A Holographic Study of Thermalization in Strongly Coupled Plasmas

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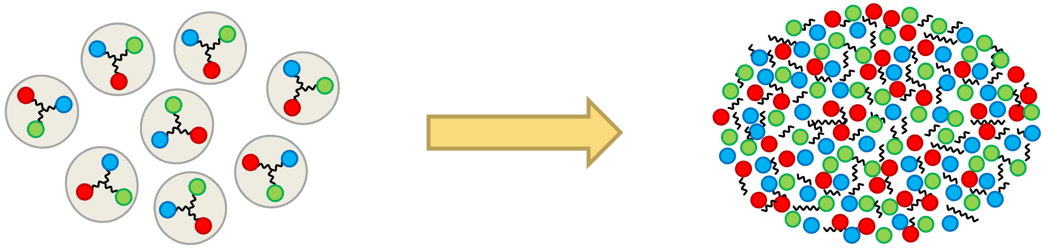
An old & new question

What happens in nuclear matter at high temperature?



Low T

High T

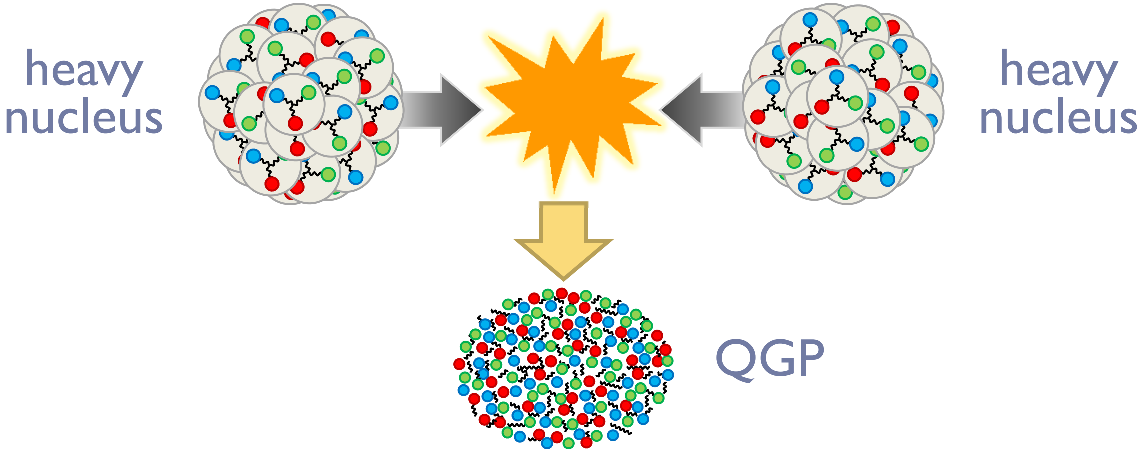
~10^{12} Kelvin

quarks, gluons: confined

quarks, gluons: deconfined

Quark-Gluon Plasma (QGP)

Heavy ion collision



heavy nucleus

heavy nucleus

QGP

- QGP created at RHIC, LHC
- "Little Bang"
- Strongly coupled
- Difficult to study

Perturbative QCD: not applicable

Lattice simulations: not always powerful enough

Holographic approach

QCD "$\sim$" $\mathcal{N} = 4$ SYM $\overset{\text{AdS/CFT}}{=}$ AdS string

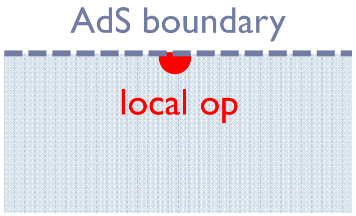
Equilibrated QGP	AdS black hole
HI collision	Energy injection?
Thermalization	BH formation

- A toy model for QCD
- General insights into strongly coupled physics

Questions we ask & (try to) answer

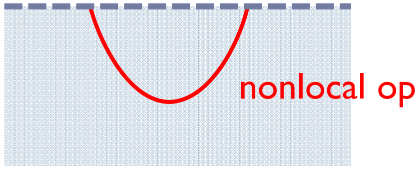
- What is the measure for thermalization in the bulk?
  - Local operators are not sufficient

    $\langle T_{\mu\nu} \rangle$, etc.



  - Non-local operators do better job

    $\langle \mathcal{O}(x)\mathcal{O}(x') \rangle$, etc.



- What is the thermalization time?

  When observables become identical to the thermal ones

Outline

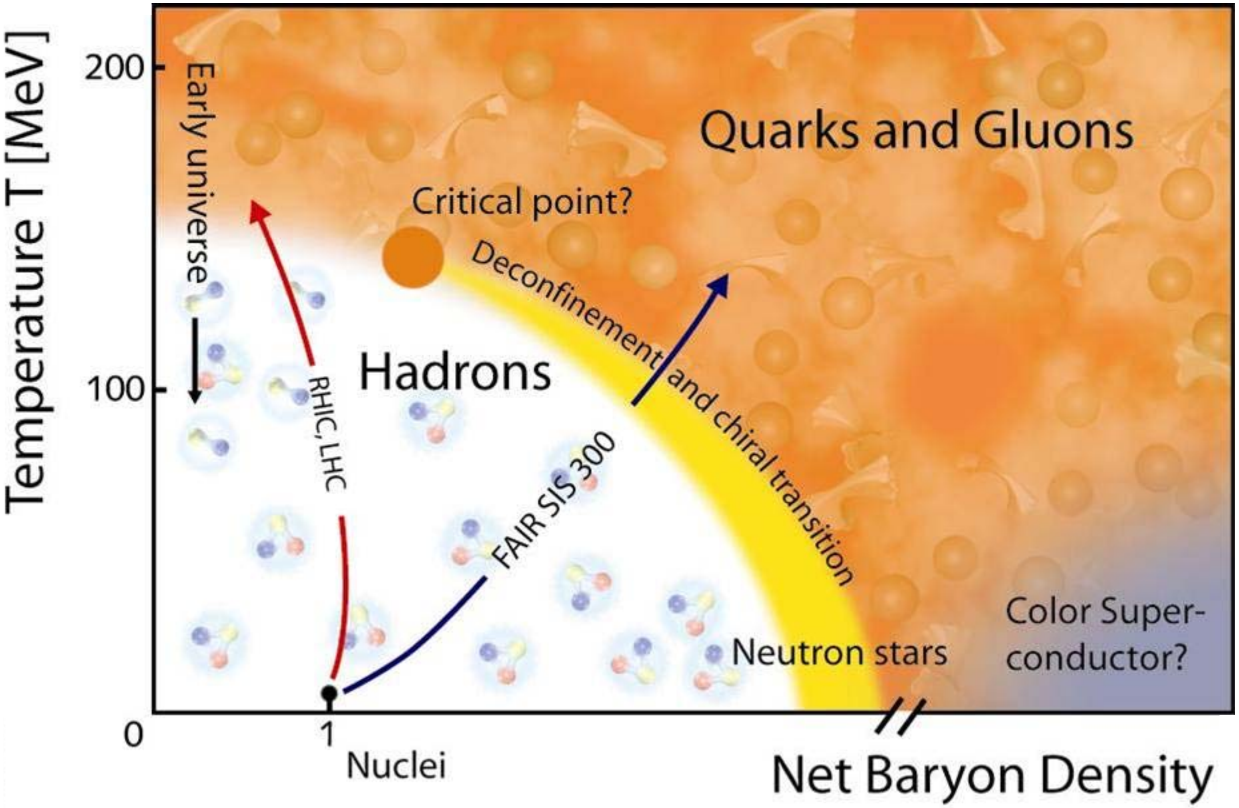
0. Intro
1. Review
    - 1.1 HI collision
    - 1.2 Lattice
    - 1.3 Holographic approach
2. Holographic thermalization
    - 2.1 Models & probes for therm'n
    - 2.2 The model & results
    - 2.3 Summary

Mostly based on Casalderrey-Solana et al. 1101.0618

1.1 QCD and heavy ion collision

QCD at high T

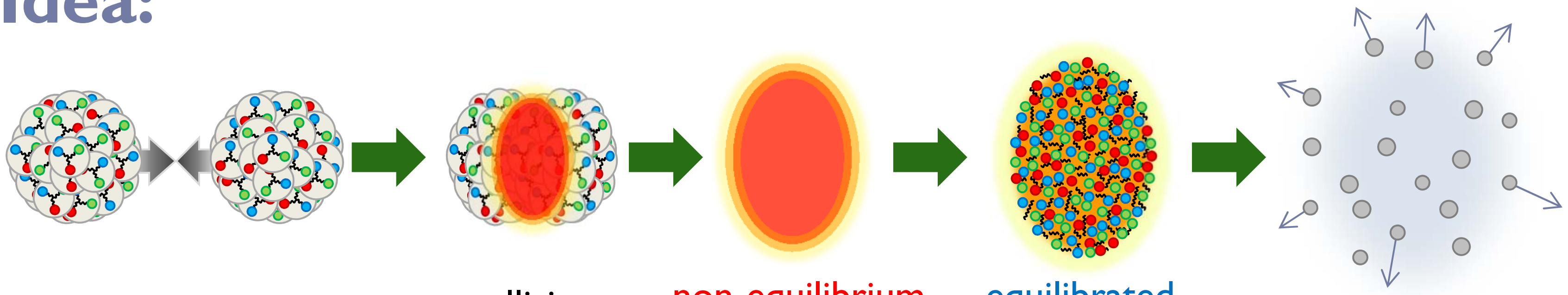
- Conjectured phase diagram



Taken from FAIR website

Heavy Ion Collision

Idea:



collision

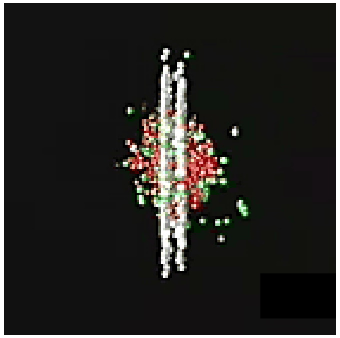
non-equilibrium state

equilibrated QGP

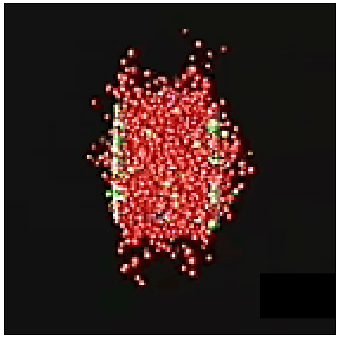
freeze-out

Actual:

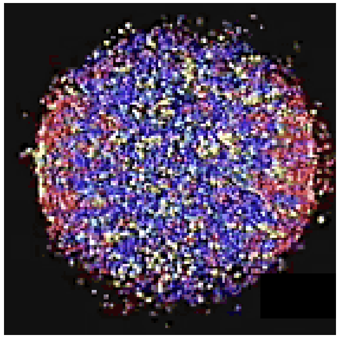
1. Ions about to collide*



2. Ion collision



3. Quarks, gluons freed



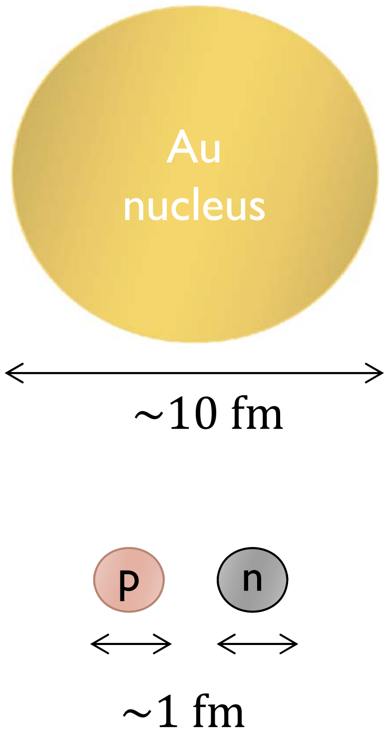
4. Plasma created

Taken from BNL website

Scales in HI collision (1)

fm = Fermi = femtometer = 10^{-15} m
$\sim 3 \times 10^{-24}$ sec = 3 yoctoseconds

m	mili	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}
f	femto	10^{-15}
a	atto	10^{-18}
z	zepto	10^{-21}
y	yocto	10^{-24}



hydrogen atom: $\sim 10^5$ fm

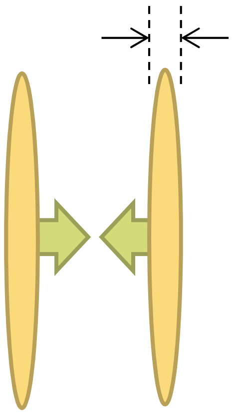
Scales in HI collision (2)

