NONLINEAR LONG-WAVELENGTH PERTURBATIONS

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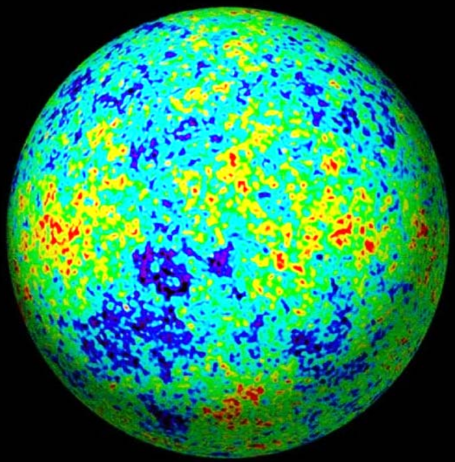


Ref: arXiv:1210.6525(PTEP)

I wish you a ***Happy New* Year**, in Kyoto！

2013（巳年：Year of Snake）

書：The Mayor of Kyoto-city:

Kadokawa（門川大作市長）→

◇ Is there the end of the (Our) Universe?

A. Perhaps Yes, it exists as the 'temporal' cosmological horizon of our Universe

Beyond the cosmological horizon,

What's the deal ?

◇ What is Physics(?) beyond the Horizon ?

◆ *Motivations*

- Cosmic Landscape (String theory)
  Various values of Cosmological Constant
- Relating to **Our origin** (Astrophysics)
  Stars, Galaxies, Clusters,..,??
  ■ Fluctuations of Inflaton

➢ **Classification** of quantum perturb (What's the Observer ?)

➢ **Classical evolution**: **Our formalism with Naruko, Mukohyama, Yokoyama, Sasaki, 2010, 2012**

◇ Classical Evolution of Long-wavelength Perturbations Beyond the Horizon ?

◆ Our formalism

✓ Einstein Gravity + Gradient Expansion

✓ Background spacetime: the FLRW universe

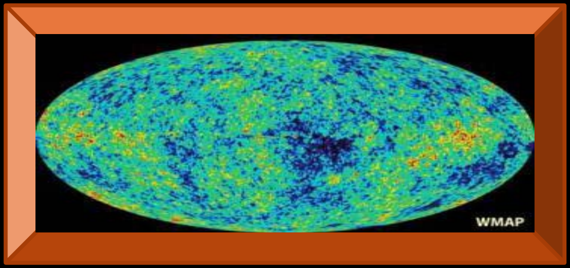
■ Applications

- Time evolution of Primordial perturbation (Fully nonlinear effect) *What is Inflaton ?*
- Observation of CMB fluctuations (also LSS)

  Nonlinearity(Non-Gaussianity) of CMB (PLANCk, data release on Apri,2013)
- Bridge to construction of Quantum Gravity (?)

□ Cosmic Microwave Background (*CMB*)

Messages from Oldest
`Cosmic Photo`



$$T = 2.73K$$

① Homogeneous and isotropic (FLRW)

- Evidence for Cosmological principle
 ＋ the *horizon* problem

$$\frac{\delta T}{T} = 10^{-5}$$

② Tiny fluctuation as a ***Seed***

- Origin of large scale Structure formation (Cluster & Galaxy)

◆ ***Key roles: Paradigm of Inflation*** ***(Sato, Guth '81)***

①Accelerated expansion + ②Primordial density perturb.

◆ What is a scalar field named *Inflaton* ?

