Recent Developments on N=2 supersymmetric gauge theory

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<u>4d N=2 supersymmetric gauge theory</u>

is very interesting framework where various interesting results have been found: e.g.

- exact effective action related with the Seiberg-Witten curve
- instanton counting (Nekrasov partition function)
- relation with integrable systems

Restriction on low energy effective theory

The low energy effective theory of N=2 supersymmetric gauge theory is highly restricted.

The effective action can be written in terms of the only one function, *the prepotential*, as

$$\mathcal{L}_{eff} = \mathrm{Im}\Big[\int d^2\theta \frac{1}{2}\frac{\partial^2\mathcal{F}(a)}{\partial a^i \partial a^j}\mathcal{W}^{\alpha i}\mathcal{W}^j_\alpha + \int d^2\theta d^2\bar{\theta}\,\mathrm{Tr}\bar{a}e^{adV}\frac{\partial\mathcal{F}(a)}{\partial a}\Big]$$

Exact solution [Seiberg-Witten ’94]

It has been found that the exact low energy prepotential is determined from

- *Seiberg-Witten curve*: $y^2 = P_N^2(z) - \Lambda^{2N}$
- *meromorphic one form* on the curve: λ_{SW}

Using these, the low energy prepotential is

$$a_i = \oint_{A_i} \lambda_{\mathrm{SW}}, \quad \frac{\partial \mathcal{F}}{\partial a_i} = \oint_{B_i} \lambda_{\mathrm{SW}}$$

M5-branes wrapped on Riemann surface

N=2, SU(n) quiver SCFTs can be induced on worldvolume of n M5-branes wrapped on Riemann surfaces $\Sigma_{g,n}$: [Witten '97, Gaiotto '09]

genus $\longrightarrow$ "genus" of quiver diagram

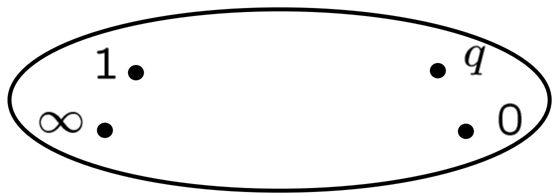
punctures $\longrightarrow$ flavor symmetries

complex structures $\longrightarrow$ gauge coupling constants

$$q_i = e^{\pi i \tau_i}$$

SU(2) gauge theory with 4 flavors

A simple example: 2 M5-branes on $\Sigma_{0,4}$, *a sphere with 4 punctures induce SU(2) gauge theory with 4 flavors*:



In this case, the curve obtained from M-theory is

$$x^2 = \left(\frac{m_0}{t} + \frac{m_1}{t-1} + \frac{m_2}{t-q}\right)^2 + \frac{1}{t(t-1)(t-q)}\left((m_4^2 - (\sum m_i)^2)t + qU\right)$$

