Codimension-2 Solutions in 5D Supergravity

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Minkyu Park & MS, arXiv:1505.05169

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CERN-CKC TH Institute on
Duality Symmetries in String and M-Theories

Introduction

Low codim branes (1)

- Branes: important for nonpertubative physics
- Low codim (≤ 2) branes:
  - Less studied
  - Non-standard features
    - Codim 2 → destroys asymptotics
    - Codim 1 → spacetime ends at finite distance

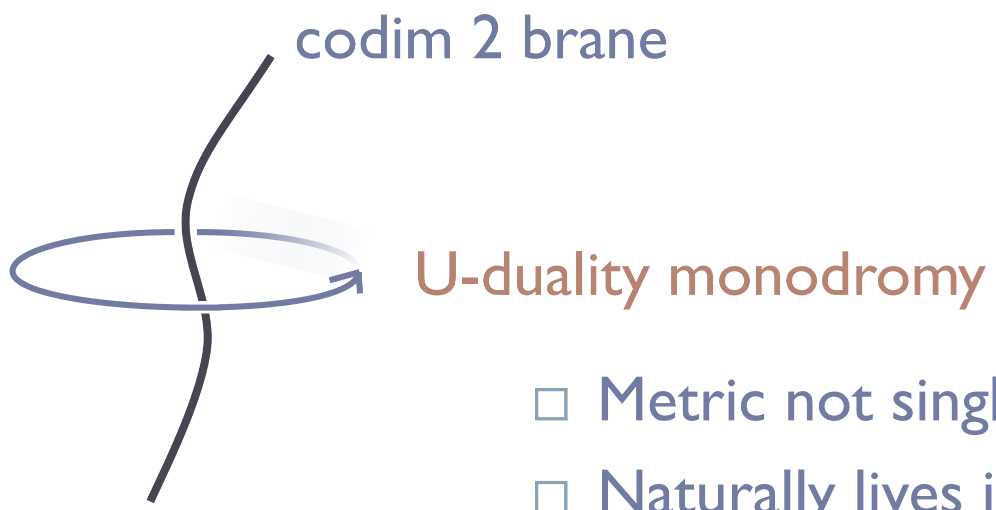
Peculiar, but all the more interesting!

Low codim branes (2)

Why interesting?

- F-theory
- Exotic, non-geometric in general $5^2_2,\ 7_3,\ 1^6_4, \ldots$

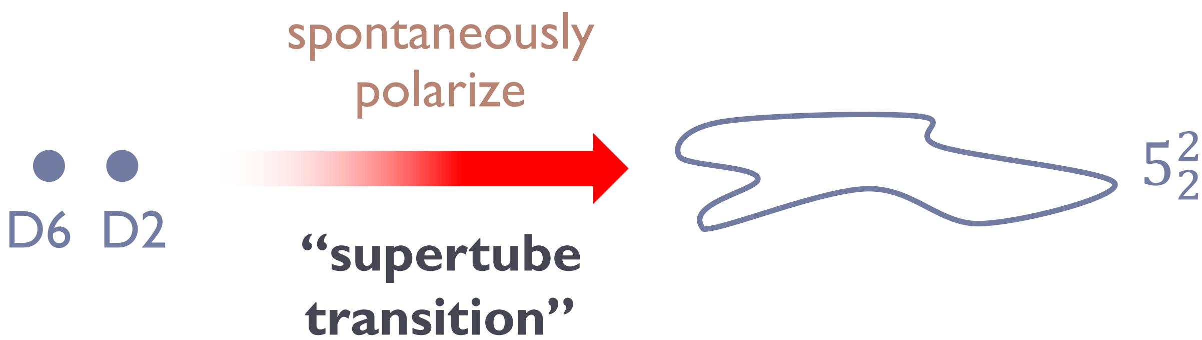
[de Boer+Shigemori '10, '12]



- Metric not single-valued
- Naturally lives in **DFT/EFT**

Low codim branes (3)

- Can be created out of ordinary branes



- More common than previously thought
- Relevance for black hole physics

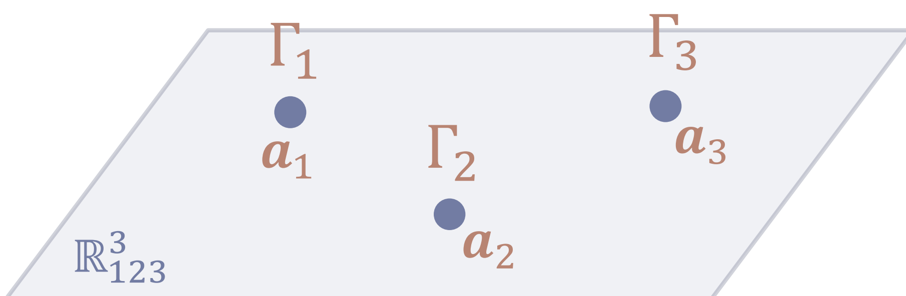
→ Cf. Fuzzball proposal,
Microstate geometry program

Multi-center solutions

IIA on $T^6 = T^2_{45} \times T^2_{67} \times T^2_{89}$

→ System of $(\mathrm{D6}, \mathrm{D4}, \mathrm{D2}, \mathrm{D0}) \equiv \Gamma$

→ Multi-center solution



codim-3 source

$$H(\boldsymbol{x}) = h + \sum_p \frac{\Gamma_p}{|\boldsymbol{x} - \boldsymbol{a}_p|}$$

Paradigm for BH physics

- Split attractor flow, wall crossing, quiver QM…
- Microstate geometry program

Codim-2 solutions

$$\mathrm{D2}(45) + \mathrm{D2}(67) \rightarrow \mathrm{NS5}(\lambda 4567)$$
$$\mathrm{D6}(456789) + \mathrm{D2}(89) \rightarrow 5^2_2(\lambda 4567{,}89)$$

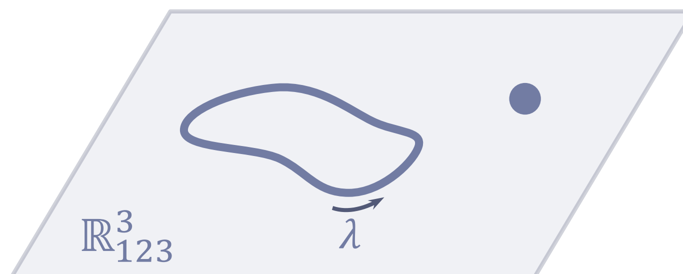
$H(x)$: includes codim-2 source.
multi-valued.

