Lunch Seminar 2015/06/10

Quantum Entanglement of Local Operators

Masahiro Nozaki

Yukawa Institute for Theoretical Physics (YITP), Kyoto University

↓

Kadanoff Center for Theoretical Physics, The University of Chicago

1. arXiv:1401.0539 [hep-th]
2. arXiv:1405.5875 [hep-th]
3. arXiv:1405.5946 [hep-th]
4.arXiv.15xx.xxxxx[hep-th]

Introduction

Recently, (Renyi) entanglement entropy ((R)EE) has a center of wide interest in a broad array of theoretical physics.

- It is useful to study the distinctive features of various quantum state in condensed matter physics. (***Quantum Order Parameter***)
- (Renyi) entanglement entropy is expected to be an important quantity which may shed light on the mechanism behind the AdS/CFT correspondence .(***Gravity*** $\leftrightarrow$ ***Entanglement***)

Introduction

Recently, (Renyi) entanglement entropy ((R)EE) has a center of wide interest in a broad array of theoretical physics.

- In the lattice gauge theory, it is expected that entanglement entropy is a new order parameter which helps us study QCD more.

- ***But*** entanglement entropy in the gauge theory is ill-defined.

Introduction

Recently, (Renyi) entanglement entropy ((R)EE) has a center of wide interest in a broad array of theoretical physics.

- In the lattice gauge theory, it is expected that entanglement entropy is a new order parameter which helps us study QCD more.

- ***But*** entanglement entropy in the gauge theory is ill-defined.

It is important to study the properties of (Renyi) entanglement entropy.

Introduction

Recently, (Renyi) entanglement entropy ((R)EE) has a center of wide interest in a broad array of theoretical physics.

- In the lattice gauge theory, it is expected that entanglement entropy is a new order parameter which helps us study QCD more.

- ***But*** entanglement entropy in the gauge theory is ill-defined.

In this work, we investigate the time dependent property of (Renyi) entanglement entropy.

The Definition of (Renyi) Entanglement Entropy

- Definition of Entanglement Entropy

We divide the total Hilbert space into A and B: $H_{tot} = H_A \otimes H_B$.

The reduced density matrix ρ_A is defined by $\rho_A \equiv Tr_B \rho_{tot}$

This means the D O F in B are traced out.

The entanglement entropy is defined by von Neumann entropy S_A.

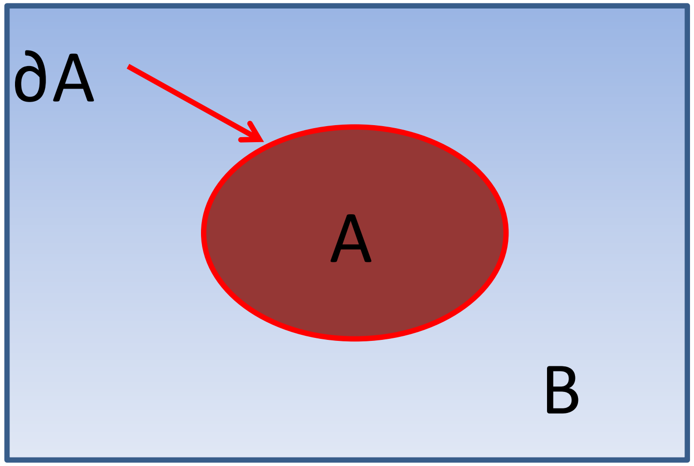
(Renyi) Entanglement Entropy (REE)

$$S_A^{(n)} = \frac{\log \mathrm{tr}[\rho_A^n]}{1-n}$$

n→1

Entanglement Entropy (EE)

$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A$$



on a certain time slice

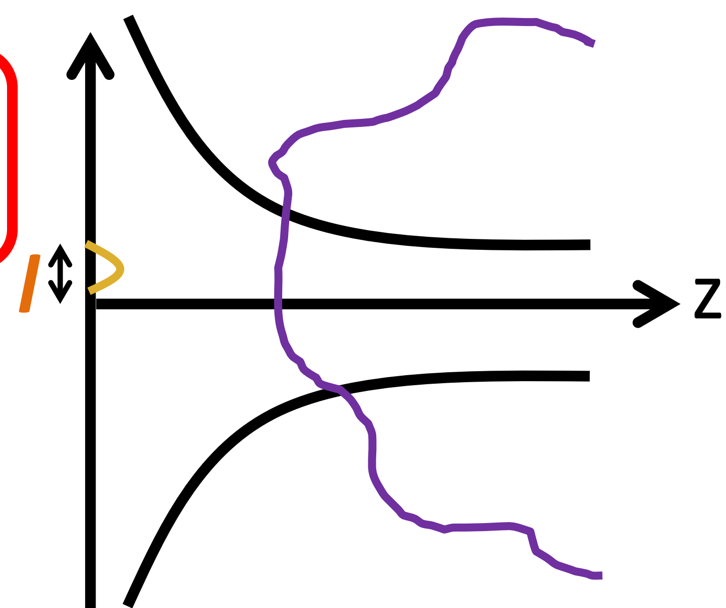
Motivation

Previously, we studied the property of EE for the subsystem whose size (*l*) is ***very small*** in d CFT.

