On "Universe" from canonical tensor model

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cf. arXiv:1203.0421

Quantum gravity

An idea – emergent space

Space is dynamically generated, not fundamental.

The challenge:

Discrete degree's of freedom

↓ Infrared

Space {
- Classical
- 3dim
- Local
- Gravity, matters

Tensor model

Matrix model M_{ab}

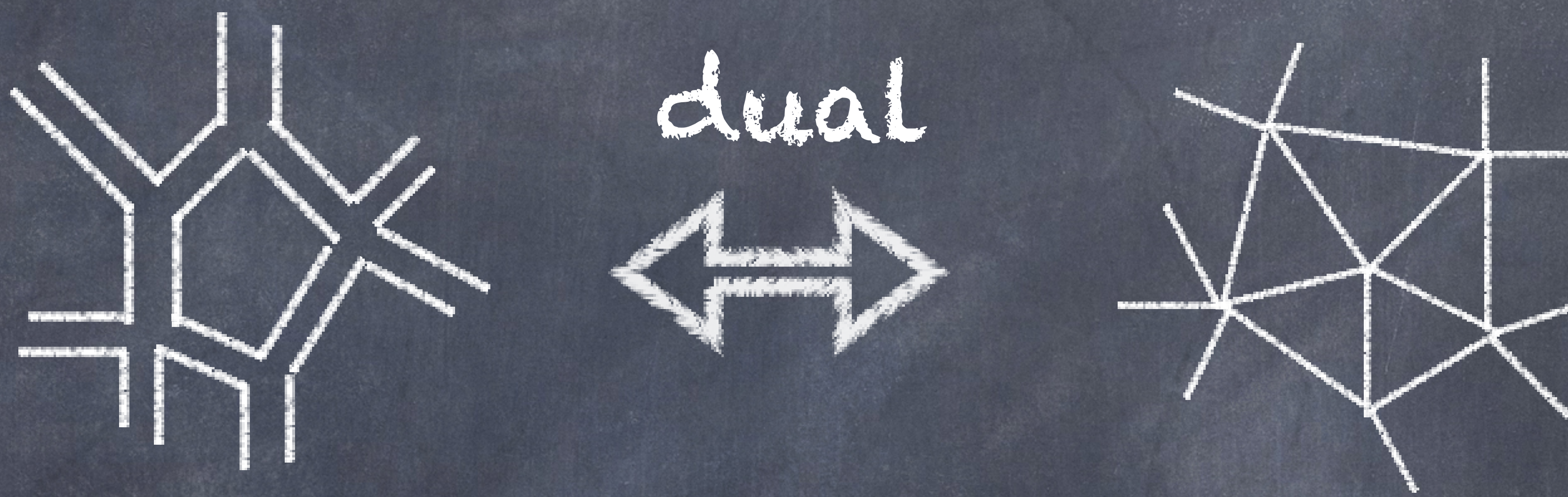
2dim quantum gravity
String theory - Irie's lunch talk
Susy theories
Statistical systems

Tensor model (Rank-three) M_{abc}

Quantum gravity in D>2
Loop quantum gravity

Interpretations

- Feynman diagrams <-> simplicial manifolds



dependent on D

- Rank-three tensor model M_{abc}

$$f_a \cdot f_b = \underline{M_{ab}}^c f_c$$

Fuzzy space - universal for any D

Local time

How can time be introduced to tensor models ?

(1) Global time $M_{abc}(t)$

Problems: hard to believe, need God

Lorentz symmetry easily broken

(2) Local time for each index $M_{(a\,t_a)(b\,t_b)(c\,t_c)}$

Problem: multiple time integrals in Lagrangian

(3) Hamilton formalism with local time

Dynamical evolution = time

Kill unphysical modes automatically

Consistency of local time evolution

$$[H_a, H_b] = \text{gauge transformation}$$

$$o(N)$$

N : number of "points"

Local Hamiltonian

1st surprise

$$H_a = P_{a(bc)} P_{bde} M_{cde}$$

( ): symmetrization

$$\{M_{abc}, P_{def}\} = \delta_{abc,def}$$

There exists one and only one
on physically reasonable assumptions

Assumptions

(i) A Hamiltonian constraint has one index.

(ii) The kinematical symmetry is given by an orthogonal group.

(iii) A consistent first class constraint algebra is formed by a Hamiltonian constraint and the generators of the kinematical symmetry.

(iv) A Hamiltonian constraint is invariant under time reversal transformation.

(v) A Hamiltonian constraint is an at most cubic polynomial function of canonical variables.

(vi) There are no disconnected terms in a constraint algebra.

