Small Black Rings

Masaki Shigemori
(Caltech)

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N. Iizuka + MS, 0506215

A. Dabholkar + N. Iizuka + A. Iqubal + MS, 0511120

A. Dabholkar + N. Iizuka + A. Iqubal + A. Sen + MS, 0611166

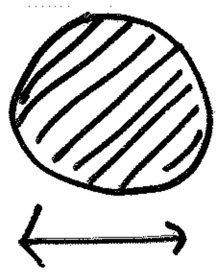
Introduction

Black holes & String Theory :

One of greatest successes in str. thy. !

- Entropy counting [Strominger - Vafa]
- AdS/CFT
- Attractor mechanism
- OSV

⋮



$S_{micro} = S_{BH}$

$r \gg l_s, l_p$

More precise counting

- Beyond sugra / Bekenstein-Hawking
- R^2 correction [CdWM]

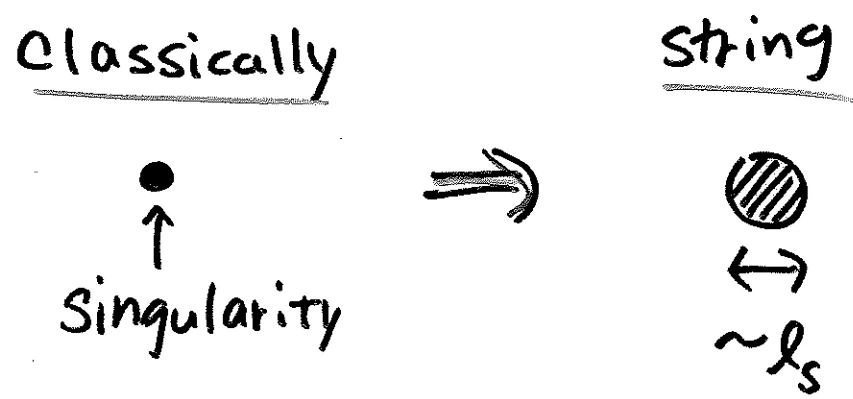
$$S = \int \sqrt{G}\,[R + \underline{\underline{\alpha' R^2}} + \cdots]$$

"Small" Black Holes (SBH)

- Classically $A = 0$, thus $S_{BH} = 0$
- But microscopically $S_{micro} \neq 0$
- Stringy effects $\Rightarrow S_{BHW} \neq 0$

  [Sen]
  [Dabholkar] [DKM] ...

- Micro counting: can be done precisely

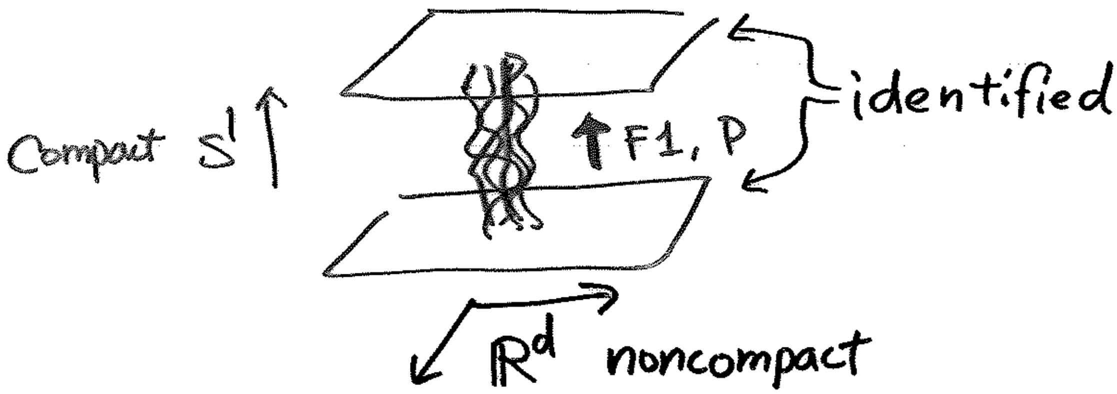


Example:

FP system (or historically Dabholkar-Harvey sys.)

