# GENERALIZED UNCERTAINTY PRINCIPLE AND BLACK HOLE TEMPERATURE

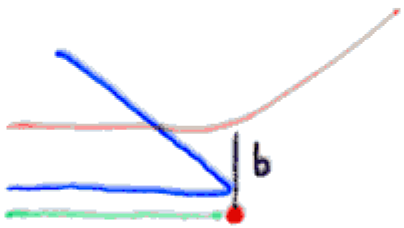
- Quantum description of spacetime should involve uncertainty

UNCERTAINTY PRINCIPLE $\xrightarrow[\text{INCLUDE}]{\text{SHOULD}}$ GRAVITATIONAL SOURCE(S) OF ERROR

- ## Three roads to GUP

① STRING THEORY (Veneziano et al. 1986-90)

- THEY STUDY ULTRA HIGH ENERGY SCATTERING OF STRINGS
- THEY FIND THAT THE LARGER MOMENTUM TRANSFER DOES NOT ALWAYS CORRESPOND TO SHORTER DISTANCES
- INFACT: THERE $\exists$ A SCATTERING ANGLE $\theta_M$ SUCH THAT:




WHEN $\theta < \theta_M \Rightarrow$ the relation between the impact parameter $b$ and the momentum transfer $\langle p \rangle$ is the classical one (à la Heinsenberg)

$$\langle p \rangle \sim \frac{\hbar}{b}$$

WHEN $\theta \gg \theta_M \Rightarrow$ a new regime appears: $\langle p \rangle \sim b$

This leads to a modification of the uncertainty relation (at the Planck scale)

$$\Delta x \sim \frac{\hbar}{\Delta p} + Y\alpha\,\Delta p$$

WHERE $Y$ = model dependent constant ; $\alpha$ = string tension

CONSEQUENCE: THERE $\exists$ A MINIMAL OBSERVABLE LENGTH OF THE ORDER OF THE STRING SIZE $\lambda_s$

## MACRO BLACK HOLE GEDANKEN EXPERIMENT (Maggiore 1993)

- MEASUREMENT OF THE RADIUS $R_h$ OF THE HORIZON OF A b.h. VIA THE PHOTONS OF THE HAWKING RADIATION EMITTED BY THE HOLE

ERRORS AFFECTING THIS MEASURE:

- RESOLVING POWER OF THE MICROSCOPE: $\Delta x_{(1)} \sim \dfrac{\lambda}{\sin\theta}$ (as in Heisenberg classical analysis)

  $\theta$ = SCATTERING ANGLE

- VARIATION OF THE MASS (i.e. RADIUS) OF THE HOLE DURING THE EMISSION PROCESS: $M \to M - \Delta M$ with $\Delta M = \dfrac{h}{\lambda c}$

  $$\Delta x_{(2)} = \frac{2G\Delta M}{c^2} = \frac{2Gh}{c^3\lambda}$$

- SINCE $\dfrac{\lambda}{\sin\theta} \geq \lambda \Longrightarrow \Delta x_{TOT} \geq \lambda + K\dfrac{2Gh}{c^3\lambda} = \dfrac{h}{\Delta p} + K\dfrac{2G}{c^3}\Delta p$

- K CANNOT IN GENERAL BE PREDICTED BY THIS MODEL-INDEPENDENT ARGUMENT

## ⊕ MICRO BLACK HOLE GEDANKEN EXPERIMENT

- WE STUDY HOW THE FORMATION OF A MICRO BLACK HOLE AFFECTS THE MEASURE PROCESS (USING ONLY HEISENBERG PRINCIPLE AND THE NOTION OF GRAVITATIONAL RADIUS)
- TO PROBE A REGION OF SIZE $\Delta x$, WE SHOULD CONCENTRATE IN THAT REGION AN ENERGY $\Delta E \simeq \frac{\hbar c}{2\,\Delta x}$
- THE GRAVITATIONAL RADIUS ASSOCIATED WITH THIS ENERGY IS

