The Microstate Geometry Program and Superstrata

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Plan

1. BH microstates
2. Microstate geom
3. Fuzzball conjecture & microstate geom program
4. Microstate geom in 5D
5. Double bubbling
6. Superstratum

Black hole microstates

black hole

=

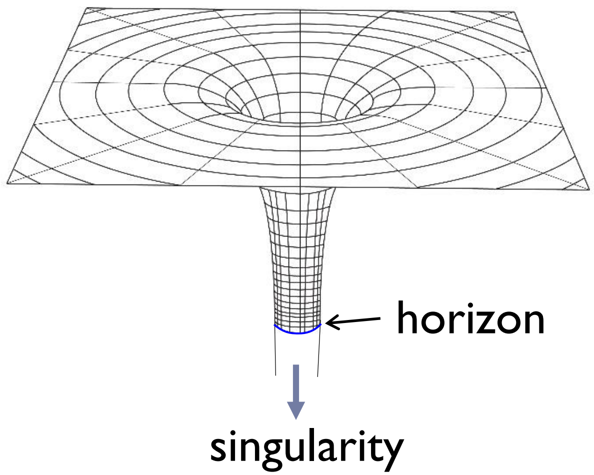
thermodyn. ensemble

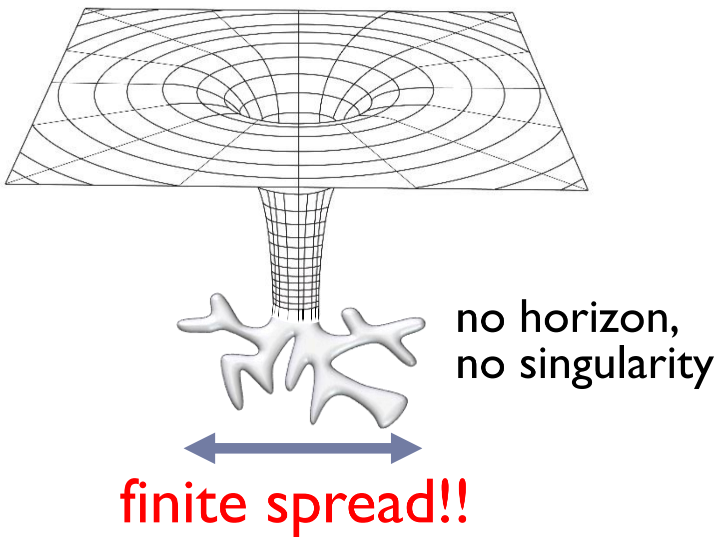
individual microstate?

$$\rho = \sum_i e^{-\beta E_i} |\psi_i\rangle \langle\psi_i|$$

$$|\psi_i\rangle$$

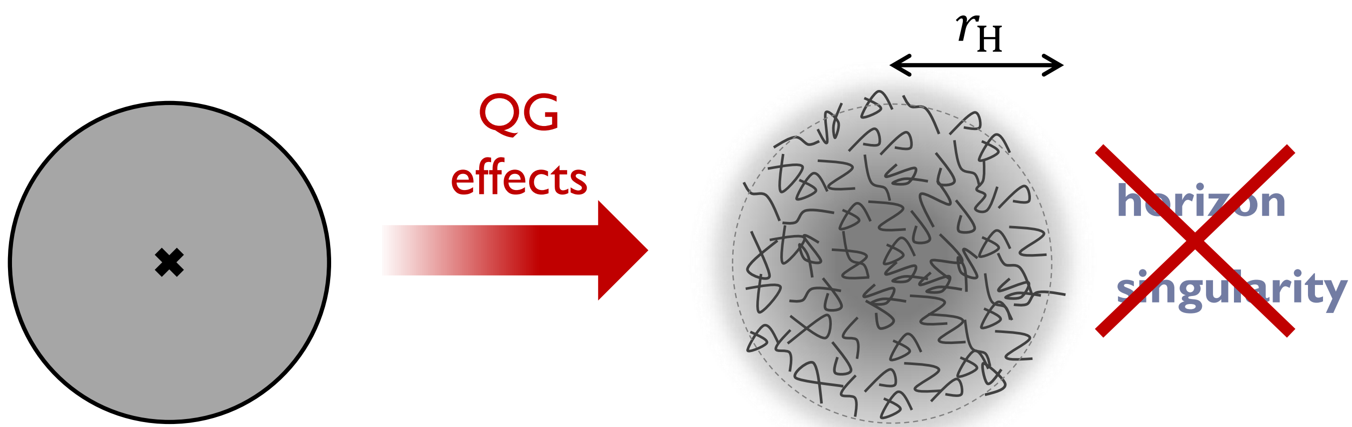
Microstate geometries





In some cases, black hole microstates are described by smooth horizonless solutions of classical gravity

Fuzzball conjecture



conventional picture

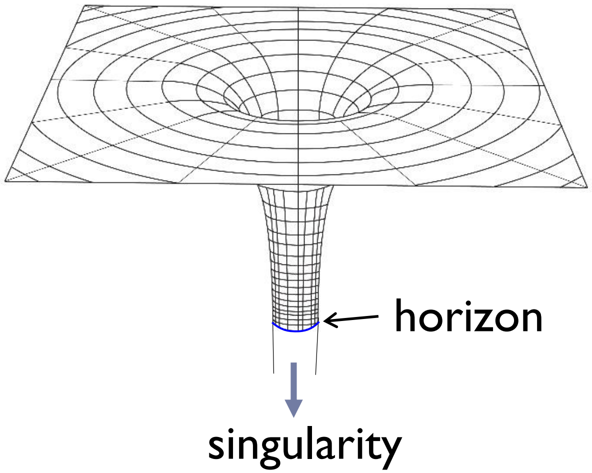
fuzzball picture

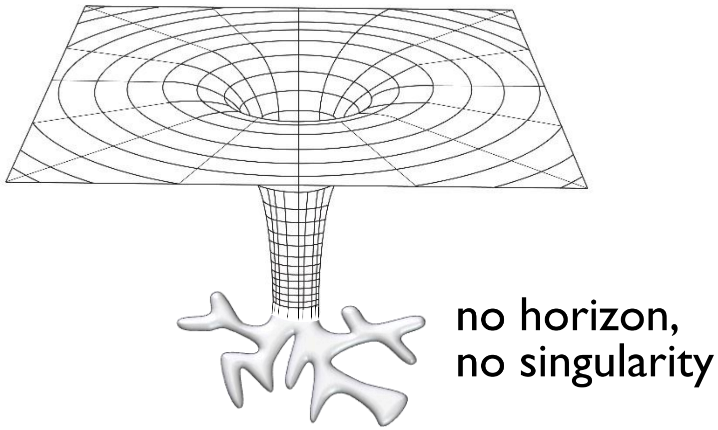
Can that be generally true?

— BH microstates are some *stringy* configurations *spreading over a wide distance?*

Microstate Geometry Program

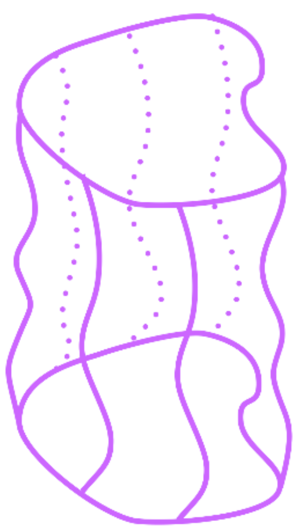
How much of black hole entropy can be accounted for by smooth, horizonless solutions of classical gravity?





Superstrata

- A new class of microstate geometries for D1-D5-P 3-charge BH.
- Most general class with known CFT dual

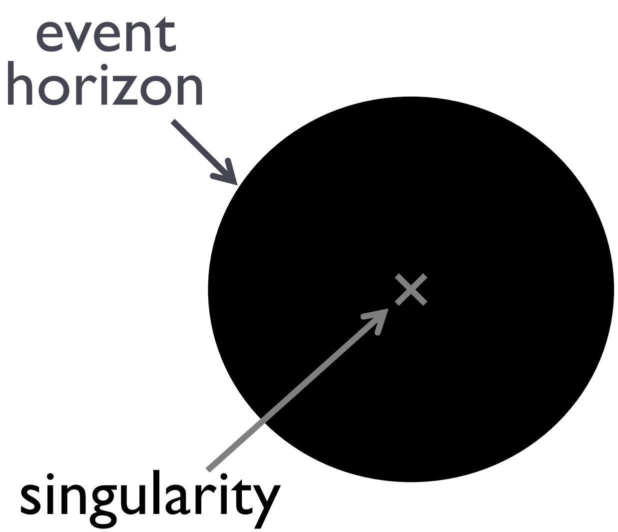


Explicitly constructed basic ones

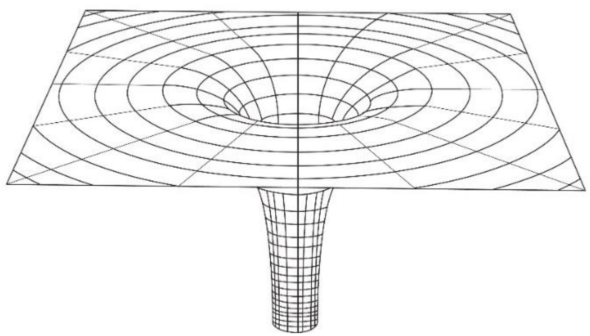
Need more general ones to reproduce BH entropy

1. Black hole microstates

Black holes



- Solution to Einstein equations
- Boundary of no return: event horizon
- Spacetime breaks down at spacetime singularity



- Classical (macro) physics: Well understood
- Quantum (micro) physics: Various puzzles

BH entropy puzzle

- BH entropy:

$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{N}}}$$

Stat mech: $\boxed{N_{\mathrm{micro}} = e^{S_{\mathrm{BH}}}}$

- BH with solar mass: $S_{BH} \sim 10^{77}$

Uniqueness theorem: only one BH solution

$$\boxed{N_{\mathrm{gravity}} = 1}$$

BH entropy puzzle

$$\frac{N_{\mathrm{gravity}}}{N_{\mathrm{micro}}} = 10^{10^{77}}$$: huge discrepancy

Cf. Cosmo. const. problem: $$\frac{\Lambda_{\mathrm{expected}}}{\Lambda_{\mathrm{observed}}} = 10^{120}$$

— ***Where are the microstates?***

- Uniqueness theorems
  → Avoided in higher dimensions
- Need quantum gravity (string theory)?
  We don't really know QG. What to do?

AdS/CFT correspondence

A way out : AdS/CFT

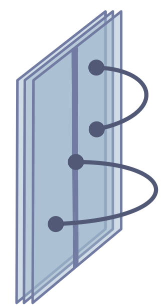
string theory / gravity in AdS space $\Longleftrightarrow$ quantum field theory (CFT)

- Defines string / QG
- Can in principle study BHs in full string / QG

BH microstate counting in AdS/CFT

string theory / gravity in AdS space ⟺ quantum field theory (CFT)

black hole → thermodyn. ensemble



Actually, SV predates AdS/CFT.

Valid for all couplings for 2D CFT

$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{N}}} \overset{!}{=} S_{\mathrm{CFT}} = \log N_{\mathrm{micro}}$$

[Strominger-Vafa '96]

→ Stat mech interpretation of BH put on firm ground

BH microstates

string theory / gravity in AdS space ⟺ quantum field theory (CFT)

black hole → thermodyn. ensemble

$\Cup$

microstate in gravity picture? ← individual microstate $|\Psi\rangle$

- Must be a state of quantum gravity / string theory in general

By "gravity" picture, I mean the bulk side of AdS/CFT and not classical gravity. Can be fully stringy / QG

Summary:

We want to know
the *gravity picture*
of BH microstates!

Again, by "gravity" picture, I
don't mean classical gravity;
it can be fully stringy / QG

2. Microstate geometries

Are any examples of gravity microstates known?

They generally require full string theory…

- Involves all string oscillators
- $g_{\mu\nu}, B_{\mu\nu}, \Phi, C$ are massless truncations

– Yes!

We know examples of microstates called *microstate geometries*.

- Solution of *classical* gravity (no massive string modes)
- Has same mass & charge as the BH
- Smooth & horizonless
- Many are susy, but some are non-susy

Example 1: LLM geometries

[Lin-Lunin-Maldacena 2004]

N D3-branes,
16 supersymmetries

AdS_5/SYM_4 in ½ BPS sector

Sugra in $AdS_5 x S^5$

D=4, $\mathcal{N}$=4 SYM

½ BPS BH: "superstar"
(singular, $A = 0$)

N D3-branes

½ BPS states (16 susy's)

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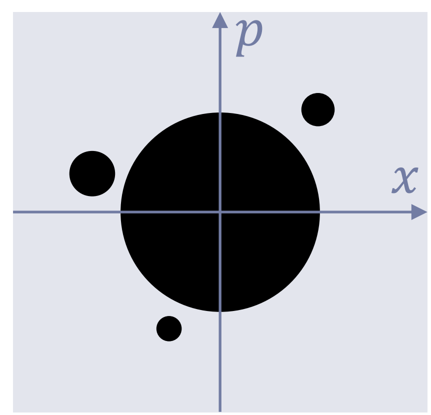
N free fermions in harmonic potential

$H = \frac{1}{2}(p^2 + x^2)$



At large N, the state is represented by droplets on the phase space

LLM (bubbling) geometries

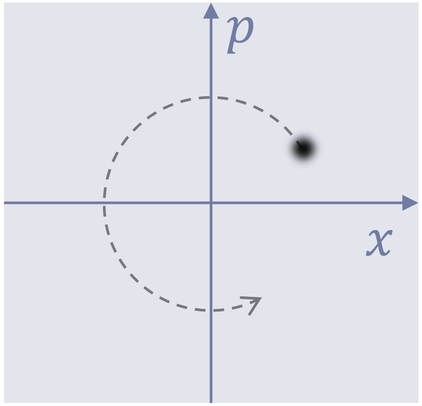
no stringy modes!

