K. Shizuya

Particle physics $\cap$ Condensed-matter physics

"Graphene"

- "Graphene" : monolayer of C atoms

Novoselov et al, Nature ('05)

Zhang, et al, Nature ('05)

"relativistic" condensed-matter system

Electrons and holes ~ massless Dirac fermions

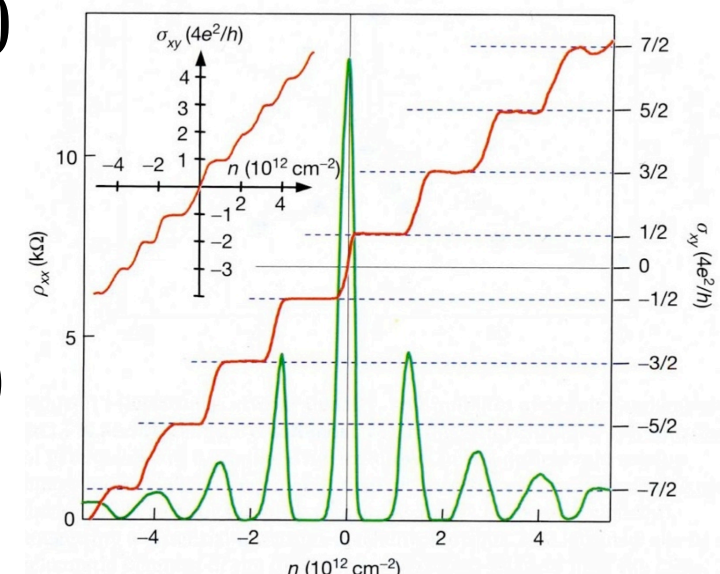
$\epsilon \approx v_F \, |\mathbf{k}| \qquad v_F \sim 10^6 \mathrm{m/s} \sim c/300$

Exotic phemonena $\sigma_{\min}$

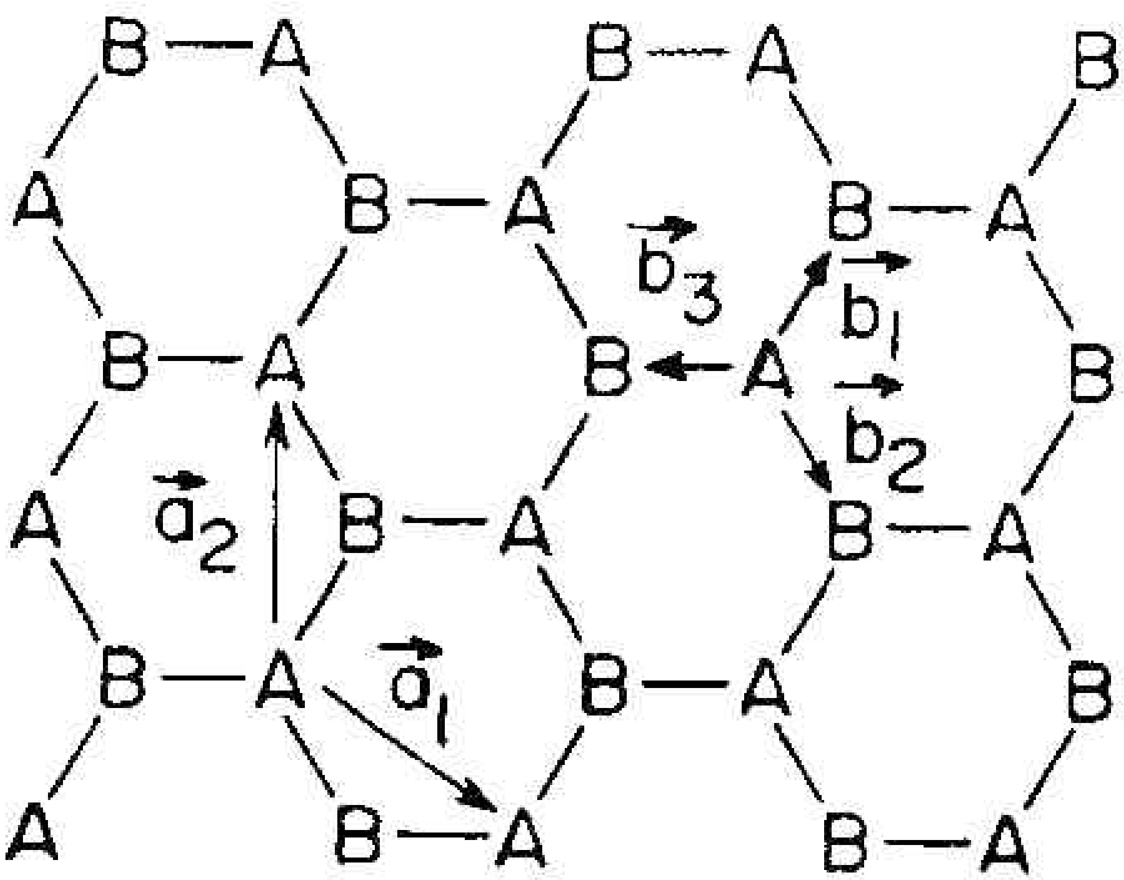
half-integer QHE $\sigma_H = (e^2/h)\, 4(n + \frac{1}{2})$

