Nuclear Fission Process

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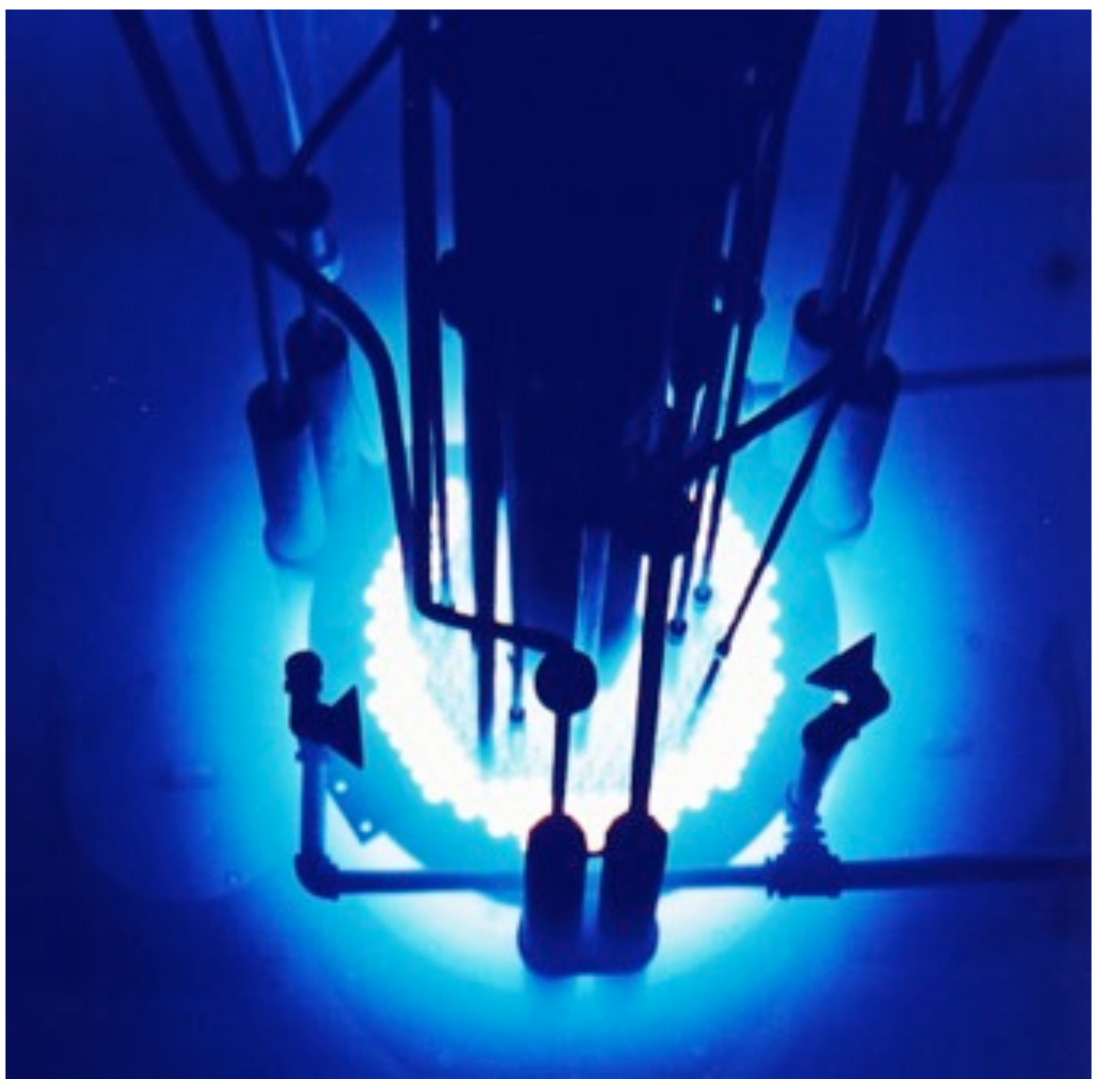
Nuclear Fission

http://www.jaea.go.jp/jaeri/jpn/bgphoto/bgphoto_main.html



Splits into two nuclei

Main fission channel

$$^{235}U + n \rightarrow {}^{95}Y + {}^{139}I + 2n$$

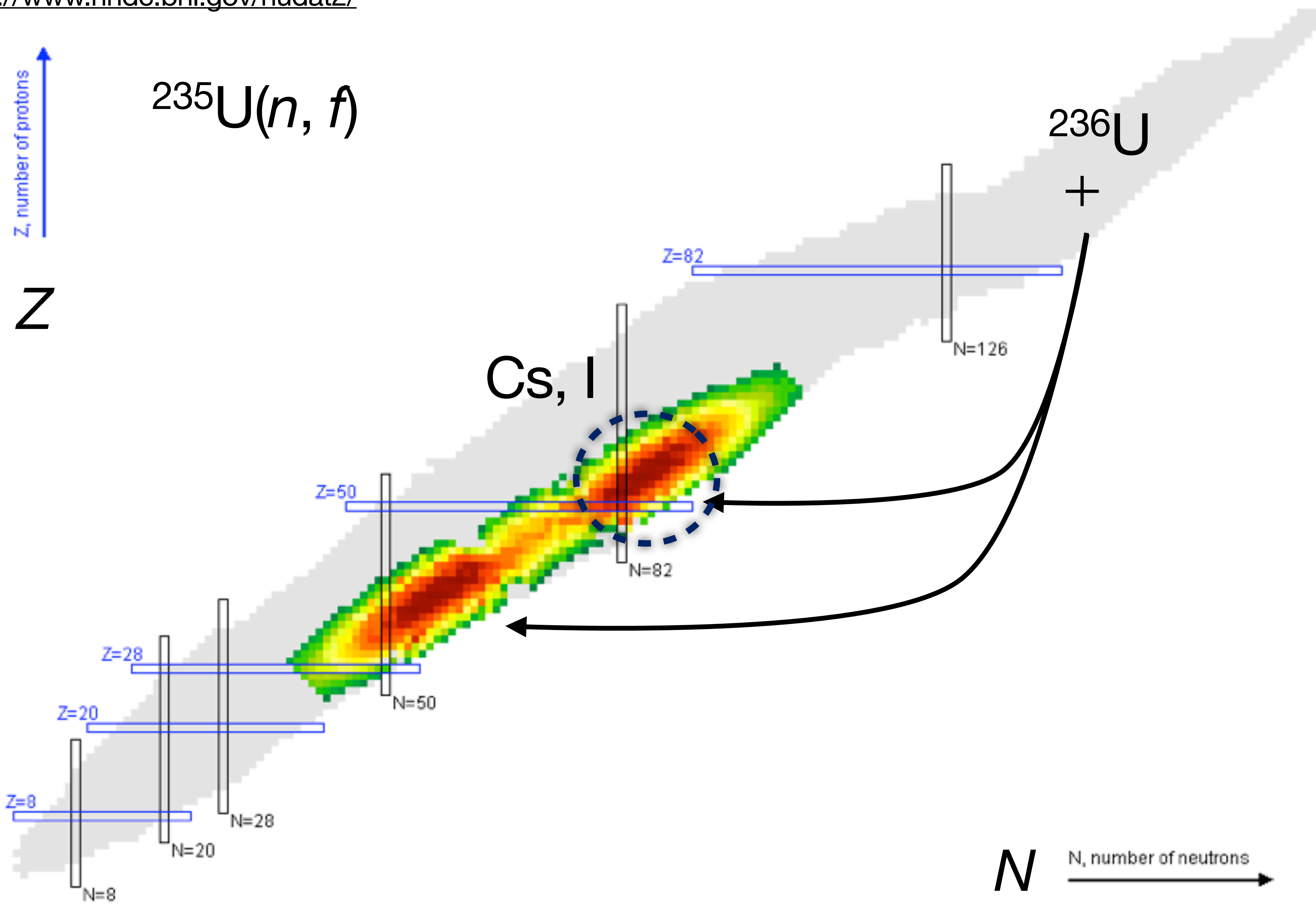
Not symmetric splits

$^{133}Cs > {}^{135}I > {}^{93}Zr > {}^{137}Cs > {}^{99}Tc > {}^{90}Sr > {}^{131}I > {}^{147}Pm > {}^{149}Sm > {}^{129}I$

