Toward Construction of Supergravity Superstrata States

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第6回静岡素粒子集中セミナー

stra·tum [stréitəm | strάː-]
〖ラテン語「広がったもの」の意から〗
—名C(複 -ta [-tə], ～s)
1 【地質】地層; 層.
2 層, 階級.

*

Based on collaboration with:

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Jan de Boer (Amsterdam)
Stefano Giusto (Padova)
Rodolfo Russo (Queen Mary)
Nicholas Warner (USC)

1412.xxxx, 1406.4506, 1307.3115,
1209.6056, 1110.2781, 1107.2650, 1004.2521

Road to Superstratum

Black hole puzzles

- Entropy (microstate) problem

$$S_{\mathrm{BH}} = \frac{A}{4G_N}$$

Schwarzschild: $S_{\mathrm{BH}} = 10^{77}\left(M/M_{\odot}\right)^2$

Cf. No-hair theorem: $e^S = 1$

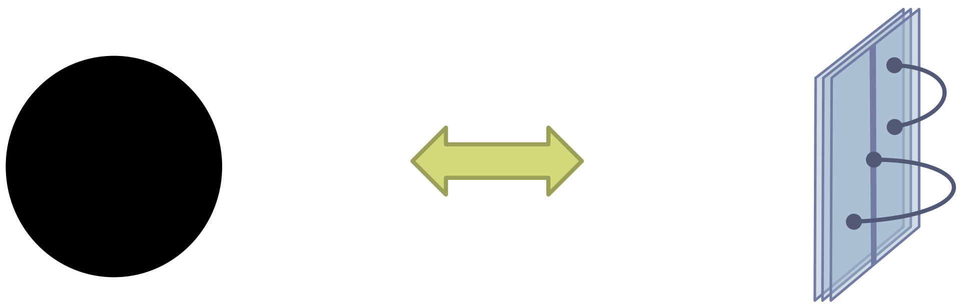
- Information paradox
- Firewall

…

Microstate counting

- Strominger-Vafa 1996:

Gravity (strong coupling) — Field theory (weak coupling)

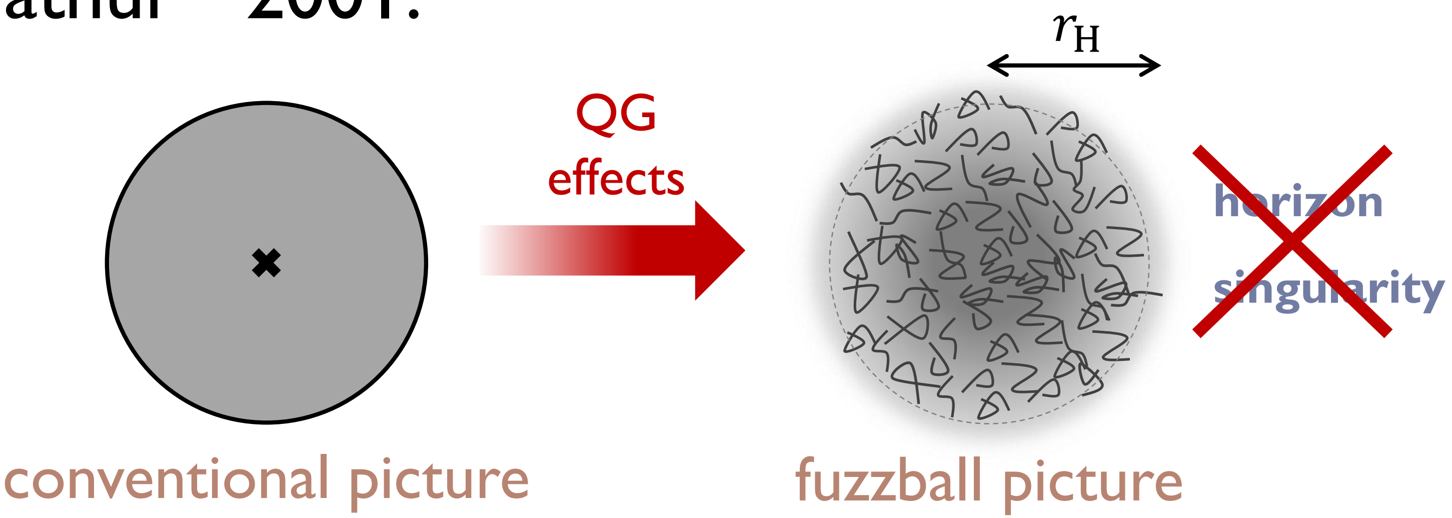


$$S_{\mathrm{BH}} = S_{\mathrm{CFT}} = \log N_{\mathrm{CFT}}$$

→ Cool, but what's gravity picture of the microstates?