RHIC: Au $\sqrt{s_{\text{nucl}}} \sim 200\ \text{GeV}$

$\sqrt{s_{\text{tot}}} \sim 40\ \text{TeV}$

LHC: Pb $\sqrt{s_{\text{nucl}}} \sim 5.5\ \text{TeV}$

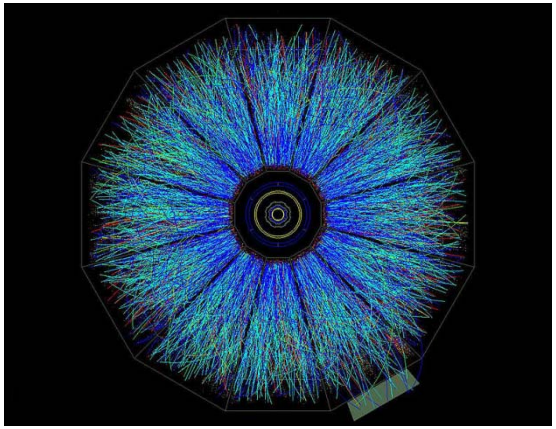
$\sqrt{s_{\text{tot}}} \sim 1000\ \text{TeV}$



$v = 0.99995c$

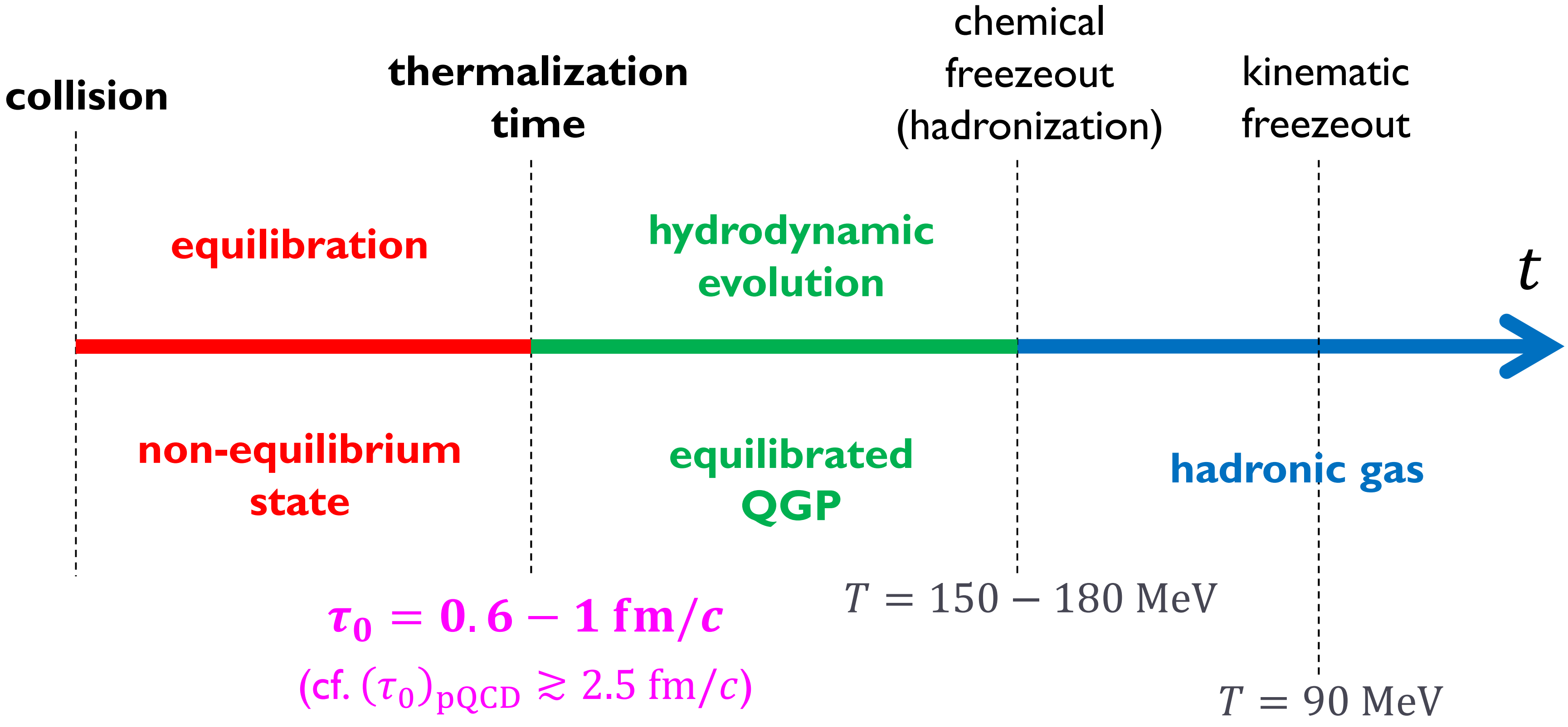
$\gamma = 100$

Energy density on collision $\sim 5\ \text{Gev/fm}^3$
(RHIC, 5x crossover energy density)



8000 hadrons produced
(RHIC)

Stages in HI collision



collision

thermalization time

chemical freezeout (hadronization)

kinematic freezeout

t

equilibration

non-equilibrium state

hydrodynamic evolution

equilibrated QGP

hadronic gas

$\boldsymbol{\tau_0 = 0.6 - 1\ \mathrm{fm}/c}$

(cf. $(\tau_0)_{\mathrm{pQCD}} \gtrsim 2.5\ \mathrm{fm}/c$)

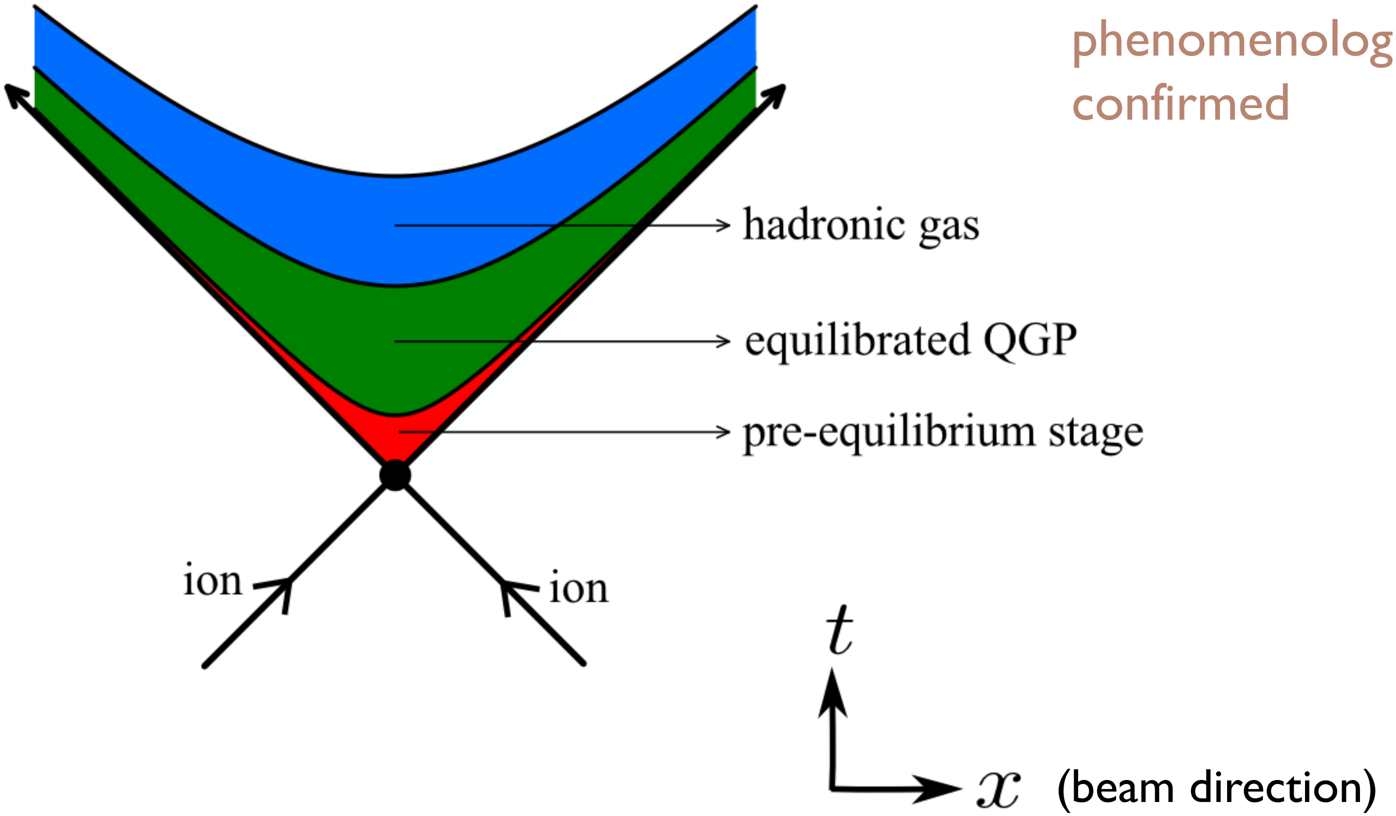
$T = 150 - 180\ \mathrm{MeV}$

$T = 90\ \mathrm{MeV}$

energy density
~10 GeV/fm^3 (RHIC, t=0.4 fm)
~60 GeV/fm^3 (LHC, t=0.25 fm)

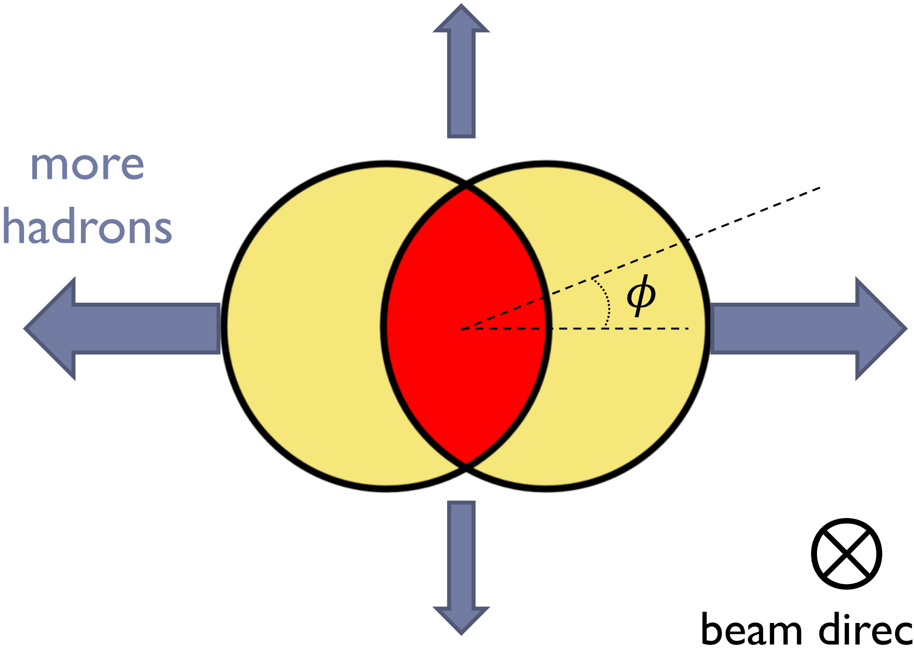
Bjorken flow

- Boost invariance (in central rapidity region)



phenomenologically confirmed

Elliptic flow



more hadrons

less

beam direction

$$\frac{dN}{d^2\mathbf{p}_T\,dy} \propto 1 + 2v_2\cos(2\phi) + \cdots$$

"elliptic flow" v_2: parametrizes asymmetry in hadron multiplicity

v_2 is important for determining hydro properties of QGP

Phenomenology of HI collision (1)

