Quark deconfinement in lattice QCD: From Hadrons to Quarks

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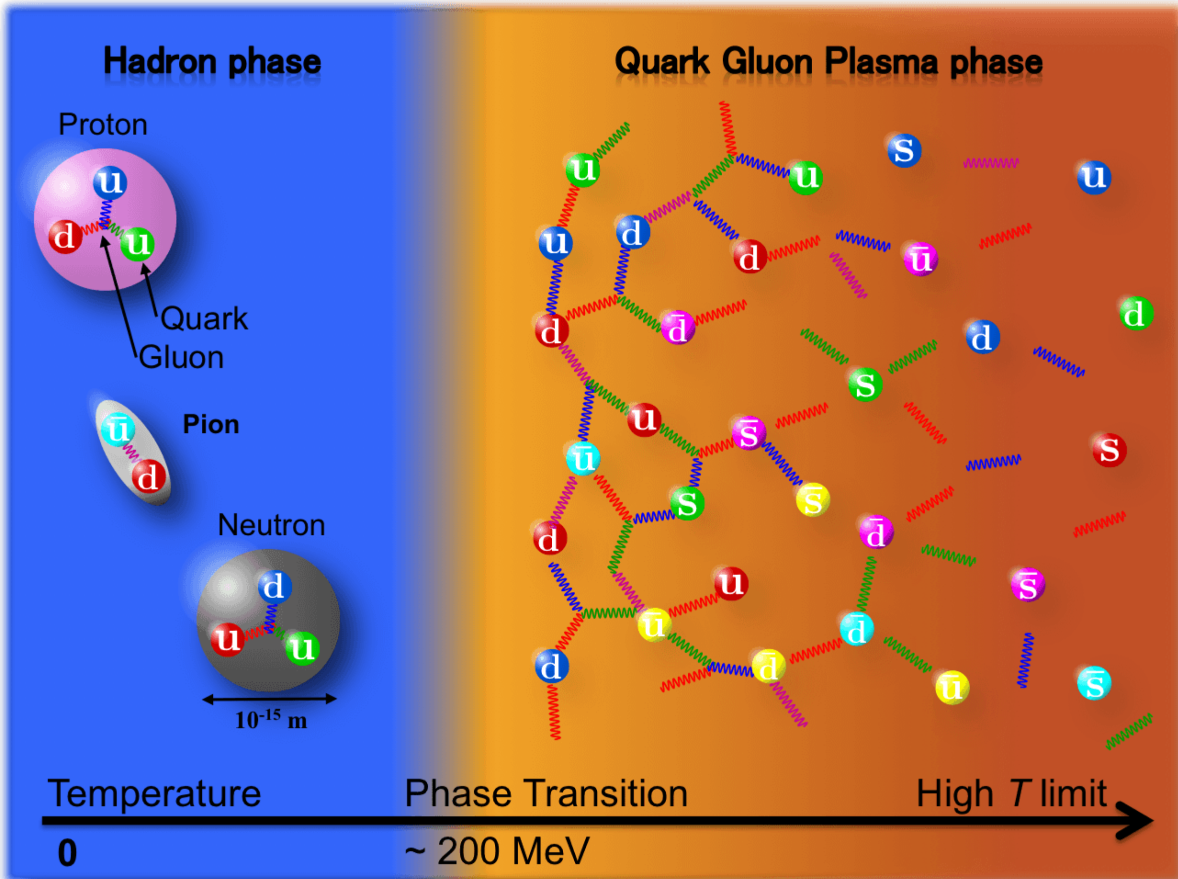
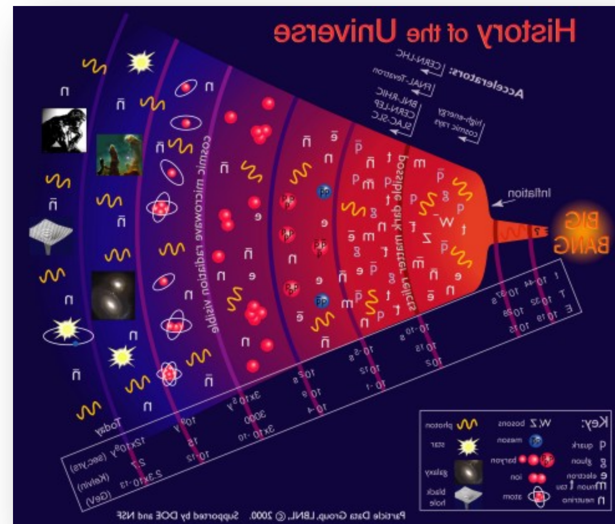
Lunch seminar @ YITP, June 24, 2015