New solutions!

- Various applications
- New microstates (MGP)

Goal:

- Demonstrate that these codim-2 solutions exist by presenting some simple but explicit examples



- We will see monodromy structure characteristic of codim-2 branes

4D/5D Solutions

Setup

- M-theory on T^6

$\Downarrow$

- $D = 5, \mathcal{N} = 1$ sugra with 2 vector multiplets

gauge fields: A_μ^I , $I = 1,2,3.$ $F^I \equiv dA^I$.

scalars: X^I, $X^1 X^2 X^3 = 1$

- Action

$$S_{\text{bos}} = \int (*_5 R - Q_{IJ} dX^I \wedge *_5\, dX^I - Q_{IJ} F^I \wedge *_5\, F^J \\ -\tfrac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K)$$

$$C_{IJK} = |\epsilon_{IJK}| \,, \qquad Q_{IJ} = \tfrac{1}{2}\mathrm{diag}(1/X^1, 1/X^2, 1/X^3)$$

BPS solutions

[Gutowski-Reall '04] [Bena-Warner '04]
[Gutowski-Gauntlett '04]

- Require susy
- Assume $U(1)$ symmetry

Solution specified by harmonic functions in $\mathbb{R}^3$:

$$H = (V, K^I, L_I, M), \qquad I = 1,2,3$$

$$\triangle H = 0 \quad \Rightarrow \quad H = h + \sum_p \frac{\Gamma_p}{|\boldsymbol{x} - \boldsymbol{a}_p|}$$

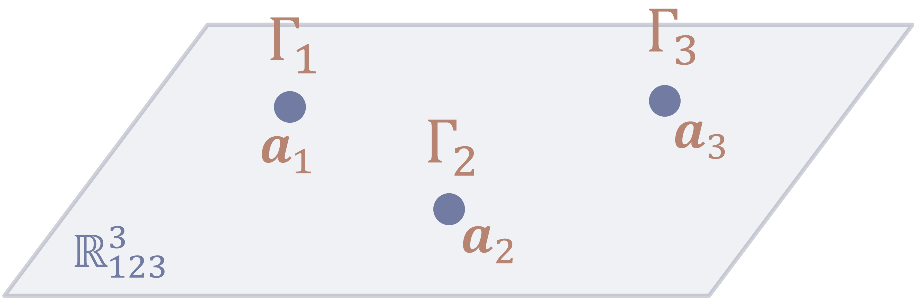
Bates-Denef/Gutowski-Gauntlett/Bena-Warner solution, or

"4D/5D solution"

Multi-center solution

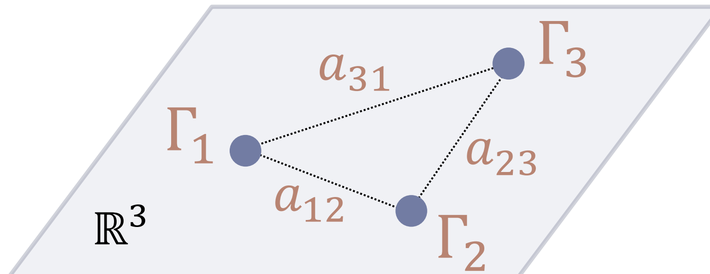
$$H = (V, K^I, L_I, M), \qquad H = h + \sum_p \frac{\Gamma_p}{|\boldsymbol{x} - \boldsymbol{a}_p|}$$

— Describes multi-center config of branes in IIA on T^6_{456789}



$$V \leftrightarrow \mathrm{D6}(456789) \qquad \begin{array}{l} K^1 \leftrightarrow \mathrm{D4}(6789) \\ K^2 \leftrightarrow \mathrm{D4}(4589) \\ K^3 \leftrightarrow \mathrm{D4}(4567) \end{array} \qquad \begin{array}{l} L_1 \leftrightarrow \mathrm{D2}(45) \\ L_2 \leftrightarrow \mathrm{D2}(67) \\ L_3 \leftrightarrow \mathrm{D2}(89) \end{array} \qquad M \leftrightarrow \mathrm{D0}$$

Integrability cond.



- Positions $\boldsymbol{a}_p$ satisfy "integrability cond"

$$\sum_{q(\neq p)} \frac{\langle \Gamma_p, \Gamma_q \rangle}{a_{pq}} = \langle h, \Gamma_p \rangle \qquad a_{pq} \equiv |\mathbf{a}_p - \mathbf{a}_q|$$

- Represents force balance
- Comes from integrability for ω

$$0 = V \triangle M - M \triangle V + \frac{1}{2} \left(K^I \triangle L_I - L_I \triangle K^I \right)$$

10D IIA fields

$$ds^2_{10,\mathrm{str}} = -\frac{1}{\sqrt{V(Z - V\mu^2)}}(dt + \omega)^2 + \sqrt{V(Z - V\mu^2)}\, dx^i dx^i$$
$$+ \sqrt{\frac{Z - V\mu^2}{V}}\left(Z_1^{-1} dx_{45}^2 + Z_2^{-1} dx_{67}^2 + Z_3^{-1} dx_{89}^2\right)$$
$$e^{2\Phi} = \frac{(Z - V\mu^2)^{3/2}}{V^{3/2} Z}\,, \qquad B_2 = \left(V^{-1} K^I - Z_I^{-1}\mu\right) J_I\,, \qquad \cdots$$

$$Z = Z_1 Z_2 Z_3 \qquad J_1 \equiv dx^4 \wedge dx^5, \quad J_2 \equiv dx^6 \wedge dx^7, \quad J_3 \equiv dx^8 \wedge dx^9$$

$$Z_I = L_I + \frac{1}{2} C_{IJK} V^{-1} K^J K^K$$
$$\mu = M + \frac{1}{2} V^{-1} K^I L_I + \frac{1}{6} C_{IJK} V^{-2} K^I K^J K^K$$

$$*_3\, d\omega = V dM - M dV + \frac{1}{2}\left(K^I dL_I - L_I dK^I\right)$$

Torus moduli

Complexified Kähler moduli for T^2_{45}:

$$\tau^1 = B_{45} + i\sqrt{\det G_{ab}}$$
$$= \left(\frac{K^1}{V} - \frac{\mu}{Z_1}\right) + \frac{i\sqrt{V(Z - V\mu^2)}}{Z_1 V}$$

