

How do diquark fluctuations and chiral soft modes affect di-lepton production in the deconfined phase

Teiji Kunihiro (YITP, Kyoto)

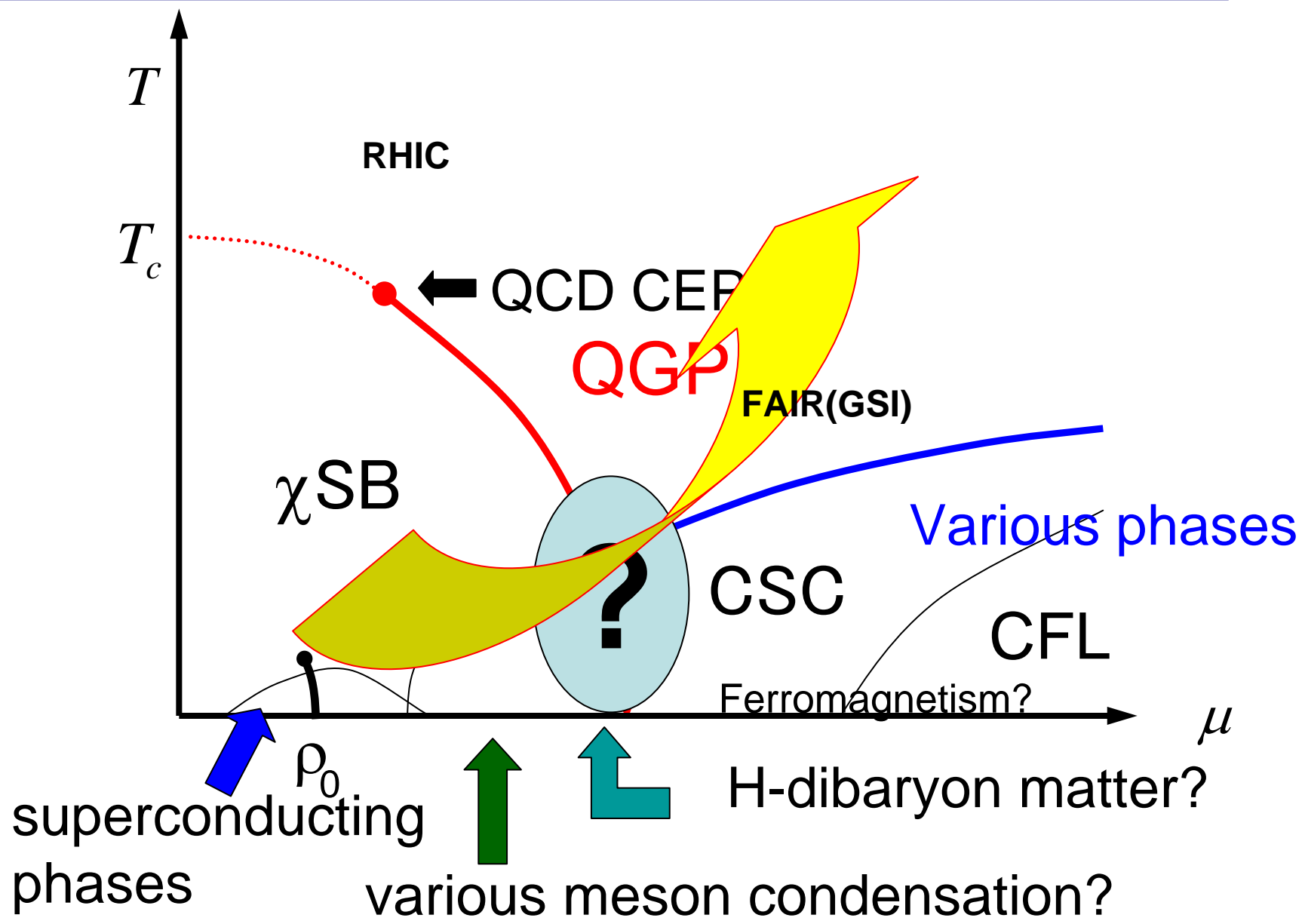
CPOD2007:

GSI, Germany

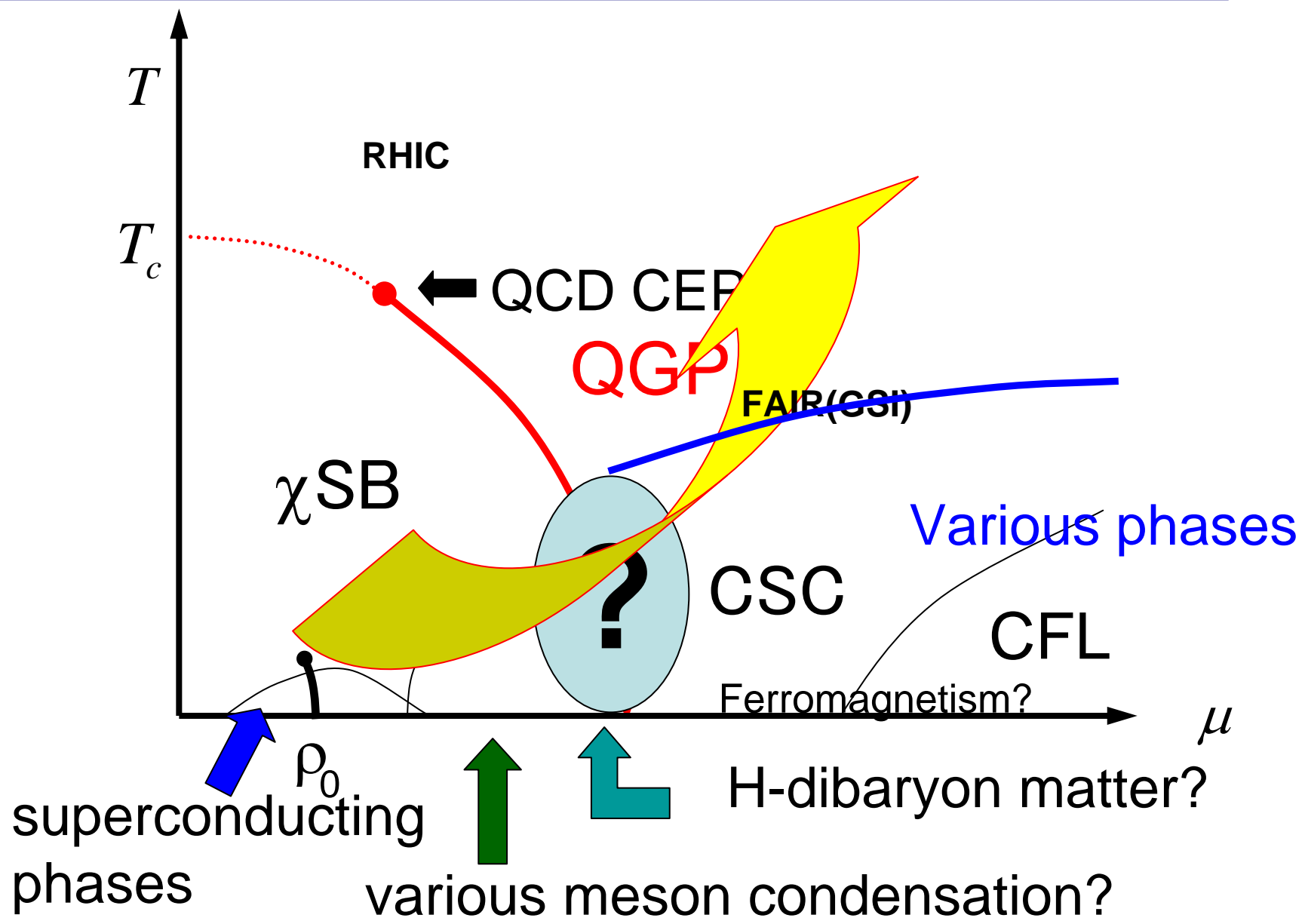
July 9 --13, 2007

in collaboration with M. Kitazawa and Y. Nemoto

A conjectured QCD phase diagram



A conjectured QCD phase diagram



Contents

- Part 1 The soft mode of CSC above T_c and lepton-pair production
- Part 2 The sigma mode above T_c at finite density and lepton-pair production

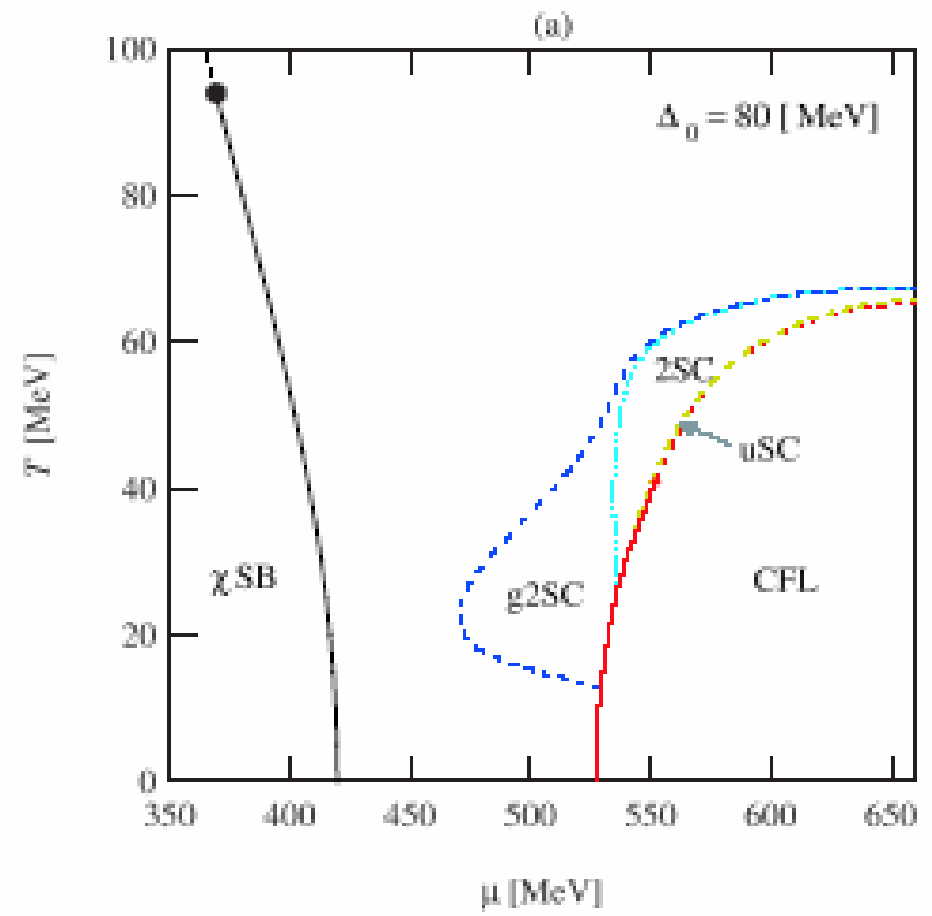
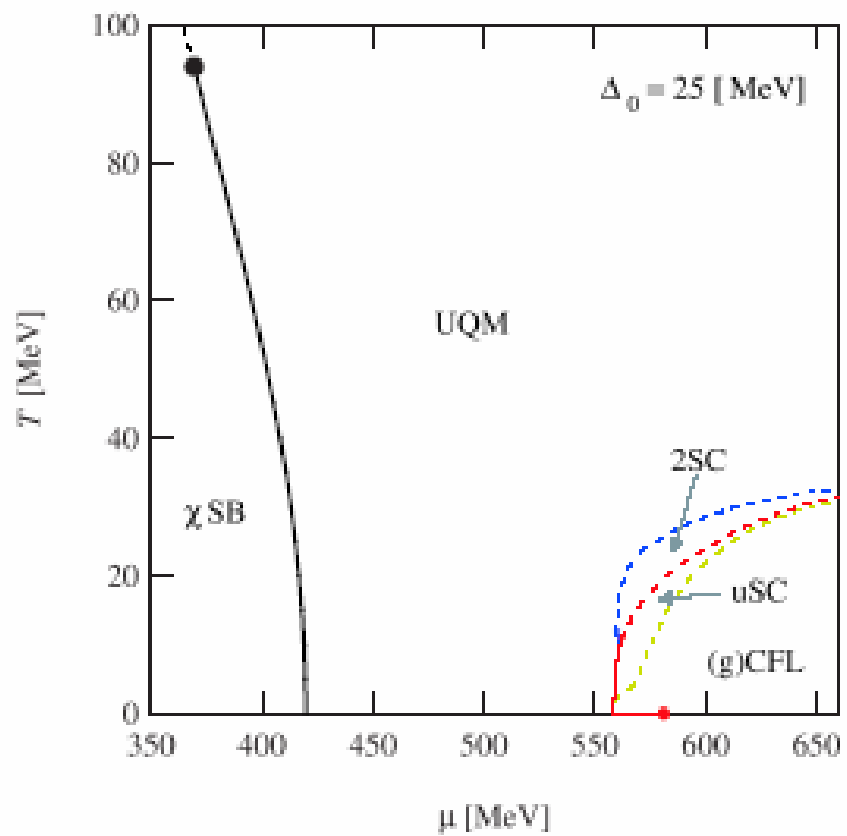
PART I

Precursory Phenomena of Color Superconductivity in Heated Quark Matter

- Ref. M. Kitazawa, T. Koide, T. K. and Y. Nemoto
Phys. Rev. D70, 956003(2004);
Prog. Theor. Phys. 114, 205(2005),
M. Kitazawa, T.K. and Y. Nemoto,
Phys. Lett.B 631(2005),157
M. Kitazawa and T. K., in preparation