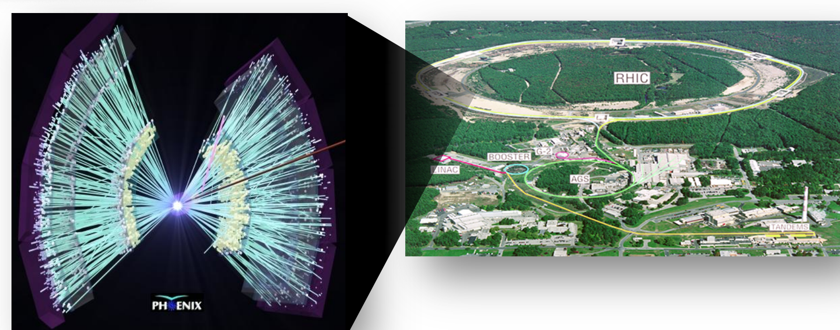
Hot QCD world



Big Bang



Little Bang



Time

Amazing plasma of quarks and gluons…

→ QCD thermodynamics

Quantum Chromo-Dynamics

$$\mathcal{L}_{\mathrm{QCD}} = \underbrace{\bar{q}^{\alpha}\left(i\gamma_{\mu}D^{\mu}_{\alpha\beta} - m\delta_{\alpha\beta}\right)q^{\beta}} - \underbrace{\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha}}$$

Quark: chiral symmetry

→ Origin of mass

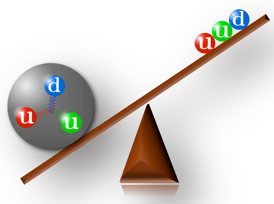
Chiral condensate: $\langle\bar{q}q\rangle$

Gluon: non-Abelian

→ Color confinement

Polyakov loop:

$$L = \langle e^{ig\oint A_4 d\tau}\rangle$$



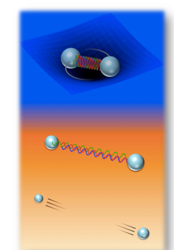
Phase transition

Vanishing quark mass

2nd order PT

in $N_f = 2$ and $m = 0$

Color deconfinement



1st order PT in $m \to \infty$

(pure Yang-Mills)

Real QCD world

$$m_u \simeq 3\ \mathrm{MeV},\ m_d \simeq 5\ \mathrm{MeV},\ m_s \simeq 95\ \mathrm{MeV},\ m_c \simeq 1275\ \mathrm{MeV},\ \cdots$$

...various quark flavors

Lattice QCD simulations

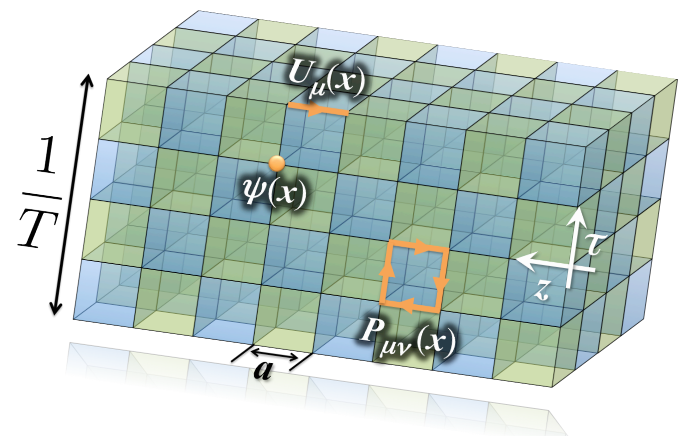
QCD: Strong non-linearity and infinite-dimensional integral

Field theory on lattice in Euclidean space

Monte-Carlo simulations based on importance sampling

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D\bar{q} D q D A \, \mathcal{O}(\bar{q}, q, A) \, e^{-S_{QCD}}$$

$$= \frac{1}{N_{\rm conf}} \sum_{\{U_i\}}^{N_{\rm conf}} \mathcal{O}(U_i) \pm O\left(\frac{1}{\sqrt{N_{\rm conf}}}\right)$$



