Covariant Dissipative Fluid-dynamical Equations that Are Consistent with Boltzmann Equation

Stable and

Teiji Kunihiro (YITP, Kyoto)

with Kyosuke Tsumura (Kyoto) and Kazuaki Ohnishi (Yonsei)

Ref. K. Tsumura, K. Ohnishi and T.K.
Phys. Lett. B646 (2007) 134-140
K. Tsumura and T.K.
in preparation

~~T.~~ Tsumura

K

INPC2007, June 3 – 8, 2007, Tokyo

Introduction

- Relativistic hydrodynamics for a perfect fluid is widely and successfully used in the RHIC phenomenology. D. Teaney et al …

.

- A growing interest in dissipative hydrodynamics.

hadron corona (rarefied states); Hirano et al …

Generically, an analysis using dissipative hydrodynamics is needed even to show the dissipative effects are small.

A.Muronga and D. Rischke; A. K. Chaudhuri and U. Heinz,; R. Baier, P. Romatschke and U. A. Wiedemann; R. Baier and P. Romatschke (2007) and the references cited in the last paper.

However,

is the theory of relativistic hydrodynamics for a viscous fluid fully established?

The answer is

No!

unfortunately.

Typical hydrodynamical equations for a viscous fluid
--- Choice of the frame and ambiguities in the form ---

Fluid dynamics = a system of balance equations

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu N^\mu = 0.$$

energy–momentum : $T^{\mu\nu}$ number : N^μ

$$T^{\mu\nu} \equiv \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \delta T^{\mu\nu} \qquad N^\mu \equiv n u^\mu + \delta N^\mu$$

Dissipative part

Eckart eq. no dissipation in the number flow; ⇒ Describing the flow of matter

$$\delta T^{\mu\nu} = u^\mu T\lambda\left(\frac{1}{T}\nabla^\nu T - Du^\nu\right) + u^\nu T\lambda\left(\frac{1}{T}\nabla^\mu T - Du^\mu\right)$$
$$+ 2\eta\frac{1}{2}\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla\cdot u\right) + \zeta\Delta^{\mu\nu}\nabla\cdot u$$
$$\delta N^\mu = 0.$$

--- Involving time-like derivative ---.

with $D \equiv u^\mu\partial_\mu \quad \nabla^\mu \equiv \Delta^{\mu\nu}\partial_\nu$

$$\Delta_p^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \equiv \Delta^{\mu\nu},$$

Landau–Lifshits no dissipation in energy flow ⇒ describing the energy flow.

$$\delta T^{\mu\nu} = 2\eta\frac{1}{2}\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla\cdot u\right) + \zeta\Delta^{\mu\nu}\nabla\cdot u$$

$$u_\mu u_\nu \delta T^{\mu\nu} = 0, u_\mu \Delta_{\sigma\nu}\delta T^{\mu\nu} = 0$$

$$\delta N^\mu = -\lambda\frac{nT}{\epsilon + p}\left(\frac{1}{T}\nabla^\mu T - \frac{1}{\epsilon + p}\nabla^\mu p\right)$$

--- Involving only space-like derivatives ---

with transport coefficients:

ς ; Bulk viscocity, η; Shear viscocity

λ ;Heat conductivity

Fundamental problems with relativistic hydro-dynamical equations for viscous fluid

a. Ambiguities in the form of the equation, even in the same frame and equally derived from Boltzmann equation: Landau frame; unique,
Eckart frame; Eckart eq. v.s. Grad-Marle-Stewart eq.; Muronga v.s. R. Baier et al

b. Instability of the equilibrium state in the eq.'s in the Eckart frame, which affects even the solutions of the causal equations, say, by Israel-Stewart.
W. A. Hiscock and L. Lindblom ('85, '87); R. Baier et al ('06, '07)

c. Usual equations are acausal as the diffusion eq. is, except for Israel-Stewart and those based on the extended thermodynamics with relaxation times,
but the form of causal equations is still controversial.

---- The purpose of the present talk ---

Instead of applying an existing equation to RHIC phenomenology,
we focus on the fundamental problems.
For analyzing the problems a and b,
we deriving hydrodynaical equations for a viscous fluid from Boltzmann equation on the basis of a mechanical reduction theory (so called the RG method) and a natural ansatz on the origin of dissipation.
We also show that the new equation in the Eckart frame is found to be stable, which would affect the existing and possible solutions to the problem c.

