Metastable Vacua in Gauge Theory and M-Theory: DV & SW in M

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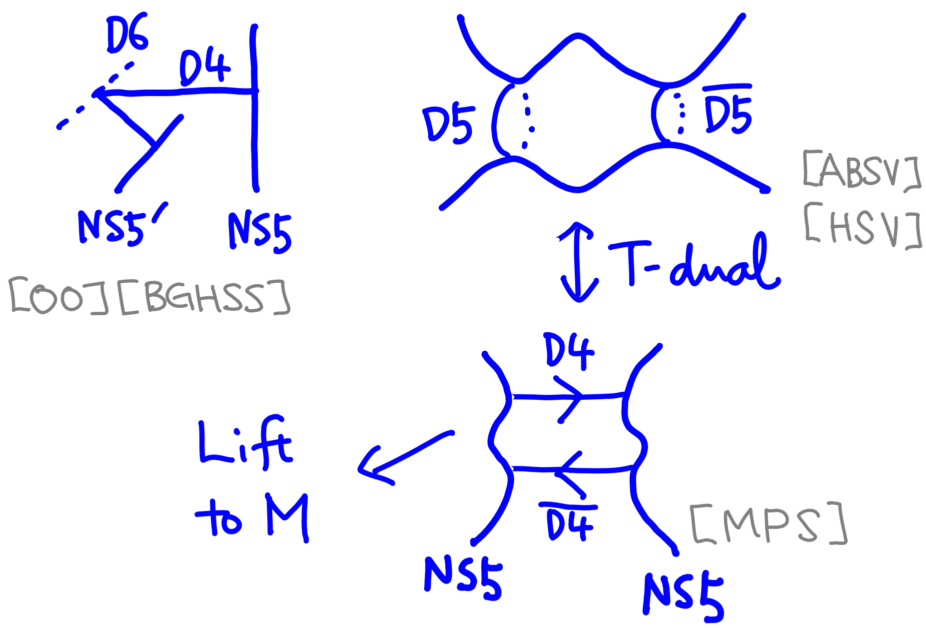
Motivation

- Nonsusy metastable configs. in gauge/string theory
  - ISS
  - Geom'ly induced metastability [Vafa et al.]
- String/M-theory : useful tool for geometric/intuitive understanding



Motivation

- Nonsusy metastable vacua in perturbed $\mathcal{N}=2$ theory
  [Ooguri-Ookouchi-Park]
  [Pastras] "OOPP vacua"

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String/M understanding?

↓

A unifying M-theory perspective of perturbed $\mathcal{N}=2$ theory, susy or nonsusy

Outline

- Gauge theory
- String/M realization: susy vacua
- Nonsusy cases
- Conclusion

Gauge theory side: review

Review: $\mathcal{N}=2$ SU(N) theory

(a.k.a. SW theory)

- Field content:

  $\mathcal{N}=1$ vector (A_μ, λ)

  $\mathcal{N}=1$ adjoint chiral (Φ, ψ)

  (no quarks)

  Φ: N×N mat. traceless

- Moduli space of vacua

  → Parametrized by eigenvalues of Φ:

$$\Phi = \begin{pmatrix} a_1 & & \\ & \ddots & \\ & & a_N \end{pmatrix}$$

standard moduli:

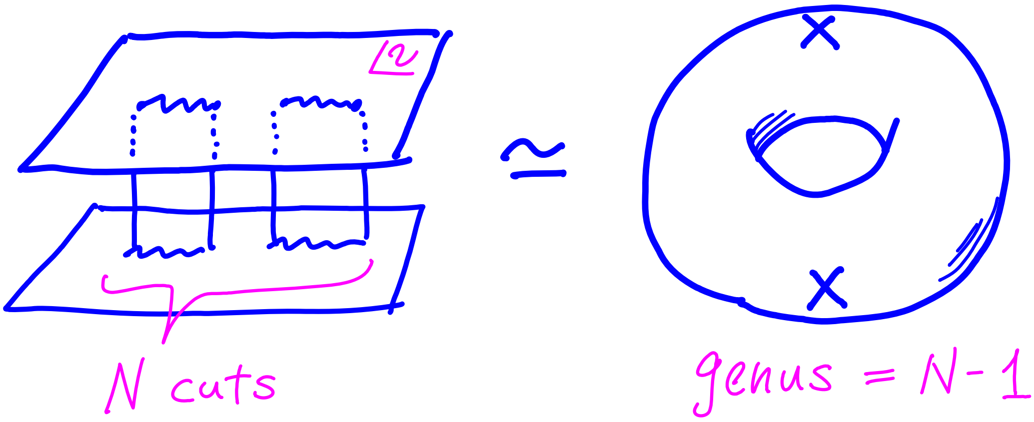
$$u_i \equiv \frac{1}{i} \mathrm{tr}(\Phi^i), \quad i \leq N$$

▶ Seiberg-Witten curve:

At each point $\{u_i\}$ in moduli space, there is an associated Riemann sfc:

$$y^2 = P_N(v)^2 - \Lambda^{2N}$$

$$P_N(v) = \det(v - \Phi)$$



$\Rightarrow$ Encodes physical quantities

<u>E.g.</u> Moduli space metric

$$ds^2 = (g^{\mathcal{N}=2})_{i\bar{j}}\, du_i\, d\bar{u}_j$$

Adding superpot: $\mathcal{N}=2 \to \mathcal{N}=1$

param

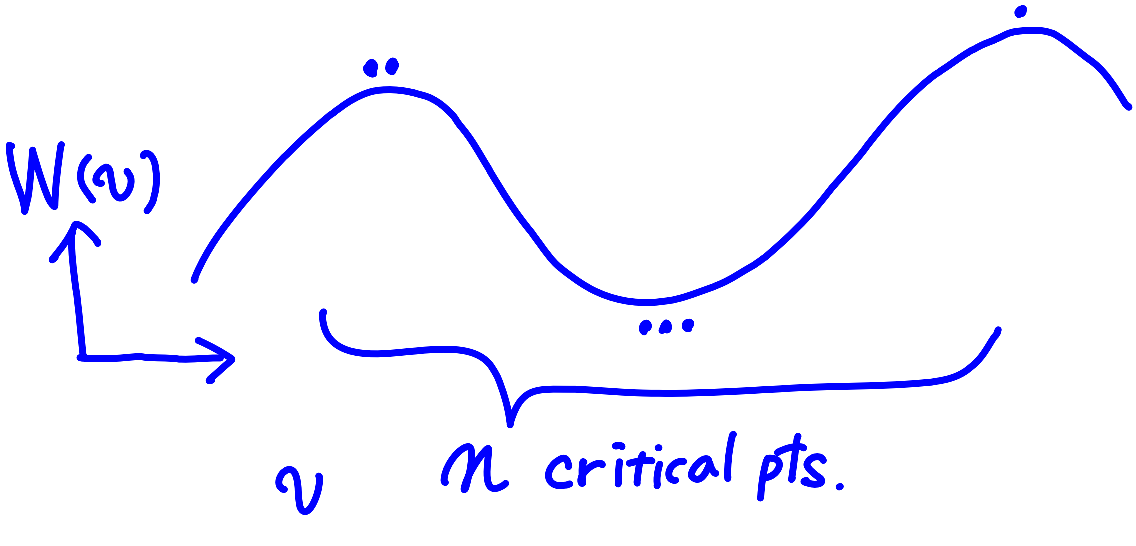
$$W_{tree} = \epsilon \, \mathrm{tr}[W(\Phi)]$$

$$W(v) = \sum_{j=1}^{n+1} \frac{g_j}{j} v^j$$

- polynomial of deg. $n+1$
- characteristic scale: m

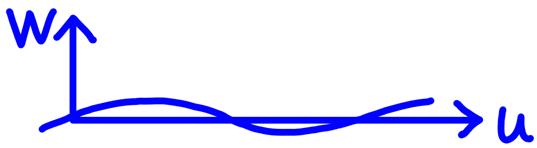
Classical vacua

$\longleftrightarrow$ How to distribute eigenvalues a_i of Φ among crit. pts.



▶ Small Superpot.: perturbed $\mathcal{N}=2$

($\Lambda \gg \epsilon m$, "SW regime")



- $\{u_k\}$: still good variables

- Susy vacua:

$$\partial_{u_i} W(u) = 0,$$

$$W(u) = \sum_j \frac{g_j}{j} \langle \mathrm{tr}\, \Phi^j \rangle = \sum_j g_j u_j$$

- Nonsusy vacua:

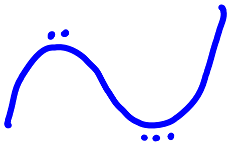
$$\partial_{u_i} V(u) = 0,$$

$$V(u) = g^{u_i u_j}_{\mathcal{N}=2}\, \partial_{u_i} W\, \overline{\partial_{u_j} W}$$

▶ Large superpot

($\Lambda \ll \epsilon m$, "DV regime")

- $SU(N) \to \prod_i U(N_i)$



- Φ gets integrated out

→ Pure $\mathcal{N}=1$ $U(N_i)$ SYM at each crit. pt.

→ $U(N_i)$'s confine. χSB.

⇓

Good variables: glueballs

$$S_i \sim \langle \lambda\lambda \rangle_{U(N_i)}$$

- DV theory: Computes glueball superpot.

$$W_{DV}(S)$$

- SUSY vacua: $\partial W_{DV}(S) = 0$

▶ Susy vacua : more details

$$SU(N) \to U(1)^k$$

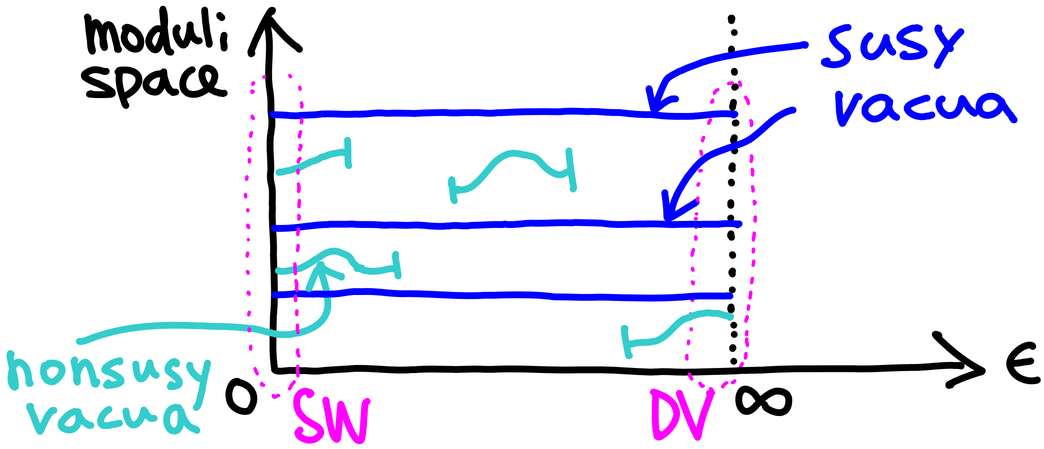
$\Downarrow$ both pert. $\mathcal{N}=2$ analysis and DV theory

