Investigation of QCD phase diagram from imaginary chemical potential

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Collaborators of related studies

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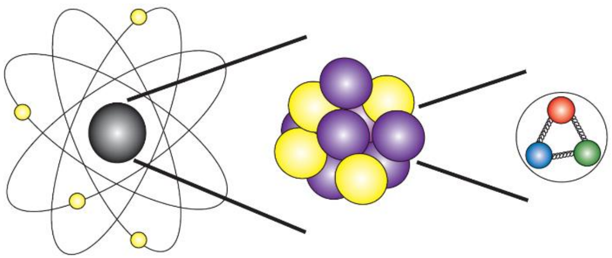
Thomas Hell (TUM)
Robert D. Pisarski (BNL, RIKEN BNL)

Purpose of this research

Understanding the phase structure of Quantum Chromodynamics



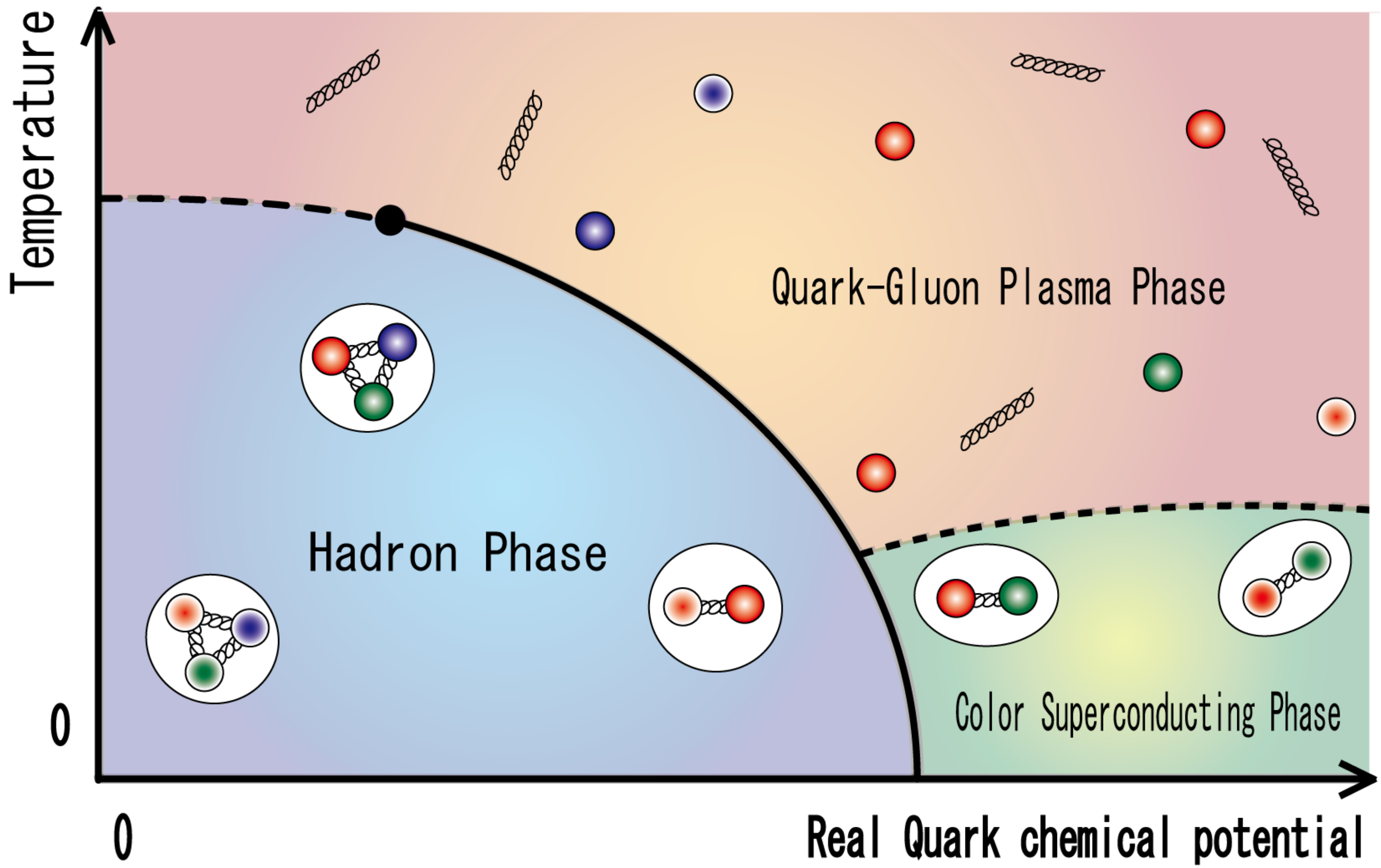
We never observe quark itself

Quarks and gluons are confined

What happen when we consider extreme condition?

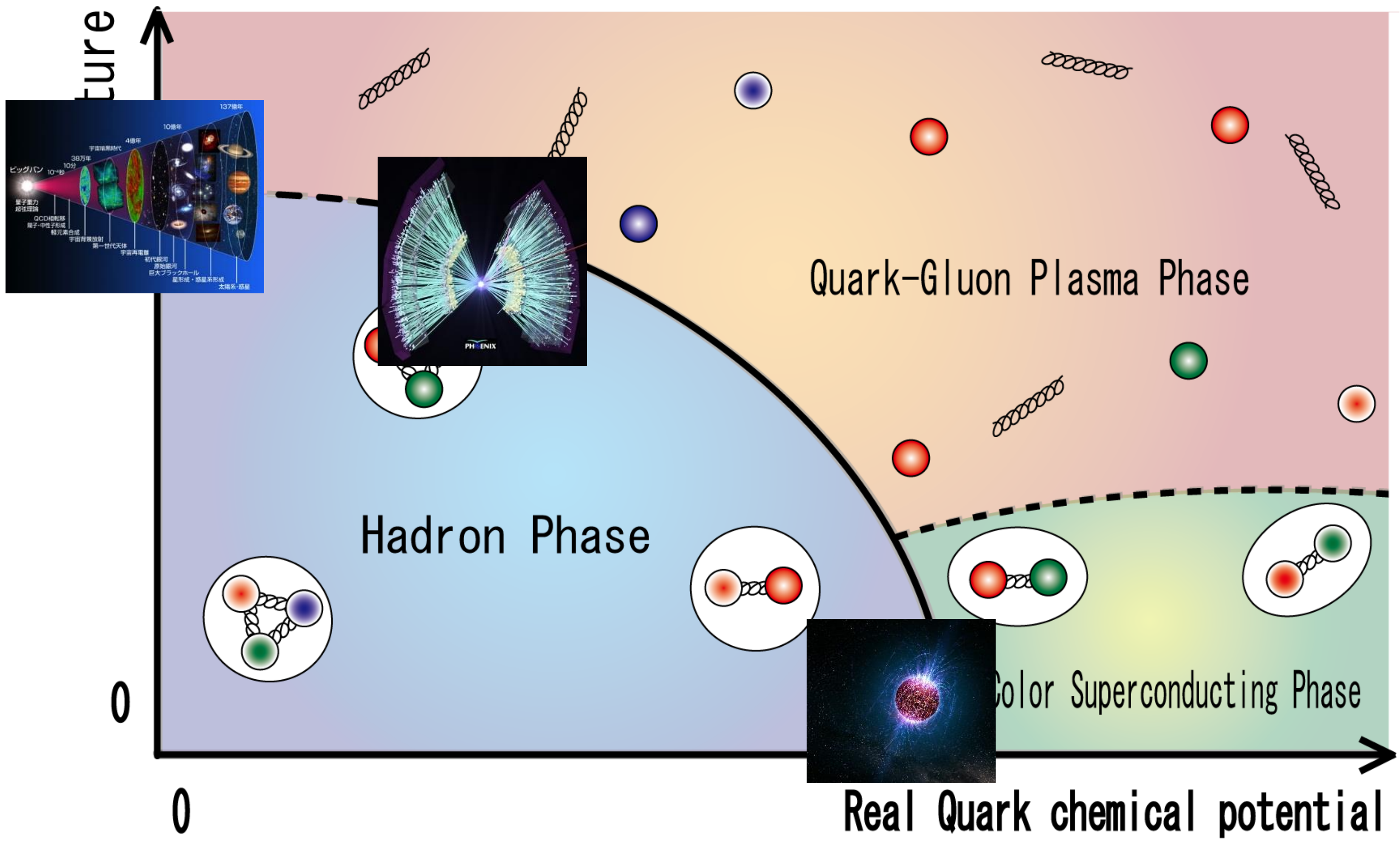
We want to know Quantum Chromodynamics (QCD) phase diagram

