Brownian Motion in AdS/CFT

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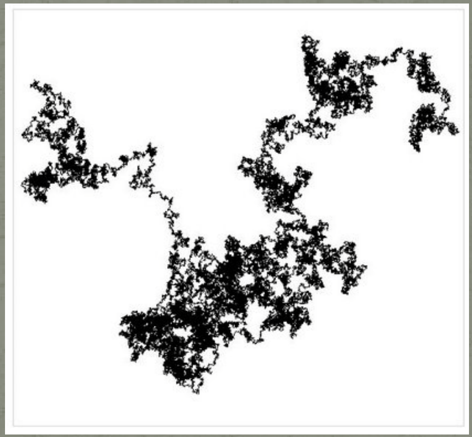
This talk is based on:

- J. de Boer, V. E. Hubeny, M. Rangamani, M.S., "Brownian motion in AdS/CFT," arXiv:0812.5112.
- A. Atmaja, J. de Boer, K. Schalm, M.S., work in progress.



Intro / Motivation

AdS/CFT and fluid-gravity

AdS

CFT

black hole in quantum gravity

plasma in strongly coupled QFT

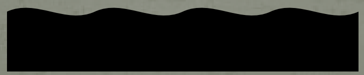
difficult

Long-wavelength approximation

horizon dynamics in classical GR

hydrodynamics Navier-Stokes eq.

easier; better-understood



Bhattacharyya+Minwalla+Rangamani+Hubeny 0712.2456

Macrophysics vs. microphysics

- Hydro: coarse-grained

→ BH in classical GR is also macro, approx. description of underlying microphysics of QG BH!

- Can't study microphysics within hydro framework (by definition)

→ want to go beyond hydro approx

Brownian motion

— Historically, a crucial step toward microphysics of nature

- 1827 Brown

Robert Brown (1773-1858)

- Due to collisions with fluid particles
- Allowed to determine Avogadro #: $N_A = 6 \times 10^{23} < \infty$
- Ubiquitous
- Langevin eq. (friction + random force)

