The Uses of Instantons in String Theory

Hirotaka Irie
Yukawa Institute for Theoretical Physics, Kyoto Univ.

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Based on collaborations with
Chuan-Tsung Chan (THU) and Chi-Hsien Yeh (NTU)
7. The uses of instantons (1977), by Sidney Coleman

This article generally discusses the basic roles of instantons in QM and QFT. It should be also applied to string theory.
The first use: *Instantons*

\[
S[X(t)] = \frac{1}{g^2} \int dt \left[ \frac{1}{2} (\dot{X})^2 - V(X) \right]
\]

Hamiltonian:

\[
H = \frac{1}{2} (\dot{X})^2 + V(X)
\]

\[
H|1\rangle = E_{pert}|1\rangle
\]

\[
H|2\rangle = E_{pert}|2\rangle
\]
The first use: *Instantons*

\[
S[X(t)] = \frac{1}{g^2} \int dt \left[ \frac{1}{2} (\dot{X})^2 - V(X) \right]
\]

**Hamiltonian:**

\[
H = \frac{1}{2} (\dot{X})^2 + V(X)
\]

\[
H|1\rangle = E_0|1\rangle + \Delta E|2\rangle
\]

\[
H|2\rangle = \Delta E|1\rangle + E_0|2\rangle
\]

**Eigenstates:**

\[
|\pm\rangle = \frac{1}{\sqrt{2}} (|1\rangle \pm |2\rangle)
\]

\[
E_\pm = E_0 \mp \Delta E
\]

\[
= E_{pert} \mp e^{-S_{inst}} |1\rangle
\]

**Vacuum structure & Non-perturb. Wave-functions**
The second use: \textit{Bounce}

\[ S[X(t)] = \frac{1}{g^2} \int dt \left[ \frac{1}{2} (\dot{X})^2 - V(X) \right] \]

Hamiltonian:

\[ H = \frac{1}{2} (\dot{X})^2 + V(X) \]

\[ \langle 1|H|1 \rangle = E_{pert} + \frac{\pm i}{2} e^{-S_{bounce}} \]

Decay rate

\[ \frac{\langle 2|2 \rangle}{\langle 1|1 \rangle} \sim e^{\beta (E_2 - E_1)} \rightarrow \infty \quad (\beta \rightarrow \infty) \]
Free-energy and chemical potentials

Free energy: \[ F \equiv \frac{1}{\beta} \ln \mathcal{Z}, \quad \mathcal{Z} = \sum_a e^{-\beta E_a} \]

1) Instanton corrections:

\[ F \sim E_{pert} - e^{-S_{inst}} \quad (\beta \to \infty) \]

2) Bounce corrections:

\[ F \sim E_{pert} + \frac{\pm i}{2} e^{-S_{bounce}} \quad (\beta \to \infty) \]

Therefore, the non-perturbative effects have universal structure:

\[ F \sim F_{pert.} + \sum_I \theta_I \times e^{F_{inst.}^{(I)}} + \ldots \]

Use of instantons \[\rightarrow\] To know the Instanton Chemical Potential

However, they are invisible from Perturbation theory!
What happens in String Theory?

“Non-perturbatively Complete” String Theory

We still don’t know yet!

\[ g \to 0 \quad \text{(string coupling)} \]

Feynman Graph = String World Sheets

\[
\mathcal{F} = \ln \mathcal{Z}(g) \sim \mathcal{F}_{\text{pert}} = \sum_{n=0}^{\infty} g^{2n-2}
\]

Then how about instanton corrections?

\[
\sum_{I} \theta_{I} \times e^{\mathcal{F}_{\text{inst}}^{(I)}} + \ldots
\]
Also, we know “solitons/instantons” in string theory

Initiate with [Polchinski ‘94] and a number of people

Whatever “instantons” we have in string theory, we obtain

\[
\mathcal{F} \approx \mathcal{F}_{\text{pert}} + \sum_I \theta_I \times \exp \left[ g^{-a_I} \sum_{n=0}^{\infty} g^n \mathcal{F}^{(I)}_n \right] + \cdots
\]

However, we cannot extract any vacuum structure of string theory, unless we know something about the chemical potentials \( \theta_I \).