$\{U_i\}$: configurations generated with the weight $\exp(-S_{QCD})$

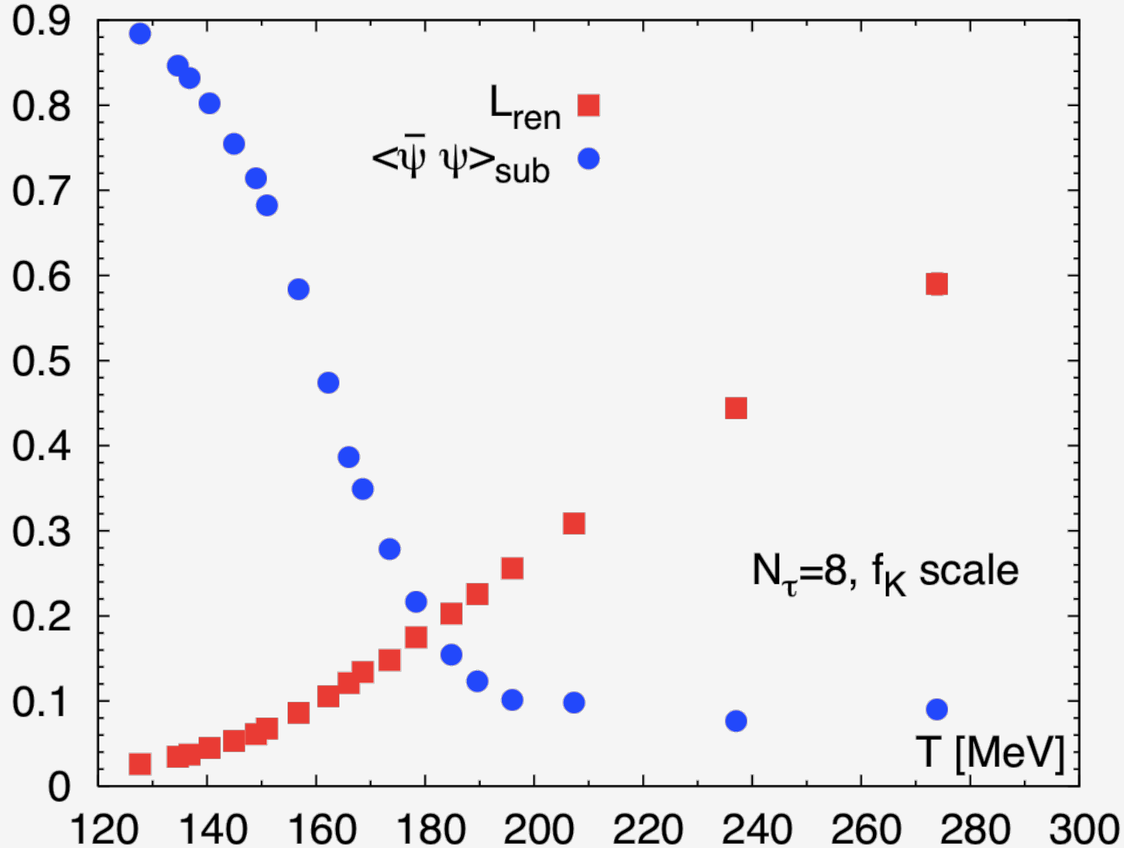
→ 1st principle and non-perturbative calculations in large-scale computational simulations

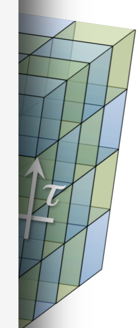
Lattice QCD simulations

Lattice results: (2+1)-flavor QCD simulations

Phase transition: **Crossover**

- Chiral: **rapid** ⟶ $T_C = 154 \pm 9$ MeV
- Deconfinement: **gradual**





Hot-QCD Coll.

