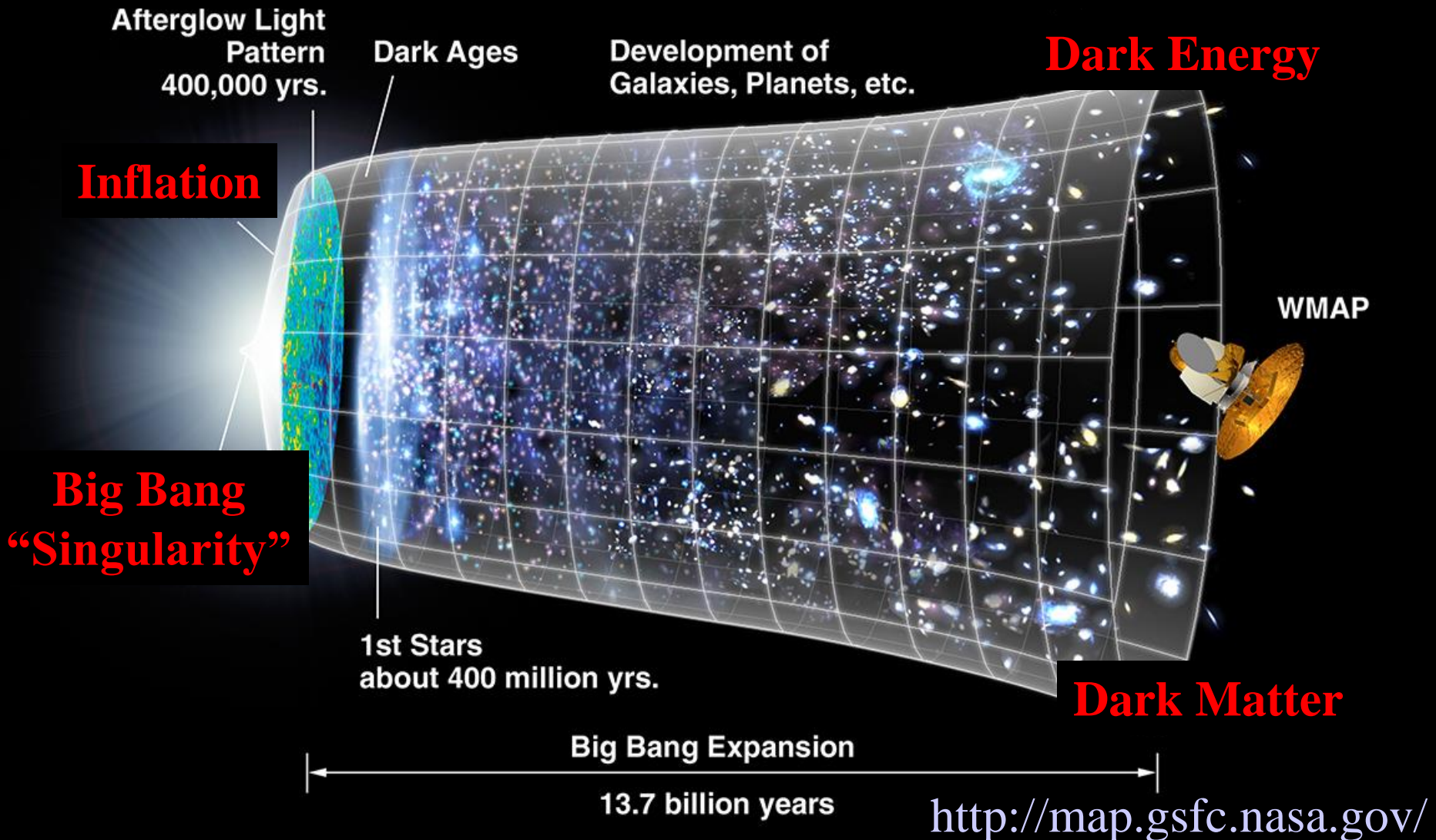


Massive gravity and cosmology

Shinji Mukohyama

Based on collaboration with
Antonio DeFelice, Garrett Goon, Emir Gumrukcuoglu, Lavinia
Heisenberg, Kurt Hinterbichler, David Langlois, Chunshan Lin,
Ryo Namba, Atsushi Naruko, Takahiro Tanaka, Norihiro
Tanahashi, Mark Trodden

Why alternative gravity theories?