CFT side: free fermions

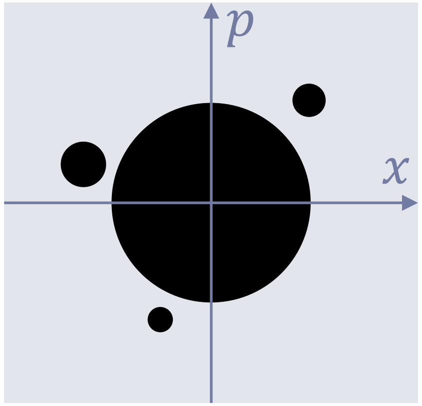
½ BPS states $=$ N free fermions in harmonic potential

[Berenstein '04] [Corley+Jevicki+Ramgoolam '02]

$$H_{1-\text{particle}} = \frac{1}{2}(p^2 + x^2)$$



1-particle
coherent state



large number (N) particles:
droplets in 1-particle phase space

LLM geometries (1)

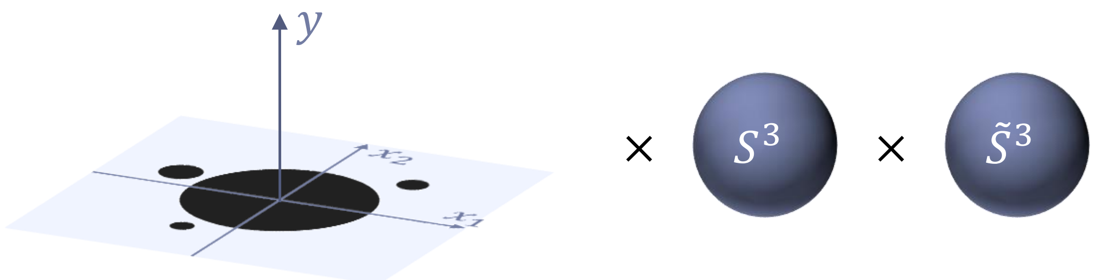
$$ds^2 = -h^{-2}(dt+V)^2 + h^2\left(dy^2 + dx_1^2 + dx_2^2\right) + ye^{G}d\Omega_3^2 + ye^{-G}d\tilde{\Omega}_3^2$$

$$h^{-2} = 2y\cosh G \qquad e^{2G} = \frac{1/2+z}{1/2-z}$$

$$[\partial_1^2 + \partial_2^2 + y\partial_y(y^{-1}\partial_y)]z(x_1,x_2,y) = 0$$

black: $z \to +\frac{1}{2}$

white: $z \to -\frac{1}{2}$



- LLM diagram encodes how S^3's shrink
- Smooth horizonless geometries
- Non-trivial topology supported by flux
- 1-to-1 correspondence with coherent states in CFT

no uniqueness thm in 10D

LLM geometries (2)

$$ds^2 = -h^{-2}(dt+V)^2 + h^2\left(dy^2 + dx_1^2 + dx_2^2\right) + ye^{G}d\Omega_3^2 + ye^{-G}d\tilde{\Omega}_3^2$$

$$h^{-2} = 2y\cosh G \qquad e^{2G} = \frac{1/2+z}{1/2-z}$$

$$[\partial_1^2 + \partial_2^2 + y\partial_y(y^{-1}\partial_y)]z(x_1, x_2, y) = 0$$

$$y\partial_y V_i = \epsilon_{ij}\partial_j z \qquad y\partial_i V_j = \epsilon_{ij}\partial_y z$$

$$F_{(5)} = F_2 \wedge d\Omega_3 + \tilde{F}_2 \wedge d\,\tilde{\Omega}_3$$

$$F_2 = dB_t \wedge (dt+V) + B_t dV + d\hat{B} \qquad \tilde{F}_2 = d\tilde{B}_t \wedge (dt+V) + \tilde{B}_t dV + d\hat{\tilde{B}}$$

$$B_t = -\frac{1}{4}y^2 e^{2G} \qquad \tilde{B}_t = -\frac{1}{4}y^2 e^{-2G}$$

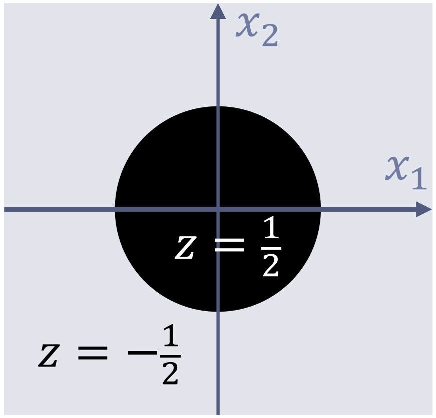
$$d\hat{B} = -\frac{1}{4}y^3 *_3 d\left(\frac{z+1/2}{y^2}\right) \qquad d\hat{\tilde{B}} = -\frac{1}{4}y^3 *_3 d\left(\frac{z-1/2}{y^2}\right)$$

Smoothness (1)

$$ds^2 = -h^{-2}(dt+V)^2 + h^2\left(dy^2 + dx_1^2 + dx_2^2\right) + ye^G d\Omega_3^3 + ye^{-G} d\widetilde{\Omega}_3^2$$

$$h^{-2} = 2y\cosh G \qquad e^{2G} = \frac{1/2+z}{1/2-z}$$

$$[\partial_1^2 + \partial_2^2 + y\partial_y(y^{-1}\partial_y)]z(x_1,x_2,y) = 0$$



Laplace eq explicitly solved:

$$z(\mathbf{x},y) = \frac{y^2}{\pi}\int \frac{z(\mathbf{x}',0)d^2\mathbf{x}}{[(\mathbf{x}-\mathbf{x}')^2+y^2]^2}$$

Behavior near the y=0 plane

$$z = \frac{1}{2} \; at \; y=0 \;\Rightarrow\; z = \frac{1}{2} - y^2 c_+(\mathbf{x})^2 + \cdots, \quad e^{-G} \sim y c_+(\mathbf{x}), \quad h^2 \sim c_+(\mathbf{x})$$

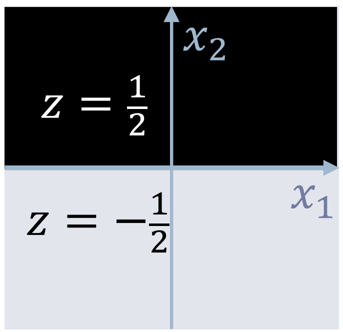
$y \to 0$ behavior of $S^3, \tilde{S}^3$:

$$h^2 dy^2 + ye^G d\Omega_3^2 + ye^{-G} d\widetilde{\Omega}_3^3 \;\sim\; c_+(\mathbf{x})\left(dy^2 + y^2 d\widetilde{\Omega}_3^2\right) + \frac{d\Omega_3^2}{c_+(\mathbf{x})}$$

- $\tilde{S}^3$ shrinks smoothly
- S^3 remains finite (radius: $1/c_+(\mathbf{x})$)

Smoothness (2)

Behavior near the black/white boundary



$$z(\mathbf{x}, y) = \frac{1}{2}\frac{x_2}{\sqrt{x_2^2 + y^2}} = \pm\left(\frac{1}{2} - \frac{y^2}{4x_2^2} + \cdots\right)$$

$$c_{\pm}(\mathbf{x}) = \frac{1}{2|\mathrm{x}_2|}$$

Exact expression:

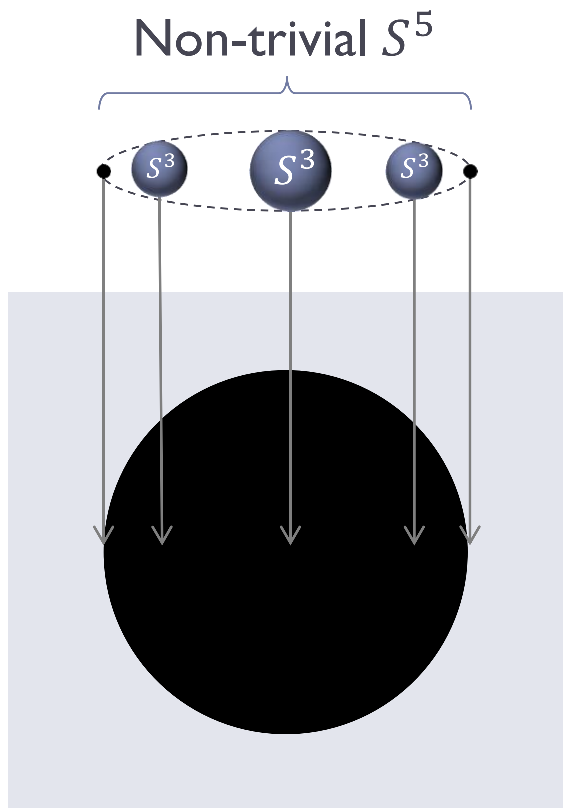
$$h^2 dy^2 + ye^G d\Omega_3^2 + ye^{-G} d\widetilde{\Omega}_3^2$$
$$= \frac{dy^2}{2\sqrt{x_2^2 + y^2}} + \left(\sqrt{x_2^2 + y^2} - x_2\right) d\widetilde{\Omega}_3^2 + \left(\sqrt{x_2^2 + y^2} + x_2\right) d\Omega_3^2$$

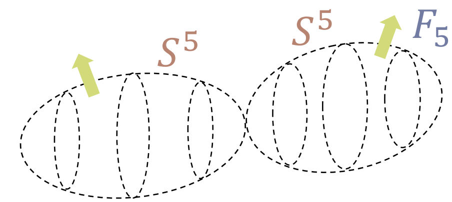
$x_2 \geq 0$:

- $\tilde{S}^3$ shrinks smoothly, S^3 remains finite
- On the black-white boundary, S^3 shrinks

Smoothness (3)

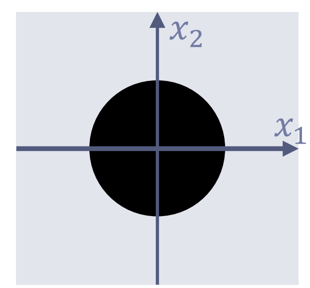
- S^3 shrinks as one approaches the boundary from inside
- S^3 over a disk is S^5
- Flux through S^5 is proportional to the area of black region





Flux supports non-trivial S^5

E.g. pure $AdS_5 x S^5$



$$N = \frac{(\text{area})}{4\pi^2 l_p^4} \qquad l_p = g_s^{1/4} l_s$$

Flux quantization

$$F_{(5)} = F_2 \wedge d\Omega_3 + \tilde{F}_2 \wedge d\,\tilde{\Omega}_3$$

$$F_2 = dB_t \wedge (dt + V) + B_t dV + d\hat{B} \qquad \tilde{F}_2 = d\tilde{B}_t \wedge (dt + V) + \tilde{B}_t dV + d\hat{\tilde{B}}$$

$$B_t = -\frac{1}{4} y^2 e^{2G} \qquad \tilde{B}_t = -\frac{1}{4} y^2 e^{-2G}$$

$$d\hat{B} = -\frac{1}{4} y^3 *_3 d\left(\frac{z + 1/2}{y^2}\right) \qquad d\hat{\tilde{B}} = -\frac{1}{4} y^3 *_3 d\left(\frac{z - 1/2}{y^2}\right)$$

$$N = -\frac{1}{2\pi^2 l_p^4} \int d\hat{B} = \frac{1}{8\pi^2 l_p^4} \int_{\Sigma_2} y^3 *_3 d\left(\frac{z + \frac{1}{2}}{y^2}\right)$$
$$= \frac{(Area)_{z=1/2}}{4\pi^2 l_p^4}$$

Classical limit

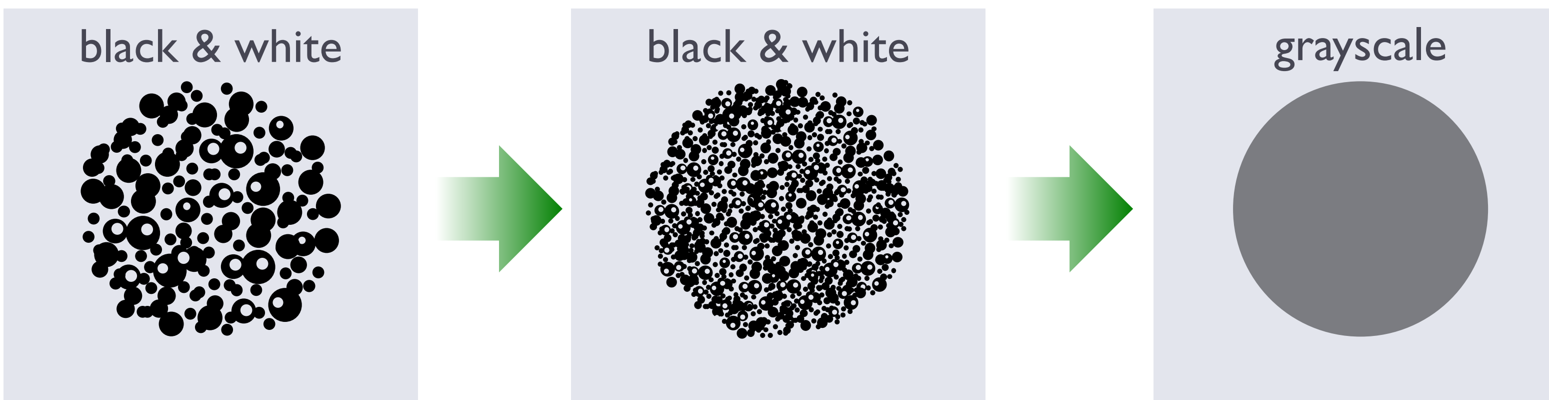
cf. what enters metric:
$H_{D3} = 1 + \frac{4\pi N g_s l_s^4}{r^4}$

How is naive singular geometry (superstar) recovered?

- Bubble area quantized

$$(\text{area}) = Nh, \qquad h = 4\pi^2 l_p^4 = 4\pi^2 g_s l_s^4$$

- Classical limit: $l_p \to 0,\ N \to \infty$ with fixed $Q = N g_s l_s^4 = Nh$



smooth

smooth

singular
(superstar)

Quantization

Why are we talking about classical sol'ns and h at the same time?

