Exotic Branes and Non-Geometric Backgrounds

Masaki Shigemori
(Amsterdam)

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J de Boer and MS: 1004.2521, 100x.nnnn

Introduction

U-duality

- Relates various objects in string/M-theory

  T: Dp ↔ D(p±1), F1 ↔ P, NS5 ↔ KKM, …

  S: F1 ↔ D1, NS5 ↔ D5, …

- Enhances in lower dims.
  - M-theory on T^k: $E_{k(k)}(Z)$ [Hull+Townsend]

k	D	G(Z)
1	10	1
2	9	$SL(2,Z) \times Z_2$
3	8	$SL(3,Z) \times SL(2,Z)$
4	7	$O(5,5,Z)$
5	6	$SL(5,Z)$
6	5	$E_{6(6)}(Z)$
7	4	$E_{7(7)}(Z)$
8	3	$E_{8(8)}(Z)$

Codimension-2 objects (1)

- U-duality on codim-2 objects produces exotic states

Known codim-2 object → Exotic states

U-duality

Known 10D/11D object wrapped on internal torus

10D/11D origin unknown!

They can have mass $\sim g_s^{-3}, g_s^{-4}$

[9707217 Elitzur+Giveon+Kutasov+Rabinovici]
[9809039 Obers+Pioline]

Codimension-2 objects (2)

- Example: Type II on T^2

	1	2	3	4	5	6	7	8	9
NS5	·	·	O	O	O	O	O	~	~

T^2 (directions 8, 9)

T-duality along x^8, x^9

$$5^2_2$$

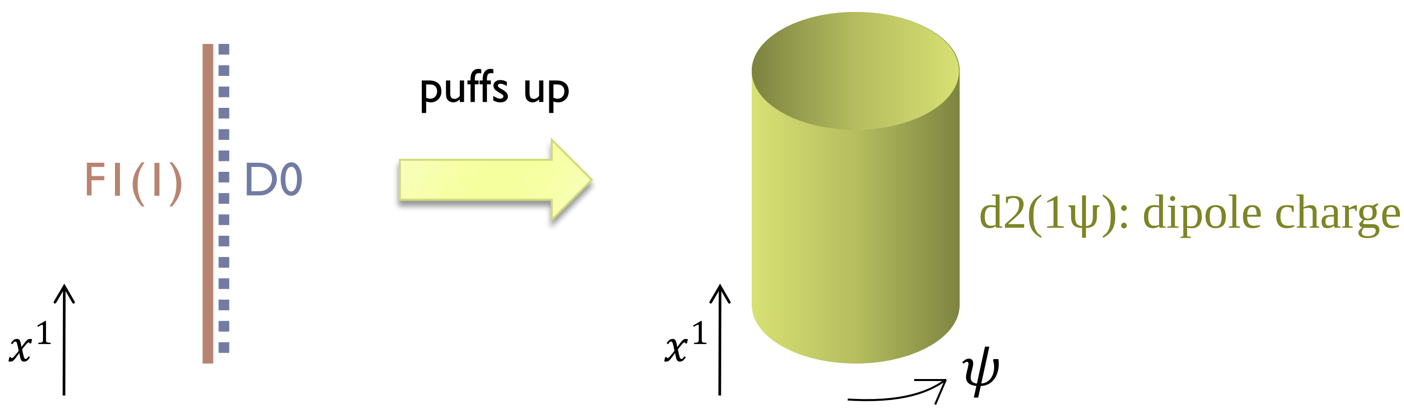
We will see that this is a "T-fold".

Supertube effect

- Codim-2 object problematic
  - Log divergences

$$V \sim \frac{1}{r^{d-2}} \xrightarrow{d=2} V \sim \log\left(\frac{\mu}{r}\right)$$

- Are they relevant? Why care?
- Supertube effect = spontaneous polarization [Mateos+Townsend]



Relevance of exotic branes

- Non-exotic branes can puff up to produce exotic dipole charges
  - → No log divergence
  - → Exotic branes are relevant to non-exotic physics!
- Black holes: bound states of branes
  - → Microstate (non-)geometries?

