Nucleon sigma term from lattice QCD

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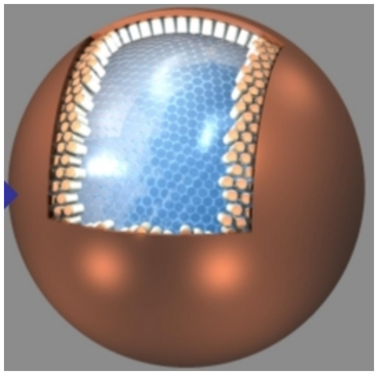
Determine the nucleon sigma term
from lattice QCD with dynamical quark
having an exact chiral symmetry on the lattice.

WMAP found existence of Cold Dark Matter

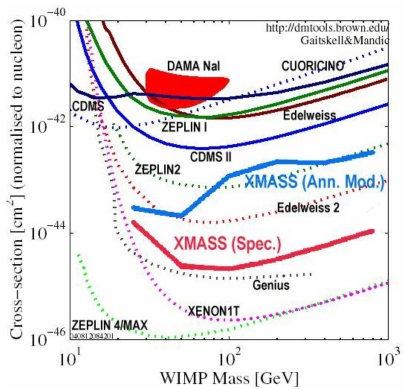
Candidates: neutralino, gravitino, axion, ….

Very promising candiditate in minimal supersymmetric standard Model (MSSM)

XMASS detector



Detection Feasibility



Direct detection of the dark matter is important for finding the property and constituents of DM.

- XMASS (Xenon detector at Kamioka)
- Super CDM

Can cover large part of the MSSM parameter space

What is the theoretical prediction of the reaction rate?

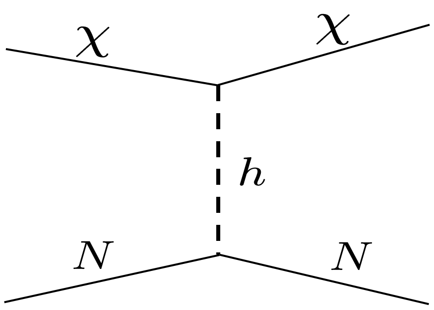
[dark matter - quark] interaction can be predicted if you specify the new physics model from LHC and ILC.
Even so, [dark matter – nucleon] interaction rate is uncertain (100% error).

We need lattice QCD calculation to convert quark interaction to nucleon interaction.

Why sigma term is important?

A crucial parameter for the neutralino dark matter detection rate.

The interaction with nucleon is mediated by the higgs boson exchange in the t-channel.



Sigma term convert the quark yukawa coupling to nucleon yukawa coupling.

$$\sigma_{\pi N} \equiv m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

related quantity

$$y \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$$

K. Griest, Phys.Rev.Lett.62,666(1988) , Phys,Rev,D38, 2375(1988)
Baltz, Battaglia, Peskin, Wizanksy,Phys. Rev. D74, 103521 (2006).

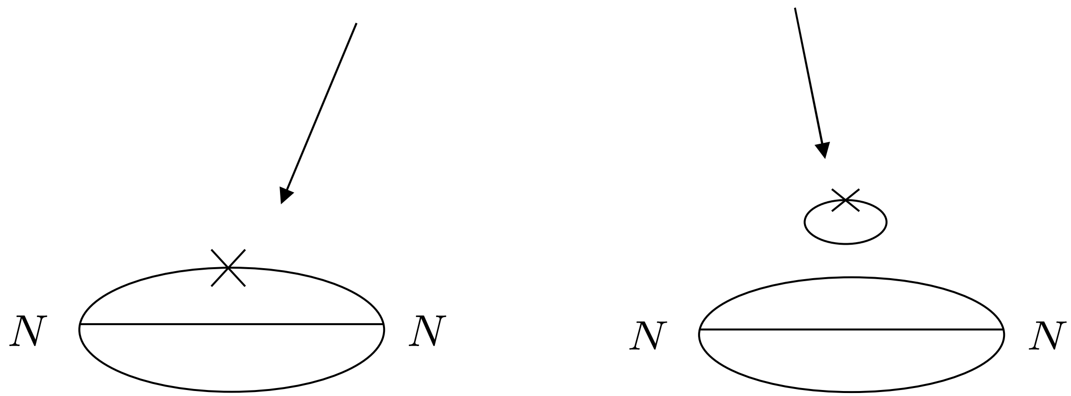
In general, all sigma terms (u, d, s, c, b, t quarks) are equally important.