Fuzzball proposal

- Mathur ~2001:



  - Microstates = QG/string "fuzz"
  - Not describable within sugra in general
    (some hope for supersymmetric states)

2-charge system (1)

- Fuzzball proposal was made based on this system
- Canonical rep:

IIB on $S^1_5 \times T^4_{6789}$

	1	2	3	4	5 (S^1)	6 (T^4)	7 (T^4)	8 (T^4)	9 (T^4)
N_1 D1	•	•	•	•	O	~	~	~	~
N_2 D5	•	•	•	•	O	O	O	O	O

- Supersymmetric (8 supercharges, ¼ BPS)
- Large microscopic entropy:

$$S_{\rm micro} = 2\sqrt{2}\pi\sqrt{N_1 N_2}$$

- Horizon vanishes classically

2-charge system (2)

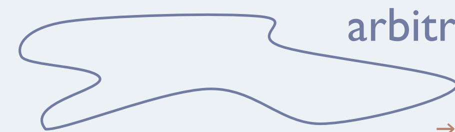
- Sugra microstates known:

  “*Microstate geometries*” [Lunin-Mathur 2001]

  - It is a *supertube*:

$$\mathrm{D1}(5) + \mathrm{D5}(56789) \rightarrow \mathrm{KKM}(\lambda 6789,5)$$

$\bullet \quad + \quad \bullet \quad \rightarrow$



arbitrary curve

$\vec{x} = \vec{f}(\lambda) \in \mathbb{R}^4_{1234}$

  - Reproduces entropy; $S \sim \sqrt{N_1 N_2}$

- Dictionary b/w CFT & sugra microstates known

  RR gnd state $\Longleftrightarrow$ curve $\vec{f}(\lambda)$

2-charge system: summary

Fuzzball works for 2-charge sys,
which however is not a black hole.

↓

Need to go to system with finite horizon
to prove / disprove fuzzball conjecture

3-charge system

- Susy BH in 5D (4 supercharges)
- Canonical rep [Strominger-Vafa 1996]

IIB on $S^1_5 \times T^4_{6789}$

	1	2	3	4	5	6	7	8	9
N_1 D1	•	•	•	•	O	~	~	~	~
N_2 D5	•	•	•	•	O	O	O	O	O
N_3 P	•	•	•	•	O	~	~	~	~

- Decoupling → $AdS_3 \times S^3 \times T^4$ / D1-D5 CFT
- Macroscopic entropy: $S \sim \sqrt{N_1 N_2 N_3}$

4-charge system

- Susy BH in 4D (4 supercharges)
- Canonical rep [Maldacena-Strominger-Witten 1997]

M on T^6_{456789}

	1	2	3	4	5	6	7	8	9	A
N_1 M5	•	•	•	~	~	O	O	O	O	O
N_2 M5	•	•	•	O	O	~	~	O	O	O
N_3 M5	•	•	•	O	O	O	O	~	~	O
N_4 P	•	•	•	~	~	~	~	~	~	O

- Decoupling → $AdS_3 \times S^2 \times T^6$ / MSW CFT
- Macroscopic entropy: $S \sim \sqrt{N_1 N_2 N_3 N_4}$

5D/4D ansatz (1)

Want to find gravity microstates
for 3- & 4-charge systems

(although there is no guarantee that they are describable within sugra…)

Start from 3-charge system

IIB / T^5_{56789} $\mathrm{D1(5), D5(56789), P(5)}$

⇩ T_5, T_6, T_7

IIA / T^5_{56789} $\mathrm{D2(67), D2(89), F1(5)}$

⇩ lift along x^{A}

M / $T^6_{56789\mathrm{A}}$ $\mathrm{M2(67), M2(89), M2(5A)}$

Nicely symmetric

5D/4D ansatz (2)

M / T^6_{56789A} $\quad \mathrm{M2}(67), \mathrm{M2}(89), \mathrm{M2}(5A)$

- Bena-Warner, Gauntlett-Gutowski 2004:

Classified all solutions in 5D preserving same susy

- 4D hyperkahler base $\mathcal{B}_4$
- Funcs & forms defined on $\mathcal{B}_4$

Technically difficult.
Assume $U(1)$ symmetry in B_4

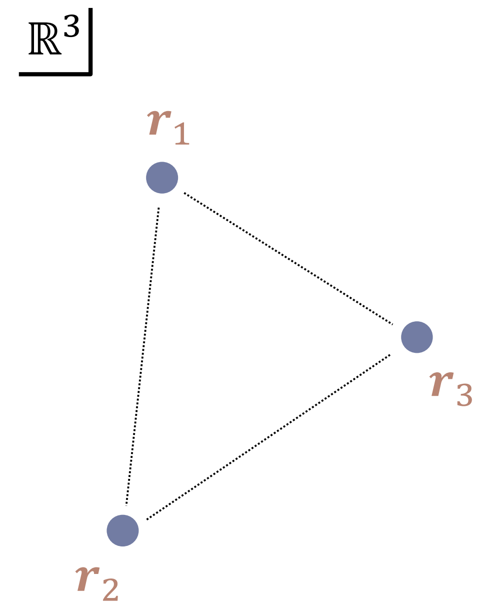
- 3D flat base $\mathbb{R}^3$
- Harmonic funcs on $\mathbb{R}^3$

$$H = (V, K^I, L_I, M), \qquad H = h + \sum_p \frac{Q_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

5D/4D ansatz (3)

- Multi-center config of BHs & BRs in 5D

$$H = h + \sum_p \frac{Q_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$



- Positions $\boldsymbol{r}_p$ satisfy "bubbling eq"
- Large family of solutions
- Reducing on $U(1)$, it becomes 4D BHs (same as Bates-Denef 2003)

$$H = (V, K^I, L_I, M)$$

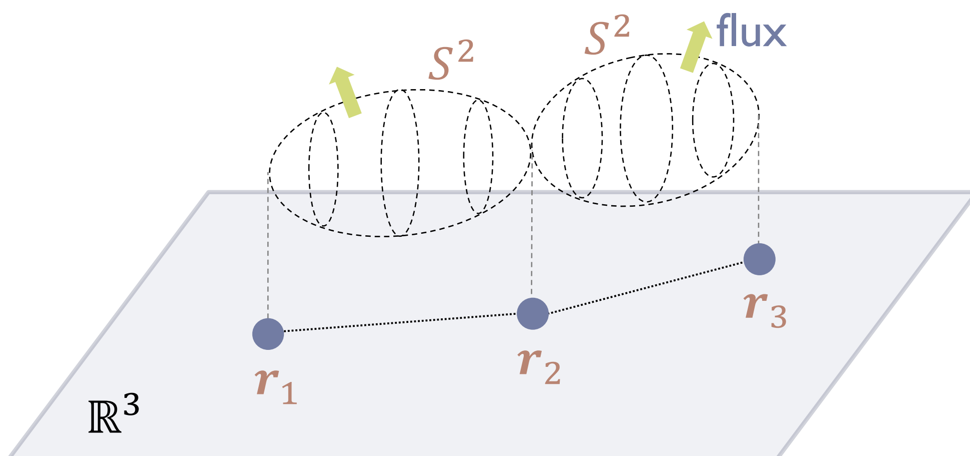
D6 D4 D2 D0

Microstates in 5D/4D ansatz (1)

- Tune charges ⇨ *Regular & horizonless* solutions!

[Bena-Warner 2006] [Berglund-Gimon-Levi 2006]