Factorization

$$P_N(v)^2 - \Lambda^{2N} = H_{N-k}(v)^2 F_{2k}(v)$$

$$W_n'(v)^2 + f_{n-1}(v) = G_{n-k}(v)^2 F_{2k}(v)$$

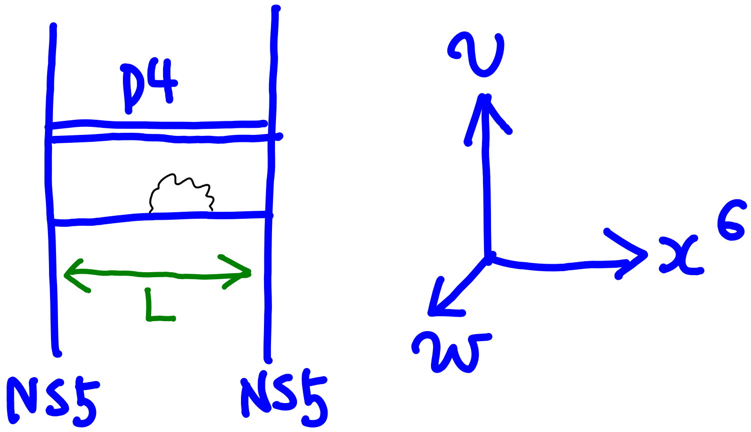
Same for any ϵ : holomorphic protection



String/M-theory realizations: susy vacua

Brane construction

▶ $\mathcal{N}=2$ Configuration



	1	2	3	4	5	6	7	8	9
NS5	o	o	o	o	o	·	·	·	·
D4	o	o	o	·	·	o	·	·	·

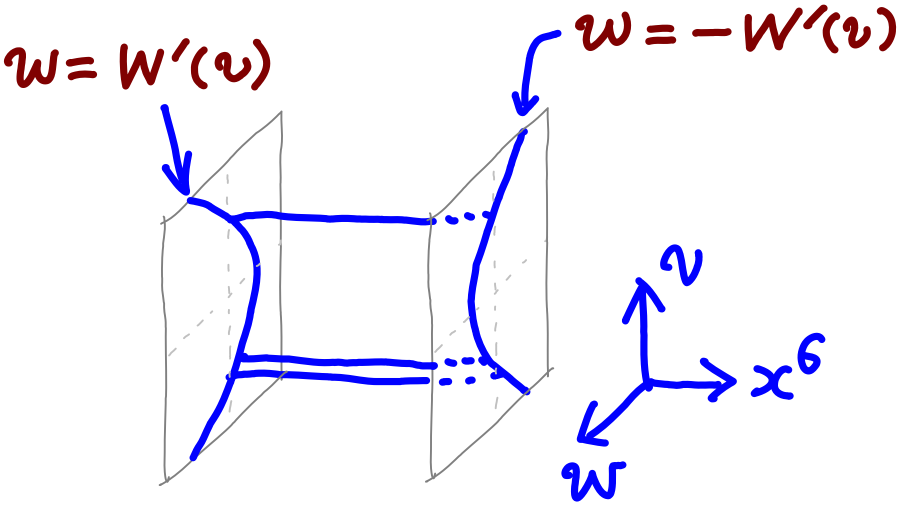
$v \equiv x^4 + i x^5$
$w \equiv x^8 + i x^9$

- $\dfrac{1}{g_{YM}^2} \sim \dfrac{L}{g_s l_s}$

- Position of D4's along v
  $\longleftrightarrow$ eigenvalues of Φ
  $\longrightarrow$ $\mathcal{N}=2$ moduli space (classical)

▶ Adding superpotential

$\leftrightarrow$ Curving NS5's in v-w plane



- Can see that eigenvalues sit at critical points, $W'(v) = 0$

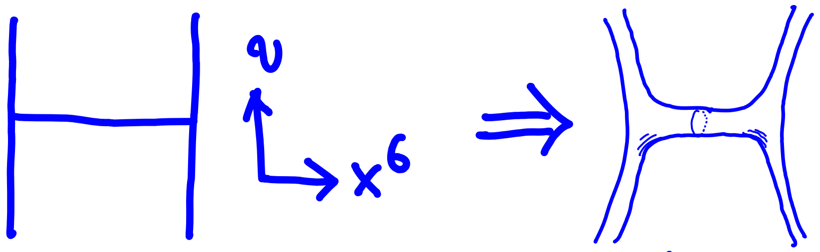
Lifting to M-theory

add 11th direction S^1_{10}

	1	2	3	4	5	6	7	8	9	10
NS5 → M5	o	o	o	o	o	·	·	·	·	·
D4 → M5	o	o	o	·	·	o	·	·	·	o

$$\begin{cases} s \equiv x^6 + i x^{10}, \quad s \simeq s + 2\pi i R_{11} \\ v = x^4 + i x^5 \\ w = x^8 + i x^9 \end{cases}$$

$\mathcal{N}=2$ config.:



Complex curve in sv space (Riemann sfc)

▶ Susy curve

- Need minimal area sfc. with desired bndy. cond.

