 h}{\partial y^2} + \frac{\lambda_x}{2} \left(\frac{\partial h}{\partial x} \right)^2 + \frac{\lambda_y}{2} \left(\frac{\partial h}{\partial y} \right)^2 + \eta(x,y,t)$$


If $\lambda_x \lambda_y < 0$

AKPZ \rightarrow EW
universality class

Wolf, PRL (1991)

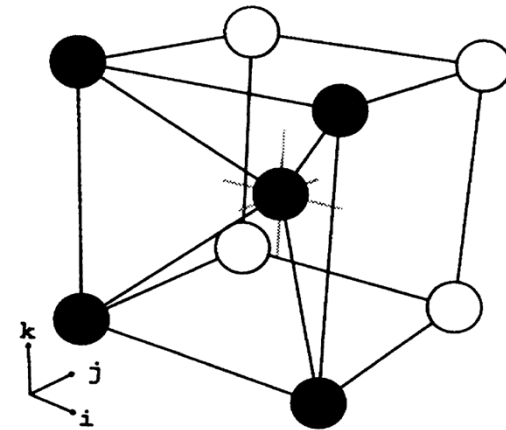
Stochastic model for the AKPZ class in weak coupling limit

- Three dimensional Toom model on a bcc lattice
- SoS model with avalanche process

Derrida, Lebowitz,
Speer, Spohn, PRL
(1991)

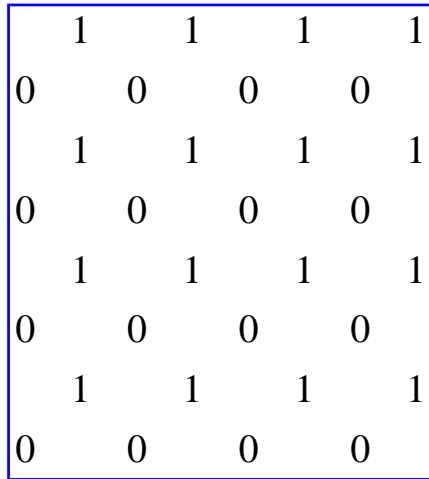
$$\sigma_{i,j,k}(t+1) = \begin{cases} \text{majority rule} & \text{with prob } 1-p-q \\ +1 & \text{with prob } p \\ -1 & \text{with prob } q \end{cases}$$

Majority rule
is applied according to
the five spins at black sites

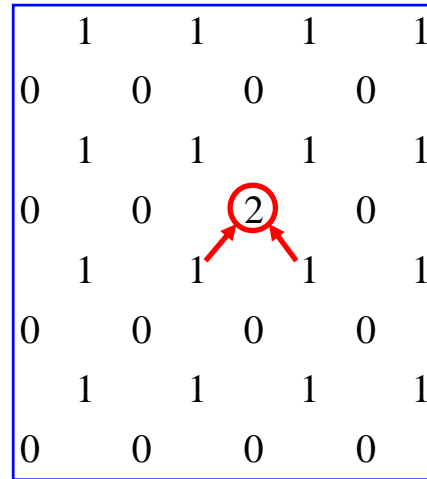


Jeong, BK and Kim, PRL (1993)