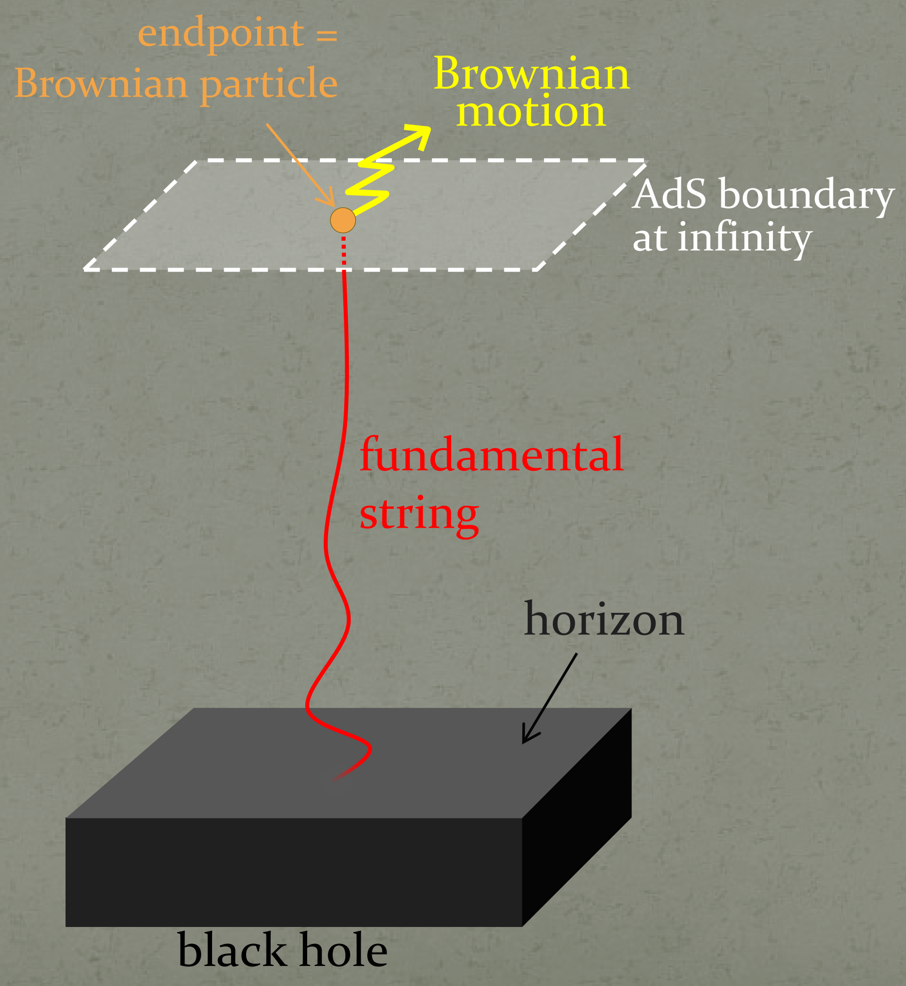
Brownian motion in AdS/CFT

→ Do the same for hydro. in AdS/CFT!

- Learn about QG from BM on boundary
- How does Langevin dynamics come about from bulk viewpoint?
- Fluctuation-dissipation theorem
- Relation to RHIC physics?

Related work:
drag force: Herzog+Karch+Kovtun+Kozcaz+Yaffe, Gubser, Casalderrey-Solana+Teaney
transverse momentum broadening: Gubser, Casalderrey-Solana+Teaney

Preview: BM in AdS/CFT



Outline

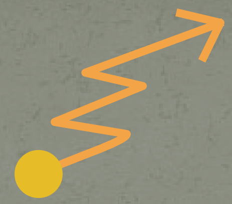
- Intro/motivation
- BM
- BM in AdS/CFT
- Time scales
- BM on stretched horizon

Brownian motion

Paul Langevin (1872-1946)

Langevin dynamics

Generalized Langevin eq:



$$\dot{p}(t) = -\int_{-\infty}^{t} dt' \underbrace{\gamma(t - t')}_{\text{delayed friction}} p(t') + \underbrace{R(t)}_{\text{random force}}$$

$$\langle R(t) \rangle = 0, \qquad \langle R(t) R(t') \rangle = \kappa(t - t')$$

General properties of BM

Displacement:

$$\langle s^2(t)\rangle \equiv \langle [x(t)-x(0)]^2\rangle$$

$$\approx \begin{cases} \dfrac{T}{m}t^2 & (t \ll t_{relax}) \\ 2Dt & (t \gg t_{relax}) \end{cases}$$

ballistic regime
(init. velocity $\dot{x} \sim \sqrt{T/m}$)

diffusive regime
(random walk)

diffusion constant $D \equiv \dfrac{T}{\gamma_0 m}$, $\gamma_0 = \int_0^\infty dt\,\gamma(t)$

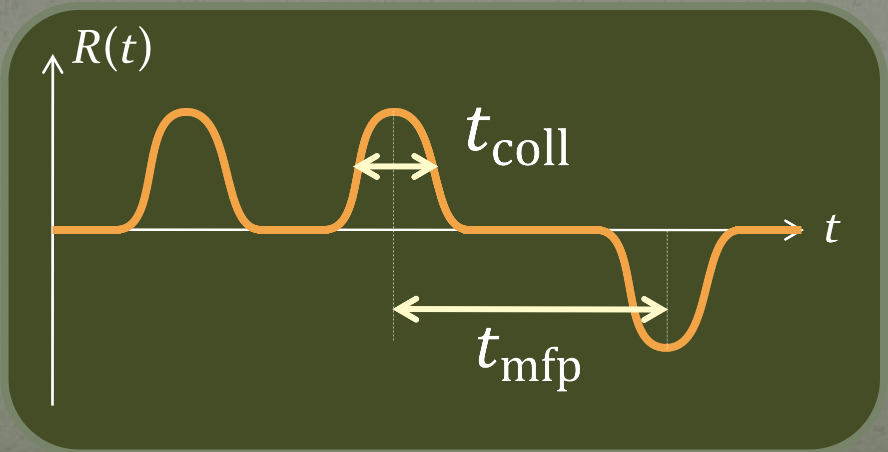
Time scales

- Relaxation time $t_{\mathrm{relax}} \equiv \frac{1}{\gamma_0}, \quad \gamma_0 = \int_0^\infty dt\, \gamma(t)$
- Collision duration time t_{coll}