Compactify hetero. string on S^1

w F1's and n P's along S^1



$\Downarrow$

From $\mathbb{R}^d$ viewpoint, pointlike

▶ Micro entropy $\boxed{S_{micro} = 4\pi\sqrt{nw}}$

▶ Macro (geometry) [Sen][Dabholkar]

$e^{\Phi} \sim \frac{1}{\sqrt{nw}} \ll 1$

$l_s \gg l_{Pl}$

cf. Susskind, Horowitz-Polchinski

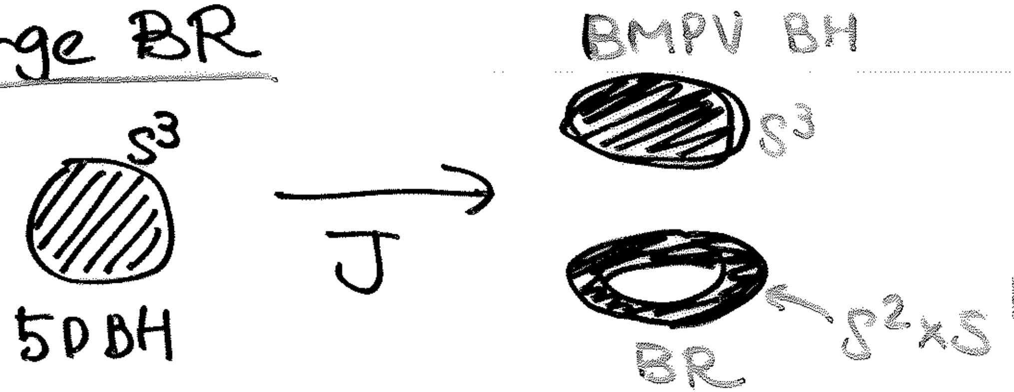
↓

A natural question: $J \neq 0$??

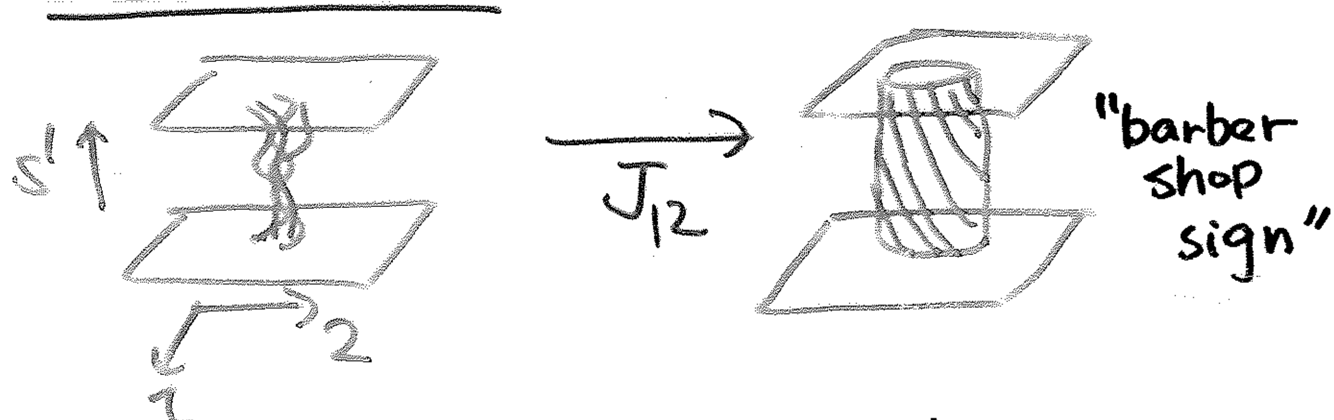
↓

Small Black Ring
(SBR)

- **Large BR**



- **Small BR**



↓ ↓

- pointlike ringlike

↓ ↓

SBH SBR

- sugra+R^2: formalism available

- micro ?
- macro ?
- precise counting ?
- help LBR ?