D6 = KKM with fluxes

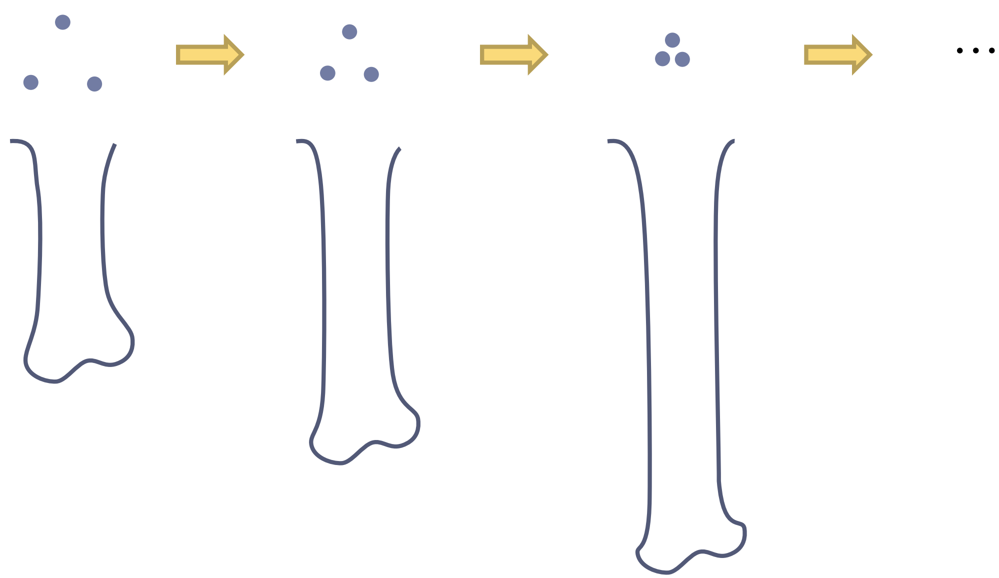


Mechanism to support horizon-sized structure!
Cf. Firewall

⇨ Microstate geometries of 3- and 4-charge black holes ☺

Microstates in 5D/4D ansatz (2)

- Various nice properties ☺
  - Scaling solutions [BW et al., 2006, 2007]



  - Gap expected from CFT: $\Delta E \sim 1/N_1 N_2$

Microstates in 5D/4D ansatz (3)

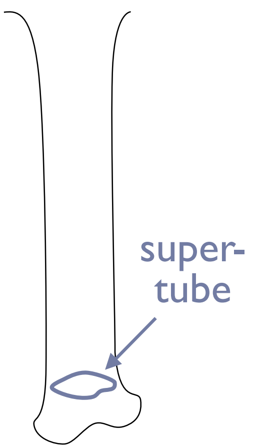
The real question: are there enough?

- 4-chage sys [de Boer et al., 2008-09]
  - Quantization of D6-$\overline{\text{D6}}$-D0 config → *much less entropy* ☹
- 3-chage sys (+ fluctuating supertube)
  - Entropy enhancement mechanism [BW et al., 2008]

    → Much more entropy?
  - An estimate [BW et al., 2010]

$$S \sim N^{\frac{5}{4}} \ll N^{\frac{3}{2}}$$

Parametrically smaller ☹



Summary

4D/5D ansatz solutions *are*
black hole microstates,
but they are *too few*.

Possibilities:

A) Sugra is not enough

B) Need more general ansatz

Superstratum

Exotic branes & double bubbling

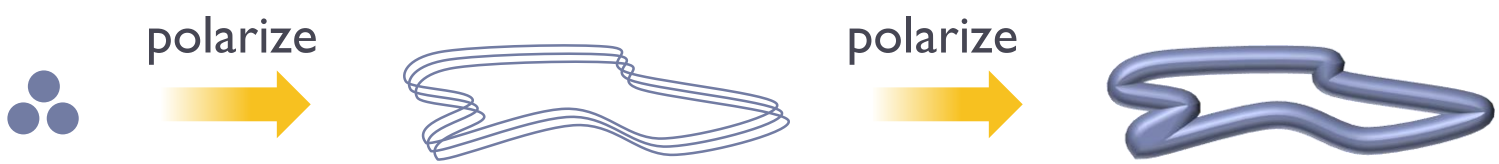
[de Boer+MS 2010]:

1. Ordinary branes can polarize into *non-geometric* (exotic) branes

   e.g., MSW: $\mathrm{M5}(6789A) + \mathrm{M5}(4589A) \rightarrow 5^3(\lambda 4567{,}89A)$

   → Microstates must involve *non-geometric* configs

2. Double bubbling



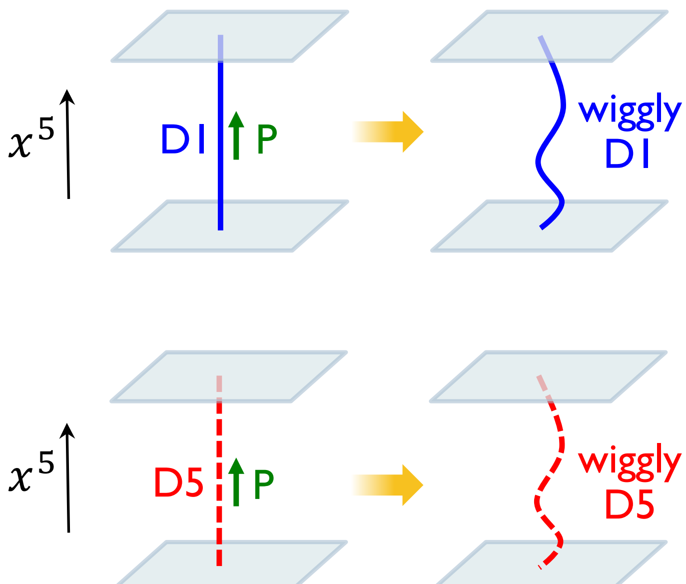
supertube
arbitrary curve $\vec{f}(\lambda)$
large entropy