Curve and one form

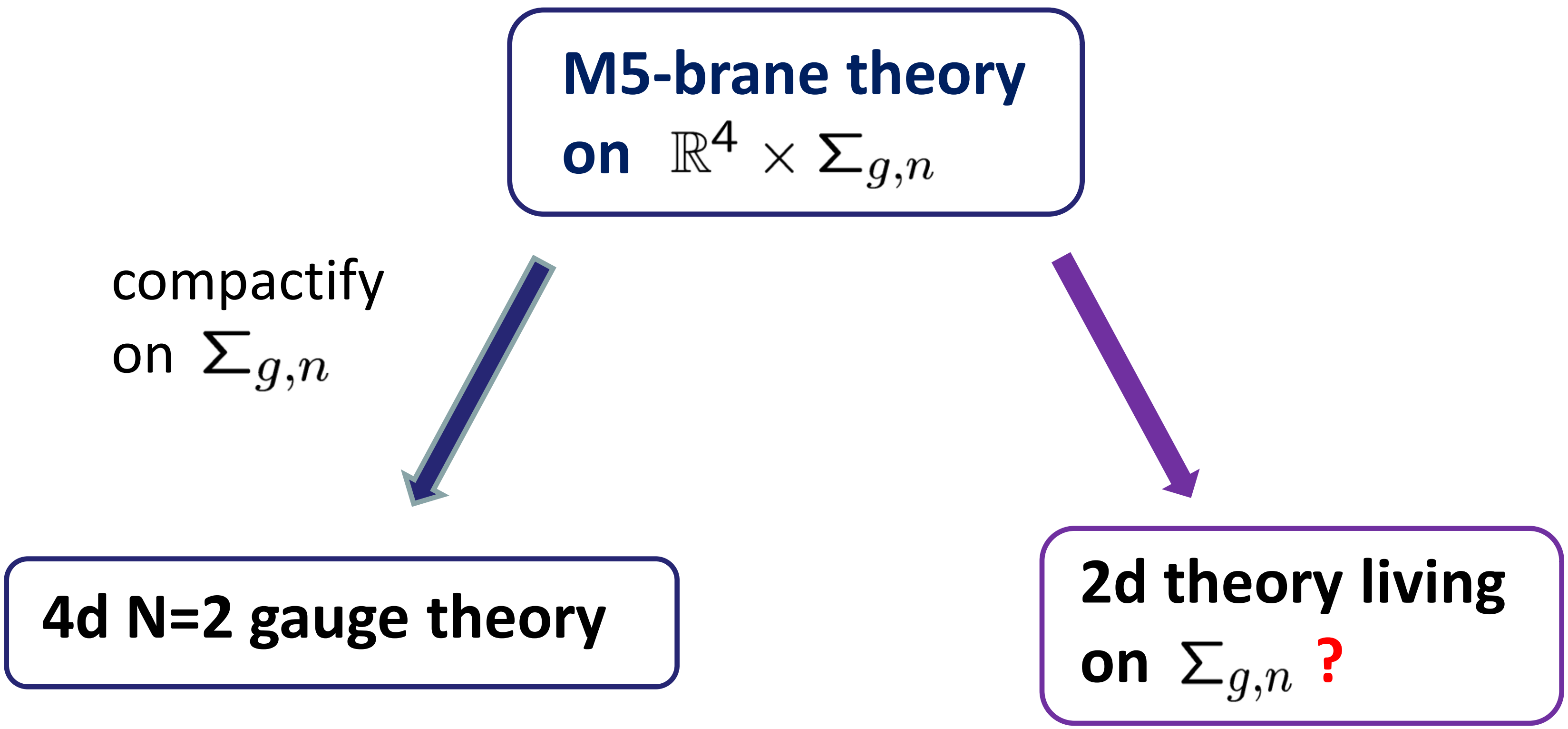
- *Seiberg-Witten curve* is double cover of the Riemann surface $\Sigma_{g,n}$ on which M5-branes wrap:

$$x^2 = \phi(t), \quad t; \text{coordinate on Riemann surface}$$

- *meromorphic one form* is given by $\lambda_{\mathrm{SW}} = x dt$

Relation with the Liouville theory

[Alday-Gaiotto-Tachikawa]



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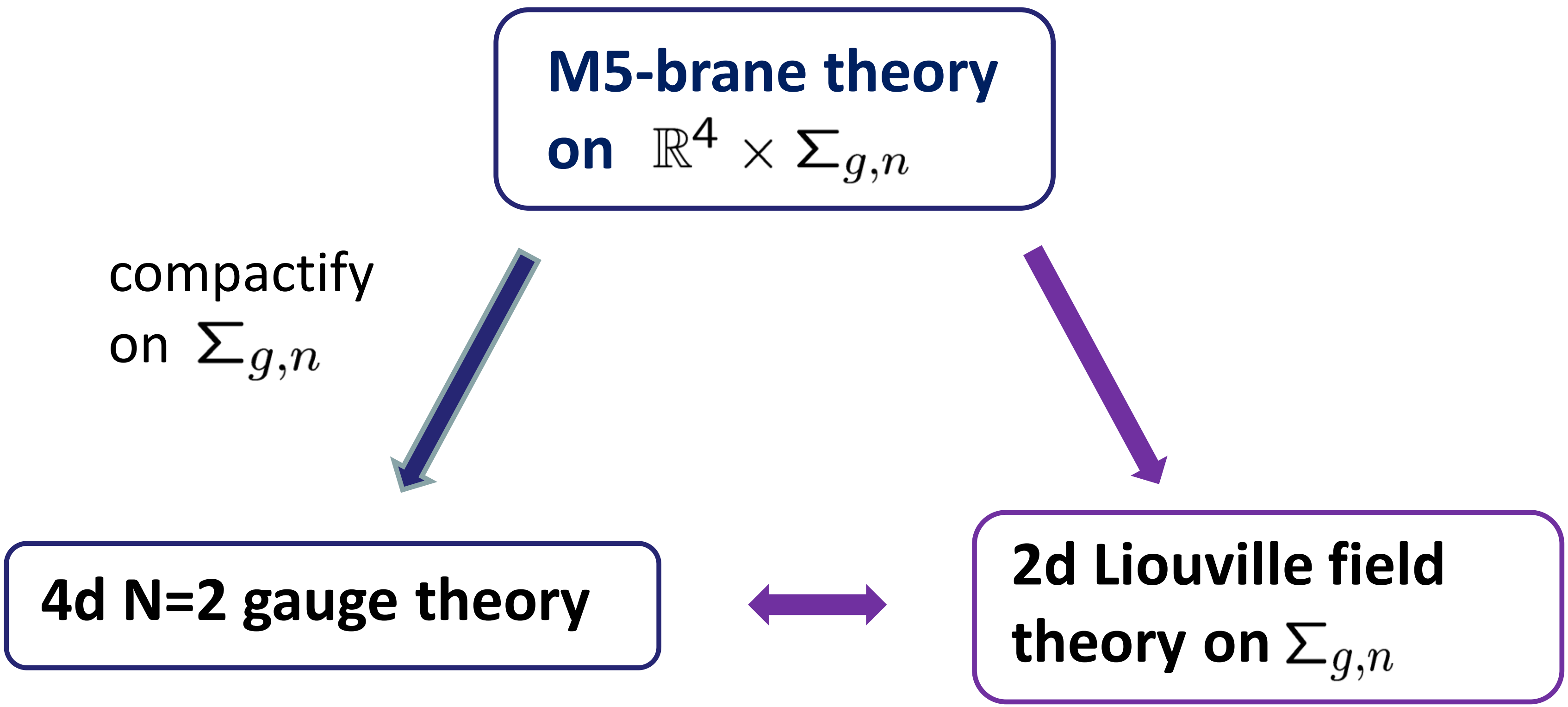