$l \ll (\text{The Excess of Energy Density})^{-d}$,

$$\Delta E_A = T_{ent} \Delta S_A$$

This temperature is universal.



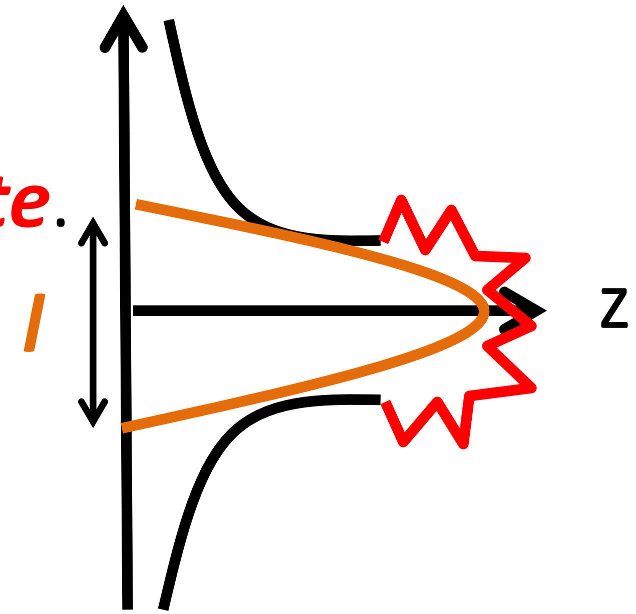
Setup

We study the property of **(R)EE** for

1. The size of subsystem is ***infinite***.

   A half of the total system:

$$x^1 \geq 0$$

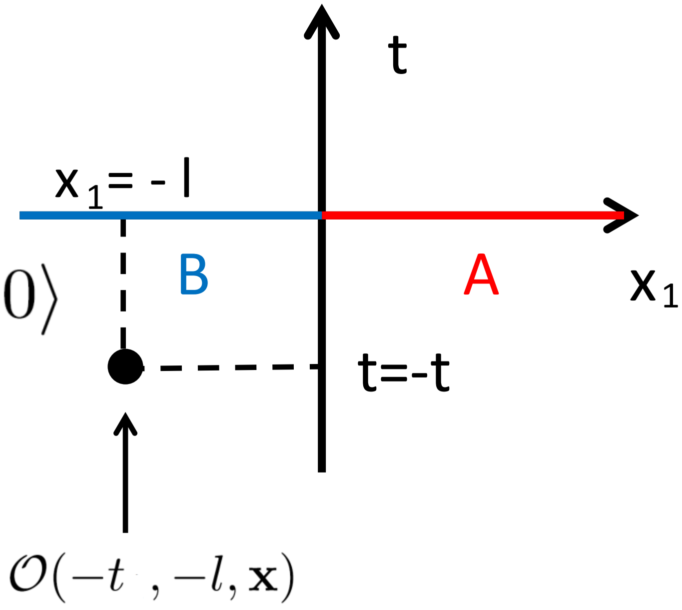


Setup

We study the property of **(R)EE** for

2. A state is defined by acting a local operator on the ground state:

$$|\Psi\rangle = \mathcal{N}\mathcal{O}(t=-t, x^1=-l, \mathbf{x})\,|0\rangle$$



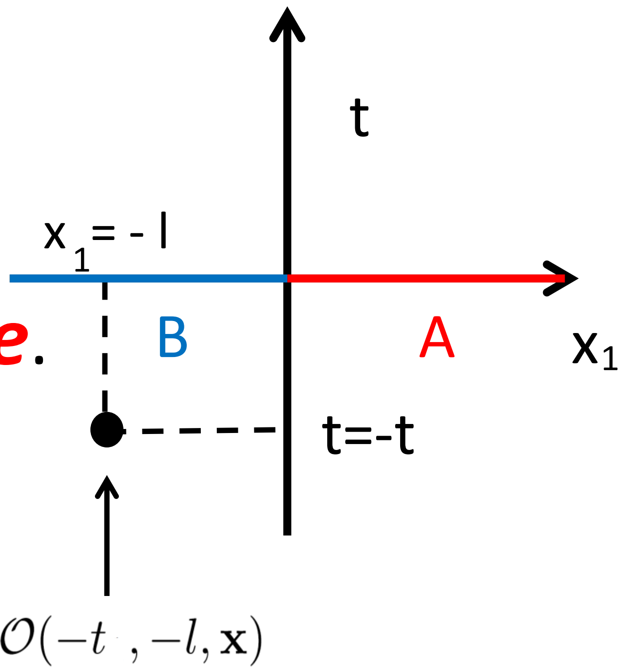
Setup

We study the property of **(R)EE** for

1. The size of subsystem is ***infinite***.

   A half of the total system:

$$x^1 \geq 0$$



2. A state is defined by acting a local operator

   on the ground state:

$$|\Psi\rangle = \mathcal{N}\mathcal{O}(t,x^i)\,|0\rangle$$

Motivation

We would like to focus on the time evolution of the (R)EE.

