Constraining Models of Inflation

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Inflation

- is the near exponential expansion of the universe at early times.
- is where the scale factor increases $\ddot{a} > 0$
- is when the comoving hubble horizon decreases

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0$$

Inflation Accounts for

- Universal:
  - ☑ homogeneity.
  - ☑ isotropy.
  - ☑ causality.
  - ☑ flatness.
  - ☑ expansion.
- ☑ The origin of structure.

Inflation Requires

- Negative pressure: $P < -\frac{\rho}{3}$
- Scalar field with:
  - Potential energy > Kinetic energy.
    - very flat potential a.k.a. canonical.
    - suppressed kinetic term a.k.a. non-canonical models

The basic picture

Properties of a scalar field

The energy density is given by

derivative w.r.t. time

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

Inflaton field

Potential

The pressure is given by

$$P = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

The equation of motion is

$$\ddot{\varphi} + 3H\dot{\varphi} + V(\varphi)_{,\varphi} = 0$$

Terms in blue are dropped in slow roll approximation.

Defining the problem

Many models (i.e. potentials) solve the main paradoxes inflation set out to solve .

So which one is it?

Overview

- A brief history of perturbations.
- Observational parameters
  - spectral index
  - running of the spectral index
  - gravitational waves
- The Wilkinson Microwave Anisotropy Probe.
- Results.
- Current and Further Work.

A brief history of perturbations

background quantity

perturbation

$$\varphi = \varphi_0 + \delta\varphi$$

Imprints on the background metric , generating what we call the curvature perturbation.

$$\zeta(\delta\phi)$$

curvature perturbation

Evolution determined by the model/potential of inflation originally chosen.

Spectrum and Spectral Index

The spectrum is defined as:

$$< \zeta(\mathbf{k})\zeta^*(\mathbf{k}') >= \delta^{(3)}(\mathbf{k}-\mathbf{k}')\frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)$$

scale on which perturbation is defined

The spectral index defines the scale dependance of the spectrum

$$n_s - 1 = \frac{d\ln \mathcal{P}_\zeta(k)}{d\ln k}$$

and the running of the spectral index is defined as its scale dependance

$$n_s' = \frac{dn_s}{d\ln k}$$

Tensor to Scalar Ratio

The spectrum of gravitational waves is found to be

$$\mathcal{P}_{grav} = \frac{8}{M_{pl}^2}\left(\frac{H}{2\pi}\right)^2_{k=aH}$$

The tensor fraction is defined as

$$r = \frac{\mathcal{P}_{grav}}{\mathcal{P}_\zeta}$$

Primordial Black Holes (PBHs)

The spectrum is well measured for θ>0.3, but not for θ<0.3

Naively: spectrum at θ<0.3 large enough to produce PBH

Astrophysical bounds

$$\mathcal{P}_{\zeta_{end}} \lesssim 10^{-2}$$

To produce PBHs

$$\mathcal{P}_{\zeta_{end}} \gtrsim 10^{-3}$$

e.g. Carr et al 1994, 2009, Kohri & Yokoyama 1999, Josan et al 2009

e-folds

This is a measure of how long inflation lasted. And is parametrised by the logarithmic ratio of the scale factor at the end of inflation to its value when scales of cosmological interest left the horizon.

$$N = \ln\left(\frac{a_{end}}{a_*}\right)$$

Small vs. Large Field Models

$$\Delta\varphi = \varphi_{end} - \varphi_*$$

Gravitational waves detectable signature --> 4 e-folds after 'start' of inflation.

$\Delta\varphi \sim 0.5 M_{pl}$ and $r \sim 0.1$

a tensor fraction of about 0.1 will be detectable by future experiments.

For this talk, large field models imply a field variation of the order of a planck mass.