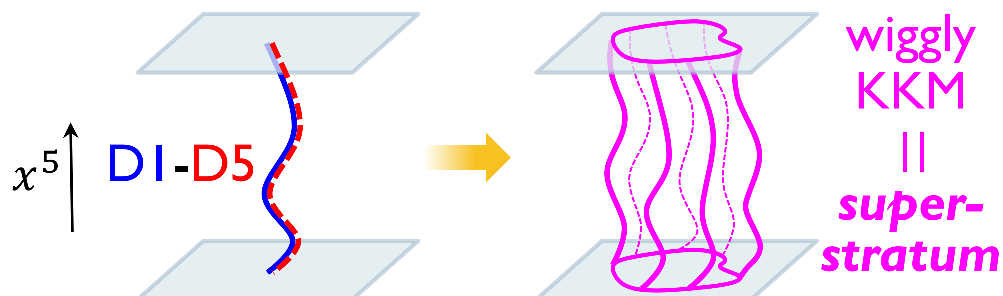
"superstratum"
arbitrary surface $\vec{f}(\lambda, \mu)$
larger entropy?

Implication for D1-D5-P

[Bena+de Boer+ MS+Warner 1107.2650]

D1(5) D5(56789) P(5)	→	D5(λ6789) D1(λ) KKM(λ6789,5)	→	$5^2_3(\mu 6789, \lambda 5)$ $1^6_3(\mu, \lambda 56789)$ KKM(μ6789, λ)





→ Need fluctuation along x^5

Recall we T-dualized along 5 to get to the frame of 4D/5D ansatz…

Implication for D1-D5-P

- Expect *geometric superstrata* sol'ns in 6D sugra

  — parametrized by a function of *two* variables

  - *Generic* solutions must be *non-geometric* superstrata
  - But we have more intuition for geometric ones

Study susy solutions in 6D sugra!

Susy solutions in 6D sugra

[Bena+Giusto+MS+Warner 1110.2781]

6D theory

- 6D $\mathcal{N} = 2$ sugra with a vector multiplet
- Bosonic fields
  - Metric $g_{\mu\nu}$
  - Dilaton ϕ
  - 2-form B_2, field strength $G_3 = dB_2$
- IIB on T^4_{6789}:

D1(5) → 1-brane coupled to B_2

D5(56789) → 1-brane coupled to $\tilde{B}_2$

Susy sol'n (1): Base

6D spacetime: (u, v, x^m)

u: isometry, $v \sim x^5$
x^m: 4D base

- 4D base $\mathcal{B}^4(v)$: almost hyper-Kähler

$$ds_4^2 = h_{mn}(x,v)dx^m dx^n, \qquad m,n = 1,2,3,4$$

$\beta(x,v)$: 1-form (↔ KKM)

$J^{(A)}(x,v),\ A = 1,2,3$: almost HK 2-forms

$$J^{(A)m}{}_n J^{(B)n}{}_p = \epsilon^{ABC} J^{(C)m}{}_p - \delta^{AB}\delta_p^m$$

$$d_4 J^{(A)} = \partial_v\left(\beta \wedge J^{(A)}\right), \qquad D \equiv d_4 - \beta \wedge \partial_v$$

Susy sol'n (2): Fields

- Fields on $\mathcal{B}^4$

Z_1: scalar $\leftrightarrow$ D1(v) $\quad$ Z_2: scalar $\leftrightarrow$ D5(v6789)

Θ_1: 2-form $\leftrightarrow$ D1(λ) $\quad$ Θ_2: 2-form $\leftrightarrow$ D5(λ6789)

ω: 1-form $\leftrightarrow$ J $\quad$ $\mathcal{F}$: scalar $\leftrightarrow$ P(v)

- 6D fields

$$ds_6^2 = \frac{2}{\sqrt{Z_1 Z_2}}(dv+\beta)\left(du+\omega+\frac{1}{2}\mathcal{F}(dv+\beta)\right) - \sqrt{Z_1 Z_2}\, ds_4^2$$

$$G_3 = d[-\frac{1}{2}Z_1^{-1}(du+\omega)\wedge(dv+\beta)] + \frac{1}{2}*_4\left(DZ_2+\dot{\beta}Z_2\right) + (dv+\beta)\wedge\Theta_1$$

$$e^{\sqrt{2}\phi} = \sqrt{Z_1/Z_2}$$

Susy sol'n (3): Linear structure

- First layer (Z, Θ)

$$D *_4 \left(DZ_I + \dot{\beta} Z_I\right) + 2D\beta \wedge \Theta_J = 0 \qquad \{I, J\} = \{1,2\}$$

$$D\Theta_J - \dot{\beta} \wedge \Theta_J - \partial_v \left[\frac{1}{2} *_4 \left(DZ_I + \dot{\beta} Z_I\right)\right] = 0 \qquad \dot{} \equiv \partial_v$$