LLM solutions: basis on which quantization is carried out

(solution space in gravity) = (phase space)

$$\Big\downarrow \; [q,p] = i\hbar$$

quantum Hilbert space

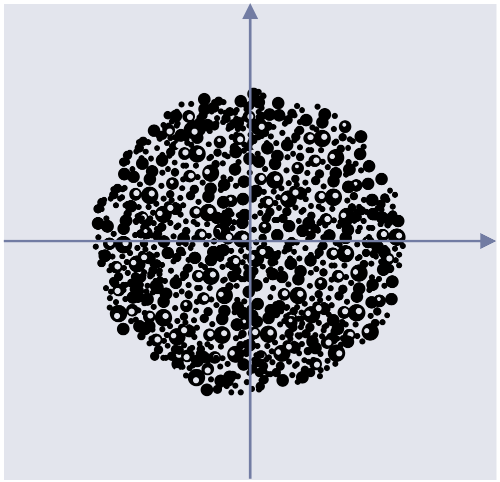
Each LLM solution is like $|x\rangle$ of a free particle

(Or, it's like classical orbit of electron in old quantum theory)

[Grant+Maoz+Marsano+Papadodimas+Rychkov '05]

Comments

- Generic states have large curvature
  - → Higher derivative corrections non-negligible
  - → LLM sol's are soln's of two-derivative gravity and are not reliable
  - → But higher derivative corrections should not change qualitative picture; DoF must be the same (no massive stringy modes needed)



smooth, but curvature large

Smooth, no singularity to be resolved

Example 2: LM geometries

[Lunin-Mathur 2001]

[Lunin-Maldacena-Maoz 2002]

N_1 D1-branes + N_5 D5-branes,
8 supersymmetries

LM geometries (1)

Sugra in AdS_3xS^3

D=2, $\mathcal{N}$=(4,4) CFT

2-charge BH
(singular, $A = 0$)

N_1 D1-branes
N_2 D5-branes

½ BPS states (8 susy's)

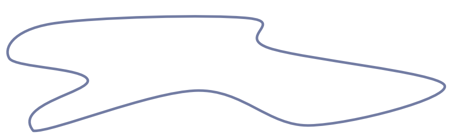
||

free bosons in 2D
$\partial X \sim \Sigma_n e^{in(\sigma-\tau)} \alpha_n$

Parametrized by integers
$n_1, n_2, n_3, \ldots$

$$\sum_k k n_k = N_1 N_2$$

"LM geometries"



LM geometries (2)

$$ds^2 = -\frac{2}{\sqrt{Z_1 Z_2}}(dv+\beta)(du+\omega) + \sqrt{Z_1 Z_2}\,dx_{1234}^2 + \sqrt{Z_1/Z_2}\,dx_{6789}^2$$

$$Z_1(\vec{x}) = 1 + \frac{Q_2}{L}\int_0^L \frac{|\dot{\vec{F}}|^2 d\lambda}{|\vec{x}-\vec{F}(\lambda)|^2}, \qquad Z_2(\vec{x}) = 1 + \frac{Q_2}{L}\int_0^L \frac{d\lambda}{|\vec{x}-\vec{F}(\lambda)|^2} \qquad \cdots$$

arbitrary curve $\vec{x} = \vec{F}(\lambda) \in \mathbb{R}^4_{1234}$

- LM curve encodes how S^1 shrinks
- Smooth horizonless geometries supported by flux
  - 1-to-1 correspondence with CFT states: $\vec{F}(\lambda) \leftrightarrow \{n_k\}$

Fourier coeffs of $\vec{F}(\lambda)$ $\Longleftrightarrow$ $\{n_k\}$

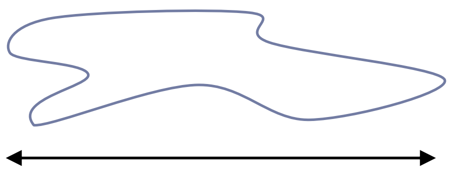
- Entropy reproduced geometrically: $S \sim \sqrt{N_1 N_2}$

[Rychkov '05]
[Krishnan+Raju '15]

Classical limit

How is naive singular geometry recovered?

smooth



$$\mathcal{R} \sim g_s^{1/3} l_s^{1/3} (Q_1 Q_2)^{1/6}$$
$$\sim g_s^{2/3} l_s (N_1 N_2)^{1/6}$$

classical limit

singular

$g_s \to 0, l_s \to 0,$

$N_{1,2} \to \infty$

Fix $Q_{1,2} \sim g_s l_s^2 N_{1,2}$

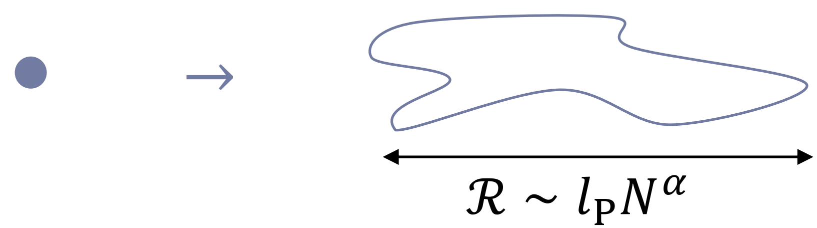
Q: what enters sugra solution

N: quantized charge

Summary:

Some "BH" microstates *are* represented by *microstate geometries*.

— Naive BH solutions are replaced by bubbling geometries with *finite spread*.



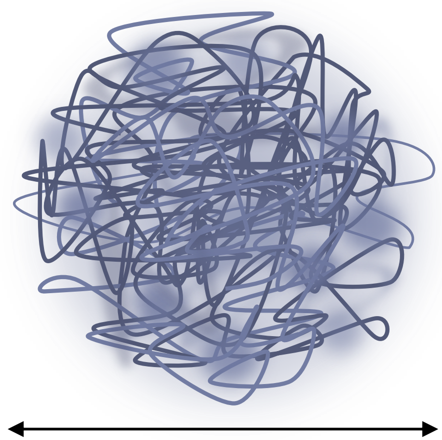
(but recall $A = 0$ so far)

The more branes you put, the larger the spread (this is against standard intuition!)

3. Fuzzball conjecture & microstate geometry program

Maybe the same is true for genuine black holes?

— BH microstates are some stringy configurations *spreading over a wide distance*?



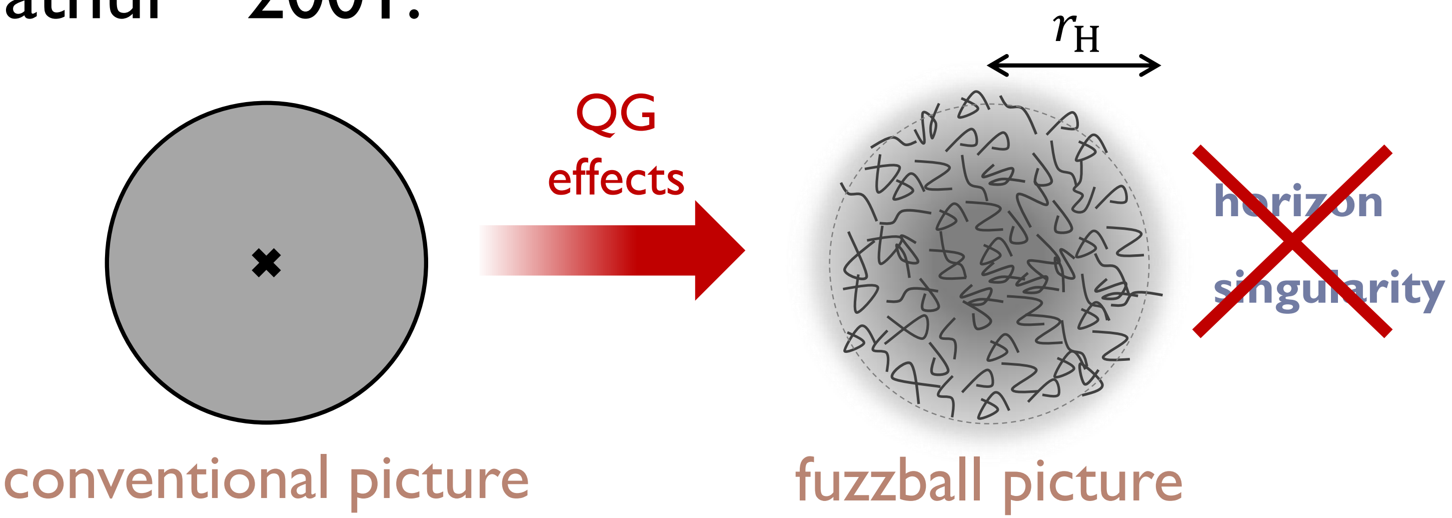
$$\mathcal{R} \sim l_{\mathrm{P}} N^{\alpha} \sim r_H ??$$

I'm talking about the size, not about whether it's describable in classical gravity.

Again, this goes against standard intuition in gravity

Fuzzball conjecture

- Mathur ~2001:



- BH microstates = QG/stringy "fuzzballs"
- No horizon, no singularity
- Spread over horizon scale

There may be singularities allowed in string theory, but scattering is unitary

Sugra fuzzballs (1)

Sugra = all fields in massless sector of string theory. $g_{\mu\nu}, B_{\mu\nu}, \Phi, C,$ fermions

Are fuzzballs describable in sugra?

- Unlikely in general
  - General fuzzballs must involve all string modes
  - Massive string modes are not in sugra

- Hope for susy (BPS) states
  - Massive strings break susy
    - → Only massless (sugra) modes allowed?
  - "Example": MSW (wiggling M5)

[Maldacena+Strominger+Witten 1997]

LLM and LM are supporting evidence, although $A = 0$

Sugra fuzzballs (2)

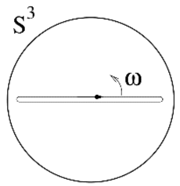
Are supersymmetric states any good?

- More tractable
  - First order PDEs
- Can tell us about *mechanism*
  - Mechanism for horizon-sized structure
- Basic string theory objects are locally susy

Sugra fuzzballs (3)

Absolutely no hope for non-susy states?

- If something is described in sugra or not is a tricky question
  - Cf. Macroscopic string [Gubser+Klebanov+Polyakov '02]
  - Large # of quanta more relevant for classicality?



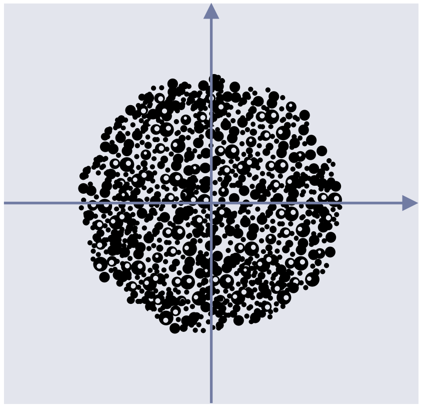
- The effect of massive modes captured by massless sector?
  - Need all orders in α' expansion?
- Some non-susy microstates known

e.g. [Jejjala-Madden-Ross-Titchener '05]

Sugra fuzzballs (3)

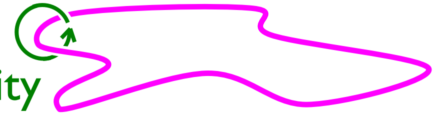
Caveats:

- Generic states have large curvature
  - Higher derivative corrections nonnegligible
  - But should not change qualitative picture; DoF must be the same



Example in LLM:
smooth, but curvature large

- Non-geometries
  - Non-geometric microstates possible [Park+MS 2015]
  - Need to extend framework (DFT, EFT)



Microstate geometry program:

What portion of the BH entropy of supersymmetric BHs is accounted for by smooth, horizonless solutions of classical sugra?

The answer may turn out to be 0, or 1.

This is my definition here. Some other people want to construct microstates for non-susy ones too.

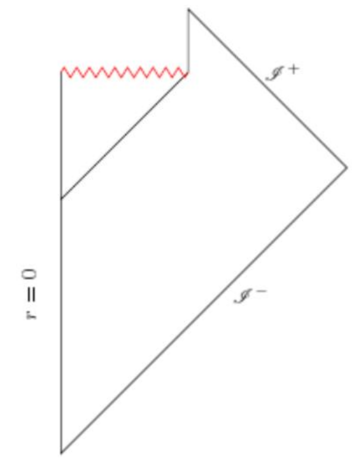
Comment: bottom up approach

[Mathur '09] $O(1)$ deviation from flat space is needed for Hawking radiation to carry information

- Based on Q info (strong subadditivity)

[AMPS '12] "Firewall"

- More arguments based on Q info (monogamy, etc.)



These arguments are "bottom-up"

→ Mechanism to support finite size not explained

Microstate geometry program is "top-down"

→ Finite size supported by topology with fluxes

I'm not saying that bottom-up approach is bad because it can't explain the size. It's a feature of the approach.

Summary of the last lecture

MGP works for certain systems:

- D3 $\leftrightarrow$ LLM geom
- 2-charge sys (D1-D5 sys) $\leftrightarrow$ LM geom

However, they are not real BHs (A=0).

Need to go to systems with a finite horizon to carry out MGP

Finite horizon

Let's review a class of *real* BH microstate geometries, including their pros & cons.

5D microstate geometries: circa 2004–09

4. Microstate geometries in 5D

3-charge system

- Susy BH in 5D (4 supercharges)
- Canonical rep [Strominger-Vafa 1996]

IIB on $S^1_5 \times T^4_{6789}$

	1	2	3	4	5	6	7	8	9
N_1 D1	•	•	•	•	O	~	~	~	~
N_2 D5	•	•	•	•	O	O	O	O	O
N_3 P	•	•	•	•	O	~	~	~	~

- Decoupling → $AdS_3 \times S^3 \times T^4$ / D1-D5 CFT
- Macroscopic entropy: $S \sim \sqrt{N_1 N_2 N_3}$

4-charge system

- Susy BH in 4D / BS in 5D (4 supercharges)
- Canonical rep [Maldacena-Strominger-Witten 1997]

M on T^6_{456789}

	1	2	3	4	5	6	7	8	9	A
N_1 M5	•	•	•	~	~	O	O	O	O	O
N_2 M5	•	•	•	O	O	~	~	O	O	O
N_3 M5	•	•	•	O	O	O	O	~	~	O
N_4 P	•	•	•	~	~	~	~	~	~	O

- Decoupling → $AdS_3 \times S^2 \times T^6$ / MSW CFT
- Macroscopic entropy: $S \sim \sqrt{N_1 N_2 N_3 N_4}$

M-theory frame

Want to find gravity microstates for 3- & 4-charge systems

Start from 3-charge system

IIB / T^5_{56789} $\quad \mathrm{D1(5), D5(56789), P(5)}$

$\Big\downarrow$ T_5, T_6, T_7

IIA / T^5_{56789} $\quad \mathrm{D2(67), D2(89), F1(5)}$

$\Big\downarrow$ lift along x^{A}

M / $T^6_{56789\mathrm{A}}$ $\quad \mathrm{M2(67), M2(89), M2(5A)}$

Nicely symmetric

$\Big\Downarrow$

Take M-theory on T^6 and go to 5D

4-charge MSW system is already in M-theory frame

Ansatz

- M-theory on $T^6_{56789\mathrm{A}}$ $\qquad\qquad$ $\mathrm{A} = 10$

$$ds^2_{11} = ds^2_5 + X^1\left(dx_5^2 + dx_6^2\right) + X^2\left(dx_7^2 + dx_8^2\right) + X^3\left(dx_9^2 + dx_{\mathrm{A}}^2\right)$$

$$\mathcal{A}_3 = \underbrace{A^1}_{\substack{\mathrm{M2}(56)\\ \updownarrow \\ \mathrm{M5}(\lambda 789\mathrm{A})}} dx_5 \wedge dx_6 + \underbrace{A^2}_{\substack{\mathrm{M2}(78)\\ \updownarrow \\ \mathrm{M5}(\lambda 569\mathrm{A})}} dx_7 \wedge dx_8 + \underbrace{A^3}_{\substack{\mathrm{M2}(9\mathrm{A})\\ \updownarrow \\ \mathrm{M5}(\lambda 5678)}} dx_9 \wedge dx_{\mathrm{A}}$$

5D theory

8 susys

- $D = 5, \mathcal{N} = 1$ sugra with 2 vector multiplets

gauge fields: $A^I_\mu\,,\ I = 1{,}2{,}3.\quad F^I \equiv dA^I.$

scalars: $X^I,\ \ X^1X^2X^3 = 1$

$X^1X^2X^3$ is a hyper scalar

- Action

$$S_{\rm bos} = \int \Big(*_5 R - Q_{IJ}\, dX^I \wedge *_5\, dX^I - Q_{IJ} F^I \wedge *_5\, F^J - \frac{1}{6} C_{IJK} \underbrace{F^I \wedge F^J \wedge A^K}_{\text{Chern-Simons interaction}} \Big)$$

$$C_{IJK} = |\epsilon_{IJK}|\,, \qquad Q_{IJ} = \tfrac{1}{2}\mathrm{diag}(1/X^1, 1/X^2, 1/X^3)$$

No solitons without topology (1)

[Gibbons-Warner '13] [Haas '14]

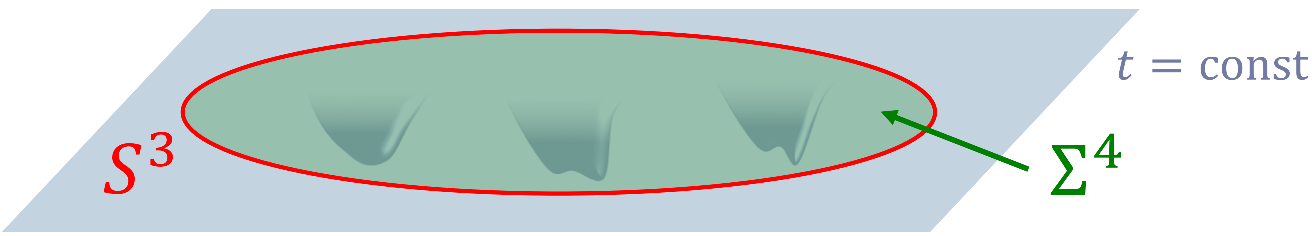
- Komar mass/Smarr formula

$V^\mu = \frac{\partial}{\partial t}$: Killing

used $\nabla^2 K^\mu = -R_{\mu\nu} K^\nu$

$$\Rightarrow \quad M \sim \int_{S^3} *_5 \, dV \sim \int_{\Sigma^4} *_5 \left(V^\mu R_{\mu\nu} dx^\nu\right),$$

if there is no internal boundary.