$a, b = 4{,}5$
$R_4 = R_5 = l_s$

We likewise have τ^2, τ^3 for T^2_{67}, T^2_{89}

Related to $SL(2,\mathbb{Z})^3$ duality of STU model

Example: 4-charge BH

- Susy BH in 4D (4 supercharges)

N^0 D6(456789)
N_1 D2(45)
N_2 D2(67)
N_3 D2(89)

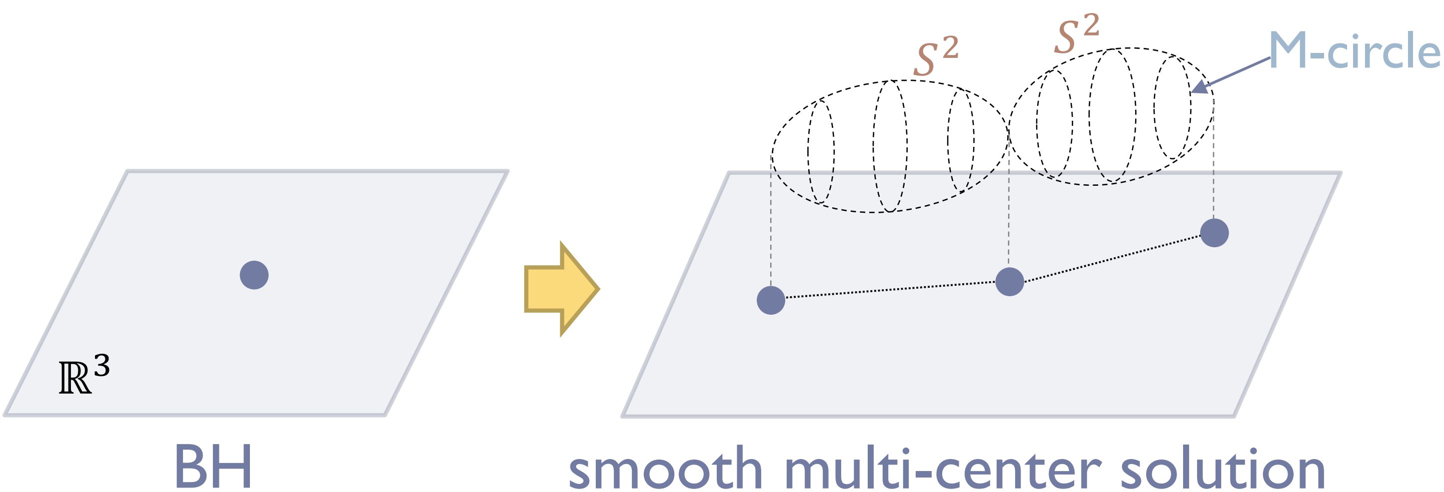
$$V = \frac{N^0}{r} \qquad K^I = 0$$

$$L_I = 1 + \frac{N_I}{r} \qquad M = 0$$

- Single-center
- Macroscopic entropy: $S \sim \sqrt{N^0 N_1 N_2 N_3}$

Microstate geometry program

What portion of the BH entropy
of supersymmetric BHs is accounted for
by smooth, horizonless solutions of classical sugra?

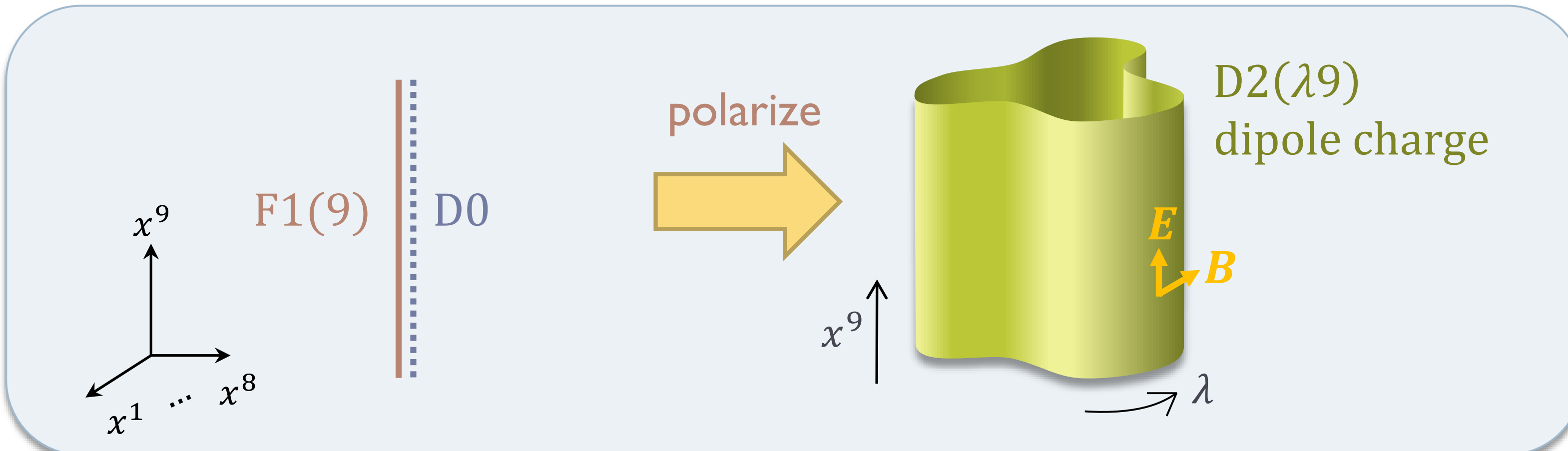


- 4D/5D solution: paradigm for MGP
  [Bena, Warner '06] [Berglund, Gimon, Levi '06]
- Not enough microstates found so far
  [Bena, Bobev, Giusto, Ruef, Warner '11] [de Boer, El-Showk, Messamah, Van den Bleeken '09]

Supertube transition

Supertube transition [Mateos+Townsend 2001]