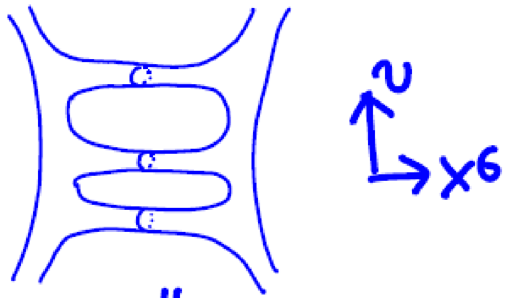
$$S_{NG} = \int d^2\sigma \sqrt{\det h}$$

- For susy = holomorphic curve, can shortcut

(all holo. curves are minimal area curves)

- $\mathcal{N}=2$ curve:

$$t^2 - 2P_N(v)t + \Lambda^{2N} = 0$$



$$t \equiv e^s$$

⇓

SW curve itself!

$$t = P_N + \sqrt{P_N^2 - \Lambda^{2N}}$$

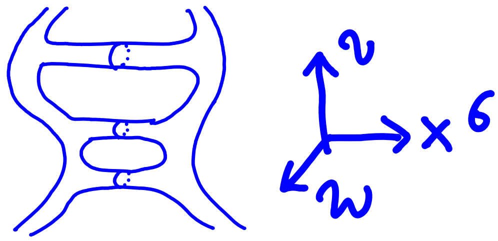
Define $y \equiv t - P_N(v)$

$$\rightarrow y^2 = P_N(v)^2 - \Lambda^{2N}$$

▶ $\mathcal{N}=1$ curve

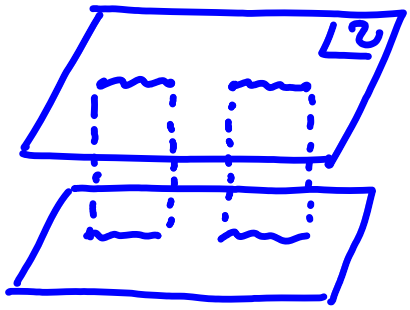
"bend" $\mathcal{N}=2$ curve in w direction

along $w = \pm W'(v)$



Think of $w(v)$ defined on the SW curve:

$$y^2 = P_N(v)^2 - \Lambda^{2N}$$



- asymtotically
  $w(v) \sim \pm W'(v)$

  $\rightarrow \; w(v) = \sqrt{W_n'(v)^2 + f_{n-1}(v)}$

- Well-def'd on SW curve

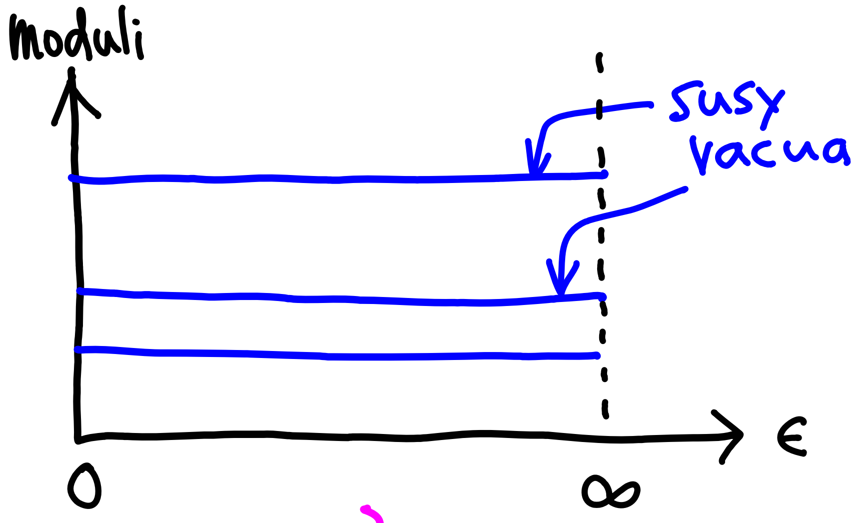
  $\rightarrow$ Factorization

  $$P_N^2 - \Lambda^{2N} = (W_n'^2 + f_{n-1}) H_{N-n}^2$$

Gives geom. interpretation to gauge theory results

Summary for susy case

- Gauge thy. & M-thy. both lead to same cond. for susy vacua (factor'n)
- M-thy. gives geom. understanding
- Doesn't care about size of superpotential, ϵ



How about nonsusy configs.?

Nonsusy configurations

Nonholo. M-theory curves

- Is there correspondence b/t gauge theory and M-theory for nonsusy curves too ?

  → Off-shell configs ?

  → OOPP vacua ?

- Boundary cond. issue

  [BGHSS]

  Holo. curves and nonholo. curves do not have same b.c. at ∞

  → One can't use M-theory to study susy & nonsusy states in the same theory ??

Nonsusy vacua: g.t. side

OOPP:

Add small superpot. $W(\Phi)$

$\mathcal{N}=2$ theory

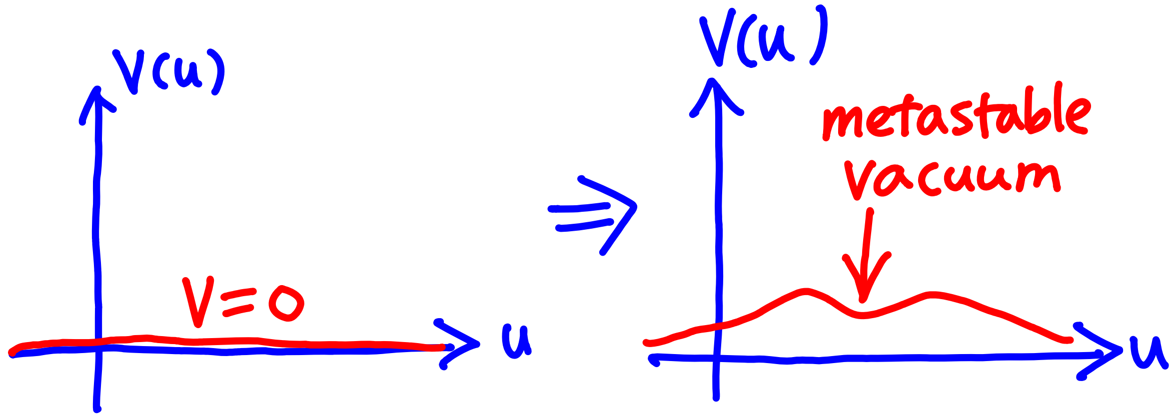
$$\Downarrow$$

$$V(u) = g^{u_i u_j}_{\mathcal{N}=2}\, \partial_{u_i} W\, \overline{\partial_{u_j} W}$$

By choosing $W(\Phi)$ appropriately,

can make <u>any</u> pt. $\{u_i\}$ a

nonsusy vacuum!



Now back to M

- **How to find M5 curves in general**

No holomorphicity to simplify things

→ Need to directly find min. area curves

$$S_{NG} = \int d^2 z \sqrt{\det h}$$

or

$$S_{Pol} = \int d^2 z \sqrt{\gamma}\, \gamma^{ab} \partial_a X^\mu \partial_b X_\mu$$

$$\Downarrow$$

$$\partial\bar{\partial} s = \partial\bar{\partial} v = \partial\bar{\partial} w = 0$$

Harmonic:

$$\begin{cases} s(z,\bar{z}) = s_H(z) + \bar{s}_A(\bar{z}) \\ v(z,\bar{z}) = v_H(z) + \bar{v}_A(\bar{z}) \\ w(z,\bar{z}) = w_H(z) + \bar{w}_A(\bar{z}) \end{cases}$$

Virasoro:

$$\partial s_H \partial s_A + \partial v_H \partial v_A + \partial w_H \partial w_A = 0$$

Need to solve these for given b.c.

Need to solve these for given b.c.

M5 curve in SW regime

Small superpot (ϵ : small)

↓

Start from $\mathcal{N}=2$ curve ($s v$ curve) and slightly turn on w

$s, v \gg w$

$$S_{NG} = \int \sqrt{h}$$
$$= \underbrace{\int \sqrt{h_{sv}}}_{\text{const.}} + \underbrace{\frac{1}{2} \int dw \wedge * d\bar{w}}_{\equiv V(u)} + \cdots$$

⇓

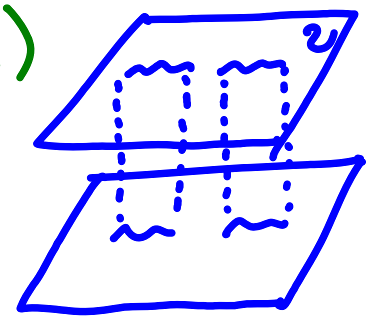
1. Consider sv curve w/ moduli $\{u_i\}$
2. Find a harmonic 1-form dw on it
3. Compute energy $V(u)$

↳ compare w/ g.t.

▶ Formulation of problem

- Riemann sfc (SW curve)

$$y^2 = P_N(v)^2 - \Lambda^{2N}$$



- Want harmonic 1-form on it

$$dw = dw_H(v) + d\bar{w}_A(\bar{v})$$

- Bndy. cond:

$$w(v) \sim \pm W'(v)$$

$$\rightarrow dw(v) \sim \pm W''(v)dv$$

$$W'(v) = \sum_j g_j v^{j-1}$$

- w : single valued:

$$\int_{\forall \text{ compact cycle}} dw = 0$$

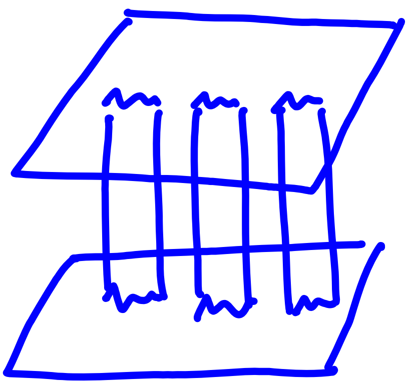
▶ 1-forms on the Riemann sfc.

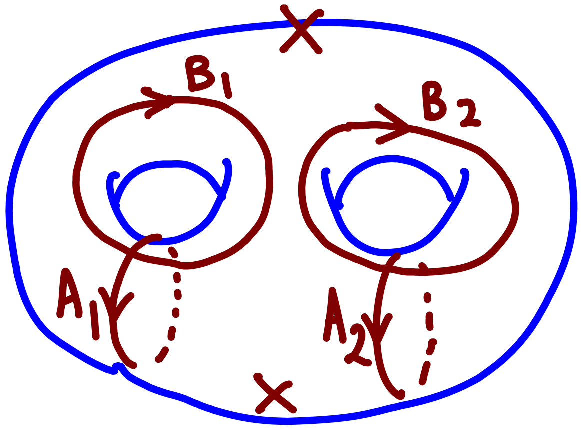
- Holo. 1-forms: $d\omega_i$, $i = 1 \dots g = N-1$

$$\int_{A_i} d\omega_j = \delta_{ij} \ , \quad \int_{B_i} d\omega_j = \tau_{ij}$$

A_i, B_i: basis of 1-cycles

$$A_i \cap B_j = \delta_{ij}$$
$$A_i \cap A_j = 0$$
$$B_i \cap B_j = 0$$





▶ 1-forms on the Riemann sfc.