Schematic QCD phase diagram



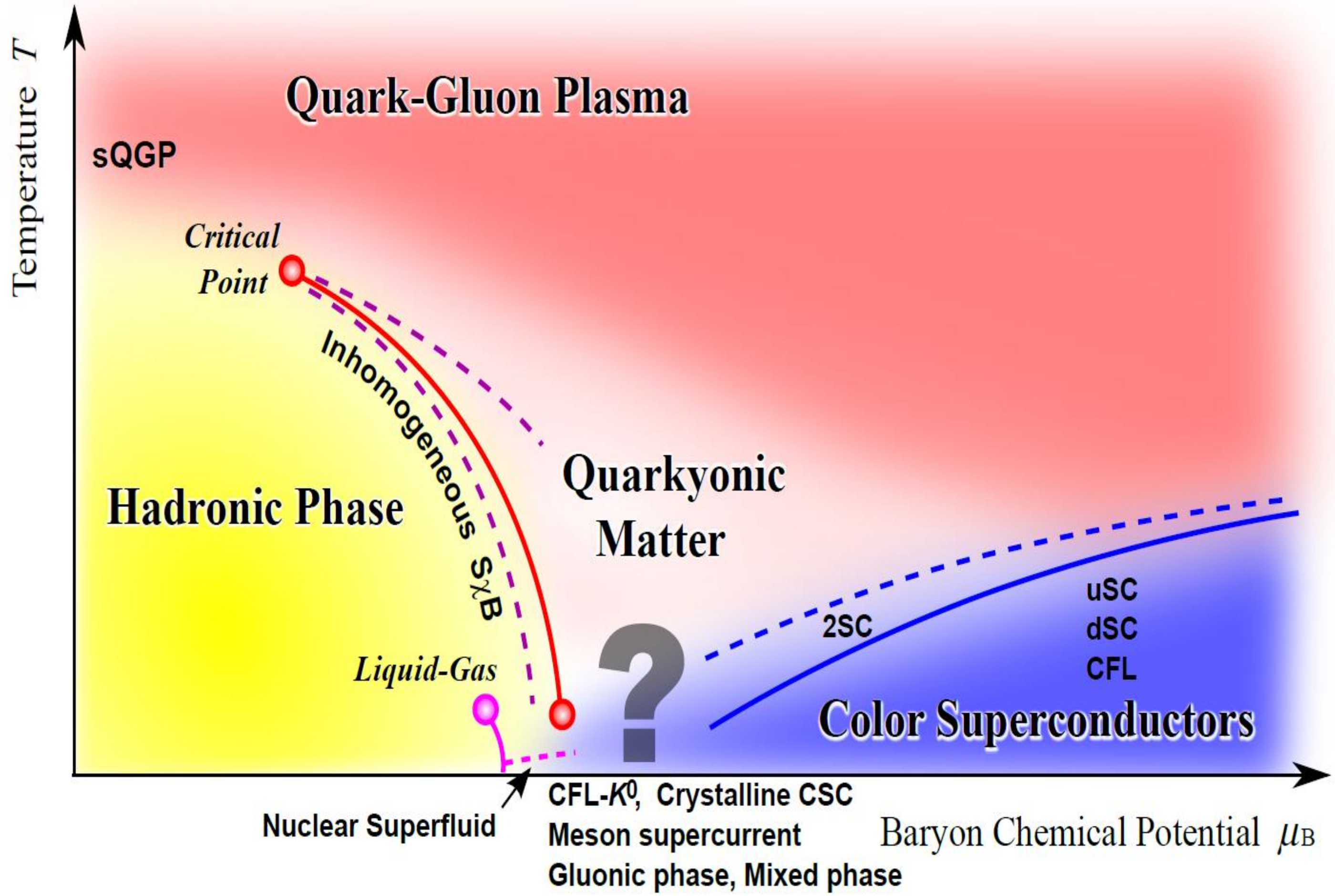
Schematic QCD phase diagram

Understand of QCD phase diagram is important!



Schematic QCD phase diagram

K. Fukushima, T. Hatsuda, Rept. Prog. Phys. 74 (2011) 014001.



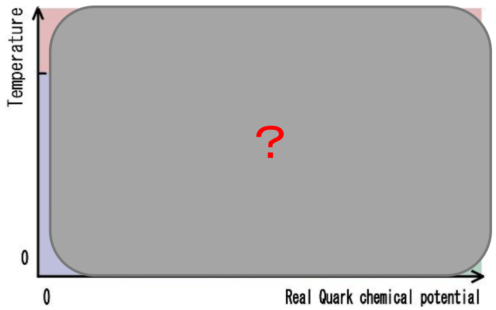
Problem of first principle approach

First principle calculation of QCD
at finite real chemical potential is **not feasible** …

Lattice QCD simulation

It is numerical problem of lattice QCD…

Sign problem



Introduction : Lattice QCD simulation and sign problem

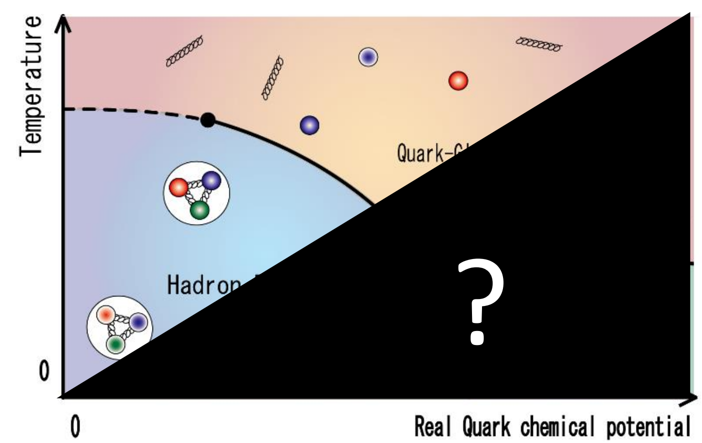
Problem of first principle approach

First principle calculation of QCD
at finite real chemical potential is **not feasible** …

Several method are proposed to circumvent the sign problem

However …

Those method are limited …
(Region $\mu/T < 1$)



Imaginary chemical potential approach

Our approach : Effective model + Lattice data

We extend effective models by using lattice data obtained at **imaginary chemical potential**

Imaginary chemical potential approach

Our approach : Effective model + Lattice data

Why **imaginary chemical potential** ??

