Cosmic censorship in a charged black hole

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Works with **Norichika Sago** and **Takahiro Tanaka**

Cosmic censorship conjecture

(R.Penrose Phys. Rev. Lett. **14** 57 (1965))

Singularity is almost inevitable in General relativity.

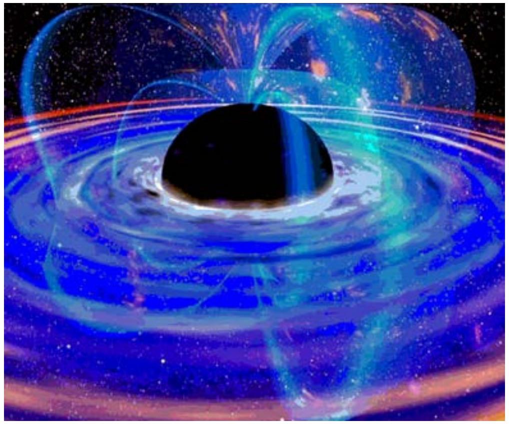
(e.g. S. Hawking and G. Ellis (1973))

Singularity: Implying breakdown of the theory.

Assure the predictability of General relativity…

Cosmic censorship conjecture

In any gravitational phenomena, all singularities are **“hidden”** within black holes, where they cannot seen be “seen” from by distant observer.

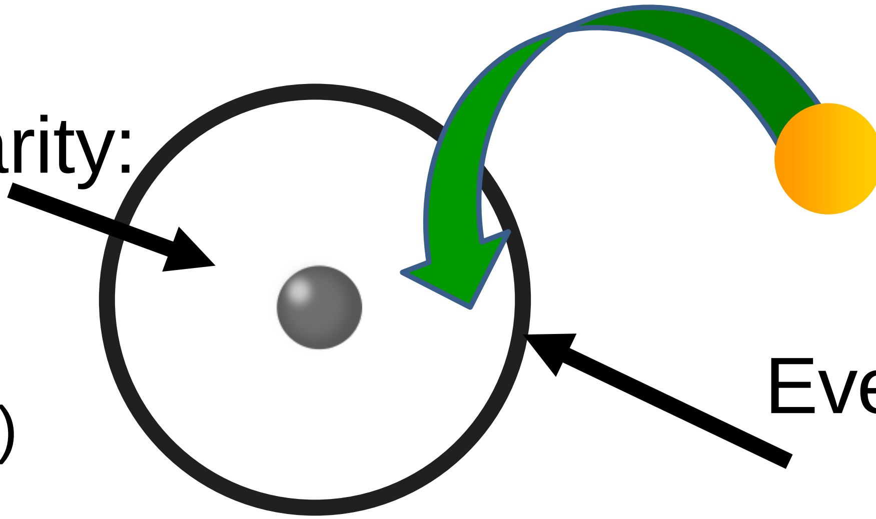


Q. Is it really the case in the classical G.R. ?

Violation of the conjecture

An example : The gedanken experiment of charging up a static charged black hole with a charged particle.

Singularity: $r = 0$



Charged particle $(E,\ q)$

(G=c=1)

Event horizon: (M > Q)

$$r = M + \boxed{\sqrt{M^2 - Q^2}}$$

① Absorption condition :

Pass the horizon of the black hole against electromagnetic repulsive force. (q > 0, Q > 0)

② Overcharging condition :

Final configuration of spacetime $\boxed{M_{\mathrm{B.H.}} + E < Q_{\mathrm{B.H.}} + q}$

Overcharging a charged black hole

(V.Hubeny Phys. Rev. **D66** 024016 (2002))

More precise formulation :

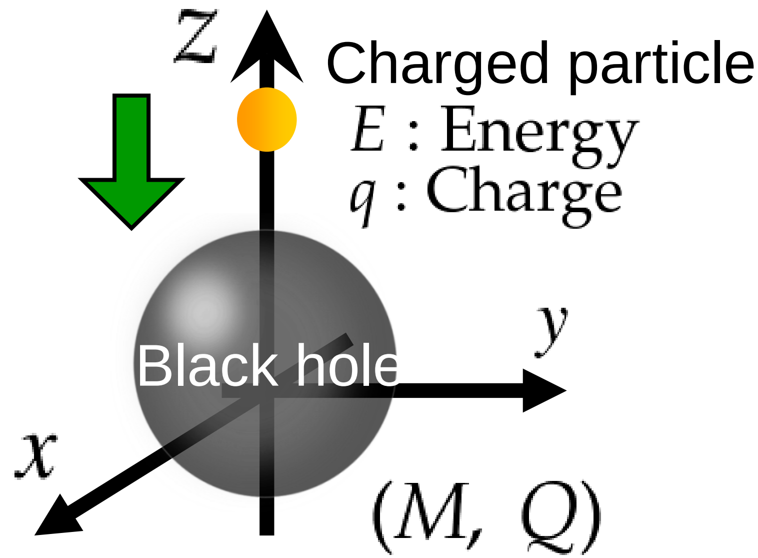
Charged particle: $\boxed{\epsilon \ll 1}$

$$E := O(\epsilon),\ \mu := O(\epsilon),\ q := O(\epsilon),$$

Charged black hole:

$$M := 1 + O(\epsilon^2),\ Q := 1,$$

(Test particle approximation) $\boxed{(Q_{\mathrm{B.H.}} + q) - (M_{\mathrm{B.H.}} + E) \approx O(\epsilon^2)}$



What are the possible attitudes for this result ?