PRD 85 (2011) 054503

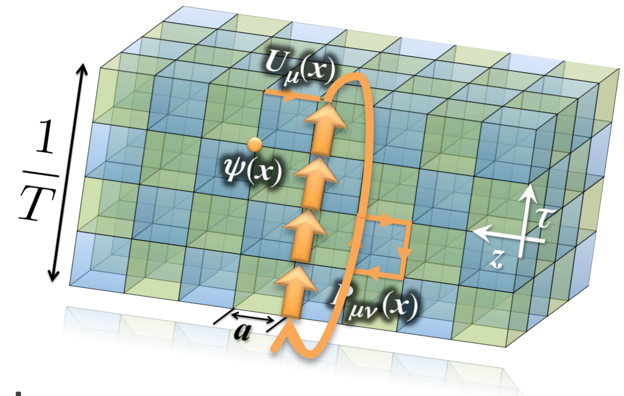
- HISQ action
- $m_l = 0.05 m_s$
- N_τ=8--12

Deconfinement of quarks

QCD phase transition at high temperature:

- Dominated by chiral transition
- Color DoF gradually deconfined

Polyakov loop: test color charge of a static quark ($m \longrightarrow \infty$)



How dynamical quarks deconfine?

→ response to chemical potential

e.g. baryon number

Baryon

$$\left.\frac{\partial^2 (P/T^4)}{\partial \mu_B^2}\right|_{\mu_B=0} \propto 1^2$$

Quark

$$\left.\frac{\partial^2 (P/T^4)}{\partial \mu_B^2}\right|_{\mu_B=0} \propto \left(\frac{1}{3}\right)^2$$

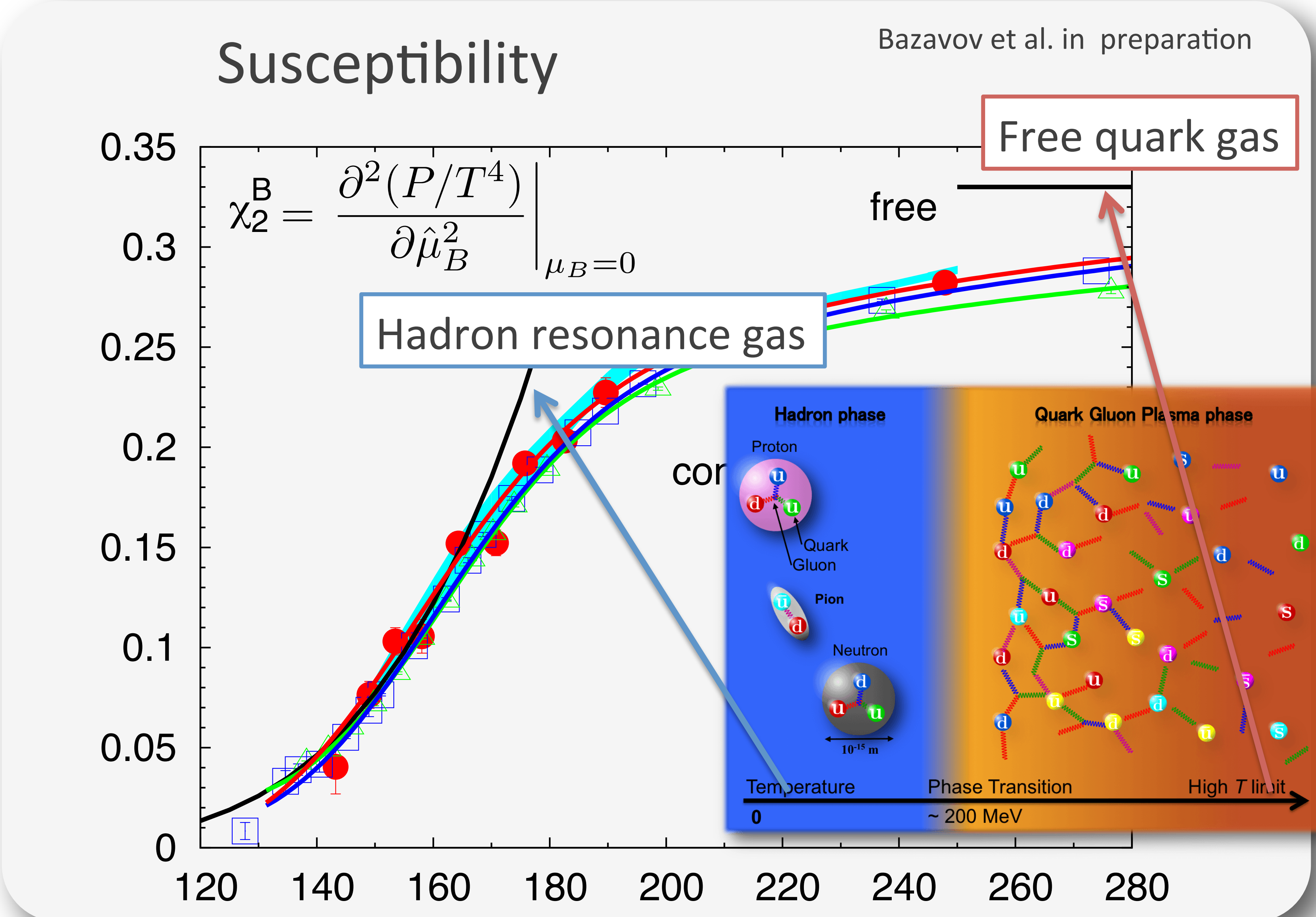
Susceptibility: $\chi_{mn}^{BS} \equiv \left.\frac{\partial^{m+n}(P/T^4)}{\partial \hat{\mu}_B^m \hat{\mu}_S^n}\right|_{\vec{\mu}=0}$