We cannot know anything about the string theory landscape!
Non-perturbative completion program

$$\mathcal{F} \simeq \mathcal{F}_{pert} + \sum_{I} \theta_I \times \exp \left[ g^{-a_I} \sum_{n=0}^{\infty} g^n \mathcal{F}_n^{(I)} \right] + \cdots$$

How to know the chemical potentials $\theta_I$ which define String Theory?

( cf. QM and QFT has path-integral! )

Additional Principle for Non-perturbatively Complete String Theory

[Chan-Irie-Yeh ’10 ~ ]

1. There are many ways to non-perturbatively complete the above asymptotic expansions (almost for arbitrary $\theta_I$)

2. Most of them are not “physically acceptable”

3. But, what is “physically acceptable?” $\Leftrightarrow$ Additional Principle
Matrix Models

non-perturbative (solvable) formulation of String Theory

$$ Z = \int dM e^{N \text{tr} V(M)} \iff 2D \text{ (Non-critical) String Theory}$$

With matrix models, we can know “physical value for $\theta_I$”

[Hanada-Hayakawa-Ishibashi-Kawai-Kuroki-Matsuo-Tada ‘04]

We should learn/extract the information from matrix models!

For, example, we succeeded:

1. *formulation of physical constraints* in terms of Stokes phenomena
   *(almost complete in non-critical string theory)* [CIY2 ‘10] [CIY4 ‘12]

2. We found *Quantum Integrability (T-systems)*
   in the physical constraint [CIY3 ‘11]
What can we say about the string theory landscape?

Most of minimal string theory (tachyonless) is meta-stable [CIY4 '12]

Analytic structure of the string theory landscape ⇔ Physical spectrum of (ghost) D-instantons [CIY4 ‘12]

\[ \mathcal{F} \simeq \mathcal{F}_{pert} + \sum_{I} \theta_{I} \times \exp \left[ g^{-a_{I}} \sum_{n=0}^{\infty} g^{n} \mathcal{F}_{n}^{(I)} \right] + \cdots \]

1. D-instanton: \( a_{I} = 1, \quad \mathcal{F}_{0}^{(I)} < 0 \)
   ghost D-instanton: \( a_{I} = 1, \quad \mathcal{F}_{0}^{(I)} > 0 \) \( \langle B_{gh} \rangle \equiv -|B\rangle \)

   [Okuda-Takayanagi ‘06]

   Exponentially large \( \Rightarrow \) Instability / break down of perturb. theory

   However, one has claimed that we can turn them off by spelling

   “it contradicts with perturbation theory!”

2. Actually, minimal string theory cannot avoid these branes
due to the physical consistency with matrix models [CIY4 ‘12]

3. Also, most of D-instantons in the worldsheet theory cannot appear!
This can be easily understood in the matrix model effective potential:

- Instability (↔ Ghost D-instanton)

Why our physical constraints are special?

1. "which non-critical string theory is (un)stable"
2. "decay rate" of string theory [identify bounce solutions]
3. "true vacuum" of string theory [universal vacuum]
4. "the string-theory landscape" in this string theory

Also with our constraints, we are now able to calculate:

- In (p,q) minimal string theory, there are a huge number of D-branes which cannot appear!
- (Topological string is also the same)

Future investigation!
Summary

• The instanton chemical potentials in string theory are a key information for non-perturbative completion of string theory.

• It is like path-integral formulation in QM and QFT. They are responsible for analytic structure of the string theory landscape.

• With this information, we actually calculate “non-perturbative instability/decay rate/true vacuum” of string theory. They are quite universal.

• More fundamental understanding of “why the matrix model is special” is missing and remained for future investigations.