Outline

- Introduction
- Exotic states & their higher-D origin
- Sugra description
- Supertube effect
- Conclusion

Exotic states and their higher-D origin

Exotic states in 3D (1)

- M on T^8 or Type II on T^7
  - → 3D N=16 sugra
  - → 128 scalars (in 3D, scalar = vector)
  - → U-duality group $E_{8(8)}(Z)$: generated by T- and S-dualities
- Particle multiplet:
  - → Start from a point-like object

    e.g. D7(3456789) wrapped on T^7
  - → Take T- and S-dualities to get other states

Exotic states in 3D (2)

- Particle multiplet:

[9707217 Elitzur+Giveon+Kutasov+Rabinovici]
[9809039 Obers+Pioline]

Type IIA	P $(\mathbf{7})$, F1 $(\mathbf{7})$, D0 $(\mathbf{1})$, D2 $(\mathbf{21})$, D4 $(\mathbf{35})$, D6 $(\mathbf{7})$, NS5 $(\mathbf{21})$, KKM $(\mathbf{42})$, 5^2_2 $(\mathbf{21})$, 0^7_3 $(\mathbf{1})$, 2^5_3 $(\mathbf{21})$, 4^2_3 $(\mathbf{35})$, 6^1_3 $(\mathbf{7})$, $0_4^{(1,6)}$ $(\mathbf{7})$, 1^6_4 $(\mathbf{7})$
Type IIB	P $(\mathbf{7})$, F1 $(\mathbf{7})$, D1 $(\mathbf{7})$, D3 $(\mathbf{35})$, D5 $(\mathbf{21})$, D7 $(\mathbf{1})$, NS5 $(\mathbf{21})$, KKM $(\mathbf{42})$, 5^2_2 $(\mathbf{21})$, 1^6_3 $(\mathbf{7})$, 3^4_3 $(\mathbf{35})$, 5^2_3 $(\mathbf{21})$, 7_3 $(\mathbf{1})$, $0_4^{(1,6)}$ $(\mathbf{7})$, 1^6_4 $(\mathbf{7})$
M-theory	P $(\mathbf{8})$, M2 $(\mathbf{28})$, M5 $(\mathbf{56})$, KKM $(\mathbf{56})$, 5^3 $(\mathbf{56})$, 2^6 $(\mathbf{28})$, $0^{(1,7)}$ $(\mathbf{8})$

240 states

- Notation for exotic states

$$b_n^c : M = \frac{R^b (R^c)^2}{g_s^n} \qquad b_n^{(d,c)} : M = \frac{R^b (R^c)^2 (R^d)^3}{g_s^n}$$

Example: $5^2_2(34567{,}89)$: $M = \dfrac{R_3 \cdots R_7\,(R_8 R_9)^2}{g_s^2 l_s^8}$

Duality rules

- Duality rules can be read off from:

$$T_y: \quad R_y \to \frac{l_2^2}{R_y}, \quad g_s \to \frac{l_s}{R_y} g_s \qquad\qquad S: \quad g_s \to \frac{1}{g_s}, \quad l_s \to g_s^{1/2} l_s$$

- Example:

$$\mathrm{NS5}(34567) \xrightarrow{\mathrm{T_8}} \mathrm{KKM}(34567{,}8) \xrightarrow{\mathrm{T_9}} 5_2^2(34567{,}89)$$

$$M = \frac{R_3 \cdots R_7}{g_s^2 l_s^6} \xrightarrow{\mathrm{T_8}} \frac{R_3 \cdots R_7}{(l_s/R_8)^2 l_s^6} = \frac{R_3 \cdots R_7 R_8^2}{g_s^2 l_s^8} \quad : 5_2^1 = \mathrm{KKM}$$

$$\xrightarrow{\mathrm{T_9}} \frac{R_3 \cdots R_7 R_8^2}{(l_s/R_8)^2 l_s^8} = \frac{R_3 \cdots R_7 (R_8 R_9)^2}{g_s^2 l_s^{10}} \quad : 5_2^2$$

Higher D origin

- 240 states for 128 gauge fields??