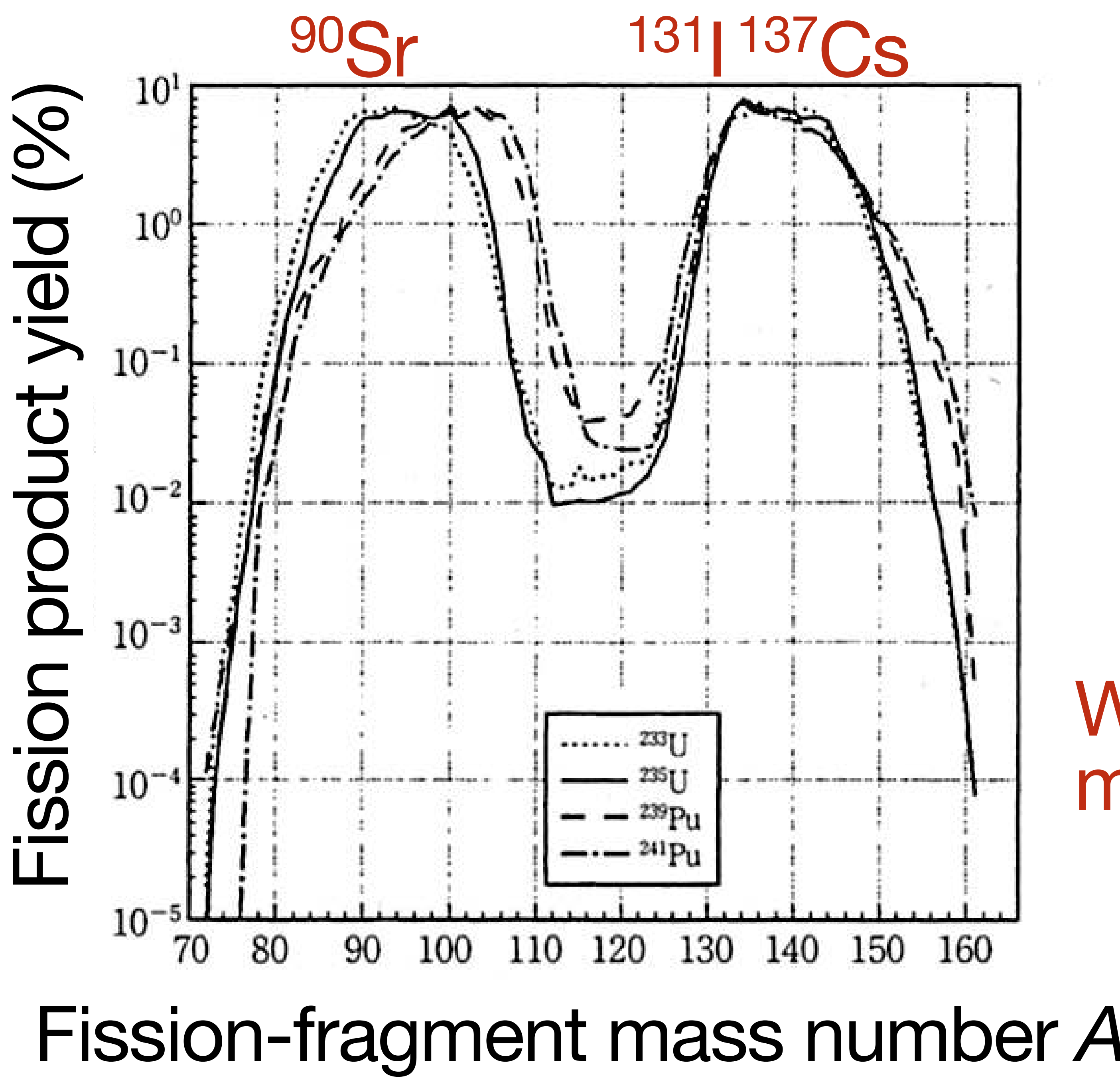
Thermal fluctuations

Fission-product yield

http://www.nndc.bnl.gov/nudat2/

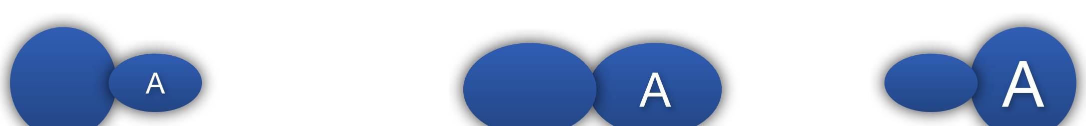


Fission-fragment mass distribution



Peak position
A ~ 140

Why fission channel is mainly mass-asymmetric divisions?



Discovery of Nuclear Fission

- In 1938

from wikipedia

Lise Meitner Otto Hahn

Naturwissenschaften **27** (1)

Über den Nachweis und das Verhalten der bei der Bestrahlung des Urans mittels Neutronen entstehenden Erdalkalimetalle[1].

Von O. Hahn und F. Strassmann, Berlin-Dahlem.

In einer vor kurzem an dieser Stelle erschienenen vorläufigen Mitteilung[2] wurde angegeben, daß bei der Bestrahlung des Urans mittels Neutronen außer den von Meitner, Hahn und Strassmann im einzelnen beschriebenen Trans-Uranen — den Elementen 93 bis 96 — noch eine ganze Anzahl anderer Umwandlungsprodukte entstehen, die ihre Bildung offensichtlich einem sukzessiven zweimaligen α-Strahlenzerfall des vorübergehend entstandenen Urans 239 verdanken. Durch einen solchen Zerfall muß aus dem Element mit Glieder beschrieben werden. Aus dem Aktivitätsverlauf der einzelnen Isotope ergibt sich ihre Halbwertszeit und lassen sich die daraus entstehenden Folgeprodukte ermitteln. Die letzteren werden in dieser Mitteilung aber im einzelnen noch nicht beschrieben, weil wegen der sehr komplexen Vorgänge — es handelt sich um mindestens 3, wahrscheinlich 4 Reihen mit je 3 Substanzen — die Halbwertszeiten aller Folgeprodukte bisher noch nicht erschöpfend festgestellt werden konnten.

^{235}U + *n*(0.025 eV) -> ^{141}Ba + ? + *n*(2.5)

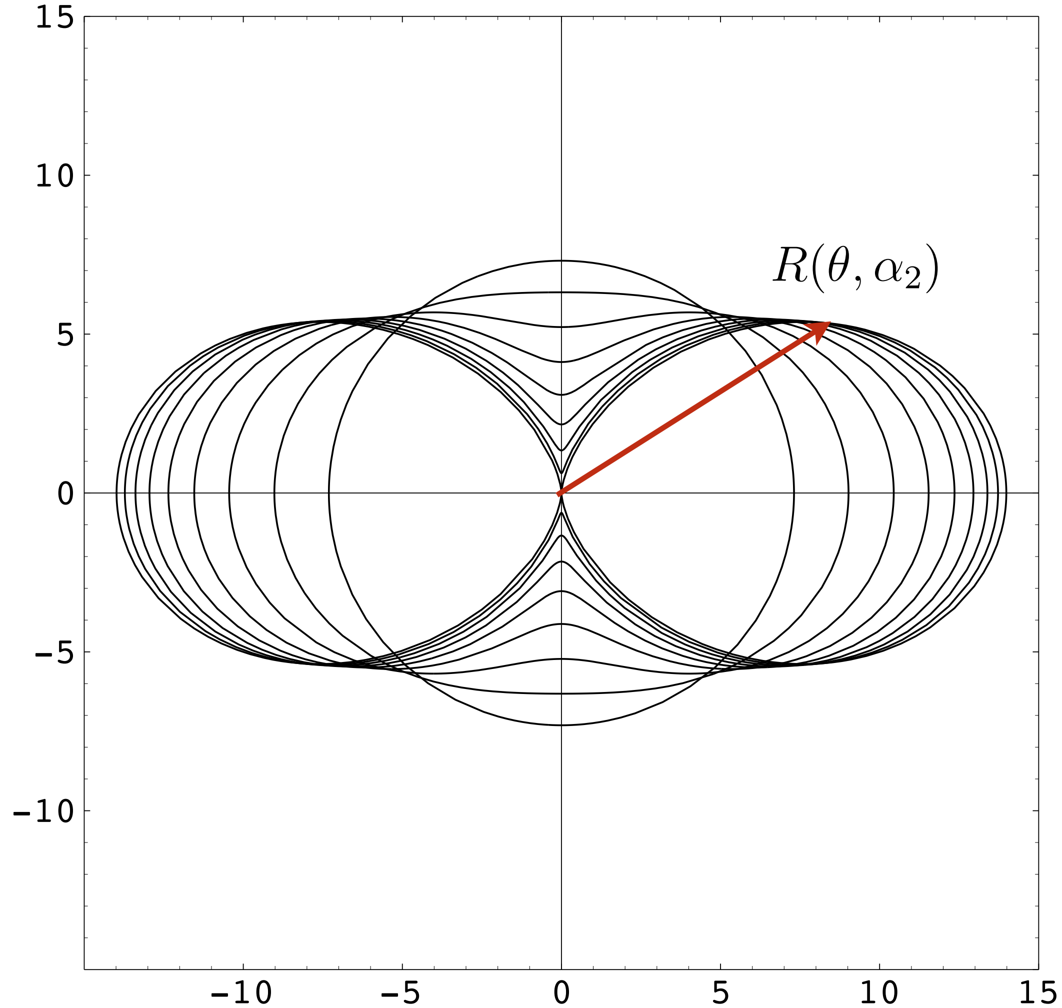
Extract Ba (Chemical separation)