① The classical GR is inconsistent. (Too radical …)

② Test particle approximation is not sufficient

Back reaction effects

Considering the existence of particle : $T_{\mu\nu}^{\text{Particle}}, \quad J_{\nu}^{\text{Particle}}$

(e.g. Gravitational radiation, particle's self energy etcetc...)

- Fields' equations

$$G_{\mu\nu}(g_{\alpha\beta}) = T_{\mu\nu}^{\text{particle}}(u^{\alpha}) + T_{\mu\nu}^{\text{EM}}(F_{\alpha\beta}) \quad \nabla^{\mu} F_{\mu\nu} = J_{\nu}^{\text{particle}}(u^{\alpha})$$

- Particle's equation of motion

$$u^{\mu}\nabla_{\mu}u^{\nu} = \mathcal{F}^{\nu}(F_{\alpha\beta})$$

We are able to handle above coupling equations with perturbation technique.

$$M_{\text{BH}} \gg \mu \sim O(\epsilon),$$

$$Q_{\text{BH}} \gg q \sim O(\epsilon)$$

$$g_{\mu\nu} = g_{\mu\nu}^{\text{BH}} + h_{\mu\nu}^{(1)} + \ldots$$

$$F_{\mu\nu} = F_{\mu\nu}^{\text{BH}} + f_{\mu\nu}^{(1)} + \ldots$$

The flavor of our analysis

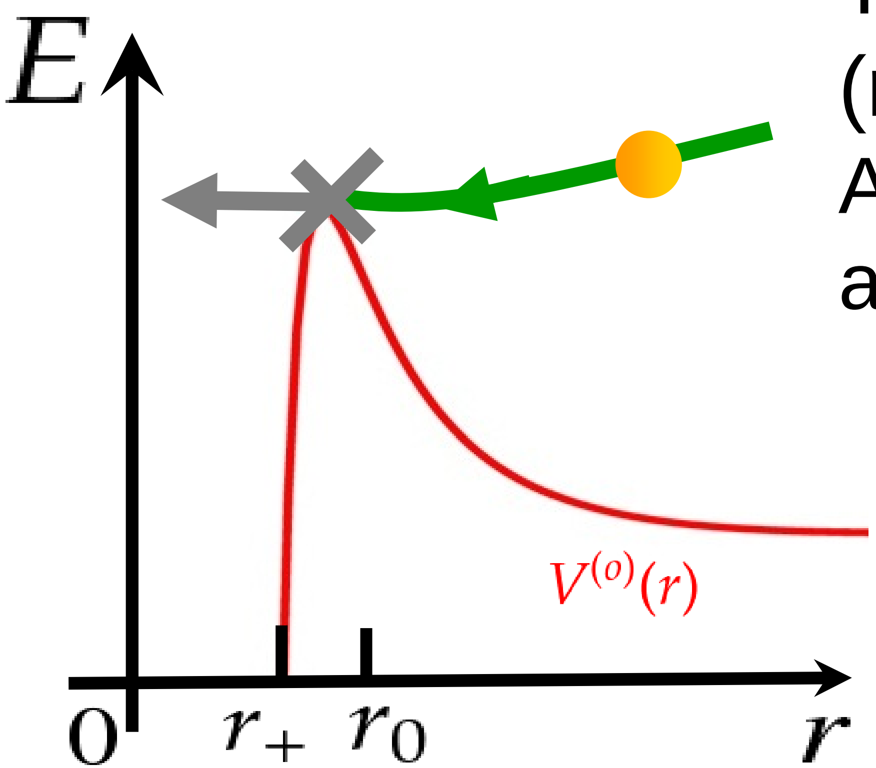
Q. How do the back reaction effects modify the mass and the charge of final space time ?

$$(Q_{\rm BH} + q)$$
$$\vee\wedge \quad ?$$
$$(M_{\rm BH} + E + {\color{red}E_{\rm B}})$$