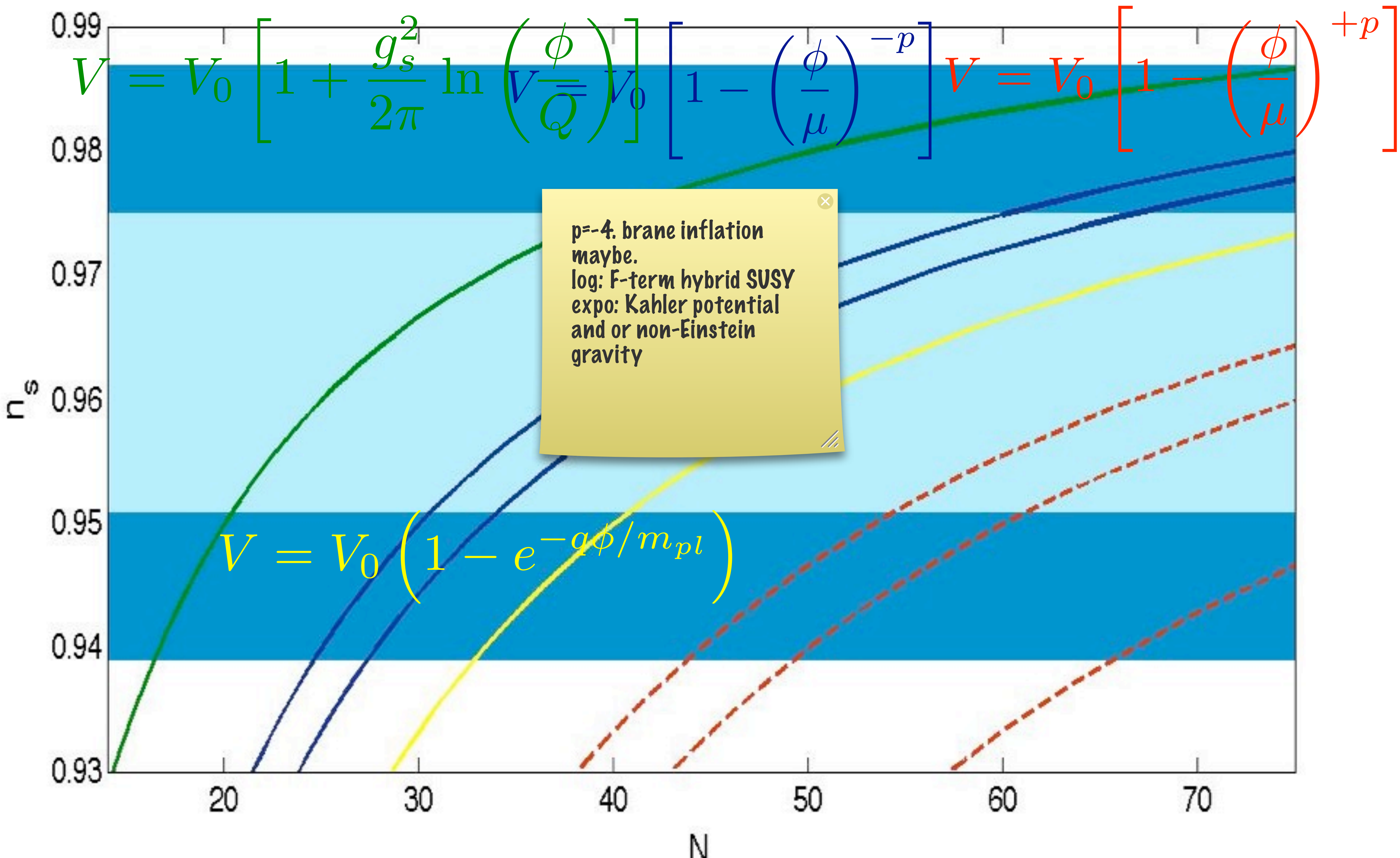
Wilkinson Microwave Anisotropy Probe

All Sky Survey of temperature anisotropies in the Cosmic Microwave Background

↓

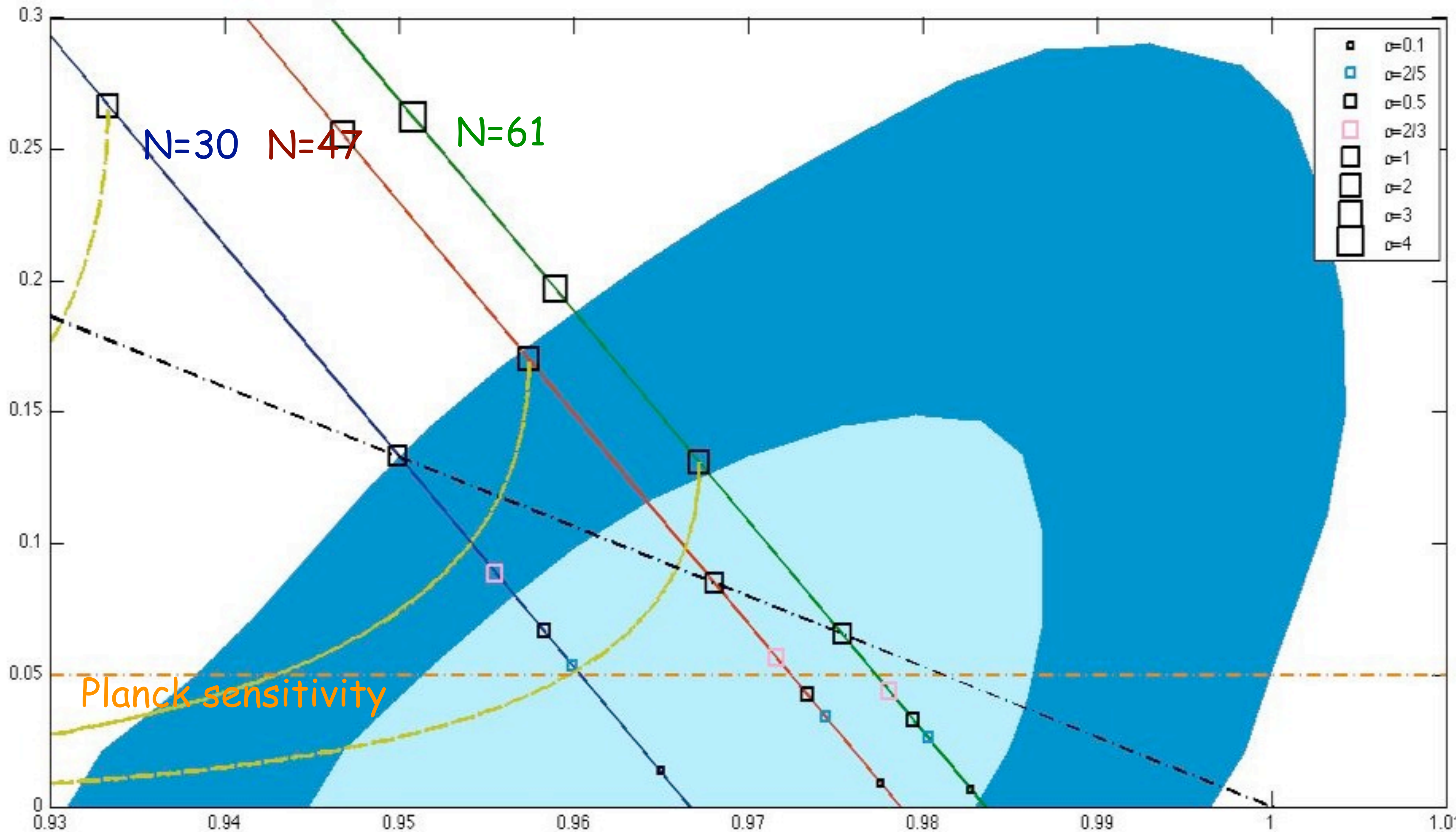
Measures the primordial density distribution.

Spectral index results


$V = V_0 \left[1 + \frac{g_s^2}{2\pi} \ln \left(\frac{\phi}{Q} \right) \right]$
$V = V_0 \left[1 - \left(\frac{\phi}{\mu} \right)^{-p} \right]$
$V = V_0 \left[1 - \left(\frac{\phi}{\mu} \right)^{+p} \right]$
$V = V_0 \left(1 - e^{-q\phi/m_{pl}} \right)$
p=-4. brane inflation maybe.
log: F-term hybrid SUSY
expo: Kahler potential and or non-Einstein gravity
n_s
0.99
0.98
0.97
0.96
0.95
0.94
0.93
20
30
40
50
60
70
N

