The physics behind topological string theory

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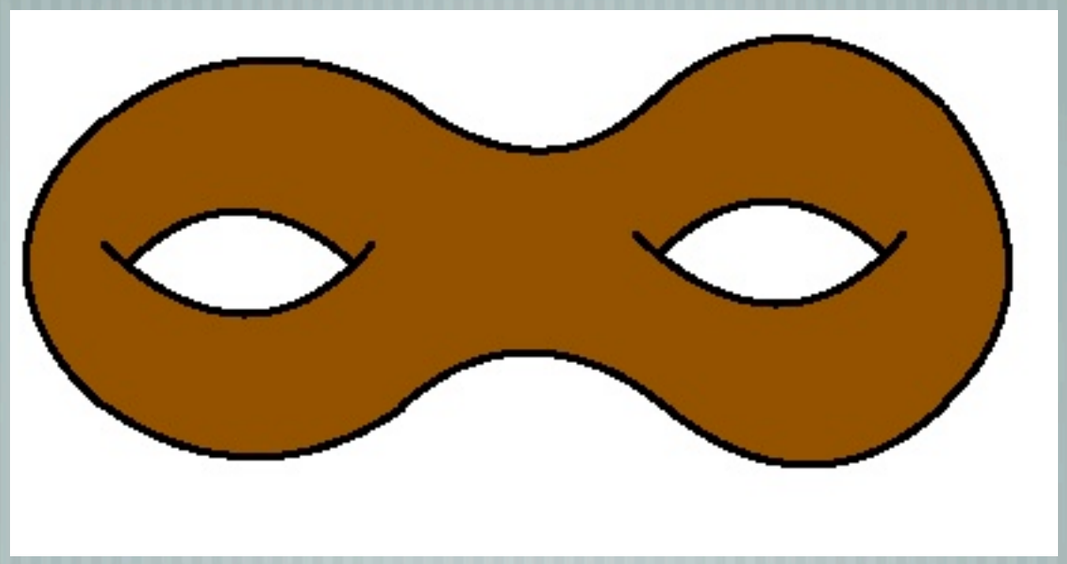
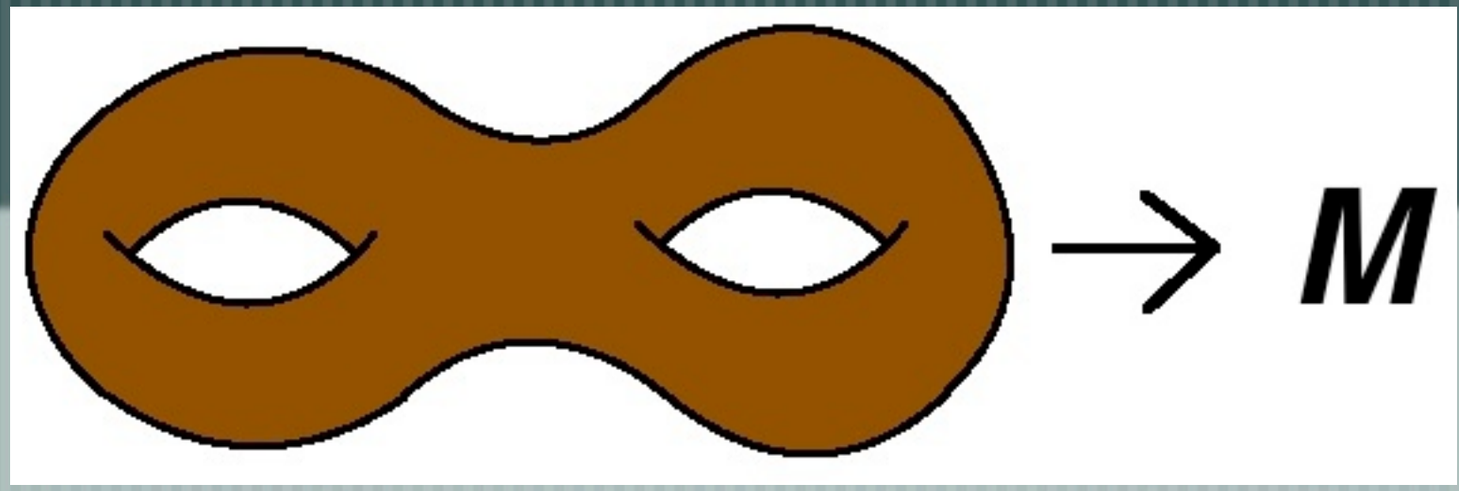


$$X : \Sigma \to M$$



$$S = t \int_\Sigma G_{i\bar{j}} \partial_a X^i \partial^a X^{\bar{j}} \quad + \quad Fermions$$