$$\langle R(t)R(0)\rangle \sim e^{-t/t_{\mathrm{coll}}}$$

→ **time elapsed in a single collision**

- Mean-free-path time t_{mfp} → **time between collisions**



Typically

$$t_{\mathrm{relax}} \gg t_{\mathrm{mfp}} \gg t_{\mathrm{coll}}$$

but not necessarily so
for strongly coupled plasma

BM in AdS/CFT

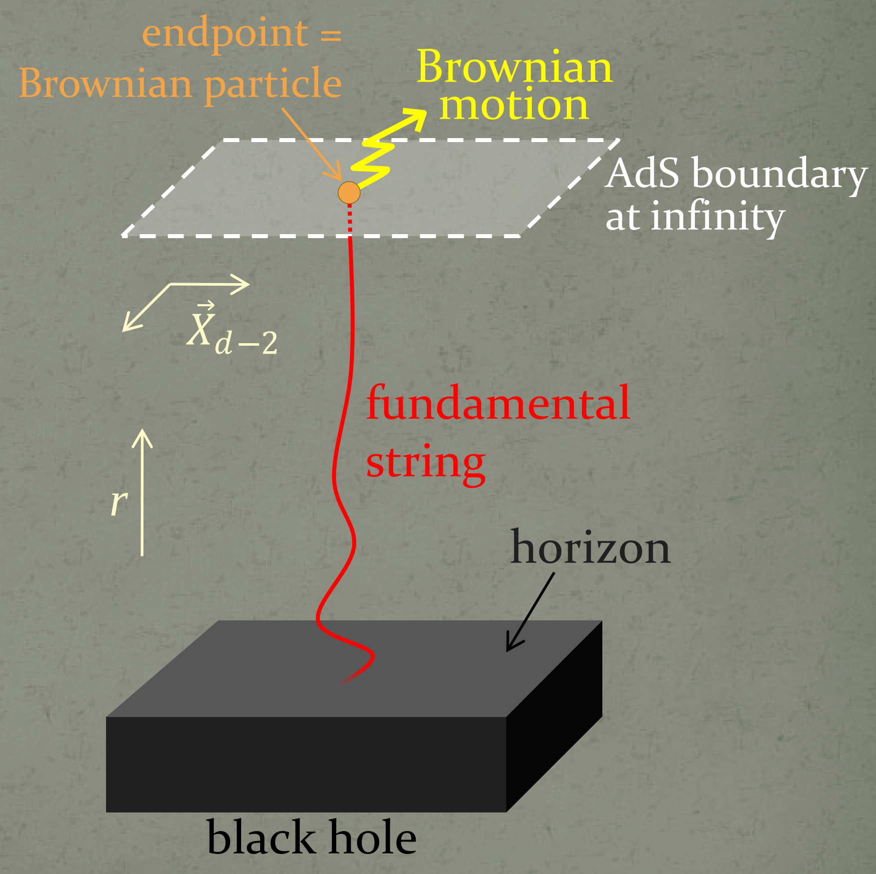
Bulk BM

- AdS Schwarzschild BH

$$ds_d^2 = \frac{r^2}{l^2}\left(-h(r)dt^2 + d\vec{X}_{d-2}^2\right) + \frac{l^2 dr^2}{r^2 h(r)}$$

