Investigation of QCD phase diagram from imaginary chemical potential

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Collaborators of related studies

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Introduction: QCD phase diagram

Purpose of this research

Understanding the phase structure of Quantum Chromodynamics

We never observe quark itself
Quarks and gluons are confined

What happen when we consider extreme condition?

We want to know Quantum Chromodynamics (QCD) phase diagram
Introduction: QCD phase diagram

Schematic QCD phase diagram

- Temperature
- Real Quark chemical potential

- Hadron Phase
- Quark-Gluon Plasma Phase
- Color Superconducting Phase
Introduction: QCD phase diagram

Understand of QCD phase diagram is important!
Introduction: QCD phase diagram

Problem of first principle approach

First principle calculation of QCD at finite real chemical potential is not feasible ...

Lattice QCD simulation

It is numerical problem of lattice QCD...
Introduction: Lattice QCD simulation and sign problem

Several methods are proposed to circumvent the sign problem.

However ...

First principle calculation of QCD at finite real chemical potential is not feasible ...

Those methods are limited ...

( Region $\mu/T < 1$ )
Our approach: Effective model + Lattice data

We extend effective models by using lattice data obtained at imaginary chemical potential.
Imaginary chemical potential approach

Our approach: Effective model + Lattice data

Why imaginary chemical potential??

1. There is no sign problem
2. Interesting behavior of QCD
Imaginary chemical potential approach

If the $\mu_I$ region has information of the $\mu_R$ region, we can construct reliable effective model!

Original ($\mu_R$)
Shadow ($\mu_I$)

O.K.!!
...

Year of the sheep
Imaginary chemical potential approach

Fortunately, the $\mu_I$ region has information of the $\mu_R$ region


Fourier transformation:

$$Z_{\text{Canonical}}(T, B) = \int_{-\infty}^{+\infty} d\left(\frac{\mu_I}{T}\right) e^{-iB\mu_I/T} Z_{\text{Grand Canonical}}(T, \mu_I)$$

Laplace transformation (Fugacity expansion):

$$Z_{\text{Grand Canonical}}(T, \mu_R) = \sum_{B=-\infty}^{+\infty} e^{B\mu_R/T} Z_{\text{Canonical}}(T, B)$$
If we can obtain reliable effective model,

we can investigate QCD phase structure, quantitatively!

Also, we may obtain reliable equation of state.
QCD at imaginary chemical potential

Phase structure at imaginary chemical potential

Perfectly different!

Roberge-Weiss (RW) periodicity

What model should we use?

Nambu–Jona–Lasinio (NJL) model

\[ L = \bar{q} (i \gamma^\mu \partial_\mu - m_0) q + G_s \left( (\bar{q}q)^2 + (\bar{q}i \gamma_5 \tau q)^2 \right) \]

This model only has $2\pi$ periodicity

We cannot use this model at imaginary chemical potential ...
What model should we use?

**Polyakov–loop extended NJL (PNJL) model** (Mean field approximation)


\[
L = \bar{q}(i\gamma^\mu D_\mu - m_0)q + G_s \left( (\bar{q}q)^2 + (\bar{q}i\gamma_5 \bar{\tau}q)^2 \right) - U(\Phi, \bar{\Phi})
\]

**Thermodynamic potential**

\[
\frac{\Omega}{V} = U + U_M - 2N_f \int \frac{d^3p}{(2\pi)^3} \left[ N_c E(p) + T \ln \left( 1 + (\Phi + \bar{\Phi} e^{-\beta E^-}) e^{-\beta E^-} + e^{-3\beta E^-} \right) \right] + T \ln \left( 1 + (\Phi + \bar{\Phi} e^{-\beta E^+}) e^{-\beta E^+} + e^{-3\beta E^+} \right) + \int \frac{d^3p}{(2\pi)^3} [N_c E(p) + T \ln \left( 1 + (\Phi + \bar{\Phi} e^{-\beta E^-}) e^{-\beta E^-} + e^{-3\beta E^-} \right)]
\]

\[
U_M = G_s \sigma^2
\]
Results: Vector interaction

Vector type interaction in PNJL model

Vector interaction: $G_v \left( q \gamma^\mu q \right)^2$

Critical point is vanished!

Results: QCD phase diagram at imaginary chemical potential


Set C

<table>
<thead>
<tr>
<th>set</th>
<th>$G_s$</th>
<th>$G_{s8}$</th>
<th>$G_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.498 GeV$^{-2}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>4.761 GeV$^{-2}$</td>
<td>403.89 GeV$^{-8}$</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>4.761 GeV$^{-2}$</td>
<td>403.89 GeV$^{-8}$</td>
<td>4.761 GeV$^{-2}$</td>
</tr>
</tbody>
</table>

TABLE II: Summary of the parameter sets in the PNJL calculations. The parameters $\Lambda$, $m_0$ and $T_0$ are common among the three sets: $\Lambda = 631.5$ MeV, $m_0 = 3.5$ MeV and $T_0 = 212$ MeV.

Lattice data:

Results: QCD phase diagram at imaginary chemical potential


This is rough estimate. We can determine the parameter set!
Results: QCD phase diagram at imaginary chemical potential

Results: QCD phase diagram at imaginary chemical potential


This is rough estimate. We can determine the parameter set!
Results: Colombia plot at imaginary chemical potential

C. Bonati, P. de Forcrand, M. D'Elia, O. Philipsen, F. Sanfilippo,

P. de Forcrand and O. Philipsen,

K.K., R. D. Pisarski,
Results: Colombia plot at imaginary chemical potential

We can obtain large model ambiguities at heavy quark mass region!


Introduction: QCD phase diagram

Boundary condition

Imaginary chemical potential can be converted to boundary condition.

Dual chiral condensate


Boundary angle dependent chiral condensate

Confinement-deconfinement transition
Introduction : QCD phase diagram

**Hosotani mechanism**  

Condensation of extra-dimensional component of $A_y$

Beyond the standard physics (Higgs phenomenology)

Relation with $\mu_1$ : K.K. and T. Misumi, JHEP 05 (2013) 042.

**$Z_3$ symmetric QCD**  
H. Kouno, T. Misumi, K.K., T. Makiyama, T. Sasaki, M. Yahiro,  

\[ q_f(x, \beta = 1/T) = -\exp[i\theta_f]q_f(x,0) \]

\[ \theta_2 = 2\pi/3 \]

\[ \theta_3 = 4\pi/3 \]

$Z_3$ transformation
The imaginary chemical potential is a good region to construct reliable effective models.

- Roberge-Weiss periodicity and transition
- Finite value of quark number density

Parameters can be determined (ex. vector interaction)

Rough estimate of phase diagram

**Imaginary chemical potential is very interesting and important region!**

However, lattice data is not enough at the present...
I look forward to working with you again this year!