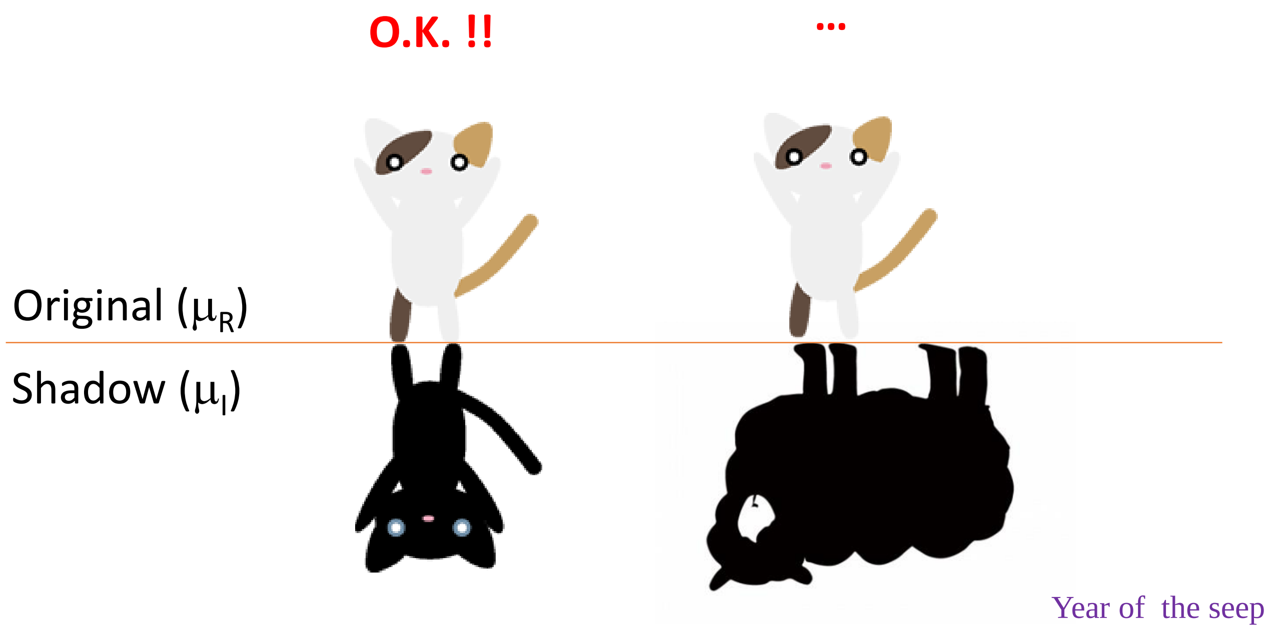
1. There is no sign problem

2. Interesting behavior of QCD

Imaginary chemical potential approach

Imaginary chemical potential approach

If the μ_I region has **information** of the μ_R region,
we can construct **reliable effective model**!



Imaginary chemical potential approach

Fortunately,

the μ_I region has **information** of the μ_R region

A. Roberge and N. Weiss, Nucl. Phys. **B** 275 (1986) 735

Fourier transformation:

$$Z_{\text{Canonical}}(T,B) = \int_{-\infty}^{+\infty} d\left(\frac{\mu_I}{T}\right) e^{-iB\mu_I/T} Z_{\text{Grand Canonical}}(T,\ \mu_I)$$

O.K. !!

Laplace transformation (Fugacity expansion):

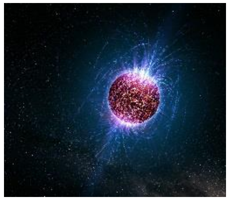
$$Z_{\text{Grand Canonical}}(T,\mu_R) = \sum_{B=-\infty}^{+\infty} e^{B\mu_R/T} Z_{\text{Canonical}}(T,B)$$

Imaginary chemical potential approach

If we can obtain reliable effective model,

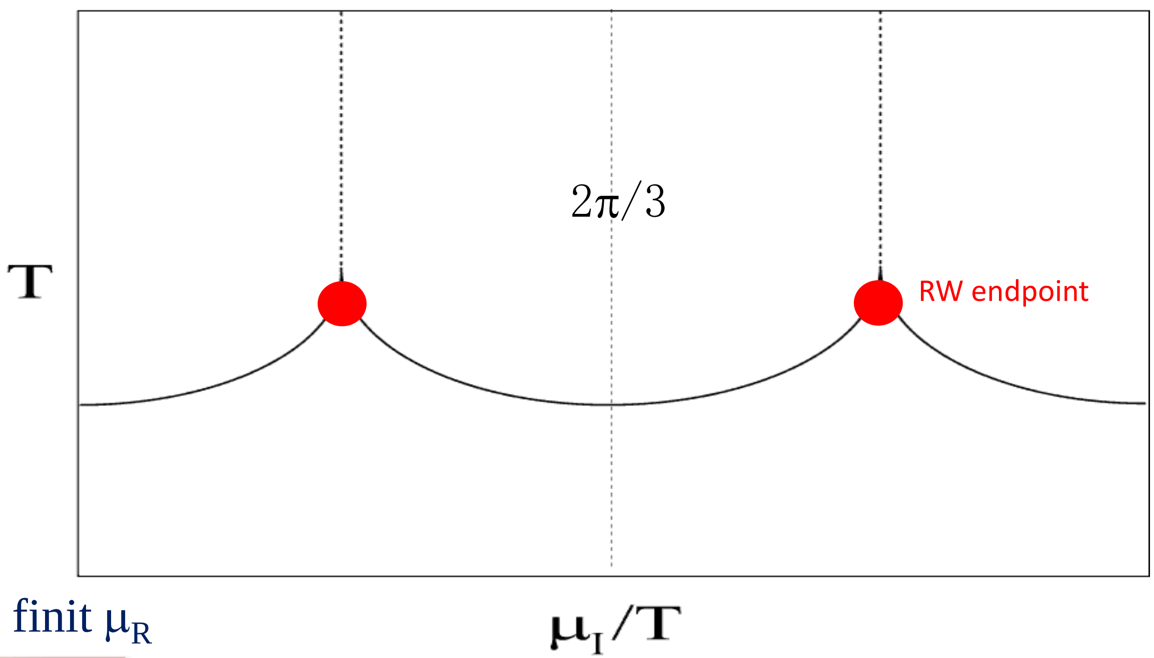
we can investigate QCD phase structure, **quantitatively**!