zero-energy Landau levels

fermion # fractionalization , chiral anomaly



- Test QED_{2+1}
- How Dirac fermions appear in a cond-mat. system



Honeycomb Lattice

The Bravais lattice is triangular and the unit cell contains two sites

Basis vectors are

$\mathbf{a}_1 = (\frac{\sqrt{3}}{2}, \frac{1}{2})a$

$\mathbf{a}_2 = (\,0\,,\,1\,)a$

The sublattices are connected by

$\mathbf{b}_1 = (\frac{1}{2\sqrt{3}}, \ \frac{1}{2}\,)a$

$\mathbf{b}_2 = (\frac{1}{2\sqrt{3}}, -\frac{1}{2})a$

$\mathbf{b}_3 = (-\frac{1}{\sqrt{3}}, \,0\,)a$

Tight-binding Hamiltonian

U : e on site A

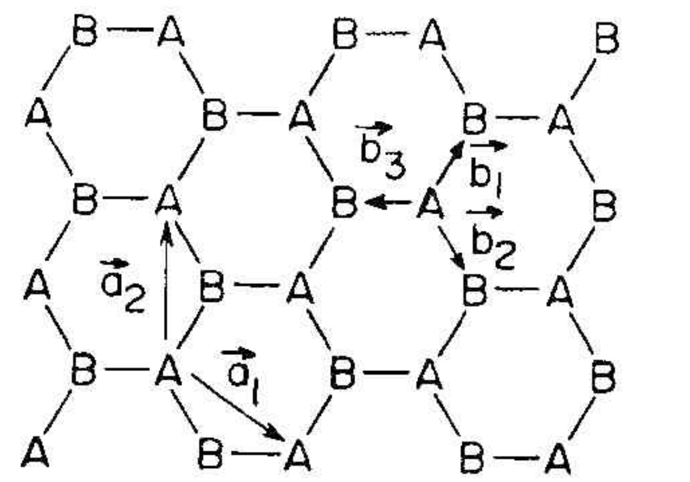
V : e on site B

$$H = \alpha \sum_{\mathbf{A}} \sum_{i} \{U^\dagger(\mathbf{A})\,V(\mathbf{A}+\mathbf{b}_i) + V^\dagger(\mathbf{A}+\mathbf{b_i})\,U(\mathbf{A})\}$$