Four-point conformal block

The four-point correlation function of Liouville theory (central charge $c = 1 + 6(b + 1/b)^2$) is the following form

$$\langle V_{\alpha_1}(\infty) V_{\alpha_2}(1) V_{\alpha_3}(q) V_{\alpha_4}(0)\rangle^{\text{full}} = \int \frac{d\alpha^{int}}{2\pi} C(\alpha_1^*, \alpha_2, \alpha^{int}) C(\alpha^{int*}, \alpha_3, \alpha_4) \times |\langle V_{\alpha_1}(\infty) V_{\alpha_2}(1) V_{\alpha_3}(q) V_{\alpha_4}(0)\rangle|^2$$

The conformal block which we will consider behaves as

$$\langle V_{\alpha_1}(\infty) V_{\alpha_2}(1) V_{\alpha_3}(q) V_{\alpha_4}(0)\rangle = q^{\Delta_\alpha^{int} - \Delta_{\alpha_3} - \Delta_{\alpha_4}} \mathcal{B}(q)$$

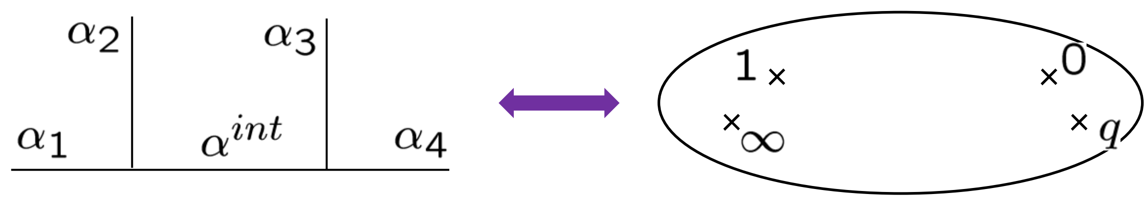
where *B* is expanded as $\mathcal{B}(q) = 1 + \mathcal{O}(q)$.

AGT relation (SU(2) theory with 4 flavors)

A simple example of the AGT relation is as follows:

the conformal block which we have seen above can be identified with the Nekrasov partition function of SU(2) theory with 4 flavors