- Ideal hydro after $\tau_0 = 0.6 - 1\ \mathrm{fm}$ explains v_2 ($\lesssim 0.2$)
  - Very fast thermalization
  - Larger τ_0 spoils agreement
  - Perturbative QCD fails: $(\tau_0)_{\mathrm{pQCD}} \gtrsim 2.5\ \mathrm{fm}/c$

Phenomenology of HI collision (2)

- QGP has very small shear viscosity - entropy density ratio:

$$\frac{\eta}{s} = \frac{1-2.5}{4\pi}$$

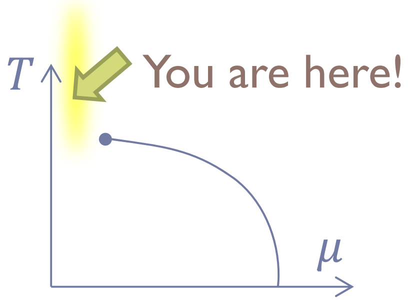
  - The smallest observed in nature (water ~380, helium ~9) (ultracold atoms at the unitarity point has similar η/s)
  - Perturbative QCD fails:

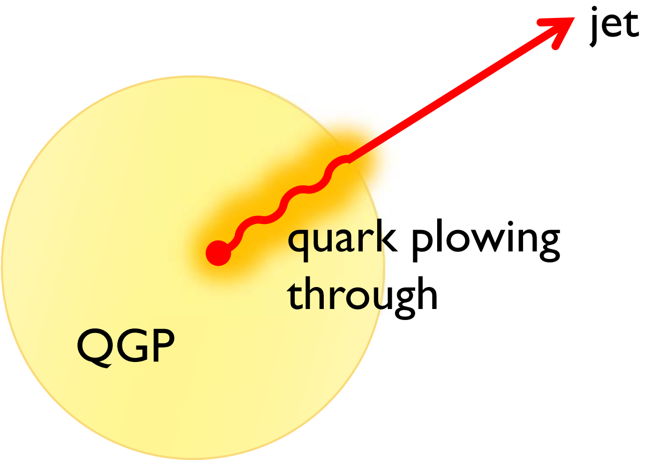
$$\frac{\eta}{s} \sim \frac{1}{\alpha_s^2 \log \alpha_s}$$

  - Short mean-free-path → no well-defined quasiparticle
  - Strongly coupled; perturbative approach invalid
  - LHC: comparably strongly coupled; η/s comparably small

Phenomenology of HI collision (3)

- All hadron species emerge from a single common fluid
- Baryon chemical potential is small: $\mu \ll T$
- Jet quenching:

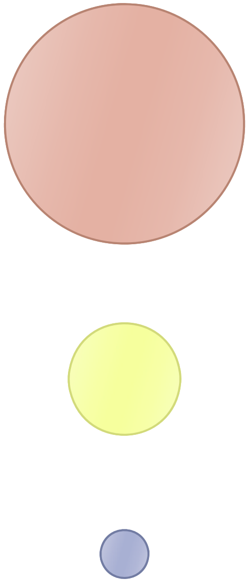




  - Medium modifies jets
  - Jets modifies medium

Phenomenology of HI collision (4)

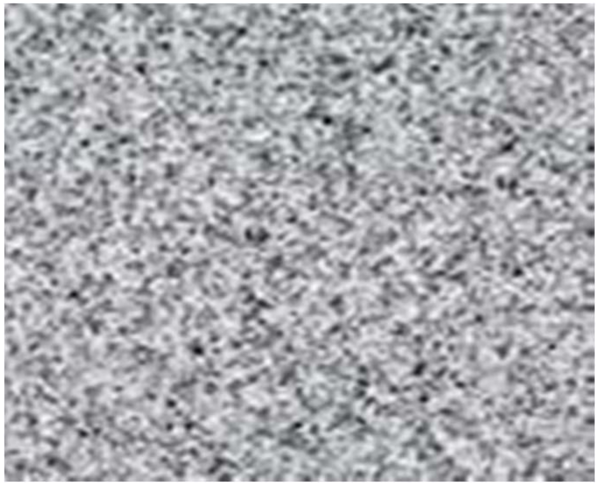
- Quarkonia: mesons made of $q\bar{q}$



proton = uud

$J/\psi = c\bar{c}$

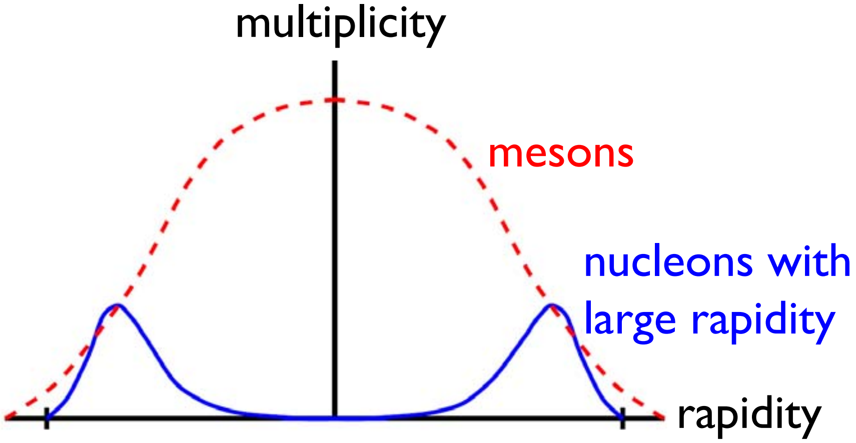
$\Upsilon = b\bar{b}$



$r > T^{-1}$ → production suppressed

A field theory model of HI collision

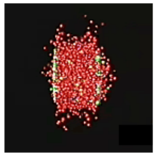
- Color glass condensate



Most particles originate from
the "wake" of the nuclei


1. Ions about to collide*


2. Ion collision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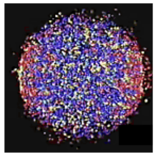
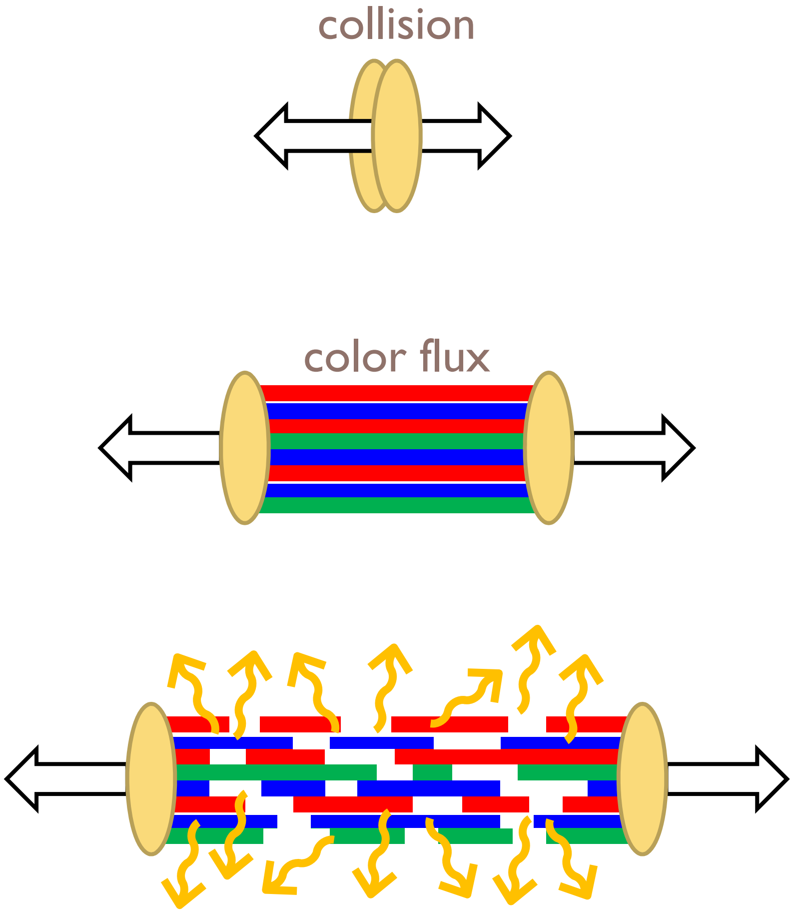

4. Plasma created



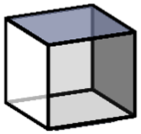
1.2 Lattice approach

Lattice QCD (1)

- Good at thermodynamics
  - Deconfinement is crossover

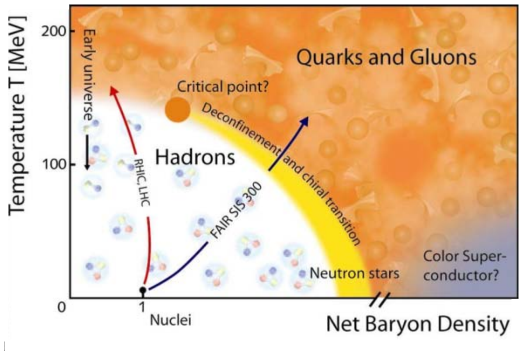
    $T_c = 150 - 170\ \mathrm{MeV}$
  - More conformal for larger T (but non-conformal at $T \sim T_c$)

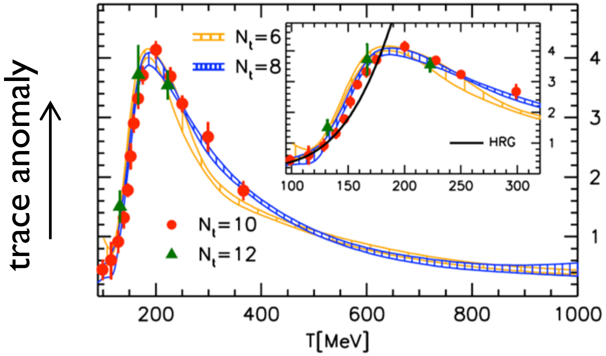
    CFT is not a crazy model
  - Still, deconfined (Polyakov loop ≠ 0)
    - Made of quarks & gluons
    - But can't be treated perturbatively



$\lesssim (4\ \mathrm{fm})^3$

$\sim$ 50 sites in one dir





Lattice QCD (2)