2.1 SBH : sugra sol'n

[Sen 0411255]

Hetero in $\mathbb{R}^4 \times S^1_4 \times T^5_{56789}$

w F1, n P along S^1_4

➤ Reduce to 4D

$$\begin{cases} ds^2_{str.10} = ds^2_{str.4} + T^2 (dx_4 + 2A^{(1)}_\mu dx^\mu)^2 \\ A^{(2)}_\mu = \frac{1}{2} B_{4\mu} , \quad S = e^{-2\Phi_4} = e^{-2\Phi_{10}} \sqrt{G_{44}} \end{cases}$$

➤ 4D metric

$$\begin{cases} ds^2_{str.4} = - \frac{g^2 \rho^2}{F(\rho)} dt^2 + g^2 (d\rho^2 + \rho^2 d\Omega_2^2) \\ F^{(1)}_{\rho t} = \frac{16 g^2 R_4^{-2} n}{(\rho + 8gn/R_4)^2} , \quad F^{(2)}_{\rho t} = \frac{g^2 w R_4^2}{16(\rho + gwR_4/2)^2} \\ T = \frac{R}{4N} \sqrt{\frac{\rho + 8gn/R_4}{\rho + gwR_4/2}} , \quad S = \frac{\sqrt{F}}{g^2 \rho} \end{cases}$$

$$F \equiv \left(\rho + \frac{gwR_4}{2}\right)\left(\rho + \frac{8gn}{2R_4}\right)$$

$\sqrt{\alpha'} = 4$ Singular at $\rho = 0$

Near-Horizon $\rho \ll \frac{gn}{R_4},\ gwR_4$

$$\begin{cases} ds^2_{str} = -\dfrac{\rho^2}{16nw}dt^2 + g^2(d\rho^2 + \rho^2 d\Omega_2^2) \\ S = \dfrac{2\sqrt{nw}}{g\rho},\ T = \sqrt{\dfrac{n}{w}},\ F^{(1)}_{\rho t} = \dfrac{1}{4n},\ F^{(2)}_{\rho t} = \dfrac{1}{4w} \end{cases}$$

$$\Big\downarrow \quad r = g\rho,\ \tau = \frac{t}{g\sqrt{nw}},\ x_5 = \sqrt{\frac{w}{n}}\, y_5$$

$$\begin{cases} ds^2_{str} = -\dfrac{r^2}{4}d\tau^2 + dr^2 + r^2 d\Omega_2^2 \\ S = \dfrac{2\sqrt{nw}}{r},\ T = 1,\ F^{(1)}_{r\tau} = F^{(2)}_{r\tau} = \dfrac{1}{4} \end{cases}$$

$$e^{\Phi_4} = S^{-1} = \frac{r}{2\sqrt{nw}} \ll 1$$

string loop negligible

- No dependence on moduli (g, R_4)
- Fully α' corrected sol'n:

$$ds^2_{str} = -f_1(r)\,d\tau^2 + f_2\,(dr^2 + r^2 d\Omega_2^2)$$

$$\phi = 2\sqrt{nw}\, f_3(r) \qquad T = f_4(r)$$

$$F^{(1)}_{r\tau} = f_5(r) \qquad F^{(2)}_{r\tau} = f_6(r)$$

- Entropy

$$S_{macro} = \int d\theta\, d\phi\, \frac{\partial \mathcal{L}}{\partial R}\, \varepsilon\varepsilon$$

$$= C\sqrt{nw}$$

agree w/ S_{micro}

2.2 SBH : Entropy func

Attractor : NH geo. is determined by charge, indep. of asympt. moduli

Entropy func : If NH is $AdS_p \times S^q$, NH geo. is obtained by extremizing "entropy func." No moduli enter

Entropy func

Assume : NH geo = $AdS_2 \times S^2$

$$ds^2 = v_1\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + v_2 \, d\Omega_2^2$$

$$S = u_S, \quad T = u_T, \quad F^{(i)}_{rt} = e_i$$

Must satisfy EoM :

$$f(u, v, e) \equiv \int d\theta \, d\phi \sqrt{-G} \, \mathcal{L}$$

elec chg.