hopping $B \leftrightarrow A$

$$\mathbf{A} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2$$

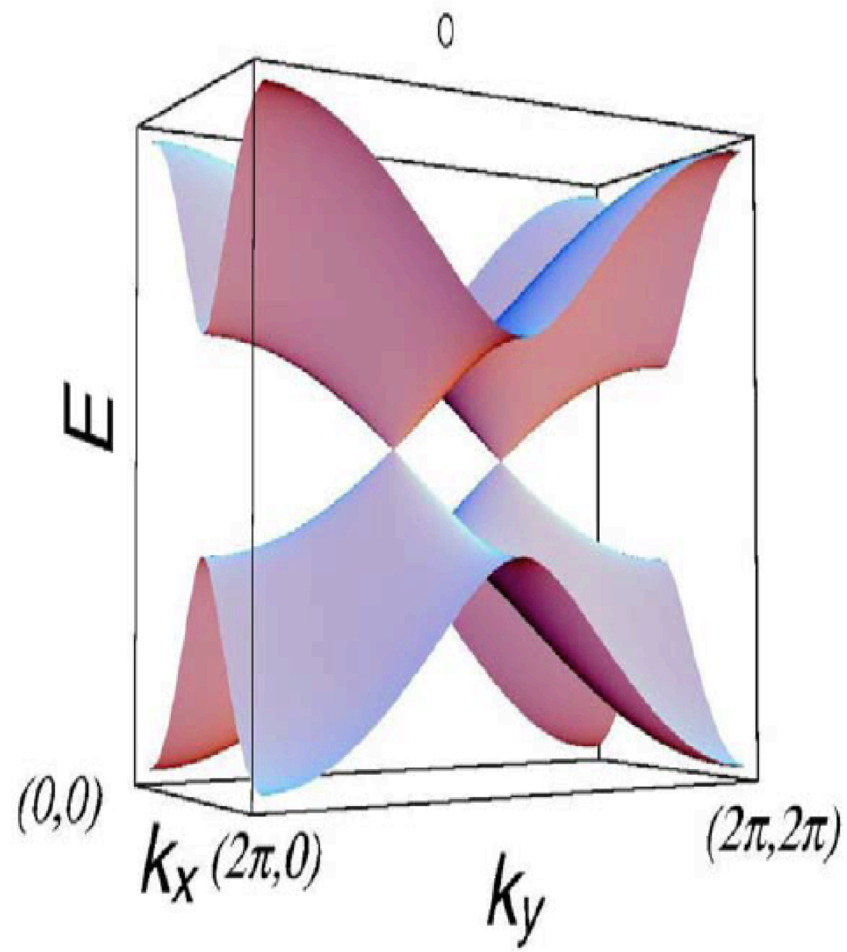
$$U(\mathbf{A}) \;=\; \int_{\Omega_B} \frac{d^2k}{(2\pi)^2}\, e^{i\,\mathbf{k}\cdot\mathbf{A}}\, U_{\mathbf{k}}, \quad V(\mathbf{B}) = \int_{\Omega_B} \frac{d^2k}{(2\pi)^2}\, e^{i\,\mathbf{k}\cdot\mathbf{B}}\, V_{\mathbf{k}},$$



$$H = \sum_{\mathbf{k}} (U^\dagger_{\mathbf{k}}, V^\dagger_{\mathbf{k}}) \begin{pmatrix} & \alpha\,\phi(\mathbf{k}) \\ \alpha\,\phi^*(\mathbf{k}) & \end{pmatrix} \begin{pmatrix} U_{\mathbf{k}} \\ V_{\mathbf{k}} \end{pmatrix}$$

$$\phi(\mathbf{k}) \;=\; e^{i\,\mathbf{k}\cdot\mathbf{b}_1} + e^{i\,\mathbf{k}\cdot\mathbf{b}_2} + e^{i\,\mathbf{k}\cdot\mathbf{b}_3}$$

$$\text{energy}: \quad E(\mathbf{k}) = \pm\alpha\,|\phi(\mathbf{k})|$$



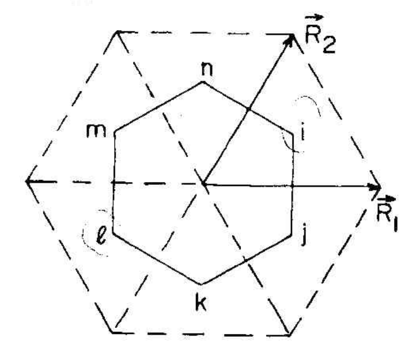
The separation of the conduction and valence bands minimized at zeros of

$$\phi(\mathbf{k}) = e^{i\,\mathbf{k}\cdot\mathbf{b}_1} + e^{i\,\mathbf{k}\cdot\mathbf{b}_2} + e^{i\,\mathbf{k}\cdot\mathbf{b}_3}$$

These occur at

$$\mathbf{q}_1 = \frac{4\pi}{3a}\,\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \text{ and } \;\mathbf{q}_2 = -\mathbf{q}_1$$

(and all other equivalent points on the corners of the Brillouin zone).