- At pioneering stage about transport properties
  - Real-time correlators difficult
  - Shear viscosity

$$\frac{\eta}{s} = \frac{1.2 - 1.7}{4\pi} \quad (T = 1.2 - 1.7T_c)$$

- Not (yet) applicable to non-equilibrium properties

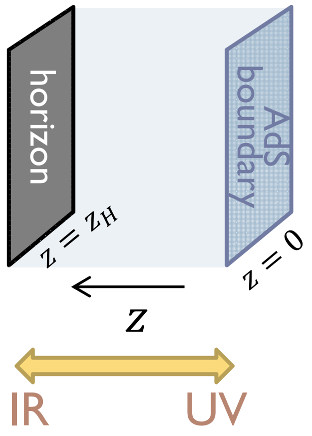
1.3 Holographic approach

Holographic approach

- Want to study strongly coupled phenomena in QCD
- Toy model: $\mathcal{N} = 4\ SU(N_c)$ SYM

$\mathcal{N} = 4$ SYM	holo. dual ⟺ AdS/CFT	String theory in AdS$_5$
Strong coupling	⟷	Weak coupling
Vacuum	⟷	Empty AdS$_5$
Thermal state (equilibrated plasma)	⟷	AdS BH
Thermalization	⟷	BH formation

$$\left(\frac{R_{AdS}}{l_s}\right)^4 = g_{\mathrm{YM}}^2 N_c \equiv \lambda$$



Difference between QCD and $\mathcal{N} = 4$

- Is $\mathcal{N} = 4$ SYM really "similar" to QCD???
  - QCD confines at low T, but $\mathcal{N} = 4$ doesn't confine

    ⇨ At $T > T_c$ QCD deconfines and is similar to $\mathcal{N} = 4$
  - $\mathcal{N} = 4$ is CFT but QCD isn't

    ⇨ QCD is near conformal at high T ($T \sim 5T_c$ achieved initially)
  - $\mathcal{N} = 4$ is supersymmetric but QCD isn't ⇨ at T>0, susy broken
  - QCD is asymptotically free

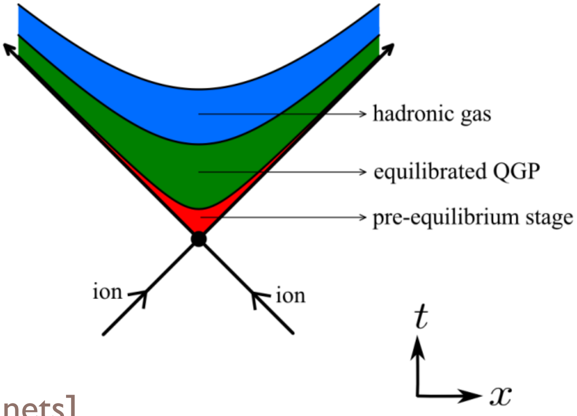
    ⇨ Experiments suggest it's strongly coupled at high T

…

Holo approach: equilibrium

- Shear viscosity

$$\frac{\eta}{s} = \frac{1-2.5}{4\pi}\ (\text{expr.}), \qquad \frac{1.2-1.7}{4\pi}\ (\text{lattice})$$



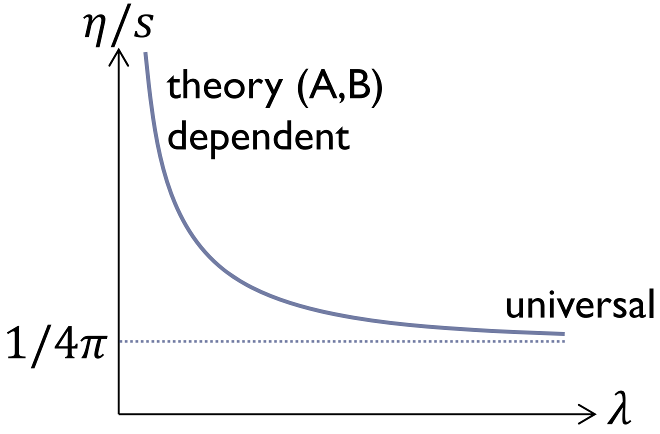
Holography:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1+\frac{15\zeta(3)}{\lambda^{3/2}}+\cdots\right)$$

[Policastro+Son+Starinets]…

Cf. weak coupling:

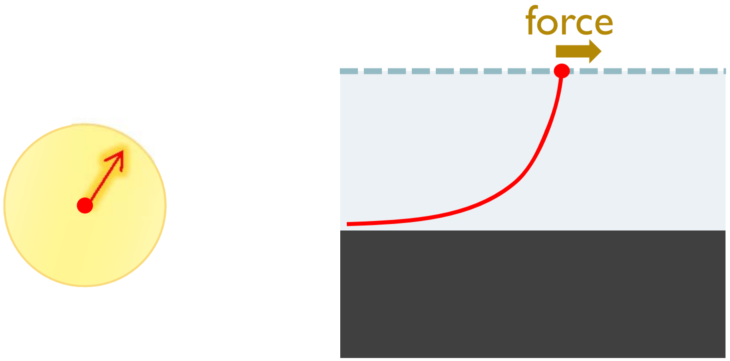
$$\frac{\eta}{s} = \frac{A}{\lambda^2 \log(B/\sqrt{\lambda})}$$



- No quasiparticles
  - No propagating light DoFs
    Only quasi-normal modes (same as QGP)

Holo approach: near-equilibrium

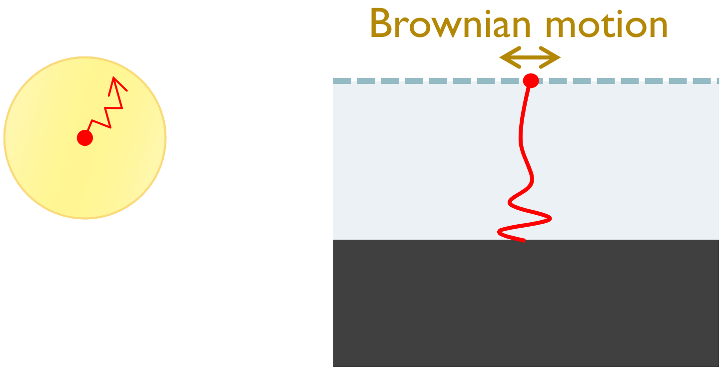
- Parton energy loss [Herzog et al.][Gubser]



  - Quark = string ending on boundary
  - Extract drag coeff / diffusion const.

- Brownian motion / transverse momentum broadening

[de Boer+Hubeny+Rangamani+MS]
[Son+Teaney]…



  - Can derive Langevin eq.

2. Probing thermalization by holography

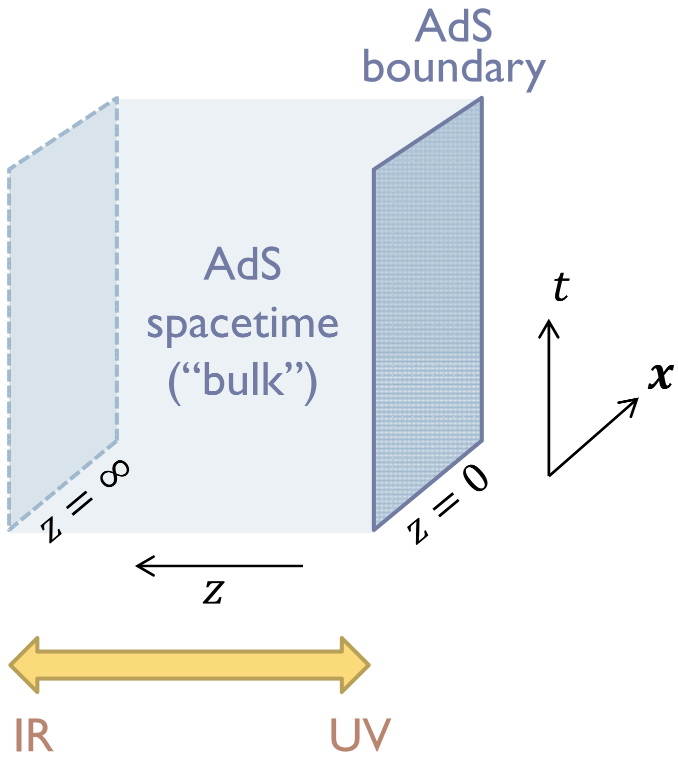
Non-equilibrium phenomena

- (Near-)equilibrium physics (dual: BH)
  - Well-studied as we saw
- Thermalization process (dual: BH formation)
  - Poorly understood
  - Occurs fast: $\tau_0 \lesssim 1\ \mathrm{fm}$ (cf. $(\tau_0)_{\mathrm{pQCD}} \gtrsim 2.5\mathrm{fm}$)
  - pQCD not applicable
  - Lattice QCD insufficient
- Non-equil. physics of strongly coupled systems: *terra incognita*

Basics of holography

- AdS spacetime

$$ds^2 = R_{AdS}^2 \frac{dz^2 - dt^2 + d\boldsymbol{x}^2}{z^2}$$



Holographic probes – local op

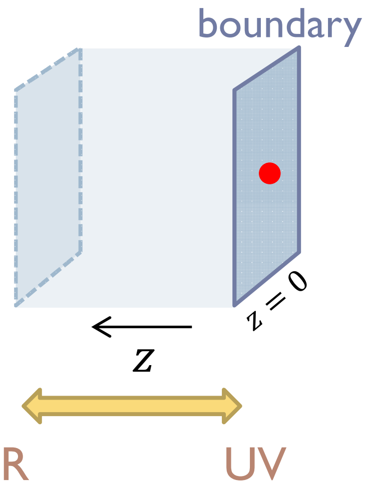
- 1-point function

boundary operator $\quad$ bulk field

$$\mathcal{O} \Longleftrightarrow \phi$$

$$\phi(z,x) = \underbrace{\alpha(x)}_{\text{perturbation}}\, z^{4-\Delta} + \cdots + \underbrace{\beta(x)}_{\text{vev}}\, z^{\Delta} + \cdots$$

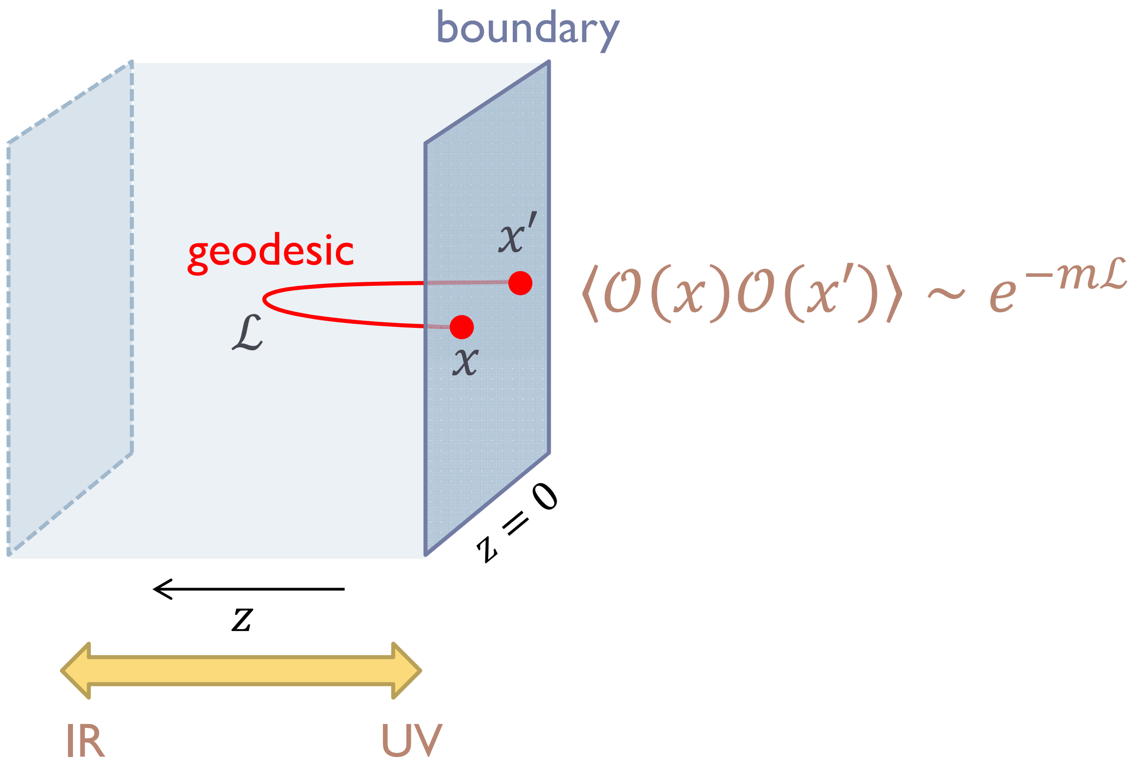
$$\Delta S = \int d^4x\, \alpha(x)\mathcal{O}(x) \qquad \langle \mathcal{O}(x) \rangle = \beta(x)$$