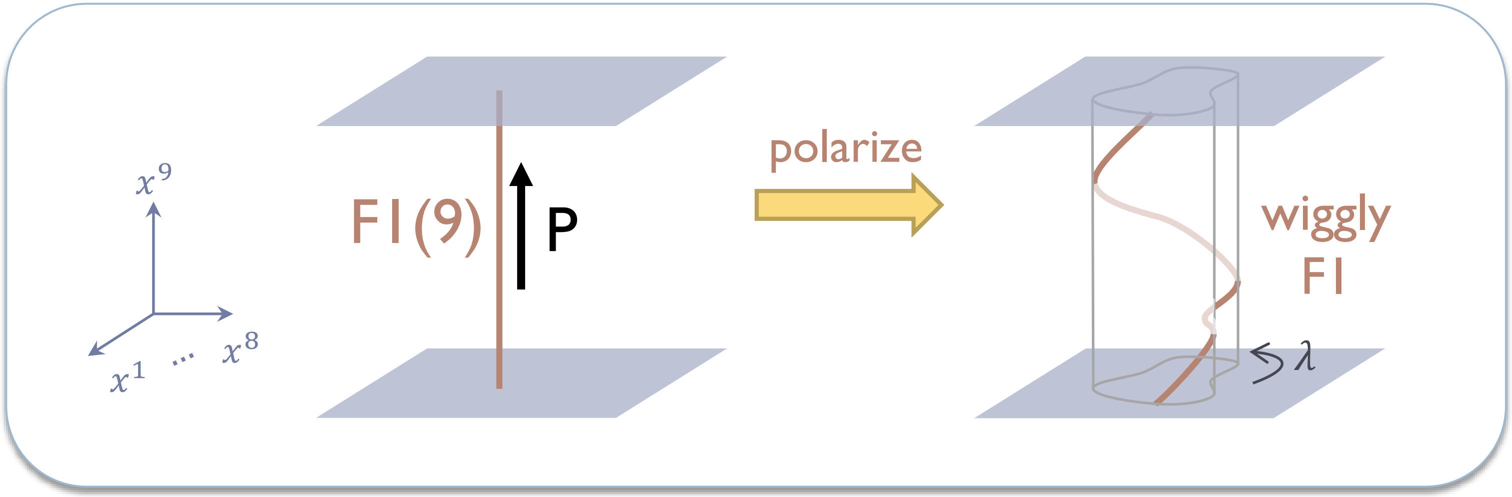
$\mathrm{D0} + \mathrm{F1}(9) \Longrightarrow \mathrm{D2}(\lambda 9)$



- Spontaneous polarization phenomenon (cf. Myers effect)
- Produces new dipole charge
- Cross section = *arbitrary* curve

F1-P frame

$\mathrm{F1}(9) + \mathrm{P}(9) \Rightarrow \mathrm{F1}(\lambda)$



- To carry momentum,
  FI must wiggle in transverse $\mathbb{R}^8$
- Projection onto transverse $\mathbb{R}^8$ is an arbitrary curve

Dualizing supertubes

Original supertube effect:

$$\mathrm{D0} + \mathrm{F1}(9) \to \mathrm{D2}(\lambda 9)$$

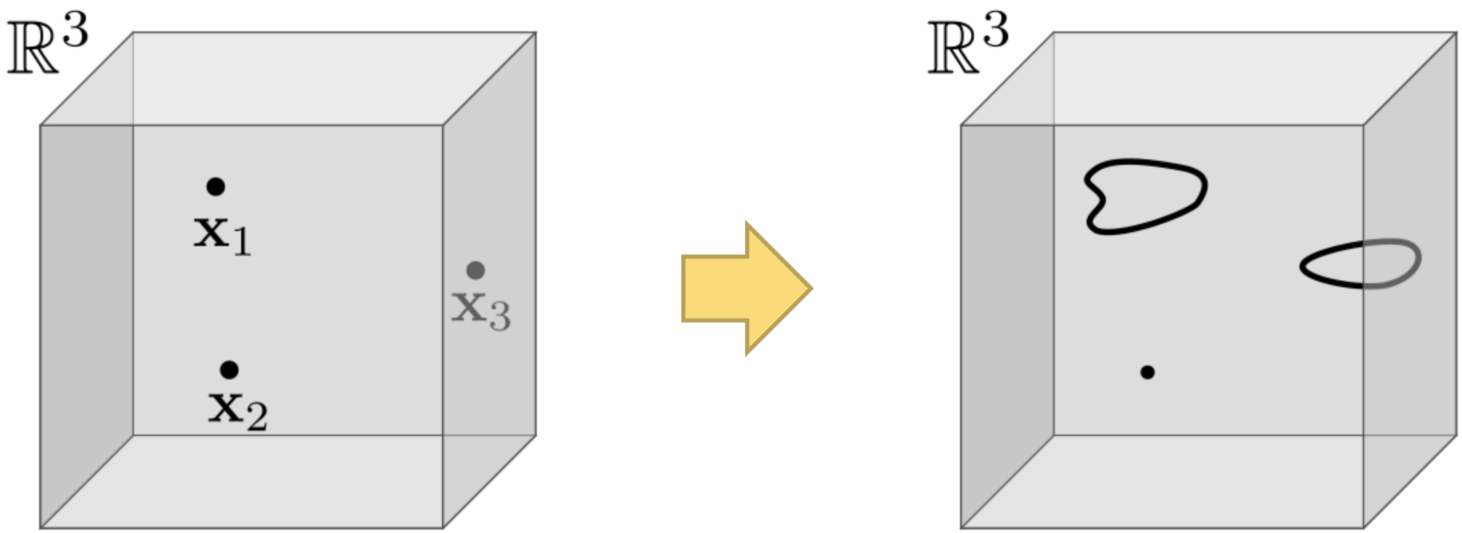
dualize!

$$\mathrm{F1}(9) + \mathrm{P}(9) \to \mathrm{F1}(\lambda 9)$$

$$\boxed{\begin{aligned} \mathrm{D2}(45) + \mathrm{D2}(67) &\to \mathrm{NS5}(\lambda 4567) \\ \mathrm{D6}(456789) + \mathrm{D2}(45) &\to 5^2_2(\lambda 6789{,}45) \end{aligned}}$$