We define $\Delta S_A^{(n)}$ the excess of the (R)EE:

$$\Delta S_A^{(n)} = S_A^{(n)Ex} - S_A^{(n)G},$$

$S_A^{(n)Ex}$: (R)EE for $\hat{\rho}_A$ (Reduced Density Matrix for $|\Psi\rangle = \mathcal{N}\mathcal{O}(t, x^i)\,|0\rangle$)

$S_A^{(n)G}$: (R)EE for the ground state

Field Theory

1. Free massless scalar field theory

2. *U(N)* or *SU(N)* free massless scalar field theory in Large N limit

3. Free massless fermionic field theory

4. Holographic field theory

Field Theory

1. Free massless scalar field theory

2. *U(N)* or *SU(N)* free massless scalar field theory
   in Large N limit

3. Free massless fermionic field theory

4. Holographic field theory

Field Theory

1. Free massless scalar field theory
2.
3. Free massless fermionic field theory
4. Holographic field theory

$$\Delta S_A^{(n)}$$

At late time

Some Constants

Field Theory

1. Free massless scalar field theory

2.

$$\Delta S_A^{(n)}$$ → ***Logarithmically grows***

At late time

3.

4. Holographic field theory

Results

$\Delta S_A^{(n)}$ At Late Time(t >> l)

F.T.	Operator	$n \geq 2$	$n = 1$
Free Massless Scalar	$:(\partial^m \phi)^k:$	$\frac{1}{1-n}\log\left(\frac{1}{2^{nk}}\sum_{j=0}^{k}({}_kC_j)^n\right)$	$k \cdot \log 2 - \frac{1}{2^k}\sum_{j=0}^{k}{}_kC_j \log {}_kC_j$
U(N),SU(N) Free Massless Scaler	$Tr(\phi_1 + i\phi_2)^J$	$\frac{2n-1}{n-1} \cdot \log 2$ ***J=2***	$\log\left(2\sqrt{2}N\right)$ ***J=2***
Free Massless Fermion	$\bar{\psi}\psi$	$\frac{1}{1-n}\log\left[\frac{2 \cdot 12^n + 4 \cdot 9^n + 4}{2^{6n}}\right]$	$\frac{3}{4}\log\left(\frac{128}{9}\right)$
AdS/CFT	Gauge invariant Operator $\mathcal{O}_\Delta$	$\frac{4n\Delta}{d(n-1)}\log t$ $1 \ll \Delta \ll c$	$\frac{c}{6}\log t$ ***2d CFT*** $\Delta \simeq c$

Results

$\Delta S_A^{(n)}$ At Late Time(t >> l)

F.T.	Operator	$n \geq 2$	$n = 1$
Free Massless Scalar	$:(\partial^m \phi)^k:$		
U(N),SU(N) **Free Massless Scaler**	$Tr(\phi_1 + i\phi_2)^J$		
Free Massless Fermion	$\bar{\psi}\psi$		
AdS/CFT	Gauge invariant Operator $\mathcal{O}_\Delta$	$\frac{4n\Delta}{d(n-1)} \log t$ $1 \ll \Delta \ll c$	$\frac{c}{6} \log t$ ***2d CFT*** $\Delta \simeq c$

Constant

Example

We consider ***free massless scalar*** field theory in ***d+1 dim.***

Especially, we focus on that in ***4 dim.***

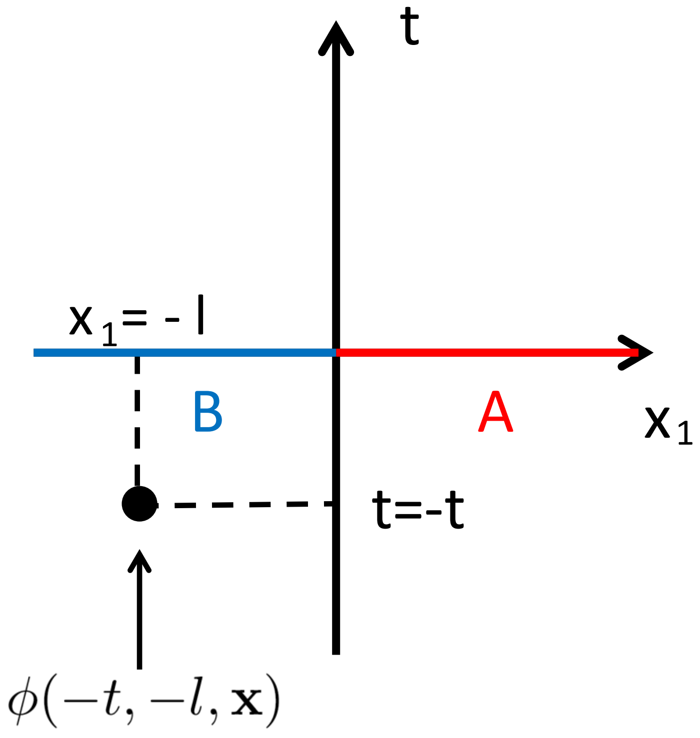
We act a local operator $\phi(-t,-l,\mathbf{x})$ on the ground state:

$$|\Psi\rangle = \mathcal{N} \quad \phi(-t,-l,\mathbf{x})\,|0\rangle \ .$$

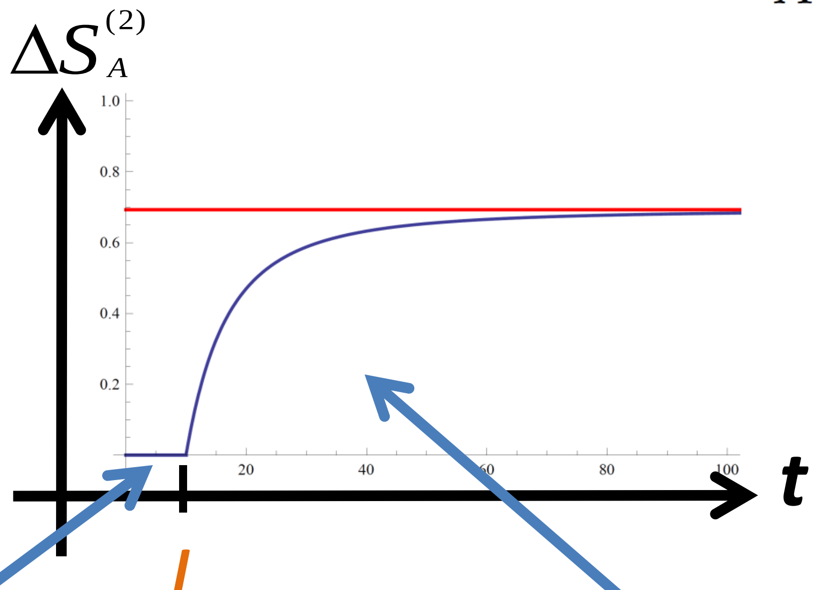
We measure the (Renyi)

entanglement entropies at t=0.