Nuclear Chemistry

Energy release ~200MeV

Bohr-Wheeler Model (1939)

- Regard nucleus as a "classical" liquid drop
  → Surface tension due to the attractive nuclear force
- Describe nuclear shapes using the Legendre polynomial



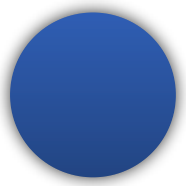
$$R(\theta, \alpha_2) = \lambda^{-1} R_0[1 + \alpha_2 P_2(\cos\theta)]$$

α_2: Shape parameter

$$\alpha_2 = \begin{cases} 0 & \text{Spherical} \\ 2 & \text{Scission} \end{cases}$$

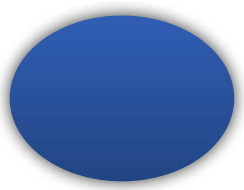
Liquid-drop model

- Calculate the change of Coulomb and surface energies from spherical shape



$$E_c^{(0)} \quad E_s^{(0)}$$

$$R(\theta, \alpha_2) = \lambda^{-1} R_0 [1 + \alpha_2 P_2(\cos\theta)]$$



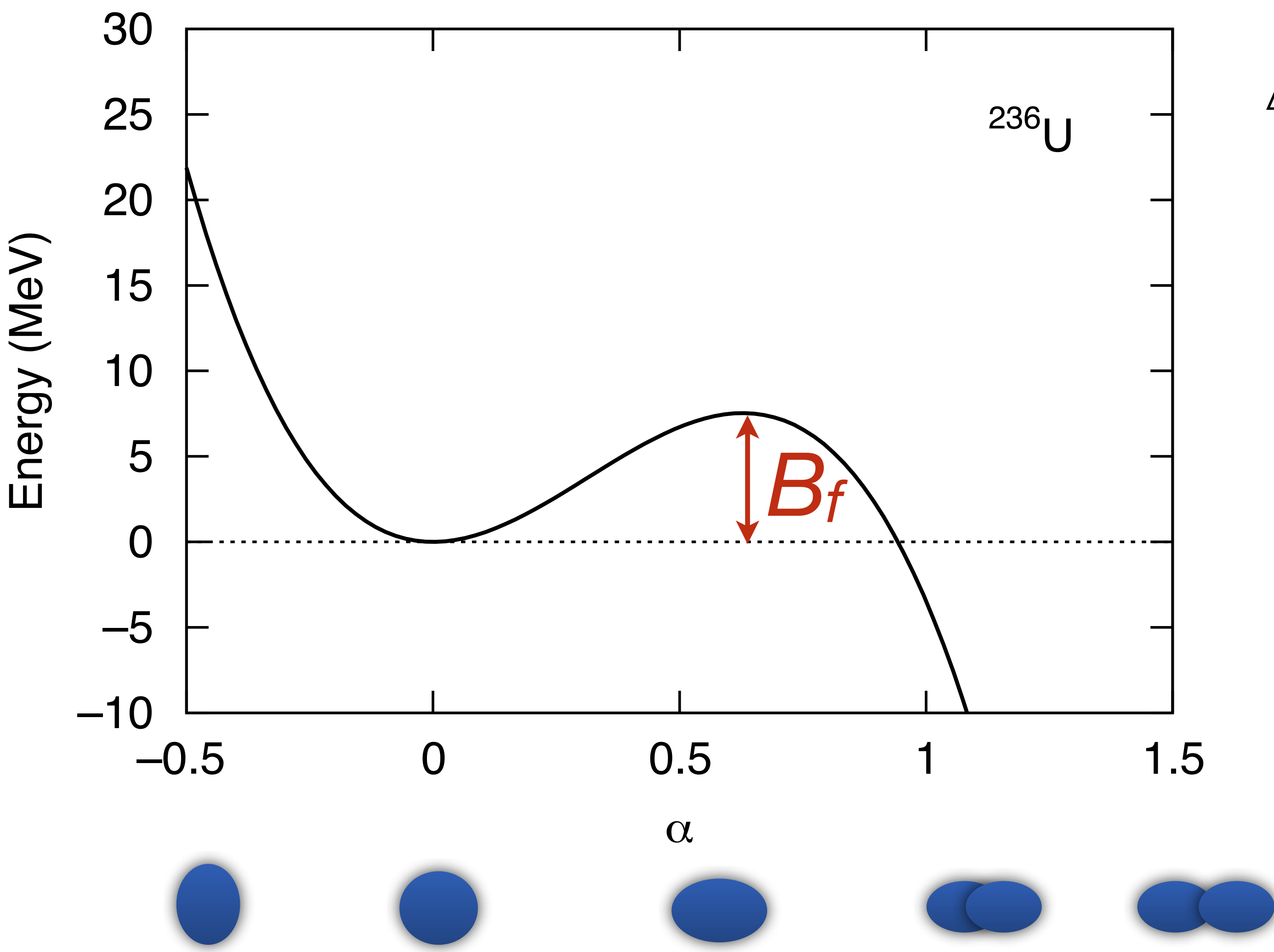
$$E_c(\alpha) = \frac{1}{2} \int \int \frac{\rho(\vec{r_1})\rho(\vec{r_2})}{|\vec{r_1} - \vec{r_2}|} dV_1 dV_2 = E_c^{(0)} \left(1 - \frac{1}{5}\alpha^2 - \frac{4}{105}\alpha^3 + \cdots \right)$$

$$E_s(\alpha) = E_s^{(0)} S(\alpha) = E_s^{(0)} \int dS = E_s^{(0)} \left(1 + \frac{2}{5}\alpha^2 - \frac{4}{105}\alpha^3 + \cdots \right)$$

$$\Delta E_s(\alpha) = (E_c(\alpha) + E_s(\alpha)) - (E_c^{(0)} + E_s^{(0)})$$

$$= E_s^{(0)} \left(\frac{2}{5}(1 - x)\alpha^2 - \frac{4}{105}(1 + 2x)\alpha^3 \right) \qquad x = \frac{E_c^{(0)}}{2E_s^{(0)}}$$

Fission barrier



$$\Delta E(\alpha) = E_s^{(0)}\left(\frac{2}{5}(1-x)\alpha^2 - \frac{4}{105}(1+2x)\alpha^3\right)$$