Derivation of the relativistic fluid dynamical equation from the rel. Boltzmann eq. --- an RG-reduction of the dynamics

K. Tsumura, T.K. K. Ohnishi; Phys. Lett. B646 (2007) 134-140

c.f. Non-rel. Y.Hatta and T.K., Ann. Phys. 298 ('02), 24; T.K. and K. Tsumura, J.Phys. A:39 (2006), 8089

Ansatz of the origin of the dissipation= the spatial inhomogeneity,

leading to Navier-Stokes in the non-rel. case .

$\boldsymbol{a}_p^\mu$ would become a macro flow-velocity ⟵ Coarse graining of space-time

$$\tau \equiv \boldsymbol{a}_p^\mu\, x_\mu, \quad \sigma^\mu \equiv \left(g^{\mu\nu} - \frac{\boldsymbol{a}_p^\mu \boldsymbol{a}_p^\nu}{\boldsymbol{a}_p^2} \right) x_\nu \equiv \boldsymbol{\Delta}_p^{\mu\nu}\, x_\nu \qquad x^\mu \Longrightarrow \tau \quad \sigma^\mu$$

$$\frac{\partial}{\partial\tau} = \frac{1}{\boldsymbol{a}_p^2}\, \boldsymbol{a}_p^\mu\, \partial_\mu \equiv \boldsymbol{D},$$ time-like derivative $$\boldsymbol{\Delta}_p^{\mu\nu} \frac{\partial}{\partial\sigma^\nu} = \boldsymbol{\Delta}_p^{\mu\nu}\, \partial_\nu \equiv \boldsymbol{\nabla}^\mu$$ space-like derivative

Rewrite the Boltzmann equation as,

$$\Longrightarrow \quad \frac{\partial}{\partial\tau} f_p(\tau\,,\,\sigma) = \frac{1}{p\cdot\boldsymbol{a}_p} C[f]_p(\tau\,,\,\sigma) - \frac{1}{p\cdot\boldsymbol{a}_p}\, p\cdot\boldsymbol{\nabla} f_p(\tau\,,\,\sigma)$$

perturbation

Only spatial inhomogeneity leads to dissipation.

1st order

$$\frac{\partial}{\partial\tau} \tilde{f}_p^{(1)} = \sum_q A_{pq}\, \tilde{f}_q^{(1)} + F_p$$

$$A_{pq} \equiv \frac{1}{p\cdot\boldsymbol{a}_p} \frac{\partial}{\partial f_q} C[f]_p \Big|_{f=f^{\mathrm{eq}}} \qquad F_p \equiv -\frac{1}{p\cdot\boldsymbol{a}_p}\, p\cdot\boldsymbol{\nabla} f_p^{\mathrm{eq}}$$

RG gives a resummed distribution function, from which $T^{\mu\nu}$ and N^μ are obtained.

Landau equation: $a_p^\mu = u^\mu$

Eckart (particle-flow) frame:

Setting $a_p^\mu = \frac{m}{p\cdot u}\, u^\mu$

$$T^{\mu\nu} = (\epsilon + 3\,\zeta\,\tilde{X})\,u^\mu\,u^\nu - (p + \zeta\,\tilde{X})\,\Delta^{\mu\nu} + \lambda\,T\,u^\mu\,\tilde{X}^\nu + \lambda\,T\,u^\nu\,\tilde{X}^\mu + 2\,\eta\,X^{\mu\nu}$$

$N^\mu = m\,n\,u^\mu$

i.e., $\delta N^\mu = 0.$

with

$$\tilde{X} \equiv -\{1/3\,(4/3 - \gamma)^{-1}\}^2\,\nabla\cdot u$$

$$\tilde{X}^\mu \equiv \nabla^\mu \ln T.$$

(i) This satisfies the GMS constraints but not the Eckart's.

(ii) Notice that only the space-like derivative is incorporated.

(iii) This form is different from Eckart's and Grad-Marle-Stewart's, both of which involve the time-like derivative.