$$R_S = \frac{2 G \Delta E}{c^4}$$

- USUALLY $\underline{R_S \ll \Delta x}$ IN ALL PRACTICAL CASES
- TO IMPROVE THE PRECISION $\Delta x$, WE SHOULD INCREASE THE ENERGY $\Delta E$, SO THAT $R_S$ IS GOING TO BECOME LARGER, UNTIL $\underline{R_S \simeq \Delta x}$
- AT THIS POINT A MICRO BLACK HOLE ORIGINATES. THIS HAPPEN WHEN

$$R_S = \Delta x \Rightarrow \left.\begin{array}{l} \Delta E\, \Delta x \sim \hbar c/2 \\ \Delta x = \frac{2 G \Delta E}{c^4} \end{array}\right] \Rightarrow \Delta x = \left(\frac{G \hbar}{c^3}\right)^{1/2} =: \ell_P$$

AND THE ASSOCIATED ENERGY IS $\mathcal{E}_P$ $\left(\mathcal{E}_P\, \ell_P \simeq \hbar c/2\right)$

- TO FURTHER DECREASE $\Delta x$ REQUIRES THE USE OF A GREATER ENERGY AND THIS ENLARGES FURTHER THE SIZE $R_S$ OF THE MICRO HOLE, HIDING MORE DETAILS OF THE REGION BEYOND THE EVENT HORIZON OF THE MICRO HOLE: THE UN-OBSERVABLE REGION WOULD INCREASE INSTEAD OF DECREASING!

• SUMMARIZING

$$\Delta x \gtrsim \begin{cases} \frac{\hbar c}{2 \Delta E} & \text{for } \Delta E < \mathcal{E}_P \\ \frac{2 G \Delta E}{c^4} & \text{for } \Delta E > \mathcal{E}_P \end{cases}$$

OR ALSO:

$$\Delta x \gtrsim \frac{\hbar c}{2\,\Delta E} + \frac{2 G \Delta E}{c^4}$$

WHICH IN TERMS OF $\Delta p$ READS ($\Delta E \sim c\Delta p$)

$$\Delta x \gtrsim \frac{\hbar}{2\Delta p} + 2\,\ell_p^2\,\frac{\Delta p}{\hbar} \qquad (*)$$

## Application of GUP to b.h. temperature (Adler Chen 200…

### ⊝ CLASSICAL HEISENBERG PRINCIPLE : HAWKING TEMPERATURE of a b.h.

- THE INTRINSIC UNCERTAINTY IN THE POSITION OF A PARTICLE CLOSE TO A BLACK HOLE HORIZON IS ABOUT $\sim R_s$ (Schw. RADIUS)
- THE CORRESPONDING MOMENTUM UNCERTAINTY IS:

$$\Delta p \sim \frac{\hbar}{\Delta x} = \frac{\hbar}{2R_s} = \frac{\hbar c^2}{4GM} \qquad (M = \text{mass of the b.h.})$$

WHICH IN ENERGY READS $\Delta E \sim c\,\Delta p = \dfrac{\hbar c^3}{4GM}$

- WE CAN IDENTIFY THIS ENERGY WITH THE CHARACTERISTIC ENERGY OF THE EMITTED PHOTONS, AND THEREFORE WITH THE TEMPERATURE

$$\Delta E = \frac{\hbar c^3}{4GM} \sim \frac{3}{2} KT \Longrightarrow T_H \sim \frac{\hbar c^3}{6GKM}$$

WHICH IS THE HAWKING TEMPERATURE OF THE HOLE.

### ⊜ GUP : MODIFIED B.H. TEMPERATURE

- SOLVING (*) FOR $\Delta p$ : $\dfrac{\Delta p}{\hbar} = \dfrac{\Delta x}{2\ell_p^2}\left(1 - \sqrt{1 - \dfrac{4\ell_p^2}{\Delta x^2}}\right)$
- IDENTIFYING THE UNCERTAINTY $\Delta x$ WITH THE SCHW. RADIUS $R_s$ WE GET

$$T_{GUP} = \frac{Mc^2}{4\pi}\left(1 - \sqrt{1 - \frac{m_p^2}{M^2}}\right)$$

WHICH AGREES WITH THE STANDARD RESULT FOR $M \to \infty$.

NOTE THAT $T_{GUP}$ BECOMES UNPHYSICAL for $M < M_P$

THEREFORE THE B.H. MASS MUST BE $M > M_P$, GREATER THAN THE PLANCK MASS.