$$x = \frac{E_c^{(0)}}{2E_s^{(0)}}$$

$$E_s^{(0)} = a_s(1-\kappa_s I^2)A^{2/3}$$

$$E_c^{(0)} = a_c Z^2/A^{1/3}$$

$a_s = 17.944$ MeV $\kappa_s = 1.7826$

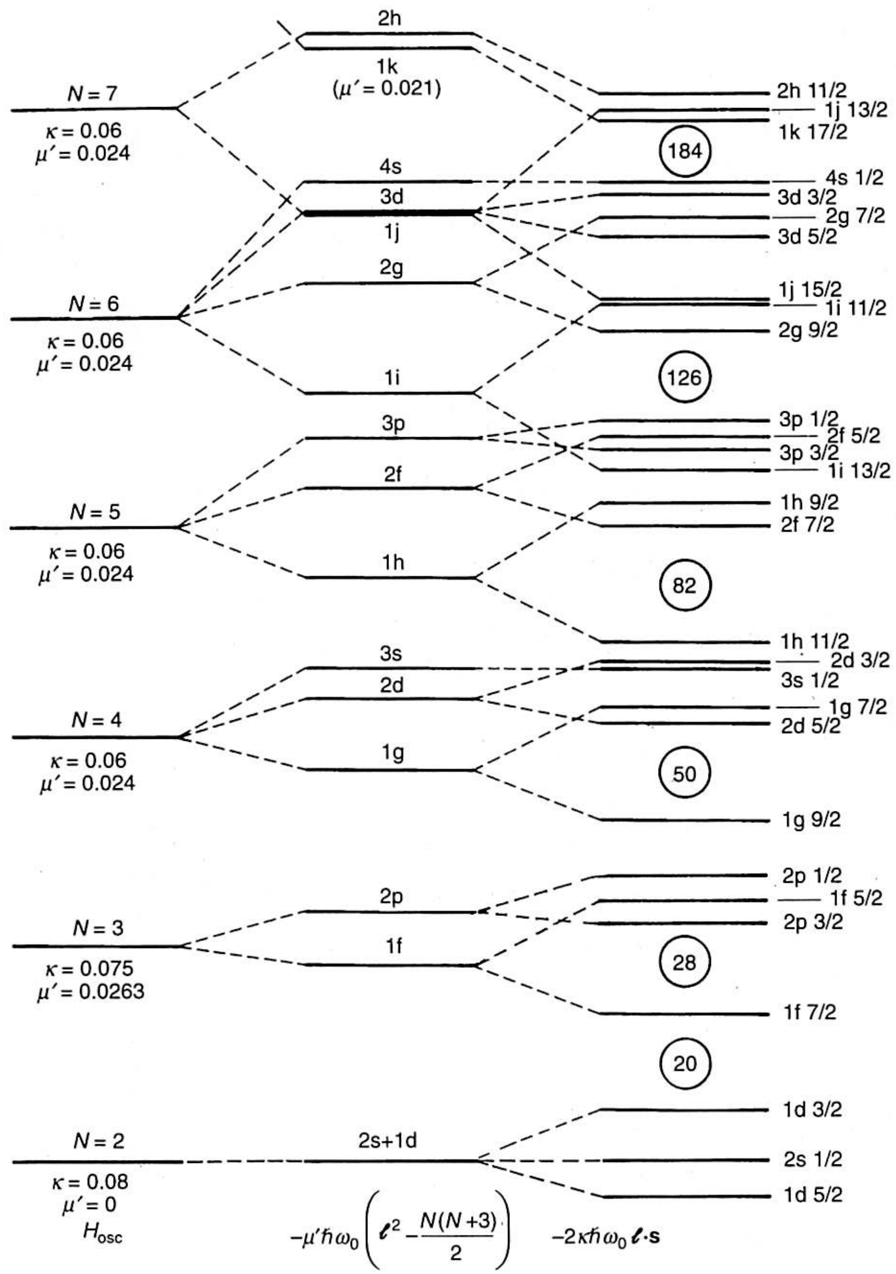
$a_c = 0.7053$ MeV

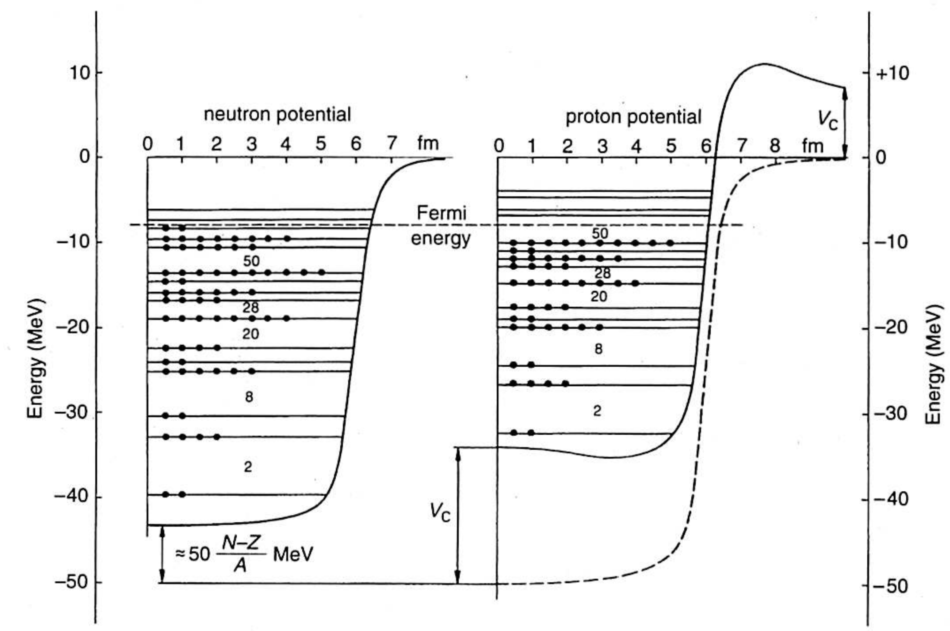
Competition between Coulomb and surface energies

Saddle point → "Transition state"