  → Higher D origin unclear

  cf. D>3: (#point particle states) = (#gauge fields)

  Higher D origin: wrapped branes

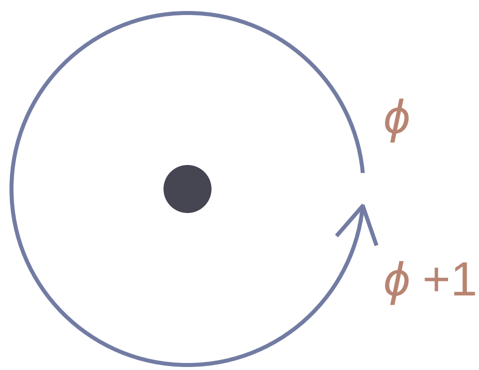
- Claim: higher D origin is

  U-fold = non-geometric background

Higher D origin = U-folds (1)

E.g. D7 on T^7

- (magnetically) coupled to RR 0-form C_0
- 3D scalar $\phi = C_0$
- Monodromy: $\phi \rightarrow \phi + 1$ (part of SL(2,Z) symmetry of IIB)



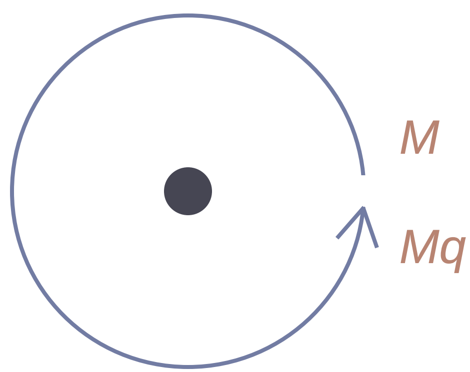
Higher D origin = U-folds (2)

- In 3D, charge = monodromy of scalar ϕ
- Shifting symmetry + S, T-dualities $\Longrightarrow$ $E_{8(8)}(\mathbb{Z})$
- ϕ gets combined with other scalars to form moduli matrix

$$M \in \mathcal{M} = SO(16)\backslash E_{8(8)}(\mathbb{R})/E_{8(8)}(\mathbb{Z})$$

- Can consider a particle with general U-duality monodromy

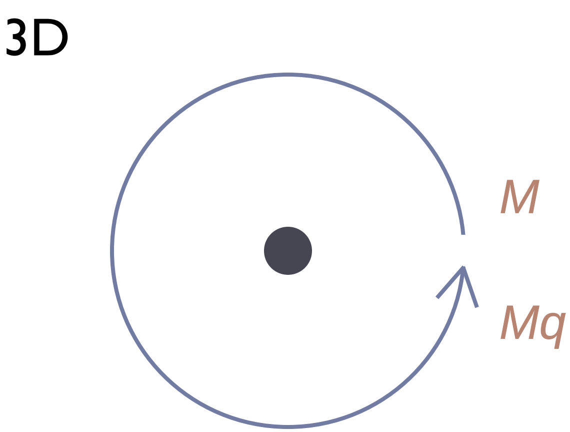
$$q \in E_{8(8)}(\mathbb{Z}) \equiv G(\mathbb{Z})$$