Three conditions for good alternative theories of gravity (my personal viewpoint)

1. Theoretically consistent
e.g. no ghost instability
2. Experimentally viable
solar system / table top experiments
3. Predictable
e.g. protected by symmetry

Some examples

- I. Ghost condensation
IR modification of gravity
motivation: dark energy/matter
- II. Nonlinear massive gravity
IR modification of gravity
motivation: “Can graviton have mass?”
- III. Horava-Lifshitz gravity
UV modification of gravity
motivation: quantum gravity
- IV. Superstring theory
UV modification of gravity
motivation: quantum gravity, unified theory

A motivation for IR modification

- Gravity at long distances

Flattening galaxy rotation curves

extra gravity

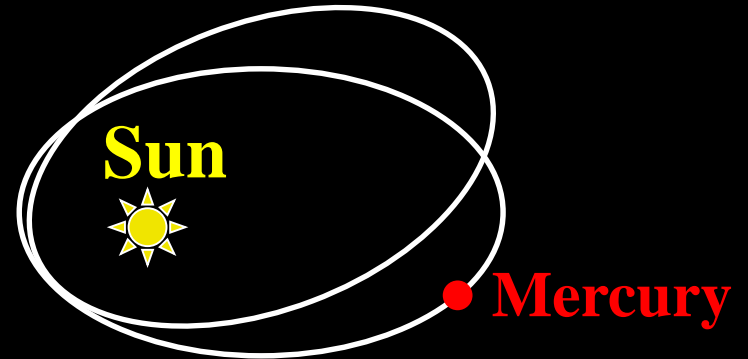
Dimming supernovae

accelerating universe

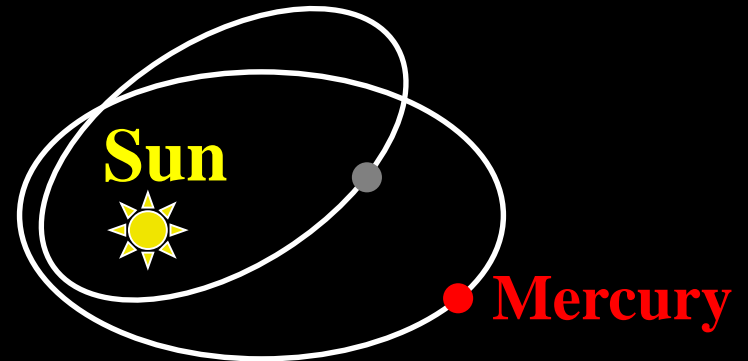
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion
observed in 1800's...



which people tried to
explain with a “dark
planet”, Vulcan,



But the right answer wasn't “dark planet”, it was
“change gravity” from Newton to GR.

Can we change gravity in IR?

➤ Change Theory?

Massive gravity

Fierz-Pauli 1939

DGP model

Dvali-Gabadadze-Porrati 2000

➤ Change State?

Higgs phase of gravity

The simplest: Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

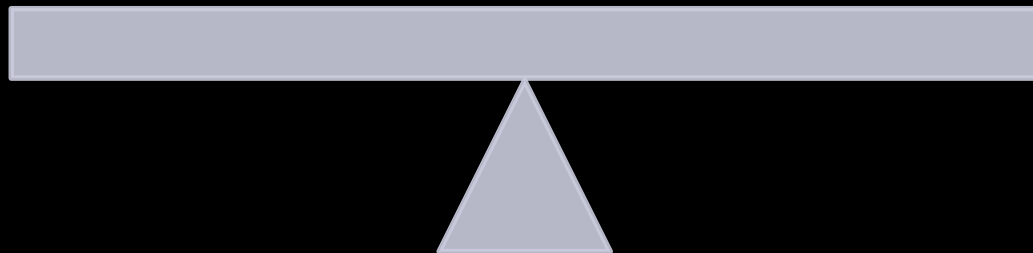
Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



Massive gravity: history

Simple question: Can graviton have mass?

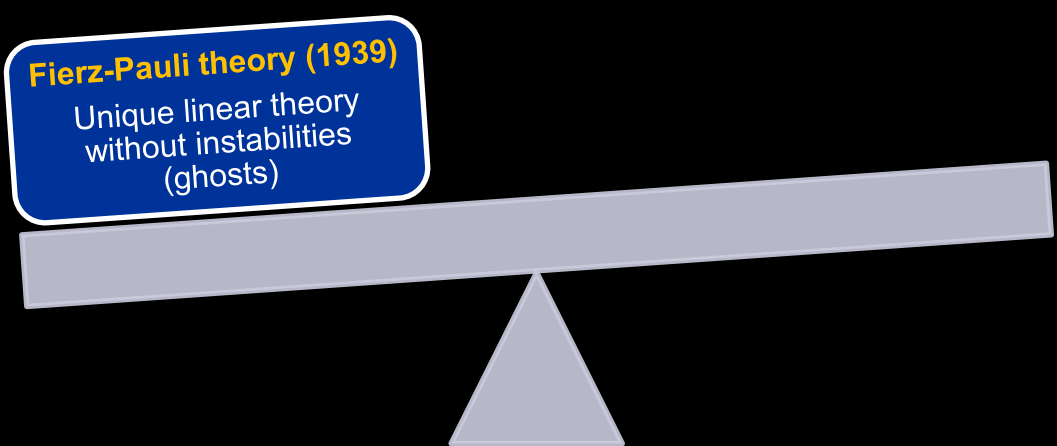
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Fierz-Pauli theory (1939)