Stage 1. Initial condition of orbit

- Particle's equation of motion :(**An assumption**)

$$\mu\left(\frac{dr}{ds}\right)=F(r)\left[E(t)-V^{(o)}(r)\right]$$

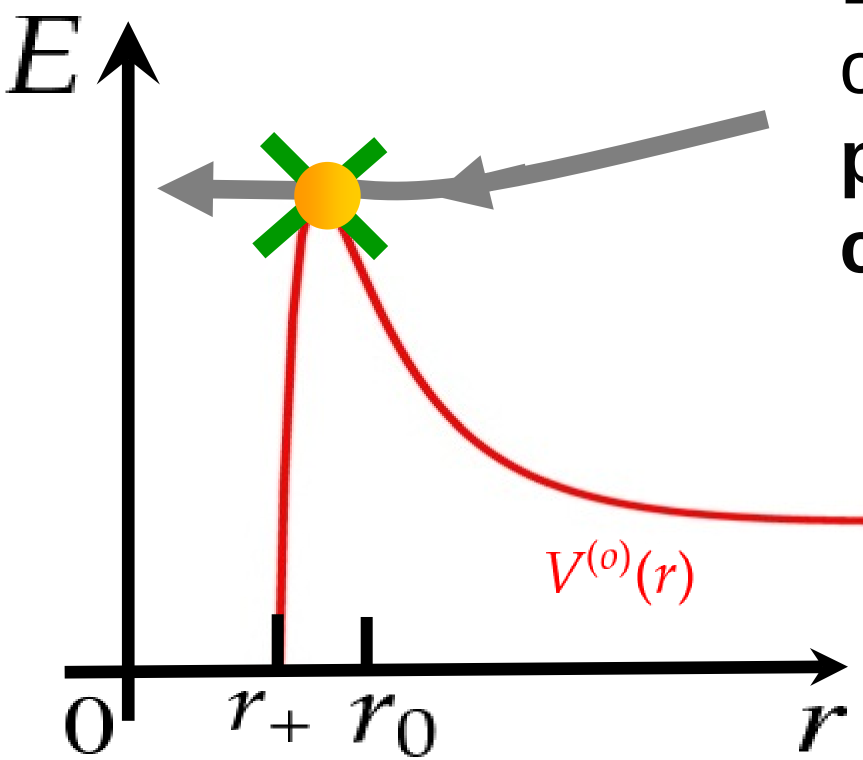


There always exists a (radial) **marginal orbit**:
A separatrix between plunge and bounce orbits

Focusing on the **marginal orbit**

Stage 2. Particle's self energy

The marginal orbit enjoys the (unstable w.r.t. higher order perturbation) **equilibrium configuration**



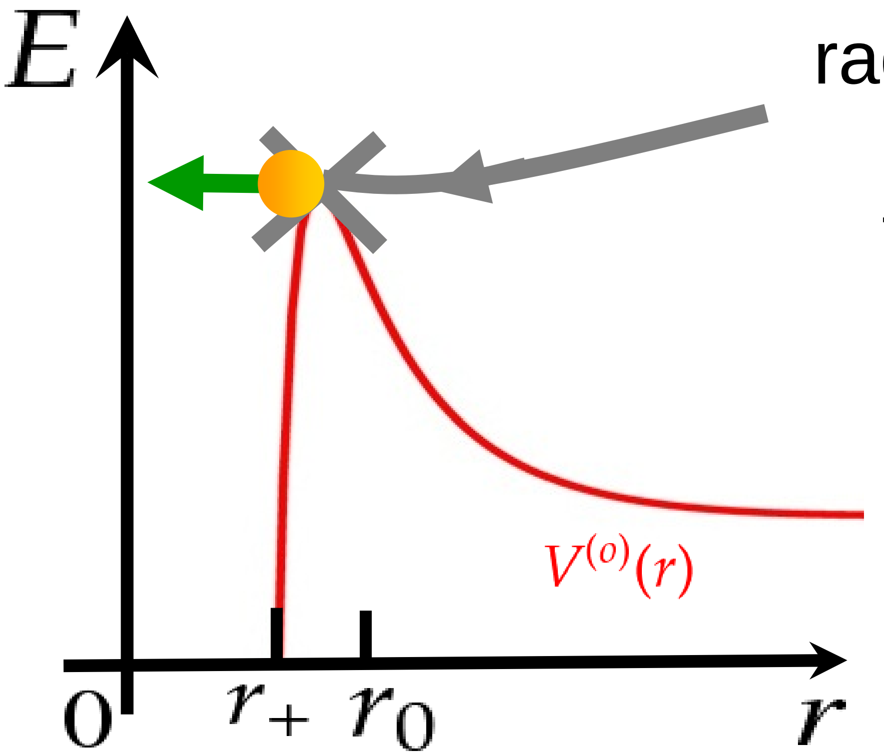
Evaluate the particle's energy of the self field, due to the **particle's own mass and charge.**

(Static contribution for system's energy.)

$$M + E + E_{\rm int} > Q + q$$

Stage 3. Radiation effects

Moving particle in a curved space time **emit the energy** to the infinity via radiation. (e.g. gravitational wave)



One must consider **the energy loss** from the system via radiation.

(Dynamical contribution for system's energy.)

$(M\mu \approx Qq)$

Almost balancing

$$M + E + E_{\rm int} - E_{\rm rad} > Q + q$$

Conclusion

We have re-investigated **an “counter” example** for the cosmic censorship conjecture : Overcharging process

Back reaction effects on system’s total energy

① self-field’s energy
② Radiation

The final spacetime after absorbing the particle is **still the black hole,** not the singularity.

Back reaction effects (non-linearity of gravitational field) might play a role of cosmic censorship.