Basic Methods

Feynman - Hellman theorem

$$\frac{dm_N}{dm_q} = \langle N|\bar{q}q|N\rangle$$

Moreover, partial derivatives with respect to the valence and sea quark masses give contributions from ‘connected’ and ‘disconnected’ diagrams.



$$\frac{\partial m_N(m_{\text{val}}, m_{\text{sea}})}{\partial m_{\text{val}}} = \langle N|\bar{q}q|N\rangle_{\text{conn.}} \qquad \frac{\partial m_N(m_{\text{val}}, m_{\text{sea}})}{\partial m_{\text{sea}}} = \langle N|\bar{q}q|N\rangle_{\text{disc.}}$$

Theoretical expectation

Down-type Higgs Yukawa coupling

$$y_{NNH_d} \propto \langle N| \underbrace{m_d\bar{d}d}_{20-30\text{MeV}} + \underbrace{m_s\bar{s}s}_{70-360\text{MeV}} + \underbrace{m_b\bar{b}b}_{\sim 60\text{MeV}} |N\rangle \times \tan(\beta)$$

Chiral perturbation theory (ChPT) + πN scattering exp. Data (connected+ disconnected)

Quark model, Lattice (disconnected)

Trace anomaly formula (disconnected)

$$\begin{aligned} m_N &= \langle N|T^\mu_\mu|N>, \\ T^{mu}_\mu &= \frac{\beta^{n_f}(\alpha)}{4\alpha}G^2_{\mu\nu} + \sum_{i=1}^{n_f} m_{q_i}\bar{q}_i q_i, \\ \lim_{m_Q\to\infty} &= \langle N|m_Q\bar{Q}Q|N\rangle_{\text{disc.}} = \frac{\beta(\alpha)^{n_f=3+1} - \beta(\alpha)^{n_f=3}}{\beta(\alpha)^{n_f=3}} m_N \end{aligned}$$

Strange quark contribution can be dominant but with large uncertainty. (Huge enhancement in the strange quark mass region?)

Naïve question: Is disconnected contribution really so big?

Lattice QCD project by JLQCD Collaboration

KEK S. Hashimoto, T. Kaneko, H. Matsufuru,
J. Noaki, E. Shintani, N. Yamada
RIKEN/Niels Bohr H. Fukaya
Tsukuba S. Aoki, T. Kanaya, N. Ishizuka,
Y. Taniguchi, A. Ukawa, T. Yoshie
Hiroshima K.-I. Ishikawa, M. Okawa
YITP H. Ohki, T. Onogi

KEK BlueGene (10 racks, 57.3 TFlops)

- Ginsparg-Wilson relation

  Ginsparg and Wilson, Phys.Rev.D 25(1982) 2649.

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

Exact chiral symmetry on the lattice (index theorem)

Hasenfratz, Laliena and Niedermayer, Phys.Lett. B427(1998) 125
Luscher, Phys.Lett.B428(1998)342.

$$\psi \to \psi + i\gamma_5(1 - aD)\psi = \psi + i\hat{\gamma}_5\psi$$
$$\bar{\psi} \to \bar{\psi} + i\bar{\psi}\gamma_5$$

- Overlap fermion (explicit construction by Neuberger)

$$D = \frac{1}{a}[1 + \gamma_5 sign(H_W)], \quad H_W \equiv \gamma_5(D_W + M_0)$$

D_W : Wilson Dirac op., M_0 : negative mass

We succeeded in the first large scale Lattice QCD simulation with dynamical overlap fermion simulation using new theoretical method.

Numerical simulation

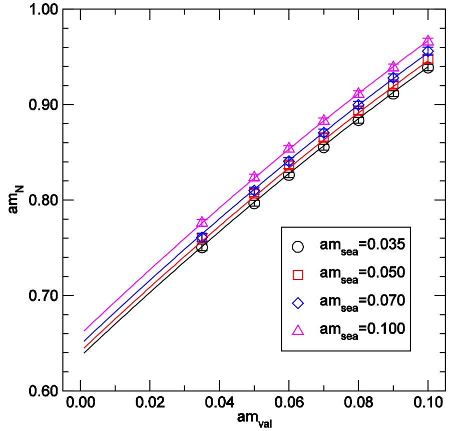
Dynamical simulation with Nf=2 overlap fermion

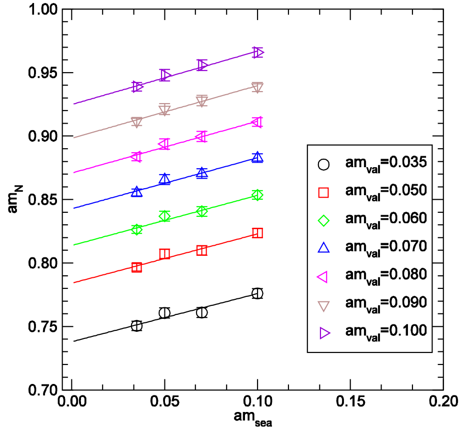
- 16^3 x 32, a=0.12 fm, L=1.9 fm
- 6 values of sea quark mass in the range of ms/6-ms
- 9 values of valence quark mass in the range of ms/6-ms
- fixed topology
- At Q=0 accumulated 10,000 trajectories

A good test for real calculation with Nf=2+1,which will finish within 1-2 months.