$\delta\phi$

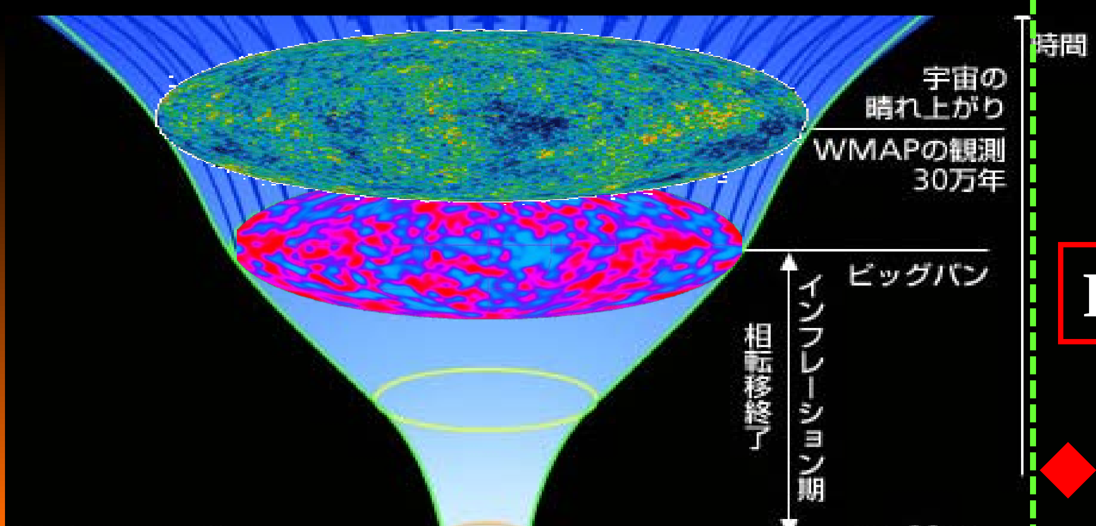
Inflaton

→ **Evolution in Superhorizon scale** →

$$\frac{\delta T}{T} = 10^{-5}$$

CMB anisotropy

◆ Primordial perturbation as a **「Window」** to ‘see’ the high energy physics before the Big Bang universe



◆ **Observation**
(CMB, Galaxies)

⇕

Primordial perturbation

⇕

◆ **Model for early universe**

◆ What is the scalar field named *Inflaton* ?

$$\delta\phi$$

Inflaton

Evolution in Superhorizon scale

$$\frac{\delta T}{T} = 10^{-5}$$

CMB anisotropy

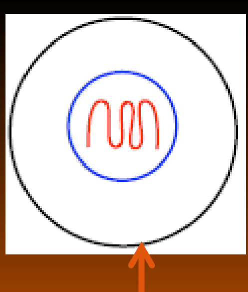
$$\zeta = \psi + H\frac{\delta\phi}{\dot\phi}$$

Gravitational Potential

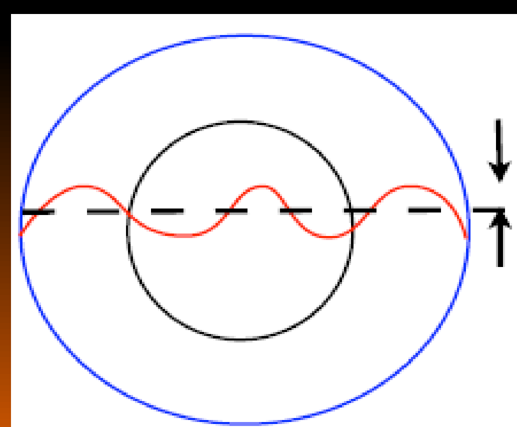
● Conserved : ζ

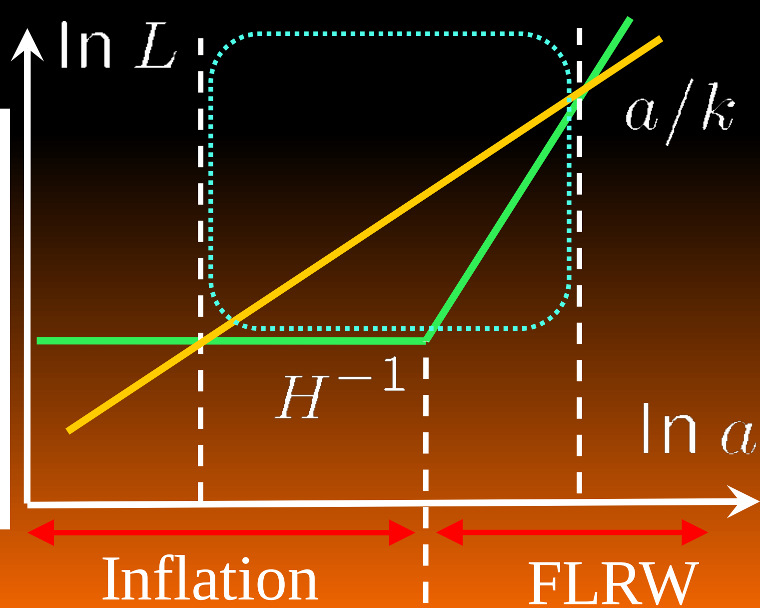
If adiabatic fluid or single slow-roll scalar

$$\frac{\delta T}{T} = \frac{1}{5}\zeta$$



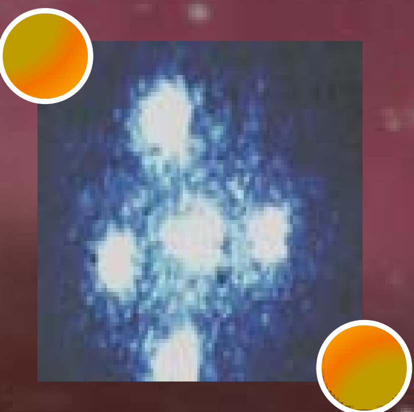
Horizon





◆Classical evolution of superhorizon perturbation

➢ *Spatial gradient approach* Salopek & Bond (90)