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial v} = 0, \quad \frac{\partial f}{\partial e_i} = q_i$$

Legendre trfm → Entropy func

$$\mathcal{E}(u, v, q, e) \equiv 2\pi\left(e_i q_i - f(u, v, e)\right)$$

$$\frac{\partial \mathcal{E}}{\partial u} = \frac{\partial \mathcal{E}}{\partial v} = \frac{\partial \mathcal{E}}{\partial e_i} = 0$$

- Extremizing $\mathcal{E}$ determines geo.
  No dep. on asympt. moduli
- From Wald's formula

$$S_{macro} = \mathcal{E}\big|_{extr.}$$

Explicitly

$$\mathcal{L} = (\text{Sugra}) + \underbrace{(R^2 \text{ corr})}_{\substack{\text{CdWM's conf. sugra} \\ \text{or} \\ \text{Gauss-Bonnet}}}$$

$\Downarrow$

Correct entropy

Finite radii

$$S_{macro} = 4\pi\sqrt{nw}$$

$$v_1 = v_2 = 8$$

cf. Kraus-Larsen

Entropy func. for SBH

$\mathcal{L}$ = Sugra + GB

$$\Delta\mathcal{L} = \frac{S}{16\pi}\left[R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2\right]$$

$$f(v_1, v_2, u_S, u_T, e_1, e_2)$$
$$= \int d\theta\, d\phi \sqrt{-G}\,\mathcal{L}$$
$$= \frac{v_1 v_2 u_S}{8}\left(-\frac{2}{v_1} + \frac{2}{v_2} + \frac{8}{v_1^2}\left(e_1^2 u_T^{-2} + e_2^2 u_T^2\right)\right) \underset{\overset{\downarrow}{GB}}{- 2u_S}$$

$\downarrow$ $\partial f/\partial e_1 = n,\quad \partial f/\partial e_2 = w$

$$\mathcal{E}(v_1 v_2 u_S n w) = \frac{\pi}{2}\left(u_S(v_2 - v_1) + \frac{v_1}{u_S v_2}\left(n^2 u_T^{-2} + w^2 u_T^2\right) + 8u_S\right)$$

$\downarrow$ $\frac{\partial f}{\partial v_1} = \frac{\partial f}{\partial v_2} = \frac{\partial f}{\partial u_S} = \frac{\partial f}{\partial u_T} = 0$

$v_1 = v_2 = 8,\quad u_S = \frac{\sqrt{nw}}{2},\quad u_T = \sqrt{\frac{n}{w}}.$

$S_{macro} = \mathcal{E} = 4\pi\sqrt{nw}$

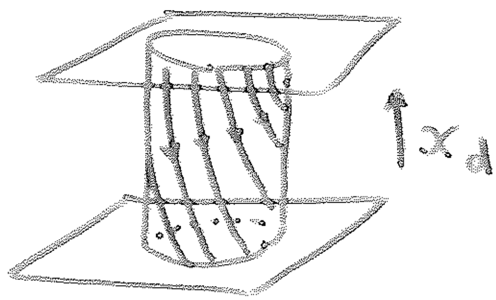
NH geo

$$\begin{cases} ds^2_{str.4D} = \frac{\alpha'}{2}\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + \frac{\alpha'}{2} d\Omega_2^2 \\ e^{\Phi_4} = S^{-1} = 2/\sqrt{nw} \ll 1 \text{ : loop suppressed} \\ G^{(10)}_{44} = T^2 = n/w \text{ : } S^1 \text{ stabilized} \end{cases}$$

2.3 SBR : sugra sol'n [DIISS 0611166]

Het. str. in $\mathbb{R}^d \times S^1 \times T^{9-d}$

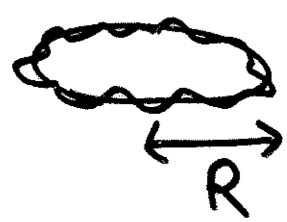
$$\begin{array}{ccc} \mathbb{R}^d & S^1 & T^{9-d} \\ \uparrow & \uparrow & \uparrow \\ t x_1 \cdots x_{d-1} & x_d & \vec{z} \end{array}$$



■ Sugra sol'n for F1 with arbitrary left-moving traveling wave on it is known [DGHW] [Callan-Maldacena-Peet]

$\downarrow$

- Take herical profile.
- Smear along x_d.
- Consider fluct.