Also, we may obtain **reliable equation of state**.

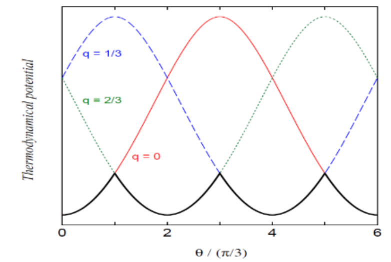


QCD at imaginary chemical potential

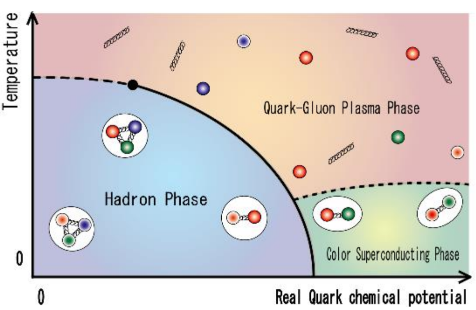
Phase structure at imaginary chemical potential



Origin of RW transition



Phase diagram at finit μ_R



Perfectly different !

Roberge-Weiss (RW) periodicity

RW transition

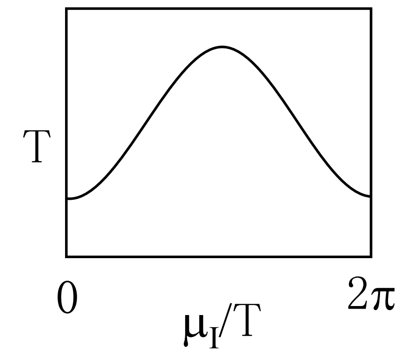
A. Roberge and N. Weiss, Nucl. Phys. **B** 275 (1986) 735

Model setting

What model should we use?

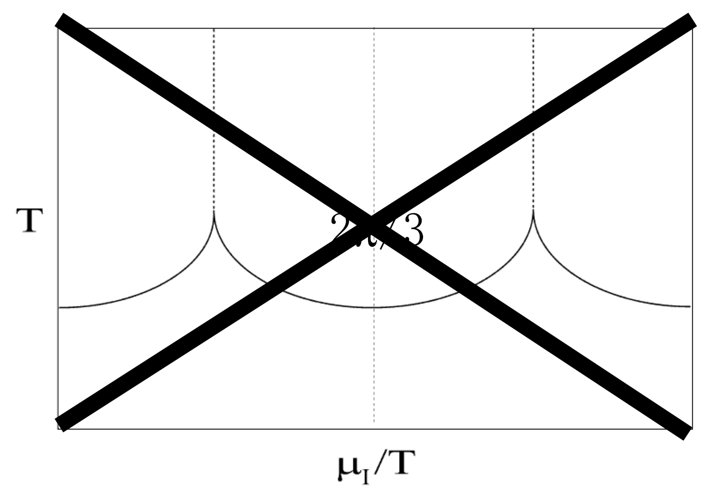
Nambu–Jona–Lasinio (NJL) model

$$L = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0)q + G_s\left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right)$$



This model only has 2π periodicity

We can not use this model
at imaginary chemical potential …



Model setting

What model should we use?

Polyakov−loop extended NJL (PNJL) model K. Fukushima, Phys. Lett. B591 (2004) 277

$$L = \bar{q}(i\gamma^{\mu} D_{\mu} - m_0)q + G_s\left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right) - U(\bar{\Phi},\Phi)$$

Gluonic contribution

Thermodynamic potential (Mean field approximation)

$$\frac{\Omega}{V} = U + U_M - 2N_f \int \frac{d^3p}{(2\pi)^3}\Big[N_c E(p) + T\ln\left(1+(\Phi+\bar{\Phi}e^{-\beta E^-})e^{-\beta E^-} + e^{-3\beta E^-}\right)$$
$$+T\ln\left(1+(\Phi+\bar{\Phi}e^{-\beta E^+})e^{-\beta E^+} + e^{-3\beta E^+}\right)\Big]$$