If there is a horizon, it will be an internal boundary and we get Smarr's formula relation M,S,T (for vacuum grav, $R_{\mu\nu} = 0$ and no bulk contribution.

No solitons without topology (2)

EOMs / Bianchi:

$$dF^I = 0$$

$$dG_I = 0, \qquad G_I \equiv *_5\, Q_{IJ}F^J + C_{IJK}F^J \wedge A^K$$

$$R_{\mu\nu} = Q_{IJ}\partial_\mu X^I \partial_\nu X^J + Q_{IJ}F^I{}_{\mu\rho}F^J{}_\nu{}^\rho + Q^{IJ}G_{I\,\mu\rho\sigma}G_{J\,\nu}{}^{\rho\sigma} \qquad (*)$$

(ignoring numerical factors)

Assume time-independent config:

$$\mathcal{L}_V X^I = \mathcal{L}_V F^I = \mathcal{L}_V G_I = 0$$

$$\rightarrow d\left(\iota_V F^I\right) = d(\iota_V G_I) = 0 \qquad (\text{used } \mathcal{L}_V = d\iota_V + \iota_V d)$$

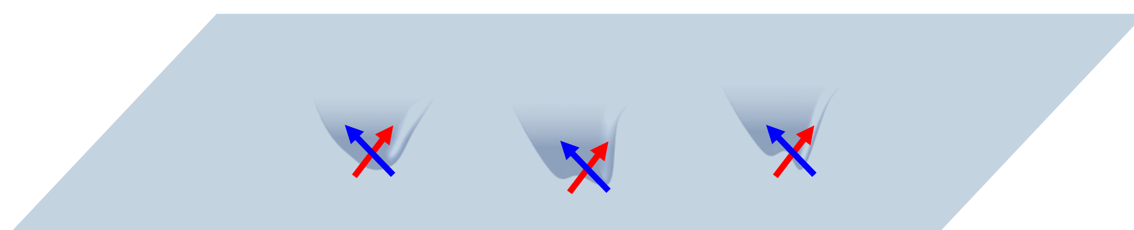
$$\rightarrow \iota_V F^I = \underbrace{f^I}_{\in H^1(\Sigma^4):\ \text{elec flux}} + (\text{exact}), \quad \iota_V G_I = \underbrace{g_I}_{\in H^2(\Sigma^4):\ \text{mag flux}} + (\text{exact}).$$

Now contract (*) with K^μ, and plug it into Komar integral

No solitons without topology (3)

$$M \sim \int_{\Sigma^4} \left(\overset{\text{elec}}{f_I} \wedge \overset{\text{mag}}{G^I} + \overset{\text{mag}}{g^I} \wedge \overset{\text{elec}}{F_I} \right)$$

- M can be topologically supported by crossing of elec & mag fluxes in the cohomology $H^*(\Sigma^4)$.
- No spatial topology → $M = 0$ → Spacetime is flat



BPS solutions [Gutowski-Reall '04] [Bena-Warner '04]

- Require susy

$$ds_5^2 = -Z^{-2}(dt+k)^2 + Z \overbrace{ds_4^2}^{\text{4D base } \mathcal{B}^4 \text{ (hyperkähler)}}$$

$$A^I = \underbrace{-Z_I^{-1}(dt+k)}_{\text{elec}} + \underbrace{B^I}_{\text{mag}}, \qquad dB^I = \Theta^I$$

* timelike class

$F \sim g \sim \Theta$

$f \sim 0$

$$Z = (Z_1 Z_2 Z_3)^{1/3}; \quad X^1 = \left(\frac{Z_2 Z_3}{Z_1^2}\right)^{1/3} \text{ and cyclic}$$

All depend only on B_4 coordinates

- BPS eqs: linear system

$$\Theta^I = *_4 \, \Theta^I,$$

$$\nabla^2 Z_I = C_{IJK} \; *_4 \left(\Theta^J \wedge \Theta^K\right)$$

$$(1 + *_4)dk = Z_I \Theta^I$$

Sol'ns with U(1) sym [Gutowski-Gauntlett '04]

Solving BPS eqs in general is difficult.
Assume $U(1)$ symmetry in $\mathcal{B}^4$

* tri-holomorphic U(1)

$$ds_4^2 = V^{-1}(d\psi + A)^2 + V\underbrace{\left(dy_1^2 + dy_2^2 + dy_3^2\right)}_{\text{flat } \mathbb{R}^3},$$

(Gibbons-Hawking space)

$dA = *_3\, dV$

V is harmonic in $\mathbb{R}^3$:

$$\Box V = 0 \quad \Rightarrow \quad V = v_0 + \sum_p \frac{v_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

Multi-center KK monopole / Taub-NUT

Complete solution

All eqs solved in terms of harmonic functions in $\mathbb{R}^3$:

$$H = (V, K^I, L_I, M), \qquad H = h + \sum_p \frac{Q_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

$$\Theta^I = d\left(\frac{K^I}{V}\right) \wedge (d\psi + A) - V *_3 d\left(\frac{K^I}{V}\right)$$

$$Z_I = L_I + \frac{1}{2V} C_{IJK} K^J K^K$$

$$k = \mu(d\psi + A) + \omega$$

$$\mu = M + \frac{1}{2V} K^I L_I + \frac{1}{6V^2} C_{IJK} K^I K^J K^K$$

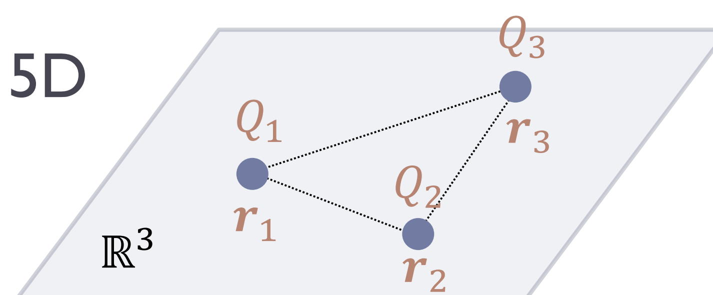
$$*_3\, d\omega = V dM - M dV + \frac{1}{2}\left(K^I dL_I - L_I dK^I\right)$$

Multi-center solution

$$H = (V, K^I, L_I, M), \qquad H = h + \sum_p \frac{Q_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

KK monopole — mag (M5) — elec (M2) — KK momentum along ψ

- Multi-center config of BHs & BRs in 5D
- Positions $\boldsymbol{r}_p$ satisfy "bubbling eq"
  (force balance)
- Reducing on ψ gives 4D BHs
  (same as Bates-Denef 2003)



Microstate geometries (1)

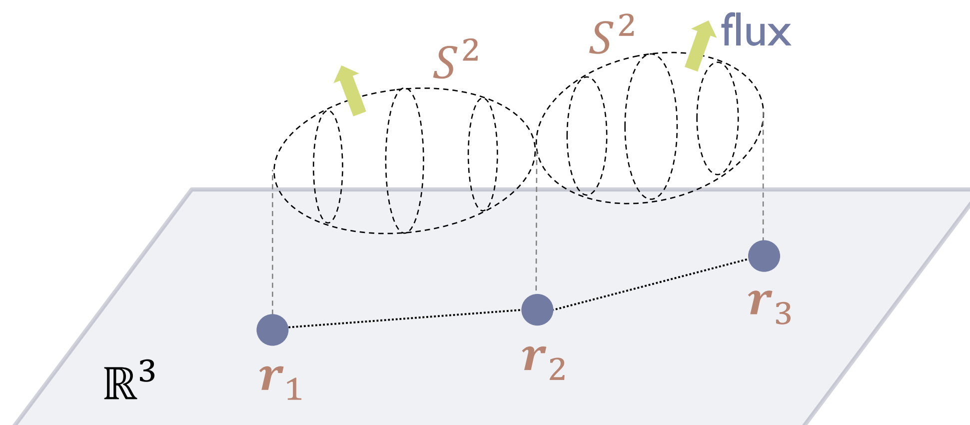
Tune charges:

$$l_p^I = -\frac{C_{IJK}}{2}\frac{k_p^J k_p^K}{v_p}$$

$$m_p = \frac{C_{IJK}}{12}\frac{k_p^I k_p^J k_p^K}{v_p^2}$$

Smooth horizonless solutions

[Bena-Warner 2006] [Berglund-Gimon-Levi 2006]

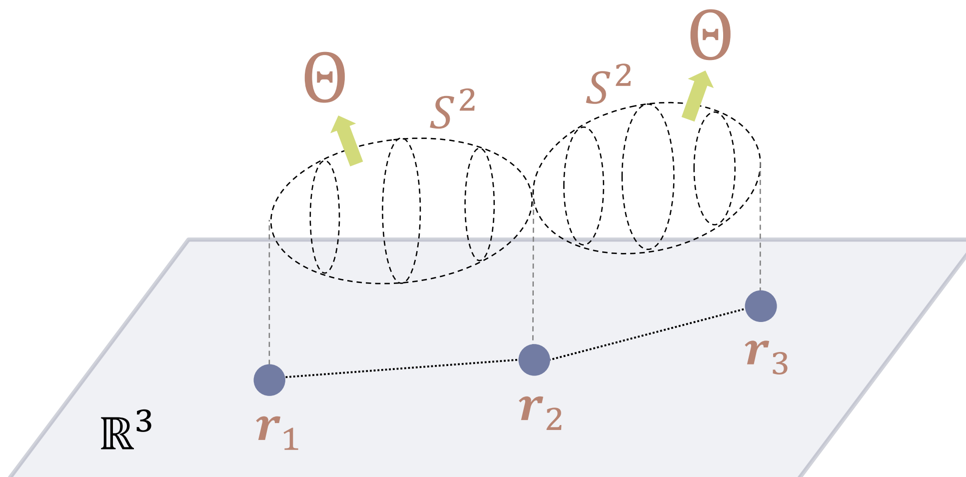


- Microstate geometries for 5D (and 4D) BHs ☺
  - ☐ Same asymptotic charges as BHs
- Topology & fluxes support the soliton
- Mechanism to support horizon-sized structure!

Komar mass/Smarr relation

$F \sim g \sim \Theta$

$f \sim 0$



$$M \sim \int F \wedge g \sim \int \Theta \wedge \Theta \sim Q_{\mathrm{elec}}$$

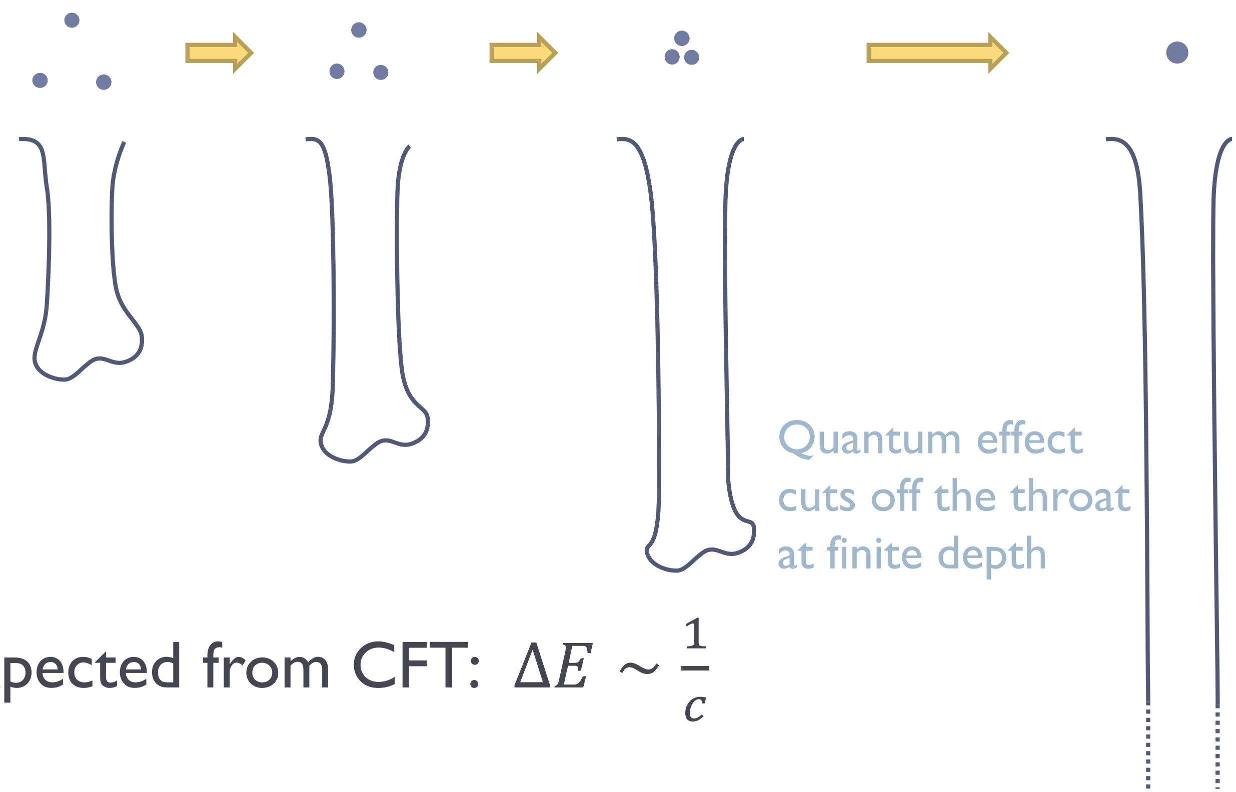
Non-trivial flux supported by magnetic fluxes

$$d * F \sim F \wedge F$$

Electric flux sourced by crossing of magnetic fluxes

Microstate geometries (2)

- Various nice properties ☺
  - Scaling solutions [BW et al., 2006, 2007]



  - Gap expected from CFT: $\Delta E \sim \frac{1}{c}$

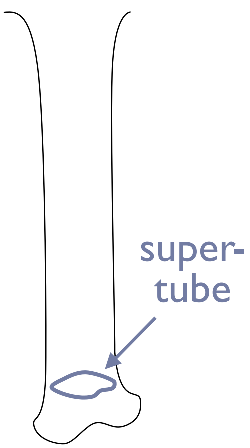
The real question:

Are there enough?