$\downarrow$

Sugra Sol'n

Sugra sol'n

$$
\begin{cases}
ds^2_{str,d+1} = f_f^{-1}\left[-(dt-A)^2+(dx_d-A)^2 \right. \\
\qquad\qquad \left. +(f_p-1)(dt-dx_d)^2\right] + d\mathbf{x}^2_{d-1} \\
e^{2\Phi_{d+1}} = g^2 f_f^{-1}, \qquad B_{td} = 1-f_f^{-1} \\
B_{ti} = -B_{di} = f_f^{-1}A_i \qquad i=1,2,\cdots d-1
\end{cases}
$$

▶ Harm. funcs:

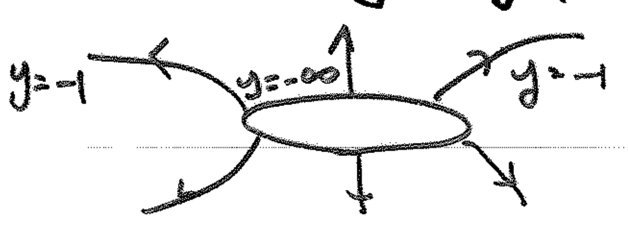
$$
\begin{cases}
f_f = 1+\dfrac{Q_f}{R^{d-3}}\left(\dfrac{x-y}{-2y}\right)^{\frac{d-3}{2}} {}_2F_1\left(\dfrac{d-3}{4},\dfrac{d-1}{4};1;1-\dfrac{1}{y^2}\right) \\
f_p = 1+\dfrac{Q_p}{R^{d-3}} \qquad \text{''} \\
A = -\dfrac{d-3}{2}\dfrac{q}{R^{d-5}}\dfrac{(y^2-1)(x-y)^{\frac{d-5}{2}}}{(-2y)^{\frac{d-1}{2}}}\,{}_2F_1\left(\dfrac{d-1}{4},\dfrac{d+1}{4};2;1-\dfrac{1}{y^2}\right)
\end{cases}
$$

▶ Charges vs quantized charges (n, w, J, p)

$$
\begin{cases}
Q_f = \dfrac{16\pi G_d R_d w}{(d-3)\Omega_{d-2}\alpha'}, \qquad Q_p = \dfrac{16\pi G_d}{(d-3)\Omega_{d-2}R_d}n \\
q = \dfrac{16\pi G_d}{(d-3)\Omega_{d-2}\alpha'}p, \qquad R^2 = \alpha'\dfrac{J}{p} \\
16\pi G_d = \dfrac{(2\pi)^{d-3}g^2\alpha'^{\frac{d-1}{2}}}{R_d}
\end{cases}
$$

▶ Coords:

$$
d\mathbf{x}^2_{d-1} = \frac{R^2}{(x-y)^2}\left[\frac{dy^2}{y^2-1}+(y^2-1)d\psi^2+\frac{dx^2}{1-x^2}+(1-x^2)d\Omega^2_{d-4}\right]
$$



$y=-\infty$: on ring
$y=-1$: asympt infini

↑

U-dual of sugra supertube
[Emparan-Mateos-Townsend]

Near-Ring: "simplifies" ($\alpha' = 1$)

$$\begin{cases} ds^2_{str.d+1} = \frac{n}{W}\frac{1}{R_d^2}(dx^d - dt)^2 + 2\frac{(d-3)\Omega_{d-2}}{c_d(2\pi)^{d-3}}\frac{1}{W}\frac{R}{g^2}\times \\ \quad \times\left(-\frac{R}{y}\right)^{d-4} dt(dx^d - dt) \\ \quad + 2\frac{J}{WR_d}d\psi(dx^d - dt) + R^2\frac{dy^2}{y^4} + R^4 d\psi^2 + \left(\frac{R}{y}\right)^2 d\Omega^2_{d-3} \\ \frac{1}{2}B_{\mu\nu}dx^\mu \wedge dx^\nu = \cdots \\ e^{2\Phi_{d+1}} = \cdots \end{cases}$$