Eckart's constraints :
$$\begin{cases} 1.\ u_\mu\,u_\nu\,\delta T^{\mu\nu} = 0, \\ 2.\ u_\mu\,\delta N^\mu = 0, \\ 3.\ \Delta_{\mu\nu}\,\delta N^\nu = 0, \end{cases} \Longleftrightarrow \begin{cases} 5.\ T^\mu{}_\mu = 0, \\ 2.\ u_\mu\,\delta N^\mu = 0, \\ 3.\ \Delta_{\mu\nu}\,\delta N^\nu = 0. \end{cases}$$

Grad-Marle-Stewart constraints

c.f. Grad-Marle-Stewart equation;

$$\delta T^{\mu\nu} = -3\,(3T^{-1}\,C_T + 1)^{-1}\,\zeta\,u^\mu\,u^\nu\,\nabla\cdot u + u^\mu\,T\,\lambda\left(\frac{1}{T}\,\nabla^\nu T - Du^\nu\right) + u^\nu\,T\,\lambda\left(\frac{1}{T}\,\nabla^\mu T - Du^\mu\right)$$
$$+ 2\,\eta\,\frac{1}{2}\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\,\Delta^{\mu\nu}\,\nabla\cdot u\right) + (3\,T^{-1}\,C_T + 1)^{-1}\,\zeta\,\Delta^{\mu\nu}\,\nabla\cdot u,$$
$$\delta N^\mu = 0.$$

Compatibility with the underlying kinetic equations?

Eckart constarints are not compatible with the Boltzmann equation. (Ch.G. van Weert('87), Proved in K.Tsumura, T.K. and K.Ohnishi ('06))

The five collision invariants forms the fluid dynamical variables and spans the invariant manifold.

Which equation is better?

The linear stability analysis around the thermal equilibrium state.

c.f. Ladau equation is stable. (Hiscock and Lindblom ('85))

The stability of the equations in the "Eckart(particle)" frame:

K.Tsumura and T.K. ; in preparation.

(i) The Eckart and Grad-Marle-Stewart equations show an instability, which has been known, and is now found to be attributed to the fluctuation-induced dissipation, proportional to Du^{μ}.

(ii) Our equation (TKO equation) can be stable, being dependent on the values of the transport coefficients and the EOS;

$$\frac{4}{3}\eta + \zeta\,(3\,\gamma - 4)^{-2}\left(1 - 3\,\frac{\{p\,,\,n\}_0}{\{\epsilon\,,\,n\}_0}\right) \geq 0$$

with $\{F\,,\,G\}_0 \equiv \left(\frac{\partial F}{\partial T}\right)_0 \left(\frac{\partial G}{\partial \mu}\right)_0 - \left(\frac{\partial G}{\partial T}\right)_0 \left(\frac{\partial F}{\partial \mu}\right)_0$

c.f. This inequality is satisfied, at least, for massless system; $\varepsilon = 3p$

Linear Stability Analysis

Def. $T(x) = T_0 + \delta T(x),\ \mu(x) = \mu_0 + \delta\mu(x)$ and $u^\mu(x) = u_0^\mu + \delta u^\mu(x)$

with $u_0 \cdot \delta u = 0 \longleftarrow u \cdot u = 1$

Actually, we will put $u_0 = 0$.

Equation of Motion:

$0 = \partial_\mu T^{\mu\nu} = \partial_\mu \delta T^{\mu\nu}$ and $0 = \partial_\mu N^\mu = \partial_\mu \delta N^\mu$

Ansatz for the solution; plane-wave solution

$$(\delta u^\mu,\ \delta T,\ \delta\mu) = (\delta\tilde{u}^\mu,\ \delta\tilde{T},\ \delta\tilde{\mu})\,\mathrm{e}^{-ik\cdot x}$$

$$\sum_{\beta=1}^{5} M_{\alpha\beta}\Phi_\beta = 0,$$ where $M_{\alpha\beta} = M_{\alpha\beta}(k^0, \vec{k})$

$\det M_{\alpha\beta} = 0$ $\Longrightarrow$

Dispersion relation; $\varpi \equiv k^0 = k^0(k)$ (generically complex.)

The stability condition: $\mathrm{Im}(k^0(k)) \leq 0 \quad \forall k$

Summary and concluding remarks

- Eckart equation, which and a causal extension of which are widely used, is not compatible with the underlying Relativistic Boltsmann equation.
- The RG method gives a consistent fluid dynamical equation for the particle (Eckart) frame as well as other frames, which is new and has no time-like derivative for thermal forces.
- The linear analysis shows that the new equation in the Eckart (particle) frame can be stable in contrast to the Eckart and (Grad)-Marle-Stewart equations which involve dissipative terms proportional to Du^{μ} .
- The RG method is a mechanical way for the construction of the invariant manifold of the dynamics and can be applied to derive a causal fluid dynamics, a la Grad 14-moment method. (c.f. A. Gorban and I. Karlin)
- According to the present analysis, even the causal (Israel-Stewart) equation which is an extension of Eckart equation should be modified.
- There are many fundamental isseus to clarify for establishing the relativistic fluid dynamics for a viscous fluid.