- 3-chage sys (+ fluctuating supertube)
  - Entropy enhancement mechanism [BW et al., 2008]

    → Much more entropy?
  - An estimate [BW et al., 2010]

    $$S \sim Q^{\frac{5}{4}} \ll Q^{\frac{3}{2}}$$ *Parametrically smaller* ☹

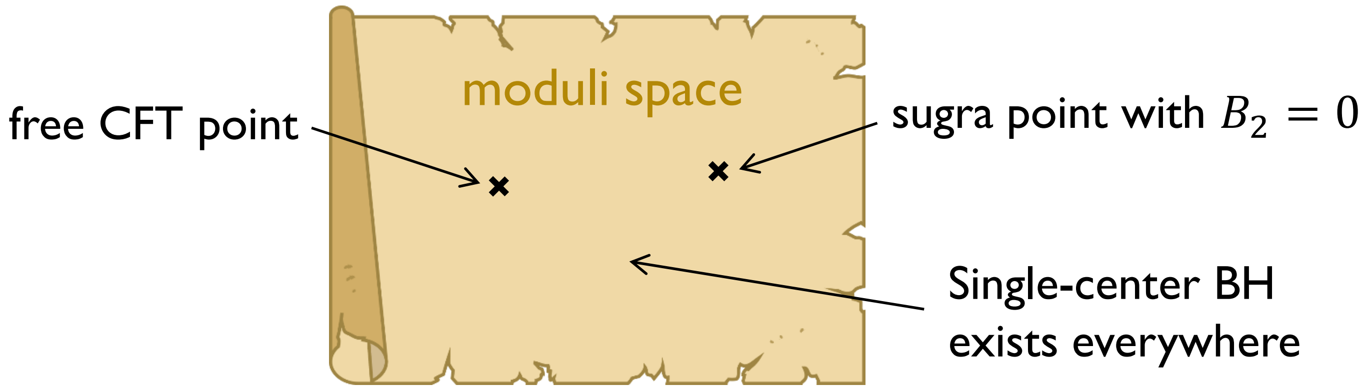


- 4-chage sys [de Boer et al., 2008-09]

  $$S \sim Q^{\frac{4}{3}} \ll Q^{2}$$
  - Quantization of D6-$\overline{\text{D6}}$-D0 config → *much less entropy* ☹

Further issues (1)

- Lifting [Dabholkar, Giuca, Murthy, Nampuri '09]



  - Single-ctr BH exists everywhere and contributes to index (elliptic genus).
  - Microstates must also exist everywhere and contribute to index.
  - But >2 center solns do not contribute to index!

    → They disappear when generic moduli are turned on?

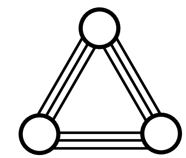
    → They are irrelevant for microstates?

  - Cf. Moulting BH [Bena, Chowdhury, de Boer, El-Showk, MS 2011]

Expect correspondence even for states that generically lift?

Further issues (2)

- Pure Higgs branch [Bena, Berkooz, de Boer, El-Showk, Van den Bleeken '12]
  - Vacua of Quiver QM (scaling regime)



Coulomb branch

- Corresponds to multi-center solutions
- Small entropy
- Generally $J \neq 0$

Pure Higgs branch

- Corresponding sugra solution unclear
- Large entropy
- $J = 0$

Cf. "supereggs"

[Denef+Gaiotto+Stroinger+van den Bleeken+Yin '07]
[Martinec '15] [Martinec+Niehoff '15]
[Reaymaekers+van den Bleeken '15]

Summary:

We found microstate geometries
for genuine BHs,
but they are *too few*.

Possibilities:

A) Sugra is not enough

B) Need more general ansatz ⇦ this talk

5. Double bubbling

2010–

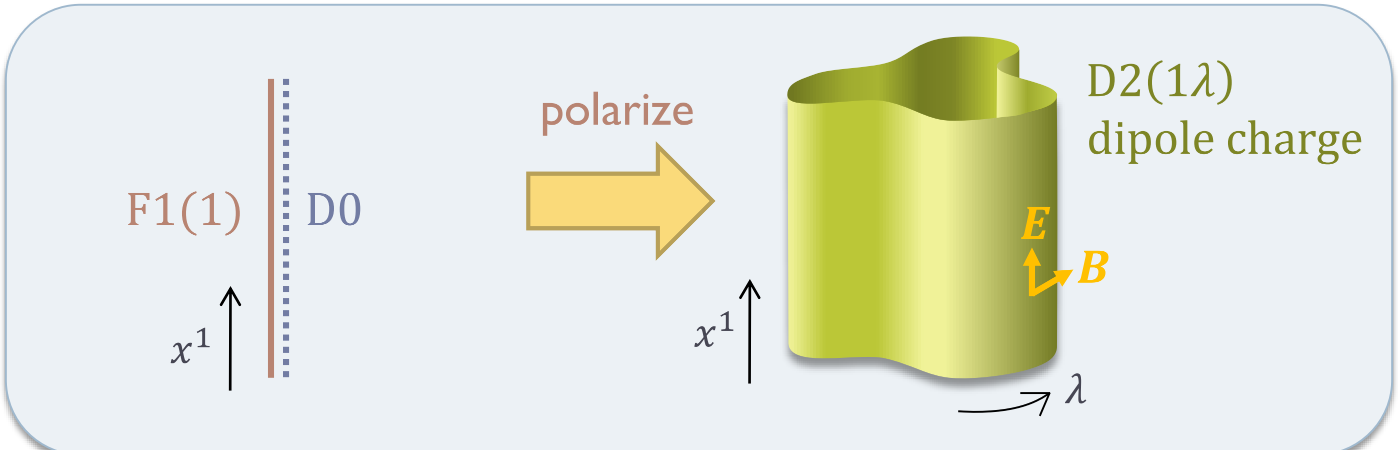
What are we missing?

— A guiding principle for constructing microstate geometries.

Revisit better understood example:
2-charge system (LM geometries)

Supertube transition [Mateos+Townsend 2001]

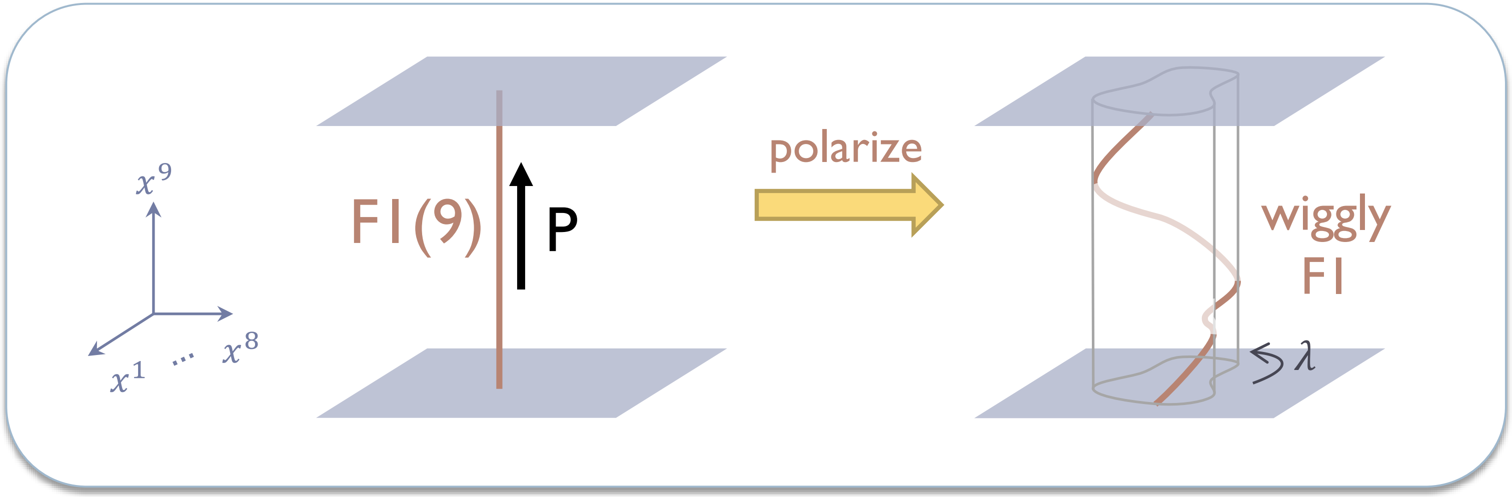
$D0 + F1(1) \longrightarrow D2(1\lambda)$



- Spontaneous polarization phenomenon
  (cf. Myers effect)
- Produces new dipole charge
- Represents *genuine bound state*
- Cross section = *arbitrary* curve

F1-P frame

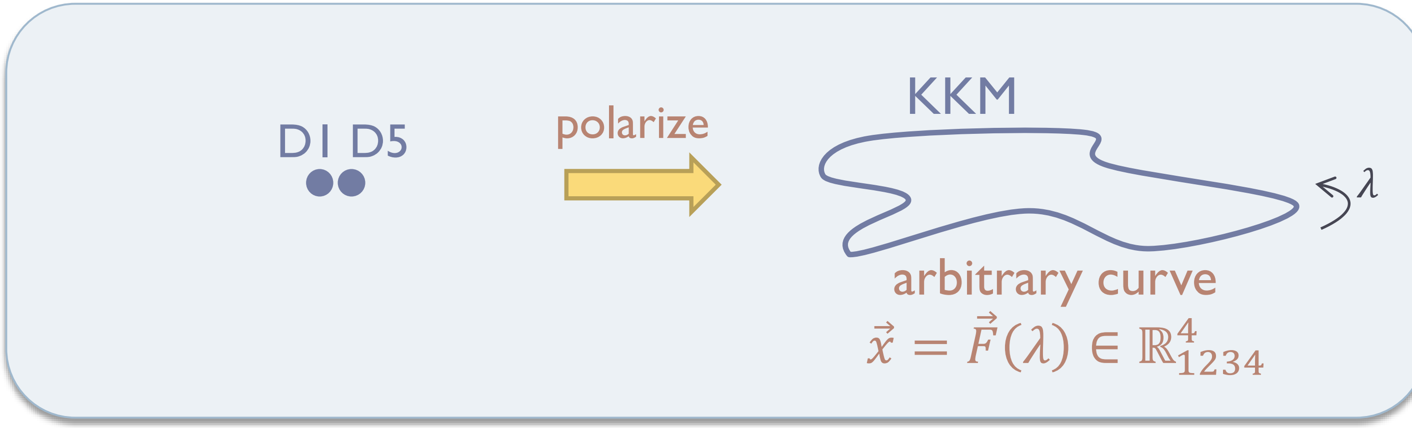
$\mathrm{F1}(9) + \mathrm{P}(9) \Rightarrow \mathrm{F1}(\lambda)$



- To carry momentum,
  F1 must wiggle in transverse $\mathbb{R}^8$
- Projection onto transverse $\mathbb{R}^8$ is an arbitrary curve

D1-D5 frame

$$\mathrm{D1(5) + D5(56789) \to KKM(\lambda 6789,5)}$$



- This is LM geometry
- Arbitrary curve → large entropy $S \sim \sqrt{N_1 N_2}$
- Explains origin of 2-charge microstate geometries

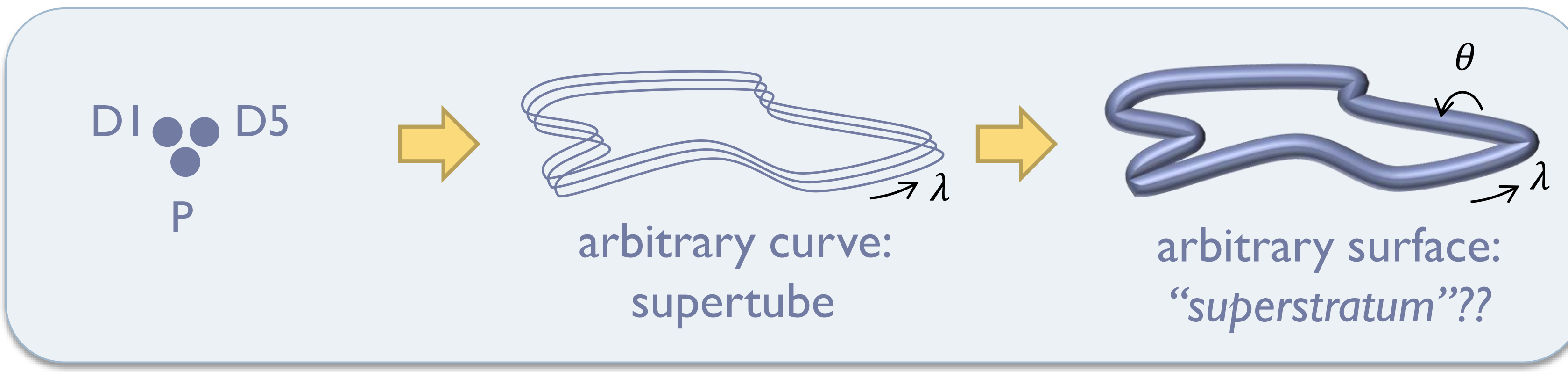
3-charge case

"Double bubbling"

D1(5)
D5(56789)
P(5)

KKM(λ6789,5)
D5(λ6789)
D1(λ)