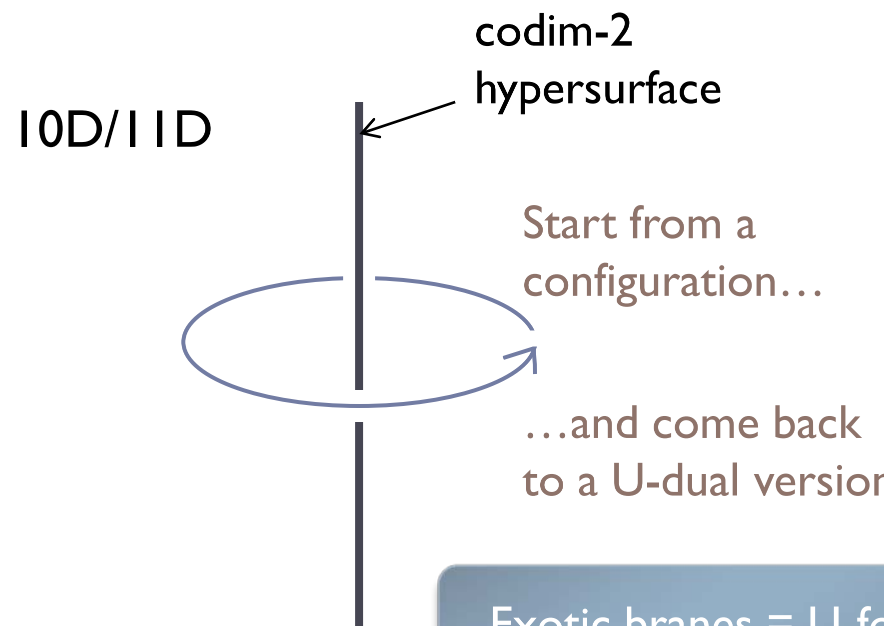


"Charge" of a 3D particle is U-duality monodromy around it!

Higher D origin = U-folds (3)

- "Charge" of a 3D particle = U-duality monodromy around it
- No relation between #(gauge fields) = #scalars and #charges
- In 10D/11D, we have a non-geometric U-fold





Exotic branes = U-folds

How many “charges” are there? (1)

1. Start from charge = monodromy $q \in G(\mathbb{Z})$, i.e.
$$M \to Mq$$
2. Go to another U-dual frame by $U \in G(\mathbb{Z})$
3. In the dual frame, charge and moduli are
$$\tilde{q} = U^{-1} q\, U \qquad \widetilde{M} = MU$$
4. Change moduli adiabatically back to M
5. In the original frame, we have charge $\tilde{q} = U^{-1} q\, U$

Starting from a charge q, can obtain other charges $\tilde{q}$ by conjugation by U-duality matrix U

Caveats:

How many “charges” are there? (2)

Example

- D7 has monodromy

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in SL(2, \mathbb{Z})$$

- Conjugate by a general SL(2,Z) matrix

$$U = \begin{pmatrix} s & r \\ q & p \end{pmatrix}, \qquad ps - qr = 1, \qquad p, s, q, r \in \mathbb{Z}$$

New charge:

$$\tilde{T} = U^{-1} T U = \begin{pmatrix} 1 + pq & p^2 \\ -q^2 & 1 - pq \end{pmatrix}$$: monodromy of (p,q) 7-brane

Note: there are only 2 parameters although *U* had 3

How many "charges" are there? (3)

- Set of all charges obtained from given q: conjugacy orbit
- "Charges" live in discrete non-Abelian "lattice" $G(\mathbb{Z})$
  - → Meaningless to ask "how many" different charges there are
- Replace $G(\mathbb{Z}) \to G(\mathbb{R})$ and compute dimension of conjugacy class in $G(\mathbb{R})$
- For 1/2 BPS objects, dimension = 58
  - 240 states are special points in this 58-dim space preserving rectangularity of internal torus

Sugra description of exotic states

Sugra solution for 5^2_2 (1)

KKM(56789,4):

$$ds^2 = dx^2_{056789} + H dx^2_{123} + H^{-1}(dx^4 + \omega)^2$$

$$e^{2\Phi} = 1, \quad d\omega = *_3 \, dH,$$

$$H = 1 + \sum_p H_p \,, \quad H_p = \frac{R_4}{2|\vec{x} - \vec{x}_p|}$$

$\vec{x}_p$: positions of centers in $\mathbb{R}^3_{123}$

compactify x^3

$$H(r) = h + \sigma \log\left(\frac{\mu}{r}\right)$$

[Sen] [Blau+O'Loughlin]

T-dualize along x^3

Sugra solution for 5^2_2 (2)

$5^2_2(56789,34)$ metric:

$$ds^2 = H(dr^2 + r^2 d\theta^2) + HK^{-1} dx_{34}^2 + dx_{056789}^2$$

$$B_{34}^{(2)} = -K^{-1}\theta\sigma, \quad e^{2\Phi} = HK^{-1}, \quad K \equiv H^2 + \sigma^2\theta^2$$

$$H(r) = h + \sigma \log\left(\frac{\mu}{r}\right) \qquad \sigma = \frac{R_3 R_4}{2\pi\alpha'}$$

Cf. [Blau+O'Loughlin]: 6^1_3

- T-fold structure:

$$\theta = 0: \quad G_{33} = G_{44} = H^{-1},$$

$$\theta = 2\pi: \quad G_{33} = G_{44} = \frac{H}{H^2 + (2\pi\sigma)^2}$$

→ x^3-x^4 torus size doesn't come back to itself!