- Second layer $(\mathcal{F}, \omega)$

$$*_4 D *_4 L = \dot{Z}_1 \dot{Z}_2 + \ddot{Z}_1 Z_2 + Z_1 \ddot{Z}_2 + \tfrac{1}{2} \partial_v (Z_1 Z_2) h^{mn} \dot{h}_{mn}$$
$$+ \tfrac{1}{2} Z_1 Z_2 \left(h^{mn} \ddot{h}_{mn} - \tfrac{1}{2} h^{mn} \dot{h}_{np} h^{pq} \dot{h}_{qm}\right) - 2\, \dot{\beta}_m L^m - 2 *_4 \left(\Theta_1 \wedge \Theta_2 - \hat{\psi} \wedge D\omega\right)$$

$$(1 + *_4) D\omega = 2(Z_1 \Theta_1 + Z_2 \Theta_2) - \mathcal{F} D\beta - 4 Z_1 Z_2 \hat{\psi}$$

$$L \equiv \dot{\omega} + \frac{1}{2} \mathcal{F} \dot{\beta} - \frac{1}{2} D\mathcal{F} \qquad \hat{\psi} = \frac{1}{16} \epsilon^{ABC} J^{(A)mn} \dot{J}^{(B)}_{mn} J^{(C)}$$

— Linear if solved in the right order

6D susy sol'ns: Summary

- 6D eqs have nice linear structure

  → can be solved in principle

- Difficult in practice

  → Need some physical intuition / organizing principle to proceed

A CFT view on superstrata

[Bena+MS+Warner 1404.4506]

Questions

- What sector of CFT states are expected to be visible in sugra?
- What is the structure of solutions?

D1-D5 CFT (1)

- D1-D5 system

S^1 (direction 5), T^4 (directions 6–9)

	1	2	3	4	5	6	7	8	9
D1	•	•	•	•	○	~	~	~	~
D5	•	•	•	•	○	○	○	○	○

$SO(4)_{1234} \cong SU(2)_L \times SU(2)_R$ $\quad$ $SO(4)_{6789} \cong SU(2)_1 \times SU(2)_2$

- $d = 2$, (large) $\mathcal{N} = (4,4)$ SCFT
- Sigma model with target space $(T^4)^N / S_N, \; N \equiv N_1 N_2$

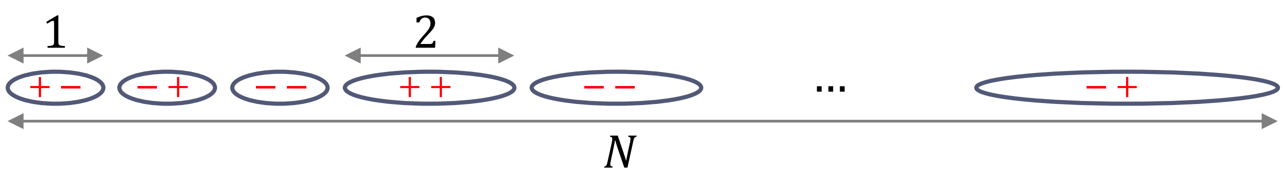
D1-D5 CFT (2)

- Matter content: 4 bosons, 4 fermions

	$SU(2)_L \times SU(2)_R$	$SU(2)_1 \times SU(2)_2$
$X^{\dot{A}A}_{(r)}(z,\bar{z})$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{2},\mathbf{2})$
$\psi^{\alpha\dot{A}}_{(r)}(z)$	$(\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$
$\tilde{\psi}^{\dot{\alpha}\dot{A}}_{(r)}(\bar{z})$	$(\mathbf{1},\mathbf{2})$	$(\mathbf{1},\mathbf{2})$

$r = 1, \dots, N$: copy index

- RR gnd states $\sim$ chiral primaries = twisting of N copies



Spins carried by $\psi, \tilde{\psi}$ zero modes

Maximally spinning:

Visible sector

Conjecture:

R-symmetry $SO(4)_{1234} = SU(2)_L \times SU(2)_R$ is visible from 6D sugra. Carriers: $\psi, \tilde{\psi}$