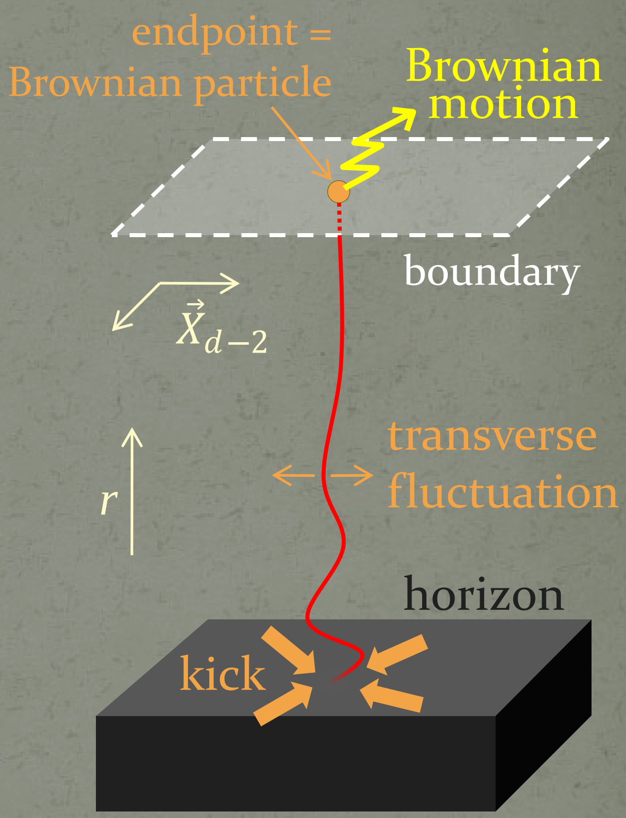
$$h(r) = 1 - \left(\frac{r_H}{r}\right)^{d-1}$$

$$T = \frac{1}{\beta} = \frac{(d-1)r_H}{4\pi l^2}$$



Physics of BM in AdS/CFT

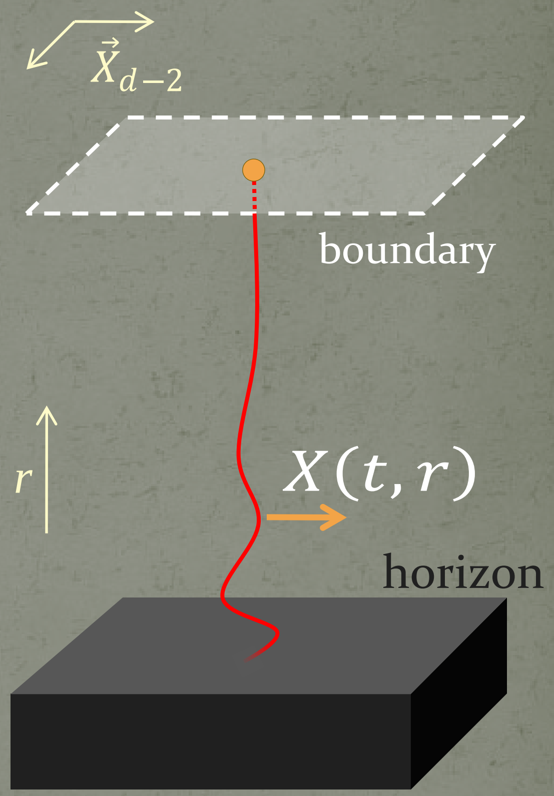
- Horizon kicks endpoint on horizon (= Hawking radiation)
- Fluctuation propagates to AdS boundary
- Endpoint on boundary (= Brownian particle) exhibits BM



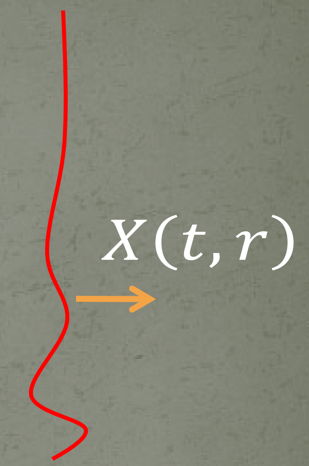
Whole process is dual to quark hit by QGP particles