1-point function knows only about UV

Holographic probes – non-local op

- 2-point function



Non-local ops. know also about IR

2.1 Models & probes of thermalization

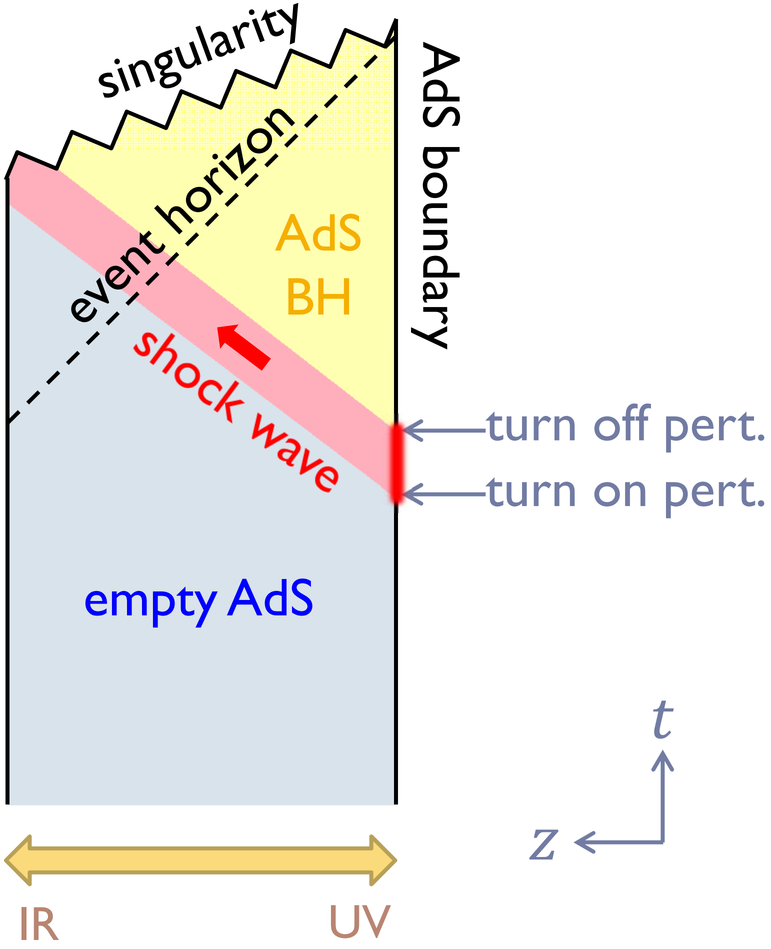
Models for holographic thermalization

Thermalization ↔ BH formation

Initial cond?

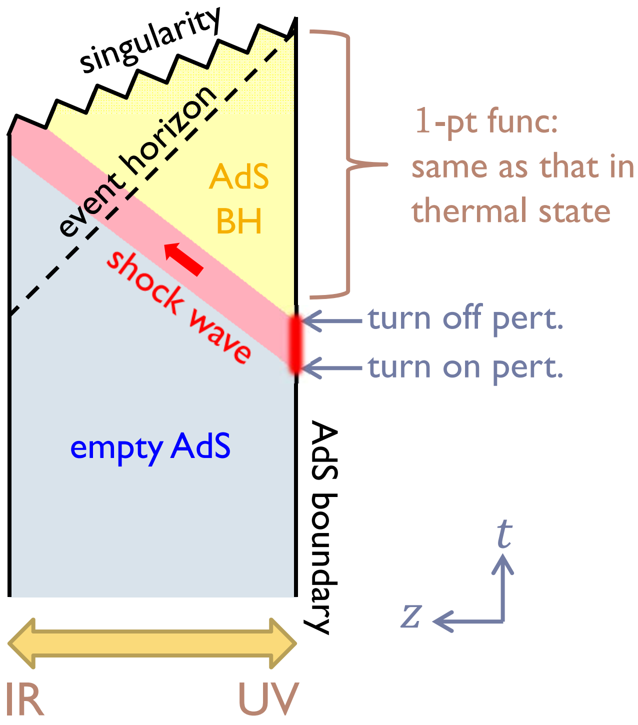
- Not clear how to implement initial cond in bulk
- Various scenarios
  - Collision of shock waves [Chesler+Yaffe]…
  - Falling string [Lin+Shuryak]…
  - Boundary perturbation [Bhattacharyya+Minwalla]…

The bulk spacetime



Probes of thermalization (1)

- Instantaneous thermalization? [Bhattacharyya+Minwalla]
  - Inside = empty AdS
    Outside = AdS BH
  - 1-point function instantaneously thermalizes

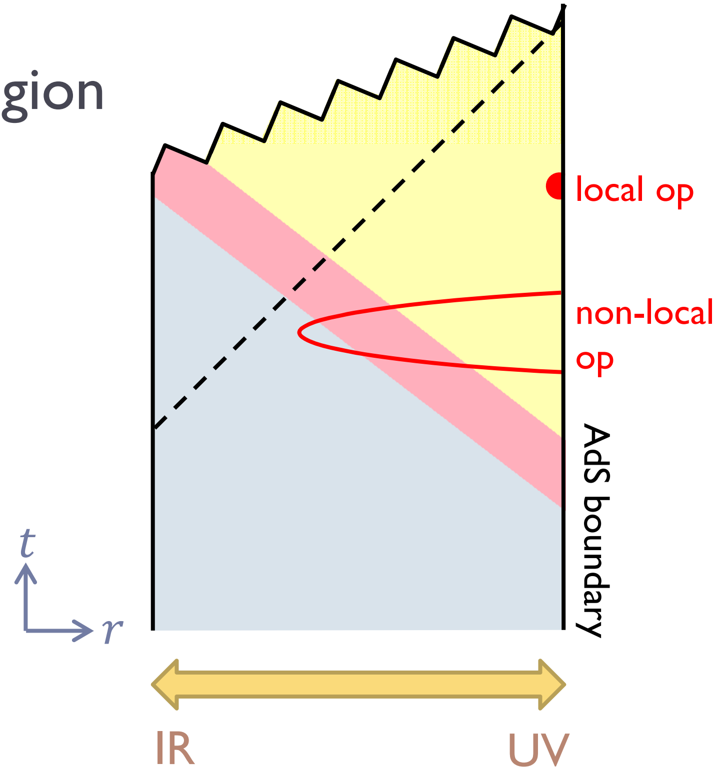


Probes of thermalization (2)

- Local operators are not sufficient
  - They know only about near-bndy region

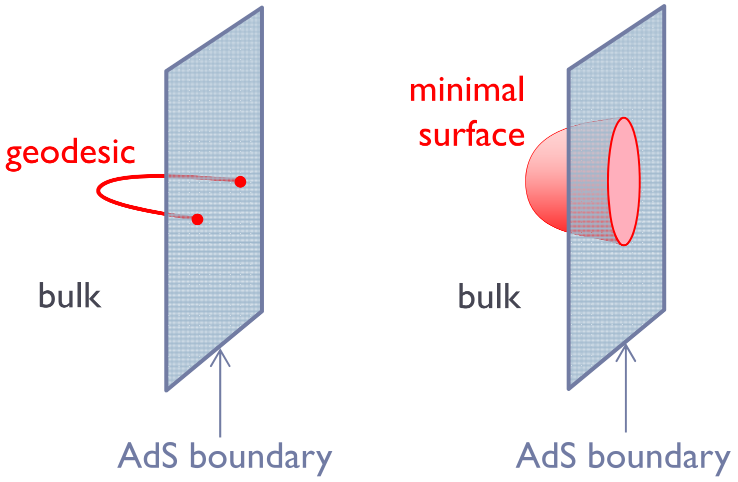
    ⇨ They probe UV only
- Non-local operators do better job
  - They reach deeper into bulk

    ⇨ They probe IR also
  - They know more detail
    about thermalization process



Non-local operators

- 2-point function
  - $\langle \mathcal{O}(x)\mathcal{O}(x)\rangle$
  - Bulk: geodesic (1D)
- Wilson line
  - $W = P\{\exp[\int_C A_\mu(x)dx^\mu]\}$
  - Bulk: minimal surface (2D)
- Entanglement entropy
  - $S_A = -\mathrm{Tr}_A[\rho_A \log \rho_A], \quad \rho_A = \mathrm{Tr}_B[\rho_{\mathrm{tot}}]$
  - Bulk: codim-2 hypersurface



⇨ Let's study these during BH formation process!

2.2 The model and results

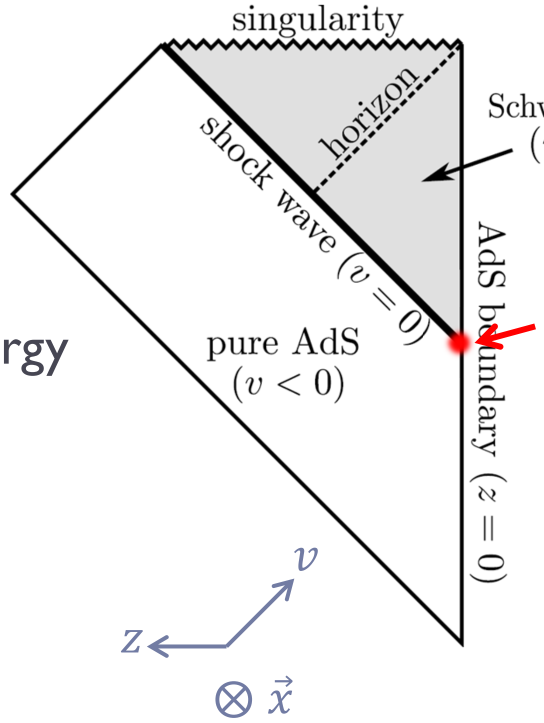
The model: Vaidya AdS

- Null infalling shock wave in AdS (Vaidya AdS spacetime)

$$ds^2 = \frac{1}{z^2}\left[-\left(1 - m(v)z^d\right)dv^2 - 2dz\,dv + d\vec{x}^2\right]$$

$z = 0 \quad \longleftrightarrow \quad$ UV (AdS boundary)