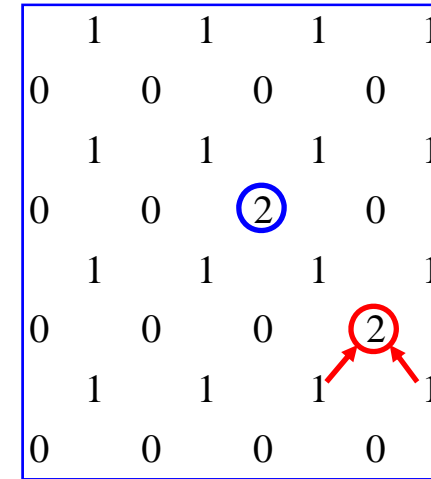
Solid-on-Solid model



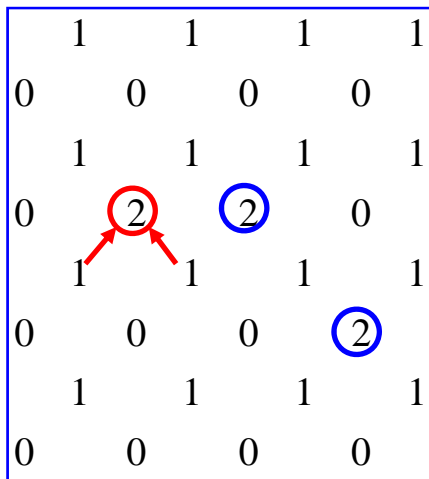
Initial state



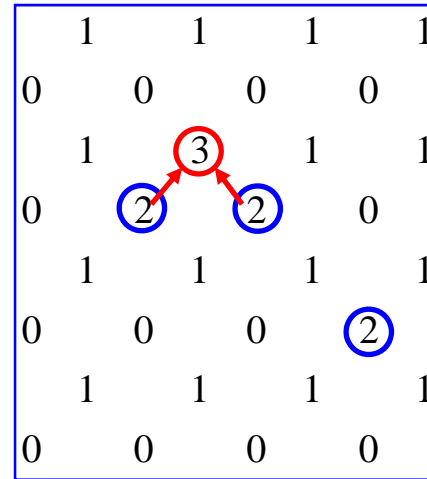
Random deposition under SOS



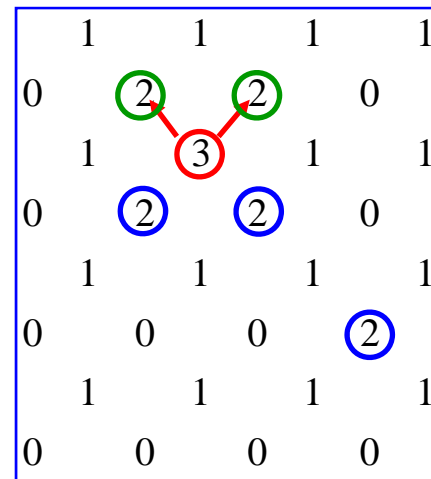
Random deposition under SOS



Random deposition under SOS



Random deposition under SOS



Avalanche for SOS

Signs of KPZ terms

Purely deposition case $p=0$

1	1	1	1
0	0	0	0
1	1	1	1
0	0	0	0
1	1	1	1
0	0	0	0
1	1	1	1
0	0	0	0

Initial state

1	1	1	1
0	2	2	0
1	3	1	1
2	2	2	2
1	1	1	1
0	0	2	0
1	1	1	1
0	0	0	0

Tilted along j direction

1	1	1	1
0	0	2	0
1	1	1	1
0	2	2	0
1	1	1	1
0	0	2	0
1	1	1	1
0	0	2	0

Tilted along i direction

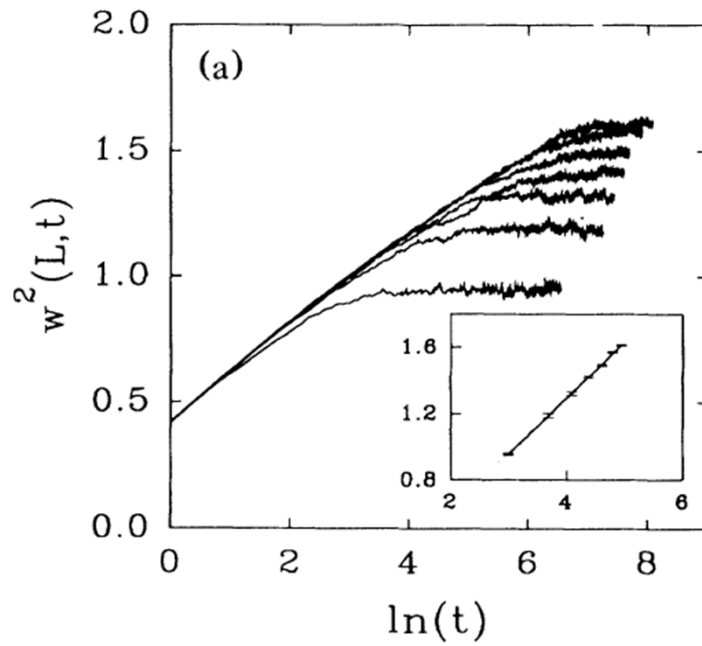
$$\langle \Delta h \rangle_0 = \frac{L^2}{2L^2} 2 = 1$$

$$\langle \Delta h \rangle_{\parallel} = \frac{L(L-1)}{2L^2} 2 + \frac{L}{2L^2} 6 = 1 + \frac{2}{L}$$