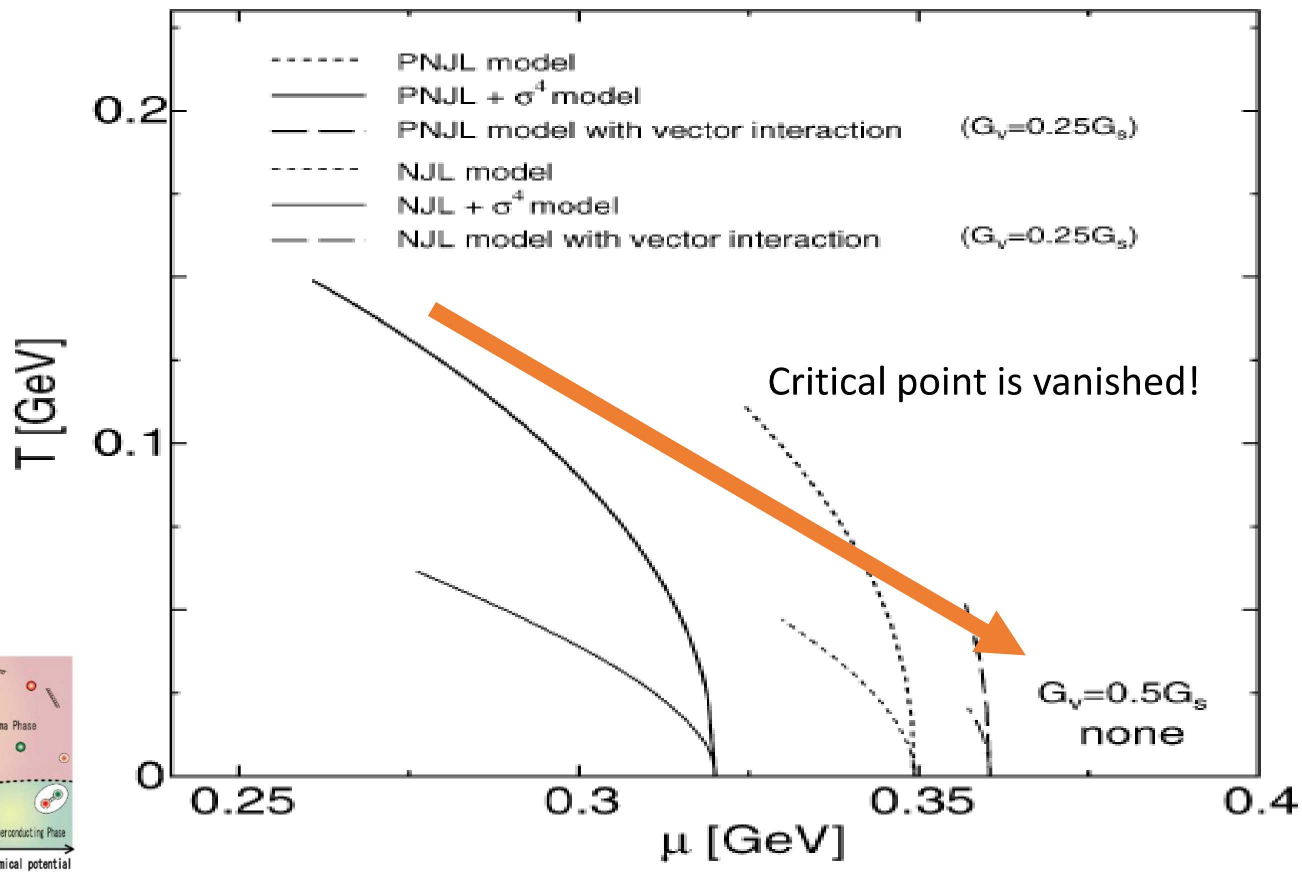
$$U_M = G_s\sigma^2$$

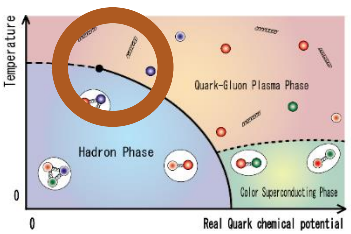
Results : Vector interaction

Vector type interaction in PNJL model

Usually it is free parameter

Vector interaction : $G_v\left(\bar{q}\gamma^\mu q\right)^2$

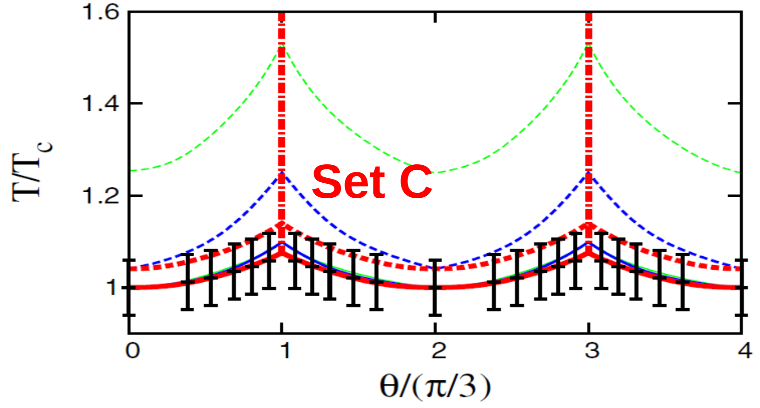


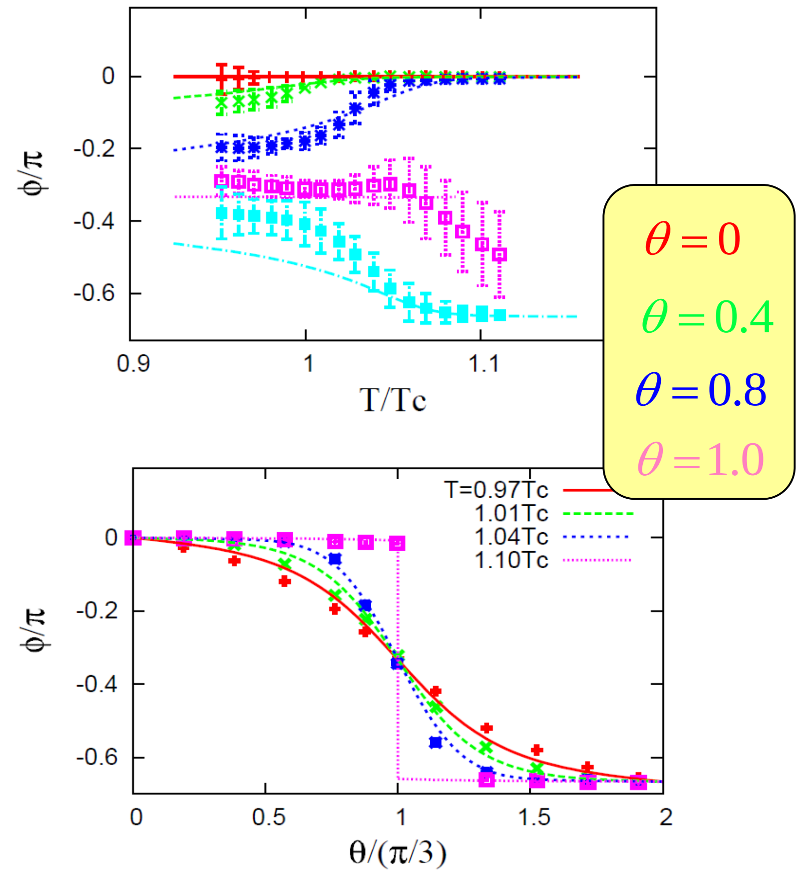


K.K., H. Kouno, M. Matsuzaki, M. Yahiro, Phys. Lett. B 662 (2008) 26

Results : QCD phase diagram at imaginary chemical potential

PNJL model : Y. Sakai, **K. K**, H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Rev. D **79** (2009) 096001