$\hat{\mu} = \mu/T$

$n + m \in \mathrm{even}$

$\mu_S \leftrightarrow \mu_C$

Deconfinement of quarks



Hadron resonance gas

Non-interacting Meson-Baryon gas system

$$\frac{P^{\mathrm{HRG}}}{T^4} = \sum_{i\in\mathrm{meson}} \frac{g_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right)\cosh(S_i\hat{\mu}_S) + \sum_{i\in\mathrm{baryon}} \frac{g_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right)\cosh(B_i\hat{\mu}_B + S_i\hat{\mu}_S)$$

$$\pi^{\pm}, \pi^0, \rho, \cdots \quad K, K^{\pm}, \bar{K} \cdots$$

$$= M_0 + M_1\cosh(-\hat{\mu}_S)$$

$$+ B_0\cosh(\hat{\mu}_B) + B_1\cosh(\hat{\mu}_B - \hat{\mu}_S) + B_2\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + B_3\cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

$$N, \Delta, N^*, \cdots \qquad \Lambda, \cdots \qquad \Xi, \cdots \qquad \Omega, \cdots$$

Relation to susceptibilities (up to 4th order)

$$\begin{array}{lclclclcl}
\chi_2^B & = & & & B_0 & + & B_1 & + & B_2 & + & B_3 \\
\chi_4^B & = & & & B_0 & + & B_1 & + & B_2 & + & B_3 \\
\chi_{11}^{BS} & = & & & & & (-1)B_1 & + & (-2)B_2 & + & (-3)B_3 \\
\chi_{31}^{BS} & = & & & & & (-1)B_1 & + & (-2)B_2 & + & (-3)B_3 \\
\chi_2^S & = & (-1)^2 M_1 & & & + & (-1)^2 B_1 & + & (-2)^2 B_2 & + & (-3)^2 B_3 \\
\chi_{22}^{BS} & = & & & & & (-1)^2 B_1 & + & (-2)^2 B_2 & + & (-3)^2 B_3 \\
\chi_{13}^{BS} & = & & & & & (-1)^3 B_1 & + & (-2)^3 B_2 & + & (-3)^3 B_3 \\
\chi_4^S & = & (-1)^4 M_1 & & & + & (-1)^4 B_1 & + & (-2)^4 B_2 & + & (-3)^4 B_3
\end{array}$$

$\chi_2^B - \chi_4^B = 0$: constraint for u, d quarks

Hadron resonance gas

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$$= M_0 + M_1\cosh(-\hat{\mu}_S)$$

$$+ B_0\cosh(\hat{\mu}_B) + B_1\cosh(\hat{\mu}_B - \hat{\mu}_S) + B_2\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + B_3\cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

Relation to susceptibilities (up to 4th order)

$$\begin{pmatrix} \chi_{11}^{BS} \\ \chi_{31}^{BS} \\ \chi_{2}^{S} \\ \chi_{22}^{BS} \\ \chi_{13}^{BS} \\ \chi_{4}^{S} \end{pmatrix} = \begin{pmatrix} 0 & -1 & -2 & -3 \\ 0 & -1 & -2 & -3 \\ 1 & 1 & 4 & 9 \\ 0 & 1 & 4 & 9 \\ 0 & -1 & -8 & -27 \\ 1 & 1 & 16 & 81 \end{pmatrix} \begin{pmatrix} M_1 \\ B_1 \\ B_2 \\ B_3 \end{pmatrix}$$

rank 4

Hadron resonance gas

Non-interacting Meson-Baryon gas system

$$\frac{P^{\mathrm{HRG}}}{T^4} = \sum_{i\in\mathrm{meson}} \frac{g_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right)\cosh(S_i\hat{\mu}_S) + \sum_{i\in\mathrm{baryon}} \frac{g_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right)\cosh(B_i\hat{\mu}_B + S_i\hat{\mu}_S)$$

$$= M_0 + M_1\cosh(-\hat{\mu}_S)$$
$$+ B_0\cosh(\hat{\mu}_B) + B_1\cosh(\hat{\mu}_B - \hat{\mu}_S) + B_2\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + B_3\cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

