♠ AdS/CFT Correspondence

Anti-de Sitter space:

$$-X_0^2 + X_1^2 + \cdots + X_{d-1}^2 - X_d^2 = -R^2$$

Poincare metric

$$ds^2 = R^2 \, \frac{d\rho^2 + \Sigma_{i=1}^{d-1} dx_i^2}{\rho^2}$$

$$U = X_{d-1} + X_d, \;\; V = X_{d-1} - X_d,$$

$$\rho = \frac{R}{U}, \;\; x_i = \frac{X_i}{U}, \;\; i = 0, 1, \cdots, d-2$$

Scale invariance

$$\rho \longrightarrow \lambda\rho, \;\; x^i \longrightarrow \lambda x^i$$

Radial variable ρ acts like a mass scale of boundary field theory.

Space-time around N overlapping D3 branes gives $AdS_5 \times S^5$ with size

$$g_s N = \left(\frac{R}{\ell_s}\right)^4 \;\; (g_s = g_{YM}^2)$$

Small g_{YM}; perturbative gauge theory
Large g_{YM}; Gravity on AdS space

A large number of examples have been worked out. For instance,

$$AdS_5 \times M^{p,q}, \;\; p,q = 1,2,\cdots$$

$M^{p,q}$ are 5-dimensional Sasaki-Einstein spaces so that

$$ds^2 = dU^2 + U^2 ds^2_{M^{p,q}}$$

is a Calabi-Yau manifold. Gravity theory on $AdS_5 \times M^{p,q}$ are dual to various kinds of quiver gauge theories. Central charge of gauge theories and the volume of Sasaki-Einstein spaces M are related as

$$c \propto \frac{1}{M}$$

By introducing additional branes or fluxes one can deform the geometry of AdS space and break conformal invariance. These theories give gravity duals of confining gauge theories. Klebanov-Strassler etc.

♠ de Sitter Space

$$-X_0^2 + X_1^2 + \cdots + X_{d-1}^2 + X_d^2 = R^2$$
$$\mathcal{R}_{ij} - \frac{1}{2} g_{ij}\mathcal{R} + \Lambda g_{ij} = 0, \;\; \Lambda = \frac{(d-2)(d-1)}{2R^2}$$

Global coordinate

$$ds^2 = -d\tau^2 + \cosh\tau d\Omega_{d-1}^2$$
$$(X_0 = \sinh\tau, \;\; X_i = \omega^i \cosh\tau)$$

Planar coordinate

$$ds^2 = -dt^2 + e^{-2t} \sum_{a=1}^{d-1} dx_a^2$$
$$(X_0 = \sinh t - \frac{1}{2}\sum x_a^2 e^{-t}, \;\; X_a = x_a e^{-t}$$
$$X_d = \cosh t - \frac{1}{2}\sum x_a^2 e^{-t})$$

Static coordinate

$$ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega_{d-2}$$

$$(X_0 = \sqrt{1-r^2}\sinh t,\ x_a = r\omega^a$$
$$X_d = \sqrt{1-r^2}\cosh t)$$

Space is expanding so fast that the light rays coming from a distance can not reach the observer. There exists a cosmological event horizon. Observer at $r = 0$ is surrounded by the horizon at $r = 1$ and is in a thermal bath of particles with temperature and entropy

$$T = \frac{1}{2\pi R},\ S = \frac{4\pi R^2}{4G}$$

No microscopic understanding of de Sitter entropy is known.

Number of degrees of freedom finite?

$$S = \log N$$

There exist no positive definite Hamiltonian or SUSY in de Sitter space. Vacua

of string theory always have $\Lambda \leq 0$. $\exists$ Difficulties in quantum mechanical treatment of de Sitter space.

Speculation:
de Sitter space is unstable and decays into Minkowski space or anti-de Sitter space.

Polyakov (arXiv:0709.2899) proposes a correspondence of de Sitter space with non-unitary conformal field theories at future/past infinity.

dS_3 dual to non-unitary 1+1 dim CFT

Example of non-unitary CFT's

$$c = 1 - \frac{6pq}{(p-q)^2}, \;\; p \neq q+1$$