In the continuum (low energy) limit ($a \to 0$),
only electron state near $\mathbf{q}_1$ and $\mathbf{q}_2$ participate in the dynamics.

(1) Near $\mathbf{k} = -\mathbf{q}_1$,

$$\begin{aligned} H &= v_{\mathrm{F}} \sum_{\mathbf{k}} (U^\dagger, V^\dagger) \begin{pmatrix} & ie^{-i\pi/3}(k_x - ik_y) \\ -ie^{i\pi/3}(k_x + ik_y) & \end{pmatrix} \begin{pmatrix} U(\mathbf{k}-\mathbf{q}_1) \\ V(\mathbf{k}-\mathbf{q}_1) \end{pmatrix} \\ &= v_{\mathrm{F}} \sum_{\mathbf{k}} (U^\dagger e^{-i\pi/6}, V^\dagger e^{i\pi/6}) \begin{pmatrix} & i(k_x - ik_y) \\ -i(k_x + ik_y) & \end{pmatrix} \begin{pmatrix} e^{i\pi/6} U(\mathbf{k}-\mathbf{q}_1) \\ e^{-i\pi/6} V(\mathbf{k}-\mathbf{q}_1) \end{pmatrix} \end{aligned}$$

$$v_{\mathrm{F}} = \sqrt{3}\alpha/2$$

Dirac Hamiltonian

$$\begin{pmatrix} & i(k_x - ik_y) \\ -i(k_x + ik_y) & \end{pmatrix} = -k_x\sigma_2 + k_y\sigma_1 = \gamma^0(-k_1\gamma^1 - k_2\gamma^2)$$

with $\gamma^\mu = (\sigma_3, i\sigma_1, i\sigma_2)$, and $k^i = (k_x, k_y)$.

$$\to H_1 = -\gamma^0 \not{k} \text{ for } \psi_1 \sim e^{i\frac{\pi}{6}\sigma_3}(U(\mathbf{k}-\mathbf{q}_1), V(\mathbf{k}-\mathbf{q}_1))^{\mathrm{t}}.$$

Similarly, (2) Near $\mathbf{k} = \mathbf{q}_1$,

$$H = v_{\rm F} \sum_{\mathbf{k}} (U^\dagger, V^\dagger) \begin{pmatrix} & ie^{i\pi/3}(k_x+ik_y) \\ -ie^{-i\pi/3}(k_x-ik_y) & \end{pmatrix} \begin{pmatrix} U(\mathbf{k}+\mathbf{q}_1) \\ V(\mathbf{k}+\mathbf{q}_1) \end{pmatrix}$$

One has

$$\begin{pmatrix} & i(k_x+ik_y) \\ -i(k_x-ik_y) & \end{pmatrix} = -k_x\sigma_2 - k_y\sigma_1 = \sigma_2(-k_x\sigma_2 + k_y\sigma_1)\sigma_2$$
$$= \sigma_2\sigma_3(-k_1\gamma^1 - k_2\gamma^2)\sigma_2$$

Let $\psi_2 = \sigma_2 e^{-i\frac{\pi}{6}\sigma_3}(U(\mathbf{k}+\mathbf{q}_1), V(\mathbf{k}+\mathbf{q}_1))^{\rm t}$..

$\rightarrow H_2 = -\gamma^0 \not{k}$

$$H = \int d^2\mathbf{x} \Big[\psi^\dagger \big(v_\mathrm{F}\, \sigma_k \Pi_k + m\sigma_3 - eA_0 \big) \psi$$
$$+ \chi^\dagger \big(v_\mathrm{F}\, \sigma_k \Pi_k - m\sigma_3 - eA_0 \big) \chi \Big]$$

$\Pi_i = -i\partial_i + eA_i$ band gap $m \to 0$

$\psi_\alpha = (\psi_1, \psi_2)^\mathrm{t} \propto (U_{\mathbf{k}-\mathbf{q}}, V_{\mathbf{k}-\mathbf{q}})^\mathrm{t}$ field near K