Relation to susceptibilities (up to 4th order)

$$\left.\begin{aligned} v_1 &\equiv \chi_{31}^{BS} - \chi_{11}^{BS} = 0 \\ v_2 &\equiv \tfrac{1}{3}(\chi_2^S - \chi_4^S) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS} = 0 \end{aligned}\right\} \text{constraints for } s \text{ quark}$$

$$M_1 = \chi_2^S - \chi_{22}^{BS}$$

$$B_1 = \frac{1}{2}\left(\chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS}\right)$$

$$B_2 = -\frac{1}{4}\left(\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}\right)$$

$$B_3 = \frac{1}{18}\left(\chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS}\right)$$

Deconfinement of u, d and s quarks

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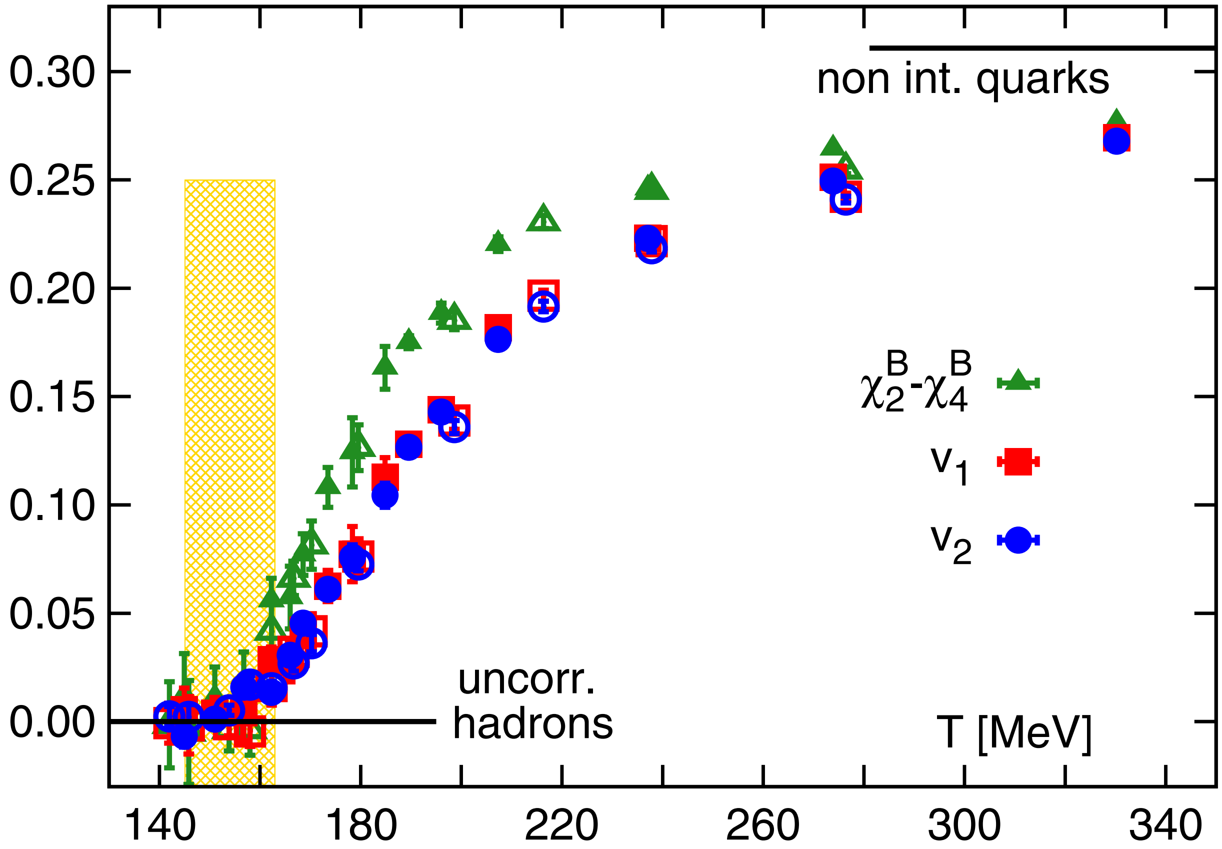
$\chi_2^B - \chi_4^B = 0$ constraints for u, d quarks