Various CSC phases in $T-\mu$ plane

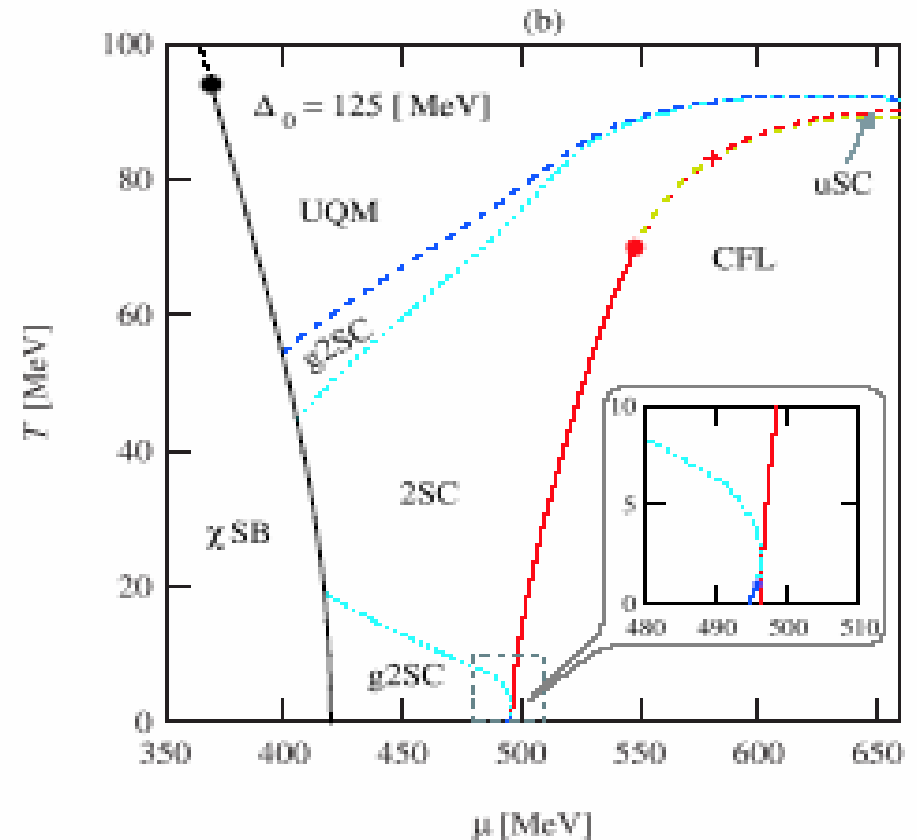
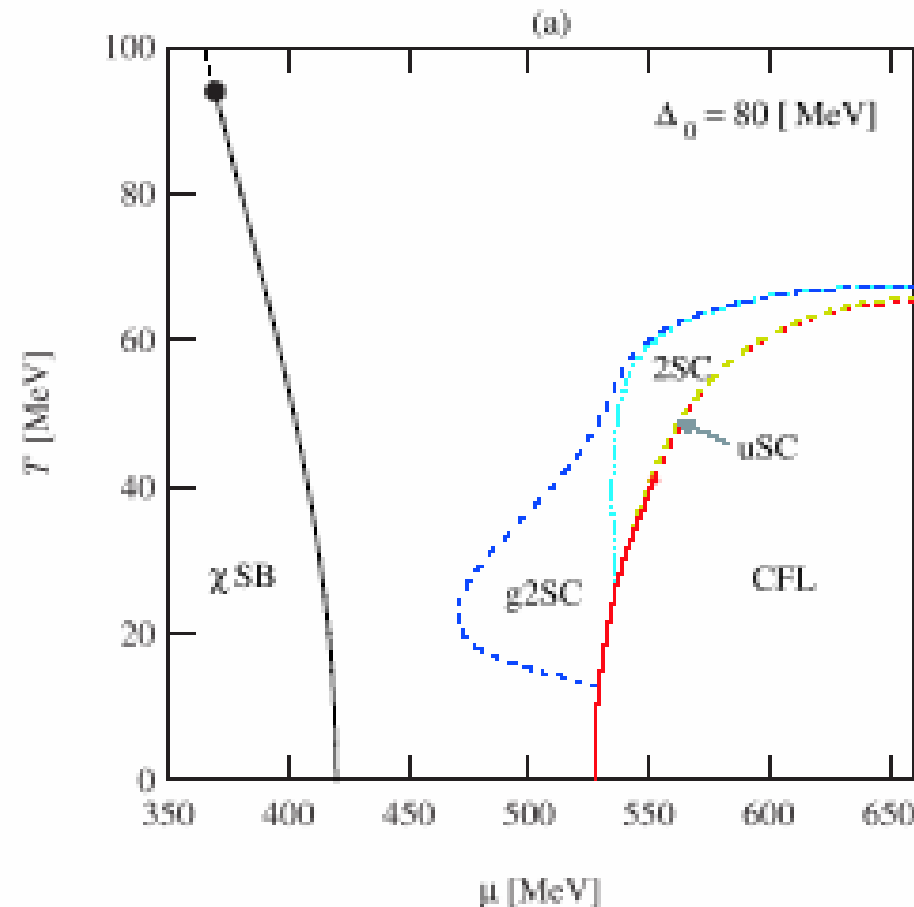
H .Abuki and T.K.
 Nucl. Phys. A, 768 (2006),118;
 with charge and color neutrality



The phase in the highest temperature is 2SC or g2SC.

Various CSC phases in $T-\mu$ plane

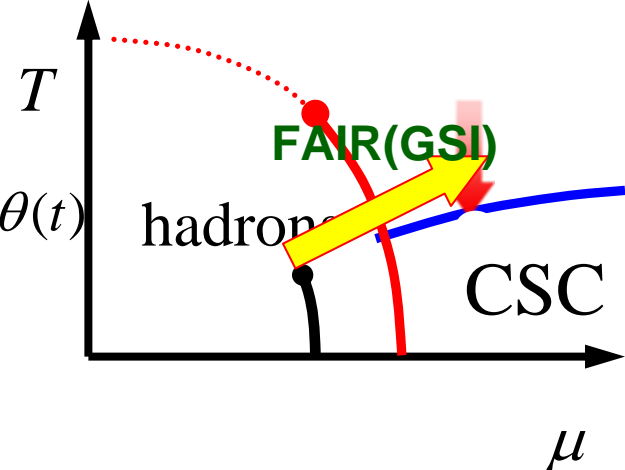
H .Abuki and T.K.
Nucl. Phys. A, 768 (2006),118;
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The phase in the highest temperature is
2SC or g2SC.

Pair Fluctuations in CSC

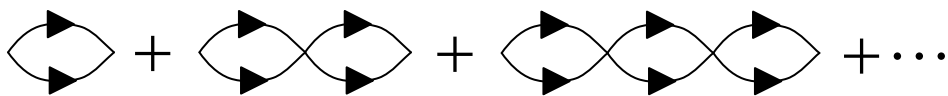
$$D^R(\mathbf{x}, t) = -2G_c \left\langle \left[\bar{\psi}(x) i\gamma_5 \tau_2 \lambda_2 \psi^C(x), \bar{\psi}(0) i\gamma_5 \tau_2 \lambda_2 \psi^C(0) \right] \right\rangle \theta(t)$$



Spectral Function of the diquark excitations

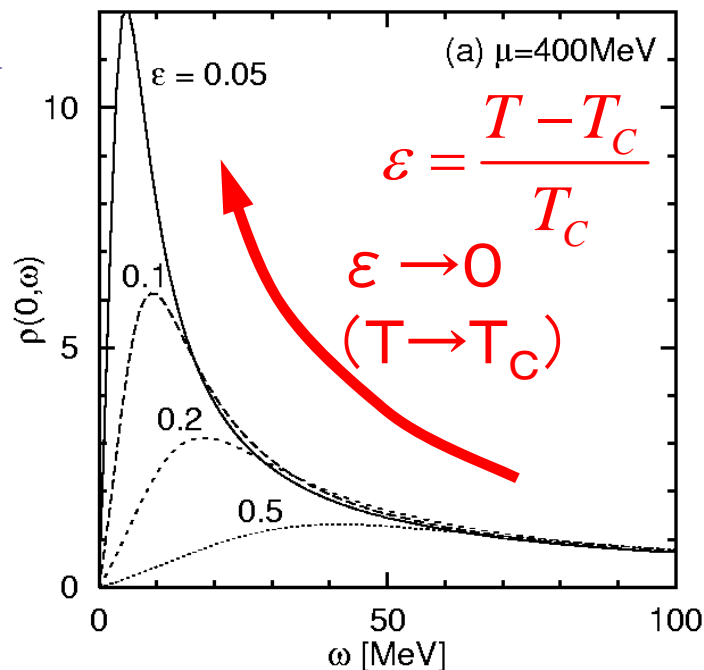
$$\rho(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} D^R(\mathbf{k}, \omega)$$

$$D^R(\mathbf{k}, \omega) =$$



Sharp peak from $\varepsilon \sim 0.2$

electric SC: $\varepsilon \sim 0.005$
even in 2d-SC



Existence of large pair fluctuations

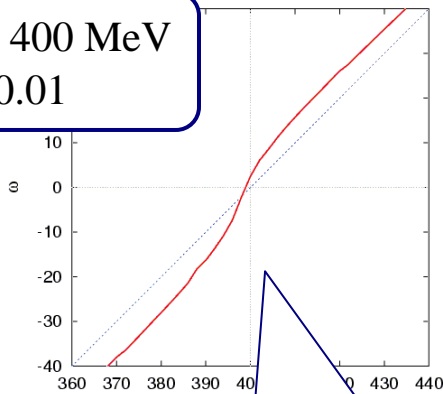
M.Kitazawa, T. Koide, T. K., Y. Nemoto, PRD **65**, 091504 (2002)

It may affect various observables even well above T_c .

Precursory Phenomena

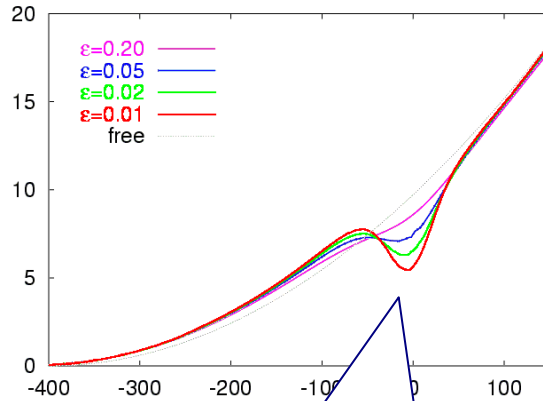
quark dispersion

$\mu = 400 \text{ MeV}$
 $\varepsilon = 0.01$



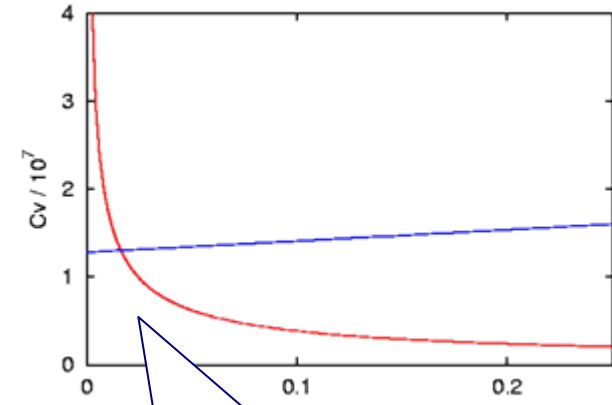
rapid increase
around p_F

density of states



formation of
the **pseudogap**

specific heat



anomalous
enhancement
as $T \rightarrow T_c$

M.Kitazawa, T.Koide, T.K., Y. Nemoto, PRD70,056003('04)

QM~ High T_c SC

maybe relevant to
the cooling process of
newly-born neutron star

More accessible observables?



dilepton-pair production.
in H-I collisions.

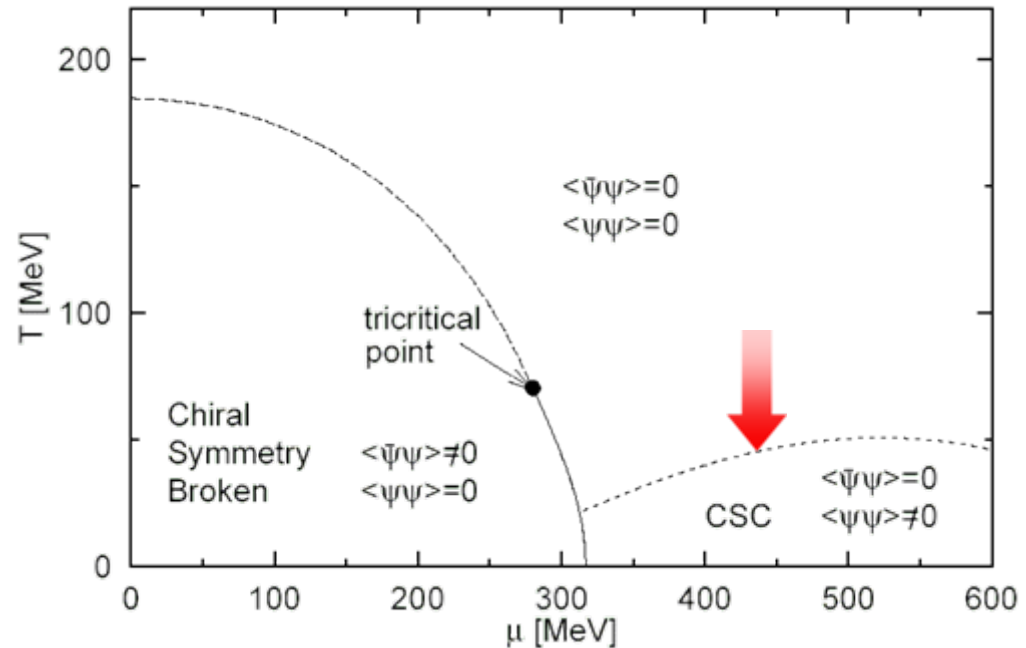
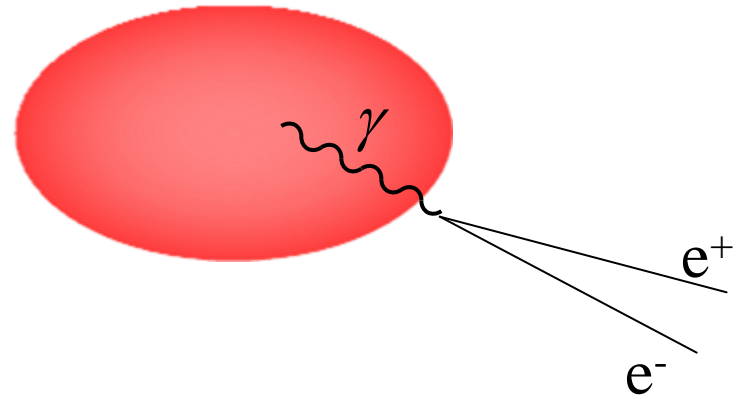
But how?

Dilepton-pair Production Rate

$$\frac{dR_{ee}}{d^4q} = -\frac{\alpha}{12\pi^4 Q^2} \text{Im} \Pi^{R\mu}_{\mu} \frac{1}{e^{q^0/T} - 1}$$

Photon Self-Energy

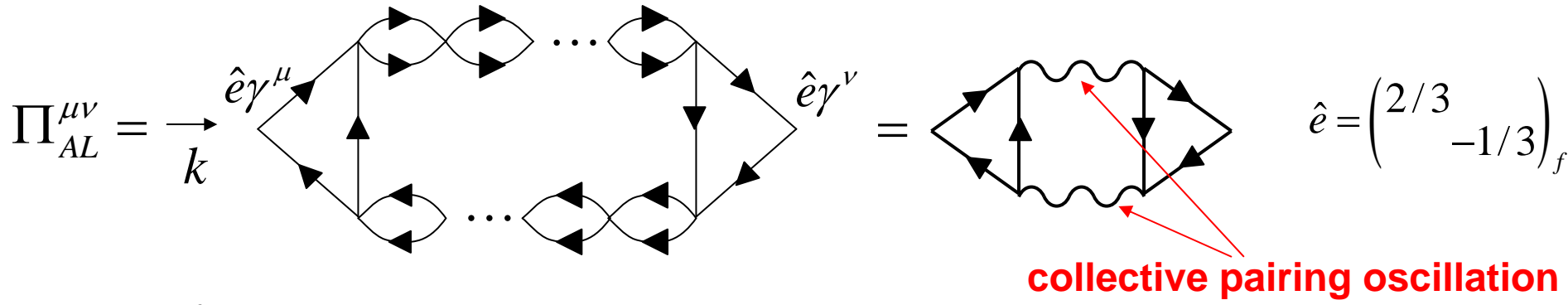
How do the diquark fluctuations affect the photon self-energy?