Unique linear theory
without instabilities
(ghosts)



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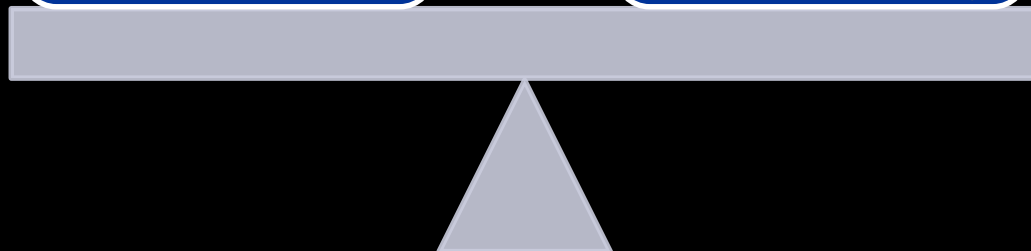
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van Dam-Veltman-
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(1970)

Massless limit \neq
General Relativity



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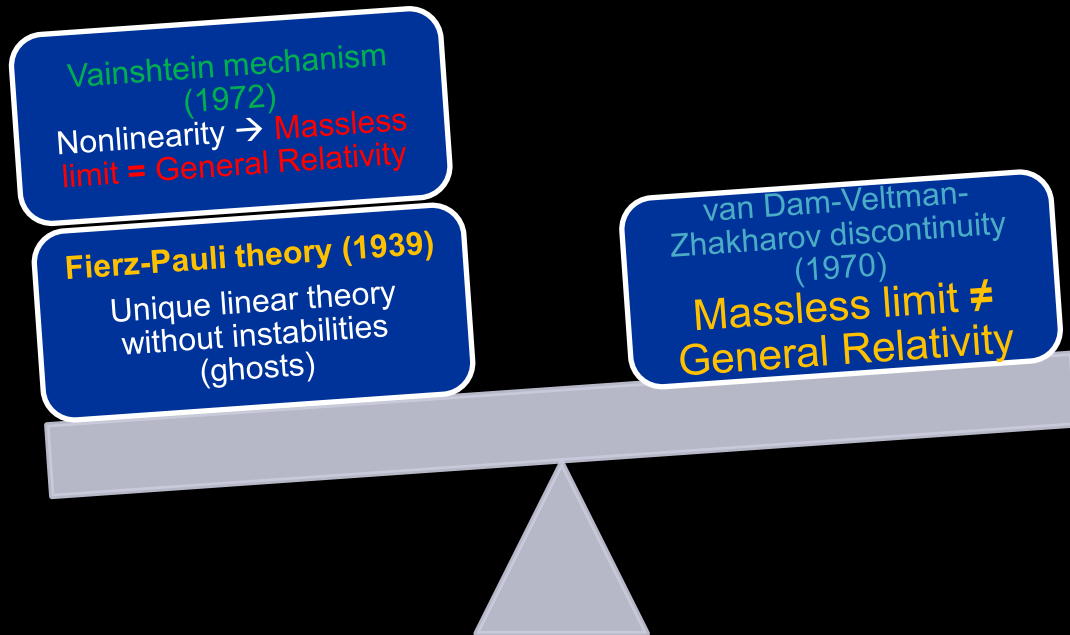
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Vainshtein mechanism
(1972)
Nonlinearity \rightarrow Massless
limit = General Relativity

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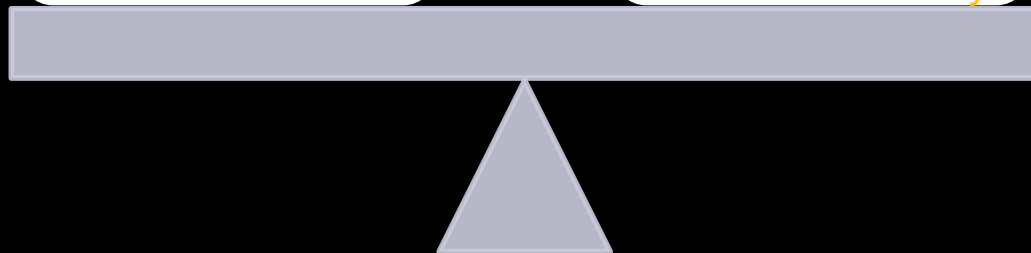
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Boulware-Deser ghost
(1972)

6th d.o.f. @ Nonlinear level
 \rightarrow Instability (ghost)

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Massless limit \neq
General Relativity