Sea and valence quark mass dependences

The valence quark mass dependence is clear,
while the sea quark mass dependence is small.



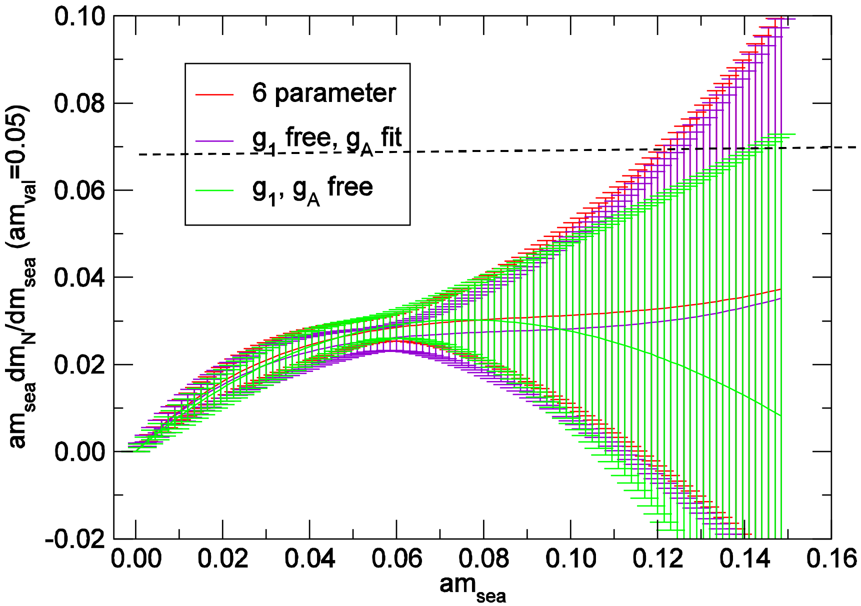


Relation between the sigma term (disconnected) and the heavy quark limit result from trace anomaly

We fix $am_{\text{val}} = 0.05$ and vary $am_{\text{sea}} = 0 - 0.15$

Heavy quark limit from trace anomaly

$\langle N|m_q\bar{q}q|N\rangle_{\text{disc}}$



Strange quark mass

There is no enhancement in the strange quark mass region at least for $am_{\text{val}} = 0.05$

Our results from the fit of the quark mass dependence

1. Fit of diagonal (unitary) points with Chiral Perturbation Theory (ChPT)

sea and valence quark masses are set equal

$$m_N = B_0 + B_1 m_q + B_2 m_q^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3$$

g_A: axial coupling of nucleon

Phenomenological value: $g_A = 1.267$

$$\sigma_{\pi N} = 48 \pm \underbrace{2}_{\text{stat.}} \pm \underbrace{{}^{+6}_{-15}}_{\text{sys.}} \text{ MeV}$$

2. Global fit using partially quenched chiral perturbation (PQChPT)

sea and valence quark masses are treated separately

$$\begin{aligned} m_N = \; & B_{00} + B_{10} m_{val} + B_{01} m_{sea} + B_{11} m_{sea} m_{val} + B_{20} m_{val}^2 + B_{02} m_{sea}^2 \\ & - \frac{1}{16\pi f_\pi^2} \{ \frac{g_A^2}{12} \left[-7(m_\pi^{vv})^3 + 16(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2) \right] \\ & + \frac{g_1^2}{12} \left[-19(m_\pi^{vv})^3 + 10(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2) \right] \\ & + \frac{g_1 g_A}{3} \left[-13(m_\pi^{vv})^3 + 4(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2) \right] \} \end{aligned}$$

g_A, g_1: axial couplings of nucleon

Phenomenological values:

$g_A = 1.267$, $g_1 = -(0.4 - 0.6)$

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = 0.05(2)_{\text{stat.}}({}^{+14}_{-0})_{\text{extrap.}}$$