Anomalous Self-energy of Photon; Aslamasov-Larkin term

Sov. Phys. SS 10,875('68)

Photon Self-Energy Π



collective pairing oscillation

$$= 3 \int d^4 q \Gamma^\mu(q, q+k) \Xi(q+k) \Gamma^\nu(q+k, q) \Xi(q)$$

factor 3 due to color degrees of freedom

● Pair field (T-matrix):

$$\Xi(q) = 1 + \text{loop} + \text{two-loop} + \dots$$

$$= \text{wavy line}$$

● Vertex:

$$\Gamma^\mu(p_1, p_2) = \text{triangle diagram with } \hat{e}\gamma^\mu$$

cf) Maki-Thompson term

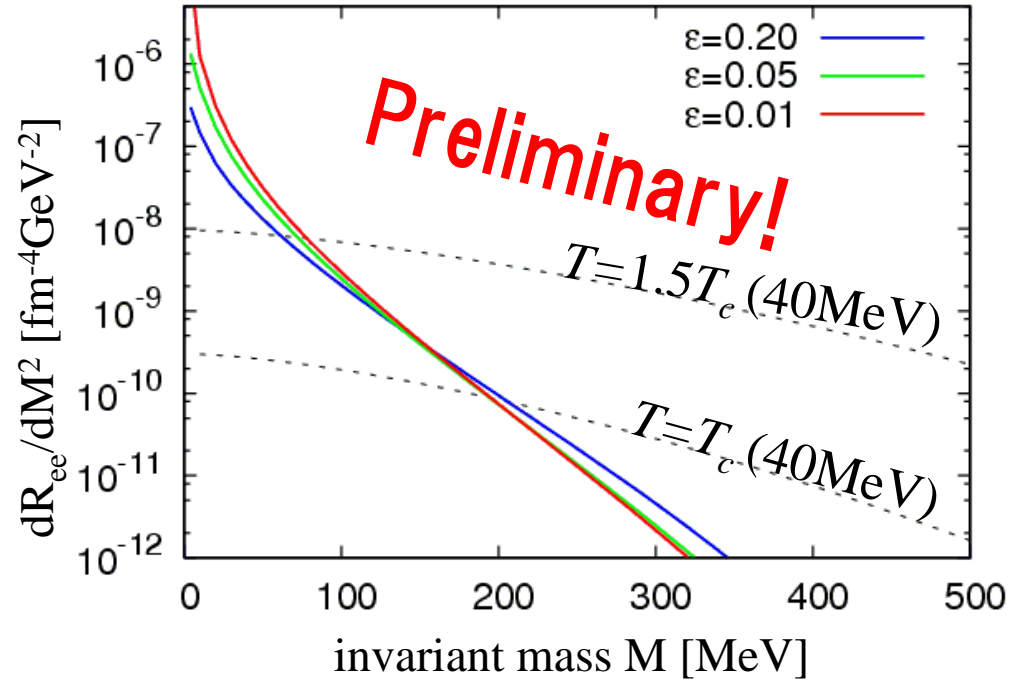
Dilepton-pair Production

$$\frac{dR_{ee}}{d^4q} = -\frac{\alpha}{12\pi^4 Q^2} \text{Im} \Pi^{R\mu}_{\mu} \frac{1}{e^{q^0/T} - 1}$$

-per invariant mass

$$\frac{dR_{ee}}{dM^2} = \int \frac{d^3q}{2q^0} \frac{dR_{ee}}{d^4q}$$

— from **AL-term**
 - - - - from free quarks



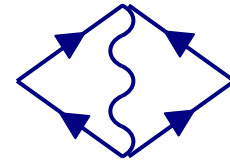
M. Kitazawa and T.K., (2005), unpublished

- Prominent enhancement at $M < 150 \text{ MeV}$.
- The peak becomes sharp as $\epsilon \rightarrow 0$.

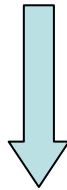
Possible experimental observable for the CSC to be seen in **FAIR(GSI)**?
 Maybe difficult, unfortunately, because of the too-low mass enhancement.

Remarks:

* Effects of the Maki-Thompson term



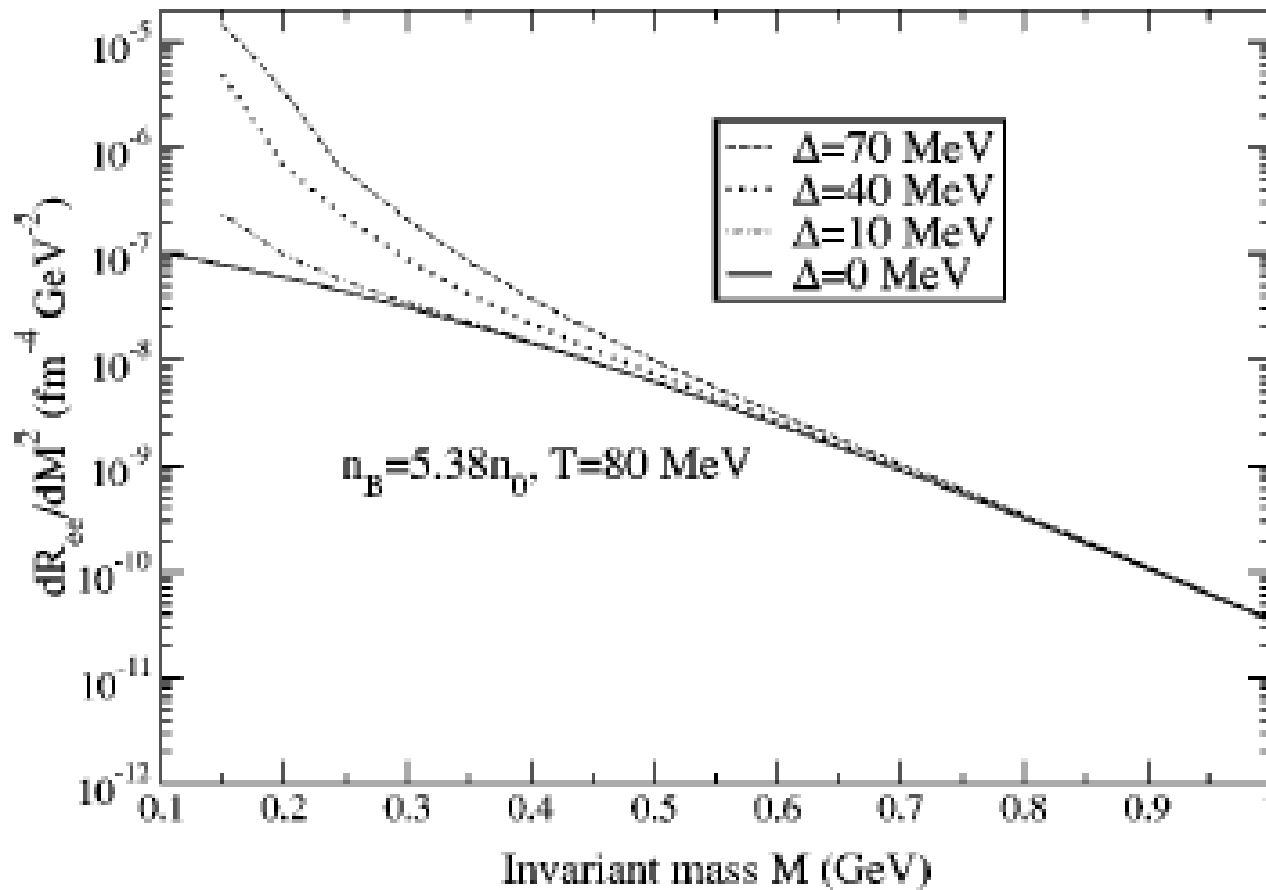
*If T_c is higher, say, 100 MeV or higher, the enhancement is more prominent.



How about, the lepton pair emission from the color-conducting phase?

Dilepton emission rate from the CFL phase

P. Jaikumar R. Rapp and I. Zahed('02)

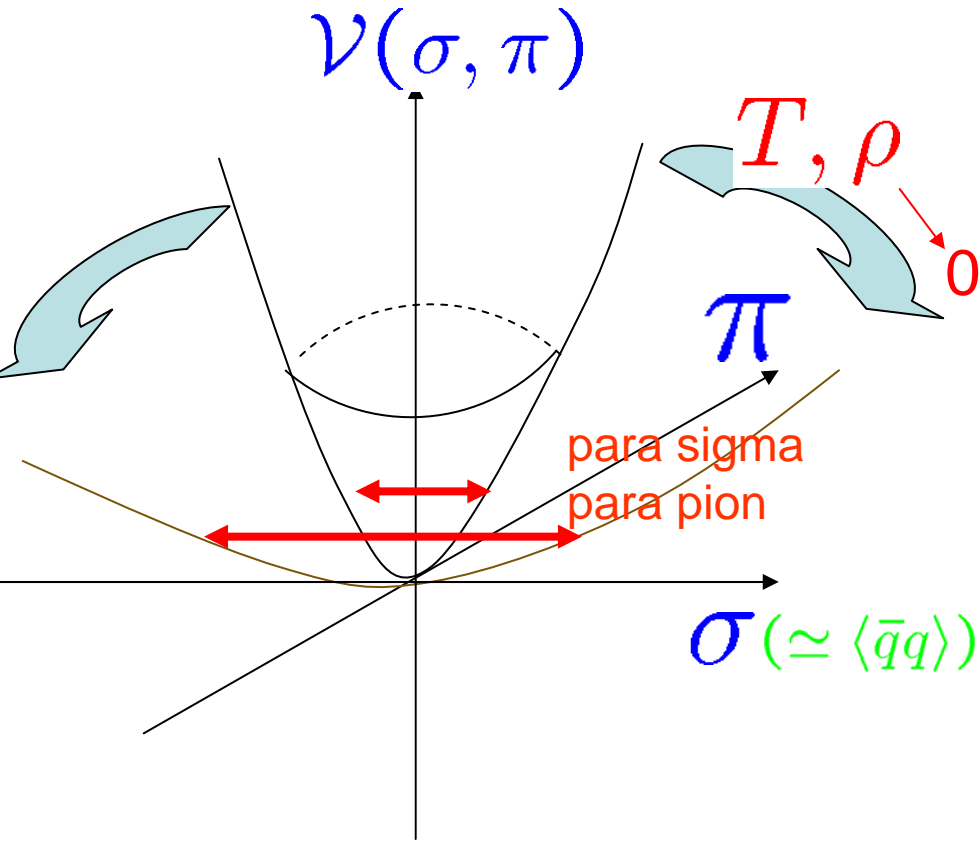


Enhancement due to photon-gluon mixing in the CFL phase, and from the generalized rho meson.(not shown here)

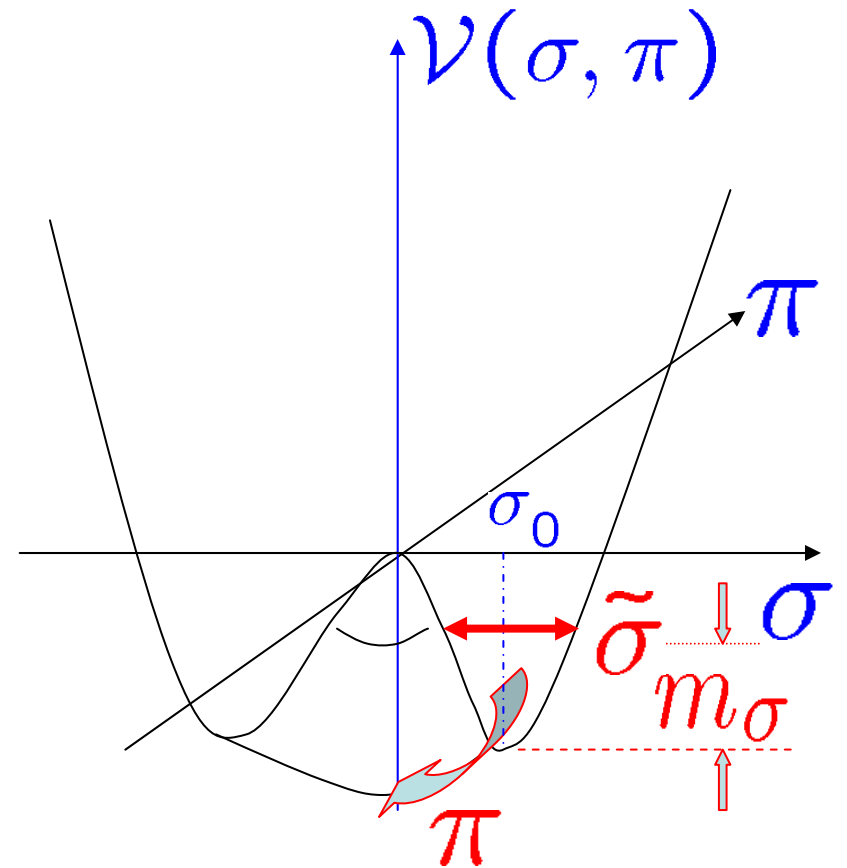
PART II

The Case of the Chiral Transition

Chiral Transition and the sigma mode (meson)



$T > T_c \quad \rho > \rho_c$



$T < T_c \quad \rho < \rho_c$

$\sigma = \sigma_0 + \tilde{\sigma}$

c.f. Higgs particle in WS model

ϕ ; Higgs field $\longrightarrow \phi = \langle \phi \rangle + \tilde{\phi}$
Higgs particle

Confinement and chiral transition in Lattice QCD

(F.Karsch, Lect. Notes 583 (2002), 209)

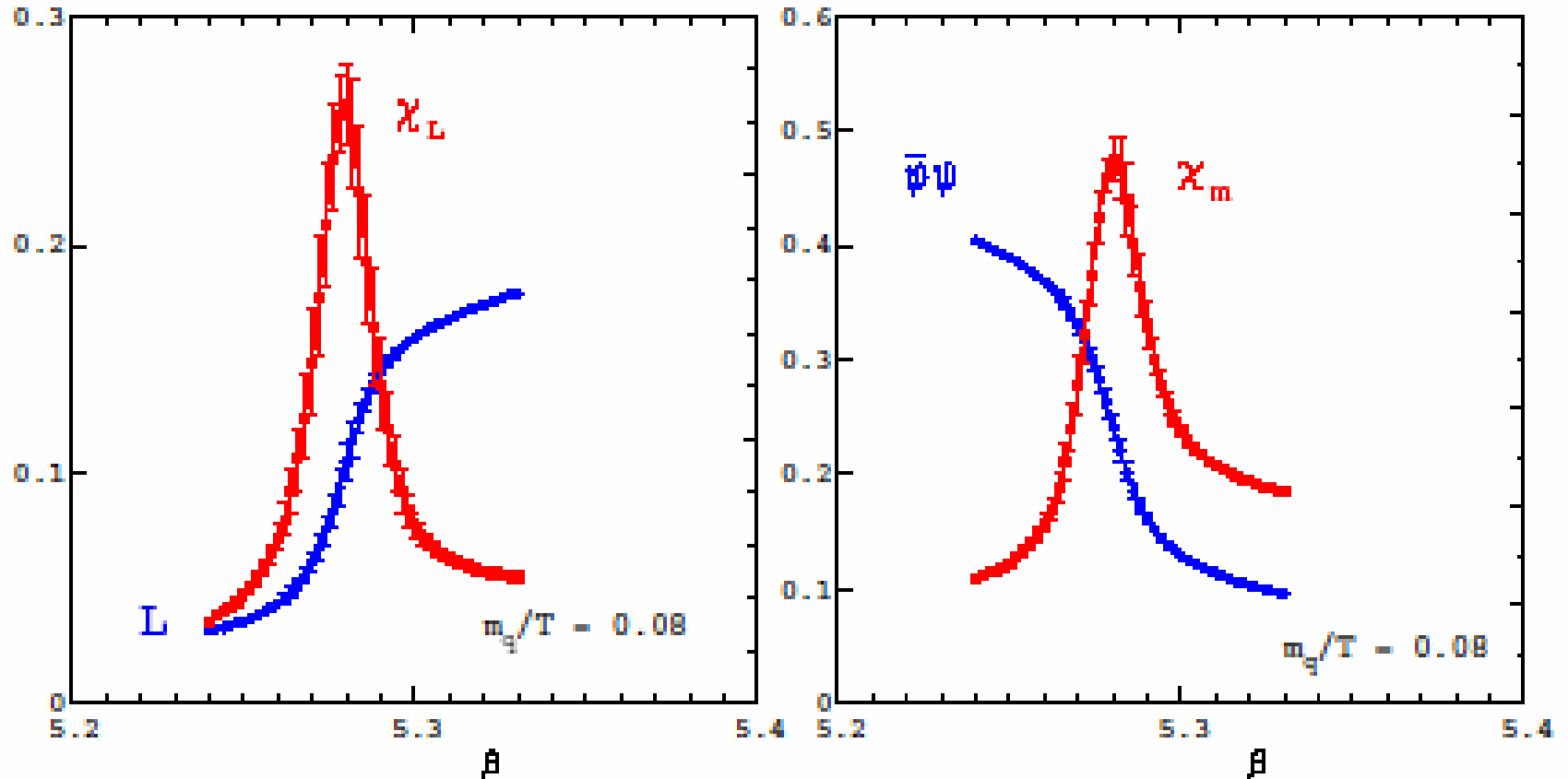


Fig. 2. Deconfinement and chiral symmetry restoration in 2-flavour QCD: Shown is $\langle L \rangle$ (left), which is the order parameter for deconfinement in the pure gauge limit ($m_q \rightarrow \infty$), and $\langle \bar{\psi}\psi \rangle$ (right), which is the order parameter for chiral symmetry breaking in the chiral limit ($m_q \rightarrow 0$). Also shown are the corresponding susceptibilities as a function of the coupling $\beta = 6/g^2$.