➡ ***Time evolution!!***



Time Evolution of $\Delta S_A^{(2)}$

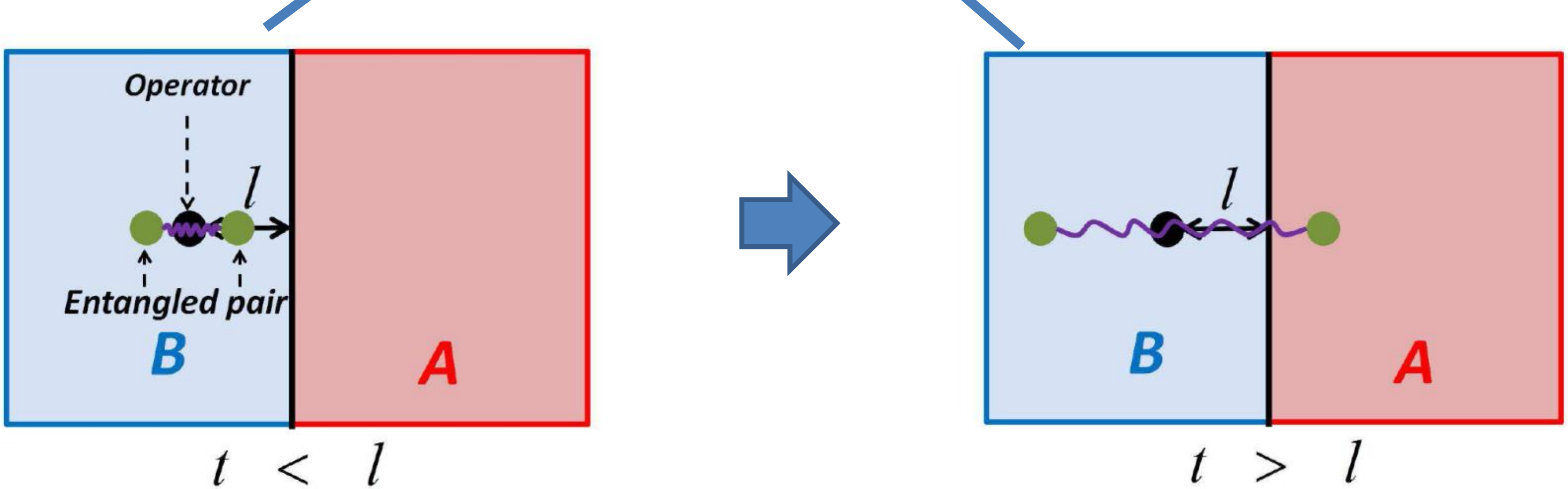


$t < l$

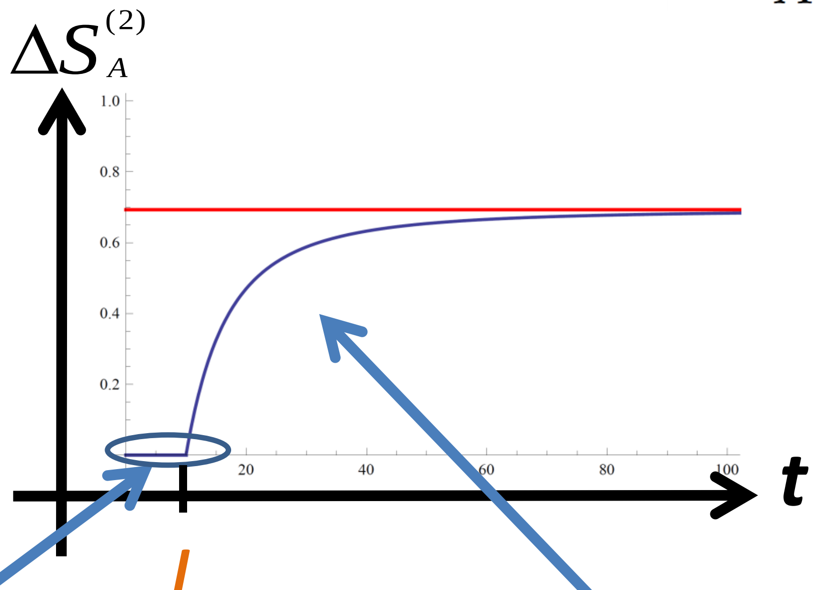
$$\Delta S_A^{(2)} = 0$$

$t \geq l$

$$\Delta S_A^{(2)} = \log\left[\frac{2t^2}{t^2 + l^2}\right]$$



Time Evolution of $\Delta S_A^{(2)}$

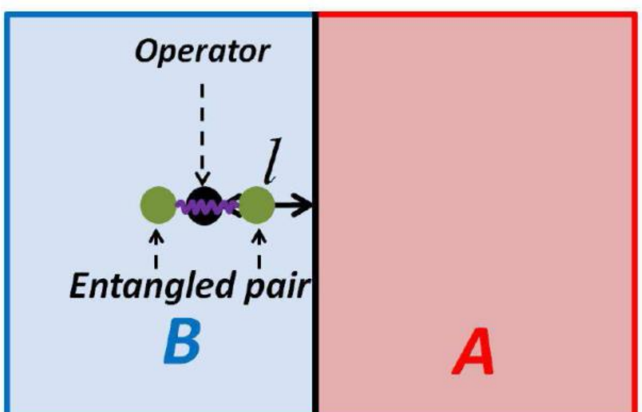


$t < l$

$$\Delta S_A^{(2)} = 0$$

$t \geq l$

$$\Delta S_A^{(2)} = \log\left[\frac{2t^2}{t^2 + l^2}\right]$$



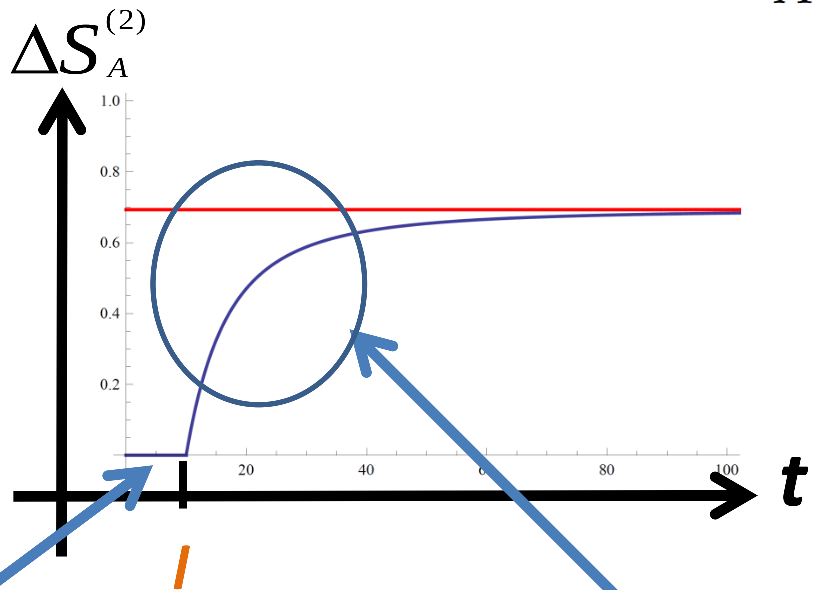
$t \quad < \quad l$

$t < l$

An entangled pair appears.

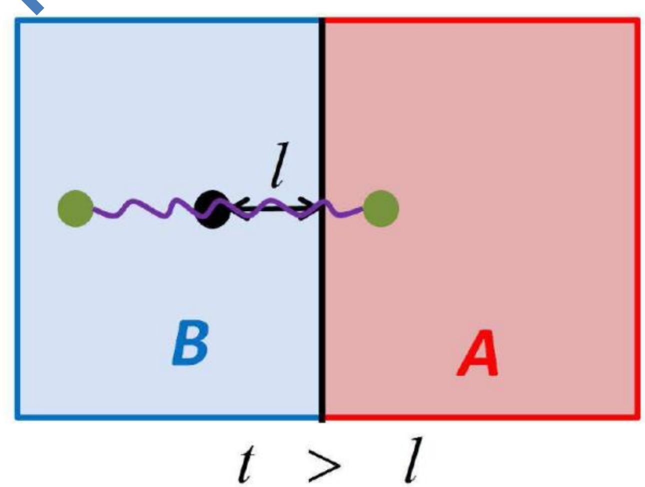
Each of pair is included in the region B.