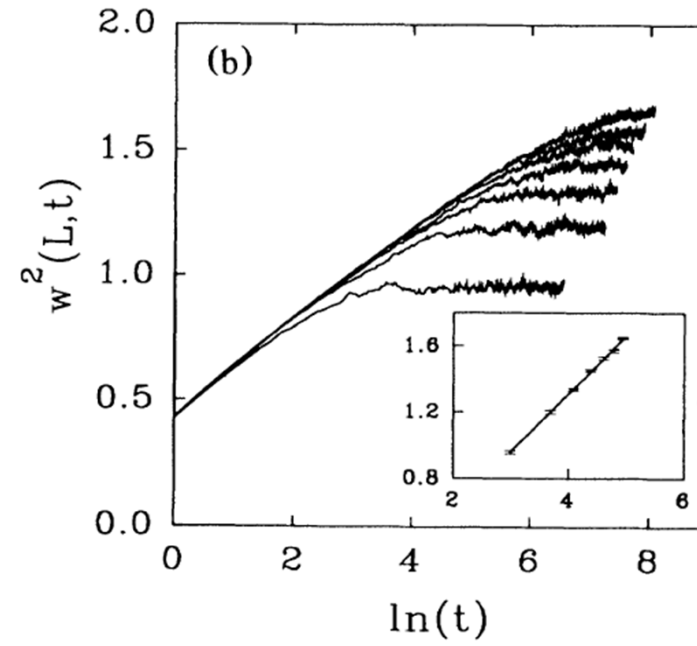
$$\langle \Delta h \rangle_{\perp} = \frac{L(L-1)}{2L^2} 2 = 1 - \frac{1}{L}$$

$$\langle \Delta h \rangle_{\parallel} > \langle \Delta h \rangle_0 \Rightarrow \lambda_y > 0$$

$$\langle \Delta h \rangle_{\perp} < \langle \Delta h \rangle_0 \Rightarrow \lambda_x < 0$$



Evaporation rate $\rho=0.5$



Evaporation rate $\rho=0.3$

EW universality class

KPZ equation with quenched noise

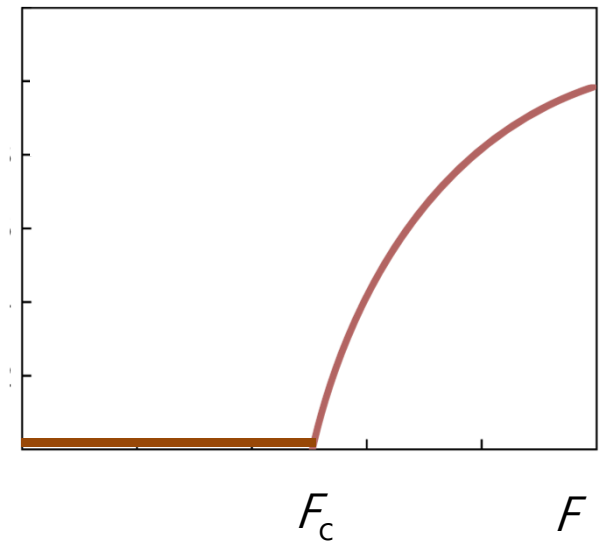
$$\frac{\partial h(x,t)}{\partial t} = v \frac{\partial^2 h}{\partial x^2} + \frac{\lambda}{2} \left(\frac{\partial h}{\partial x} \right)^2 + F + \eta(x,h)$$

$$\langle \eta(x,h) \rangle = 0 \quad \langle \eta(x,h) \eta(x',h') \rangle = 2D \delta(x-x') \delta(h-h')$$

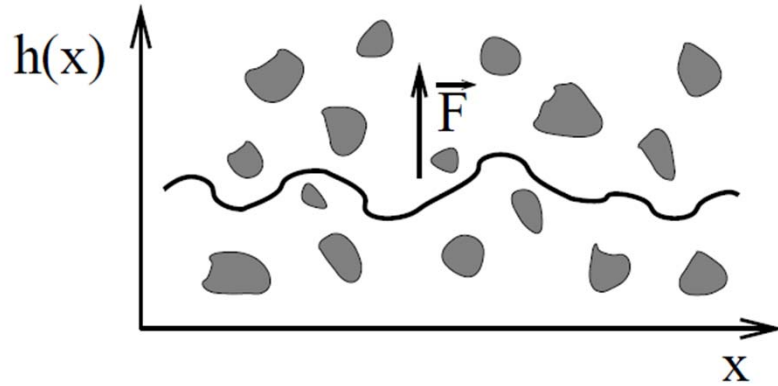
Pinning-depinning (PD) transition: **When $\lambda > 0$**

$$v(m=0) \sim (F - F_c)^\theta, \quad \theta = v_{\parallel} - v_{\perp} \approx 0.64$$