Nuclear shell effect

Single-particle picture





Strong spin-orbit force

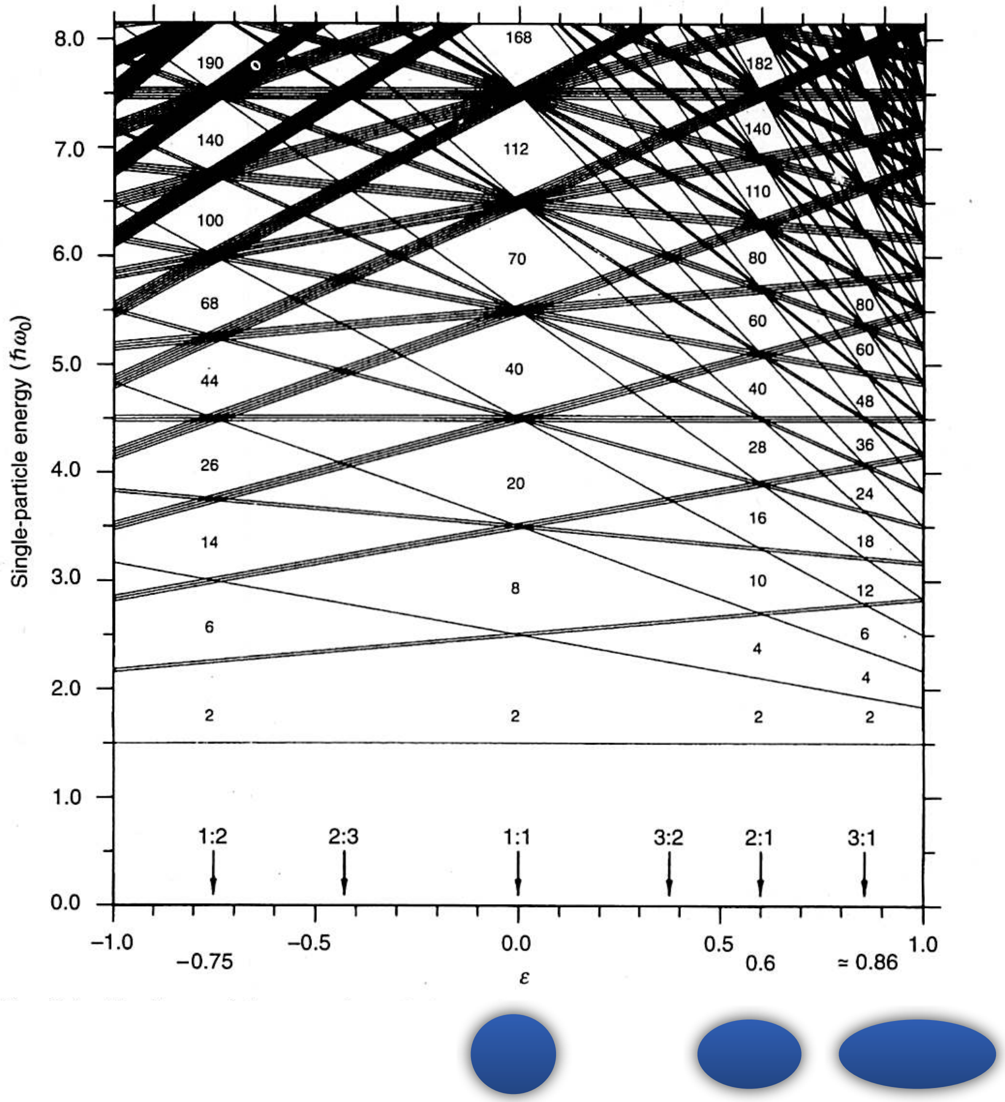
Nuclear Magic Number

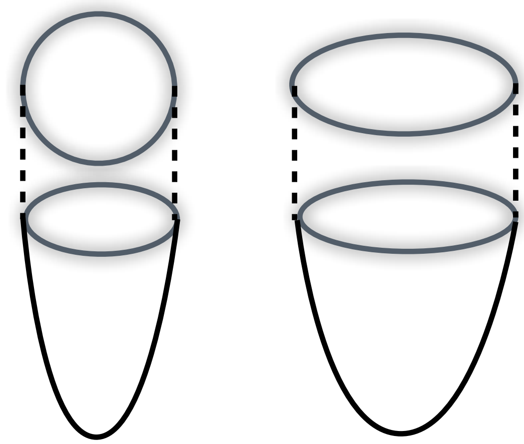
$Z = 50, N = 82$ ^{132}Sn

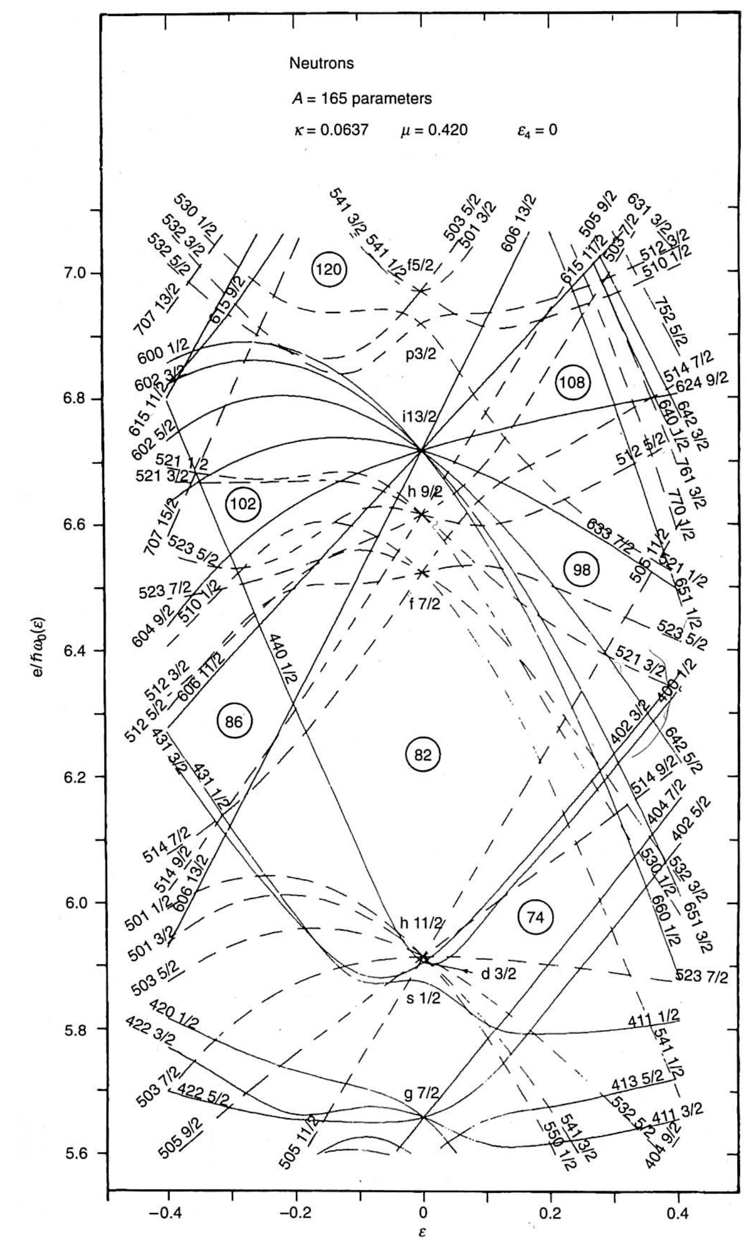
→ A = 132 ~140???

Nuclear deformations

- 3D harmonic oscillator potential





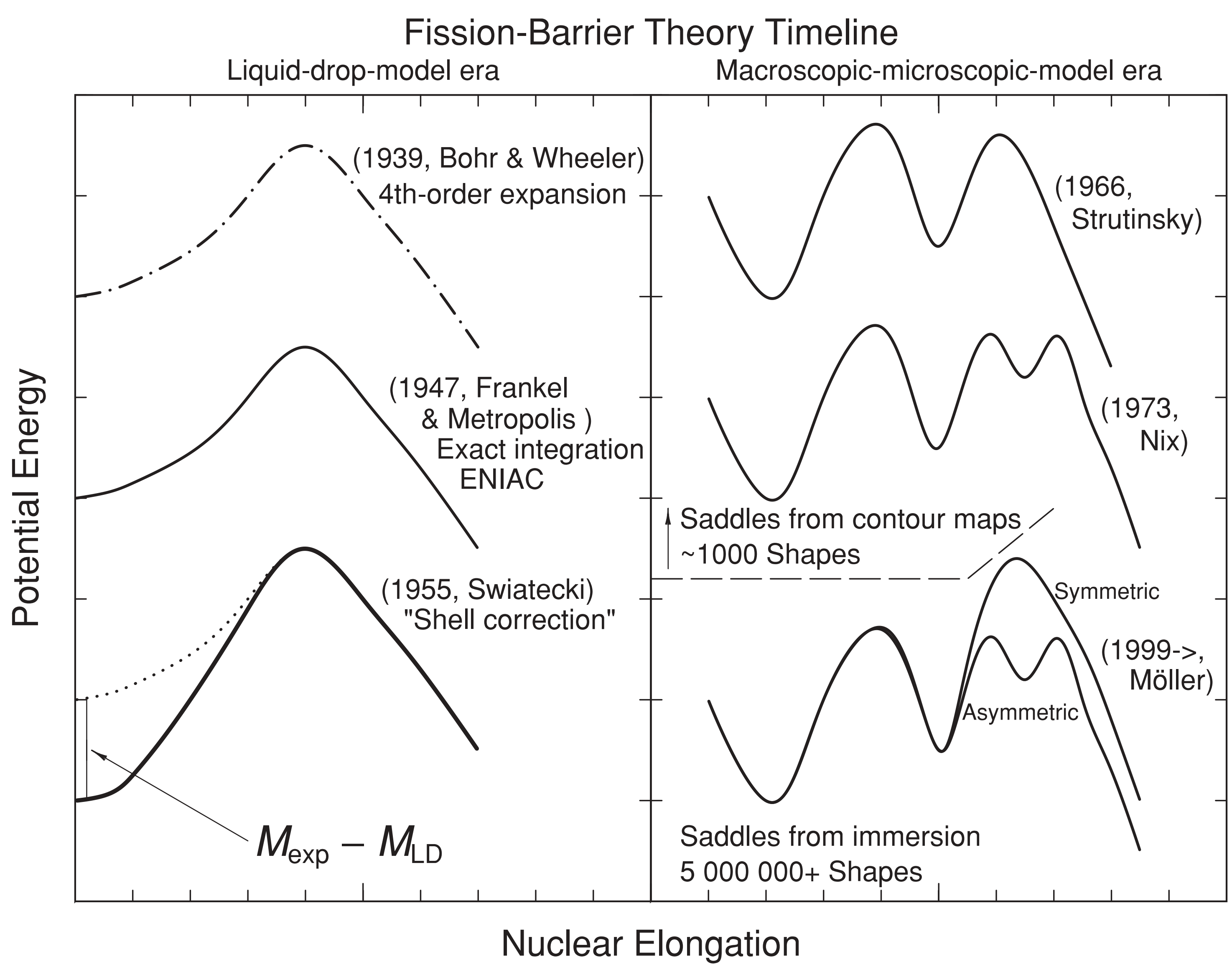


With spin-orbit force

S.P. levels are sensitive to deformations

Shell correction energy

P. Möller, A.J. Sierk, TI, A. Iwamoto, R. Bengtsson, H. Uhrenholt, and S. Åberg, Phys. Rev. C **79**, 064304 (2009)



Strutinsky Method

Liquid drop energy
+
"Shell correction energy"

Finite-range liquid-drop model (FRLDM2002)

■ Total Energy

Quantum correction term

$$E_{\text{Total}} = E_{\text{Vol}} + E_{\text{Coul}}(\delta) + E_{\text{YPE}}(\delta) + E_{\text{Shell}}(\delta)$$

δ : shape parameter

■ Macroscopic part

- Coulomb term

$$E_{\text{Coul}} = \frac{\rho_0^2}{2} \int \int_V d\vec{r_1} d\vec{r_2} \frac{1}{\left|\vec{r_1} - \vec{r_2}\right|} - \frac{\rho_0^2}{2} \int \int_V d\vec{r_1} d\vec{r_2} \frac{1}{\left|\vec{r_1} - \vec{r_2}\right|} e^{-\left|\vec{r_1} - \vec{r_2}\right|/a} \left(1 + \frac{1}{2} \frac{\left|\vec{r_1} - \vec{r_2}\right|}{a}\right)$$