BM in AdS/CFT

- Probe approximation
- Small g_s
    - No interaction with bulk
    - The only interaction is at horizon
- Small fluctuation
    - Expand Nambu-Goto action to quadratic order
    - Transverse positions $\vec{X}_{d-2}(t,r)$ are similar to Klein-Gordon scalars



Brownian string



- Quadratic action

$$S_{\mathrm{NG}} = \mathrm{const} + S_2 + S_4 + \cdots$$

$$S_2 = -\frac{1}{4\pi\alpha'}\int dt\,dr\left[\frac{\dot{X}^2}{h(r)} - \frac{r^4 h(r)}{l^4}X'^2\right]$$

- Mode expansion

$$X(t,r) = \int_0^\infty d\omega\left(f_\omega(r)e^{-i\omega t}a_\omega + \mathrm{h.c.}\right)$$

$$\left[\omega^2 + \frac{h(r)}{l^4}\partial_r\left(r^4 h(r)\partial_r\right)\right]f_\omega(r) = 0$$

d=3: can be solved exactly
d>3: can be solved in low frequency limit

Bulk-boundary dictionary

Near horizon:

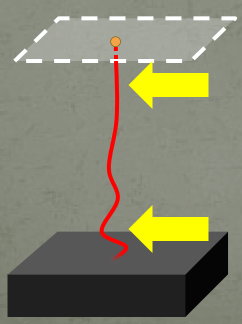
$$X(t,r) \sim \int_0^\infty \frac{d\omega}{\sqrt{2\omega}} \Big[\Big(\overbrace{e^{-i\omega(t-r_*)}}^{\text{outgoing mode}} + \underbrace{e^{i\theta_\omega}}_{\text{phase shift}} \overbrace{e^{-i\omega(t+r_*)}}^{\text{ingoing mode}}\Big) a_\omega + \mathrm{h.c.}\Big]$$

r_* : tortoise coordinate

Near boundary

$$X(t,r_\mathrm{c}) \equiv x(t) = \int_0^\infty d\omega \left[f_\omega(r_c) e^{-i\omega t} a_\omega + \mathrm{h.c.}\right]$$

r_c : cutoff



$$\langle x(t_1)x(t_2)\cdots\rangle \leftrightarrow \langle a_{\omega_1} a^\dagger_{\omega_2} \cdots\rangle$$

observe BM in gauge theory ⟺ **correlator of radiation modes**

Can learn about quantum gravity in principle!

Semiclassical analysis

- Semiclassically, NH modes are thermally excited:

$$\langle a_\omega a_\omega^\dagger \rangle \propto \frac{1}{e^{\beta\omega} - 1}$$

➡ Can use dictionary to compute $x(t)$, $s^2(t)$ (bulk → boundary)

$$s^2(t) \equiv \langle : [x(t) - x(0)]^2 : \rangle \approx \begin{cases} \frac{T}{m} t^2 & (t \ll t_{relax}) \quad \textbf{ballistic} \\ \frac{\alpha'}{\pi l^2 T} t & (t \gg t_{relax}) \quad \textbf{diffusive} \end{cases}$$

