



Effective Field Theory of Spatial Diffeomorphisms Breaking

Chunshan Lin
YITP@Kyoto
Lunch Seminar

Ref: 1501.xxxxxx
To appear soon

Motivations

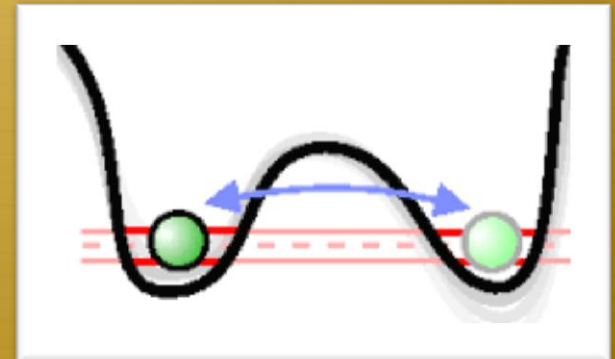
- ✧ Space time diffeomorphisms are local symmetries In GR

$$t \rightarrow t + \xi^0, \quad x^i \rightarrow x^i + \xi^i$$

- ✧ Temporal diff breaking – e.g. FRW universe
- ✧ Spatial diff breaking $\rightarrow ?$
- ✧ Three examples
 - ✧ Monopole solution in $SU(2) \rightarrow U(1)$

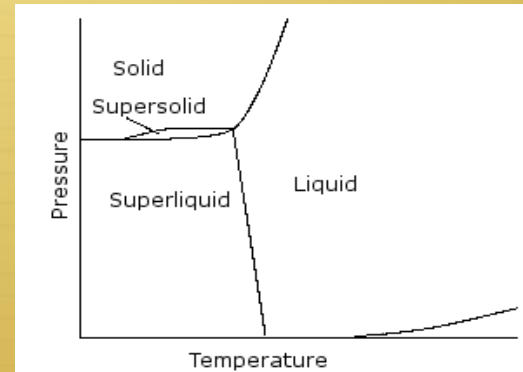
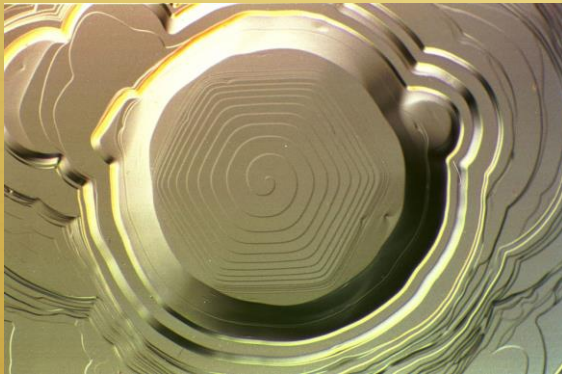
$$\phi^a = f(r) \frac{x^a}{r}, \quad a = 1, 2, 3,$$

Spatial diff is broken!



Motivations

✧ Solid and Supersolid in condense matter physics



- Atoms flow without friction through a solid block – supersolid
- At low energy, d.o.f of supersolid arise from the broken spatial translation symmetry

$$\langle \phi^a \rangle = x^a, \quad a = 1, 2, 3.$$

Spatial diff is broken too!

Motivations



✧ Massive gravity

- Graviton is a massless spin-2 particle in GR;
- Do we have a Higgs mechanism in gravity?

Massless spin 2 particle, 2 degrees of freedom



Massive spin 2 particle, 5 degrees of freedom

Fierz-Pauli (1939), dRGT (2010), Quasi-dilaton (2013)...

Motivations

- 3 spatial diffeomorphisms breaking



3 extra degrees, eaten by graviton in the unitary gauge



2 gravitational wave modes



5 degrees of freedom in gravity

2 tensor, 2 vector, 1 scalar modes, all of them are massive



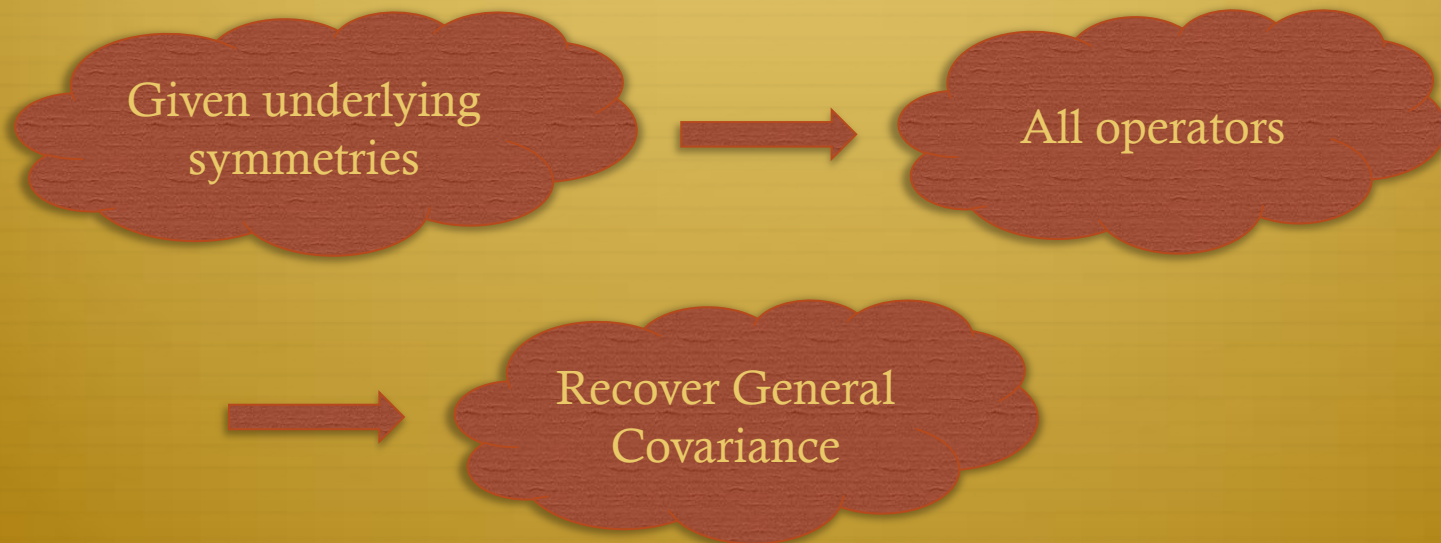
A massive gravity theory!