- ASSUMING AN ENERGY LOSS DOMINATED BY PHOTONS, WE MAY USE THE STEFAN-BOLTZMAN LAW TO ESTIMATE THE MASS OUTPUT AND THE RADIATION RATE AS A FUNCTION OF TIME

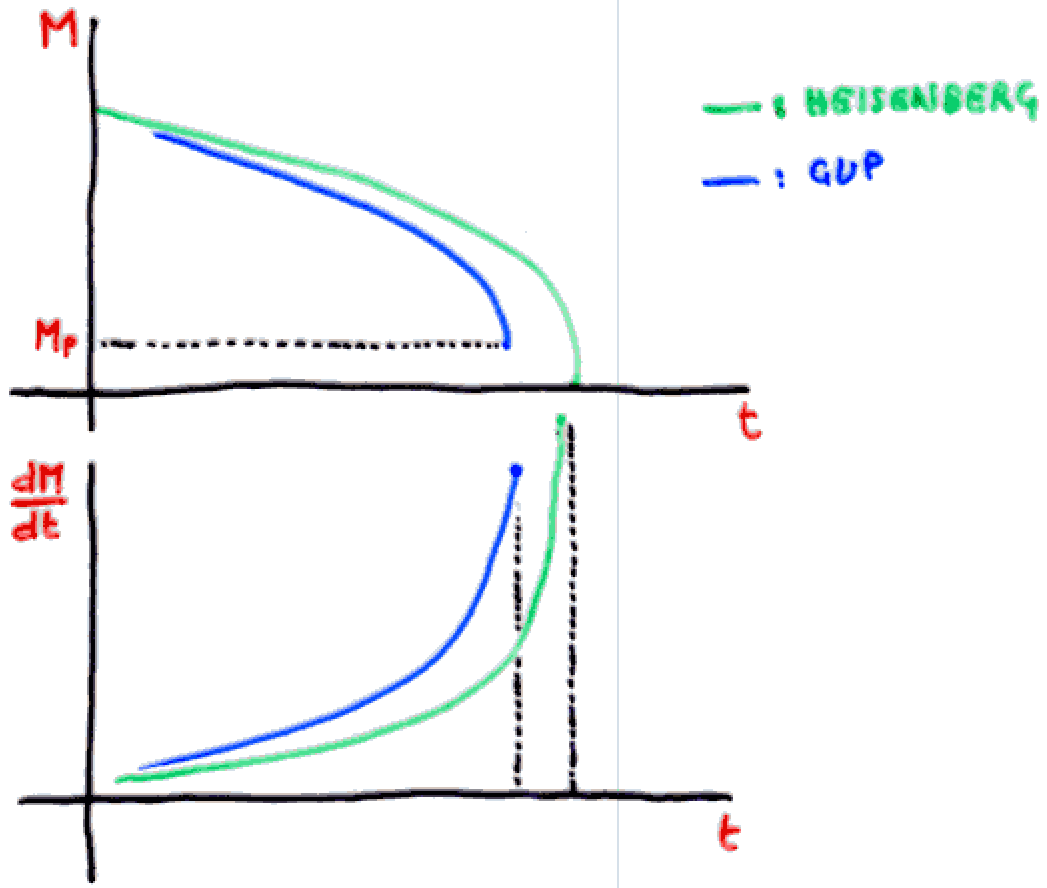
$$\frac{dM}{dt} = -\frac{\sigma T^4 \, 4\pi R_s^2}{c^2}$$

⊖ CLASSICAL HEISENBERG PRINCIPLE : $\frac{dM}{dt} = -\frac{\alpha}{M^2}$

⊖ G.U.P. : $\frac{dM}{dt} = -\alpha M^6 \left[ 1 - \sqrt{1 - \frac{M_P^2}{M^2}} \right]^4$ $\boxed{M > M_P \, !}$

MASS OUTPUT VS TIME

RADIATION RATE VS TIME




THE GUP STOPS THE EVAPORATION WHEN M REACHES THE PLANCK MA

PREDICTION OF LONG LIFETIME REMNANTS ⇒ CANDIDATE FOR DARK MATTER