$$\delta X^i = i\alpha_- \psi^i_+ + i\alpha_+ \psi^i_-$$

$$\delta X^{\bar{j}} = i\tilde{\alpha}_- \psi^{\bar{j}}_+ + i\tilde{\alpha}_+ \psi^{\bar{j}}_-$$

$$\delta \psi^i_+ = -\tilde{\alpha}_- \partial_z X^i + \ldots$$

$$\delta \psi^{\bar{j}}_+ = -\alpha_- \partial_z X^{\bar{j}} + \ldots$$

$$\vdots$$

There are four fermion supercharges on the worldsheet

$$Q^+, Q^-, \tilde{Q}^+, \tilde{Q}^-$$

We redefine the energy momentum tensor

$$T_{zz} \to T_{zz} + \frac{1}{2}\partial_z J_z$$

$$\tilde{T}_{\bar{z}\bar{z}} \to \tilde{T}_{\bar{z}\bar{z}} \pm \frac{1}{2}\tilde{\partial}_{\bar{z}} \tilde{J}_{\bar{z}}$$

The result is that fields charged under the R-symmetry modify their spin, fermions become either scalars or one forms

$$Q^+ + \tilde{Q}^+ = Q_B$$

$$Q^+ + \tilde{Q}^- = Q_A$$

A model:

1- localizes on holomorphic maps

2- depends only on the Kahler parameters of the target space M

3- $\mathcal{O}_W = W(X)_{i_1 \dots i_n \bar{j}_1 \dots \bar{j}_m} \chi^{i_1} \dots \chi^{i_n} \chi^{\bar{j}_1} \dots \chi^{\bar{j}_m} \equiv \mathcal{H}_d^{n,m}(M)$

B model:

1- localizes on constant maps

2- depends only on the complex structures of M

3- $\mathcal{O}_W \equiv \mathcal{H}_{\bar{\partial}}^{n,m}(M)$

Localization:

Suppose it exists a globally well defined symmetry F without fixed points

$$\int_K e^{-S}\mathcal{O} = Vol(F)\int_{K/F} e^{-S}\mathcal{O}$$

If the symmetry F is fermionic the volume of F is zero

$$Vol(F) = \int d\theta = 0$$

Then the path integral picks up contributions only from the fixed points under F

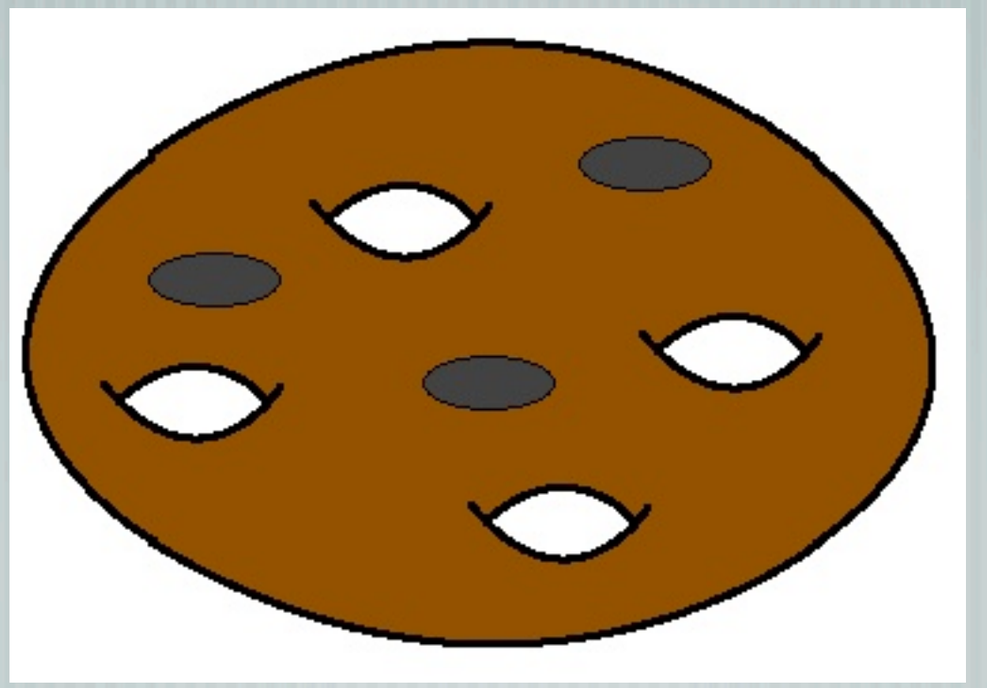
$$\delta\phi = 0$$

Mirror symmetry:
(T duality for a torus)

