The Uses of Instantons in String Theory

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Based on collaborations with

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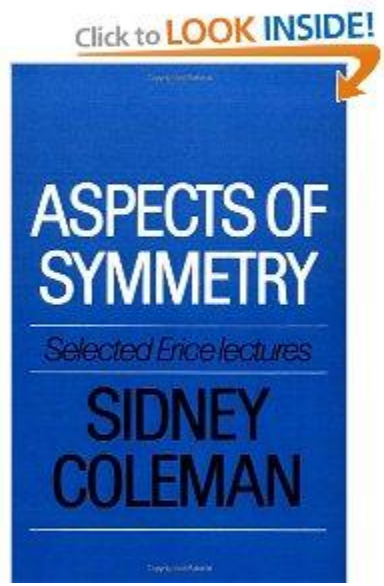
7. The uses of instantons (1977),
by Sidney Coleman

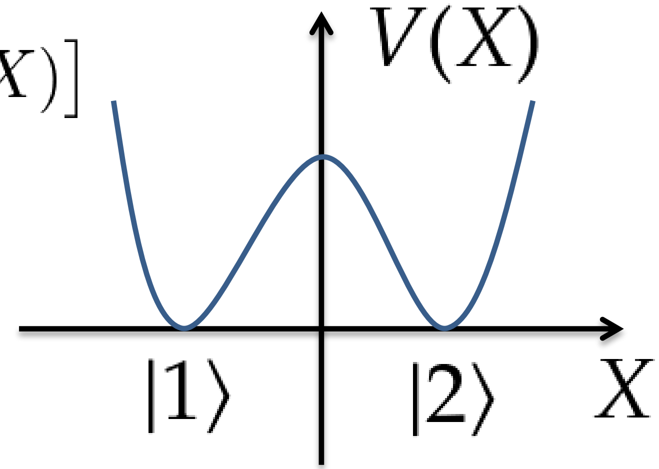


This article generally discusses
the basic roles of instantons in QM and QFT

it should be also applied to string theory

The first use: *Instantons*

$$S[X(t)] = \frac{1}{g^2}\int dt\,[\frac{1}{2}(\dot{X})^2 - V(X)]$$