T-fold structure of 5^2_2

- Package 3-4 part of G, B:

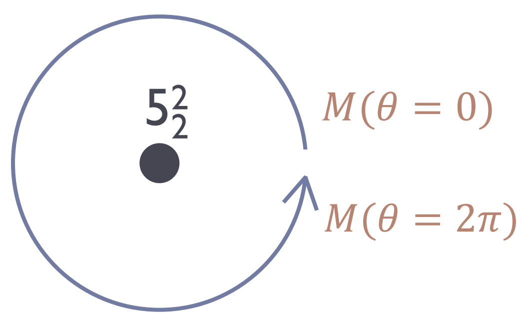
$$M = \begin{pmatrix} G^{-1} & G^{-1}B \\ -BG^{-1} & G - BG^{-1}B \end{pmatrix}$$

T-duality acts as

$$M \to M' = \Omega^t M \Omega, \qquad \Omega \in SO(2,2,\mathbb{R})$$

T-duality monodromy around 5^2_2:

$$\Omega = \begin{pmatrix} 1 & 0 \\ 2\pi\sigma & 1 \end{pmatrix}: \qquad M(\theta = 0) \to M(\theta = 2\pi)$$



Comments

- Can compute mass by putting it in flat space
- Not well-defined as stand-alone objects
  - Log divergence
    - → Superpositions (cf. F-theory 7-branes)
    - → Configs with higher codims. (next topic)
- Easy to get sugra metric for other exotic branes
  - Questionable for states with $M \sim g_s^{-3}, g_s^{-4}$

Supertube effect and exotic branes

Dualizing supertube effect

Original supertube effect:

$$\mathrm{D0} + \mathrm{F1}(1) \to \mathrm{D2}(1\psi)$$

dualize

Various other known puff-ups:

$$\mathrm{F1}(1) + \mathrm{P}(1) \to \mathrm{F1}(\psi) \qquad \text{FP sys}$$

$$\mathrm{D1}(1) + \mathrm{D5}(12345) \to \mathrm{KKM}(2345\psi, 1) \qquad \text{LM geom}$$

$$\mathrm{M2}(12) + \mathrm{M2}(34) \to \mathrm{M5}(1234\psi) \qquad \text{black ring}$$

Puff-ups involving exotic charges

$$\mathrm{D0} + \mathrm{F1}(1) \to \mathrm{D2}(1\psi)$$

$$\Big\downarrow \ \text{dualize}$$

Exotic puff-ups:

$$\mathrm{D4}(6789) + \mathrm{D4}(4589) \to 5^2_2(4567\psi, 89)$$

: part of 4D BH system

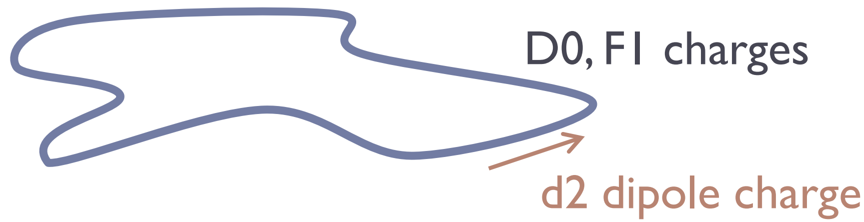
$$\mathrm{D3}(589) + \mathrm{NS5}(46789) \to 5^2_3(4567\psi, 89)$$