Cf. Lattice Calculation of the *generalized masses*

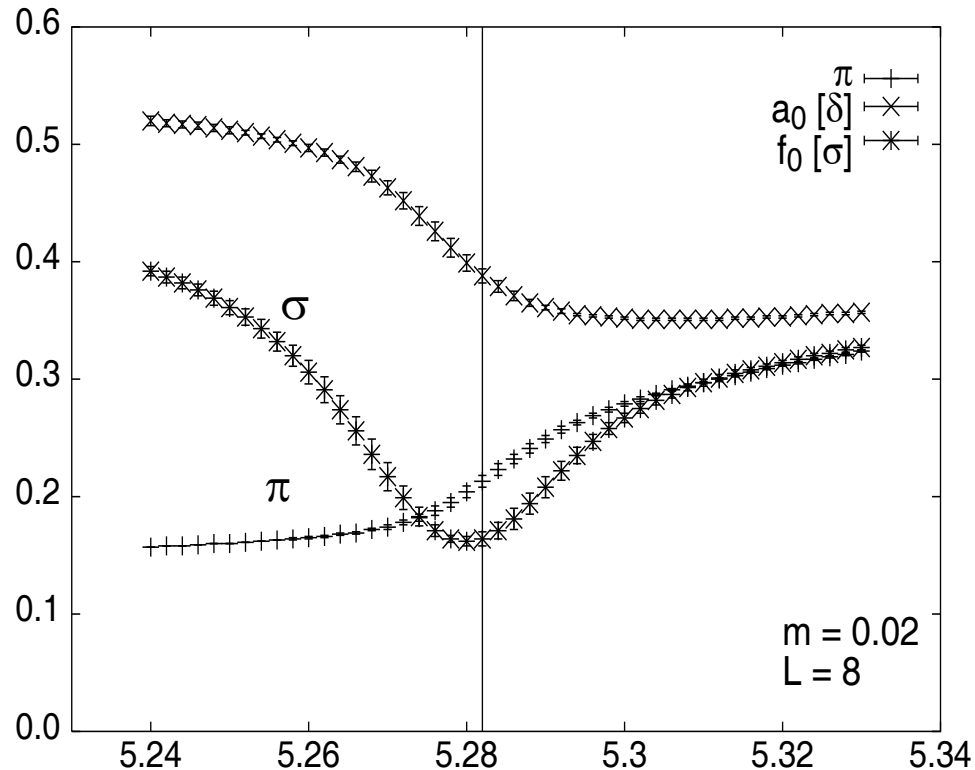
F. Karsch, Lect. Note Phys. **583** (2002), 209. $N_f = 2$, $8^3 \times 4$; Staggered fermion

$$m_\sigma^2 = \chi_\sigma^{-1}$$
$$\chi_\sigma = \langle (\bar{q}q)^2 \rangle$$

the **softening** of the σ with increasing T

and

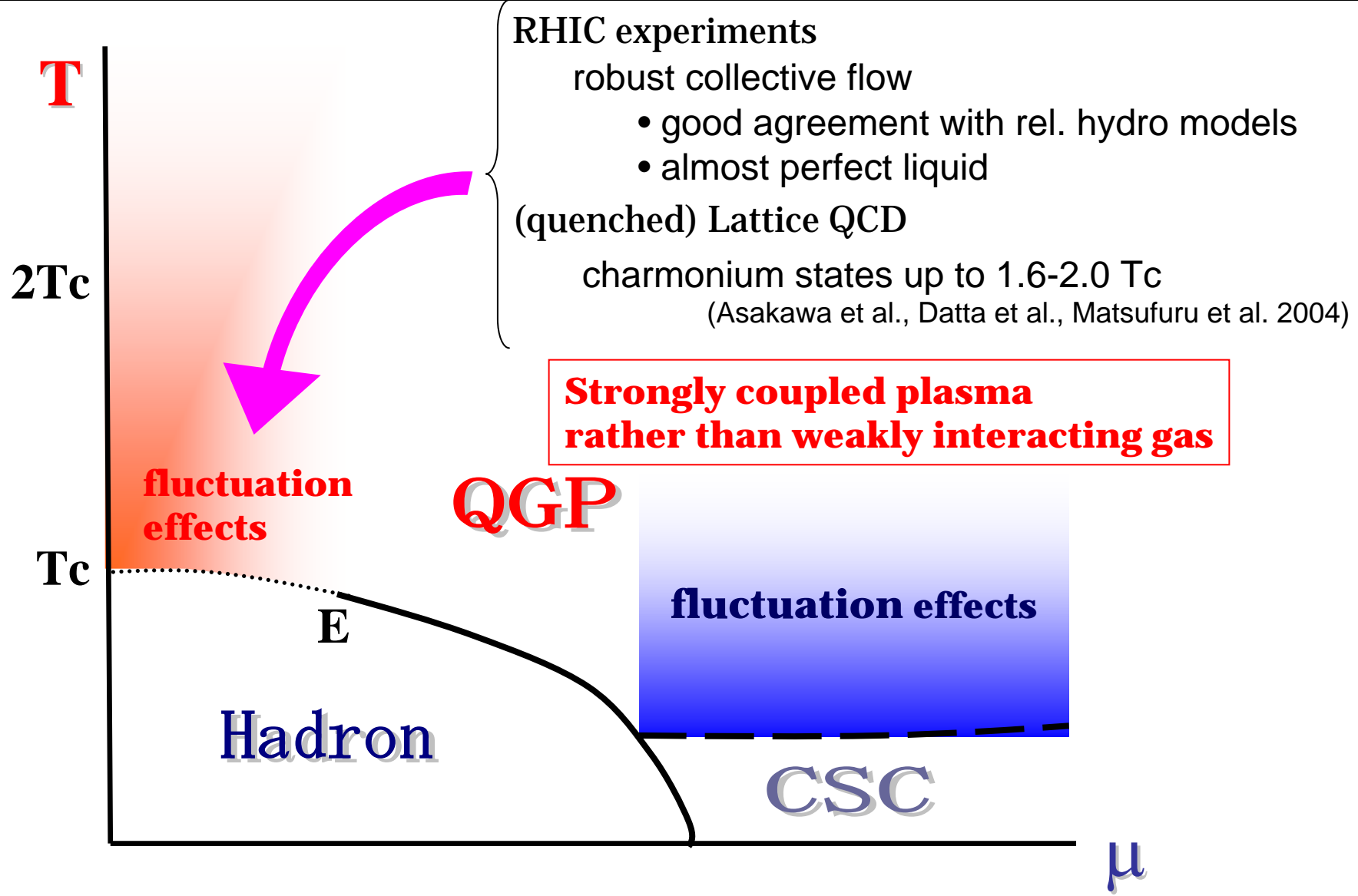
a degeneracy of the σ and π at high T



What is the significance of the σ in hadron physics?

How about above T_c ?

Interest in the nature of elementary modes in 'QGP' phase



The spectral function of the degenerate hadronic "para-pion" and the "para-sigma" at $T > T_c$ for the chiral transition: $T_c = 164$ MeV

T. Hatsuda and T.K. (1985)

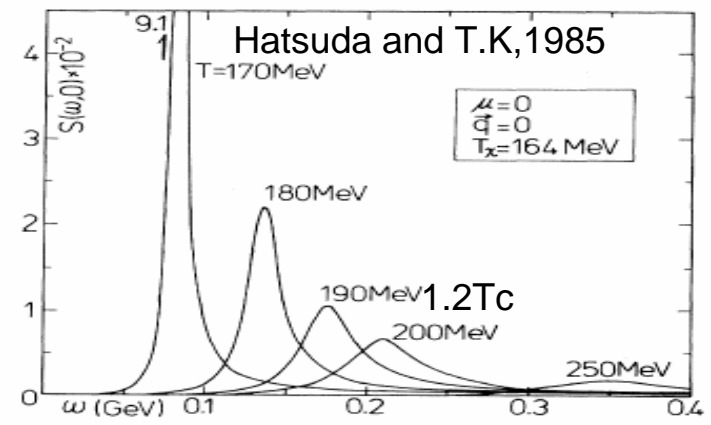
- response function in RPA

$$D(\mathbf{k}, \omega) = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \dots$$

- spectral function

$$A(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} D(\mathbf{k}, \omega)$$

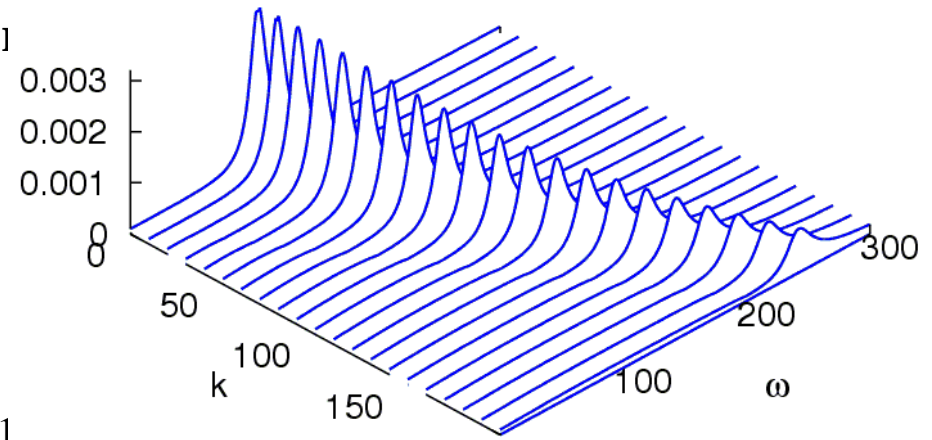
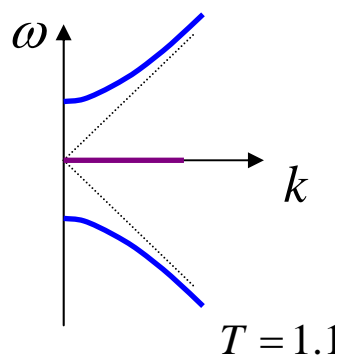
$T \rightarrow T_c$, they become elementary modes with small width!



sharp peak in time-like region:

1. two γ decay
2. modified quark spectrum

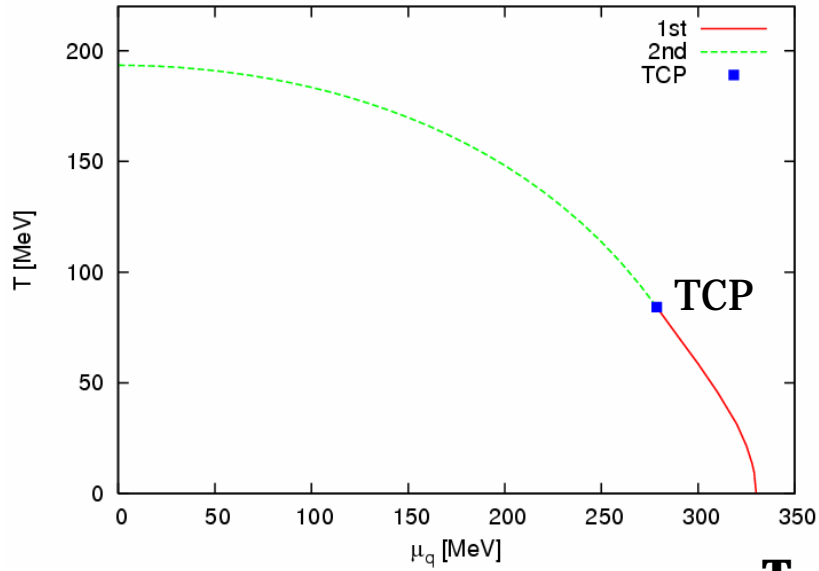
M.Kitazawa,
Y.Nemoto and
T.K. (05)