$1_3^6(\theta,\lambda 56789)$
KKM(λ6789, θ)
$5_3^2(\theta 6789,\lambda)$

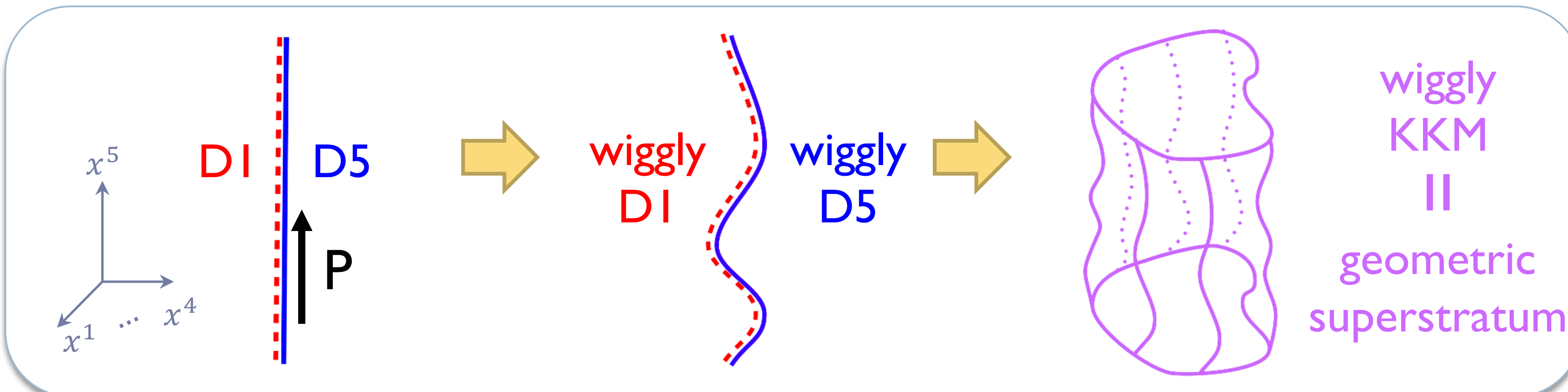


- Multiple transitions can happen in principle
- Arbitrary surface → larger entropy?
- Non-geometric in general

[de Boer+MS 2010, 2012]
[Bena+de Boer +Warner+MS 2011]

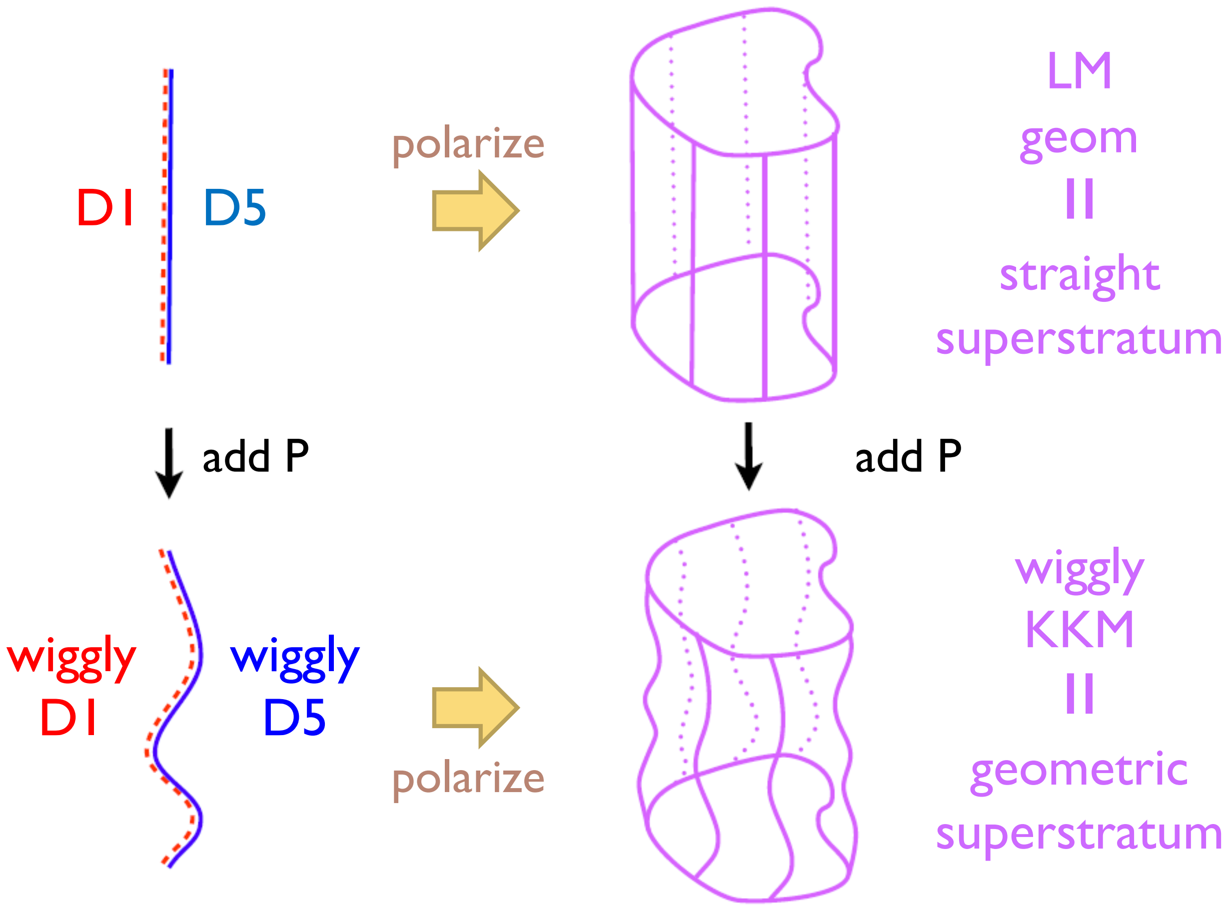
A geometric channel

$$\begin{matrix} \mathrm{D1}(5) \\ \mathrm{D5}(56789) \\ \mathrm{P}(5) \end{matrix} \Rightarrow \begin{matrix} \mathrm{D5}(\lambda 6789) \\ \mathrm{D1}(\lambda) \end{matrix} \Rightarrow \mathrm{KKM}(\lambda 6789, \theta)$$



- Dependence on x^5 is crucial
- Must live in *6D*
- Possibility to recover $S \sim \sqrt{N_1 N_2 N_3}$ [Bena+ +Warner+MS 2014]

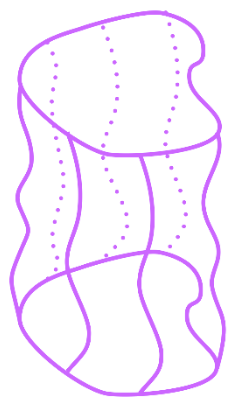
Two routes to superstratum



Summary:

Existence of superstrata
depending on functions of two variables
is a necessary condition for

$$S_{\mathrm{BH}} \sim S_{\mathrm{geom}}$$



6. Microstate geometries in 6D (sugra superstratum)

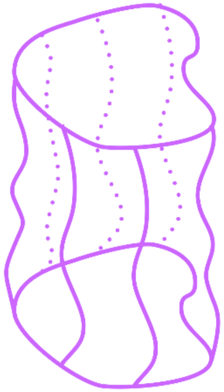
2011–

Sugra side

Goal:

Explicitly construct "superstrata" or wiggly KKM in 6D

They must depend on functions of two variables: $F(v,w)$



Susy solutions in 6D

- IIB sugra on T^4_{6789}
- No dependence on T^4 coordinates
- Require same susy as preserved by D1-D5-P
- Expected charges / dipole charges:

D1(v)	D1(λ)	KKM(λ6789, v)
D5(v6789)	D5(λ6789)	
P(v)		

$$u = \frac{t - x^5}{\sqrt{2}}, \quad v = \frac{t + x^5}{\sqrt{2}}$$

x^5: compact

[Gutowski+Martelli+Reall 2003]
[Cariglia+Mac Conamhna 2004]
[Bena+Giusto+MS+Warner 2011]
[Giusto+Martucci+Petrini+Russo 2013]

The sol'n is characterized by…

scalars

$Z_1 \leftrightarrow$ D1(v)

$Z_2 \leftrightarrow$ D5(v6789)

$\mathcal{F} \leftrightarrow$ P(v)

$Z_4 \leftrightarrow$ NS5(v6789)+F1(v)

2-forms

$\Theta_1 \leftrightarrow$ D1(λ)

$\Theta_2 \leftrightarrow$ D5(λ6789)

$\Theta_4 \leftrightarrow$ NS5(λ6789)+F1(λ)

1-forms

$\beta \leftrightarrow$ KKM(λ6789, v)

$\omega \leftrightarrow$ P(λ)

Explicit form of BPS solution

$$ds_{10}^2 = -\frac{2\alpha}{\sqrt{Z_1 Z_2}}(dv+\beta)\left(du+\omega+\frac{1}{2}\mathcal{F}(dv+\beta)\right) - \sqrt{Z_1 Z_2}\,ds^2(\mathcal{B}^4) + \sqrt{\frac{Z_1}{Z_2}}\,ds^2(T^4)$$

$$e^{2\Phi} = \frac{\alpha Z_1}{Z_2} \qquad \alpha \equiv \frac{Z_1 Z_2}{Z_1 Z_2 - Z_4^2} \qquad \mathcal{D} \equiv d_4 - \beta \wedge \partial_v \qquad \dot{\ } \equiv \partial_v$$

$$H_3 = -(du+\omega)\wedge(dv+\beta)\wedge\left(\mathcal{D}\left(\frac{\alpha Z_4}{Z_1 Z_2}\right) - \frac{\alpha Z_4}{Z_1 Z_2}\dot{\beta}\right)$$
$$+(dv+\beta)\wedge\left(\Theta_4 - \frac{\alpha Z_4}{Z_1 Z_2}\mathcal{D}\omega\right) + \frac{\alpha Z_4}{Z_1 Z_2}(du+\beta)\wedge\mathcal{D}\beta + *_4\left(\mathcal{D}Z_4 + Z_4\dot{\beta}\right)$$

$$F_1 = \mathcal{D}\left(\frac{Z_4}{Z_1}\right) + (dv+\beta)\wedge\partial_v\left(\frac{Z_4}{Z_1}\right)$$

$$F_3 = -(du+\omega)\wedge(dv+\beta)\wedge\left(\mathcal{D}\left(\frac{1}{Z_1}\right) - \frac{1}{Z_1}\dot{\beta} + \frac{\alpha Z_4}{Z_1 Z_2}\mathcal{D}\left(\frac{Z_4}{Z_1}\right)\right)$$
$$+(dv+\beta)\wedge\left(\Theta_1 - \frac{Z_4}{Z_1}\Theta_4 - \frac{1}{Z_1}\mathcal{D}\omega\right) + \frac{1}{Z_1}(du+\beta)\wedge\mathcal{D}\beta + *_4\left(\mathcal{D}Z_2 + Z_2\dot{\beta}\right) - \frac{Z_4}{Z_1} *_4\left(\mathcal{D}Z_4 + Z_4\dot{\beta}\right)$$

… … … …

0^{th} layer: 4D base

6D spacetime: (u, v, x^m)

u: isometry
$v \sim x^5$ (compact)
x^m: 4D base

- 4D base $\mathcal{B}^4(v)$: almost hyper-Kähler

$$ds^2(\mathcal{B}^4) = h_{mn}(x,v)dx^m dx^n, \quad m,n = 1,2,3,4$$

$\beta(x,v)$: 1-form (↔ KKM)

$J^{(A)}(x,v),\ A = 1,2,3$: almost HK 2-forms

$$J^{(A)m}{}_n J^{(B)n}{}_p = \epsilon^{ABC} J^{(C)m}{}_p - \delta^{AB}\delta^m_p$$

$$d_4 J^{(A)} = \partial_v\big(\beta \wedge J^{(A)}\big), \qquad D \equiv d_4 - \beta \wedge \partial_v$$

BPS equations

- First layer (Z, Θ)

$$\mathcal{D} *_4 \left(\mathcal{D}Z_1 + \dot{\beta} Z_1\right) = -\mathcal{D}\beta \wedge \Theta_2$$

$$\mathcal{D}\Theta_2 - \dot{\beta} \wedge \Theta_2 = \partial_v\left[*_4\left(\mathcal{D}Z_1 + \dot{\beta} Z_1\right)\right] \qquad \ldots$$

$$\Theta_2 - Z_1\psi = *_4\left(\Theta_2 - Z_1\psi\right)$$

$$\psi \equiv \frac{1}{8}\epsilon^{ABC} J^{(A)mn} \dot{J}^{(B)}_{mn} J^{(C)}$$

- Second layer $(\omega, \mathcal{F})$

$$(1 + *_4)\mathcal{D}\omega + \mathcal{F}\mathcal{D}\beta = Z_1 *_4 \Theta_1 + Z_2\Theta_2 - Z_4(1 + *_4)\Theta_4$$

$$\begin{aligned} *_4 \mathcal{D} *_4 L + 2\dot{\beta}_i L^i = {} & \dot{Z}_1\dot{Z}_2 + \ddot{Z}_1 Z_2 + Z_1\ddot{Z}_2 - \dot{Z}_4^2 - 2Z_4\ddot{Z}_4 + \tfrac{1}{2}\partial_v\left(Z_1Z_2 - Z_4^2\right)h^{mn}\dot{h}_{mn} \\ & + \tfrac{1}{2}\left(Z_1Z_2 - Z_4^2\right)\left(h^{mn}\ddot{h}_{mn} - \tfrac{1}{2}h^{mn}\dot{h}_{np}h^{pq}\dot{h}_{qm}\right) \\ & - \tfrac{1}{2} *_4 \left((\Theta_1 - Z_2\psi) \wedge (\Theta_2 - Z_1\psi) - (\Theta_4 - Z_4\psi) \wedge (\Theta_4 - Z_4\psi) + \tfrac{Z_1Z_2}{\alpha}\psi \wedge \psi - 2\psi \wedge \mathcal{D}\omega\right) \end{aligned}$$

$$L \equiv \dot{\omega} + \mathcal{F}\dot{\beta} - \mathcal{D}\mathcal{F}$$

— Linear if solved in the right order

— *Very* complicated!
Hard to find general superstrata

Strategy:

To prove concept,
construct simple superstrata
depending on functions of two variables