$$t_{relax} \sim \frac{\alpha' m}{l^2 T^2}$$

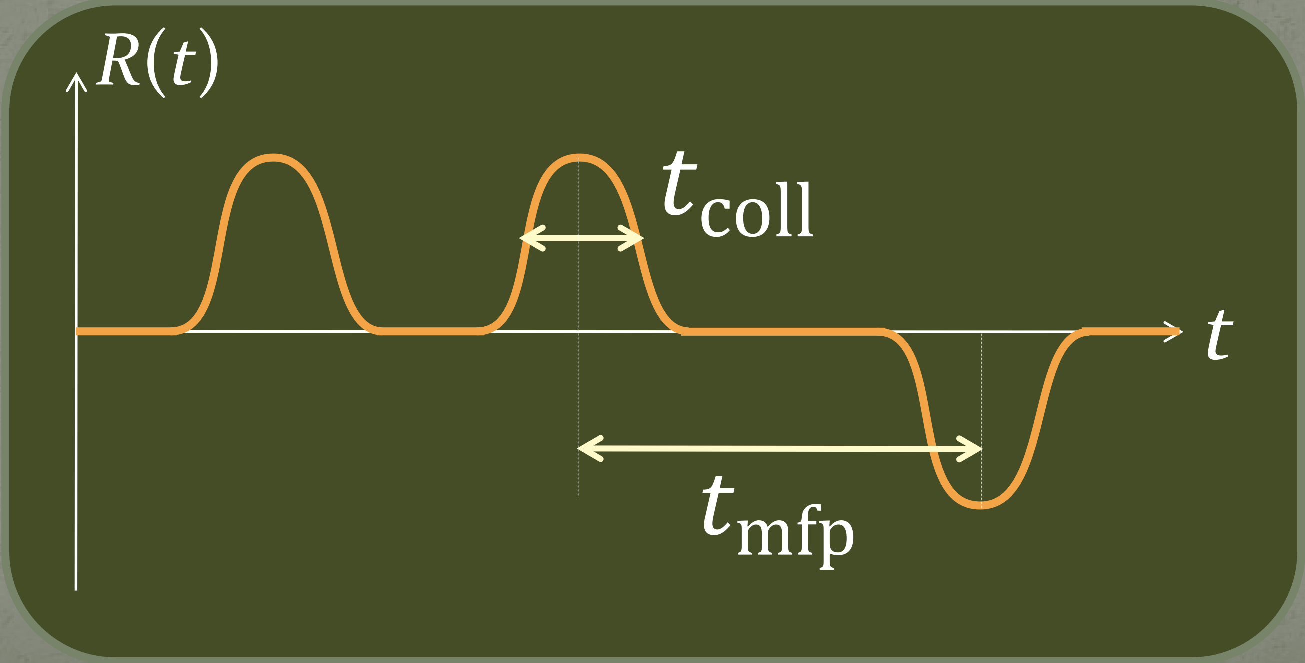
➡ **Does exhibit Brownian motion**

Time scales

Time scales

t_{relax} t_{mfp} t_{coll}

information about
plasma constituents



Time scales from R-correlators

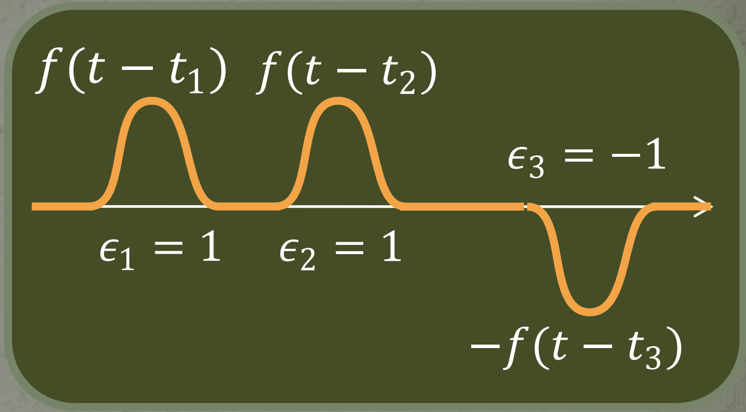
Simplifying assumptions:

- $R(t)$: consists of many pulses randomly distributed

$$R(t) = \sum_{i=1}^{k} \epsilon_i f(t - t_i)$$

$f(t)$: shape of a single pulse

$\epsilon_i = \pm 1$: random sign



- Distribution of pulses = Poisson distribution

μ : number of pulses per unit time, $\sim 1/t_{\text{mfp}}$

Time scales from R-correlators

- 2-pt func $\langle R(t)R(0)\rangle \to t_{\text{coll}}$
- Low-freq. 4-pt func $\langle \tilde{R}(\omega_1)\tilde{R}(\omega_2)\tilde{R}(\omega_3)\tilde{R}(\omega_4)\rangle \to t_{\text{mfp}}$