$$R \gg |y| \gg \frac{R}{(g^2 p)^{1/(d-4)}}, \left(\frac{R_d^2}{g^2 p}\right)^{\frac{1}{d-4}} R, 1$$

$$c_d = \frac{\Gamma((d-4)/2)}{2\sqrt{\pi}\,\Gamma((d-3)/2)}$$

➤ Aside: metric in $\psi - x^d$ plane will have negative eignenv. unless

$$nw - Jp \geq 0$$

Regge bnd ⟷ Absence of CTC

Furthor define

$$\sigma = \sqrt{\frac{n}{w} - \frac{Jp}{w^2}\frac{x^d - t}{R_d}}\,, \qquad \rho = -\frac{R}{y}$$

$$\tau = \frac{(d-3)\Omega_{d-2}}{c_d(2\pi)^{d-3}}\frac{R_d}{\sqrt{nw - Jp}}\frac{R}{y^2}t\,, \qquad \chi = \sqrt{\frac{J}{p}}\psi + \frac{\sqrt{Jp}}{w}\frac{x^d - t}{R_d}$$

$$\Downarrow$$

$$\left\{\begin{aligned} ds^2_{str;\,d+1} &= d\sigma^2 + d\chi^2 + 2\rho^{d-4}d\tau\, d\sigma + d\rho^2 + \rho^2 d\Omega^2_{d-3} \\ B &= -\rho^{d-4}d\tau\wedge d\sigma + \left(\frac{nw}{Jp} - 1\right)^{-\frac{1}{2}} d\sigma\wedge d\chi \\ e^{2\Phi_{d+1}} &= \frac{(d-3)\Omega_{d-2}}{c_d(2\pi)^{d-3}}\frac{1}{w\sqrt{\frac{J}{p}}}\rho^{d-4} \end{aligned}\right.$$

<u>Most (n,w,J,p) dependence disappeared</u>

- $\rho \gg 1$: curvature small, $e^{\Phi_{d+1}}$ small

  $\Rrightarrow$ α'. g_s negligible

- $\rho \sim 1$: curvature ~ 1, $e^{\Phi_{d+1}}$ small

  $\Rrightarrow$ α' correction : important
  g_s still negligible

How can α' corrections enter?

- $SO(d-2)$ of S^{d-3} preserved
- No dependence of metric on chg.
- $B_{\sigma x}$: can be gauged away
- Mult. factor of $e^{\Phi_{d+1}}$ irrelevant at tree level ;

$$S_{\text{tree}} = \int e^{-2\Phi_{d+1}} (\dots \partial\Phi \dots)$$

$$\Downarrow$$

Fully α' corrected sol'n must be

$$\begin{cases} ds^2 = g_{\alpha\beta}(\rho)\, d\xi^\alpha d\xi^\beta + f_1(\rho)\, d\Omega_{d-3}^2 + d\rho^2 \\ B = b_{\alpha\beta}(\rho)\, d\xi^\alpha \wedge d\xi^\beta + \left(\frac{nw}{Jp} - 1\right)^{-1/2} d\sigma \wedge d\chi \\ e^{2\Phi_{d+1}} = \frac{1}{w}\sqrt{\frac{J}{p}}\, f_2(\rho) \end{cases}$$

Funcs. $g_{\alpha\beta}$, f_1, etc. are <u>indep</u> of n, w, J, p

Entropy:

$$\begin{pmatrix}\text{periodicity} \\ \text{of } \chi\text{-}\sigma\end{pmatrix} \times (\text{dilaton}) \times \begin{pmatrix}\text{some} \\ \text{number}\end{pmatrix}$$