- Nuclear-energy term

  Yukawa-plus-exponential model

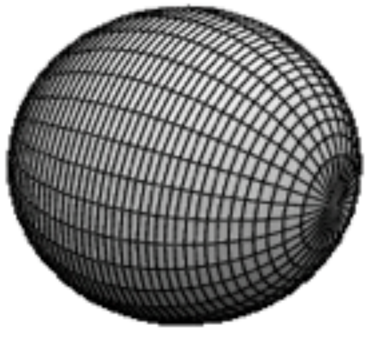
$$E_{\text{YPE}} = -\frac{c_s}{8\pi^2 r_0^2 a^3} \int \int_V d\vec{r_1} d\vec{r_2} \left(\frac{\left|\vec{r_1} - \vec{r_2}\right|}{a} - 2\right) \frac{e^{-\left|\vec{r_1} - \vec{r_2}\right|/a}}{\left|\vec{r_1} - \vec{r_2}\right|}$$

Three-quadratic surface (3QS) parametrization

Generate macroscopic density

- Five shape parameters
  - Elongation
    Quadrupole moment Q_2
  - Neck parameter η
  - deformation (left fragment) ε_L
  - deformation (right fragment) ε_R
  - Mass asymmetry α_g

$$\alpha_g = \frac{M_L - M_R}{M_L + M_R}$$

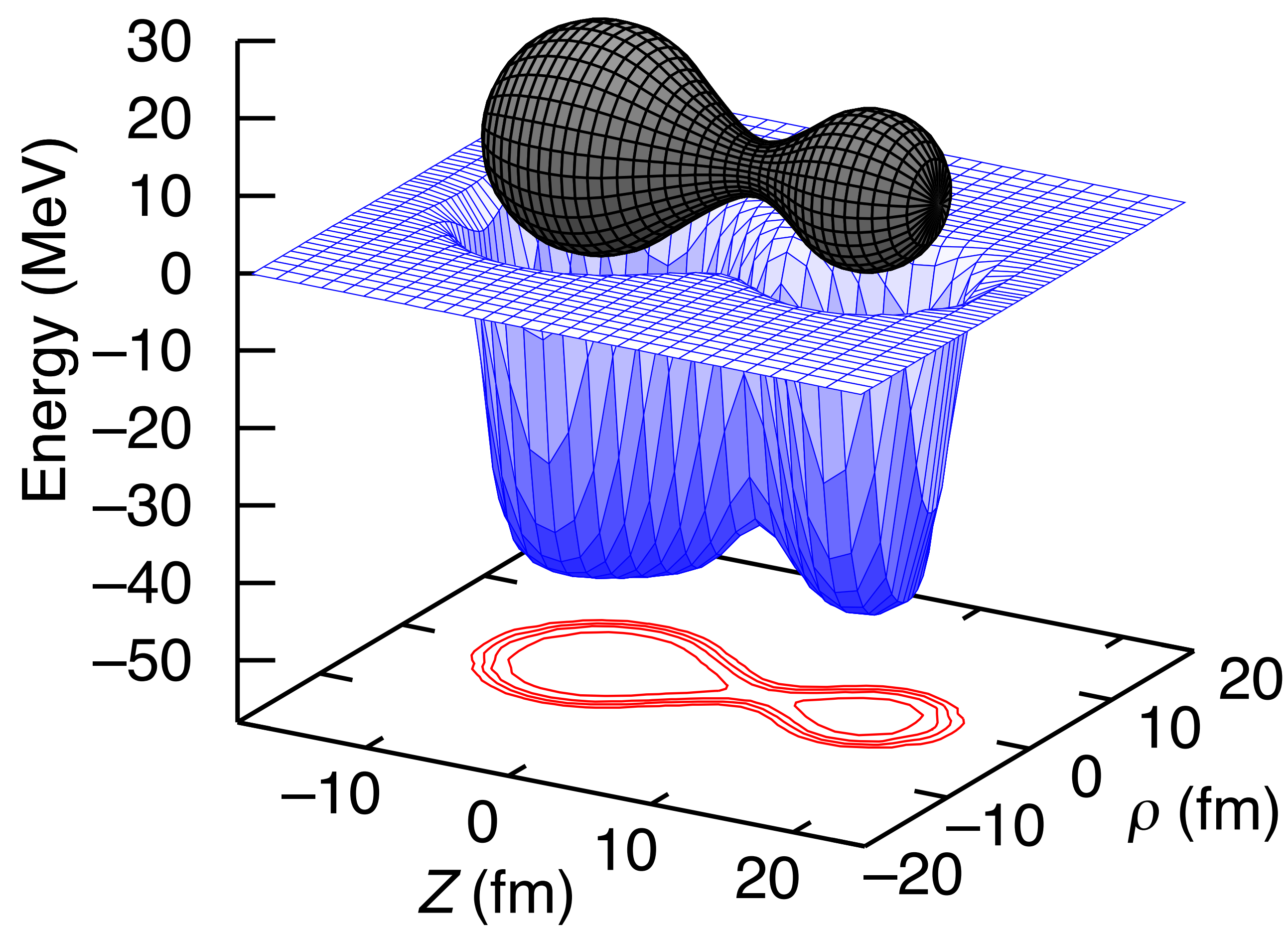


Generate mean-field single-particle potential

generate arbitrary mean-field potential

■ Yukawa-type folding potential

$$V_{\mathrm{N}}(\vec{r}) = -\frac{V_0}{4\pi a_{\mathrm{pot}}} \int_V \frac{e^{-|\vec{r}-\vec{r'}|/a_{\mathrm{pot}}}}{|\vec{r}-\vec{r'}|/a_{\mathrm{pot}}} d\vec{r'}$$



Finite-range liquid-drop model (FRLDM2002)

■ Microscopic part

$$H = -\frac{\hbar^2}{2m}\Delta + V_N(\vec{r}) + V_{\mathrm{S.O.}}(\vec{r}) + V_C(\vec{r})(1-\tau_3)/2$$

expanded by deformed harmonic-oscillator basis

- Mean-field potential:
  Folded Yukawa potential

$$V_{\mathrm{N}}(\vec{r}) = -\frac{V_0}{4\pi a_{\mathrm{pot}}}\int_V \frac{e^{-\left|\vec{r}-\vec{r'}\right|/a_{\mathrm{pot}}}}{\left|\vec{r}-\vec{r'}\right|/a_{\mathrm{pot}}}d\vec{r'}$$

- Spin-orbit potential:

$$V_{\mathrm{S.O.}} = -\lambda\left(\frac{\hbar}{2m_{\mathrm{nuc}}c}\right)^2 \frac{\vec{\sigma}\cdot\nabla V\times\vec{p}}{\hbar}$$

- Strutinsky method: Shell correction energy
- Pairing correction energy: Lipkin-Nogami model