Time Evolution of $\Delta S_A^{(2)}$

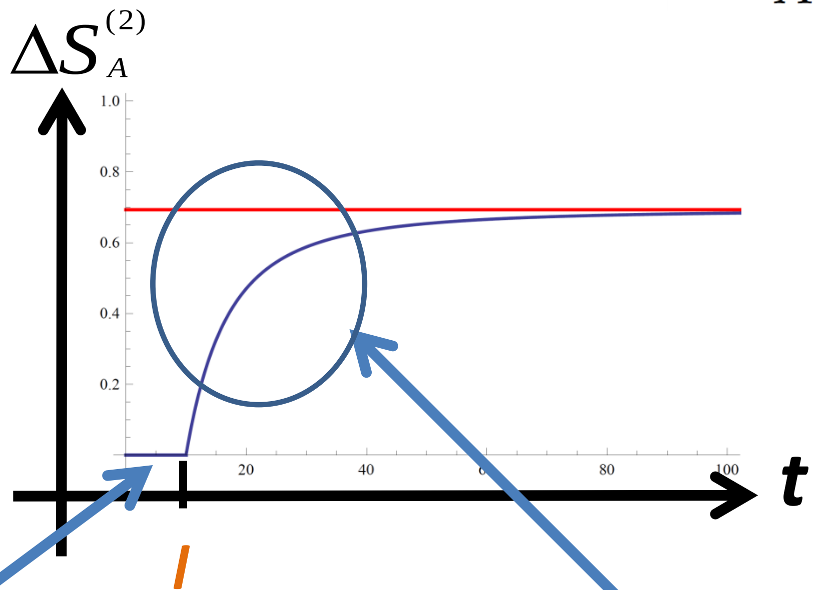


$t \geq l$

In this region, two quanta is included in A and B respectively .