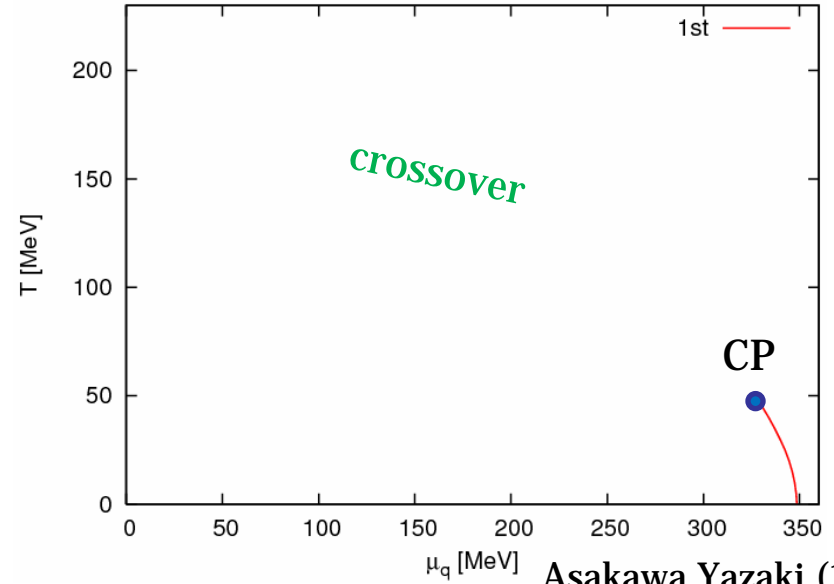
Finite T and μ with finite quark mass

Phase diagram

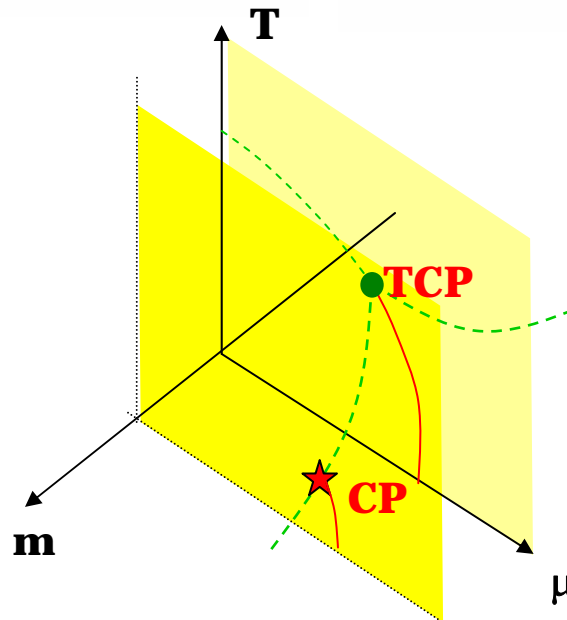
$$m_{\varphi_0} = 0$$



$$m_{\varphi_0} = 5.5 \text{ MeV}$$

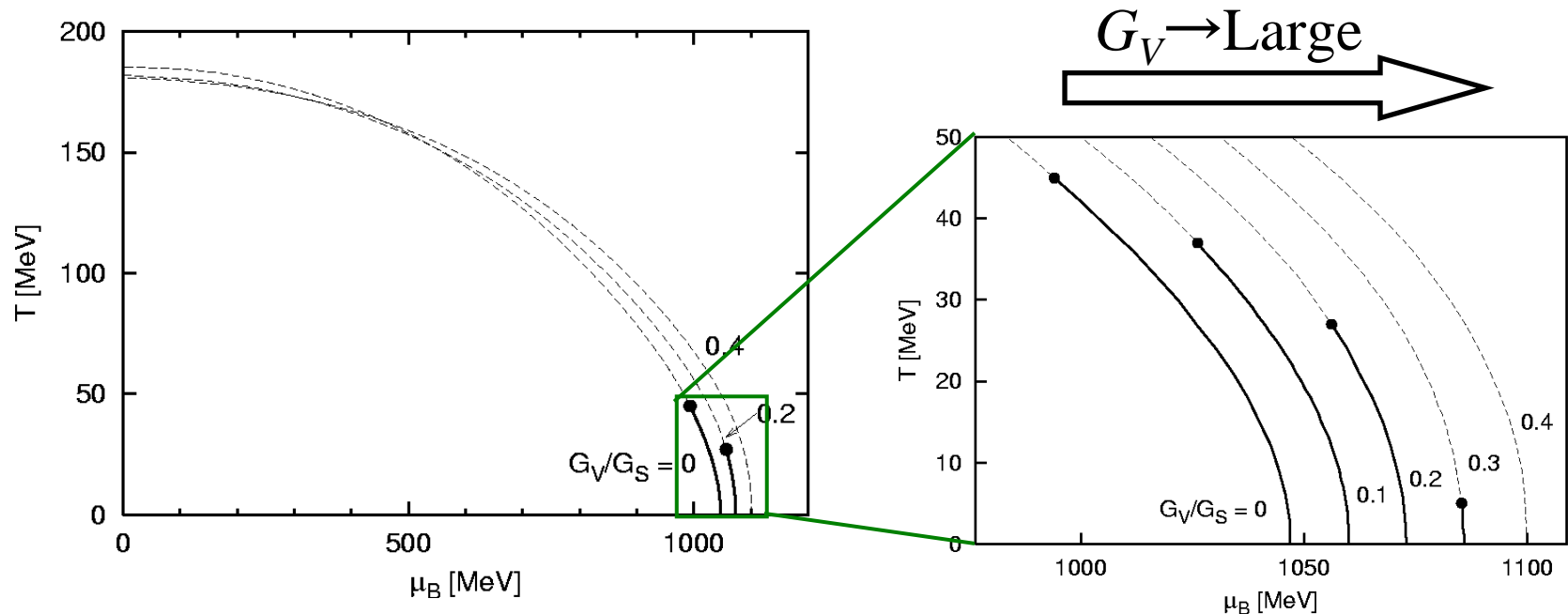


Asakawa, Yazaki, (1989)



Caveats

★ Effects of G_V on Chiral Restoration



T.Hatsuda and T.K.('85); without vector coupling

As G_V is increased,

- Chiral restoration is shifted to higher densities.
- The phase transition is weakened.

Asakawa, Yazaki '89 /Klimt,Lutz,&Weise '90 /T.K. '90/ Buballa,Oertel '96

What would happen when the CSC joins the game?

With color superconductivity transition incorporated

M. Kitazawa et al ('02)

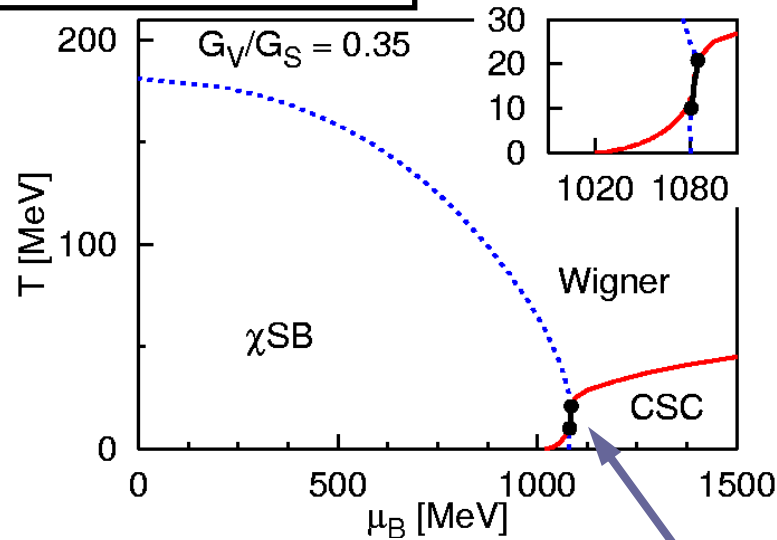
(2) The **first order transition** between χ SB and CSC phases is **weakened** and eventually **disappears**.

(3) The region of the **coexisting phase becomes broader**.

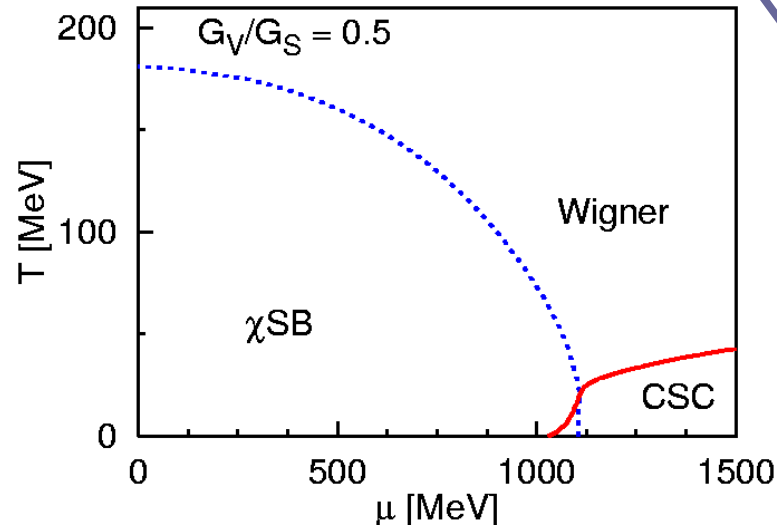
Appearance of the coexisting phase becomes robust.

(4) Another end point appears from lower temperature, and hence **there can exist two end points** in some range of G_V !
 $0.33 \sim G_V \sim 0.38$

$$G_V / G_S = 0.35$$



$$G_V / G_S = 0.5$$



Smooth variation of the quark condensate with baryon density?

S. Klimt, M. Lutz and W. Weise, PLB249 ('90)

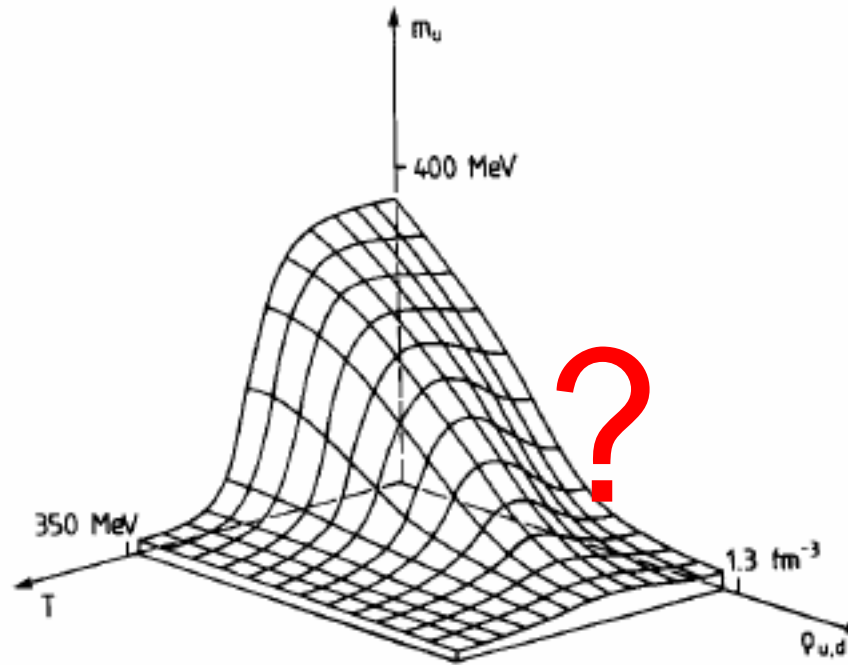
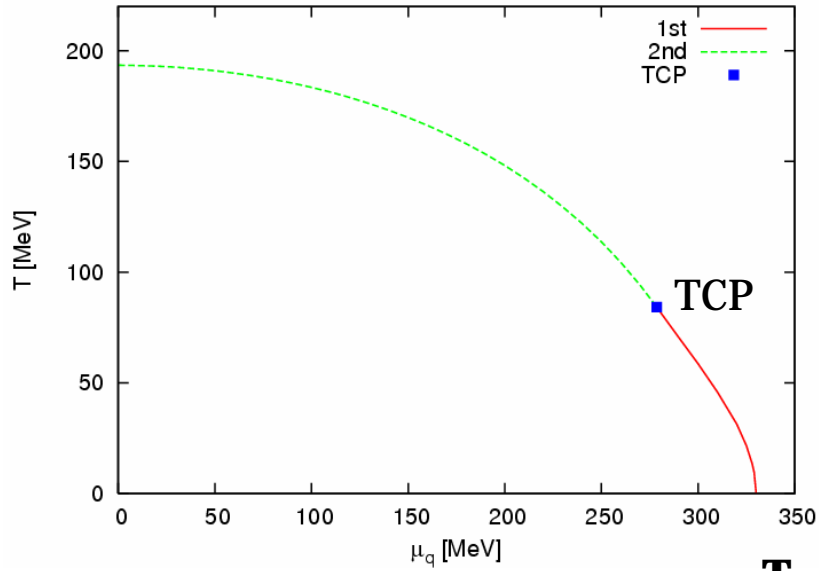


Fig. 1. Quark mass m_u as a function of temperature T and quark density ρ_u at $\rho_s=0$.

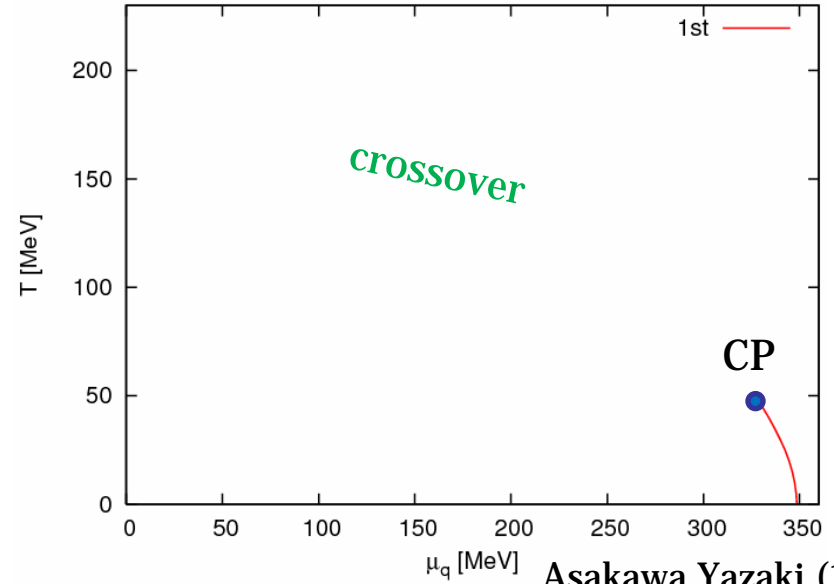
So strong vector coupling making the chiral transition crossover at finite density!

Phase diagram

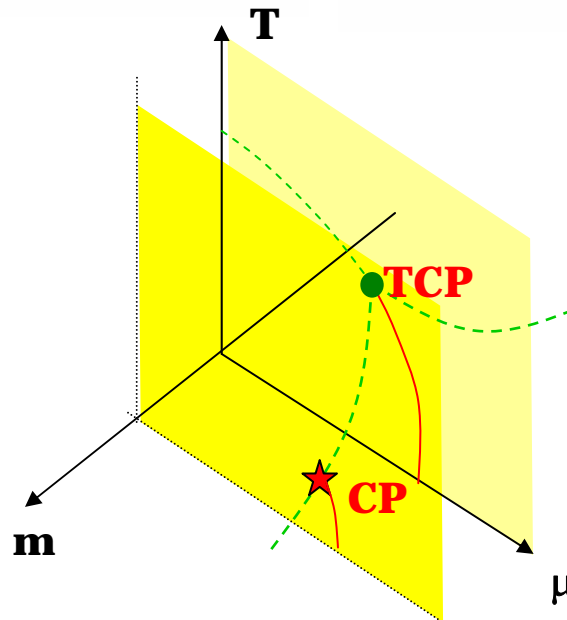
$$m_{\varphi_0} = 0$$



$$m_{\varphi_0} = 5.5 \text{ MeV}$$



Asakawa, Yazaki, (1989)



What is the soft mode at CP?

Sigma meson has still a non-zero mass at CP.

This is because the chiral symmetry is explicitly broken.

What is the soft mode at CP?

Phonon mode in the space-like region softens at CP.