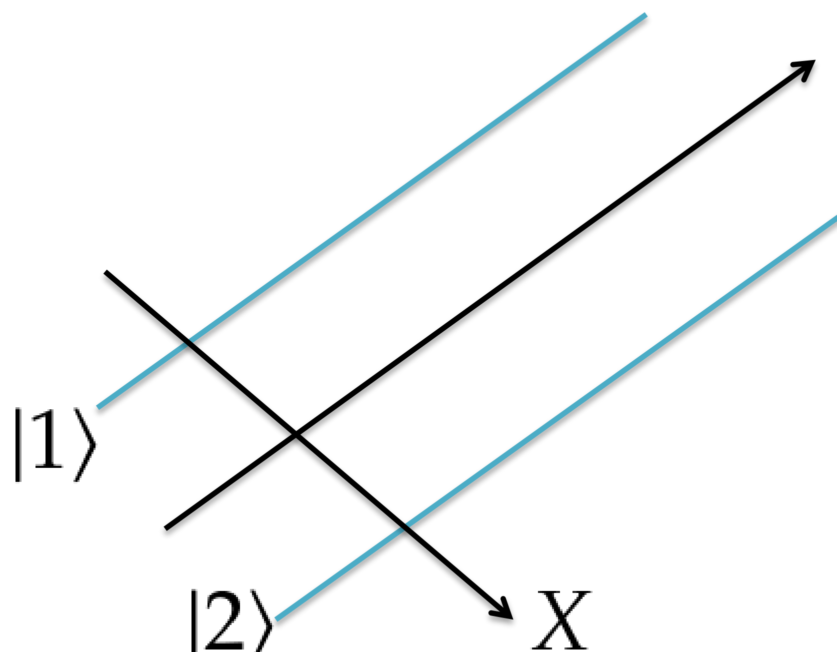


<u>Hamiltonian:</u>

$$H = \frac{1}{2}(\dot{X})^2 + V(X)$$

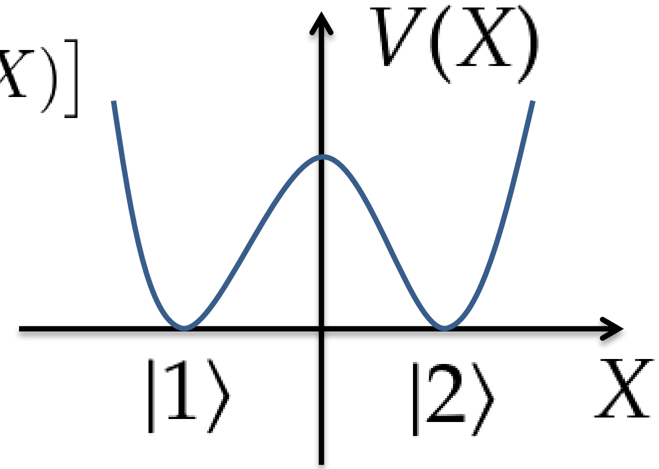
$$H|1\rangle = E_{pert}|1\rangle$$

$$H|2\rangle = E_{pert}|2\rangle$$



The first use: *Instantons*

$$S[X(t)] = \frac{1}{g^2}\int dt\left[\frac{1}{2}(\dot{X})^2 - V(X)\right]$$



Hamiltonian:

$$H = \frac{1}{2}(\dot{X})^2 + V(X)$$

$$H|1\rangle = E_0|1\rangle + \Delta E|2\rangle$$

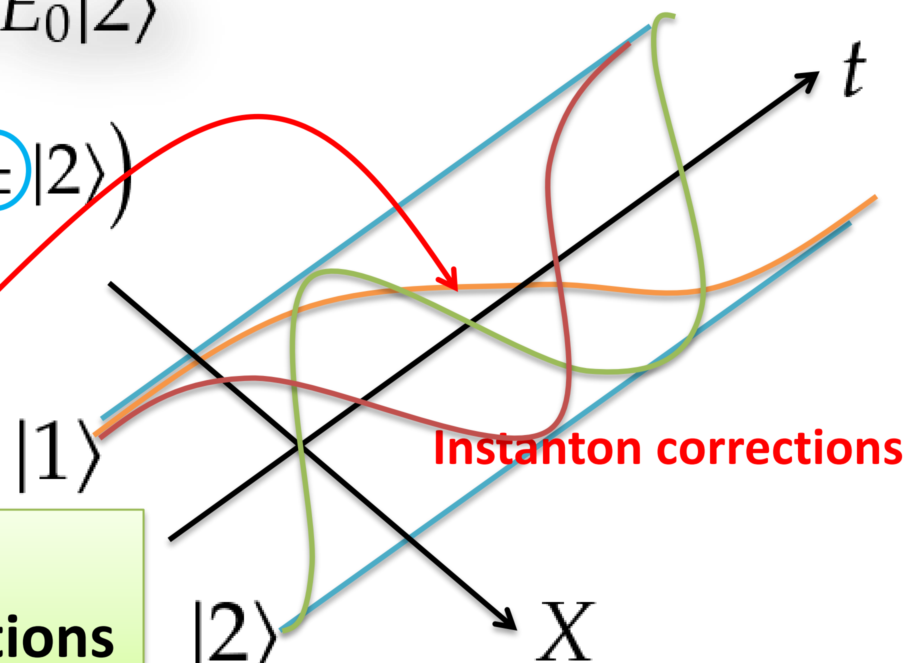
$$H|2\rangle = \Delta E|1\rangle + E_0|2\rangle$$

Eigenstates:

$$|\pm\rangle = \frac{1}{\sqrt{2}}\Big(|1\rangle \pm |2\rangle\Big)$$

$$E_\pm = E_0 \mp \Delta E$$

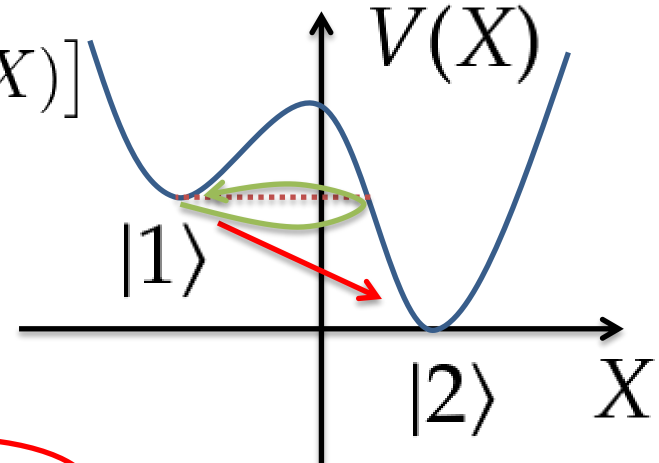
$$= E_{pert} \mp e^{-S_{inst}}$$



Vacuum structure
& Non-perturb. Wave-functions

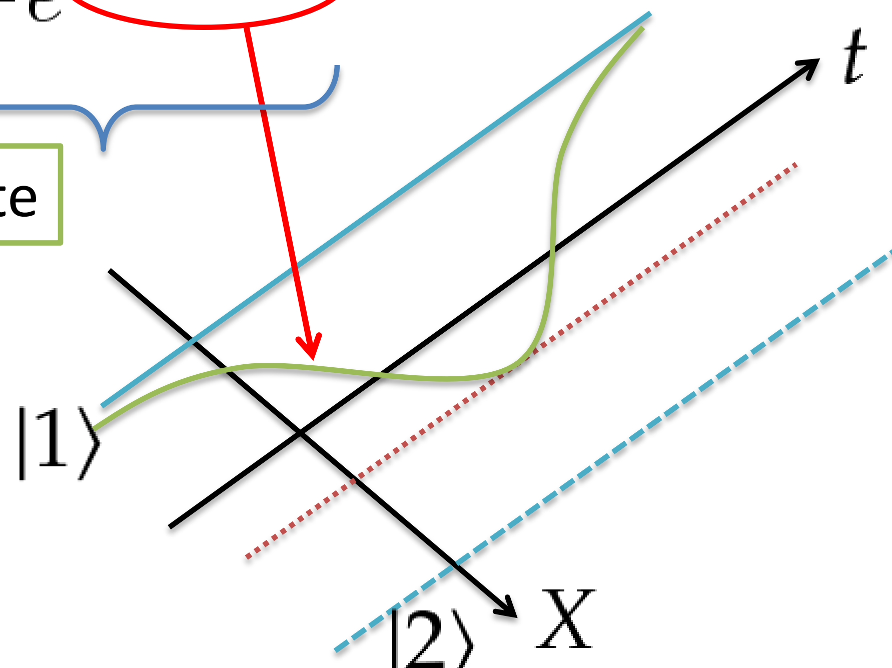
The second use: *Bounce*

$$S[X(t)] = \frac{1}{g^2}\int dt\left[\frac{1}{2}(\dot{X})^2 - V(X)\right]$$



Hamiltonian: $H = \frac{1}{2}(\dot{X})^2 + V(X)$

$$\langle 1|H|1\rangle = E_{pert} + \underbrace{\frac{\pm i}{2} e^{-S_{bounce}}}_{\text{Decay rate}}$$



instability is caused by $|2\rangle$

$$\frac{\langle 2|2\rangle}{\langle 1|1\rangle} \sim e^{\beta(E_2 - E_1)} \to \infty$$

$$(\beta \to \infty)$$

Free-energy and chemical potentials

Free energy: $$\mathcal{F} \equiv \frac{1}{\beta} \ln \mathcal{Z}, \quad \mathcal{Z} = \sum_a e^{-\beta E_a}$$

1) Instanton corrections:

$$\mathcal{F} \simeq E_{pert} - e^{-\mathcal{S}_{inst}} \quad (\beta \to \infty)$$

2) Bounce corrections:

$$\mathcal{F} \simeq E_{pert} + \frac{\pm i}{2} e^{-\mathcal{S}_{bounce}} \quad (\beta \to \infty)$$

Therefore, the non-perturbative effects have universal structure:

$$\mathcal{F} \simeq \mathcal{F}_{pert.} + \sum_I \theta_I \times e^{\mathcal{F}^{(I)}_{inst.}} + \cdots$$

Use of instantons → To know the Instanton Chemical Potential

However, they are invisible from Perturbation theory!

What happens in String Theory?

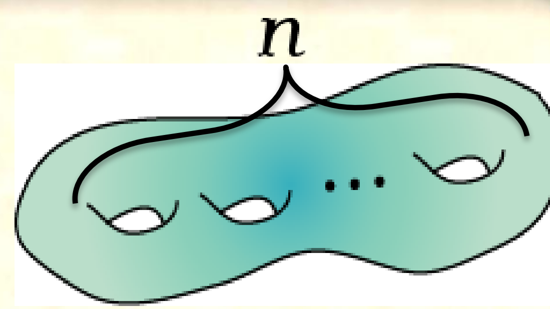
"Non-perturbatively Complete"
String Theory

$g \to 0$
(string coupling)

We still don't know yet!