First class constraint algebra

$$\{H(T_1), H(T_2)\} = D([\tilde{T}_1, \tilde{T}_2])$$

$$\{D(V), H(T)\} = H(VT)$$

$$\{D(V_1), D(V_2)\} = D([V_1, V_2])$$

$$H(T) = T^a H_a,\ D(V) = V^{ab} D_{ab}$$

$D_{ab} = P_{cd[a} M_{b]cd}$: o(N) generators

$$\tilde{T}_{bc} = T^a \underline{P_{a(bc)}}$$

Open algebra - closes only on constraints

Localized limit

2nd surprise

Formally replacing a->x and assuming local configurations,

$$M_{xyz},\ P_{xyz} \approx 0, \text{ unless } x \sim y \sim z$$

reproduces the DeWitt algebra of GR

$$\{H(T_1), H(T_2)\} = D(T_1\partial^i T_2 - T_2\partial^i T_1)$$

$$\{D(V^i), H(T)\} = H(V^i\partial_i T)$$

$$\{D(V_1^i), D(V_2^i)\} = D(V_1^i\partial_i V_2^j - V_2^i\partial_i V_1^j)$$

$$H(T) = \int dx\, T(x)\, \mathcal{H}(x),\ D(V^i) = \int dx\, V^i(x)\, \mathcal{H}_i(x)$$

Quantization
(preliminary)

No space -> not a field theory
Basically a quantum mechanical system

Quantization can be done by methods in literatures.

Faddeev 1969

$$\int DPDM\, \delta(H_a)\delta(D_{ab})\delta(\chi_a)\delta(\chi_{ab})\mathrm{Det}(\{HD,\chi\})e^{i\int dt\, P\dot{M}}$$

χ_a, χ_{ab} : gauge fixing condition

Example of gauge fixing condition

$$\chi_a = P_{abb} - t = 0$$ cf. York time

$$\chi_{ab} = P_{abb} - P_{aab} = 0$$

By solving constraints and gauge fixing conditions

$$\int dt\, P\dot{M} = \int dt\, P^* \dot{M}^* - H^*(P^*, M^*)$$

Shrödinger equation

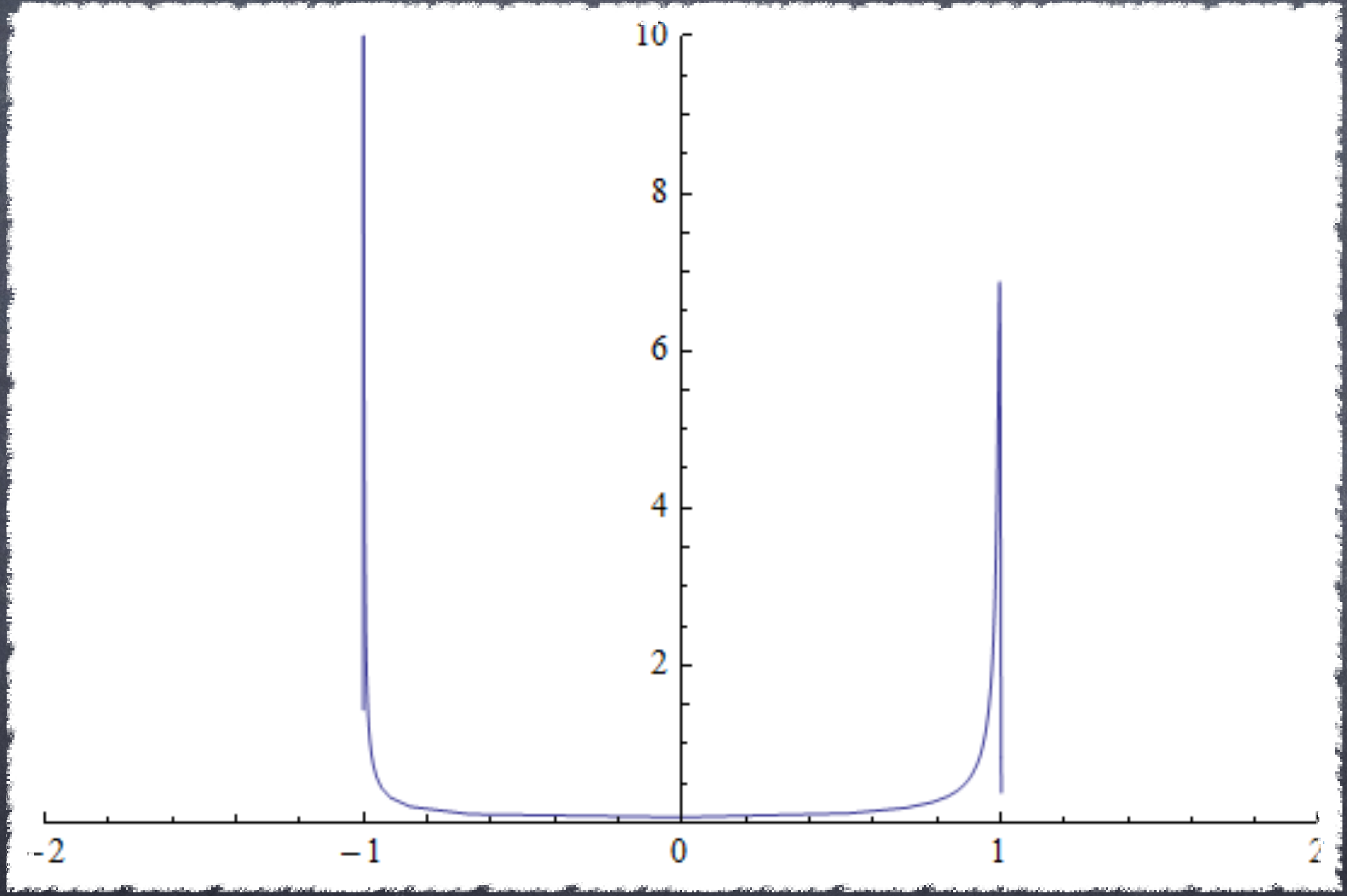
$$i \frac{\partial \psi}{\partial t} = H^* \psi$$