Nonlinear massive gravity

de Rham, Gabadadze 2010

de Rham, Gabadadze & Tolley 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical (but technically natural)
- Properties of 5 d.o.f. depend on background

- **4 scalar fields ϕ^a ($a=0,1,2,3$)**

- **Poincare symmetry in the field space:**

$$\phi^a \rightarrow \phi^a + c^a, \quad \phi^a \rightarrow \Lambda_b^a \phi^b$$



$$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

Pullback of
Minkowski metric in field space
to spacetime

Systematic resummation

de Rham, Gabadadze & Tolley 2010

$$I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_{Pl}^2 m_g^2 \int d^4x \sqrt{-g} (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4)$$

$$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

$$\mathcal{K}_\nu^\mu = \delta_\nu^\mu - \left(\sqrt{g^{-1} f} \right)^\mu_\nu$$

$$\mathcal{L}_2 = \frac{1}{2} ([\mathcal{K}]^2 - [\mathcal{K}^2])$$

$$\mathcal{L}_3 = \frac{1}{6} ([\mathcal{K}]^3 - 3 [\mathcal{K}] [\mathcal{K}^2] + 2 [\mathcal{K}^3]) \quad [\mathcal{A}] \equiv Tr \mathcal{A}$$

$$\mathcal{L}_4 = \frac{1}{24} ([\mathcal{K}]^4 - 6 [\mathcal{K}]^2 [\mathcal{K}^2] + 3 [\mathcal{K}^2]^2 + 8 [\mathcal{K}] [\mathcal{K}^3] - 6 [\mathcal{K}^4])$$

No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \quad \Rightarrow \quad \mathcal{L}_{2,3,4} = (\text{total derivative})$$

No BD ghost away from decoupling limit (Hassan&Rosen)

Massive gravity: history

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Consistent Theory

found in 2010

Does it have

Viable Cosmology?

de Rham-Gabadadze-Tolley (2010)
First example of nonlinear
massive gravity with flat
BTZ gauge invariance

Vainshtein mechanism
(1972)
Nonlinearities → massive
limit = General Relativity

Fierz-Pauli theory (1939)
Unique linear theory
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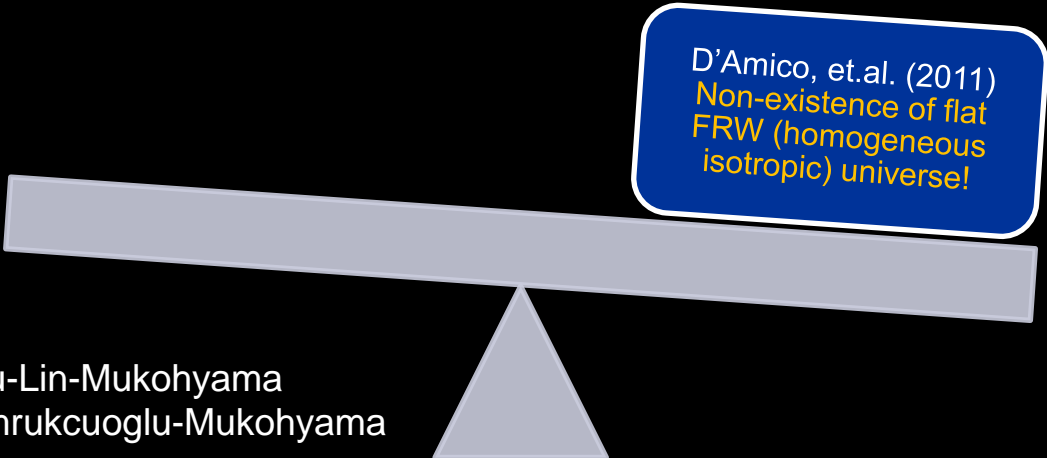
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Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

Bad?



D'Amico, et.al. (2011)
Non-existence of flat
FRW (homogeneous
isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

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Open universes with self-
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GLM (2011a)

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More general fiducial
metric $f_{\mu\nu}$
closed/flat/open FRW
universes allowed
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GLM (2011a)

NEW
Nonlinear instability of
FRW solutions
DGM (2012)

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NEW Class of Solutions
Anisotropic FRW universe
GLM (2012)

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metric $f_{\mu\nu}$
closed/flat/open FRW
universes allowed
GLM (2011b)

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acceleration
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Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

Bad?

Extended theories:
Extended quasidilatation
Bimetric theory
Rotation-invariant
massive gravity

More general fiducial
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Summary

- Nonlinear massive gravity
free from BD ghost
- Open FLRW solutions exist but are unstable
- New class of cosmological solutions:
anisotropic FLRW \rightarrow statistical anisotropy
(suppressed by small m_g^2)
- Extended theories: extended quasidilaton,
bimetric theory, rotation-invariant massive
gravity...
- New matter coupling leads to stable cosmology

Why alternative gravity theories?