$v_1 = 0,\ v_2 = 0$ constraints for s quark



- The constraints satisfied below T_C: HRG system
- Deviate from 0 at $T \sim T_C$: not only u,d but s quarks deconfined
- Close to free quark gas at high temperature

Deconfinement of charm quarks

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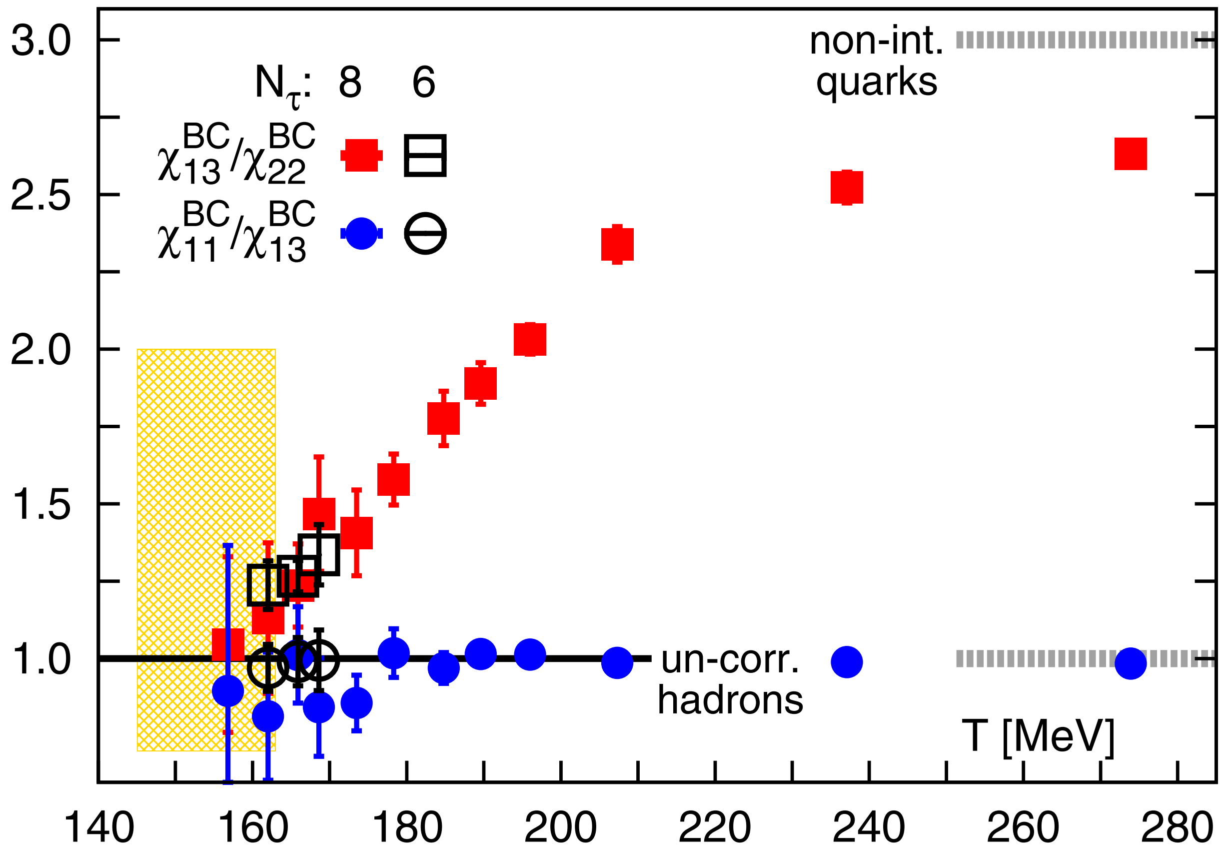
Similar procedure for charm: $\mu_S \leftrightarrow \mu_C$