$z = \infty \quad \longleftrightarrow \quad$ IR



sudden, spatially homogen. injection of energy

- Simple setup amenable to detailed study
- Sudden & spatially homog. injection of energy
- BH forms at late time
- Thin shell limit

$$m(v) = M\theta(v)$$

- We study AdS$_D$ with D=3,4,5
  → field theory in d=2,3,4

Nonlocal probes we consider

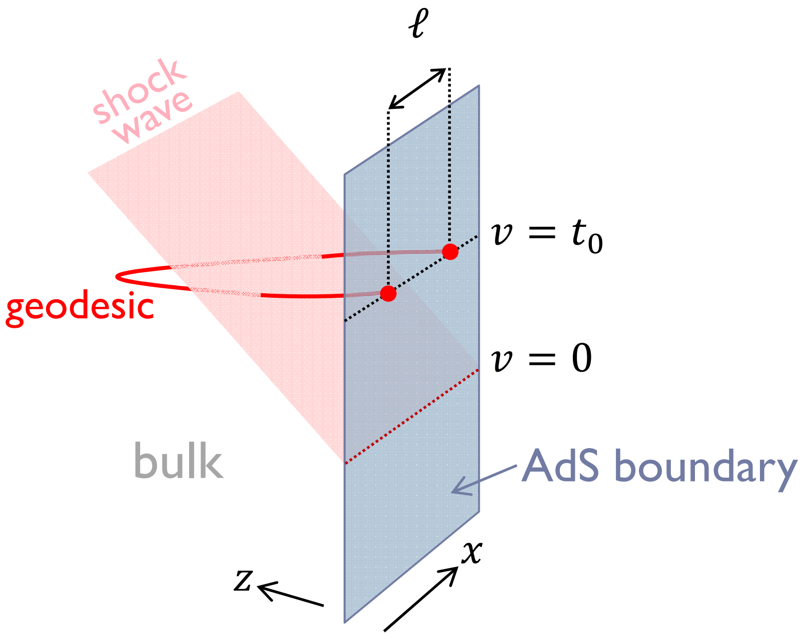
Bulk spacetime	Dim of bulk probe	Operator
AdS3	1	Geodesic = EE
AdS4	1	Geodesic
	2	Wilson line = EE
AdS5	1	Geodesic
	2	Wilson line
	3	EE

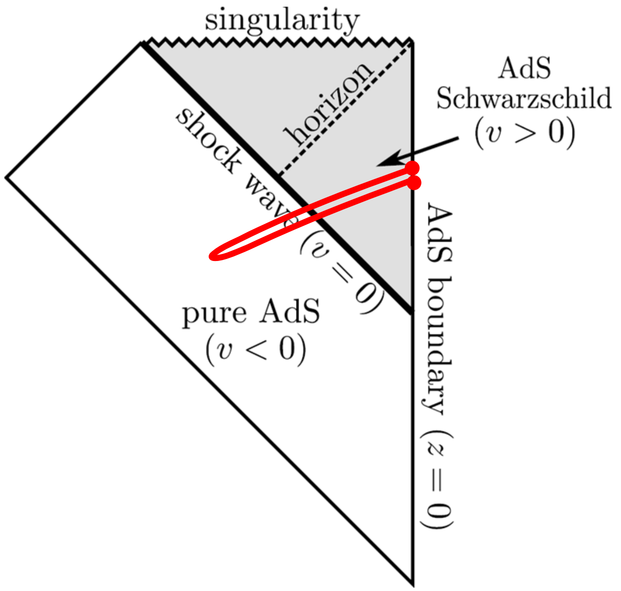
What we computed

- AdS3
  - Can be solved analytically in thin shell limit
    Cf. Numerically done in [Abajo-Arastia+Aparicio+Lopez 1006.4090]
- AdS4
  - Numerical
    Cf. Partly done in [Albash+Johnson 1008.3027]
- AdS5
  - Numerical

Geodesics in AdS_3 (1)

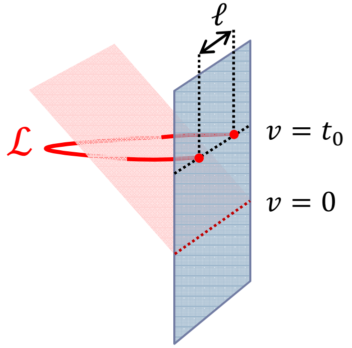
- Equal-time geodesics





  - Geodesic connecting two boundary points at distance ℓ, at time t_0 after energy was deposited into system
  - Refraction cond across shock wave

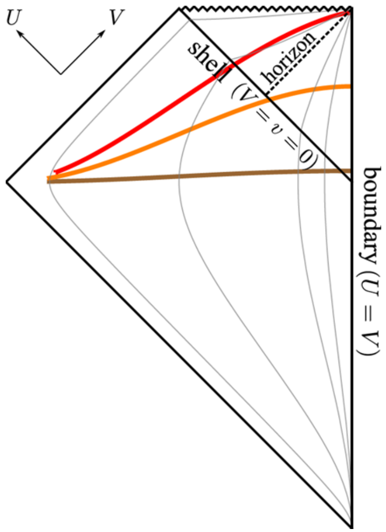
Geodesics in AdS$_3$ (2)



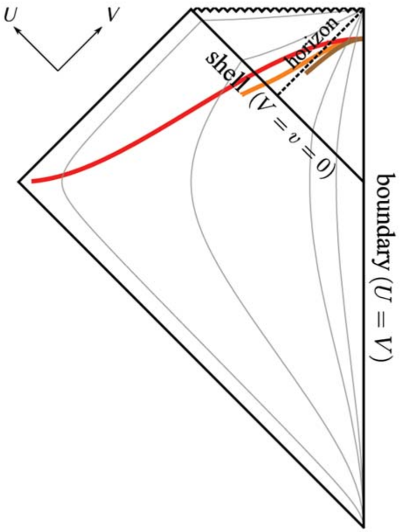
- Analytic expression

$$\mathcal{L} - \mathcal{L}_{\text{therm}} = 2\log\left[\frac{\sinh(r_H t_0)}{r_H\sqrt{1-c^2}}\right], \qquad r_H \equiv \sqrt{M},$$

$$\ell = \frac{1}{r_H}\left[\frac{2c}{s\rho} + \ln\left(\frac{2(1+c)\rho^2 + 2s\rho - c}{2(1+c)\rho^2 - 2s\rho - c}\right)\right], \qquad \rho = \frac{1}{2}\coth(r_H t_0) + \frac{1}{2}\sqrt{\coth^2(r_H t_0) - \frac{2c}{c+1}}$$

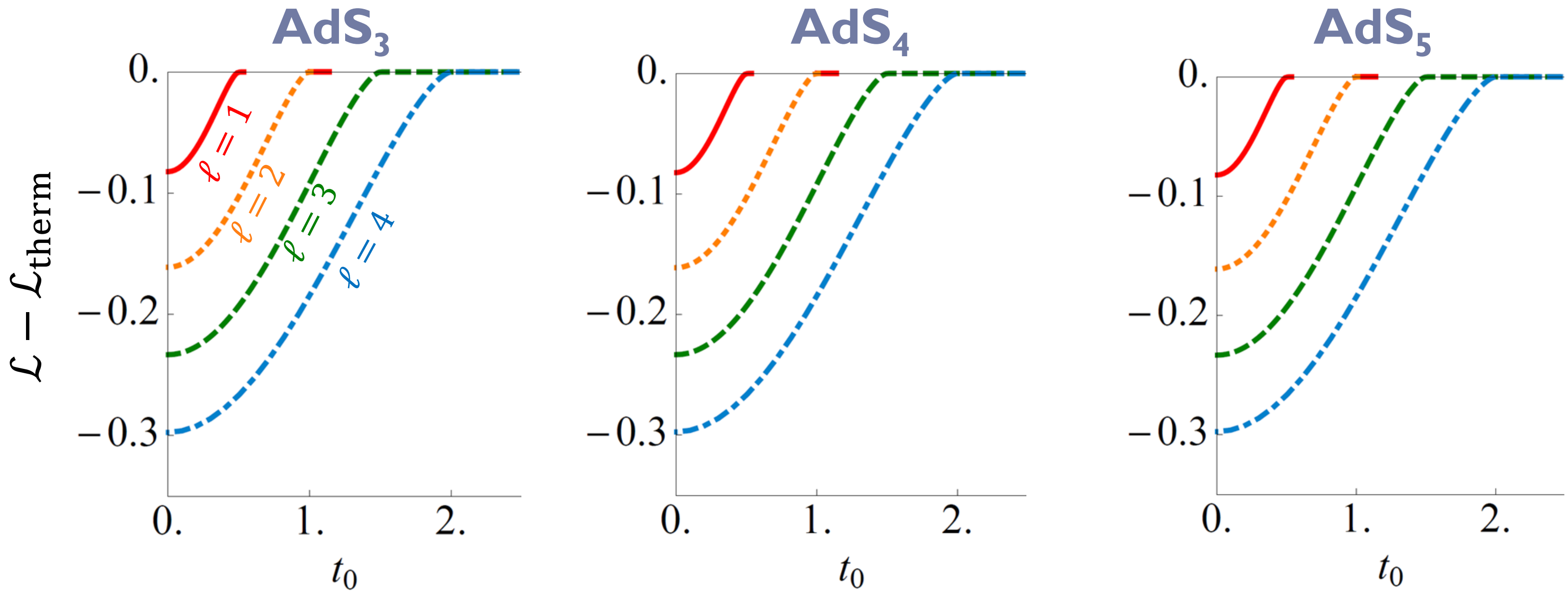


Equal-time geodesics for fixed $\ell = 21.3$ and $t_0 = 0.1, 1.0, 4.0$

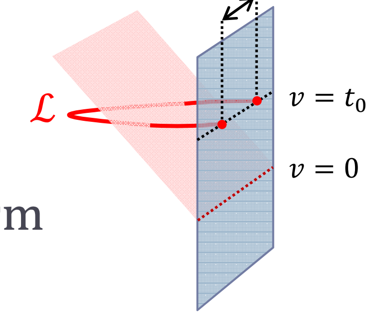


Equal-time geodesics for fixed $t_0 = 2$ and $\ell = 3.0, 4.6, 68.2$

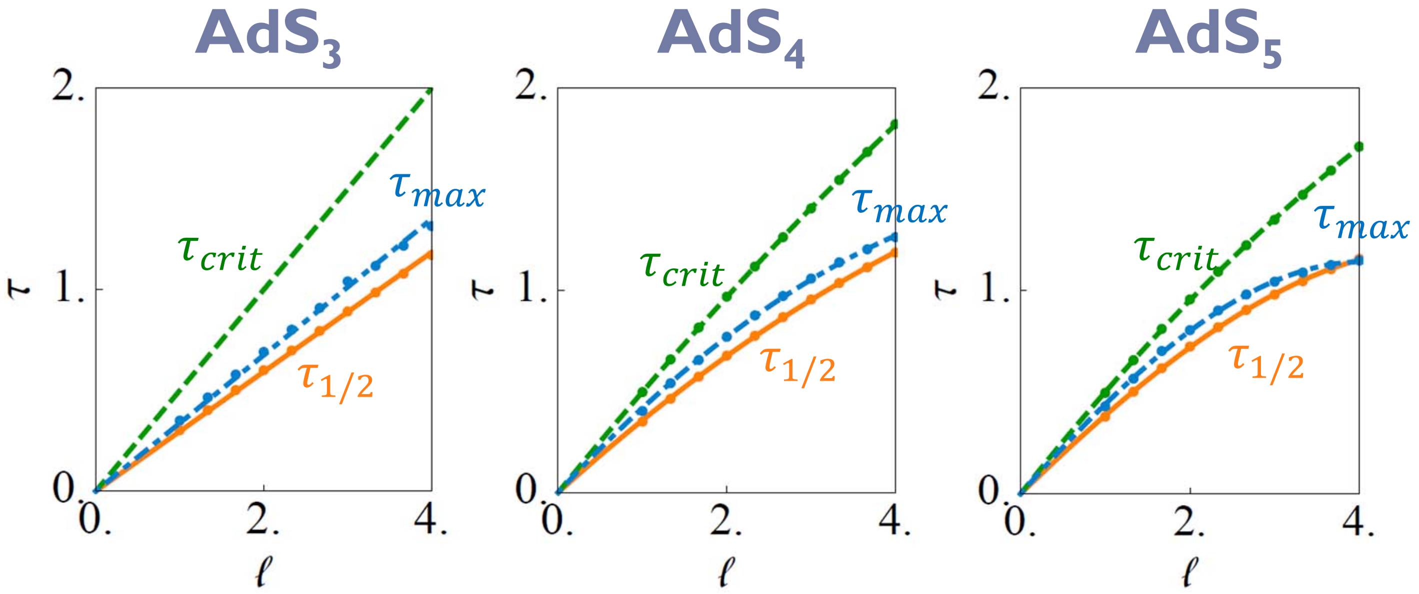
Result: geodesic length



- Given fixed ℓ, compute $\mathcal{L}$ as function of t_0
- Renormalize $\mathcal{L}$ by subtracting thermal value $\mathcal{L}_\text{therm}$
- If we wait long enough, $\mathcal{L} \to \mathcal{L}_\text{therm}$ (thermalize)
- Larger ℓ ⇨ It takes longer to thermalize — "Top-down"