In particular, sector generated by $SU(2)$ currents:

$$J^{\alpha\beta}_{(r)}(z) = \frac{1}{2}\epsilon_{\dot{A}\dot{B}}\psi^{\alpha\dot{A}}_{(r)}(z)\psi^{\beta\dot{B}}_{(r)}(z), \qquad \tilde{J}^{\dot{\alpha}\dot{\beta}}_{(r)}(\bar{z}) = \cdots$$

namely, Sugawara CFT

$$[SU(2)_L \times SU(2)_R]^N/S_N, \qquad c = N$$

must be visible (the rest has $c = 5N$)

Expected entropy from strata

By considering multiple superstrata,
expect in the bulk to see entropy for $c = N$:

$$S_{\text{strata}} = 2\pi\sqrt{\frac{NN_3}{6}}$$

Instead of the full entropy for a $c = 6N$ system:

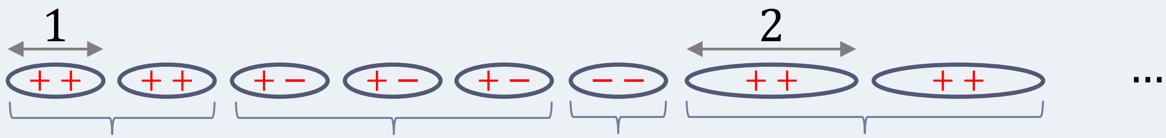
$$S_{\text{full}} = 2\pi\sqrt{NN_3}$$

Correct scaling must be reproducible!

(To get numerical factor right, need to look into compact T^4.)

Evidence: 2-chg states (RR gnd states)

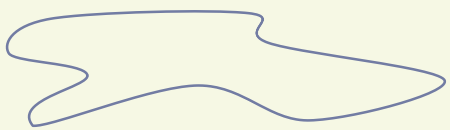
CFT: $n_k^{\alpha\dot{\alpha}}$: number of component strings with length k, spin $\alpha, \dot{\alpha}$



$n_1^{++} = 2$ $n_1^{+-} = 3$ $n_1^{--} = 1$ $n_2^{++} = 2$

Note: J_0^- (+ +) = (− +) etc.

Sugra: LM geometry



Profile $\vec{x} = \vec{f}(w) \in \mathbb{R}^4$

$$f^{\alpha\dot{\alpha}}(w) \sim \sum_{k\neq 0} a_k^{\alpha\dot{\alpha}}\, e^{2\pi i k w}$$

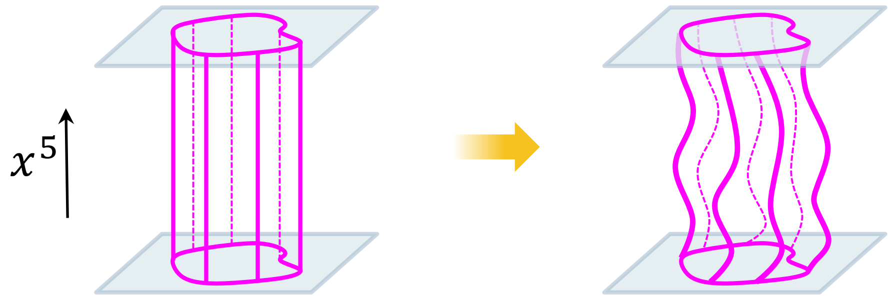
Dictionary: $n_k^{\alpha\dot{\alpha}} \leftrightarrow \left|a_k^{\alpha\dot{\alpha}}\right|^2$ [Lunin-Mathur] [Kanitscheider-Skenderis-Taylor]

$SU(2)_L \times SU(2)_R$ current sector is precisely visible

3-charge states

- Add $\mathrm{P}(v)$ → fluctuation along v
  - Expect: $f^{\alpha\dot{\alpha}}(w) \to f^{\alpha\dot{\alpha}}(w, v)$

    — Depend on two variables

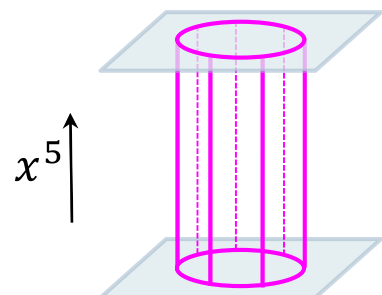


- CFT: (any, gnd)

v-dep fluct. for 3-charge states (1)

Consider:

Circular superstratum = Maximally spinning RR gnd state
= pure $AdS_3 \times S^3$



Claim:

Fluctuations around this state are parametrized by functions of two variables.

v-dep fluct. for 3-charge states (2)