- Charm quarks also deconfined at $T \sim T_C$
- Similar to strange susceptibility

Hadronic description in strangeness

$$M_1(c_1, c_2) = \chi_2^S - \chi_{22}^{BS} + c_1 v_1 + c_2 v_2$$

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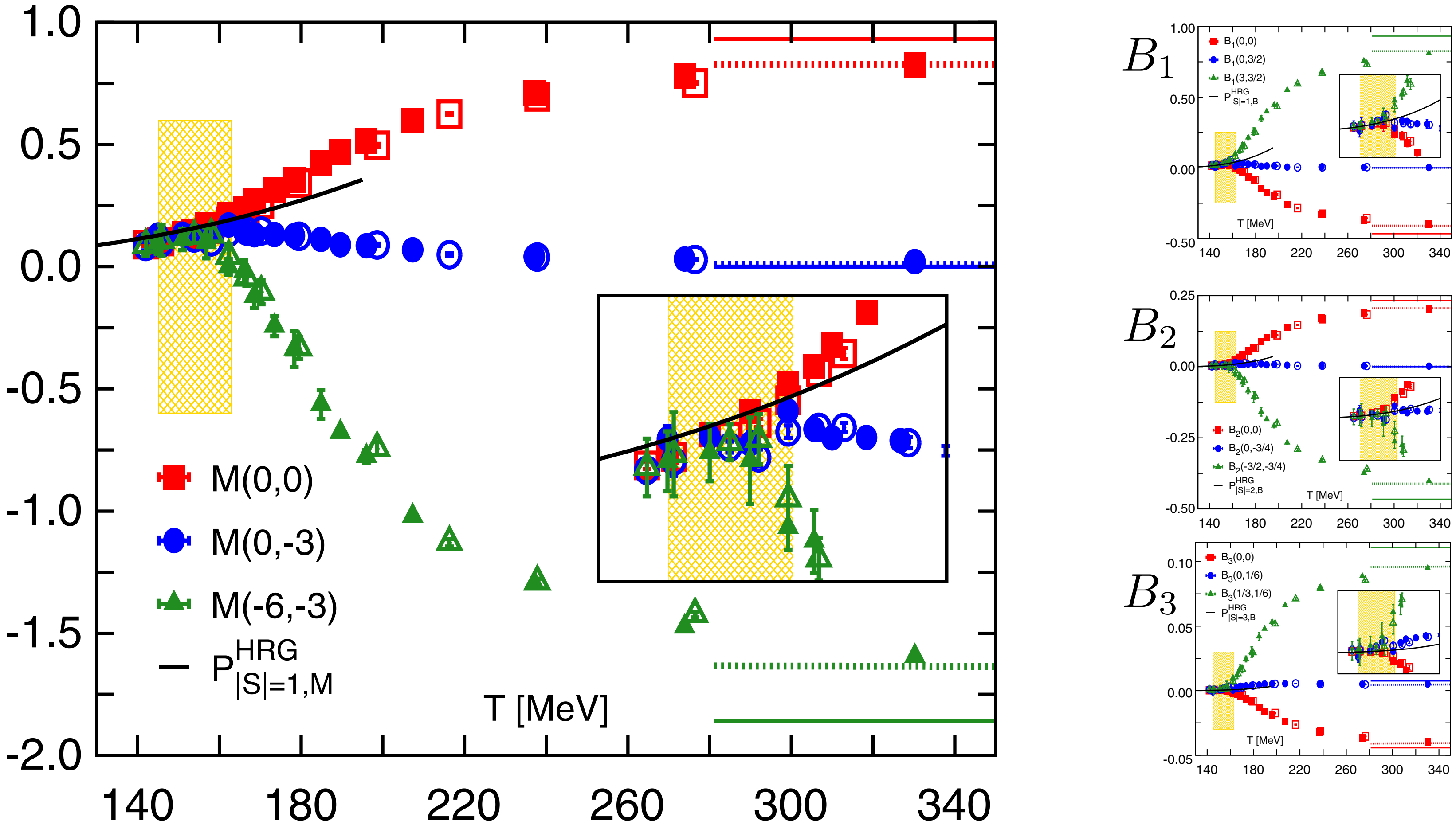

- Hadronic description breaks down at $T \sim T_C$
- Hard thermal loop perturbation theory (dotted lines) at $T > 2T_C$

Andersen *et al*. (2013)

Summary: Hot QCD world