Wave function dynamics
(preliminary)

N=2

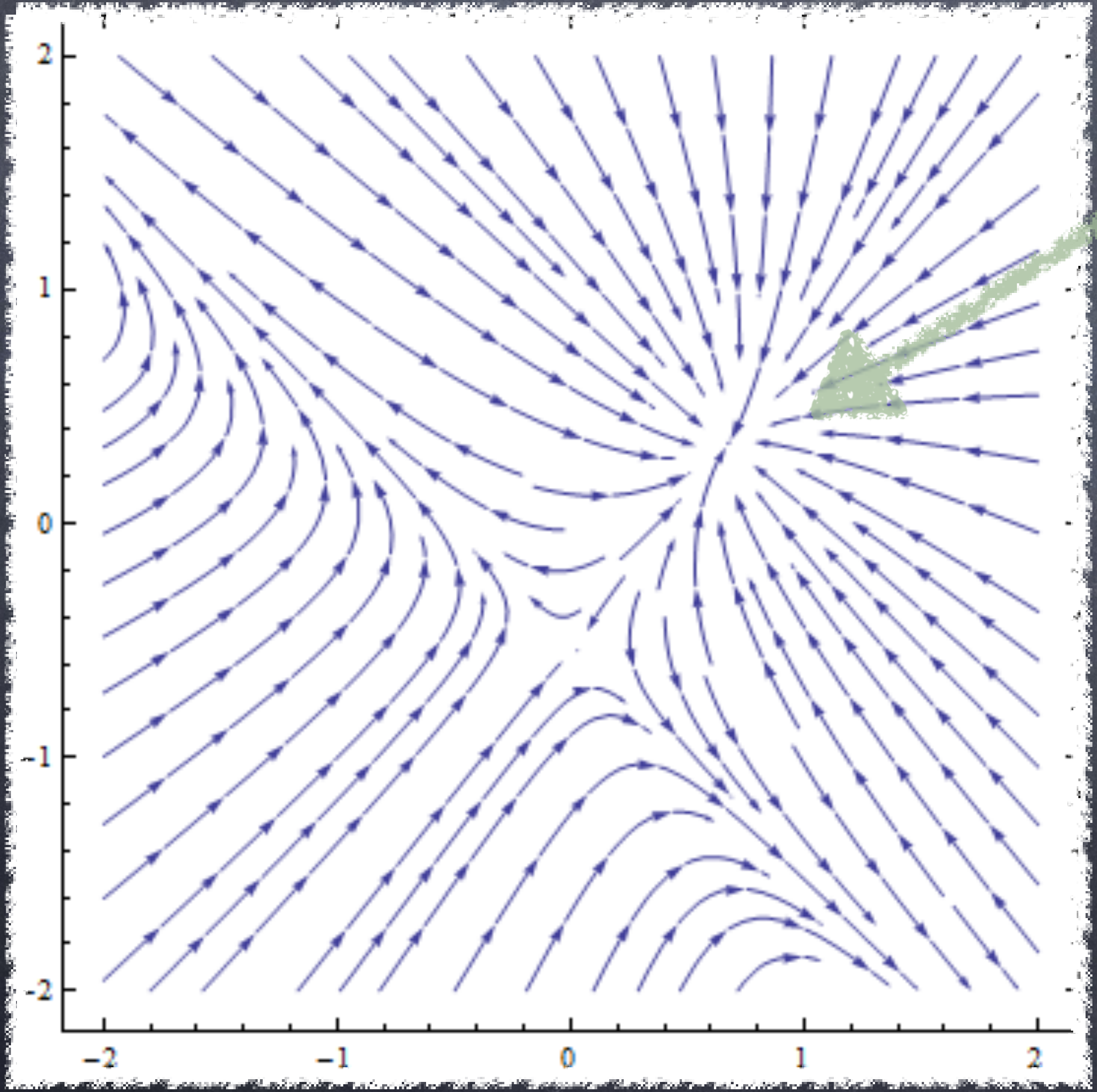
"Universe" consisting of two "points"

$\psi(p)$



Emergence of classicality

N=3



Attractor

Locality emerges

$$\frac{M_{aab}}{M_{aaa}} = -\frac{1}{6}$$

$$M_{abc} = 0$$

3rd surprise

Summary

(1) Canonical tensor model with consistent local time is proposed.
(2) Contains some surprises.
(3) Interesting rich quantum dynamics.

Future problems

Large N. Clarify quantum dynamics.
Wheeler-DeWitt (with Prof. Freidel)
Gravity.
merely "Universe" or our Universe ?