- Mero. 1-forms of 2nd kind: $d\Omega_{m\geq 1}$

$$d\Omega_m = \pm\left[\frac{v^{m-1}}{m} + \mathcal{O}\left(\frac{1}{v^2}\right)\right]dv$$

$$\int_{A_i} d\Omega_m = 0 \,, \quad \int_{B_i} d\Omega_m = K_{im}$$

→ Useful for $dw \sim \pm W''(v)dv$

- Mero. 1-forms of 3rd kind: $d\Omega_0$

$$d\Omega_0 = \pm\left[\frac{1}{v} + \mathcal{O}\left(\frac{1}{v^2}\right)\right]$$

→ Log bending
(irrelevant for SU(N))

Constructing $d\omega$

Asympt b.c.

$$d\omega \sim W''(v)\,dv \equiv \sum_m \frac{I^m}{m} v^m$$

$$\downarrow$$

$$d\omega \overset{?}{=} T^m d\Omega_m$$

Harmonic $d\omega$:

$$d\omega = T^m d\Omega_m + h^i d\omega_i + \bar{\ell}^i d\bar{\omega}_i$$

Require vanishing periods:

$$0 = \int_{A_i} d\omega = h^i - \bar{\ell}^i$$

$$0 = \int_{B_i} d\omega = K_{im} T^m + \tau_{ij} h^j + \bar{\tau}_{ij} \bar{\ell}^j$$

Just right # of vars. $h^i, \bar{\ell}^i$ to satisfy these for any value of moduli $\{u_i\}$

→ Off-shell M5 curve.

cf. susy case: $\bar{\ell}^i \equiv 0$

▶ Energy from tension

Energy increase from the w part:

$$V(u) \sim \int dw \wedge * d\bar{w}$$

In an appropriate regularization procedure,

$$= V(u)_{\text{gauge theory}}$$

- M-theory reproduces gauge-theory potential even off-shell!

- Nonsusy vacua are guaranteed to be reproduced too

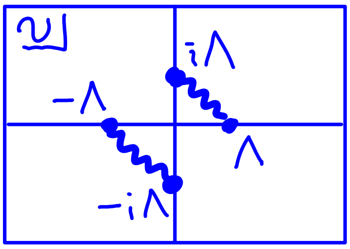
▶ OOPP vacua (SU(2))

- Order 6 superpot.

$$W(u) = g\left(\frac{v^2}{2} + \frac{\beta}{6}v^6\right)$$

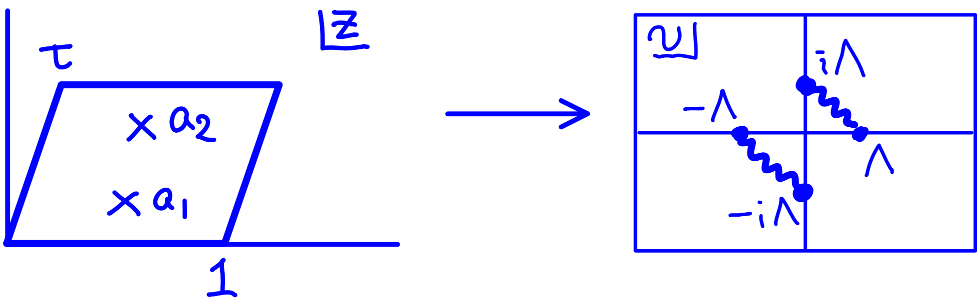
- Origin of mod. space

$$P_2(v) = v^2 - u_2 = v^2 \quad (u_2 = 0)$$



What's the corresponding M5 curve?

Easier to go to z-torus
to present it



• The explicit curve:

$$ds = 2(F_1^{(1)} - F_2^{(1)} + i\pi)dz$$
$$dv = b_2(F_1^{(2)} - F_2^{(2)})dz$$
$$dw = A_6(F_1^{(6)} + F_2^{(6)})dz + A_2\left[(F_1^{(2)} + F_2^{(2)})dz + \frac{4\pi i}{\tau - \overline{\tau}}(dz - d\overline{z})\right]$$

$$F(z) = \ln\theta(z - \tilde{\tau})$$

$$\theta(z) = \sum_{n=-\infty}^{\infty} e^{i\pi n^2\tau + 2\pi i n z}, \qquad \tilde{\tau} = \frac{1}{2}(\tau + 1).$$

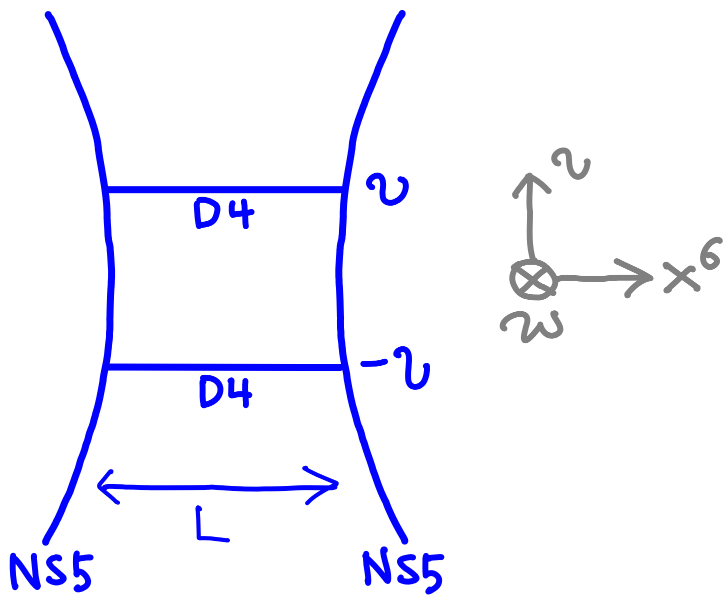
$$F_i^{(n)} = \left(\frac{\partial}{\partial z}\right)^n F(z - a_i).$$