Grid points of the 5D potential-energy surface

$$E_{\mathrm{Pot}}(Q_2, \eta, \epsilon_1, \epsilon_2, \alpha)$$

Q_2 quadrupole moment: → 45 grid

η Neck parameter : 0 (scission) ~ 1 → 15 grid

ϵ_1 deformation (left) : -0.2 ~ 0.5 → 15 grid

ϵ_2 deformation (right): -0.2 ~ 0.5 → 15 grid

α Mass asymmetry : 0 ~ 0.45 → 35 grid

45 × 15 × 15 × 15 × 35 = 5,315,625 grid points
for each nucleus

Structure of multi-dimensional potential-energy surface

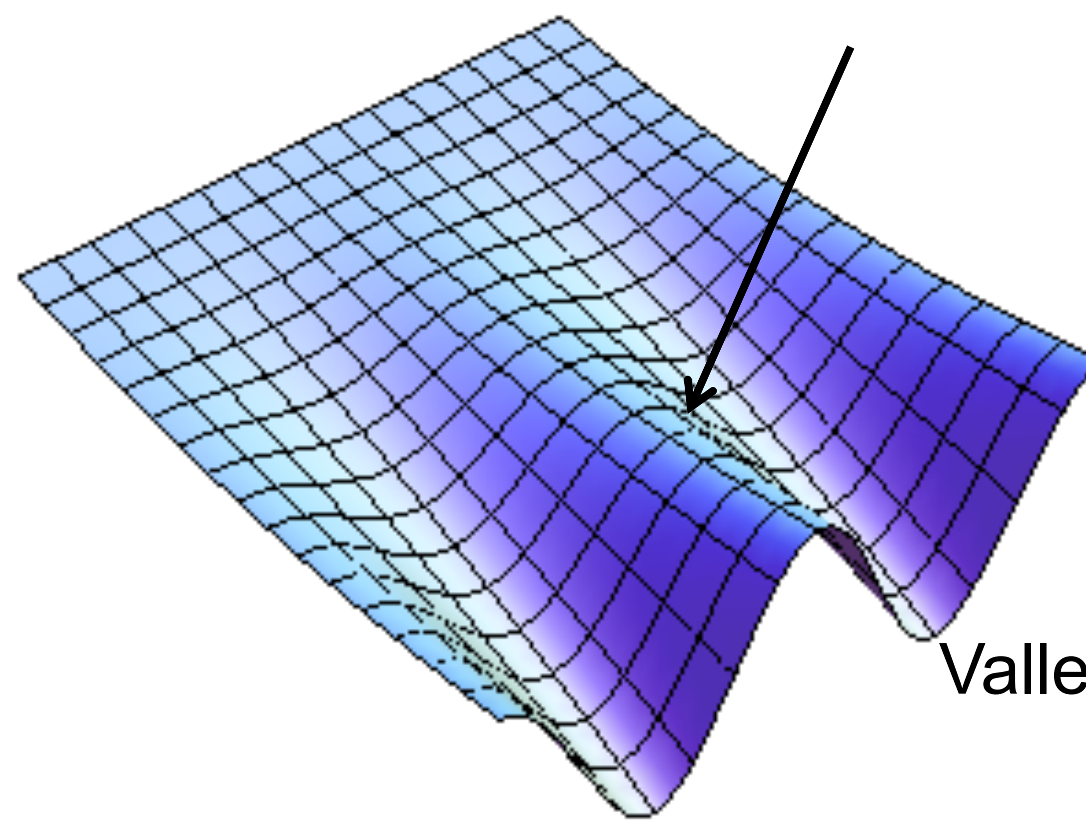
How can we explore multi-dimensional potential-energy surface?

$$E_{\mathrm{Total}}(Q_2, \eta, \epsilon_1, \epsilon_2, \alpha)?$$

- Local minima and Saddle points
- Potential-energy valley

  leading to an exit channel
- Separating ridge

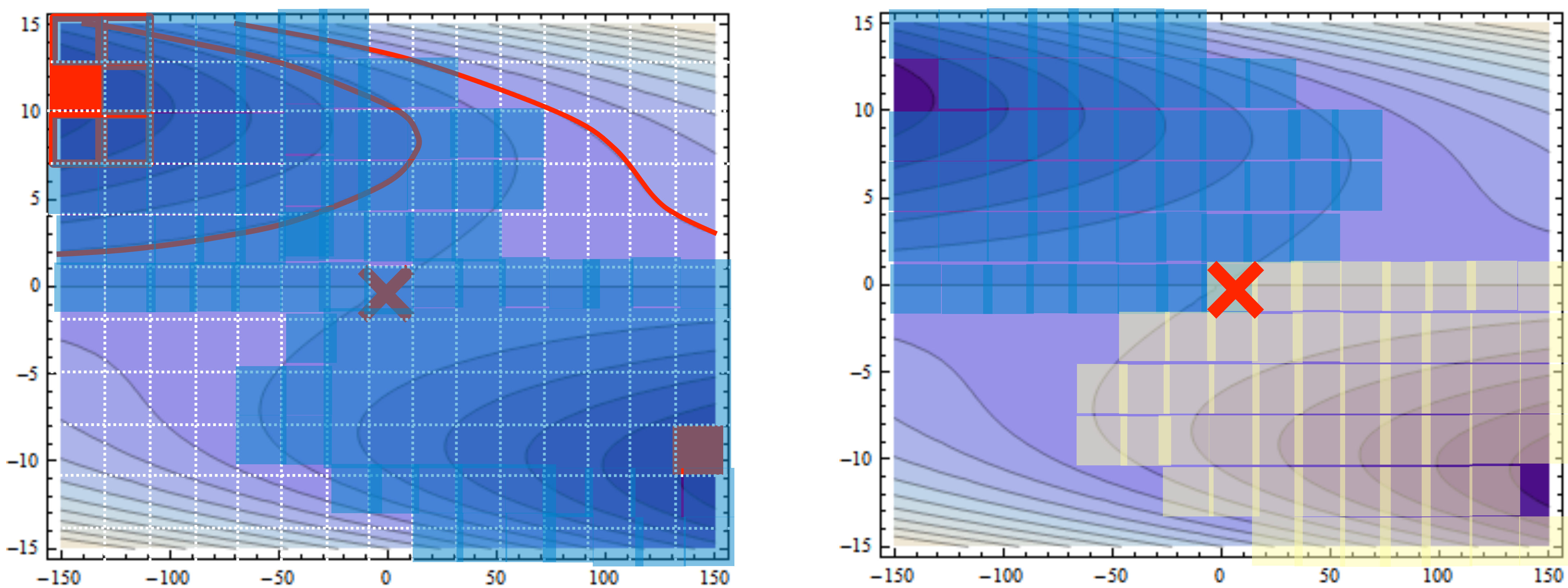
  Separate between two valleys



Immersion method

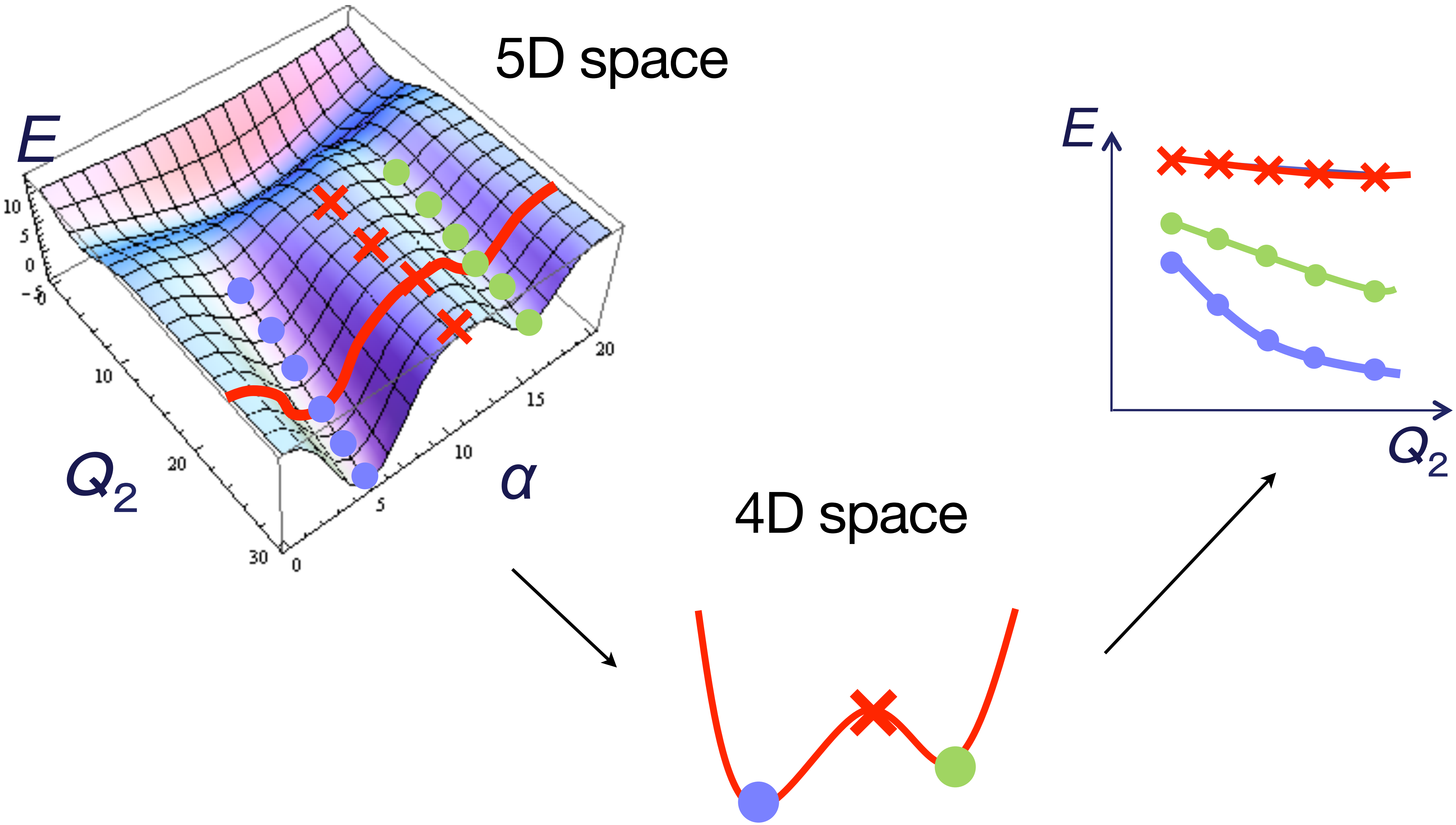
Fill multi-dimensional PES with imaginary water

Wet or Dry?