H. Fujii (2003)

H. Fujii and M. Ohtani (2004)

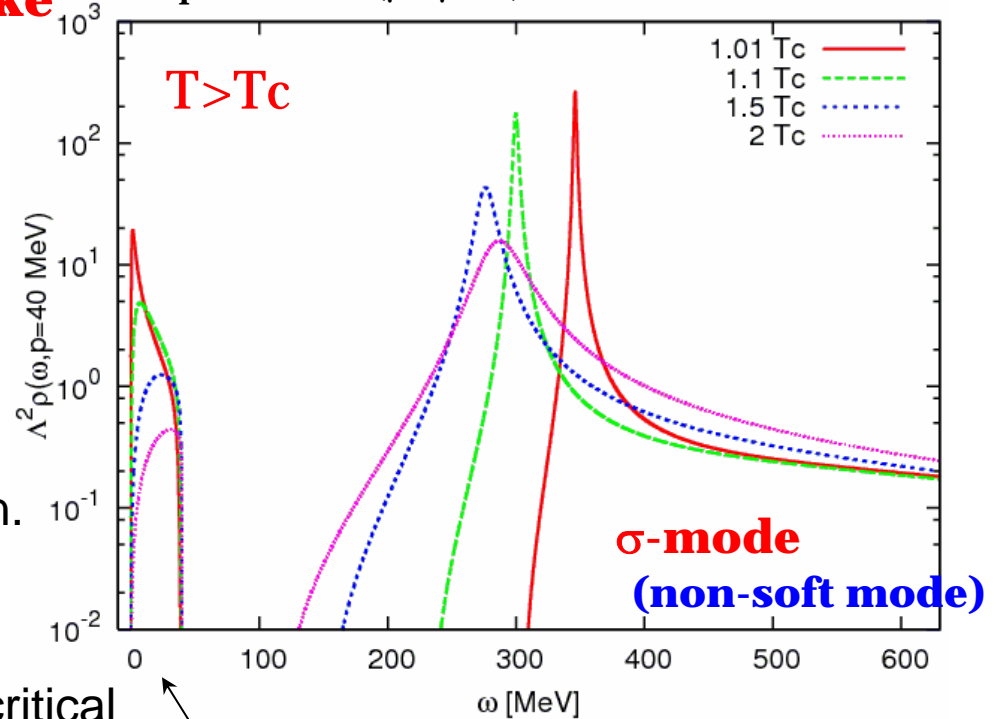
See also, D. T. Son and
M. Stephanov (2004)

does not affect particle
creation in the time-like region.

**It couples to hydrodynamical
modes,**

leading to interesting dynamical critical
phenomena.

Spectral function of the chiral condensate
T-dependence ($\mu = \mu_{CP}$)



**Space-like region
(the soft modes)**

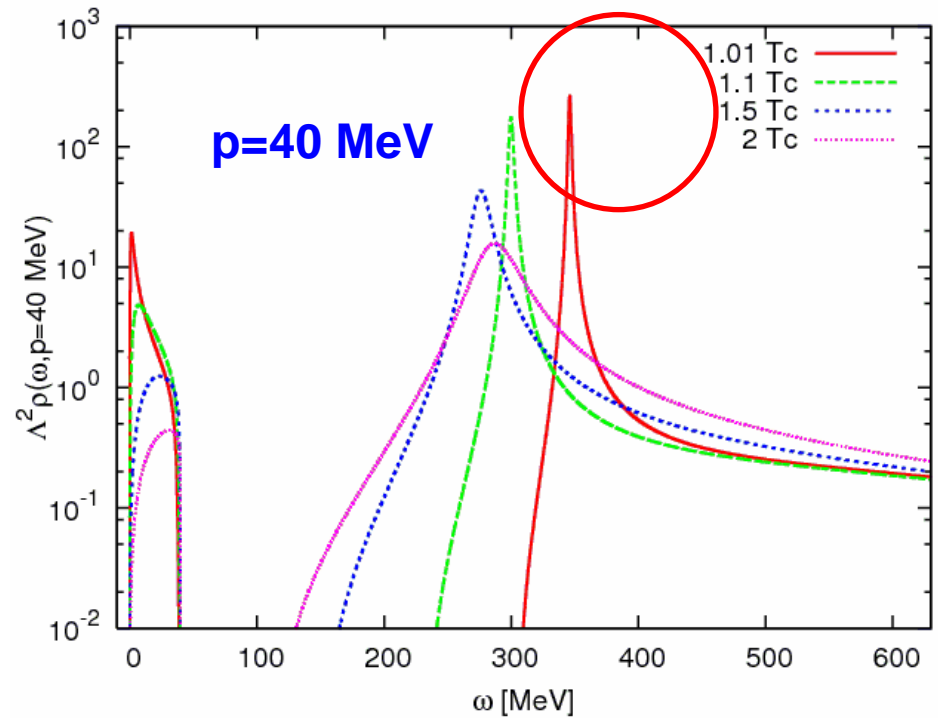
Dilepton production rate from the sigma mode at $T > T_c$

s
p

What is contributions of the sigma mode?

Enhancement around

$$m_\sigma \approx 2M_q .$$



Spectral function of the sigma mode

Vector-scalar mixing

$$\Pi_{\mu\nu}(q_0, \vec{q}) = \text{Diagram}$$

The above vector-scalar mixing exists if

$$\left\{ \begin{array}{l} \mu \neq 0 \\ m_q \neq 0 \\ q \neq 0 \\ \text{Not SU(3) limit } (m_u = m_d = m_s) \end{array} \right.$$

ex: SU(2) symmetry ($m_q = m_u = m_d$)

cf: quark number susceptibility through the vs-mixing

Kunihiro, 1991

cf: dilepton production due to $\sigma-\omega$ and $\sigma-\gamma$ mixings in hadronic matter

Weldon, 1992

Wolf, Friman, Soyeur, 1998

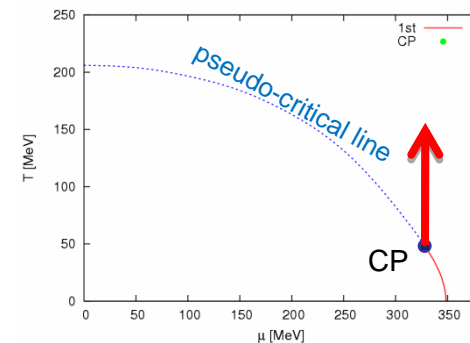
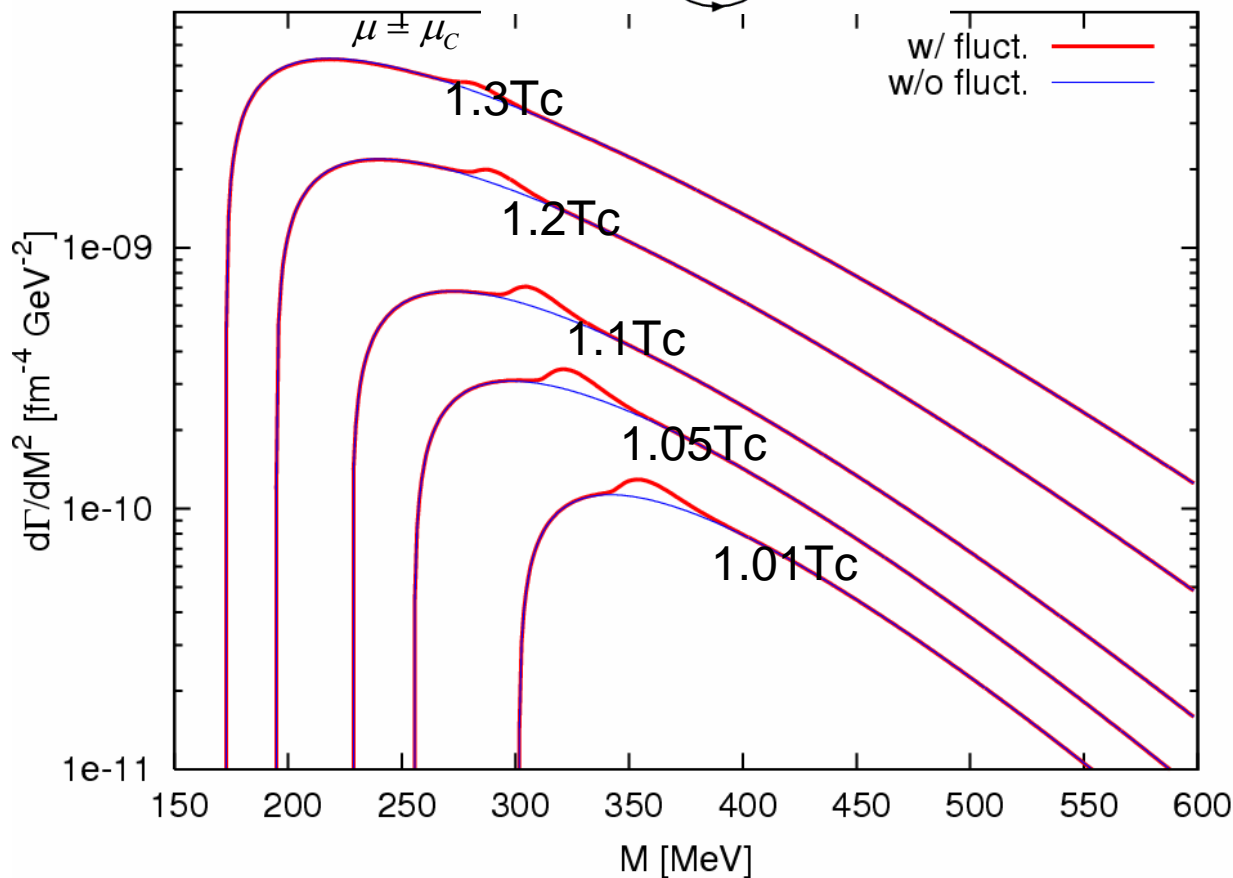
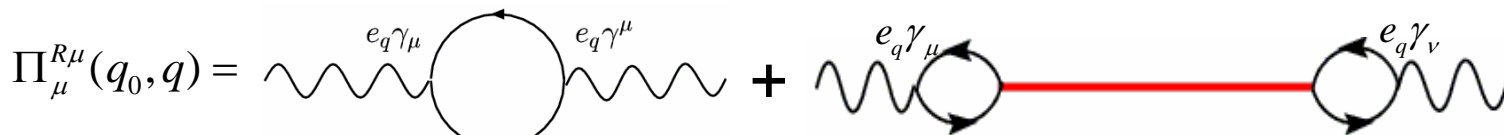
without the notion of chiral transition, nor softening of the sigma

$$\text{Loop} \propto N_C (e_u + e_d) m_q \int d^3k F(q_\mu, \vec{k}) [f(E_k - \mu) - f(E_k + \mu)]$$

Di-electron Production Rate

along T-axis

$$\frac{d\Gamma}{dM^2} = \int \frac{d^3q}{2q_0} \frac{d\Gamma}{d^4q} \quad \frac{d\Gamma}{d^4q} = \frac{-\alpha \text{Im} \Pi_\mu^{R\mu}(q_0, q)}{12\pi^4 q^2 (\exp[q_0/T] - 1)}$$

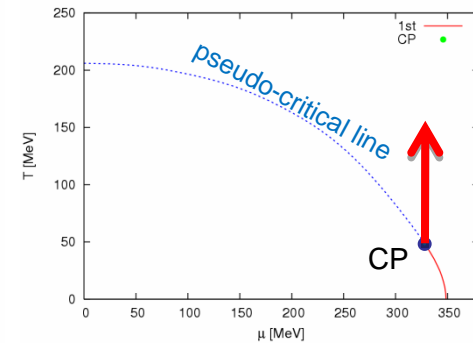
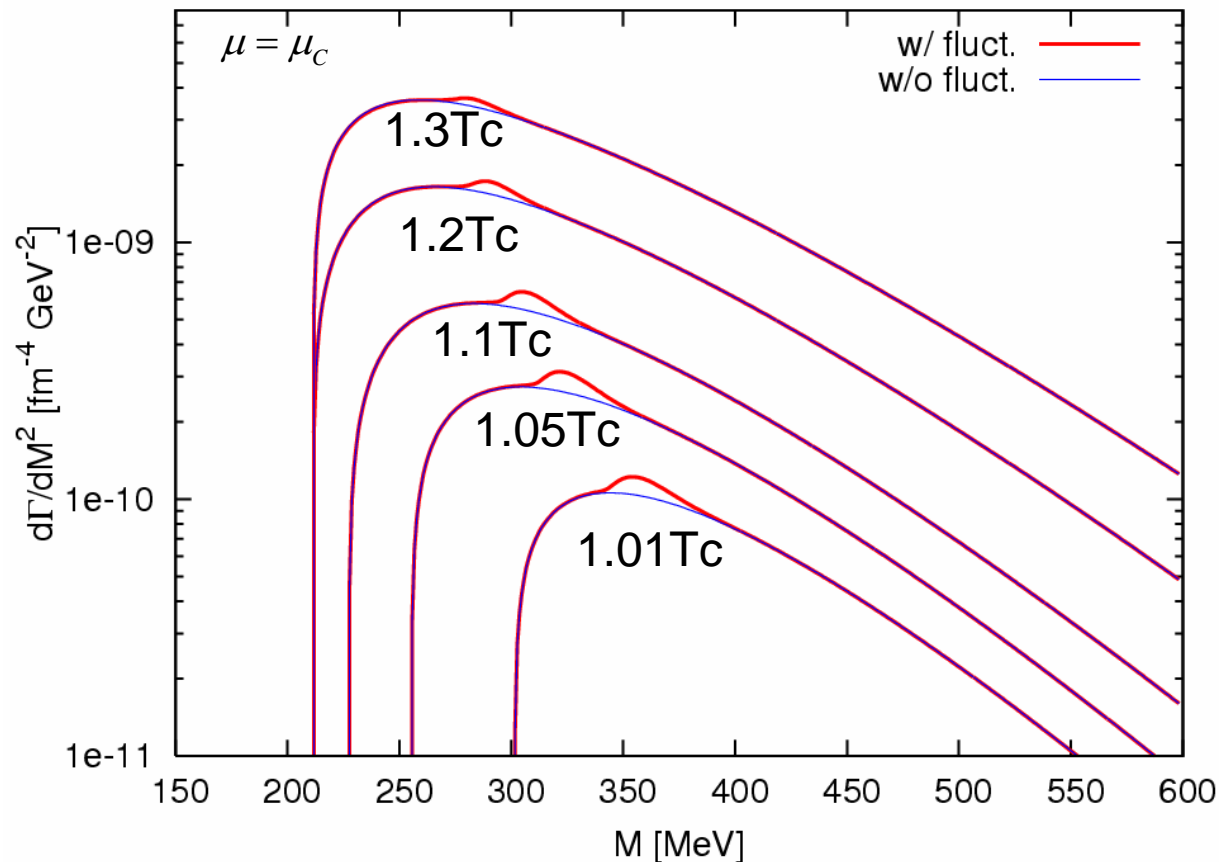
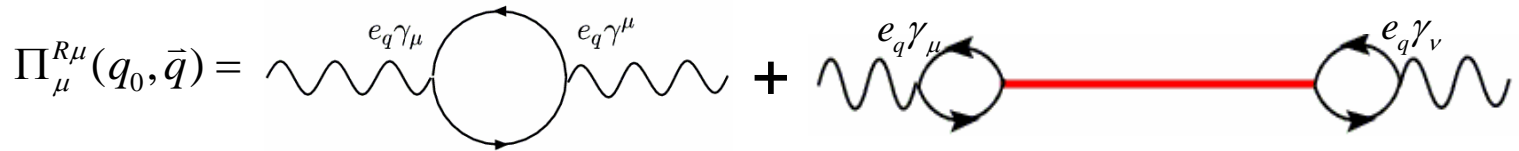