- Constituents of 4-chg BH!
- Need to incorporate codim-2 objects in 4D/5D sol'n

Sol'n with codim-2 centers



5^2_2-brane

	1	2	3	4	5	6	7	8	9
NS5	·	·	○	○	○	○	○	~	~

$$\tau \to \tau + 1$$

$$\tau \equiv B_{89} + i\sqrt{\det G_{ab}}$$

T-duality along x^8

	1	2	3	4	5	6	7	8	9
KKM	·	·	○	○	○	○	○	⊙	~

T-duality along x^9

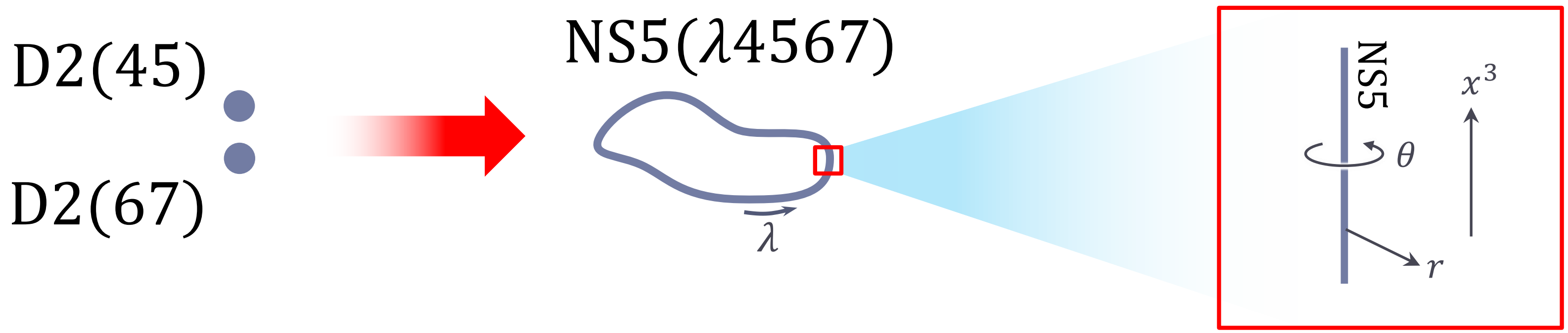
	1	2	3	4	5	6	7	8	9
$\mathbf{5^2_2}$	·	·	○	○	○	○	○	⊙	⊙

$$\tau \to \frac{\tau}{-\tau + 1}$$

$$\tau' \to \tau' + 1, \qquad \tau' \equiv -\frac{1}{\tau}$$

Codim-2 solutions (1)

Straight config (1)



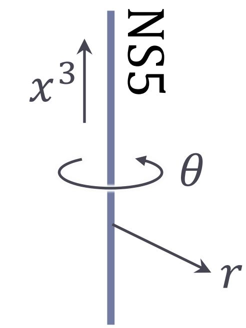
$$V = 1, \qquad K^1 = K^2 = 0, \quad K^3 = q\theta$$

$$L_1 = 1 + Q_1 \log\frac{\Lambda}{r}, \quad L_2 = 1 + Q_2 \log\frac{\Lambda}{r}, \quad L_3 = 1; \quad M = -\frac{1}{2} q\theta$$

Straight config (2)

$L_{1,2} = 1 + Q_{1,2} \log \frac{\Lambda}{r}$: D2 source along x^3 axis

$K^3 = q\theta$: monodromy around x^3 axis



⇨ $B_2 = \frac{\theta}{2\pi} dx^8 \wedge dx^9$ ⇨ NS5(34567) charge

- Metric is single-valued (no exotic brane, NS5 only)

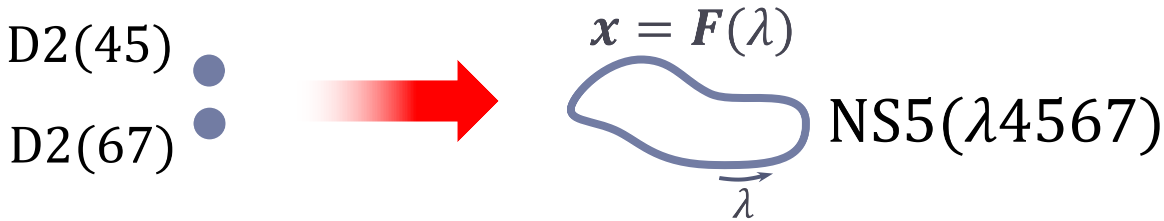
4D modulus:

$$\tau^3 = B_{89} + i\sqrt{\det G_{ab}} = \frac{\theta}{2\pi}$$

$\tau^3 \to \tau^3 + 1$ $SL(2,\mathbb{Z})$ monodromy

D2+D2→NS5 (1)

NS5 along general curve:



$$V = 1; \quad K^{1,2,3} = 0; \quad L_1 = 1 + \frac{Q_1}{r}, \quad L_2 = 1 + \frac{Q_2}{r}, \quad L_3 = 1; \quad M = 0$$

$$\begin{aligned} V = 1, \quad K^1 = 0, \quad K^2 = 0, \quad K^3 = \gamma \\ L_1 = f_2, \quad L_2 = f_1, \quad L_3 = 1, \quad M = -\frac{\gamma}{2} \end{aligned}$$

$$f_1 = 1 + \frac{Q_1}{L} \int_0^L \frac{d\lambda}{|\boldsymbol{x} - \boldsymbol{F}(\lambda)|}, \qquad f_2 = 1 + \frac{Q_1}{L} \int_0^L \frac{|\dot{\boldsymbol{F}}(\lambda)|^2 d\lambda}{|\boldsymbol{x} - \boldsymbol{F}(\lambda)|}$$

$$d\gamma = *_3 \, d\alpha, \qquad \alpha_i = \frac{Q_1}{L} \int_0^L \frac{\dot{F}_i(\lambda) d\lambda}{|\boldsymbol{x} - \boldsymbol{F}(\lambda)|}$$

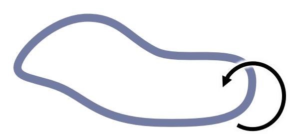
D2+D2→NS5 (2)

$$V = 1, \quad K^1 = 0, \quad K^2 = 0, \quad K^3 = \gamma$$
$$L_1 = f_2, \quad L_2 = f_1, \quad L_3 = 1, \quad M = -\frac{\gamma}{2}$$

- Obtained by dualizing known supertube solution

  E.g. [Emparan+Mateos+Townsend '01]

- γ multi-valued → multi-valued harmonic functions



$\gamma \to \gamma + \mathrm{const.}$

- Integrability condition satisfied

$$V\triangle M - M\triangle V + \frac{1}{2}\left(K^I \triangle L_I - L_I \triangle K^I\right) = -\triangle\gamma \equiv 0$$

no δ func sourse

D2+D2→NS5 (3)