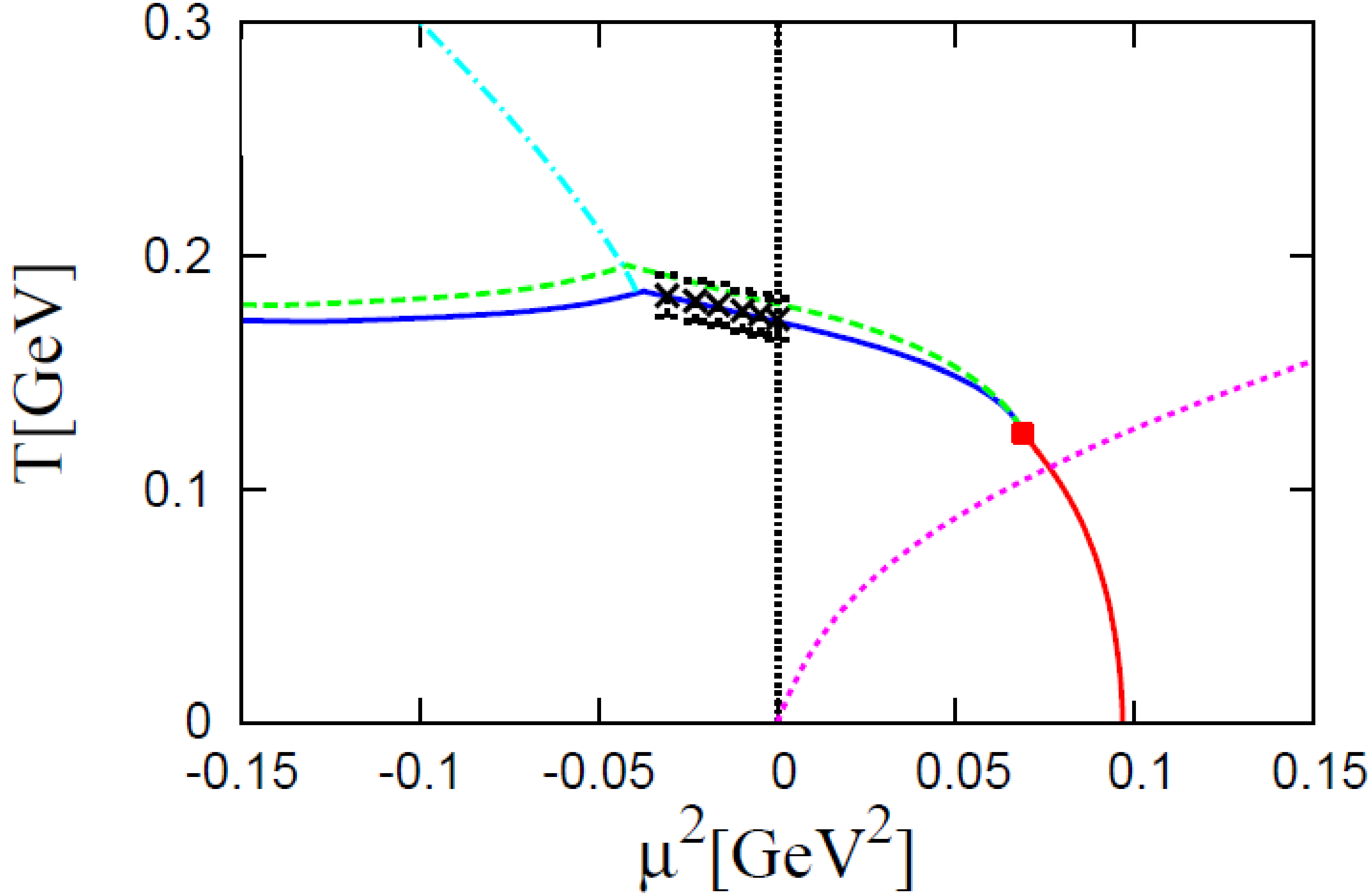
set	G_s	G_{s8}	G_v
A	5.498GeV^{-2}	0	0
B	4.761GeV^{-2}	403.89GeV^{-8}	0
C	4.761GeV^{-2}	403.89GeV^{-8}	4.761GeV^{-2}

TABLE II: Summary of the parameter sets in the PNJL calculations. The parameters Λ, m_0 and T_0 are common among the three sets; $\Lambda = 631.5$ MeV, $m_0 = 5.5$ MeV and $T_0 = 212$ MeV.

Lattice data:

P. de Forcrand and O. Philipsen, Nucl. Phys. **B 642** (2002) 290
L. K. Wu, X. Q. Luo and H. S. Chen, Phys. Rev. D **76** (2007) 034505

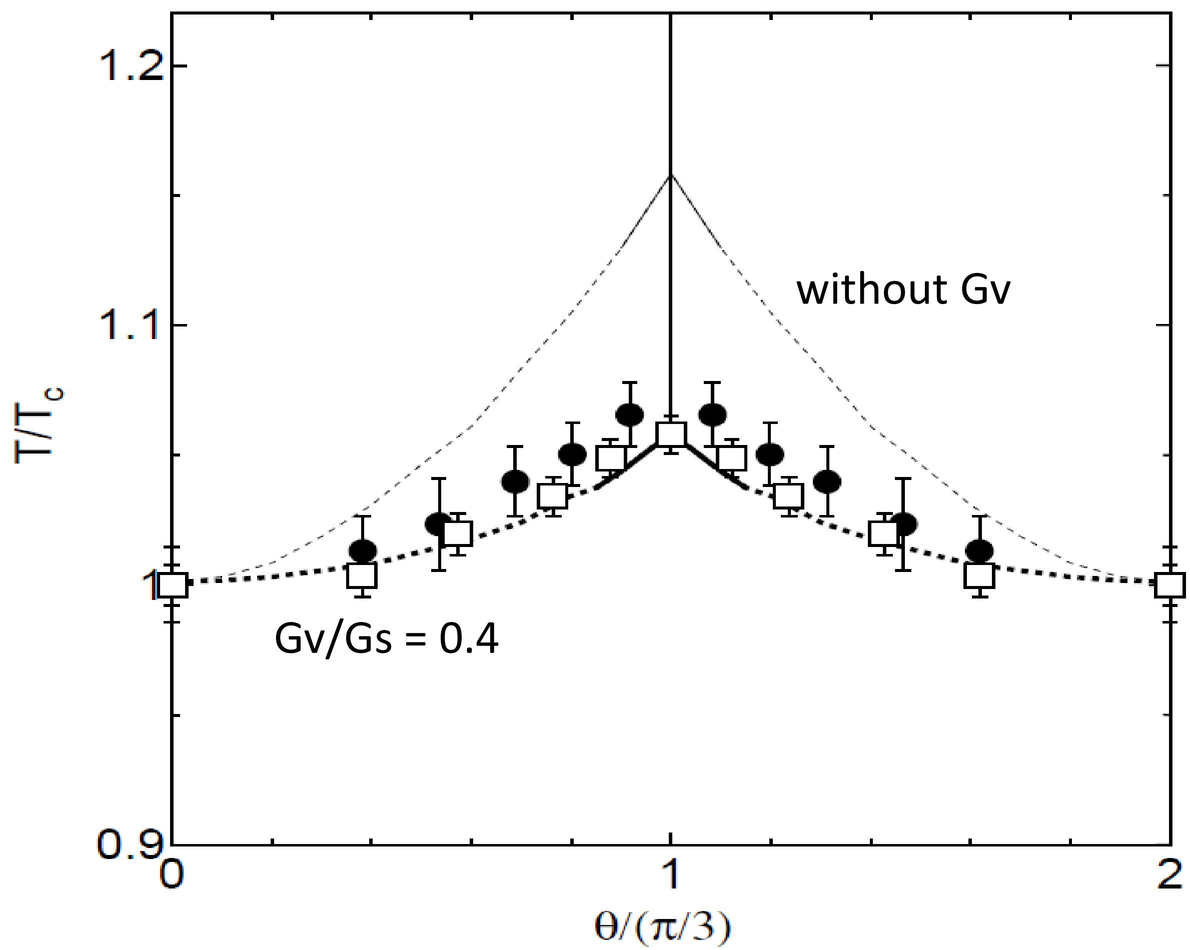
PNJL model : Y. Sakai, **K. K**, H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Rev. D **79** (2009) 096001



This is rough estimate. We can determine the parameter set!