Enhancement around
 $m_\sigma \approx 2M_q$ at $T > T_c$

Di-muon Production Rate

along T-axis

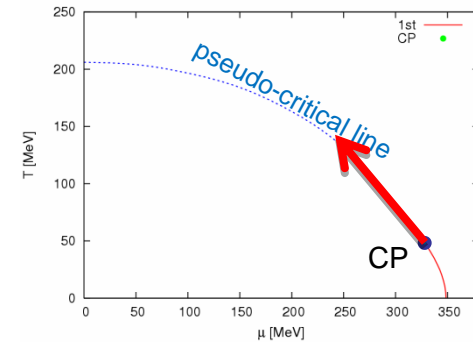
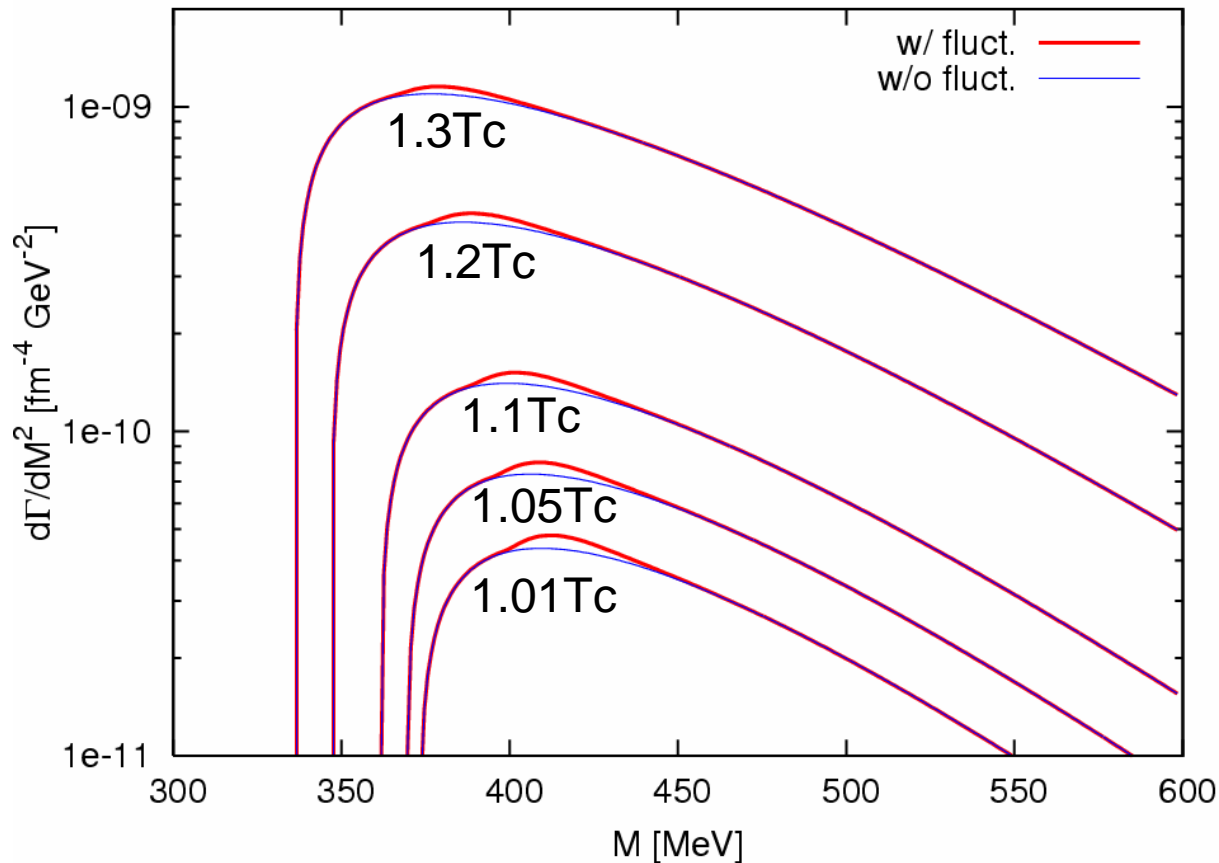
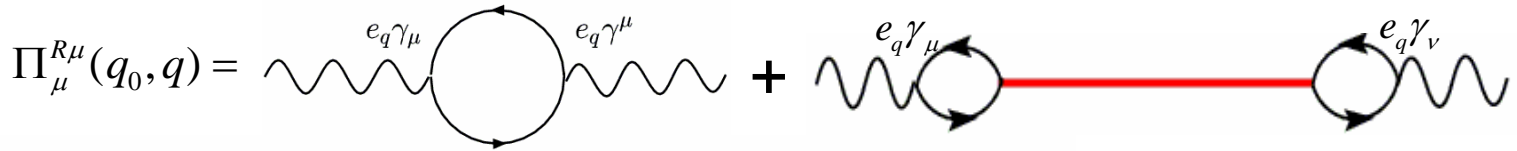
$$\frac{d\Gamma}{dM^2} = \int \frac{d^3q}{2q_0} \frac{d\Gamma}{d^4q} \quad \frac{d\Gamma}{d^4q} = \frac{-\alpha \text{Im} \Pi_{\mu}^{R\mu}(q_0, q)}{12\pi^4 q^2 (\exp[q_0/T] - 1)} \left(1 + \frac{2m_{\mu}^2}{q^2}\right) \left(1 - \frac{4m_{\mu}^2}{q^2}\right)^{1/2}$$



Di-electron Production Rate

along a pseudo-critical line

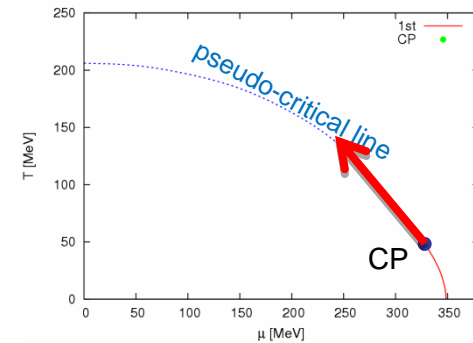
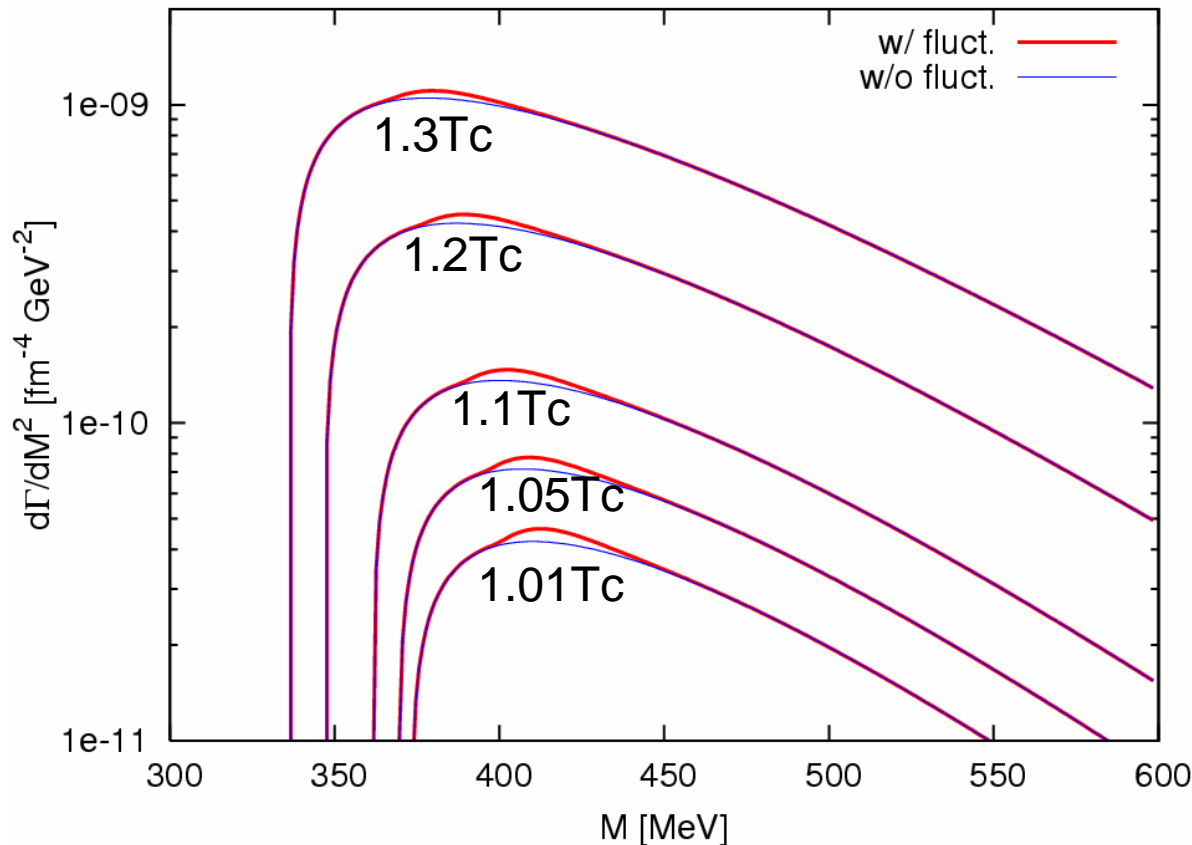
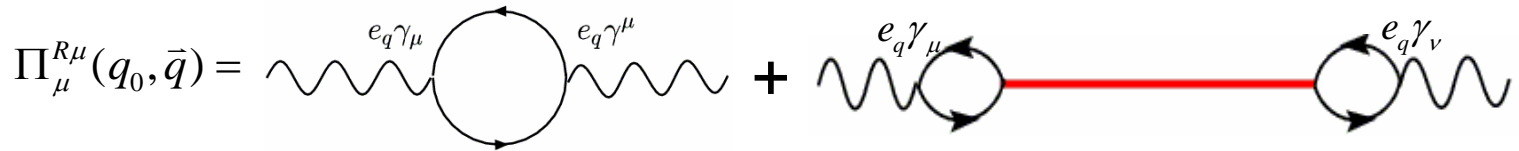
$$\frac{d\Gamma}{dM^2} = \int \frac{d^3q}{2q_0} \frac{d\Gamma}{d^4q} \quad \frac{d\Gamma}{d^4q} = \frac{-\alpha \text{Im} \Pi_{\mu}^{R\mu}(q_0, q)}{12\pi^4 q^2 (\exp[q_0/T] - 1)}$$



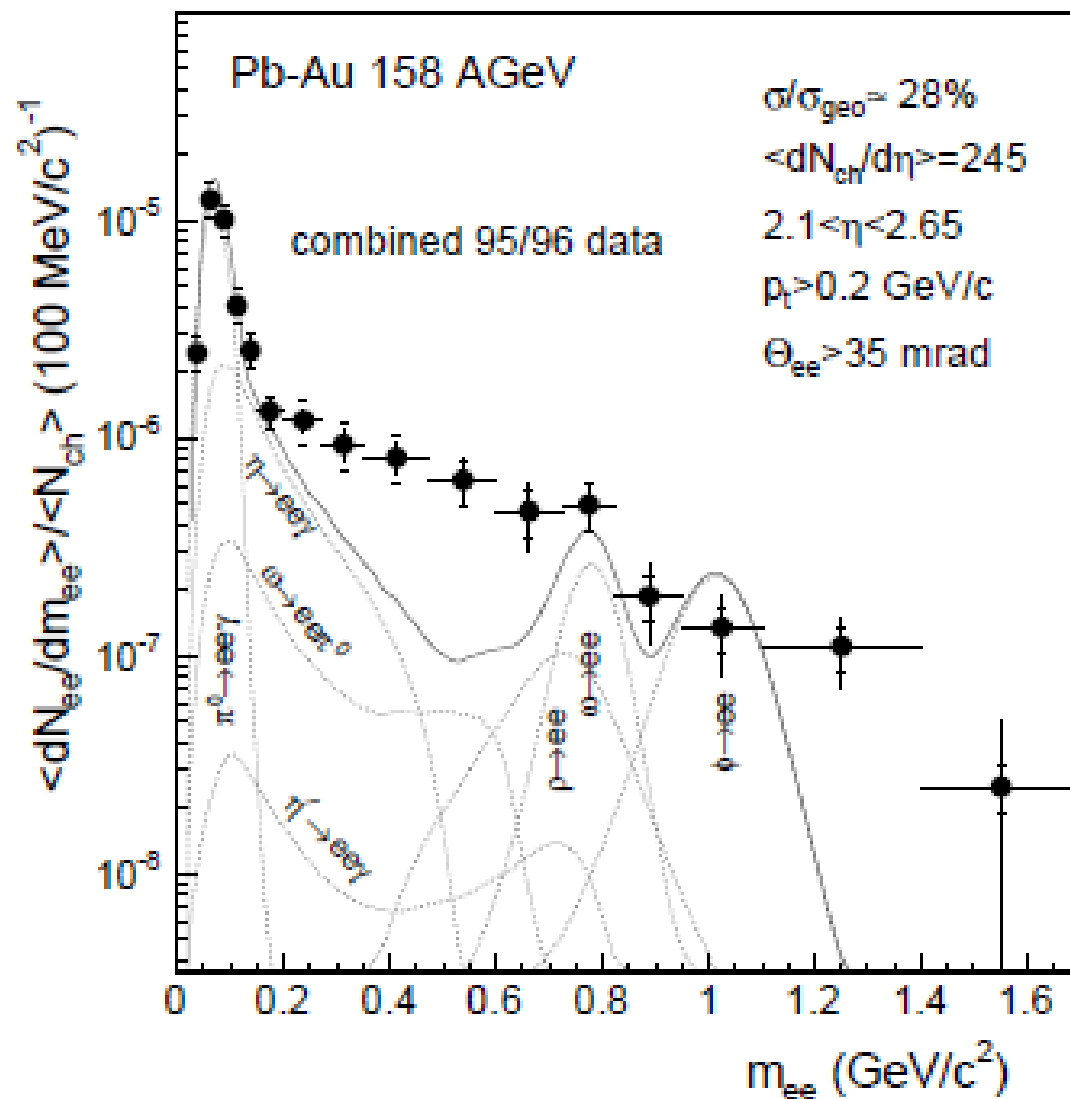
Di-muon Production Rate

along a pseudo-critical line

$$\frac{d\Gamma}{dM^2} = \int \frac{d^3q}{2q_0} \frac{d\Gamma}{d^4q} \quad \frac{d\Gamma}{d^4q} = \frac{-\alpha \text{Im} \Pi_{\mu}^{R\mu}(q_0, q)}{12\pi^4 q^2 (\exp[q_0/T] - 1)} \left(1 + \frac{2m_{\mu}^2}{q^2}\right) \left(1 - \frac{4m_{\mu}^2}{q^2}\right)^{1/2}$$