◆ ***δN formalism***

(Starobinsky85, Sasaki &Stewart96, Sasaki & Tanaka99, Nambu & Taruya96 Lyth & Rodoriguez05)

$$\zeta(t, \mathbf{x}) = \text{const}$$

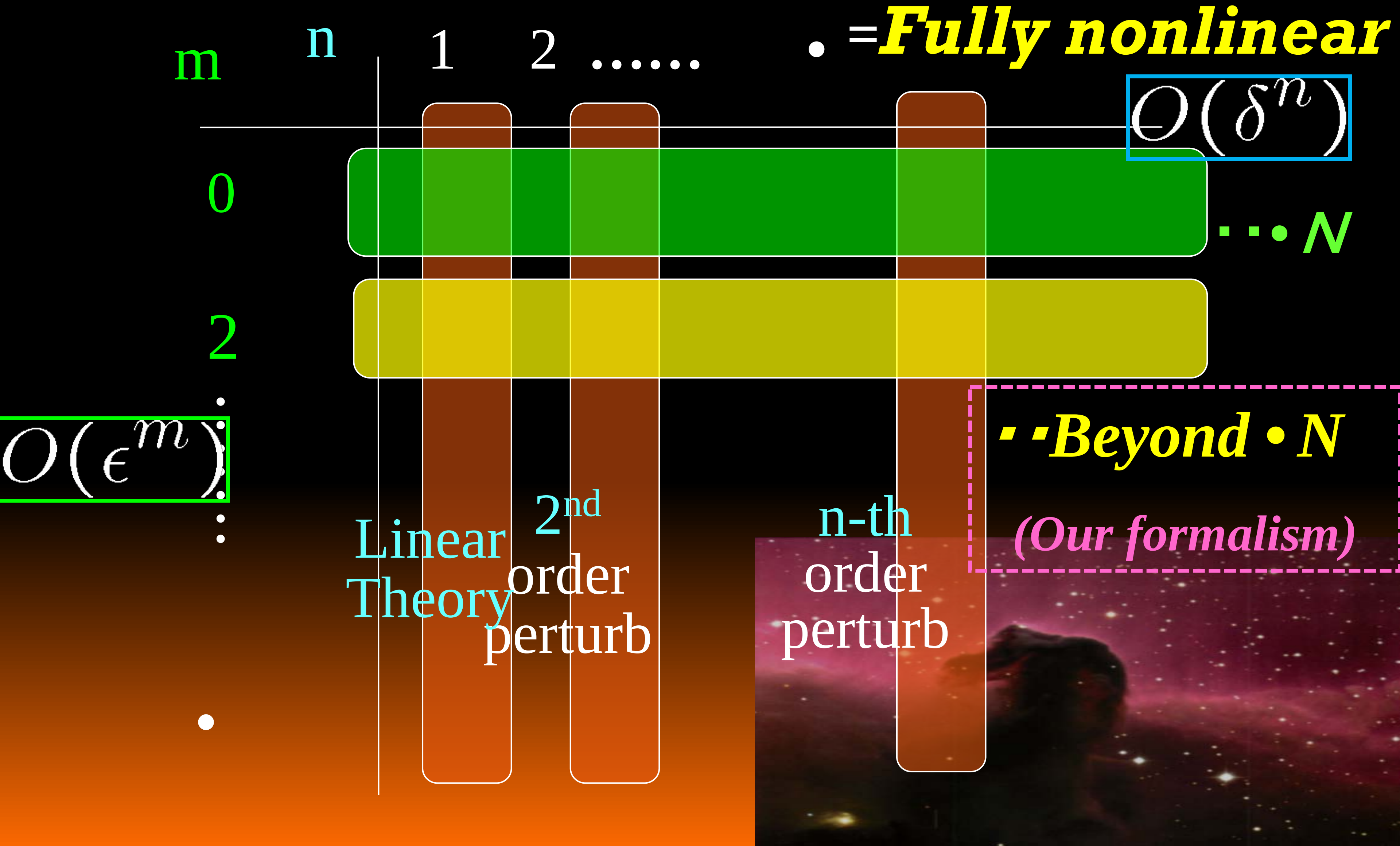
Next-leading order

◆ ***Beyond δN formalism***

$$\zeta(t, \mathbf{x}) \neq \text{const}$$

- **Single Inflaton** (YT, Mukohyama, Sasaki & Tanaka 10, YT and Yokoyama 11)
- **Multi-Inflaton** (Naruko, YT & Sasaki, arXiv: 1210.6525)

Nonlinear perturbations on superhorizon scales up to *Next-leading order* in the expansion



Time evolution of superhorizon perturbation

- *Spatial gradient approach* : $\epsilon = k/(aH)$ Salopek & Bond (90)

At Leading order $O(\epsilon^0)$

(Starobinsky85, Sasaki, Stewart & Tanaka , Nambu & Taruya96 Lyth & Rodoriguez05)

δN formalism

$$\zeta(t, \mathbf{x}) = \text{const}$$

- ***Adiabatic growing mode is constant on the comoving hyp.***

Formulation valid for Multi-field , but adiabatic field at last final time hys. as **Single field**

- **Ignore temporal enhancement of decaying modes**

(relating to Slow-roll conds.)

