Supersymmetric Solutions
in
Supergravity

Lunchtime Talk on 09 Jan. 2008

Supersymmetric Solutions

~classical solutions of supergravity perserving some of supersymmetries

They are useful to study

- string compactification (with fluxes)
- exotic solutions like black hole, black ring …
- Holographic dual of supersymmetric gauge theories

etc

The conditions preserving supersymmetry

$$\begin{aligned} \delta(\text{fermions}) &= \langle 0|\{Q, \text{fermions}\}|0\rangle \\ &= 0 \end{aligned}$$

$[\because \quad \delta(\text{bosons}) \equiv 0 \quad \text{if } \langle 0|\text{fermion}|0\rangle = 0 \ ]$

For Supergravity,

$$\boxed{\begin{aligned} \hat{\nabla}\epsilon &= 0 \\ M\epsilon &= 0 \end{aligned}} \qquad \begin{aligned} &(\text{KSE}) \\ &(*) \end{aligned}$$

where, $\hat{\nabla} = \nabla + (\text{fluxes})\cdot\gamma$ and $M = (\text{fluxes})\cdot\gamma$.

Key Point

The integrability condition of KSE includes

$$
\begin{aligned}
0 &= \gamma^{\nu}[\hat{\nabla}_{\mu}, \hat{\nabla}_{\nu}]\epsilon \\
&= (R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - T_{\mu\nu})\gamma^{\nu}\epsilon + \cdots
\end{aligned}
$$

where ・・・ vanish if we impose

- KSE's (including (*))
- EOM of fluxes

i.e. $[\mathrm{For}\ K^{\mu}K_{\mu} \neq 0\ (K_{\mu} = \bar{\epsilon}\gamma_{\mu}\epsilon)]$

KSE's (*1-st order and linear*) + EOM's of fluxes

~ Einstein's Eq. (*2-nd order and nonlinear*)

G(roup)-structure -characterization of geometry with fluxes-

e.g. For 6-dimensional compact space X_6, SU(3)-structure

If there is a Killing spinor (solution of KSE's) $\eta_{\pm}\ (\gamma_7 \eta_{\pm} = \pm\eta)$,

we can construct

$$J_{mn} = -i(\eta_+^\dagger \gamma_{mn} \eta_+), \quad \text{almost complex structure or (1,1)-form}$$

$$\Omega_{mnp} = (\eta_-^\dagger \gamma_{mnp} \eta_+), \quad \text{(3,0)-form}$$

which satisfy, due to Fierz identity,

$$J_i{}^j J_j{}^k = -\delta_i^k,$$

$$(\Pi^+)_m{}^n \Omega_{npq} = \Omega_{mpq}, \quad (\Pi^-)_m{}^n \Omega_{npq} = 0,$$

$$\Omega \wedge J = 0, \quad \Omega \wedge \bar{\Omega} = \frac{4i}{3} J \wedge J \wedge J,$$

where $(\Pi^{\pm})_m{}^n = \frac{1}{2}(\delta_m^n \mp i J_m{}^n)$

(J, Ω) completely specify SU(3)-structure on X_6.

All the tensors can be decomposed into irreps. of SU(3).

Then geometry of X_6 is characterized by the intrinsic torsion class, which parametrize the failure of the manifold to be of Calabi-Yau:

$$
\begin{aligned}
dJ &= -\frac{3}{2} Im(\mathcal{W}_1 \bar{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3, \\
d\Omega &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \bar{\mathcal{W}}_5 \wedge \Omega.
\end{aligned}
$$

e.g.

complex	$\mathcal{W}_1 = \mathcal{W}_2 = 0$	hermitian
	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$	balanced
	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	special hermitian
	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Kähler
	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Calabi-Yau
non-complex	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	nearly-Kähler
	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	almost-Kähler
	$Re\mathcal{W}_1 = Re\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	half-flat