- Fluctuation around $AdS_3 \times S^3$ is in some $SU(2)_L \times SU(2)_R$ rep.
- Other RR ground state: $(\ell, \ell; \tilde{\ell}, \tilde{\ell})$, $|\ell - \tilde{\ell}| \leq 2$

  → *One* quantum number

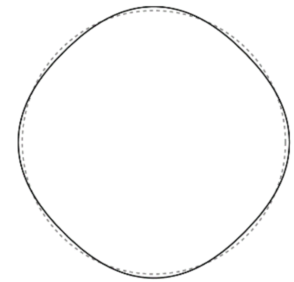
  → *One* variable (LM profile function $f(w)$)
- Act by J (but not $\tilde{J}$) modes → $(\ell, m; \tilde{\ell}, \tilde{\ell})$

  → *Two* quantum numbers

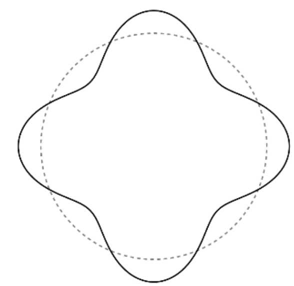
  → *Two* variables (superstratum, $f(w, v)$)

Comments

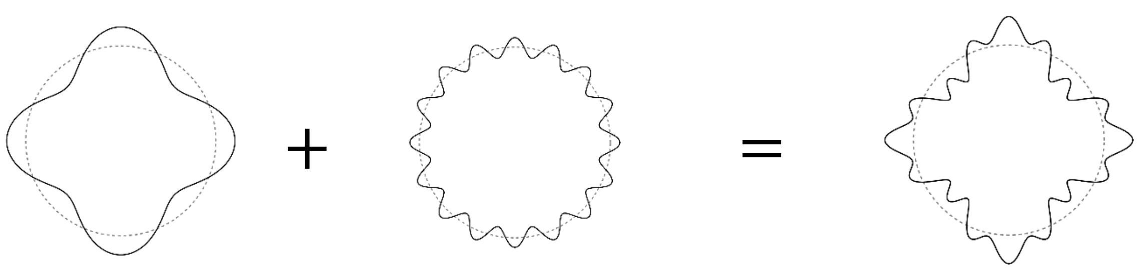
- At linear level, can realize bulk action of J on linear fluctuation around $AdS_3 \times S^3$



- Can use linear structure of 6D eqs to nonlinearly complete it (work in progress).

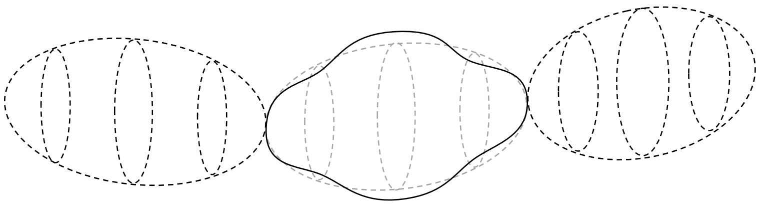


- Superposing multiple modes in bulk $\leftrightarrow$ considering non-chiral primaries in CFT



Multiple superstrata

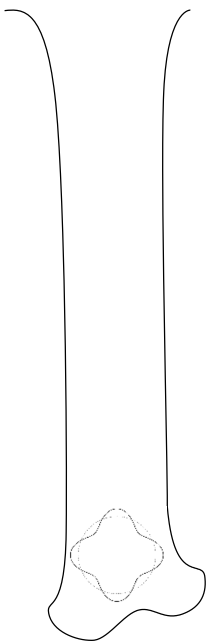
- More generally, one has multiple S^3's
- Can fluctuate each S^3 — multi-superstratum



- Can use $AdS_3 \times S^3$ as local model
- Large redshift in scaling geometries

→ entropy enhancement

→ $S \sim \sqrt{N_1 N_2 N_3}$?



Summary

- $SU(2)$ current algebra sector:
  expected to be visible in sugra
- Sugra states: superstrata depending on 2 vars.
- Have to solve 6D system
  by nonlinearly completing linear fluctuations
  (work in progress)
- Multiple superstrata, scaling solution
  → entropy enhancement?

Conclusions

Conclusions

- Superstrata: conjectural microstate (non)geometries
- They live in 6D sugra (or generalization thereof)
- $SU(2)$ current algebra of D1-D5 CFT describes their fluctuations
- A LOT more stuff to do, more fun to enjoy!
  - Construct
  - Count
  - Hit them to death

— Stay tuned! ☺

Thanks!