Continuous transition



Surface roughness: $w \sim L^\alpha$ ($\alpha = v_{\perp} / v_{\parallel} \approx 0.633$) **Directed percolation class**



KPZ equation

$$\frac{\partial h(x,t)}{\partial t} = \nu \frac{\partial^2 h}{\partial x^2} + \frac{\lambda}{2} \left(\frac{\partial h}{\partial x} \right)^2 + F + \eta(x,h)$$

Positive λ (field) assists
the interface motion

finite $|\partial h/\partial x|$

Negative λ (current) impedes
the interface motion

Strong pinning effect
to interface motion

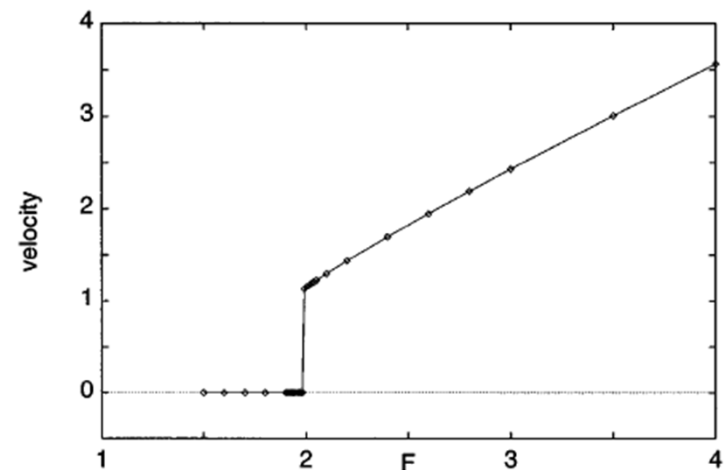
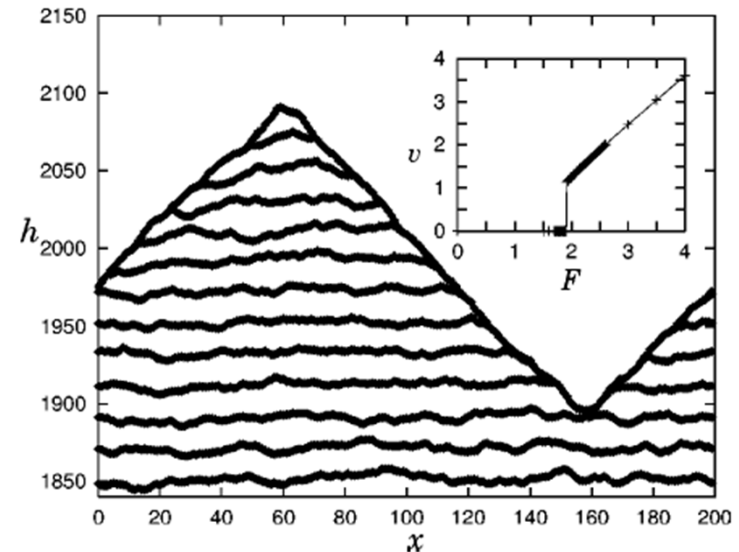
$$\frac{\partial h(x,t)}{\partial t} = v \frac{\partial^2 h}{\partial x^2} + \frac{\lambda}{2} \left(\frac{\partial h}{\partial x} \right)^2 + F + \eta(x, h)$$