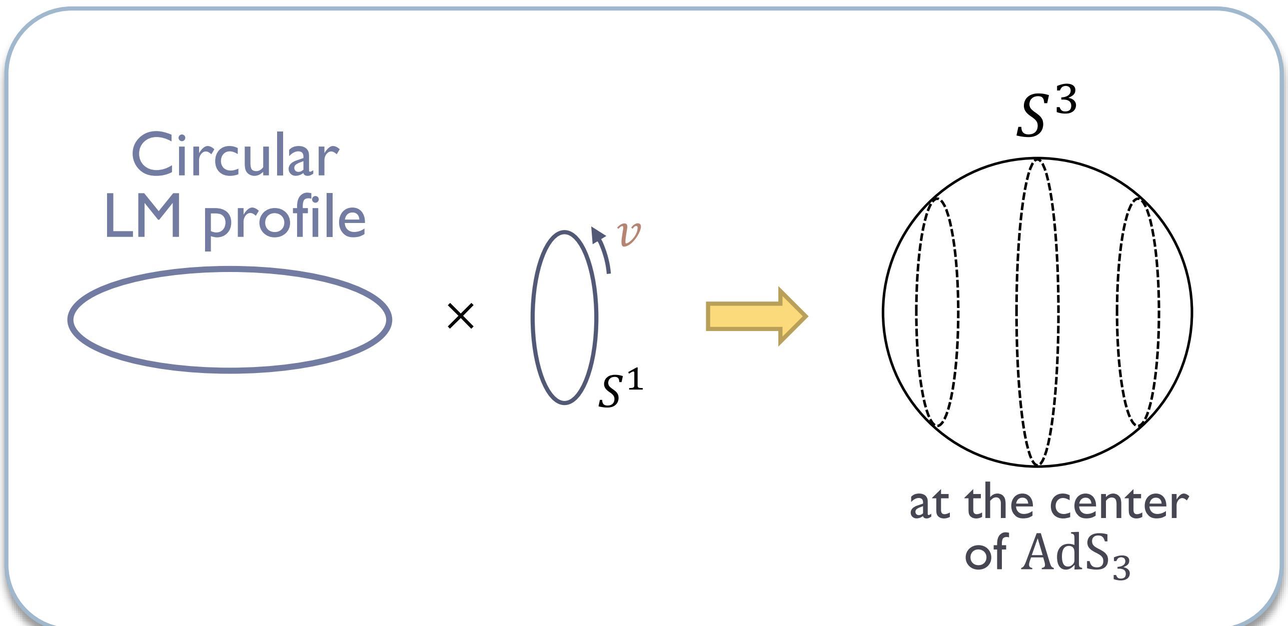
[Bena-Giusto-Russo-MS-Warner '15]

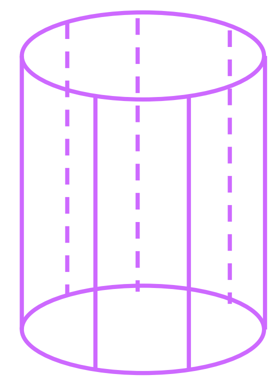
Background (1)

Starting point: simplest D1-D5 configuration (no P yet):

circular LM geom $=$ pure $\mathrm{AdS}_3 \times S^3$

$=$ “round” superstratum with no wiggle (yet)



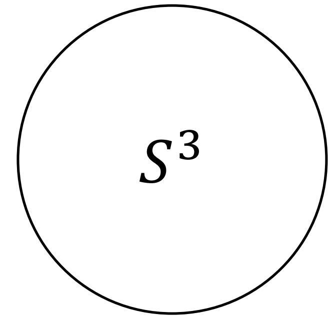


(previous naive picture)

Background (2)

Circular profile:

$$F_1 + iF_2 = a\,\exp(2\pi i\lambda/L)$$



Explicit solution:

Flat base ($\mathcal{B}^4 = \mathbb{R}^4$)

$$ds^2(\mathbb{R}^4) = \Sigma\left(\frac{dr^2}{r^2 + a^2} + d\theta^2\right) + \left(r^2 + a^2\right)\sin^2\theta\, d\phi^2 + r^2\cos^2\theta\, d\psi^2$$

$$\Sigma \equiv r^2 + a^2\cos^2\theta \qquad \beta = \frac{R_5 a^2}{\sqrt{2}\Sigma}\left(\sin^2\theta d\phi - \cos^2\theta d\psi\right)$$

Other data:

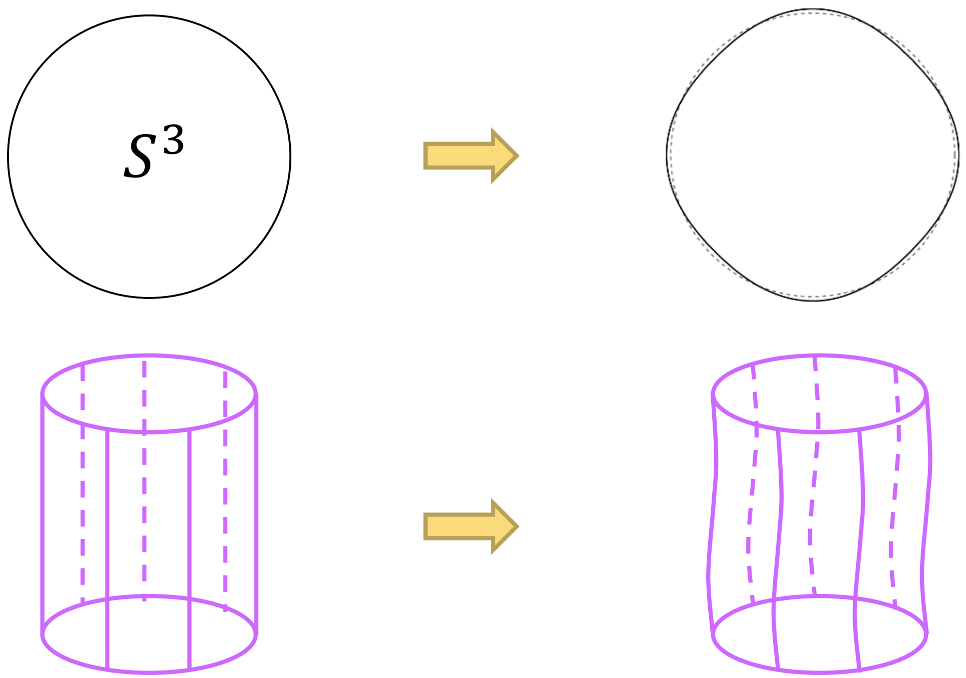
$$Z_1 = 1 + \frac{Q_1}{\Sigma} \qquad Z_2 = 1 + \frac{Q_2}{\Sigma} \qquad \omega = \frac{R_5 a^2}{\sqrt{2}\Sigma}\left(\sin^2\theta d\phi + \cos^2\theta d\psi\right)$$

$$Z_4 = \mathcal{F} = \Theta_1 = \Theta_2 = \Theta_4 = 0$$

Putting momentum

Now we want to add P

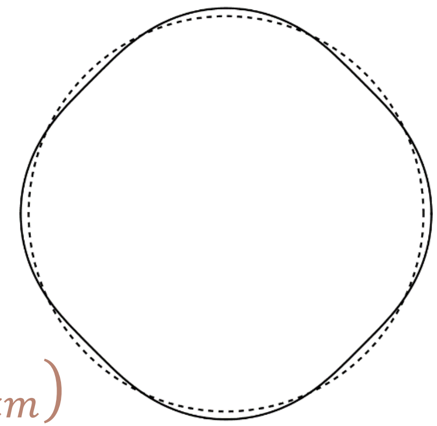
Putting momentum deforms
the round superstratum = S^3
by putting wiggles on it



Linear fluctuation

Certain *linear* solutions can be found by solution generating technique

[Mathur+Saxena+Srivastava 2003]



$$Z_4 = b\frac{R_5\Delta_{km}}{\Sigma}\cos\hat{v}_{km}$$

$$\Theta_4 = -\sqrt{2}bm\Delta_{km}\left(r\sin\theta\,\Omega^{(1)}\sin\hat{v}_{km} + \Omega^{(2)}\cos\hat{v}_{km}\right)$$

$$\Delta_{km} \equiv \left(\frac{a}{\sqrt{r^2+a^2}}\right)^k \sin^{k-m}\theta\,\cos^m\theta \qquad \hat{v}_{km} \equiv \frac{m\sqrt{2}}{R_5}v + (k-m)\phi - m\psi$$

v dependence (P)

$ds^2(\mathcal{B}^4),\ Z_{1,2,},\ \beta,\ \omega,\ \Theta_{1,2}$: unchanged at $\mathcal{O}(b)$

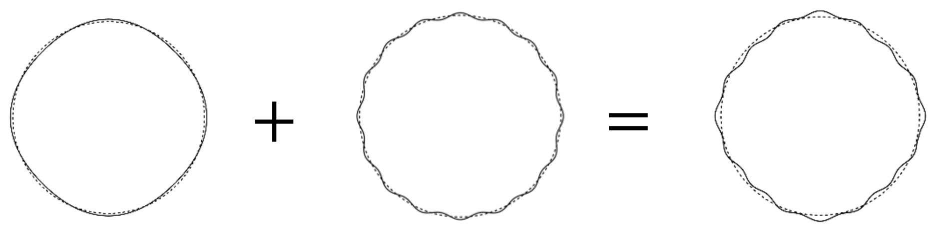
- Depends on two params (k,m)
- CFT dual: descendants of *chiral primary*

How to get function of two variables

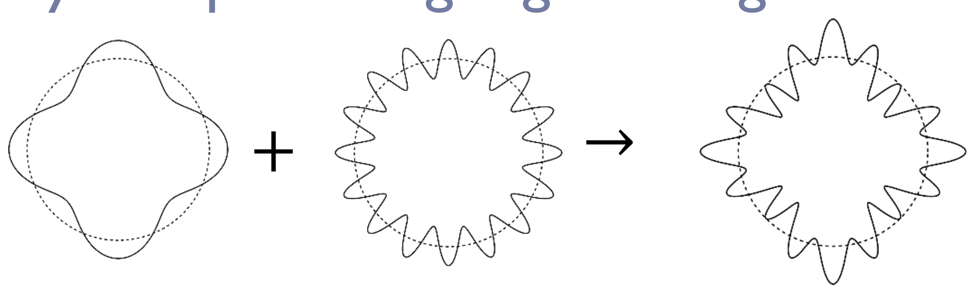
- Regard solution with (k, m) as Fourier modes on S^3

$$f(S^3) = \sum_{k,m} b_{k,m} Y_{k,m} \qquad S^3 : \underbrace{SU(2)_L}_{\text{BPS}} \times SU(2)_R$$

b_{km} independent ⟺ function of two variables!



- Non-linearly complete to get genuine geometric superstratum



Non-linear completion

Use linear structure of BPS eqs to nonlinearly complete

- Assume 0$^{\text{th}}$ data $\mathcal{B}^4, \beta$ are unchanged
- Regard Z_4, Θ_4 as non-linear sol'n of 1$^{\text{st}}$ layer

$$\mathcal{D} *_4 \mathcal{D} Z_4 = -\mathcal{D}\beta \wedge \Theta_4 \qquad \mathcal{D}\Theta_4 = \partial_v *_4 \mathcal{D} Z_4$$

- Find $\omega, \mathcal{F}$ as non-linear sol'n of 2$^{\text{nd}}$ layer

$$(1 + *_4) d\omega + \mathcal{F} d\beta = Z_1 \Theta_1 + Z_2 \Theta_2 - 2 Z_4 \Theta_4$$

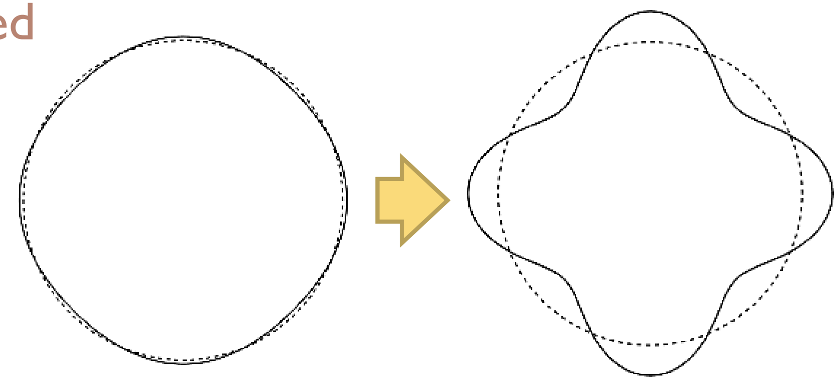
$$*_4 \mathcal{D} *_4 \left(\dot{\omega} - \tfrac{1}{2} d\mathcal{F}\right) = \dot{Z}_1 \dot{Z}_2 + \ddot{Z}_1 Z_2 + Z_1 \ddot{Z}_2 - \dot{Z}_4^2 - 2 Z_4 \ddot{Z}_4$$

  - Enough to do it for each pair of modes

- Regularity determines solution
  - It also determines $Z_{1,2}, \Theta_{1,2}$

Ex 1: $(k_1, m_1) = (k_2, m_2)$

$Z_4 \sim b\dfrac{\Delta_{k_1 m_1}}{\Sigma}\cos\hat{v}_{k_1 m_1},$ Z_2: unchanged



$Z_1 \supset b^2 \Delta_{2k_1,2m_1}\cos\hat{v}_{2k_1,2m_1}$

: needed to make ω regular

$\mathcal{F} = (2m_1)^2 F^{(0,0)}_{2k_1,2m_1}$

$\omega = \mu(d\psi + d\phi) + \zeta(d\psi - d\phi)$

undetermined

$$\mu = \frac{R_5}{4\sqrt{2}}\left(-\frac{\Delta_{2k_1,2m_1}}{\Sigma} + (2k_1 - 2m_1)^2 F^{(0,0)}_{2k_1,2m_1+2} - (2m_1)^2\frac{r^2 + a^2\sin^2\theta}{\Sigma} F^{(0,0)}_{2k_1,2m_1}\right) + \frac{x}{\Sigma}$$

Regularity ⇒ $\omega = 0$ at $r = \theta = 0$ ⇒ $x = \dfrac{R_5}{4\sqrt{2}}\left(\dfrac{k_1}{m_1}\right)^{-1}$

$\zeta = \cdots$

→ NL completed, with coeff fixed by regularity

Ex 2: (k_1, m_1): any, $(k_2, m_2) = (1,0)$

$$Z_4 \sim b_1 \frac{\Delta_{k_1 m_1}}{\Sigma} \cos \hat{v}_{k_1 m_1} + b_2 \frac{\Delta_{10}}{\Sigma} \cos \hat{v}_{10} \,, \qquad Z_2\text{: unchanged}$$

$$Z_1 \supset b_1 b_2 \left(\frac{\Delta_{k_1+1,m_1}}{\Sigma} \cos \hat{v}_{k_1+1,m_1} + \underset{\text{undetermined}}{c} \frac{\Delta_{k_1-1,m_1}}{\Sigma} \cos \hat{v}_{k_1-1,m_1} \right),$$