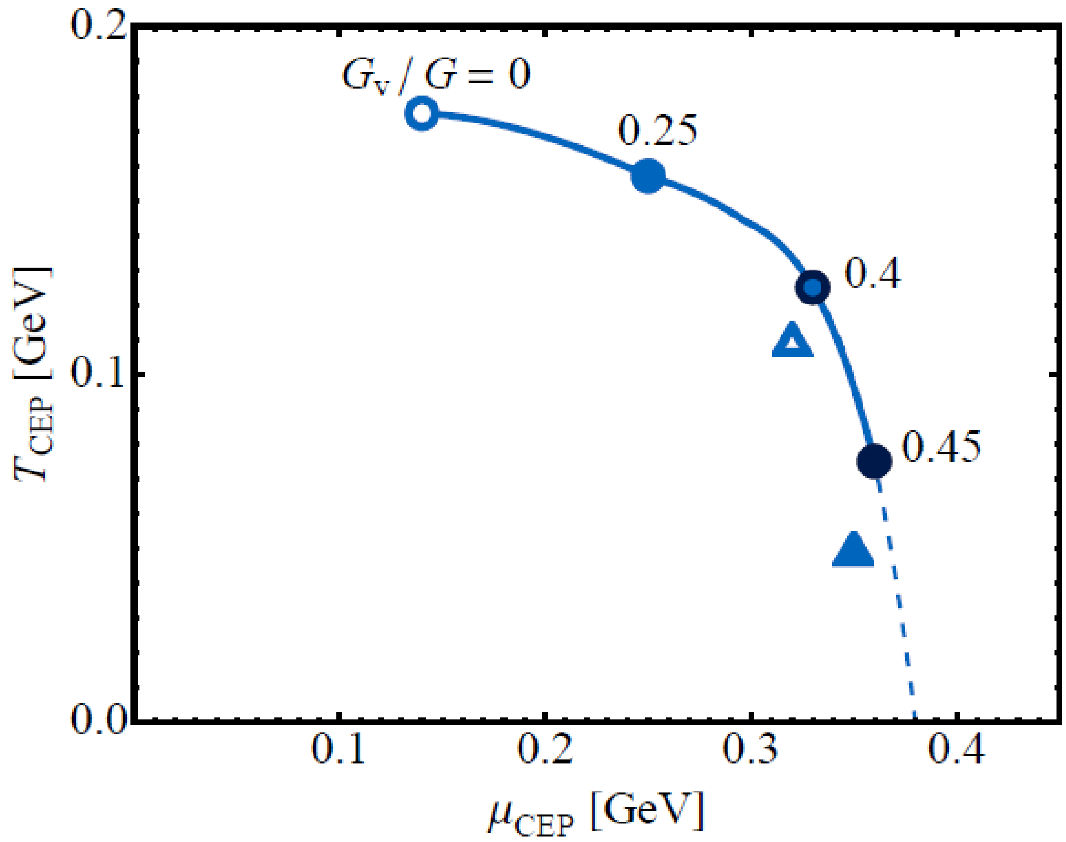
Results : QCD phase diagram at imaginary chemical potential

Nonlocal PNJL model : **K. K**, T. Hell, W. Weise, Phys. Rev. D **84** (2011) 056010.



Results : QCD phase diagram at imaginary chemical potential

Nonlocal PNJL model : T. Hell, **K. K**, W. Weise, J. Mod. Phys. 4 (2013) 644.

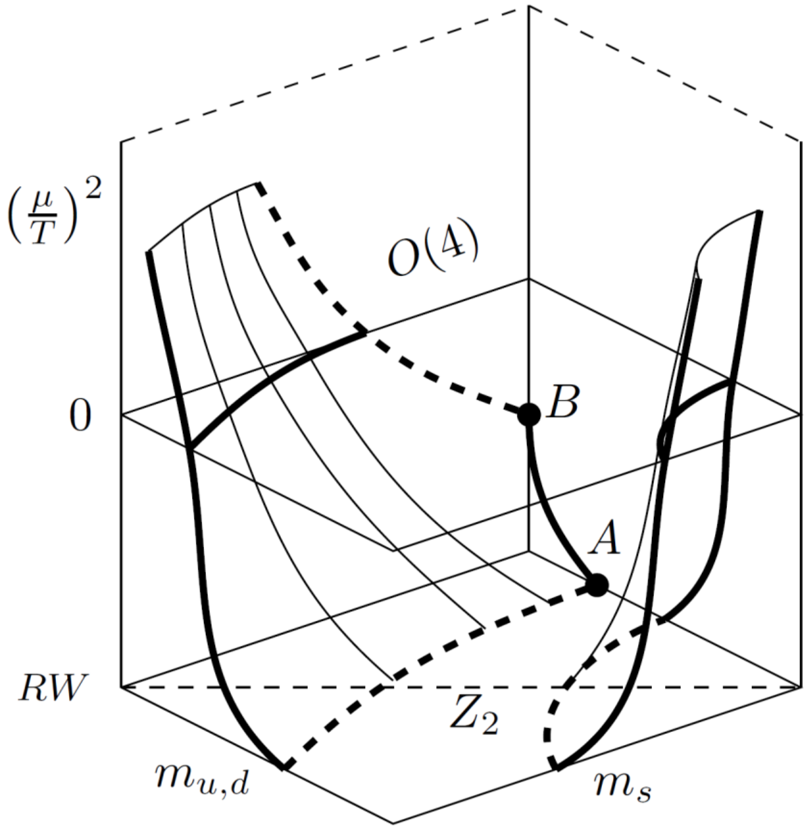


This is rough estimate. We can determine the parameter set!

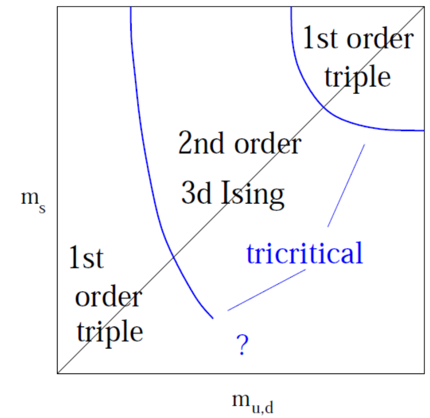
Results : Colombia plot at imaginary chemical potential

Colombia plot

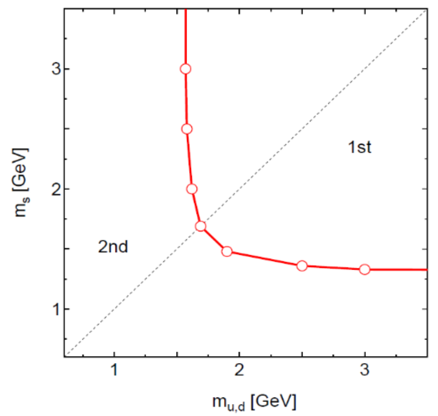
Phase diagram as a function of quark masses