When $\lambda < 0$

Discontinuous PD transition

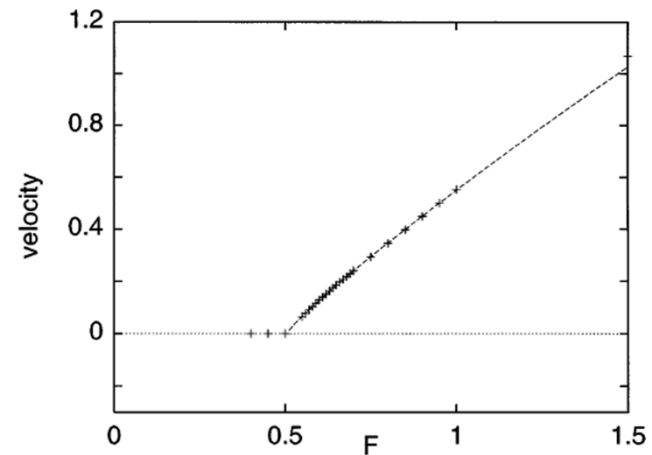
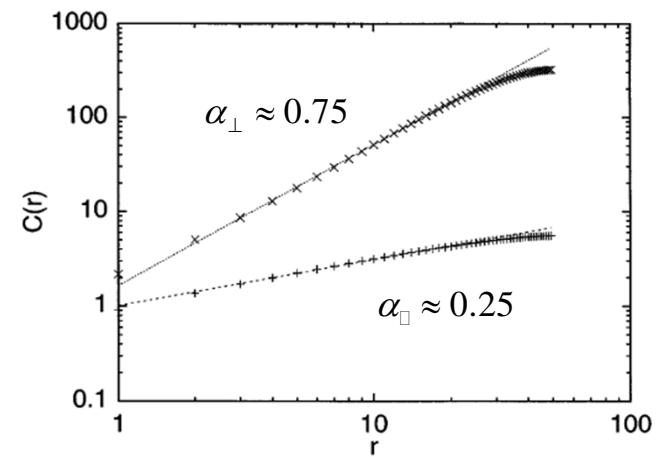
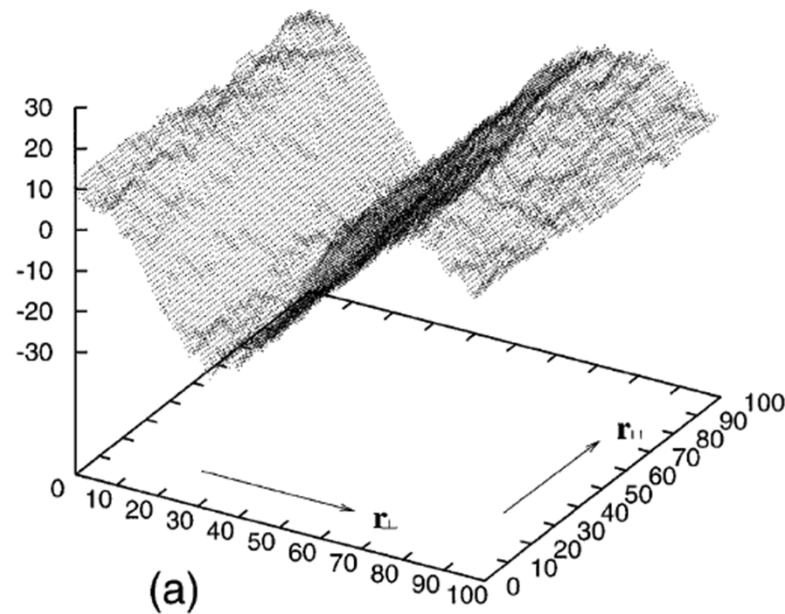
Facet formation

Jeong, BK and Kim,
PRL (1996), PRE (1999)



Anisotropic KPZ equation with quenched noise

$$\frac{\partial h(x, y, t)}{\partial t} = v_x \frac{\partial^2 h}{\partial x^2} + v_y \frac{\partial^2 h}{\partial y^2} + \frac{\lambda_x}{2} \left(\frac{\partial h}{\partial x} \right)^2 + \frac{\lambda_y}{2} \left(\frac{\partial h}{\partial y} \right)^2 + F + \eta(x, y, h)$$



Jeong, BK, Kim, PRL (1996)

Conclusion

Theory part

- Anisotropic KPZ equation with thermal noise in 2+1 dimensions: in weak coupling limit
- Anisotropic KPZ equation with quenched noise in 2+1 dimensions
- KPZ equation with negative nonlinear term in 1+1 dimensions

Experiment part

- Domain wall motion in disordered media driven by magnetic field
- Domain wall motion in disordered media driven by electric current