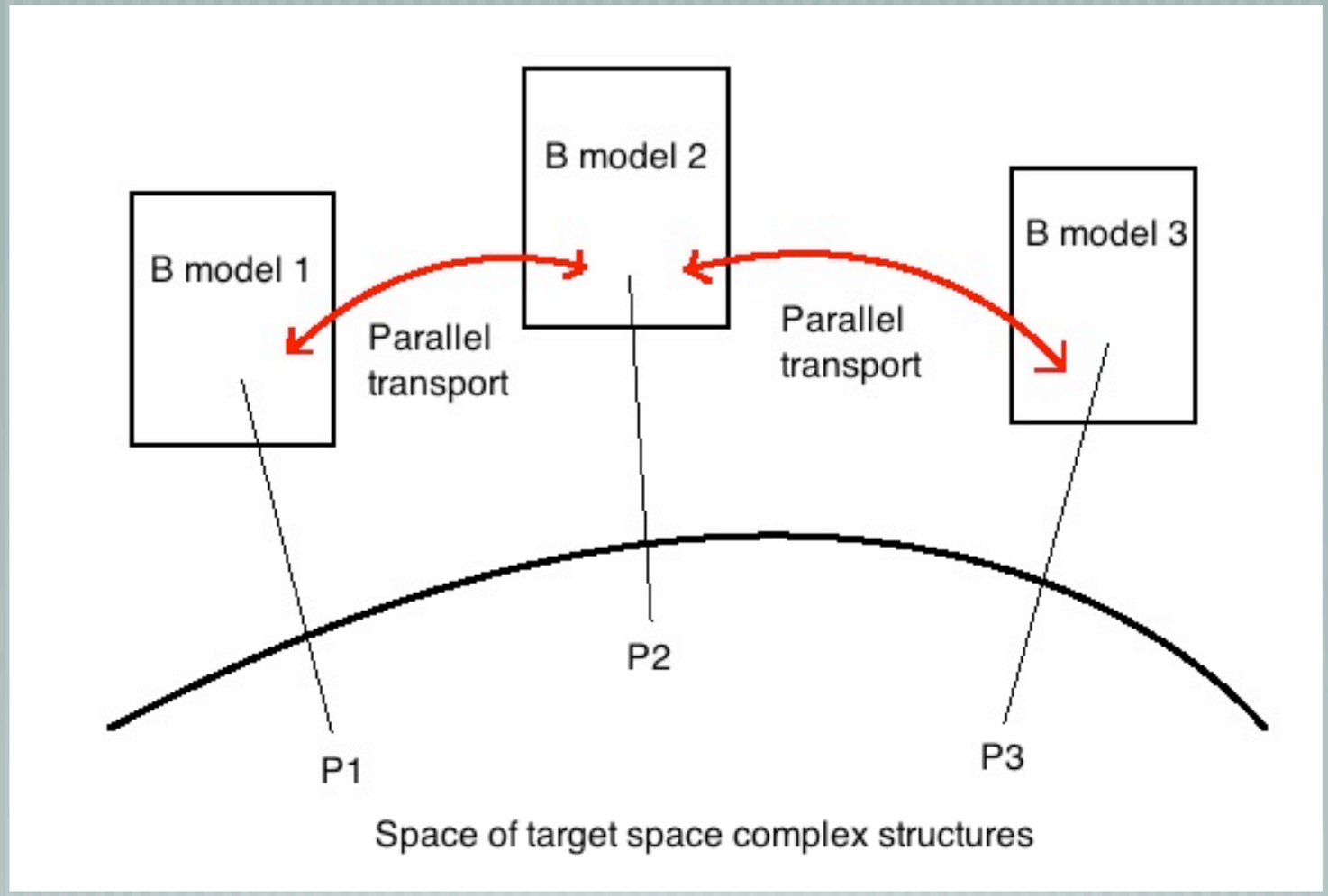
A model on a Calabi Yau M $\longleftrightarrow$ B model on a Calabi Yau W

W is called the mirror of M

Open-closed large N duality: $\sum_{g,h} g_s^{2g-2} t^h F^{g,h} = \sum_g g_s^{2g-2} F^g(t)$



" Quantum " background independence:
Consider the B model. It depends on the background only through its complex structure



Space of target space complex structures

The requirement for parallel transport is equivalent to the holomorphic anomaly equation which has been independently derived

" Physical " superstring models have a low energy 4 dimensional quantum field theory description

It contains some F-terms like

$$F^g R^2 T^{2g-2}$$

After compactification on a 6 dimensional manifold M, this term is computed by a certain superstring amplitude

This amplitude reduces to a much simpler topological amplitude at genus g, on the same internal space M