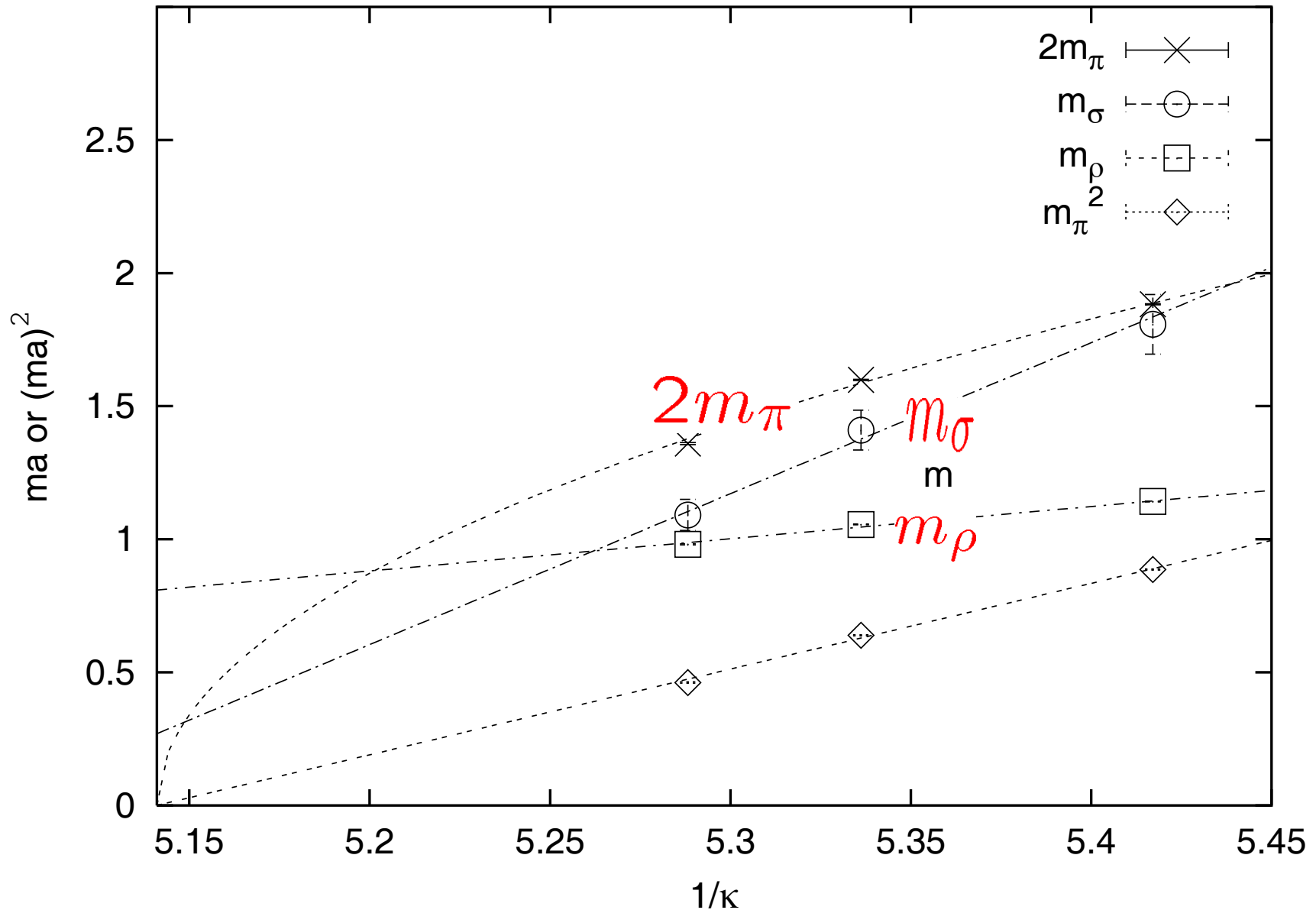
CERES ('05)



What is the origin of an enhancement around .0.3 GeV?

Remarks on the sigma mode
in the hadronic phase at finite T and density.

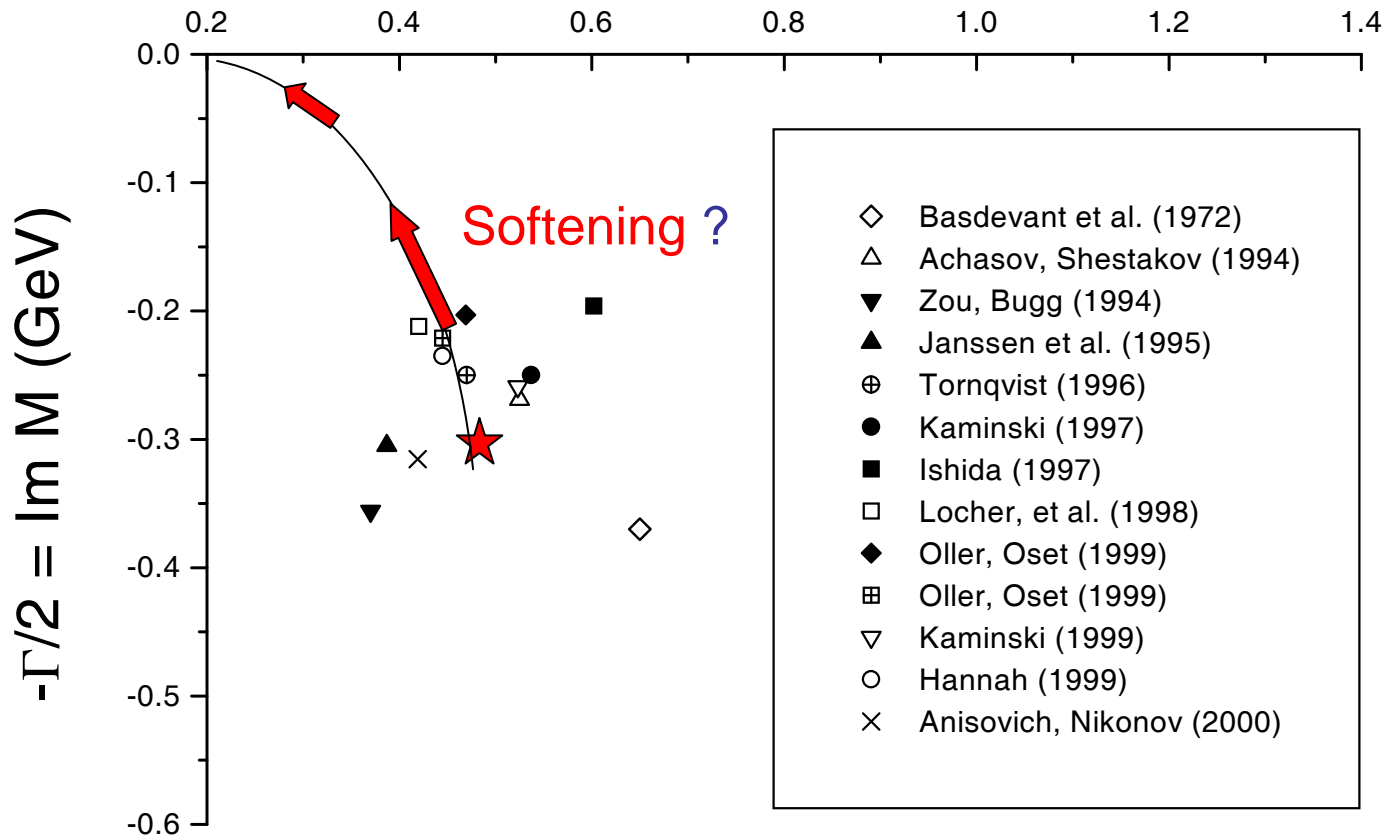
Lattice Calculations in full QCD of the sigma mass



The SCALAR collaboration, PRD70, 034504(2004)

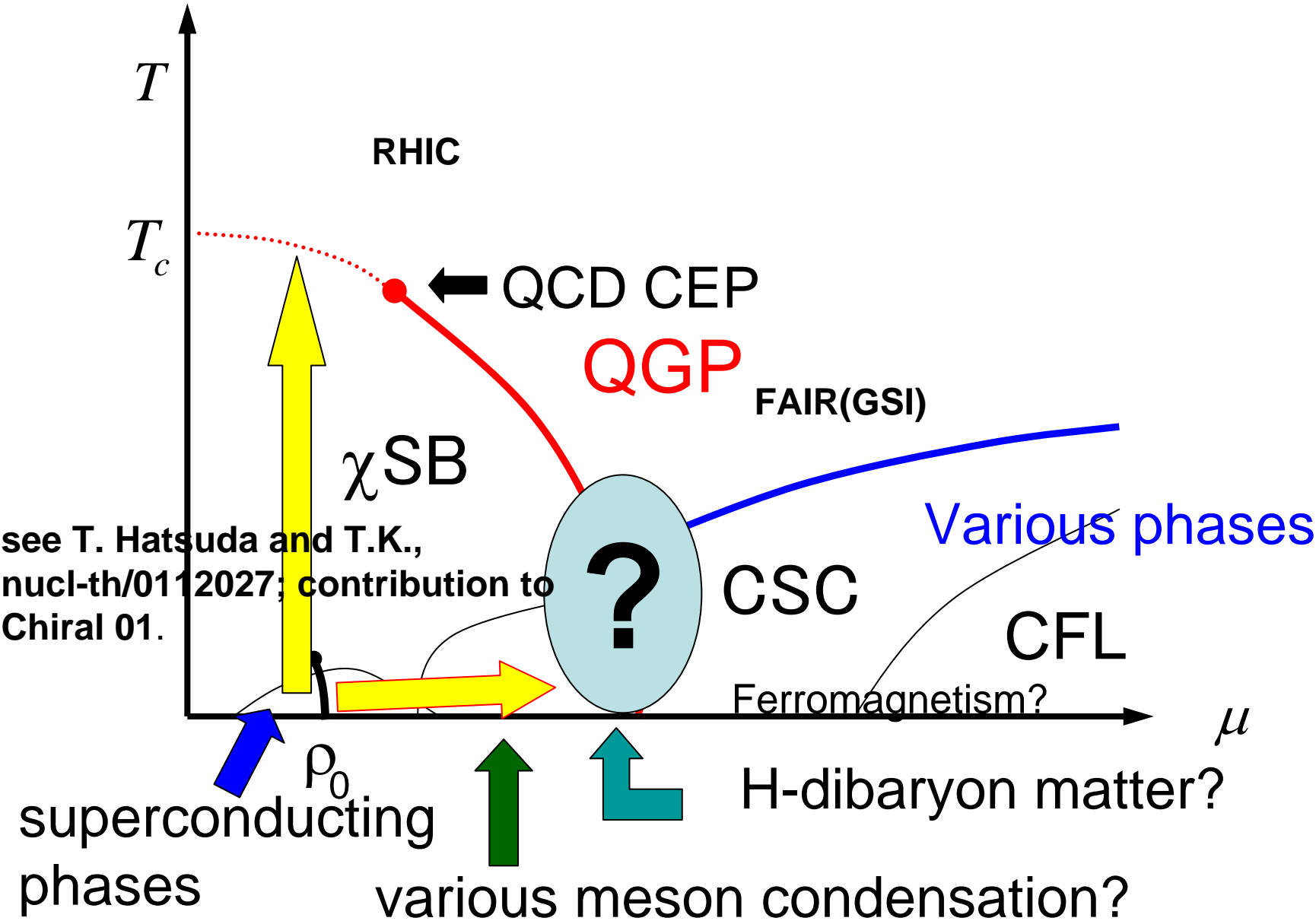
The poles of the S matrix in the complex mass plane for the sigma meson channel:

compiled in **Z. Xiao and H.Z. Zheng (2001)** M (GeV)



See also, I. Caprini, G. Colangelo and H. Leutwyler, PRL(2006);
H. Leutwyler, hep-ph/0608218 ; $M_{\text{sigma}}=441 - i 272 \text{ MeV}$

A conjectured QCD phase diagram



K. Saito et al (1998)

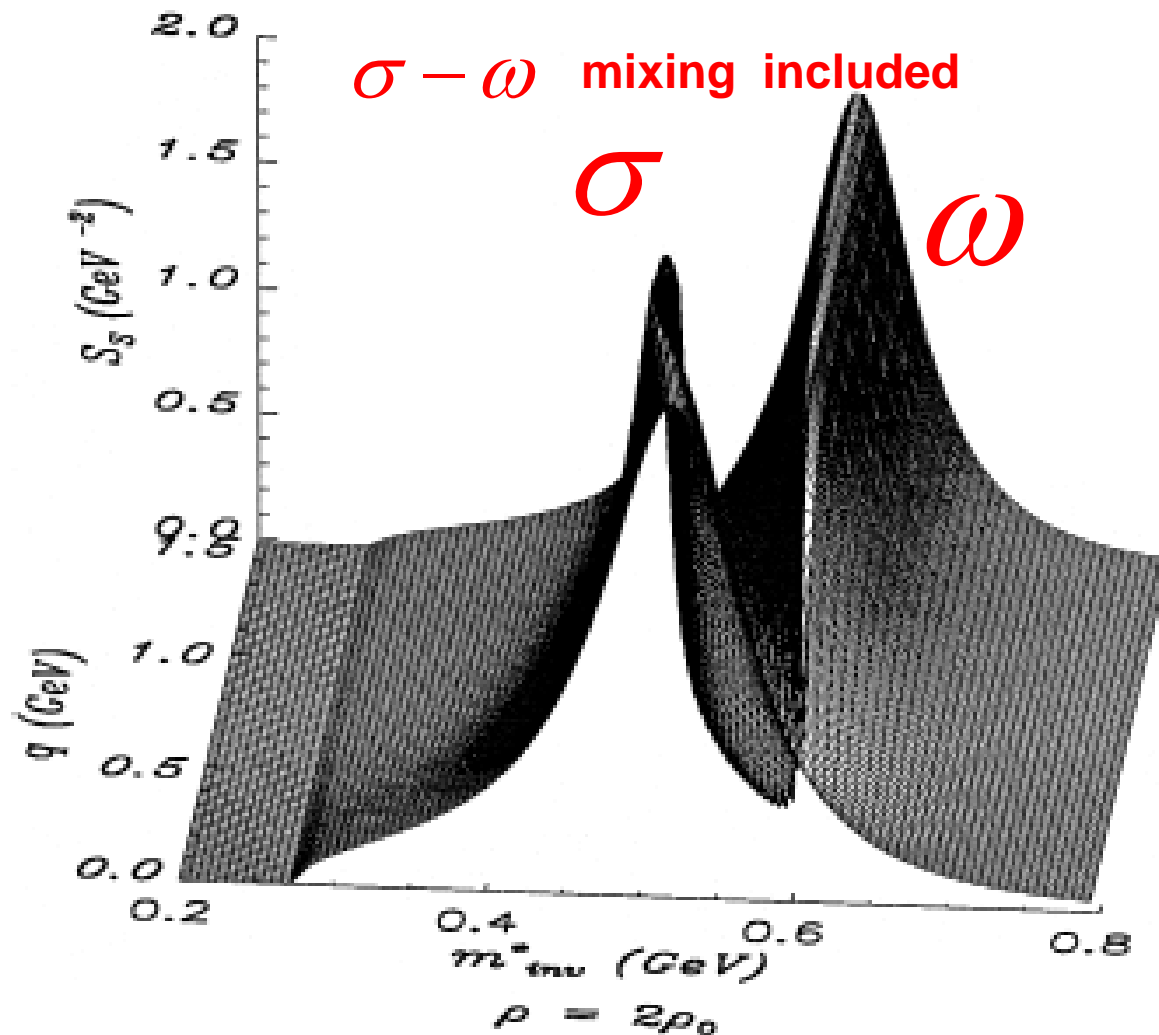


Fig. 6. Spectral function for the σ at $\rho_B / \rho_0 = 2$.

see also, G. Wolf et al(1998);
O. Teodorescu et al (2001)

Summary and concluding remarks

- The notion of the soft modes of QCD phase transitions was emphasized; they may be **hadronic excitations above T_c** .
- The soft modes of color-superconductivity above T_c may cause an enhancement of the electron-pair production in very-low mass region.
- Off the chiral limit, the lepton-pair production due to the specific mode around the QCD critical point is enhanced around **$2 M_q \sim 300-400\text{MeV}$, which might account (at least partly) for the excess of lepton pairs seen in CERES experiment.**