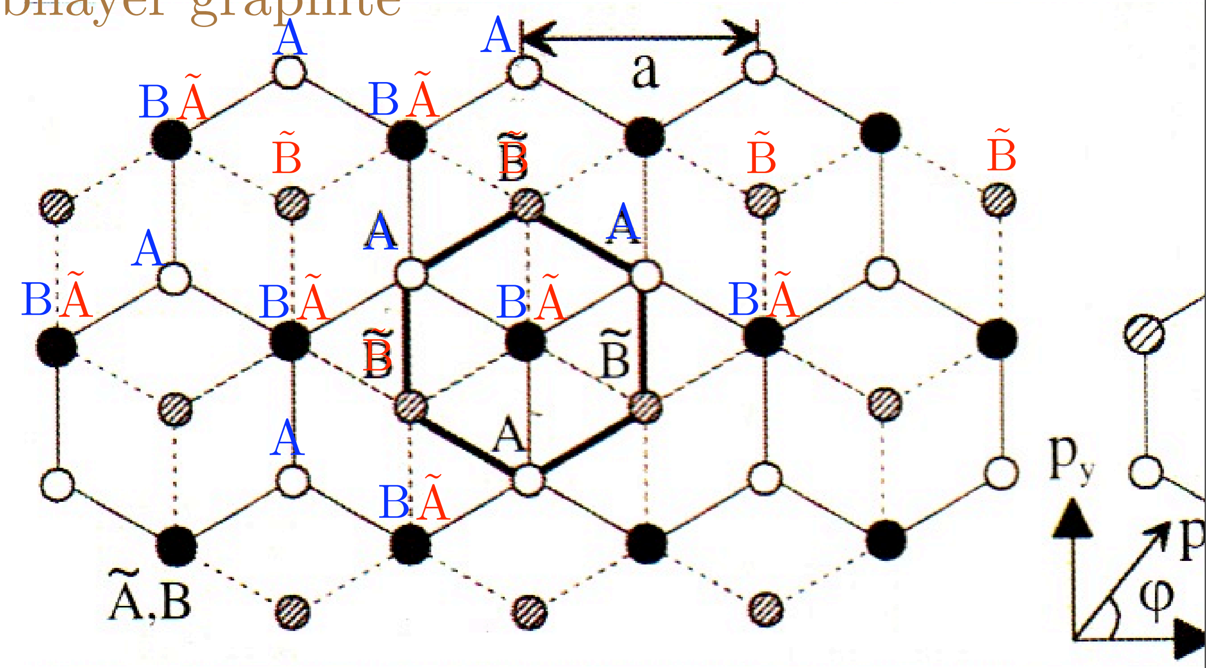
$\chi_\alpha = (\chi_1, \chi_2)^\mathrm{t} \propto (-V_{\mathbf{k}+\mathbf{q}}, U_{\mathbf{k}+\mathbf{q}})^\mathrm{t}$ field near K'

$v_\mathrm{F} \sim 10^6 \mathrm{m/s} \sim c/300$

Spinor structure <—— sublattices

Bilayer graphene

bilayer graphite



$$H^{\rm bi} = \begin{pmatrix} & v\,p^\dagger & & \\ v\,p & & \gamma_1 & \\ & \gamma_1 & & v\,p^\dagger \\ & & v\,p & \end{pmatrix} \begin{matrix} \psi_A \\ \psi_B \\ \psi_{\tilde A} \\ \psi_{\tilde B} \end{matrix}$$

$$\hat H^{\rm bi} = \begin{pmatrix} & & & v\,p^\dagger \\ & & v\,p & \\ & v\,p^\dagger & & \gamma_1 \\ vp & & \gamma_1 & \end{pmatrix} \begin{matrix} \psi_A \\ \psi_{\tilde B} \\ \psi_{\tilde A} \\ \psi_B \end{matrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

$$H_{\rm eff} = H_{11} + H_{12}\frac{1}{H_{22} - i\partial_t}H_{21} \approx \frac{1}{2m^*}\begin{pmatrix} & (p^\dagger)^2 \\ p^2 & \end{pmatrix} \qquad m^* = \frac{\gamma_1}{v^2}$$

chiral Schroedinger eq.

positive- and negative-energy states w. quadratic dispersion

gapless