- 10D metric is single-valued

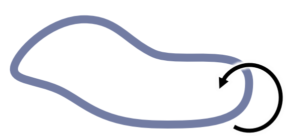
$$Z_1 = f_2,\ \ Z_2 = f_1,\ \ \ Z_3 = 1,\ \ \ \ \mu = 0$$

$$ds^2_{10} = -(f_1 f_2)^{-1/2}(dt-\alpha)^2 + (f_1 f_2)^{1/2} dx^i dx^i$$
$$+ (f_1/f_2)^{1/2} dx^2_{45} + (f_2/f_1)^{1/2} dx^2_{67} + (f_1 f_2)^{1/2} dx^2_{89}$$

Asymptotically $\mathbb{R}^{1,3} \times T^6$

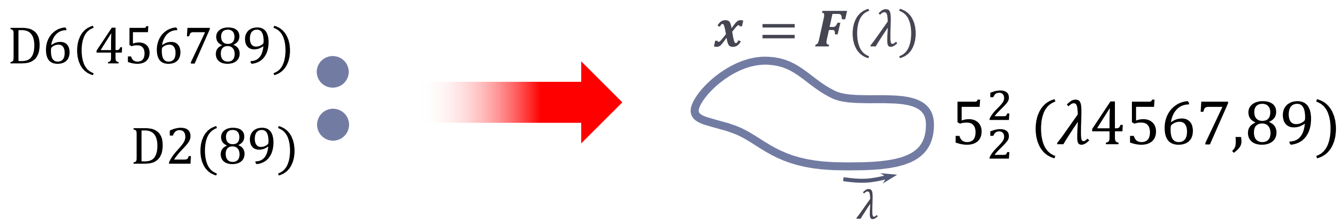
- Torus moduli

$$\tau^1 = i\sqrt{\frac{f_1}{f_2}}\,,\quad \tau^2 = i\sqrt{\frac{f_2}{f_1}}\,,\quad \tau^3 = \gamma + i\sqrt{f_1 f_2}$$



$\tau^3 \to \tau^3 + 1$ → NS5 charge

D6+D2→ 5^2_2 (1)



$$V = f_2, \quad K^1 = \gamma, \quad K^2 = \gamma, \quad K^3 = 0$$

$$L_1 = 1, \quad L_2 = 1, \quad L_3 = f_1, \quad M = 0$$

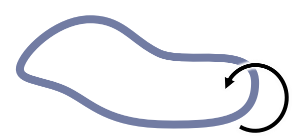
D6+D2→ 5_2^2 (2)

- 10D metric: multi-valued

$$Z_1 = Z_2 = 1, \;\; Z_3 = f_1 F, \quad \mu = f_2^{-1}\gamma \qquad F \equiv 1 + \frac{\gamma^2}{f_1 f_2}$$

$$ds_{10}^2 = -(f_1 f_2)^{-1/2}(dt-\alpha)^2 + (f_1 f_2)^{1/2} dx^i dx^i + (f_1/f_2)^{1/2}\left(dx_{4567}^2 + f_1^{-1}F^{-1}dx_{89}^2\right)$$
$$e^{2\Phi} = f_1^{1/2} f_2^{-3/2} F^{-1}, \qquad B_2 = -\frac{\gamma}{f_1 f_2 F}\, dx^8 \wedge dx^9,$$

  - Asymptotically $\mathbb{R}^{1,3} \times T^6$
  - Non-geometric



T-duality
twist

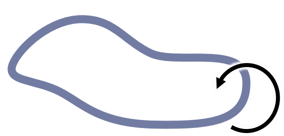
D6+D2$\rightarrow 5^2_2$ (3)

- Torus moduli

$$\tau^3 = B_{89} + i\sqrt{\det G_{ab}} = -\frac{\gamma}{f_1 f_2 F} + \frac{i}{\sqrt{f_1 f_2 F}}$$

T-dual modulus:

$$\tau'^3 \equiv -\frac{1}{\tau^3} = \gamma + i\sqrt{f_1 f_2}$$



$\tau'^3 \rightarrow \tau'^3 + 1$

$\rightarrow 5^2_2$ brane

Summary so far

- Codim-2 solutions exist
- Constituents of BH systems

$$\left.\begin{array}{l}\mathrm{D6}(456789)\\ \mathrm{D2}(45)\end{array}\right\}\ \text{can polarize into NS5}$$

$$\left.\begin{array}{l}\mathrm{D2}(67)\\ \mathrm{D2}(89)\end{array}\right\}\ \text{can polarize into } 5_2^2$$

→ must be important for BH physics

Codim-2 solutions (2)

More general solutions?

- Need to find harmonic functions such that…
  - Give desired monodromy for moduli

$$\tau^1 = \left(\frac{K^1}{V} - \frac{\mu}{Z_1}\right) + \frac{i\sqrt{V(Z - V\mu^2)}}{Z_1 V} \qquad \text{etc.}$$

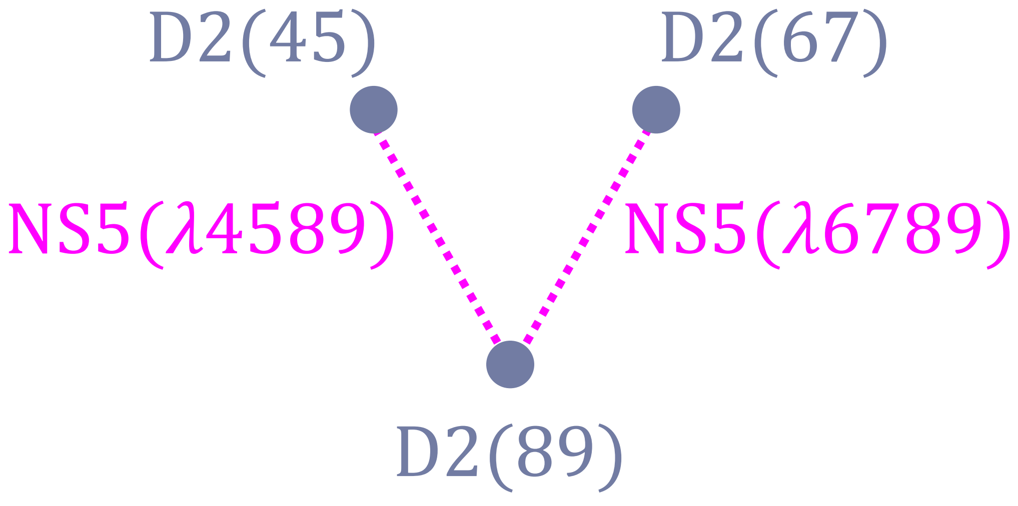
Cf. F-theory

  - Give single/multi-valued metric as necessary
  - Satisfies integrability condition