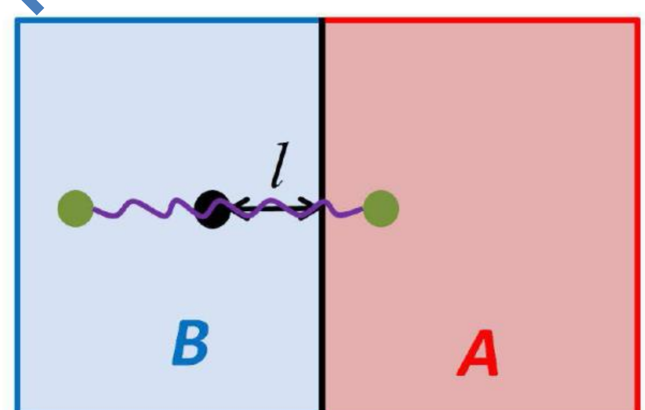
Time Evolution of $\Delta S_A^{(2)}$



$$\Delta S_A^{(2)} = \log\left[\frac{2t^2}{t^2+l^2}\right]$$

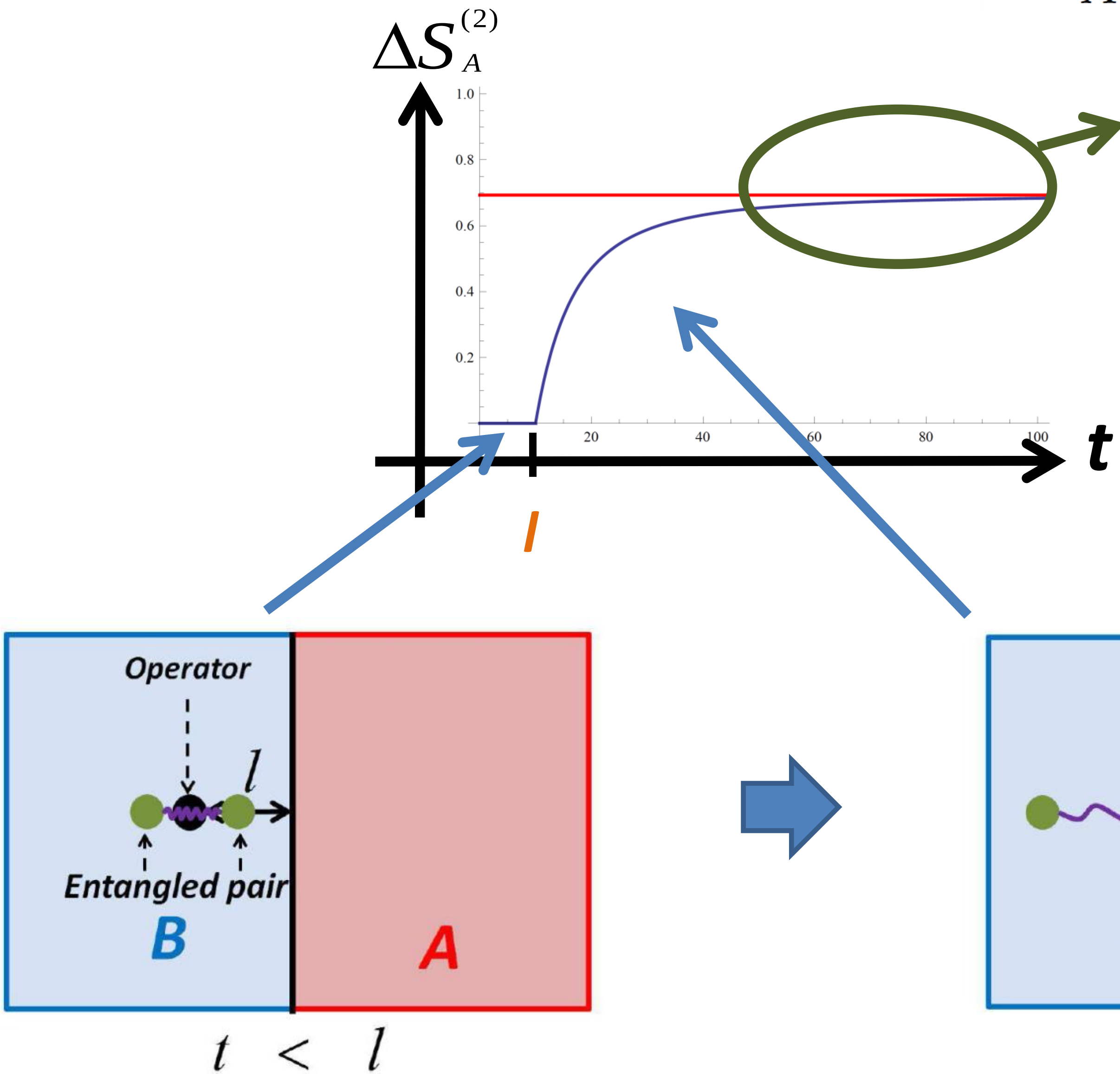
$t \geq l$

In this region, two quanta is included in A and B respectively .



Entanglement between quanta can contribute to $\Delta S_A^{(2)}$.

Time Evolution of $\Delta S_A^{(2)}$



Subsystem

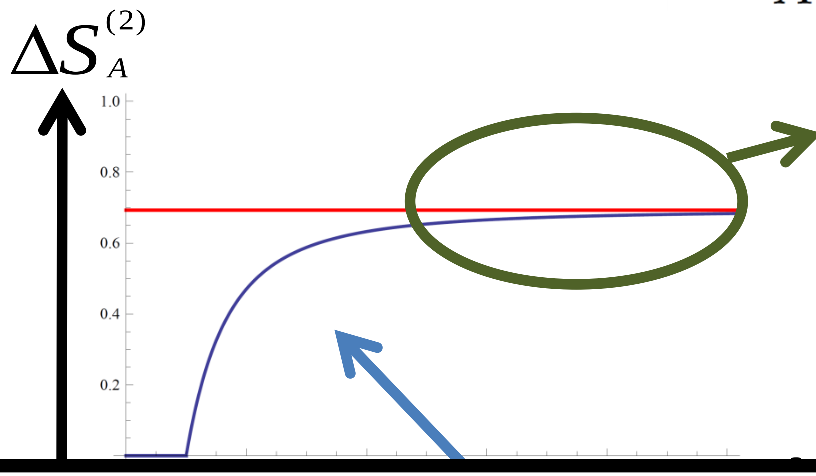
= a half of the

total space

$\Delta S_A^{(2)}$ approaches

Constant!!