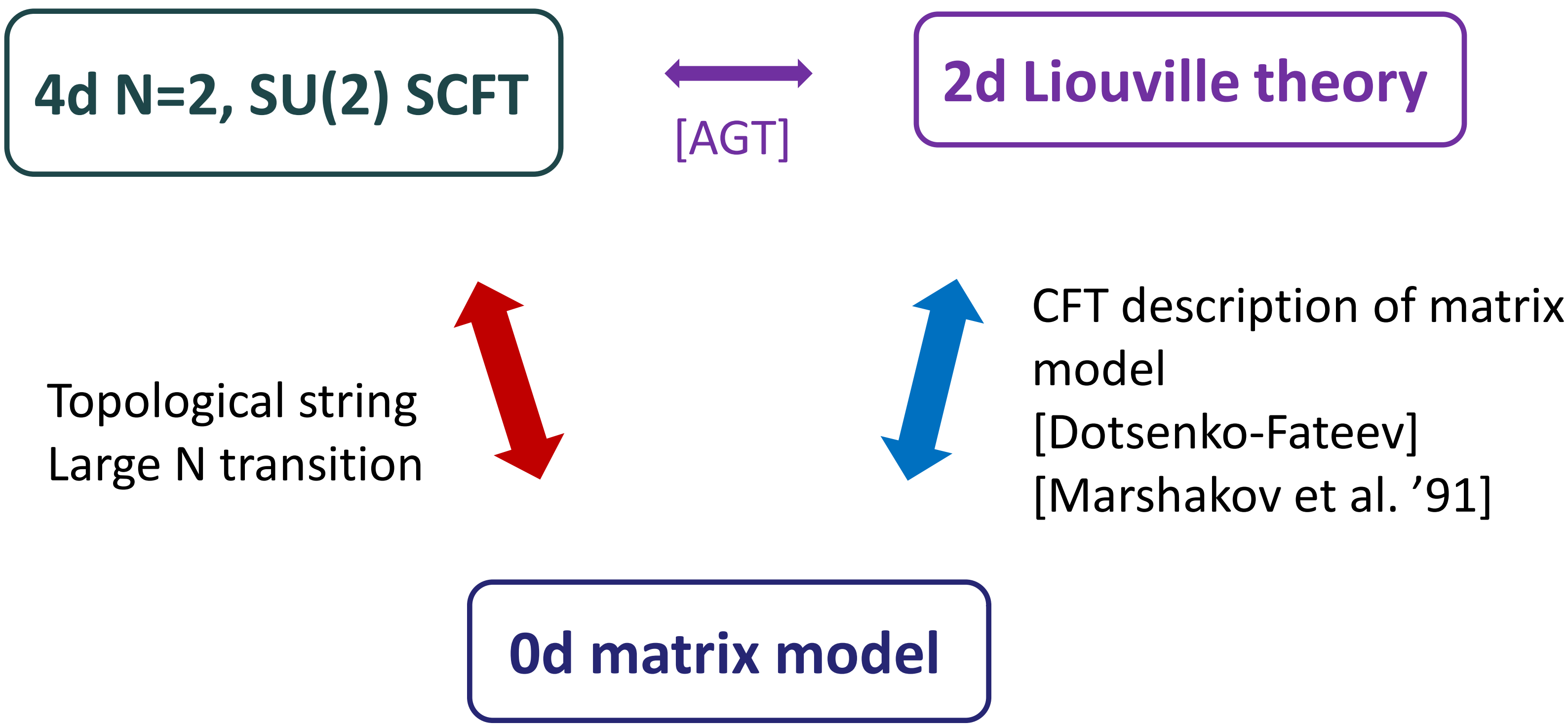
$$\mathcal{B}(\alpha^{int}, \alpha_i, q; b) = Z_{\text{inst}}^{N_f=4}(a, m_i, q; \epsilon_1, \epsilon_2)$$



$$b = \frac{\epsilon_1}{\sqrt{\epsilon_1 \epsilon_2}}, \quad b^{-1} = \frac{\epsilon_2}{\sqrt{\epsilon_1 \epsilon_2}},$$

Relation with matrix model

[Dijkgraaf-Vafa '09]



Penner type matrix model

The Dotsenko-Fateev integral representation of conformal block leads to the following matrix model:

$$Z_{matrix} = \left(\prod_{I=1}^{N} \int d\lambda_I \right) \prod_{I<J} (\lambda_I - \lambda_J)^2 e^{\frac{1}{g_s} \sum_I W(\lambda_I)}$$

where the potential is of "Penner type"

$$W(\lambda) = m_0 \log \lambda + m_1 \log(\lambda - 1) + m_2 \log(\lambda - q)$$

Large N limit

[Eguchi-Maruyoshi '09; '10]

Large N limit: $N \to \infty$
with keeping g_sN fixed

in the matrix model

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Gauge theory limit:
$\epsilon_1, \epsilon_2 \to 0$

in the Nekrasov partition function

Therefore, *we expect to obtain the gauge theory result by the large N limit of the matrix model.*

Conclusion

We have seen recent development on N=2 supersymmetric gauge theory.

We have reviewed Gaiotto's construction of N=2 supersymmetric gauge theory from M-theory.

We have seen the AGT relation and the matrix model which correctly reproduces the gauge theory results.

Thank you very much for your attention!