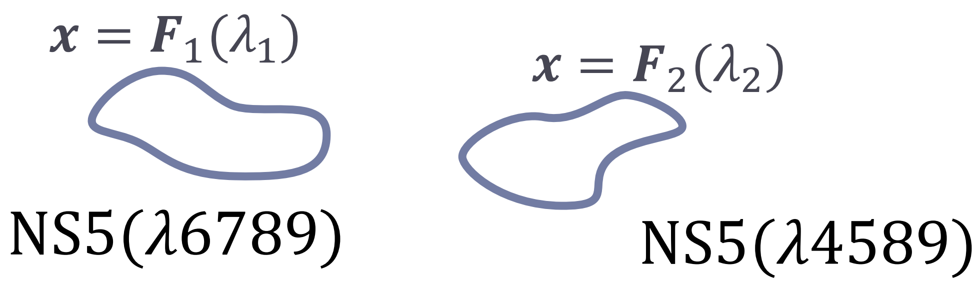
$$0 = V\triangle M - M\triangle V + \frac{1}{2}\left(K^I \triangle L_I - L_I \triangle K^I\right)$$

- Difficult in general

2-dipole solution



D2+D2+D2→NS5+NS5 (1)



$$V = 1, \qquad K^1 = \gamma_2, \quad K^2 = \gamma_1, \quad K^3 = 0,$$

$$L_1 = 1 + Q_1 \int \frac{d\lambda_1}{R_1}, \qquad L_2 = 1 + Q_2 \int \frac{d\lambda_2}{R_2}$$

$$L_3 = 1 + Q_1 \int \frac{|\dot{\boldsymbol{F}}_1|^2 d\lambda_1}{R_1} + Q_2 \int \frac{|\dot{\boldsymbol{F}}_2|^2 d\lambda_2}{R_2}$$

$$+ Q_1 Q_2 \iint d\lambda_1 d\lambda_2 \left(\frac{\dot{\boldsymbol{F}}_1 \cdot \dot{\boldsymbol{F}}_2}{2R_1R_2} - \frac{\dot{F}_{1i}\dot{F}_{2j}(R_{1i}R_{2j} - R_{1j}R_{2i})}{F_{12}R_1R_2(F_{12} + R_1 + R_2)} \right) - K^1K^2$$

$$M = \frac{1}{2} Q_1 Q_2 \iint d\lambda_1 d\lambda_2 \frac{\epsilon_{ijk}\dot{F}_{12i}R_{1j}R_{2k}}{F_{12}R_1R_2(F_{12}+R_1+R_2)} - \frac{1}{2}(K^1L_1 + K^2L_2)$$

$\boldsymbol{R}_p(\lambda_p) \equiv \boldsymbol{x} - \boldsymbol{F}_p(\lambda_p)$, $\boldsymbol{F}_{12} \equiv \boldsymbol{F}_1(\lambda_1) - \boldsymbol{F}_2(\lambda_2)$, $R_p \equiv |\boldsymbol{R}_p|$, $F_{12} \equiv |\boldsymbol{F}_{12}|$

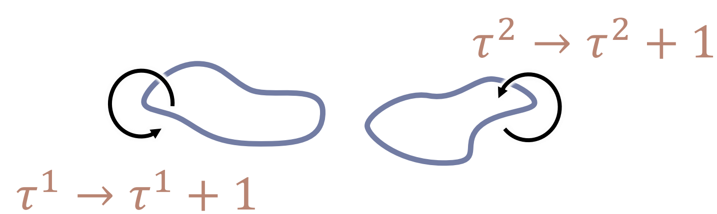
D2+D2+D2→NS5+NS5 (2)

$$V = 1, \qquad K^1 = \gamma_2, \quad K^2 = \gamma_1, \quad K^3 = 0, \quad L_1 = 1 + Q_1 \int \frac{d\lambda_1}{R_1}, \qquad L_2 = 1 + Q_2 \int \frac{d\lambda_2}{R_2}$$

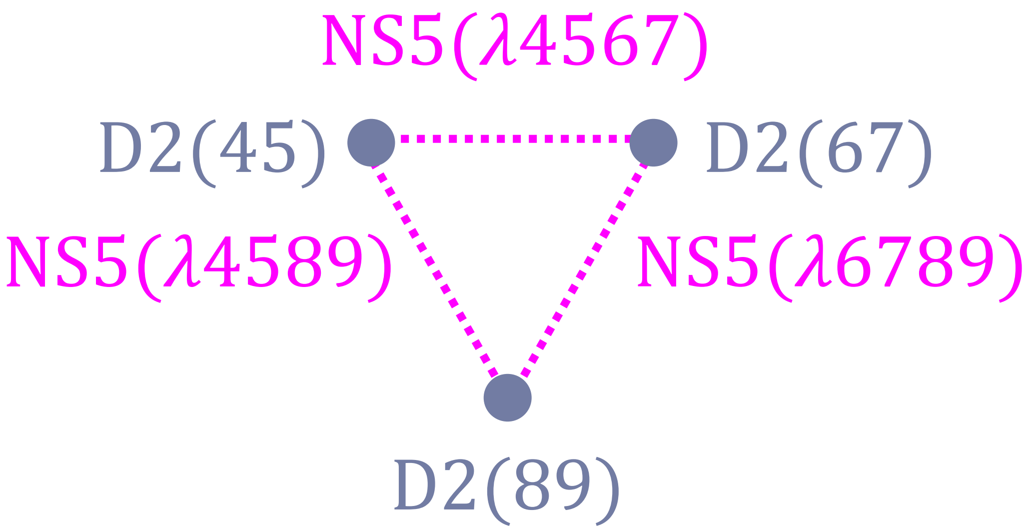
$$L_3 = 1 + Q_1 \int \frac{|\dot{F}_1|^2 d\lambda_1}{R_1} + Q_2 \int \frac{|\dot{F}_2|^2 d\lambda_2}{R_2} + Q_1 Q_2 \iint d\lambda_1 d\lambda_2 \left(\frac{\dot{F}_1 \cdot \dot{F}_2}{2R_1 R_2} - \frac{\dot{F}_{1i} \dot{F}_{2j} (R_{1i} R_{2j} - R_{1j} R_{2i})}{F_{12} R_1 R_2 (F_{12} + R_1 + R_2)} \right) - K^1 K^2$$

$$M = \frac{1}{2} Q_1 Q_2 \iint d\lambda_1 d\lambda_2 \frac{\epsilon_{ijk} \dot{F}_{12i} R_{1j} R_{2k}}{F_{12} R_1 R_2 (F_{12} + R_1 + R_2)} - \frac{1}{2} (K^1 L_1 + K^2 L_2)$$