"Thermalization time" from geodesics



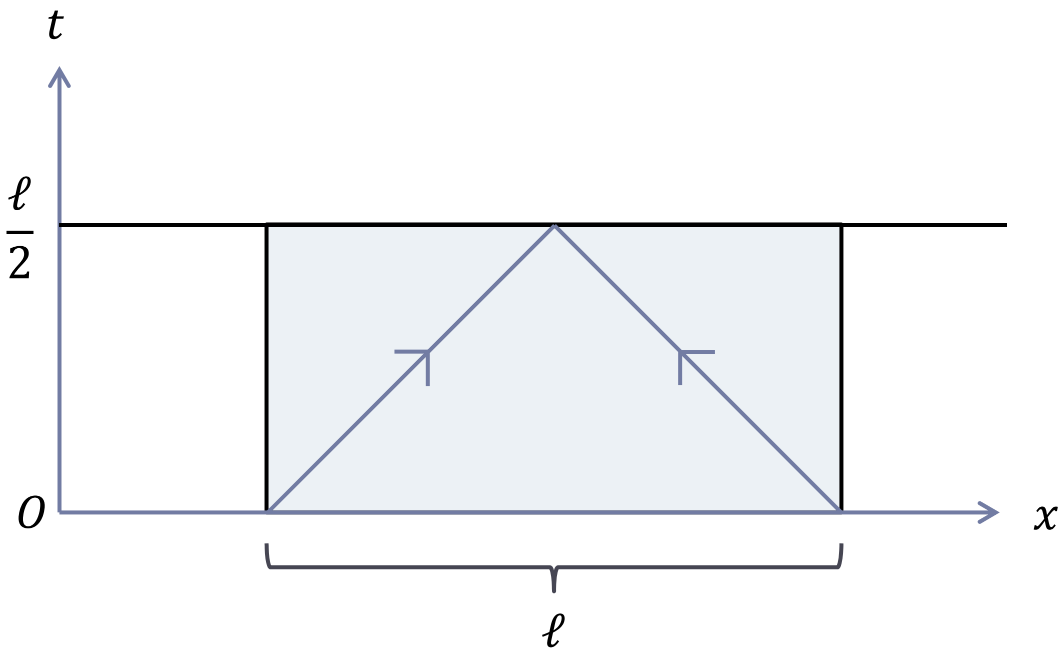
τ_{crit} : $\mathcal{L}$ becomes equal to the thermal value

τ_{max}: $\mathcal{L}(t_0)$ has steepest slope $\tau_{1/2}$: $\mathcal{L}$ becomes half the thermal value

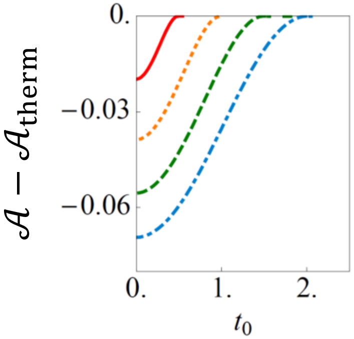
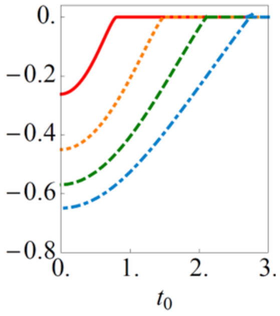
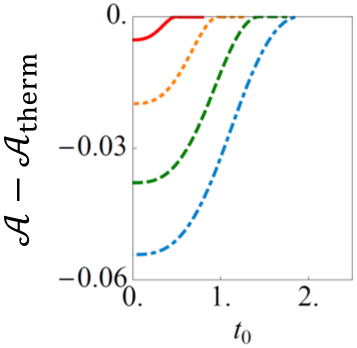
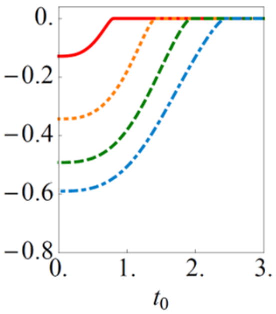
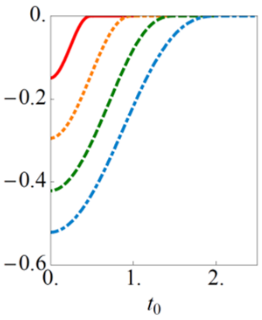
- $\tau_{\text{crit}}^{\text{AdS3}} = \ell/2$ is as expected from causality bound
- Others would indicate $\tau < \ell/2$: superluminous propagation??

⟹ Should look at observable that gives largest τ

"Causality bound"



Other non-local observables

<table>
<tr><th></th><th>Circular Wilson loop</th><th>Rectangular Wilson loop</th><th>Entanglement Entropy</th></tr>
<tr><td>AdS$_4$</td><td>
</td><td>
</td><td>N/A</td></tr>
<tr><td>AdS$_5$</td><td>
</td><td>
</td><td>
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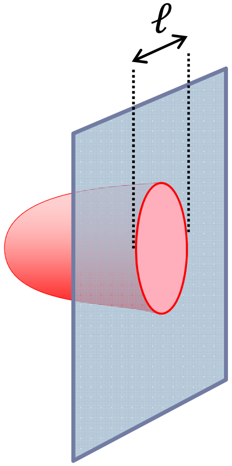
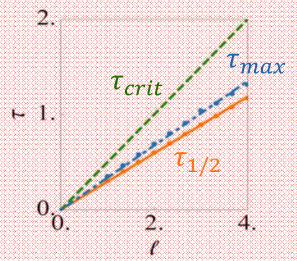
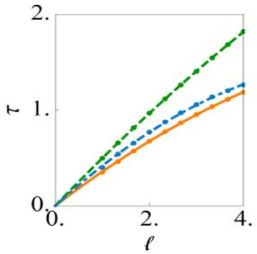
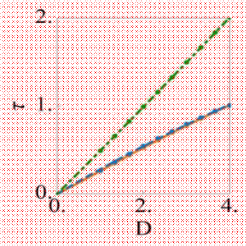
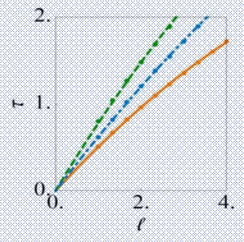
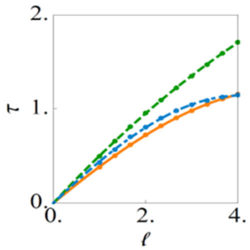
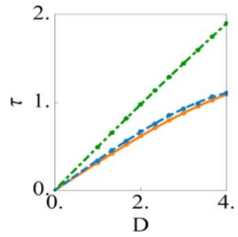
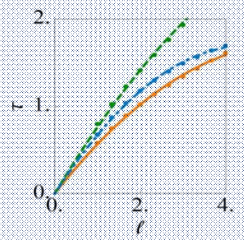
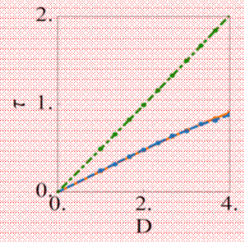


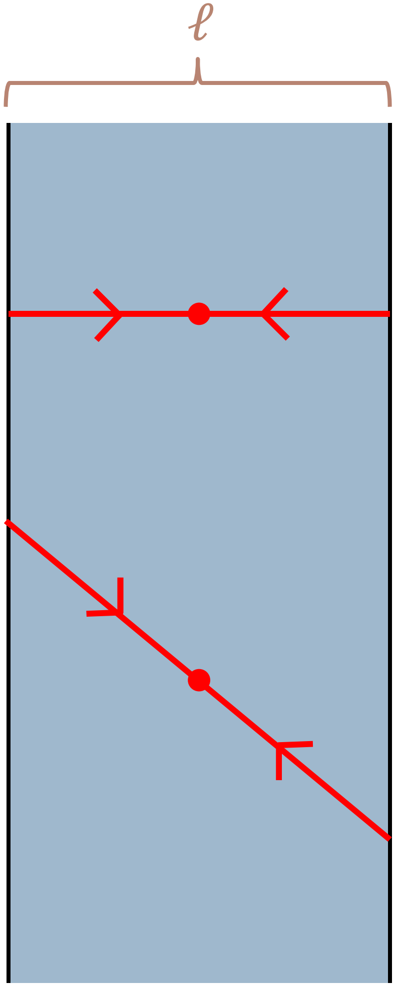
- Similar behavior for all probes – “Top-down” thermalization

Thermalization time

	geodesic	**circular Wilson loop**	**Rectangular Wilson loop**	**EE for a sphere**
AdS$_3$		N/A	N/A	N/A
AdS$_4$				N/A
AdS$_5$				

- EE for disk/sphere saturates the causality bound $\tau \geq \ell/2$
- Infinite rectangle doesn’t have one scales – reason for larger τ ?

Infinite rectangle



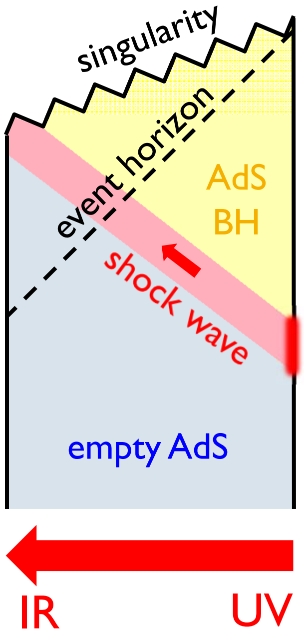
2.3 Summary

Summary

- Studied various nonlocal probes during thermalization after sudden injection of energy

- Different probes show different thermalization time
  - Largest τ for codim-2 probes
  - Equilibration propagates at speed of light

- "Top-down" thermalization
  - Thermalization proceeds from UV to IR
  - "Built-in" in the bulk,
    but not in weakly-coupled field theory



An “estimate” of therm’n time in HI collision

$$\tau_{\mathrm{crit}} = \ell/2$$

$$\downarrow \quad \ell \sim T = 300 - 400\ \mathrm{MeV}$$

$$\tau_{\mathrm{crit}} \sim 0.3\ \mathrm{fm}/c$$

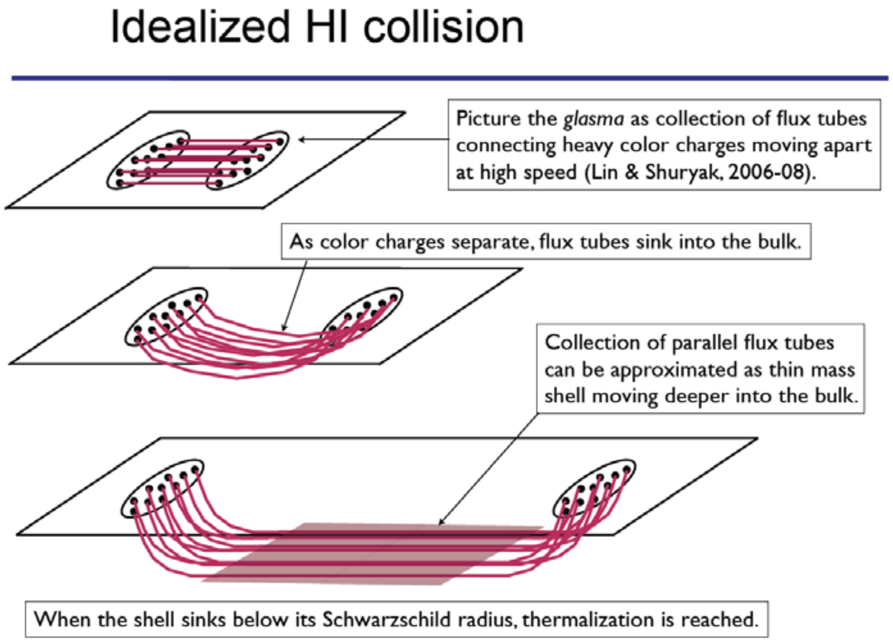
Reasonably short!