Perturbation theory!

$$\mathcal{F} = \ln \mathcal{Z}(g) \simeq \mathcal{F}_{pert} = \sum_{n=0}^{\infty} g^{2n-2}$$

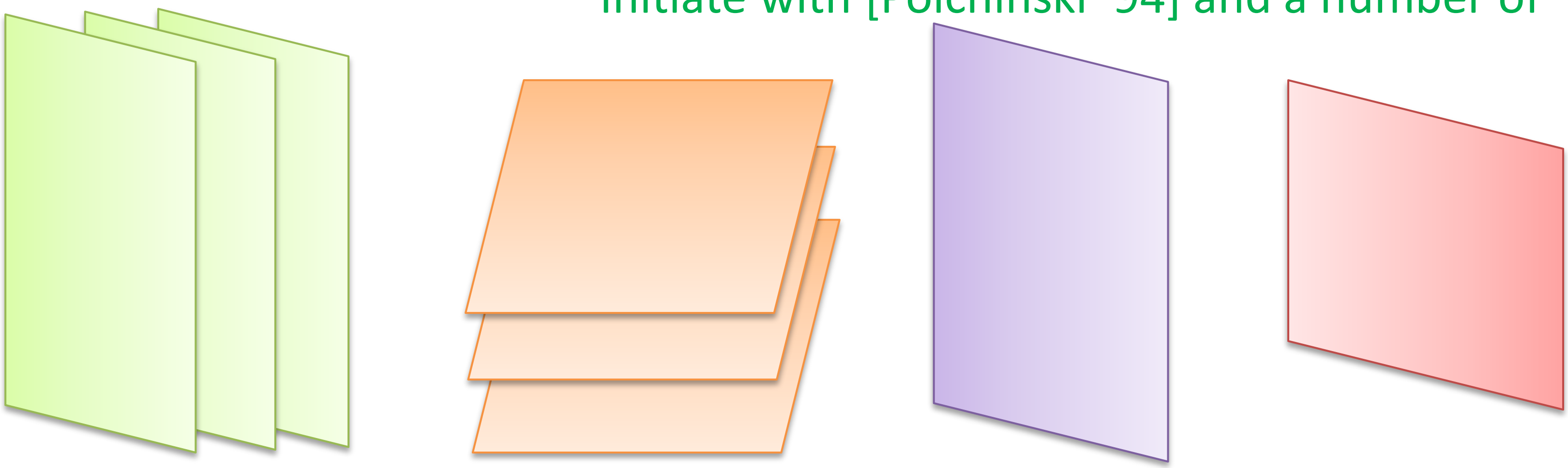


Feynman Graph = String World Sheets

Then how about instanton corrections? $\boxed{\sum_I \theta_I \times e^{\mathcal{F}^{(I)}_{inst.}} + \cdots}$

Also, we know “solitions/instantons” in string theory

Initiate with [Polchinski ‘94] and a number of people



D-branes S-branes NS-branes Membranes …

Whatever “instantons” we have in string theory, we obtain

$$\mathcal{F} \simeq \mathcal{F}_{pert} + \sum_I \theta_I \times \exp\left[g^{-a_I} \sum_{n=0}^{\infty} g^n \mathcal{F}_n^{(I)} \right] + \cdots$$

However, we cannot extract **any vacuum structure of string theory**, unless we know something about **the chemical potentials** θ_I

We cannot know anything about the string theory landscape!

Non-perturbative completion program

$$\mathcal{F} \simeq \mathcal{F}_{pert} + \sum_I \theta_I \times \exp\left[g^{-a_I} \sum_{n=0}^{\infty} g^n \mathcal{F}_n^{(I)} \right] + \cdots$$

How to know the chemical potentials θ_I which define String Theory?

(cf. QM and QFT has path-integral!)

Additional Principle for Non-perturbatively Complete String Theory

[Chan-Irie-Yeh '10 ~]

1. There are many ways to non-perturbatively complete the above asymptotic expansions (almost for arbitrary θ_I)
2. Most of them are ***not*** "physically acceptable"
3. But, what is "physically acceptable?" ⇔ Additional Principle

Matrix Models

non-perturbative (solvable) formulation of String Theory

$$\mathcal{Z} = \int dM e^{N \mathrm{tr} V(M)} \Leftrightarrow$$ 2D (Non-critical) String Theory

With matrix models, we can know “physical value for θ_I ”

[Hanada-Hayakawa-Ishibashi-Kawai-Kuroki-Matsuo-Tada ‘04]

We should learn/extract the information from matrix models!

For, example, we succeeded:

1. *formulation of physical constraints* in terms of Stokes phenomena (almost complete in non-critical string theory) [CIY2 ’10] [CIY4 ‘12]

2. We found Quantum Integrability (T-systems) in the physical constraint [CIY3 ‘11]

What can we say about the string theory landscape?