Previous results

Theory	Group		method	$\sigma_{\pi N}$ [MeV]	y
ChPT	Gasser et al.(91)	1-loop	spectrum	44(9)	~ 0.2
	Borasoy-Meissner(96)	1-loop	spectrum	48(10)	0.2(2)
	Borasoy-Meissner(97)	2-loop	spectrum	36(7)	0.21(20)
Lattice	Kuramashi et al(95)	$n_f = 0$ Wilson	3pt/2pt	40-60	0.66(15)
	Dong et al(96)	$n_f = 0$ Wilson	3pt/2pt	50(3)	0.36(3)
	SEASAM(99)	$n_f = 2$ Wilson	3pt/2pt	18(5)	0.59(13)
	UKQCD(02)	$n_f = 2$ Clover	spectrum(unsubtracted)		0.53(12)
			spectrum(subtracted)		-0.30(34)
ChPT+Lattice	Procura et al.(04)	NNLO, $n_f = 2$ Clover	spectrum	49(3)	

$\sigma_{\pi N}$=30-50 MeV from ChPT

y=0-0.2 from ChPT, y=0-0.5 from lattice QCD

- y from ChPT : Large uncertainty from Low Energy Constants.
- y from Lattice: In previous lattice caculation disconnected contribution is large. Why? mixing with wrong chirality?

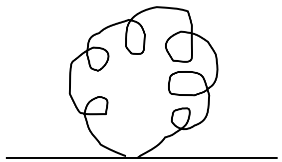
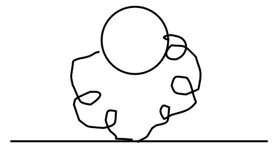
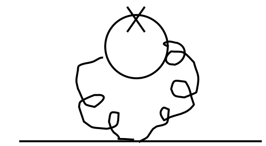
Operator mixing due to Wilson fermion artifact

If there is an additive mass shift there can be operator mixing which should be subtracted (but not subtracted except for UKQCD) for disconnected diagram.

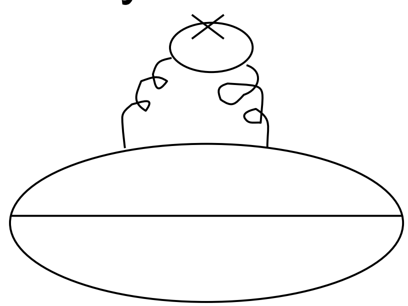
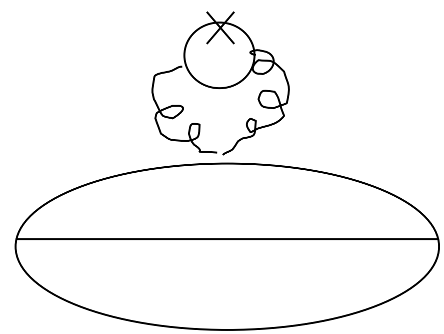
$$\left(\bar{\psi}_{\mathrm{sea}}\psi_{\mathrm{sea}}\right)^{\mathrm{lat}}$$
$$= \; C_0 I + Z_S \left[\left(\bar{\psi}_{\mathrm{sea}}\psi_{\mathrm{sea}}\right)^{\bar{M}S} + \frac{\partial \Delta m_q}{\partial m_{\mathrm{sea}}}\left(\bar{\psi}_{\mathrm{val}}\psi_{\mathrm{val}}\right)^{\bar{M}S}\right] + O(a)$$

Subtracting this mixing effect by using the sea quark mass dependence of the quark mass shift, the disconnected contribution becomes tiny (consistent with zero).

Additive mass shift and sigma term

$\Delta m_q =$  $+$  $\Longrightarrow \dfrac{d\Delta m_q}{dm_{\text{sea}}} =$ 

Wilson fermion has an additive mass shift through tadpole diagram which mimics mass operator insertion to the valence quark due to the lack of chiral symmetry

$\langle N|\,(\bar{q}q)^{\text{lat}}\,|N\rangle_{\text{disc.}} =$  $+$ 

Normal disconnected contribution

(long distance effect)

Operator mixing induced connected contribution

(short distance effect)

Summary

- Previous lattice calculation overlooked the lattice artifact.
- Our nf=2 study suggests that there is no enhancement of the sigma term in the strange quark mass region. We should wait for the nf=2+1 result for final conclusion.
- Bad news:

  We found higgs-nucleon yukawa coupling can be smaller by factor about 2 compared to the present lower bound of the model/lattice prediction.

  Smaller detection rate…..
- Good news:

  Hadronic uncertainty can be reduced drastically.

  Please wait for our nf=2+1 QCD result !