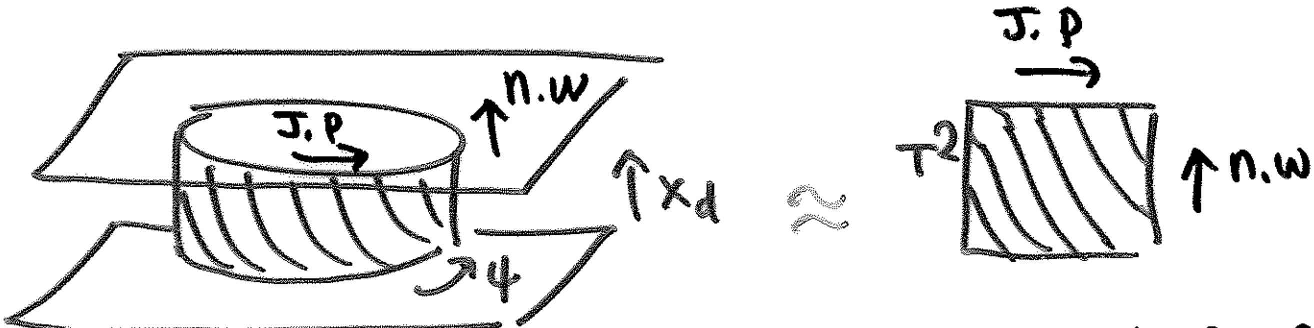
$$4\pi^2\sqrt{\frac{Jn}{pw}}\sqrt{1-\frac{Jp}{nw}} \qquad \sqrt{\frac{p}{J}}$$

$$\boxed{S_{macro} = c\sqrt{nw - Jp}}$$

Agrees with S_{micro} up to a factor!

- Can't determine C with this method.

- Near ring, d-dim SBR $\simeq$ $(d-1)$ dim SBH



Het. in $\mathbb{R}^d \times S^1 \times T^{9-d}$

Het. in $\mathbb{R}^{d-1} \times T^2 \times T^{9-d}$

$SO(2,2)$ T-duality

$\downarrow$

$nw - Jp$ is invariant.

2.4 SBR: Entropy func

NH: SBR in d-dim $\approx$ SBH in (d-1)-dim

$\downarrow$

Het. in $\mathbb{R}^{d-1} \times S^1 \times S^1 \times T^{9-d}$

J, P — coord: ψ $\qquad$ n, w — coord: $y_d = \frac{x_d - t}{R_d}$

Reduce (d+1)-dim fields to (d-1)-dim on $S^1_\psi \times S^1_d$:

$$ds^2_{str, d+1} = \hat{G}_{mn} d\xi^m d\xi^n + R^2 (dy^d + A^{(1)}_m d\xi^m)^2 + \tilde{R}^2 (d\psi + A^{(2)}_m d\xi^m)^2 + 2S (dy^d + A^{(1)}_m d\xi^m)(d\psi + A^{(2)}_m d\xi^m)^2$$

$$\frac{1}{2} B^{d+1}_{\mu\nu} dx^\mu \wedge dx^\nu = (\hat{B}_{mn}, C, A^{(3)}_M, A^{(4)}_m)$$

(d-1) dim fields:

$\hat{G}_{mn}, \hat{B}_{mn}, \Phi_{d+1}$

$R, \tilde{R}, S, C$

$m, n = 0 \dots d-2$

Entropy func

Assume $AdS_2 \times S^{d-3}$:

$$\hat{G}_{mn} d\xi^m d\xi^n = v_1\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + v_2 \, d\Omega_{d-3}^2$$

$R, \tilde{R}, S, C, \Phi_{d+1}$: const

$F^{(i)}_{rt} = e_i \quad , \quad i = 1 .. 4$

Entropy func is

$$\mathcal{E}(n, J, w, p, R, \tilde{R}, S, C, \Phi, e_i)$$
$$= 2\pi\left(ne_1 + Je_2 - we_3 + pe_4 - \int_{S_{d-3}} \sqrt{-\hat{G}} \, \mathcal{L}_{d-1}\right)$$

Need to know the form of $\mathcal{L}_{d-1}$ to obtain extremum.

But ....