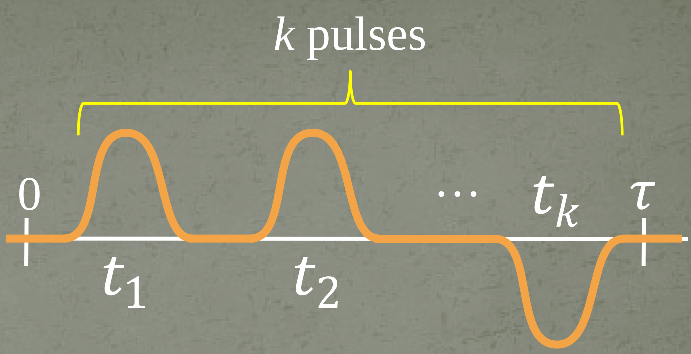
$$\langle \tilde{R}(0)^2\rangle = \mu T \tilde{f}(0)^2$$

$$\langle \tilde{R}(0)^4\rangle = 3\langle \tilde{R}(0)^2\rangle^2 + \mu T \tilde{f}(0)^4$$

$T \equiv 2\pi\delta(0)$, tilde = Fourier transform

⇒ Can determine μ, thus t_{mfp}

Sketch of derivation



$$R(t) = \sum_{i=1}^{k} \epsilon_i f(t - t_i)$$

Probability that there are *k* pulses in period [0,τ]:

$$P_k(\tau) = e^{-\mu\tau} \frac{(\mu\tau)^k}{k!} \qquad \text{(Poisson dist.)}$$

2-pt func:

$$\langle R(t) R(t') \rangle = \sum_{k=1}^{\infty} P_k(\tau) \sum_{i,j=1}^{k} \langle \epsilon_i \epsilon_j f(t - t_i) f(t - t_j) \rangle_k$$

$$\epsilon_i = \pm 1 : \text{ random signs} \rightarrow \langle \epsilon_i \epsilon_j \rangle = \delta_{ij}$$

$$\langle f(t - t_i) f(t' - t_i) \rangle_k = \frac{k}{\tau} \int_0^{\tau} du\, f(t - u) f(t' - u)$$

Sketch of derivation

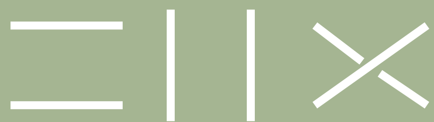
$$\Rightarrow \quad \langle R(t)R(t')\rangle = \mu \int_{-\infty}^{\infty} du\, f(t-u)f(t'-u)$$

$$\langle \tilde{R}(\omega)\tilde{R}(\omega')\rangle = 2\pi\mu\delta(\omega+\omega')\tilde{f}(\omega)\tilde{f}(\omega')$$

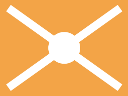
Similarly, for 4-pt func,

$$\langle \tilde{R}(\omega)\tilde{R}(\omega')\tilde{R}(\omega'')\tilde{R}(\omega''')\rangle$$
$$= \langle \tilde{R}(\omega)\tilde{R}(\omega')\rangle\langle \tilde{R}(\omega'')\tilde{R}(\omega''')\rangle + (\text{2 more terms})$$
$$+2\pi\mu\delta(\omega+\omega'+\omega''+\omega''')\tilde{f}(\omega)\tilde{f}(\omega')\tilde{f}(\omega'')\tilde{f}(\omega''')$$

"disconnected part"



"connected part"



$$\Rightarrow \quad \langle \tilde{R}(0)^2\rangle = \mu T \tilde{f}(0)^2$$
$$\langle \tilde{R}(0)^4\rangle = 3\langle \tilde{R}(0)^2\rangle^2 + \mu T \tilde{f}(0)^4$$