C. Bonati, P. de Forcrand, M. D'Elia, O. Philipsen, F. Sanfilippo, Phys. Rev. D 90 (2014) 074030.



P. de Forcrand and O. Philipsen, Phys. Rev. Lett. 105 (2010) 152001.

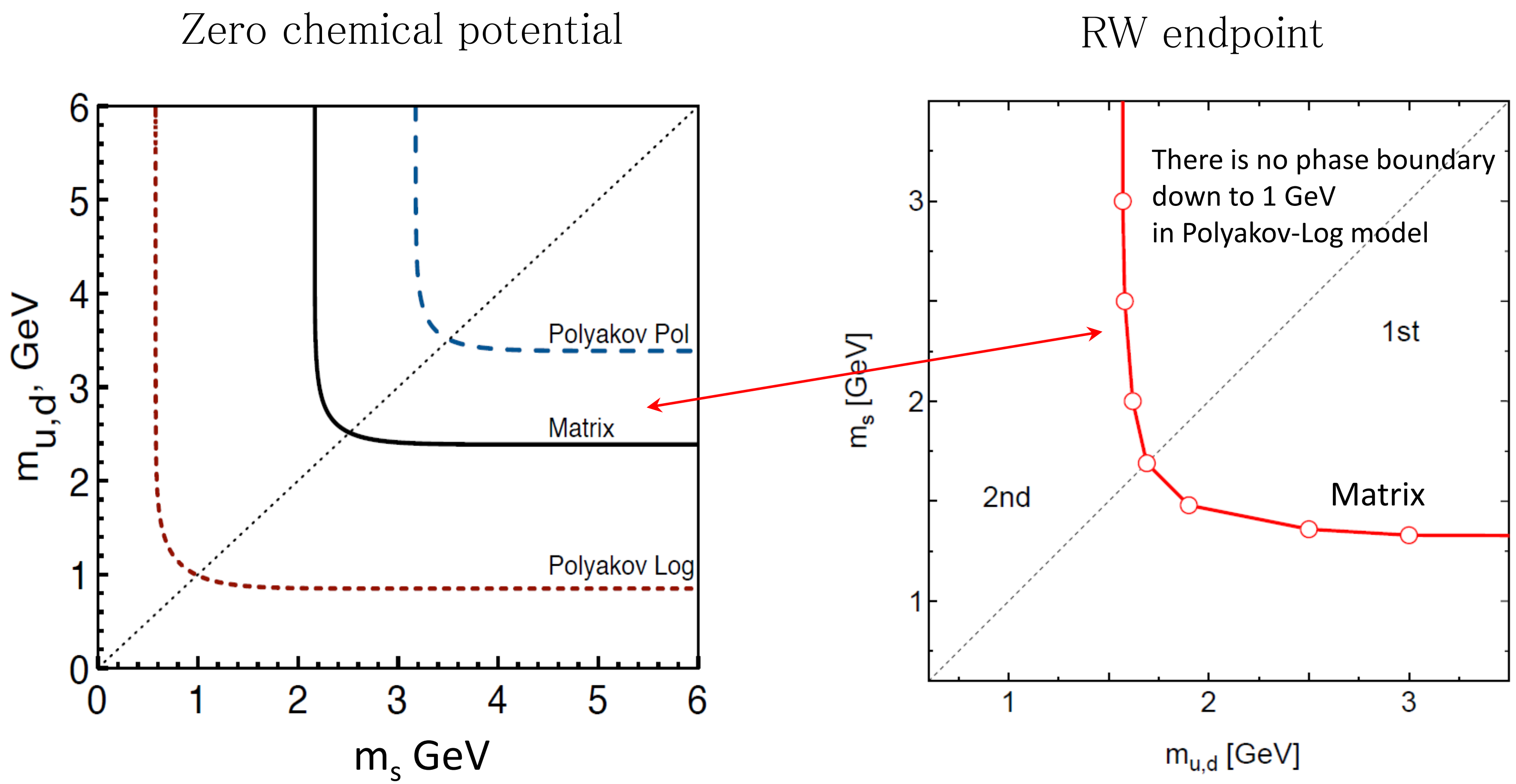


K.K., R. D. Pisarski, Phys. Rev. D 87 (2013) 096009.

Results : Colombia plot at imaginary chemical potential

Colombia plot

We can obtain large model ambiguities at heavy quark mass region!



K.K., V. V. Skokov, R. D. Pisarski, Phys. Rev. D85 (2012) 114029.

K.K., R. D. Pisarski, Phys. Rev. D87 (2013) 096009.

Boundary condition

Imaginary chemical potential can be converted to boundary condition.

Polyakov-loop like quantity

Dual chiral condensate

Lattice : E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, Phys. Rev. D77 (2008) 094007.

PNJL model : **K.K**, H. Kouno and M. Yahiro, Phys. Rev. **D** 80 (2009) 117901.

Boundary angle dependent chiral condensate

⇒ Confinement-deconfinement transition

Introduction : QCD phase diagram

Boundary condition

Hosotani mechanism

Y. Hosotani, Phys.Lett.B 126 (1983) 309.

Condensation of extra-dimensional component of A_y

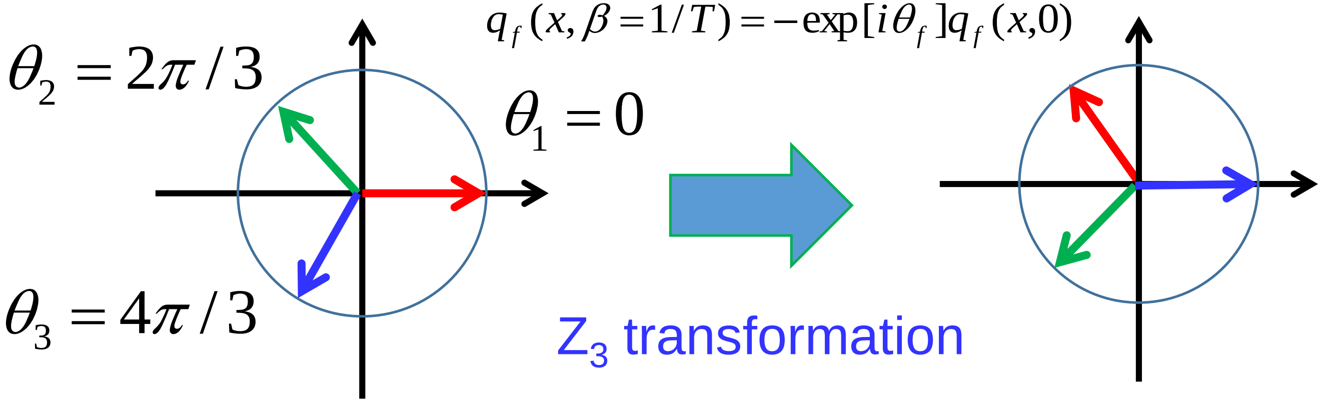
→ Beyond the standard physics (Higgs phenomenology)

Relation with μ_I : **K.K.** and T. Misumi, JHEP 05 (2013) 042.

Z_3 symmetric QCD

H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro, Phys. Rev .D 88 (2013) 016002.

$$q_f(x, \beta = 1/T) = -\exp[i\theta_f] q_f(x, 0)$$



Summary

The imaginary chemical potential is good region to construct reliable effective models

- Roberge-Weiss periodicity and transition
- Finite value of quark number density

⇒ Parameters can be determined (Ex. vector interaction)

Rough estimate of phase diagram

Imaginary chemical potential is very interesting and important region!

However, lattice data is not enough at the present…

I look forward to working with you again this year!