Most of minimal string theory (tachyonless) is meta-stable [CIY4 '12]

Analytic structure of the string theory landscape
⇔ Physical spectrum of (ghost) D-instantons [CIY4 '12]

$$\mathcal{F} \simeq \mathcal{F}_{pert} + \sum_I \theta_I \times \exp\left[g^{-a_I} \sum_{n=0}^{\infty} g^n \mathcal{F}_n^{(I)} \right] + \cdots$$

1. D-instanton: $a_I = 1, \quad \mathcal{F}_0^{(I)} < 0$

   ghost D-instanton: $a_I = 1, \quad \mathcal{F}_0^{(I)} > 0 \quad \left(|B_{gh}\rangle \equiv -|B\rangle \right)$

   [Okuda-Takayanagi '06]

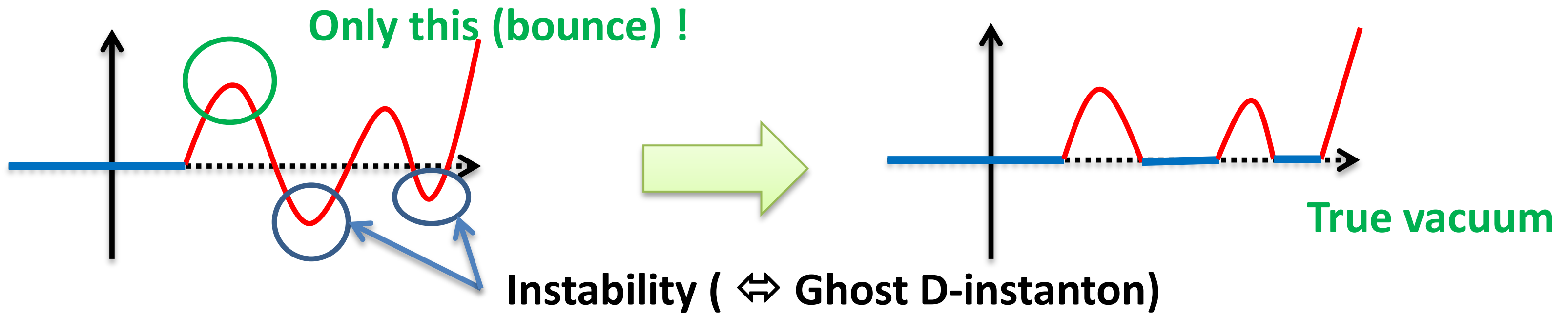
   Exponentially large → **Instability / break down of perturb. theory**

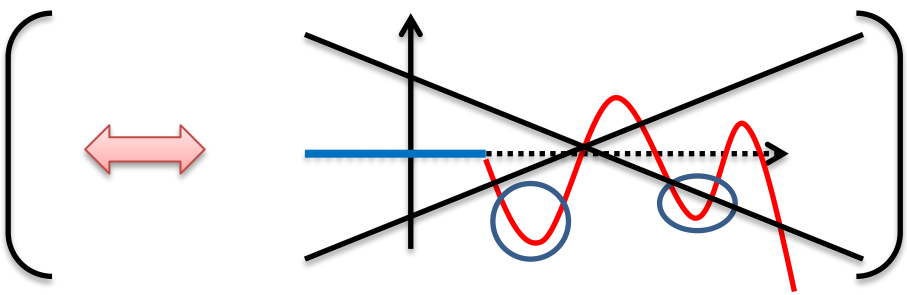
   However, one has claimed that we can turn them off by spelling

   "it contradicts with perturbation theory!"
2. Actually, minimal string theory ***cannot avoid these branes***

   due to **the physical consistency** with matrix models [CIY4 '12]
3. Also, most of D-instantons in the worldsheet theory *cannot appear*!

This can be easily understood in the matrix model effective potential:





In (p,q) minimal string theory,
there are **a huge number of D-branes** **which cannot appear**!
(Topological string is also the same)

Also with our constraints, we are now able to calculate

1. "**which non-critical string theory is (un)stable**"
2. "**decay rate**" of string theory **[identify bounce solutions]**
3. "**true vacuum**" of string theory **[universal vacuum]**
4. "**the string-theory landscape**" in this string theory

Why our physical constraints are special? → Future investigation!

Summary

- The **instanton chemical potentials** in string theory are a key information for non-perturbative completion of string theory
- It is like path-integral formulation in QM and QFT. They are responsible for **analytic structure** of the string theory landscape
- With this information, we actually calculate "non-perturbative instability/decay rate/true vacuum" of string theory. They are quite universal.
- More fundamental understanding of *"why the matrix model is special"* is missing and remained for future investigations.