Note: this curve has same bndy. cond. as susy curve (cf. [BGHSS])

Effect of nonholomorphicity on bndy. cond. is negligible for small w

Geom. understanding of stability

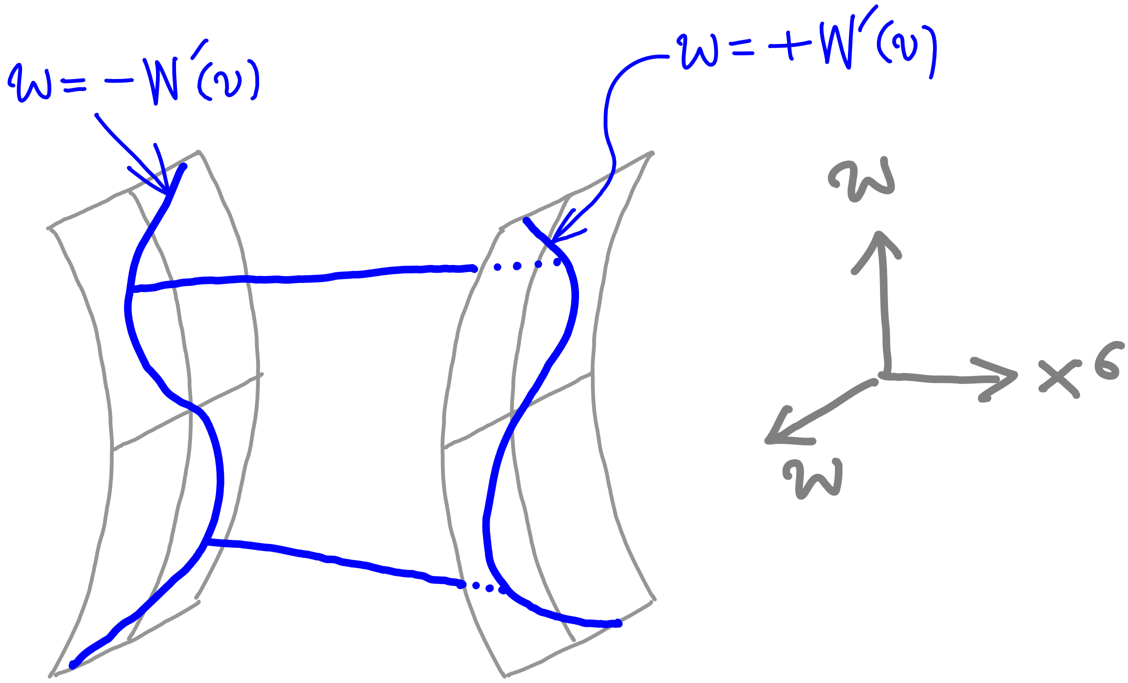
▶ $\mathcal{N}=2$ config.



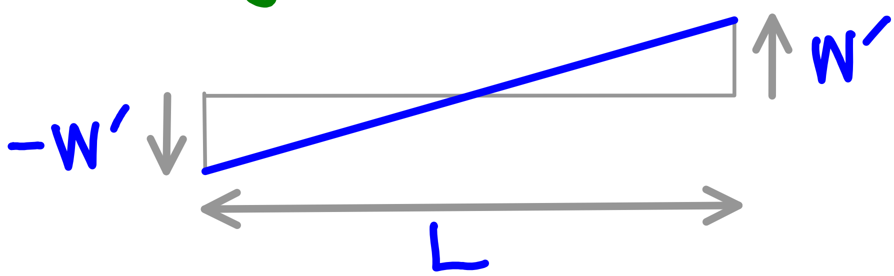
- D4's pull NS5's
  → log bending in x^6
- x^6 log bending
  → attraction $V_T(v) \sim 2T_4 L(v)$
- Coulomb → repulsion $V_c(v)$

$$V_T + V_c = 0.$$

▶ Add superpot.



- Coulomb force unchanged.
- D4's longer :



$$\text{length} = \sqrt{L^2 + |2W'|^2}$$

$$\sim L + \frac{2|W'|^2}{L}$$

⇓

V_C and V_T don't cancel

$$V_{tot} \sim \frac{|W'|^2}{L}$$

- agree with gauge theory
- EOM:

$$F \propto \underset{①}{\frac{W''}{W'}} - \underset{②}{\frac{L'}{L}} = 0$$

① Pushes D4's toward smaller W'. Tunable.

② Pushes D4's toward larger L (larger v). Log.

⇒ By tuning W', can make any v stable (OOPP)

Finite superpotential

So far we were in the SW regime;
w : small

$$S_{NG} = \int \sqrt{h}$$
$$= \int \sqrt{h_{SV}} + \frac{1}{2}\int dw \wedge * d\bar{w} + \cdots$$

Can we remove this assumption and find a curve that reduces to OOPP in w: small limit?