Effective field theory

✧ Traditionally



✧ EFT, an “up-side-down” approach



EFT of spatial diff breaking

✧ We are interested in such theory:

- 3 spatial diffeomorphisms
- Temporal diffeomorphism
- There are only 5 modes
- SO(3) internal rotations

We show that, the most generic effects among these 5 modes is (in unitary gauge)

Fixed by equation of motion, to cancel out the tadpole terms

Quadratic order operator, differences among different models are encoded in it!

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 \mathcal{R} + m_H^2 \sum_i G^{ii} + \Lambda + c \sum_i g^{ii} - \left[M_p^2 M^2 \bar{\delta} g^{ij} \bar{\delta} g^{ij} + \dots \right] \right],$$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 \mathcal{R} + m_H^2 \sum_i G^{ii} + \Lambda + c \sum_i g^{ii} - M_p^2 M^2 \bar{\delta} g^{ij} \bar{\delta} g^{ij} + \dots \right],$$

- The Einstein equation yields

$$\begin{aligned} \Lambda &= -3M_p^2 (H^2 + \dot{H}) - 3m_H^2 \frac{\dot{H}}{a^2}, \\ c &= M_p^2 a^2 \dot{H} + m_H^2 (\dot{H} - H^2). \end{aligned}$$

- It characterizes the most general theory of spatial diff breaking (with only 5 d.o.f), the action is an effective action of these 5 dynamical d.o.f, and it starts from quadratic order.

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 \mathcal{R} + m_H^2 \sum_i G^{ii} + \Lambda + c \sum_i g^{ii} - M_p^2 M^2 \bar{\delta} g^{ij} \bar{\delta} g^{ij} + \dots \right],$$

- Recover the general covariance by

$$x^i \rightarrow x^i + \xi^i$$

$$\xi^i \rightarrow \pi^i, \quad \pi^i \rightarrow \pi^i - \xi^i.$$

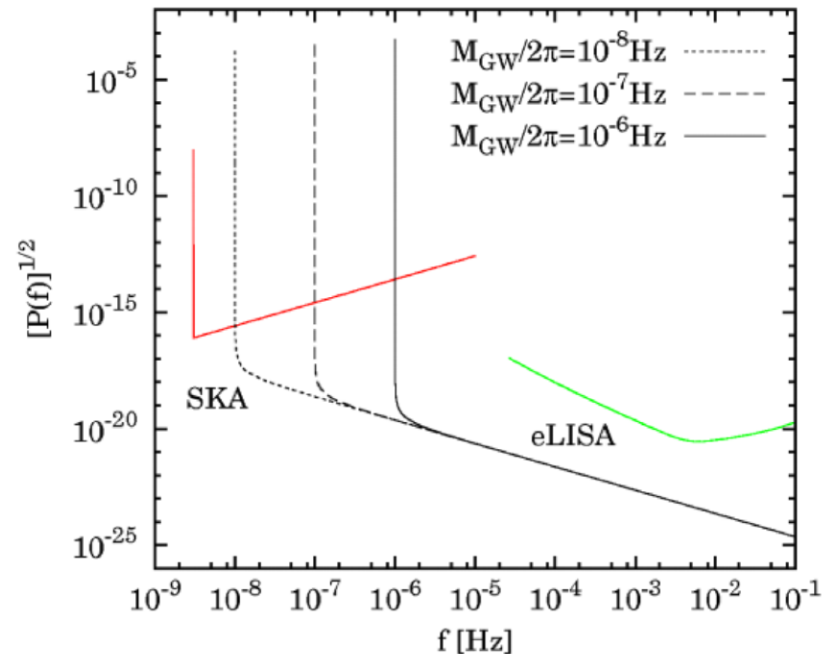
- In the unitary gauge, the Goldstone bosons, nonlinearly recover general covariance, the bosons are eaten by graviton, and a massive spin 2 particle, with $S(2, 2+1)$ on the spectrum.

Several Remarks

- ✧ The prediction of theory is characterized by a six-parameter set $\{H, \epsilon, \eta, s, M^2, m_H^2\}$.
- ✧ Massive tensor mode

There is a sharp peak on the stochastic gravitational waves spectrum, due to the graviton mass!

(1208.5975, by Emir, Sachiko, CL, Shinji, Norihiro)



Several Remarks



- ✧ In Minkowski space time, these 5 degrees are strongly coupled, and effective field theory breaks down;
- ✧ However, we can still find a stable self-accelerating solution;
- ✧ The effective theory with matter taken into account?
- ✧ Extend to non-linear perturbation theory?
- ✧ Possible observational phenomena on CMB, LLS, solar system test, binary pulsar...

Thank You!

**Please look left and right, up
and down, front and back. If
they are different, it means our
work is not meaningless!**