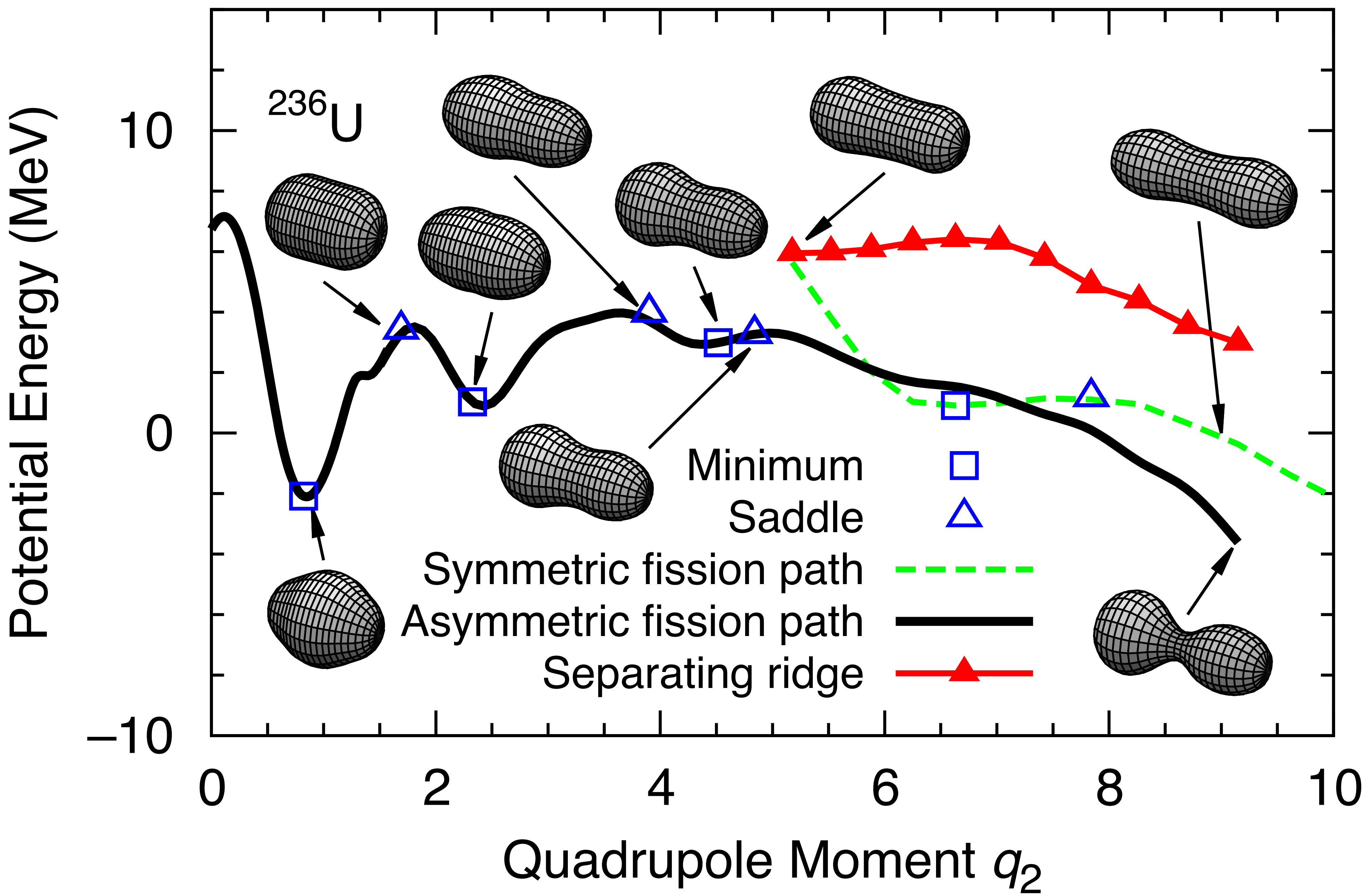


Overlap point is the saddle point

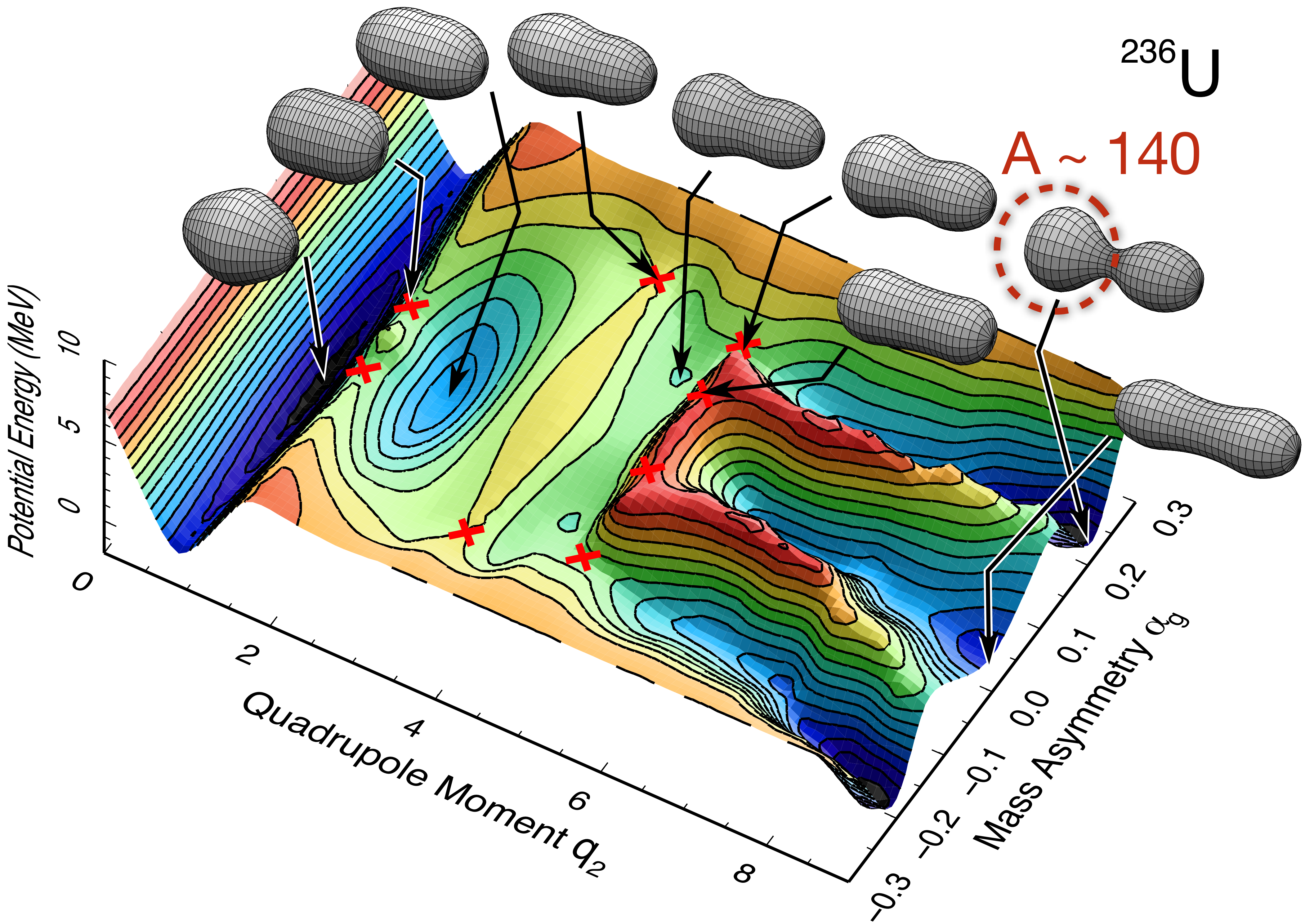
Immersion method



Calculated results



Schematic picture of 2D fission potential-energy surface for ^{236}U



Summary

- We calculate the five-dimensional fission potential energy surface for ^{236}U and investigate the energy-optimum fission path
- We find two deep valleys leading to the mass-asymmetric and -symmetric fission channels
- The mass-asymmetric fission valleys appear due to the strong competition between nuclear deformation shell and Coulomb energies
- Radioactive products such as Cs, I, and Zr are due to the thermal fluctuations in the mass-asymmetric valley

http://arxiv.org/abs/1203.2011