Time Evolution of $\Delta S_A^{(2)}$

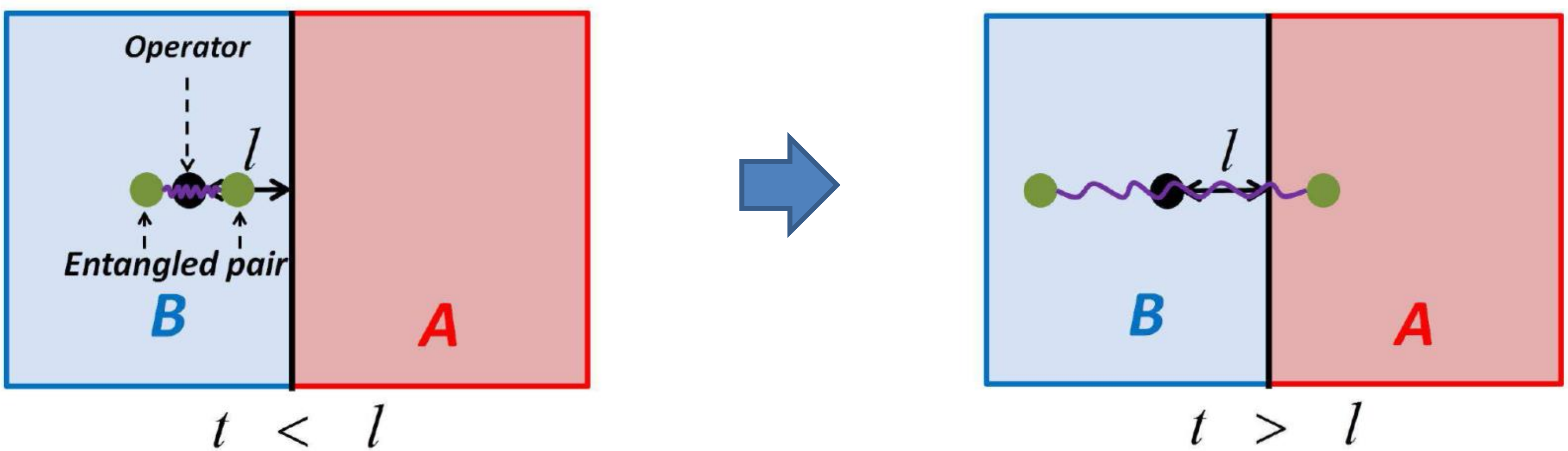


Subsystem
= a half of the
total space

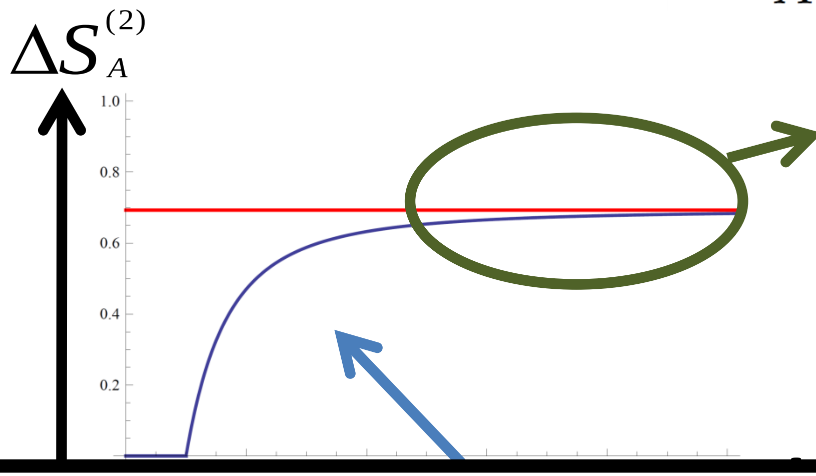
$\Delta S_A^{(2)}$ approaches

t!!

$\Delta S_A^{(2)}$ ***= Entanglement between Quasi-particles***



Time Evolution of $\Delta S_A^{(2)}$



Subsystem

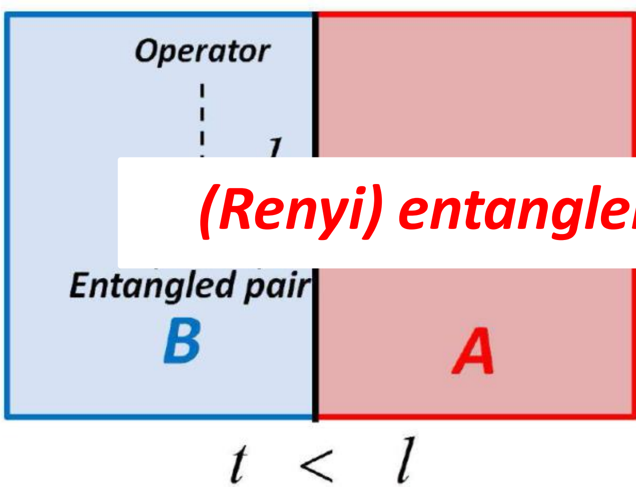
= a half of the

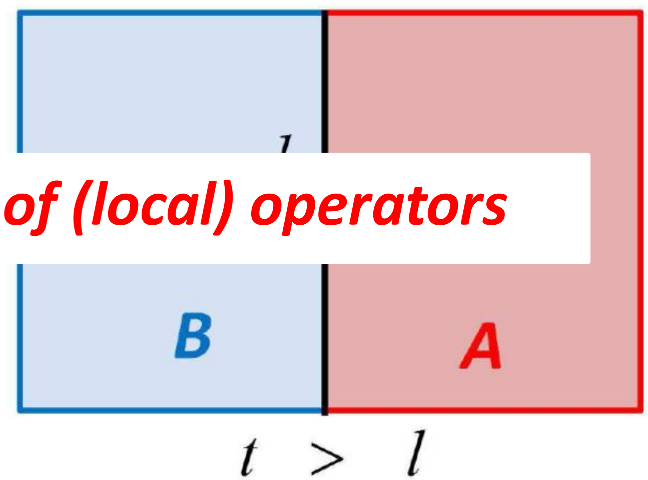
total space

$\Delta S_A^{(2)}$ approaches

t!!

$\Delta S_A^{(2)}$ = ***Entanglement between Quasi-particles***





(Renyi) entanglement entropies of (local) operators

Results

$\Delta S_A^{(n)}$ At Late Time(t >> l)

F.T.	Operator	$n \geq 2$	$n = 1$
Free Massless Scalar	$:(\partial^m \phi)^k:$		
U(N),SU(N) Free Massless Scaler	$Tr(\phi_1 + i\phi_2)^J$	***Constant***	
Free Massless Fermion	$\bar{\psi}\psi$		
AdS/CFT	Gauge invariant Operator $\mathcal{O}_\Delta$	***Logarithmically Grows***	

Summary

- If non-perturbative effect for ***c*** or ***N,*** $\Delta S_A^{(n)}$ can possibly approach constant.

•Free field theory (Infinite conserved charges) → Quasi-particle picture holds

⇒ $\Delta S_A^{(n)}$ → ***Constant***

Holographic CFT→ Quasi-particle picture breaks down

⇒ $\Delta S_A^{(n)}$ → ***Not constant***

Future Problems

- In non-relativistic case, the time evolution of $\Delta S_A^{(n)}$
- In gauge field theory, $\Delta S_A^{(n)}$
- The (Renyi) entanglement entropies of operators in the interacting field theory . (also massive and charged Renyi.)
- Beyond large N, we investigate $\Delta S_A^{(n)}$.

  - approach constant?
  - diverge?