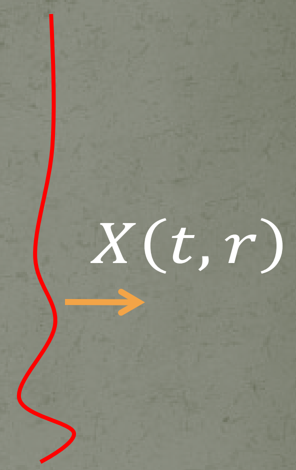
$T \equiv 2\pi\delta(0)$

R-correlators from BM in AdS/CFT

- Can compute t_{mfp} from correction to 4-pt func.
- Expansion of NG action to higher order:

$$S_{\mathrm{NG}} = \mathrm{const} + S_2 + S_4 + \cdots$$

$$S_4 = \frac{1}{16\pi\alpha'}\int dt\, dr \left[\frac{\dot{X}^2}{h(r)} - \frac{r^4 h(r)}{l^4} X'^2\right]^2$$



Can compute $\langle RRRR \rangle_c$ and thus t_{mfp}

Times scales from AdS/CFT

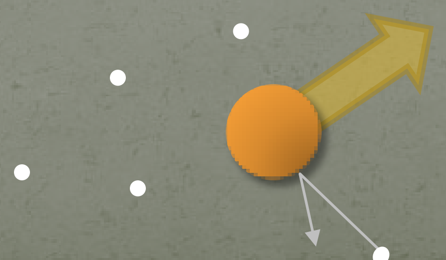
Resulting timescales:

$$t_{\text{relax}} \sim \frac{m}{\sqrt{\lambda}\, T^2} \qquad t_{\text{coll}} \sim \frac{1}{T} \qquad t_{\text{mfp}} \sim \frac{1}{\sqrt{\lambda}\, T} \qquad \lambda = \frac{l^4}{\alpha'^2}$$

- weak coupling $\lambda \ll 1$

⇒ $t_{\text{relax}} \gg t_{\text{mfp}} \gg t_{\text{coll}}$

⇒ conventional kinetic theory is good



Times scales from AdS/CFT

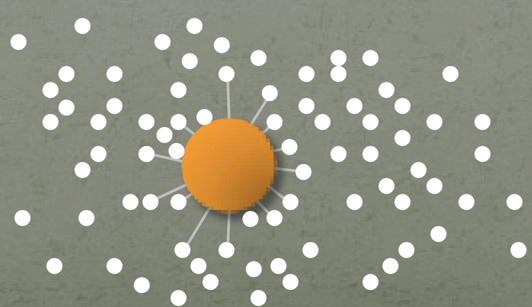
Resulting timescales:

$$t_{\rm relax} \sim \frac{m}{\sqrt{\lambda}\, T^2} \qquad t_{\rm coll} \sim \frac{1}{T} \qquad t_{\rm mfp} \sim \frac{1}{\sqrt{\lambda}\, T} \qquad \lambda = \frac{l^4}{\alpha'^2}$$

- strong coupling $\lambda \gg 1$

⇨ $t_{\rm mfp} \ll t_{\rm coll}$. $t_{\rm relax} \ll t_{\rm coll}$ is also possible.

⇨ Multiple collisions occur simultaneously.



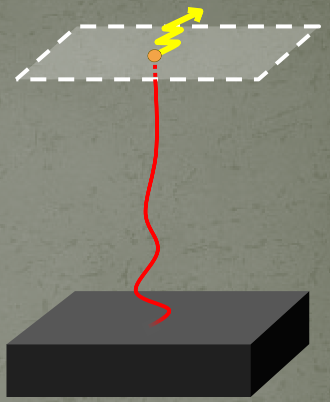
Cf. "fast scrambler"

BM on stretched horizon

(Jorge's talk)

Conclusions

Conclusions



- Boundary BM ↔ bulk "Brownian string"

  can study QG in principle

- Semiclassically, can reproduce Langevin dyn. from bulk

  random force ↔ Hawking rad. (kicking by horizon)

  friction ↔ absorption

- Time scales in strong coupling QGP: t_{relax} , t_{mfp} , t_{coll}
- BM on stretched horizon (Jorge's talk)
- Fluctuation-dissipation theorem

Thanks!