$$\mathrm{NS5}(46789) + \mathrm{KKM}(46789{,}5) \to 1^6_4(\psi, 456789)$$

$$g_s{}^{-a} + g_s{}^{-b} \to g_s{}^{-a-b}$$

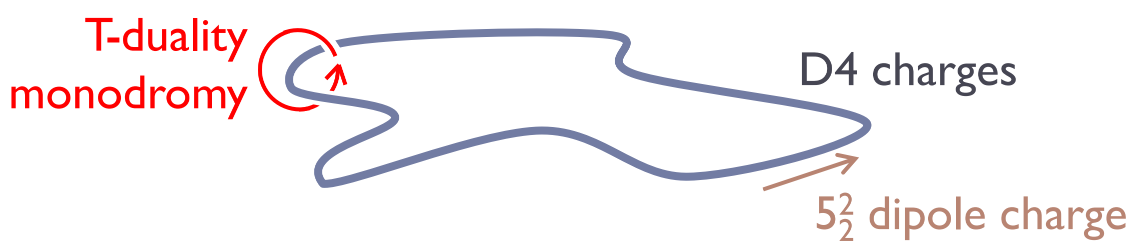
Sugra solution for D4+D4$\rightarrow 5_2^2$

- Basic sugra supertube



Dualize

- Exotic 2-charge solution



D4(6789)+D4(4589)→5^2_2 (4567ψ,89)

$$ds^2 = -\frac{1}{\sqrt{f_1 f_2}}(dt - A)^2 + \sqrt{f_1 f_5}\, dx_{123}^2 + \sqrt{\frac{f_1}{f_2}}\, dx_{45}^2 + \sqrt{\frac{f_2}{f_1}}\, dx_{67}^2 + \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}\, dx_{89}^2,$$

f_i , A : sourced along curve

$$f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{dv}{|\vec{x} - \vec{F}(v)|}, \quad f_2 = 1 + \frac{Q_1}{L}\int_0^L \frac{|\dot{\vec{F}}(v)|^2}{|\vec{x} - \vec{F}(v)|} dv, \quad A_i = -\frac{Q_1}{L}\int_0^L \frac{\dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|}$$

$$d\beta_I = *_3\, df_I, \qquad d\gamma = *_3\, dA$$

- β_i, γ have monodromy around curve

$$\beta_I \to \beta_I - 2Q_I, \quad \gamma \to \gamma - 2q,$$

→ T-fold structure just as before

- Asymptotically flat 4D

D4(6789)+D4(4589)$\rightarrow 5_2^2$(4567ψ,89)

Other fields:

$$e^{2\Phi} = \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}, \qquad B_{89}^{(2)} = \frac{\gamma}{f_1 f_2 + \gamma^2}, \qquad C^{(3)} = -\gamma\rho + \sigma$$

$$\rho = (f_2^{-1} + dt - A) \wedge dx^4 \wedge dx^5 + (f_1^{-1} + dt - A) \wedge dx^6 \wedge dx^7$$

$$\sigma = (\beta_1 - \gamma dt) \wedge dx^4 \wedge dx^5 + (\beta_2 - \gamma dt) \wedge dx^6 \wedge dx^7$$

Circular D4+D4$\rightarrow 5^2_2$

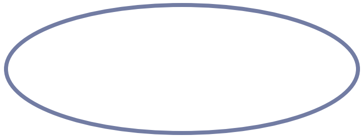
For circular profile, all functions can be explicitly written down

$$dx_{123}^2 = \frac{R^2}{(\cos\phi - y)^2}\left[\frac{dy^2}{y^2-1} + (y^2-1)d\psi^2 + d\phi^2\right]$$

$$f_I = 1 + \frac{Q_I}{R}\sqrt{\frac{\cos\phi - y}{-2y}}\,F\left(\frac{1}{4},\frac{3}{4};1;z^2\right),\quad A_\psi = -\frac{qR}{2}\frac{y^2-1}{(\cos\phi - y)^{1/2}(-2y)^{3/2}}\,F\left(\frac{3}{4},\frac{5}{4};2;z^2\right)$$

$$\begin{aligned}\gamma = -\frac{q\sqrt{1-y}}{4\sqrt{2}(-y)^{3/2}}\Bigg\{&(1+y)\mathbf{F}\left(\frac{\phi}{2}\middle|\frac{2}{1-y}\right)F\left(\frac{3}{4},\frac{5}{4};2;z^2\right)\\ &+u\mathbf{E}\left(\frac{\phi}{2}\middle|\frac{2}{1-y}\right)\left[3F\left(\frac{3}{4},\frac{1}{4};2;z^2\right)+F\left(\frac{3}{4};\frac{5}{4};2;z^2\right)\right]\Bigg\}\end{aligned}$$

$$\beta_I = \cdots$$



$$z = 1 - y^{-2}$$

Puff-ups and BH microstates (1)