Need to solve full NG

$$\begin{cases} s(z,\bar{z}) = s_H(z) + \bar{s}_A(\bar{z}) \\ v(z,\bar{z}) = v_H(z) + \bar{v}_A(\bar{z}) \\ w(z,\bar{z}) = w_H(z) + \bar{w}_A(\bar{z}) \end{cases}$$

$$\partial s_H \partial s_A + \partial v_H \partial v_A + \partial w_H \partial w_A = 0$$

complicated, but can do it

▶ Exact lift of OOPP curve

$$
\begin{aligned}
ds &= (2+\nu)(F_1^{(1)} - F_2^{(1)} + i\pi)dz + \nu(\overline{F}_1^{(1)} - \overline{F}_2^{(1)} - i\pi)d\overline{z} \\
dv &= b_2(F_1^{(2)} - F_2^{(2)})dz + \bar{c}_4(\overline{F}_1^{(4)} - \overline{F}_2^{(4)})d\overline{z} \\
dw &= A_6(F_1^{(6)} + F_2^{(6)})dz + A_2\left[(F_1^{(2)} + F_2^{(2)})dz + \frac{4\pi i}{\tau - \overline{\tau}}(dz - d\overline{z})\right] \\
A_6 &= \frac{1}{48 g_2}\left(A_2 + \frac{g_s^2 \overline{\nu}(2+\nu)}{2\overline{A}_2 \pi}\right), \qquad c_4 = -\frac{40\overline{A}_2 A_6 \pi}{b_2}.
\end{aligned}
$$

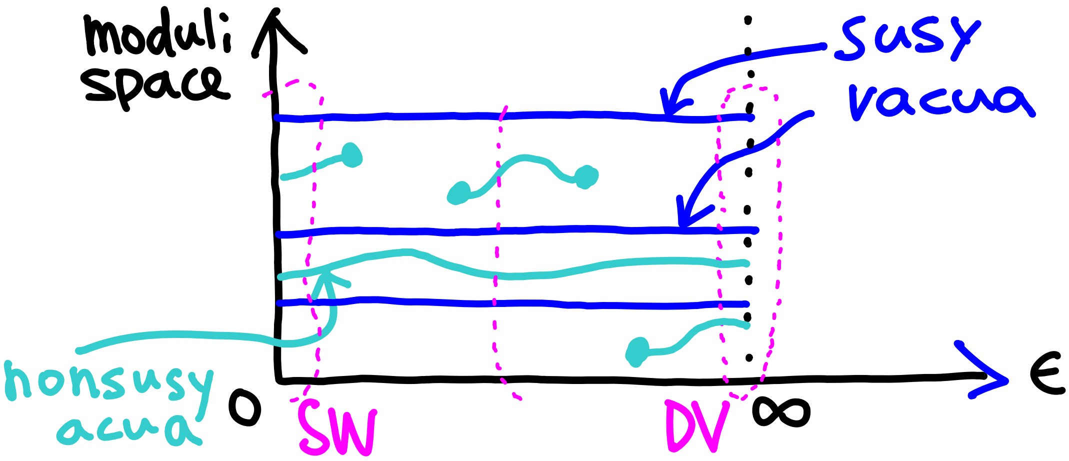
Bndy. cond is no longer holomorphic ($c_4 \neq 0$)

↓

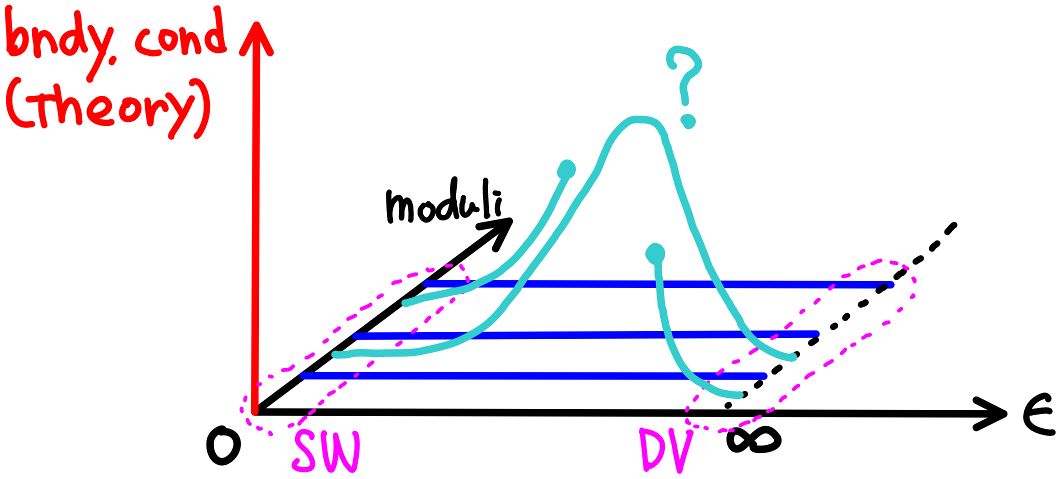
Susy curve and nonsusy curve have different b.c.

- **Structure of vacua:**

Gauge theory



M-theory



Two agrees:

1. Completely in SW and DV regimes
2. everywhere for susy vacua

Conclusion

- M-theory "lift" of perturbed $\mathcal{N}$=2 theory gives a unifying perspective

- M-theory reproduces gauge theory physics, susy and nonsusy, in SW and DV limits

- M-theory provides geometric, intuitive understanding of (meta)stability of nonsusy vacua

Thanks!