Scaling properties

- Shift dilaton

$$e^{-2\Phi} \to \lambda e^{-2\Phi} \Rightarrow \mathcal{L}_{d-1} \to \lambda \mathcal{L}_{d-1}$$

If we further scale

$$p \to \lambda p, \quad n \to \lambda n, \quad w \to \lambda w, \quad J \to \lambda J$$

then

$$\mathcal{E} \to \lambda \mathcal{E}$$

- Scale y^d

$$A^{(1)} \to \kappa^{-1} A^{(1)}, \quad A^{(3)} \to \kappa A^{(3)}$$
$$R \to \kappa R \quad S \to \kappa S \quad C \to \kappa C$$
$$\mathcal{L}_{d-1} \to \kappa \mathcal{L}_{d-1}$$

$$e_1 \to \kappa^{-1} e_1, \quad n \to \kappa^2 n$$
$$J \to \kappa J \quad e_3 \to \kappa e_3, \quad p \to \kappa p$$

$$\mathcal{E} \to \kappa \mathcal{E}$$

- Scale ψ

$$A^{(2)} \to \eta^{-1} A^{(2)}, \quad A^{(4)} \to \eta A^{(4)}$$

....

$\downarrow$

Scaling of $\mathcal{E}$:

$$n \to \lambda \kappa^{-1} \eta n \qquad J \to \lambda \kappa \eta^2 J$$

$$w \to \lambda \eta w \qquad p \to \kappa p$$

$$\mathcal{E} \to \lambda \kappa \eta \mathcal{E}$$

$$\Downarrow$$

$$S_{macro} = \mathcal{E} = \sqrt{nw}\, f\left(\frac{Jp}{nw}\right)$$

• From $SO(2,2)$ T-duality inv.,

$$\boxed{S_{macro} = C\sqrt{nw - Jp}}$$

Can all scalars be determined?

$\to$ No, since S depends only on area of T^2

$$G_{dd} G_{44} - G_{d4} = R\tilde{R} - S'$$

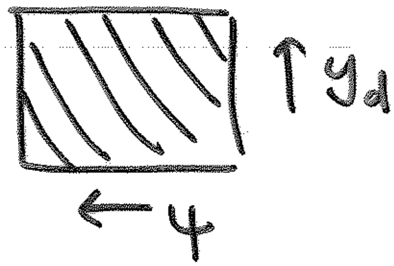
$$\downarrow$$

Two moduli unfixed

(can be checked)

Unfixed moduli …

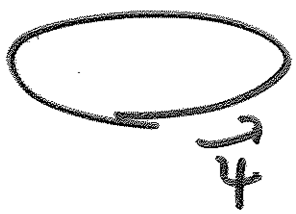
- Entropy func knows only about NH
  Same as BH.



transv. "radius" unfixed

- Need info. away from NH

Sugra knows better



ψ circle shrinks at center

- Sugra sol'n has no moduli NH
- Can fix moduli by comparing w/ sugra sol'n.

Attractor geom. is

$$ds^2_{str,\,d+1} = v_1\left(-r^2 d\bar{t}^2 + \frac{dr^2}{r^2}\right) + v_2\, d\Omega^2_{d-3}$$
$$+ R^2 (dy^d + e_1 r\, d\bar{t})^2$$
$$+ \tilde{R}^2 (d\psi + e_2 r\, d\bar{t})^2$$
$$+ 2S (dy^d + e_1 r\, d\bar{t})(d\psi + e_2 r\, d\bar{t})$$

$B = \cdot\cdot$

v_1, etc. involve unfixed moduli, even after extremization of $\mathcal{E}$

$\Downarrow$ Go to σ, χ coord., combine sugra results

$$ds^2_{str,\,d+1} = c_1\left(-r^2 d\bar{t}^2 + \frac{dr^2}{r^2}\right) + c_2\, d\Omega^2_{d-3}$$
$$+ c_3 (d\sigma + c_4 r\, d\bar{t})^2$$
$$+ c_5 (d\chi + c_6 r\, d\bar{t})^2$$
$$+ 2c_7 (d\sigma + c_4 r\, d\bar{t})(d\chi + c_6 r\, d\bar{t})$$

$B = \cdot\cdot$

c_1, etc are indep. of charges & moduli.

Conclusion

- SBH + J = SBR
- Studied micro/macro

  micro: WS

  macro: sugra, entropy func
- May help to understand LBR