- Standard 4D BH system

$$\mathrm{D0},\ \ \mathrm{D4}(6789),\ \ \mathrm{D4}(4589),\ \ \mathrm{D4}(4567)$$

: Well studied for microstate counting [MSW]

- Possible puff-ups:

$$\begin{array}{ll} & \mathrm{D4}(6789) \\ \mathrm{D0} & \mathrm{D4}(4589) \\ & \mathrm{D4}(4567) \end{array} \xrightarrow{\text{puff up}} \begin{array}{ll} \mathrm{NS5}(6789\psi) & 5^2_2(6789{,}45\psi) \\ \mathrm{NS5}(4589\psi) & 5^2_2(4589{,}67\psi) \\ \mathrm{NS5}(4567\psi) & 5^2_2(4567{,}89\psi) \end{array}$$

non-geometric

$\xrightarrow{\text{puff up}}$ More exotic charges?

Puff-ups and BH microstates (2)

- 5D BH system

$$\mathrm{M2}(56),\ \ \mathrm{M2}(78),\ \ \mathrm{M2}(9\mathrm{A})$$

: Well studied for microstate geometry

[Mathur] [Bena+Warner] [Berglund+Gimon+Levi]
[de Boer+El-Showk+Messamah+Van de Bleeken]

- Possible puff-ups:

$$\begin{array}{ccccc} \mathrm{M2}(56) & & \mathrm{M5}(789\mathrm{A}\psi) & & 5^3(789\mathrm{A}\phi, 56\psi) \\ \mathrm{M2}(78) & \xrightarrow{\text{puff up}} & \mathrm{M5}(569\mathrm{A}\psi) & \xrightarrow{\text{puff up}} & 5^3(569\mathrm{A}\phi, 78\psi) \\ \mathrm{M2}(9\mathrm{A}) & & \mathrm{M5}(5678\psi) & & 5^3(5678\phi, 9A\psi) \end{array}$$

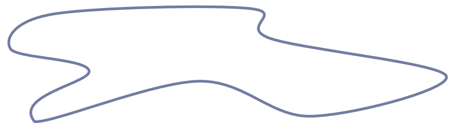
cf. black ring

non-geometric

Puff-ups and BH microstates (3)

2-charge system

- Worldvolume theory:
  - Higgs branch coming from intersection of two stacks
- Gravity:
  - Fluctuation of 1-dimensional object

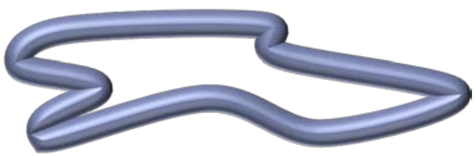


$$S_{brane} = S_{gravity}$$

Puff-ups and BH microstates (4)

3-charge system

- Worldvolume theory:
  - More complicated Higgs branch from triple intersection
- Gravity:



  - Fluctuation of 2-dimensional object?
    - — Exotic branes has just the right dimensionality
  - Explains missing entropy in sugra microstate geometries?

$$S_{\text{brane}} = S_{\text{gravity}} \ ??$$

[Bena+Warner] [Berglund+Gimon+Levi]
[de Boer+El-Showk+Messamah+Van den Bleeken]

Conclusions

Conclusions

- Exotic branes = non-geometries (U-folds)
- Exotic charges = U-duality monodromies
- Relevant even for non-exotic physics by supertube effect
- Unexplored exotic land out there awaiting us!
  - Classification of exotic branes (bound states, etc.)
  - Non-Abelian anyon statistics
  - AdS/CFT
  - Microstate non-geometries
  - 4D black ring??