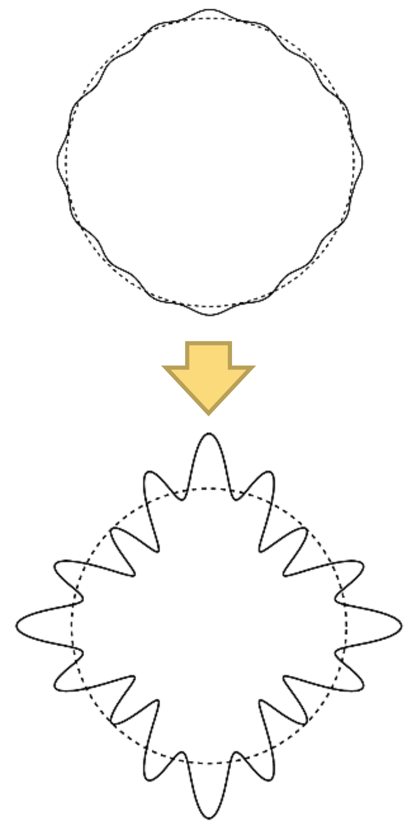
$$\mathcal{F} = 0$$

$$\omega = c\omega^{(1)} + \omega^{(2)}$$

$$\omega^{(1)} = \frac{R_5}{\sqrt{2}} \Delta_{k_1-1,m_1} \left(-\frac{dr}{r(r^2+a^2)} \sin \hat{v}_{k_1-1,m_1} + \frac{¥\sin^2\theta d\phi + \cos^2\theta d\psi}{\Sigma} \cos \hat{v}_{k_1-1,m_1} \right)$$

$$\omega^{(2)} = -\frac{R_5}{\sqrt{2}} \frac{\Delta_{k_1-1,m_1}}{r^2+a^2} \left[\left(\frac{m_1-k_1}{k_1} \frac{dr}{r} - \frac{m_1}{k_1} \tan\theta d\theta \right) \sin \hat{v}_{k_1-1,m_1} + \left(\frac{r^2+a^2}{\Sigma} \sin^2\theta d\phi + \left(\frac{r^2+a^2}{\Sigma} \cos^2\theta - \frac{m_1}{k_1} \right) d\psi \right) \cos \hat{v}_{k_1-1,m_1} \right]$$

Regularity ⇨ $\omega = 0$ at $r = \theta = 0$ ⇨ $c = \dfrac{k_1 - m_1}{k_1}$



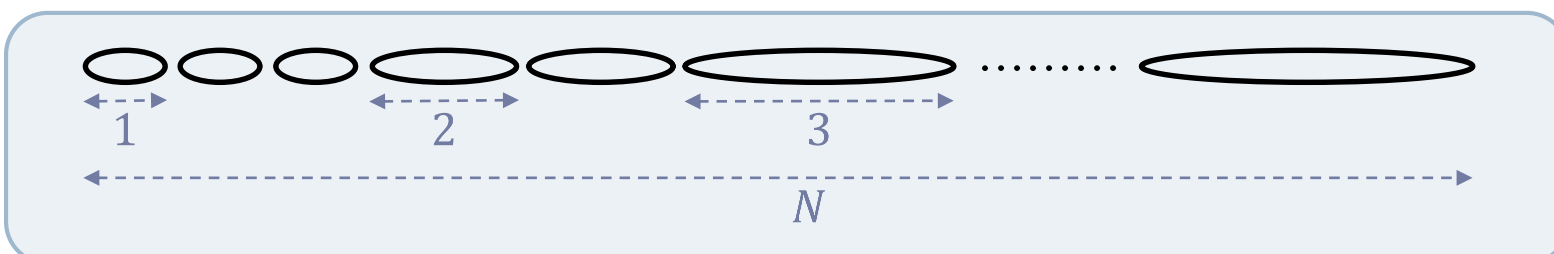
→ NL completed, with coeff fixed by regularity

CFT side

BOUNDARY CFT

- D1-D5 CFT
  - → $d = 2,\ \mathcal{N} = (4,4)$ SCFT
  - → Bosonic sym: $SL(2,\mathbb{R})_L \times SU(2)_L \times SL(2,\mathbb{R})_R \times SU(2)_R$
  - → Bosonic currents: $T(z), J^i(z), \tilde{T}(\bar{z}), \tilde{J}^i(\bar{z})$

    $L_n \quad J^i_n \quad \tilde{L}_n \quad \tilde{J}^i_n$
- Orbifold CFT
  - → There are "component strings" with total length $N = N_1 N_2$

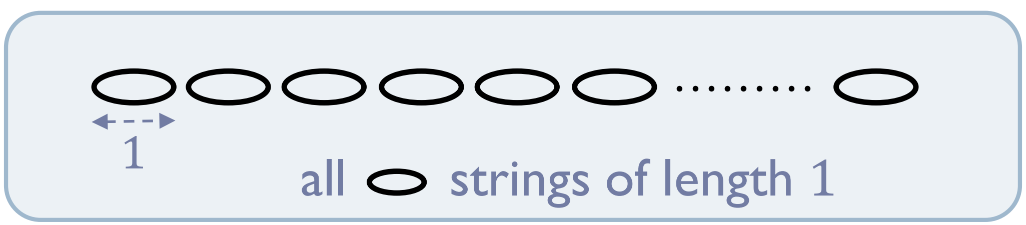


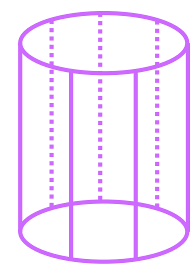
2-CHARGE STATES (I)

- Component strings come with “flavors”

related to $SU(2)_L \times SU(2)_R$ charge

- Round LM geom ⟺ NS vacuum

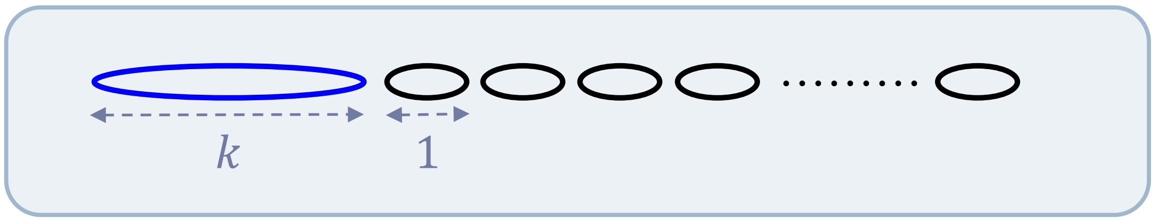


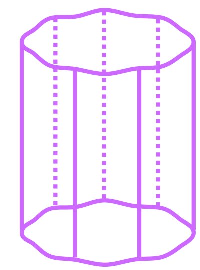


2-CHARGE STATES (2)

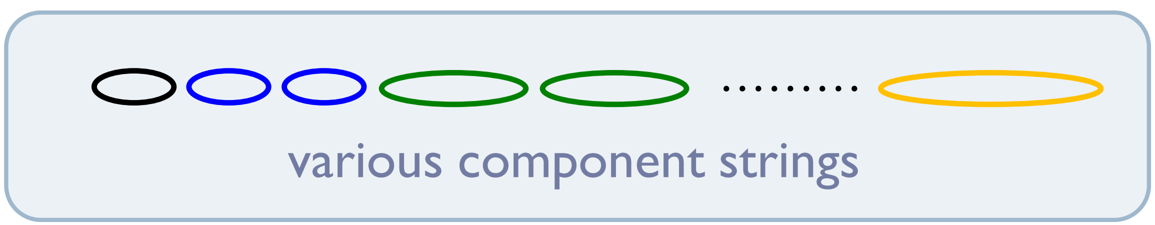
- Linear fluct around circular LM

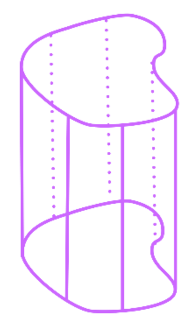
  ⇔ “single-trace” chiral primary





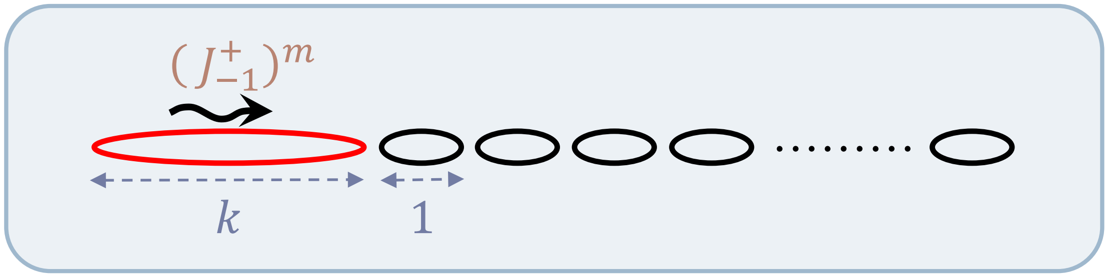
- General LM geom ⇔ general chiral primary

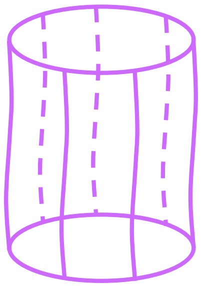




3-CHARGE STATES (1)

- P-carrying linear fluct around circular LM
  ⇔ descendant of chiral primary

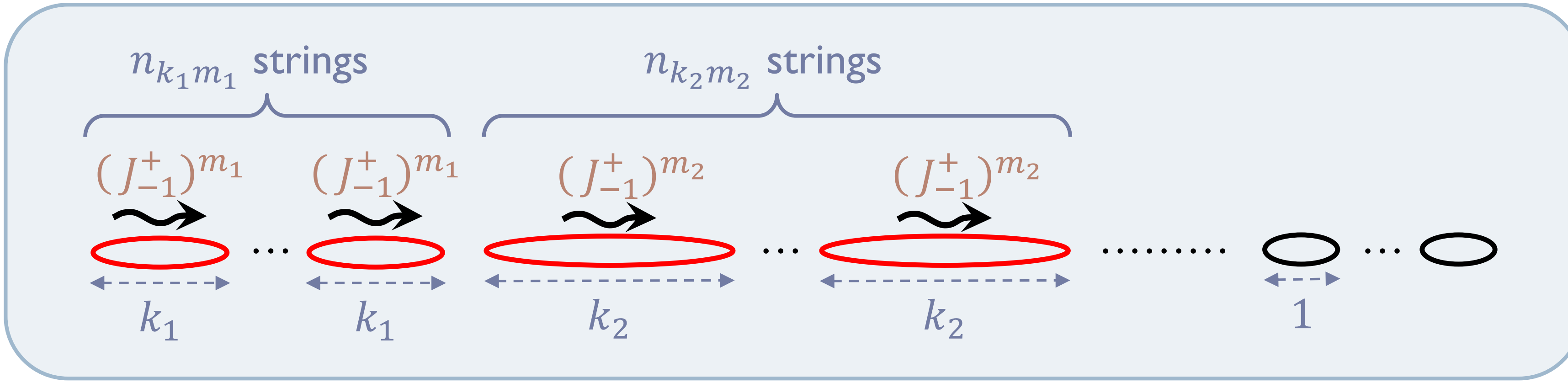


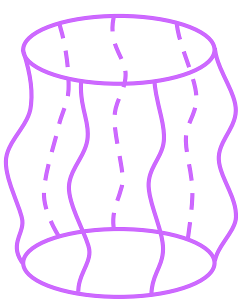


- → Single chiral primary acted on with J^+_{-1}
- → Labeled by (k, m)
- → State of a single supergraviton
  with quantum numbers (k, m)

3-CHARGE STATES (2)

- General P-carrying fluct around circular LM
  ⟺ descendant of *non-chiral* primary





→ Various modes (k, m) excited with arbitrary amp.

→ The most general microstate geometry with known CFT dual

→ Individual J^+_{-1} act independently

→ State of supergraviton gas

Where are we?

Summary

∃ Superstratum depending of two variables

⇔ Having modes with different (k, m)

⇔ NL completion for pair of modes

- Succeeded in NL completion for various pairs of modes
  - → *Constructive proof of existence of superstrata!*
  - → Big step toward general 3-charge microstate geometries
- Correspond to *non-chiral* primaries in CFT
  - → Most general microstate geom with known CFT dual

Toward more general superstrata

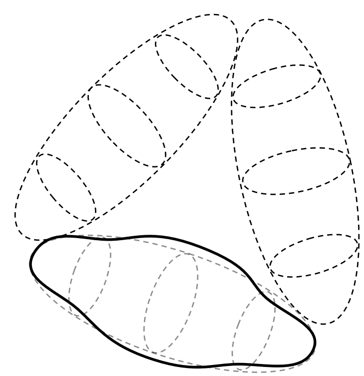
- Does this class of superstrata reproduce S_{BH}?
  - → No. These correspond to coherent states of graviton gas. Entropy is parametrically smaller.

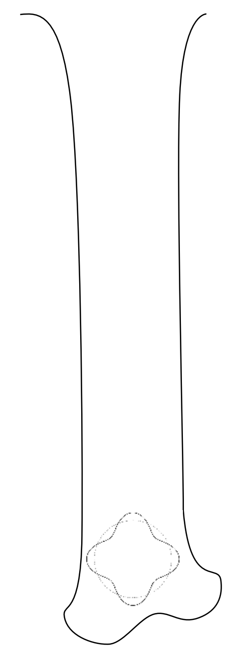
$$S_{\mathrm{geom}} \sim N^{\frac{3}{4}} \ll N^1 \sim S_{\mathrm{BH}}$$
$$(N_1 N_5 \sim N_p \sim J \sim N)$$

- Need more general superstrata
  - → In CFT language, we only considered rigid generators of $SU(1,1|2)_L \times SU(1,1|2)_R$ e.g. L_0, L_1, L_{-1}, J_0^-
  - → Need higher and fractional modes e.g. $J^-_{-\frac{1}{k}}$
  - → They probably correspond to multiple superstrata

Multiple superstrata

- More generally, one has multiple S^3's
- Can fluctuate each S^3 — multi-superstratum





- Can use $AdS_3 \times S^3$ as local model
- Large redshift in scaling geometries

→ entropy enhancement?

→ $S \sim N^1$?

Comment on “issues”

- Lifting
  - Not directly applicable to 6D configuration
- Pure Higgs branch
  - Superstratum reminiscent of Higgs branch

⇩

Maybe only states that have $J = 0$
survive when moduli are turned on?

Conclusions

Conclusions

- Microstate geometry program
  - Interesting enterprise elucidating micro nature of BHs, whether answer turns out to be yes or no
- Microstate geometries in 5D sugra
  - Have properties expected from CFT, but too few
- Superstratum
  - A new class of microstate geometries
  - CFT duals precisely understood
  - More general superstrata are crucial to reproduce $S_{\rm BH}$

Future directions

- Superstratum
  - More general solution, multi-strata
  - Clarify issues (lifting, pure Higgs)
  - Count states, reproduce entropy (or not)
- Non-geometric microstates
  - Exotic branes, DFT
  - Novel ways to store information
- More
  - Non-extremal BHs
  - Information paradox
  - Observational consequences?
  - Early universe
  - …

Thanks!

Appendix

Some formulas

$$F_{k,m}^{(p,q)} = -\frac{1}{4k_1k_2(r^2+a^2)} \sum_{s=0}^{\min\{k_1,k_2\}-1} \sum_{t=0}^{s} \binom{s}{t} \frac{\binom{k_1-s-1}{m_1-t-1}\binom{k_2-s-1}{m_2-t-1}}{\binom{k_1-1}{m_1-1}\binom{k_2-1}{m_2-1}} \Delta_{k-2s-2,m-2t-2}$$