Go next-leading order $O(\epsilon^2)$

in order to study time evolution on superhorizon scales.

going

① beyond slow-roll conds.
② beyond single field

◆ ***Beyond δN formalism***

$O(\epsilon^2)$

~~$\zeta(t, \mathbf{x}) = \mathrm{const}$~~

■ Single with non-slow-roll stage (relating to ①)

(YT, Mukohyama, Sasaki, Tanaka 10, YT and Yokoyama 11)

➢ **Enhancement of Decaying modes** play an important role in this case

■ Develop formalism for Multi-field with non-slow-roll phase (**relating to** ①&②)

Gradient expansion approach to multi-field inflation valid through $O(\epsilon^2)$

(Naruko , YT & Sasaki 12)

$$S_m = \int d^4x\sqrt{-g}P(X^{IJ},\phi^K)\,;\;\; X^{IJ} \equiv -g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J$$

◆ **Application**：In inflationary scenario, **Temporal violation of slow-roll cond.**

Ex) Double inflation

Different masses of inflatons: Non-slow-roll stage may appear between both inflation stages

● *Multi-field* Motivated by Supergravity where many flat-directions playing a role of multi-field inflation

If Observations *(CMB,Galaxies)* tell us signal of Non-Gaussianity, we can distinguish between single and multi-field inflation

(3+1)-decomposition & Gradient expansion

$$ds^2 = -\alpha^2 dt^2 + \hat{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

Spatial metric; $\hat{\gamma}_{ij} = a^2(t) e^{2\psi} \gamma_{ij}$ • Det =1

◆ Gradient expanded Einstein eq and
Solve First-order diff. eqs in each order of ε

Gauge (spatial) : NO shift $\beta^i = 0$

$\epsilon = k/(aH)$

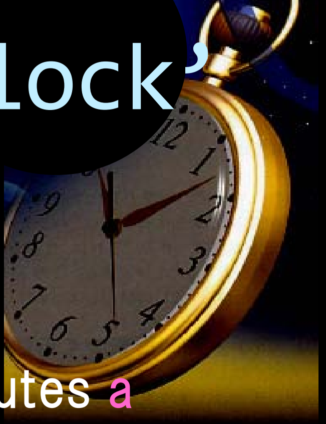
Gauge (temporal) : Extrinsic curvature is uniform

$$\Rightarrow \quad \partial_t \psi = H(t)[\alpha(t, x^i) - 1]$$

gives time evolution of superhorizon curvature perturb.

- Fluctuation of Lapse : $\alpha - 1$
- Non-adiabatic pressure

Various time slices: 'time clock'



◆ Uniform expansion, K

$$\partial_t \psi = H(t)[\alpha(t, x^i) - 1]$$

- For Single, can derive the general solution (YT & Mukohyama 09)
- For Multi, since Lapse contributes a nonzero in leading order, There is a difficulty to solving eq.

◆Uniform e-folding number, N

$$\partial_t \psi = 0$$

- `Homogeneous time clock` ⇒ ψ is Constant with time
- For Multi, can construct the solutions

➤ It is necessary to compare a quantity that can be directly related to **observable quantities** as the one **in uniform K**

So we have constructed a general method for Nonlinear Gauge transformation laws, *successfully !*

◆ Summary

- We develop a theory of nonlinear cosmological perturbations on superhorizon by employing spatial Gradient Expansion approach valid up to **Next-to-leading order** in the expansion.

- We provide a **general formalism** to obtain the solution in the multi-component scalar field with a general kinetic term and a general form of the potential. It is important for the case where **a time evolution** caused by **Decaying mode** relating to **temporal Violation of slow-roll cond.**

- **Fully nonlinear gauge transformation laws** play an essential role

- **Beyond δN formalism**; can transform **Uniform N** (deriving solution)• **Uniform K** (last time hyps.) to compare the observational signature.

  - Consistency : Re-construct the known result for single field
  - Application : As analytically solvable example, construct the solution for canonical multi-field with exponential potential
  - Future work : **Construct General classification** of Single- or Multi- scalar as **Origin of Inflaton** beyond Suyama-Yamaguchi inequality under more general situations.