Cf. $\tau_{\mathrm{expr}} = 0.6 - 1\ \mathrm{fm}/c$

$\tau_{\mathrm{pQCD}} \gtrsim 2.5\ \mathrm{fm}/c$

Future directions

- More realistic backgrounds
- Toward emergent boost invariance
  - Falling string [Lin-Shuryak]
  - Approach to Janik-Peschanski solution



Idealized HI collision

Picture the *glasma* as collection of flux tubes connecting heavy color charges moving apart at high speed (Lin & Shuryak, 2006-08).

As color charges separate, flux tubes sink into the bulk.

Collection of parallel flux tubes can be approximated as thin mass shell moving deeper into the bulk.

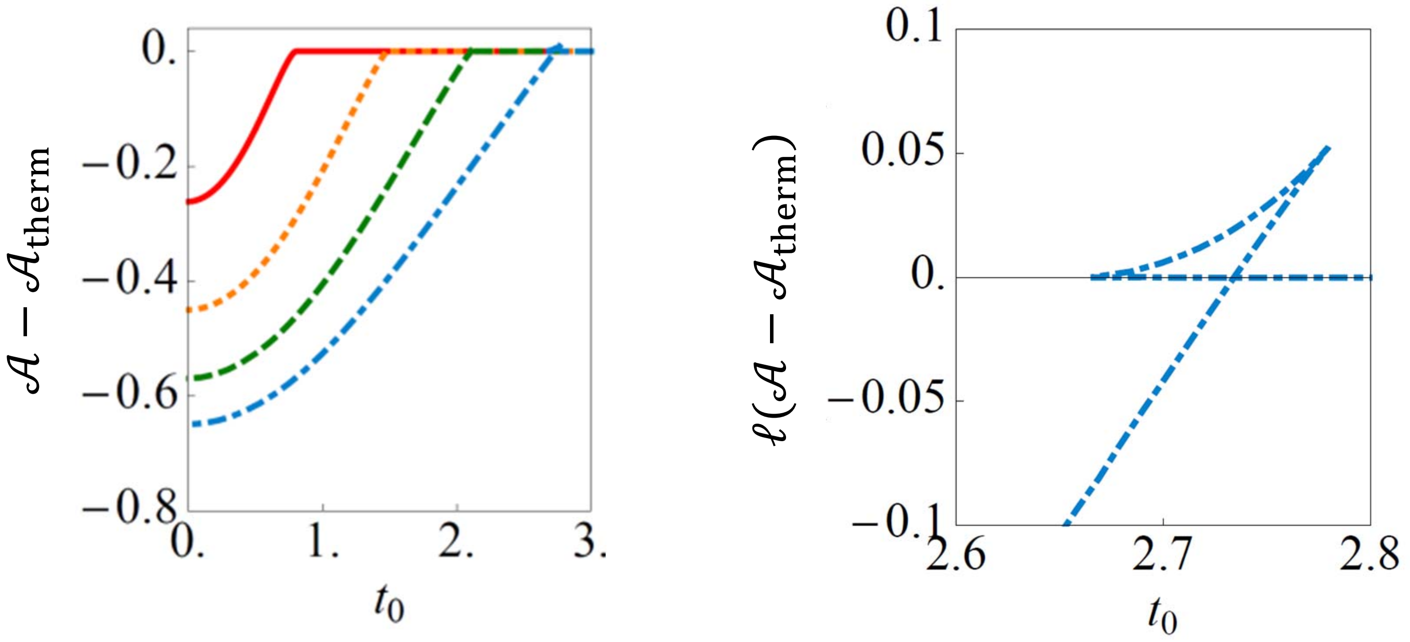
When the shell sinks below its Schwarzschild radius, thermalization is reached.

Thanks!

Extra material

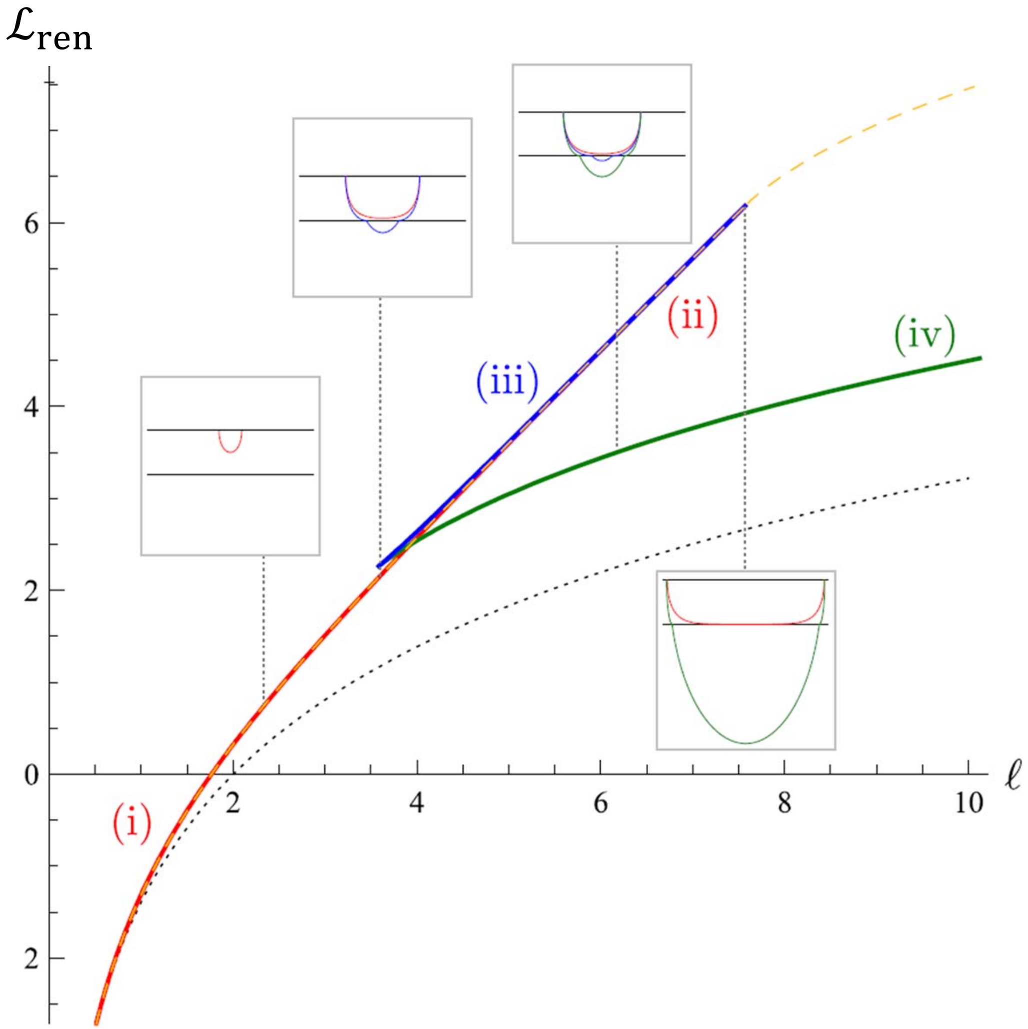
“Swallowtail” phenomenon [Albash+Johnson]

- Rectangular Wilson loop for AdS4

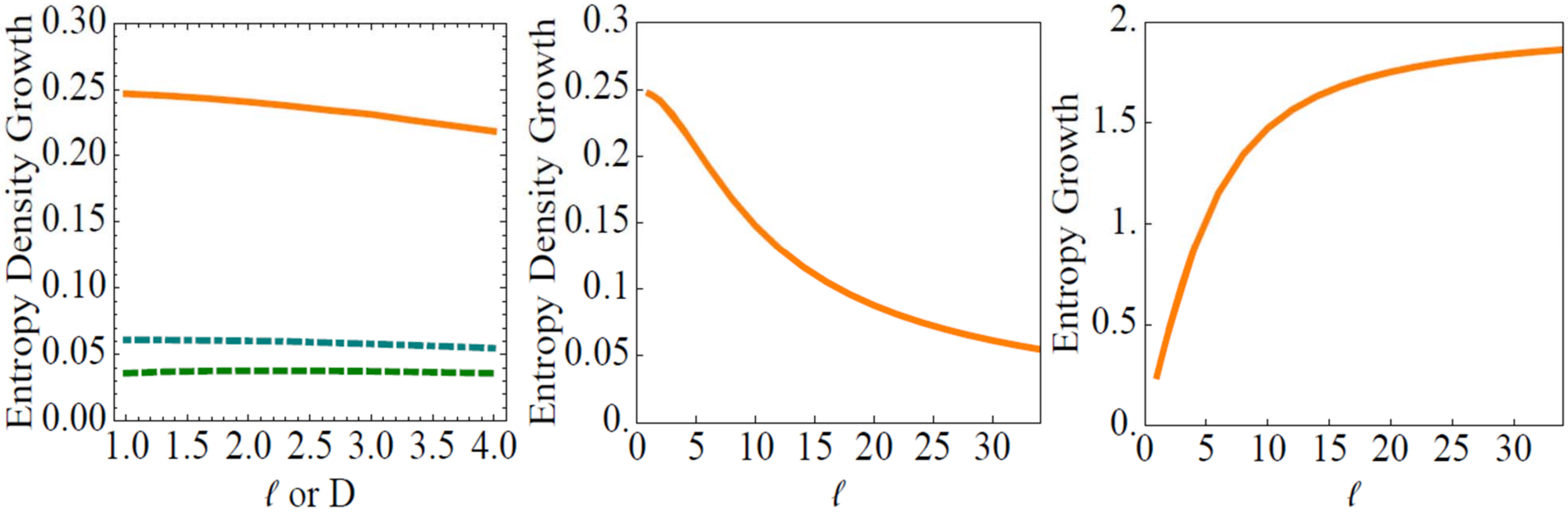


- For given t_0, there are three possible minimal area surfaces

“Swallowtail” in quasi-static shell



EE as "coarse-grained" entropy



(Left) Maximal growth rate of entanglement entropy density vs. diameter of entangled region for d = 2; 3; 4 (top to bottom).
(Middle) Same plot for d = 2, larger range of ℓ.
(Right) Maximal entropy growth rate for d = 2.