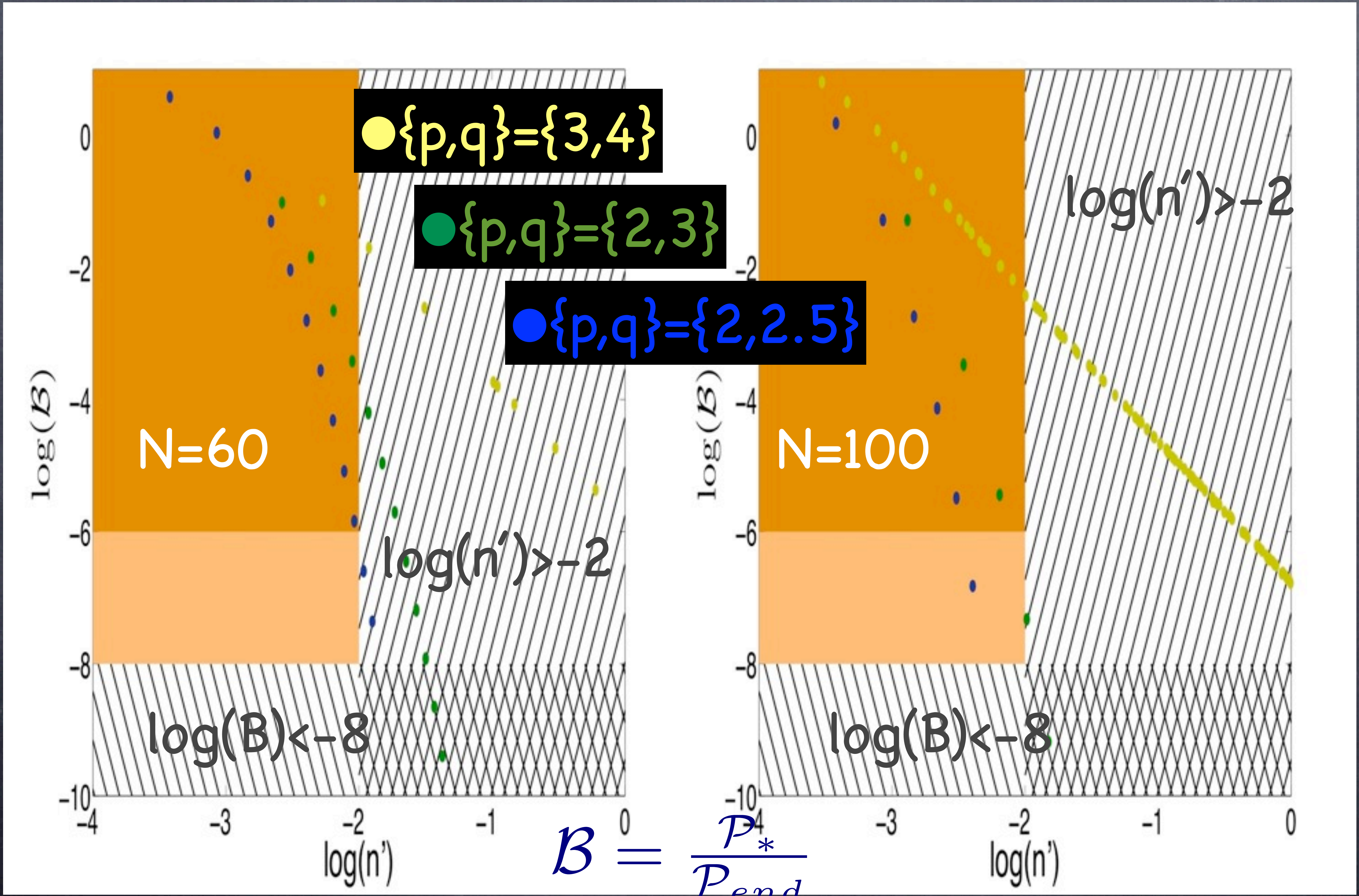
Tensor fraction results

$$V \propto \phi^{\alpha}$$



PBH results

$$V = V_0 \left(1 + \eta_p \phi^p - \eta_q \phi^q\right)$$



$$\mathcal{B} = \frac{\mathcal{P}_*}{\mathcal{P}_{end}}$$

Conclusions

Degeneracy between models.

Some can be alleviated by future data sets, but not all.

Look for further signatures and techniques for discrimination.

Going further and asking about the naturalness of models...

Further work

Second order gravitational waves

Tensor to scalar ratio sensitivity: $r \sim 10^{-3}$ at best.

Many models predict r much less than this.

Second source of gravitational waves from Inflation. Sourced by the interaction of first order scalar perturbations.

Mollerach et. al (2004): this second source may overtake the linear order source if $r < 10^{-6}$

PBH bounds give bounds on the spectrum of these second order gravitational waves (e.g. Saito and Yokoyama, Bugaev and Klimai etc.).

Hilltop inflation leads to a larger spectrum at the end of inflation. Investigating the resulting spectrum of second order gravitational waves.

Non-gaussianity

Observation: spectrum Gaussian.

There is evidence for some small deviation.

Work has shown that in canonical models, large deviations require some fine tuning.

Investigating the physical meaning of the fine tuning required.

Thank you!