- Obtained by smearing and dualizing "superthread" solution
  [Niehoff+Vasilakis+Warner '12]
- L_3, M are multi-valued but harmonic by non-trivial cancellations
- Z_3, μ are single-valued → metric is single-valued
- Correct monodromy for NS5

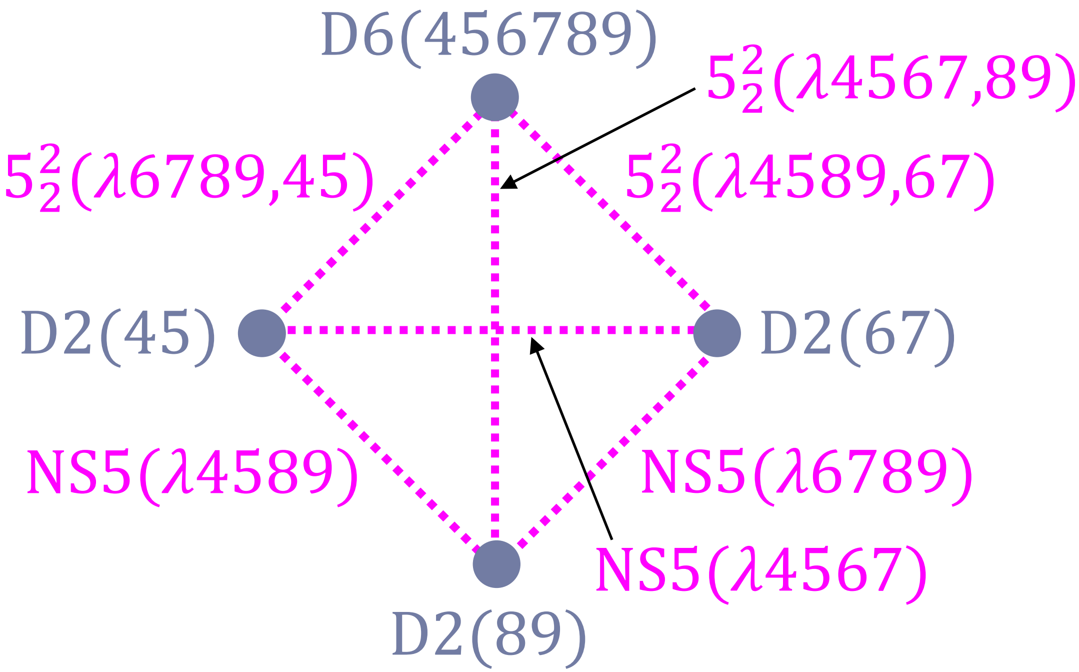


3-dipole solution



- Can be found, but more complicated and implicit

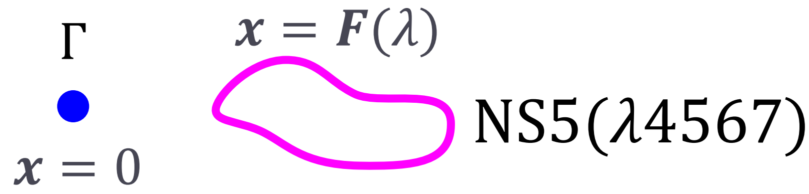
More dipoles



- Represents microstates of 4-charge BH
- Difficult to find…not successful so far

Mixed configurations

Codim-2 & codim-3



$$V = n_0 + \frac{n}{r}$$

$$K^1 = k_0^1 + \frac{k^1}{r}, \qquad K^2 = k_0^2 + \frac{k^2}{r}, \qquad K^3 = k_0^3 + \gamma + \frac{k^3}{r}$$

$$L_1 = l_1^0 + f_2 + \frac{l_1}{r}, \qquad L_2 = l_2^0 + f_1 + \frac{l_2}{r}, \qquad L_3 = l_3^0 + \frac{l_3}{r}$$

$$M = m_0 - \frac{\gamma}{2} + \frac{m}{r}$$

Integrability condition

(a) $$0 = n_0 m - m_0 n + \frac{1}{2}(k_0^I l_I - l_I^0 k^I) - \frac{1}{2}\frac{Q}{L}\int_0^L d\lambda \frac{k^1|\dot{\mathbf{F}}(\lambda)|^2 + k^2}{|\mathbf{F}(\lambda)|},$$

(b) $$0 = n + l_3,$$

(c) $$0 = k_0^2 + \frac{k^2}{|\mathbf{F}(\lambda)|} + |\dot{\mathbf{F}}(\lambda)|^2 \left(k_0^1 + \frac{k^1}{|\mathbf{F}(\lambda)|}\right) \qquad \text{for each value of } \lambda.$$

(a): total force from tube to the $r = 0$ brane is 0

→ Easy to satisfy

(c): force by the $r = 0$ brane on each point along tube is 0

→ Constrain curve $\boldsymbol{F}(\lambda)$

→ Only 2 components in $\boldsymbol{F}(\lambda)$ are free

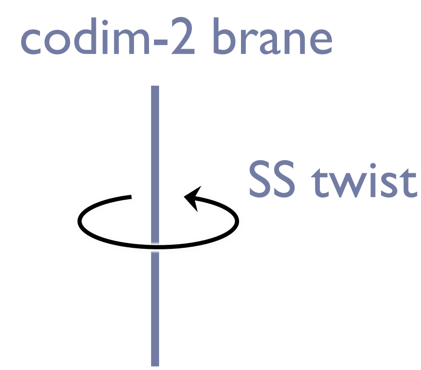
Conclusions

Conclusions

- 4D/5D solution: a paradigm for BH research
- Only codim-3 solutions studied so far
- Codim-2 solutions also important
  - Scratched the surface
  - Interesting dynamics
  - General solutions?

Future directions

- Applications
  - Split attractor flow, wall crossing, quiver QM
  - Microstate Geometry Program
- Lower codimension
  - Superstratum
- Connect to DFT/EFT
  - Brane charge and flux formulation
  - More general non-geometric sol'ns?
  - Add your own



Thanks!