Massless Black Holes & Black Rings as Effective Geometries of the D1-D5 System

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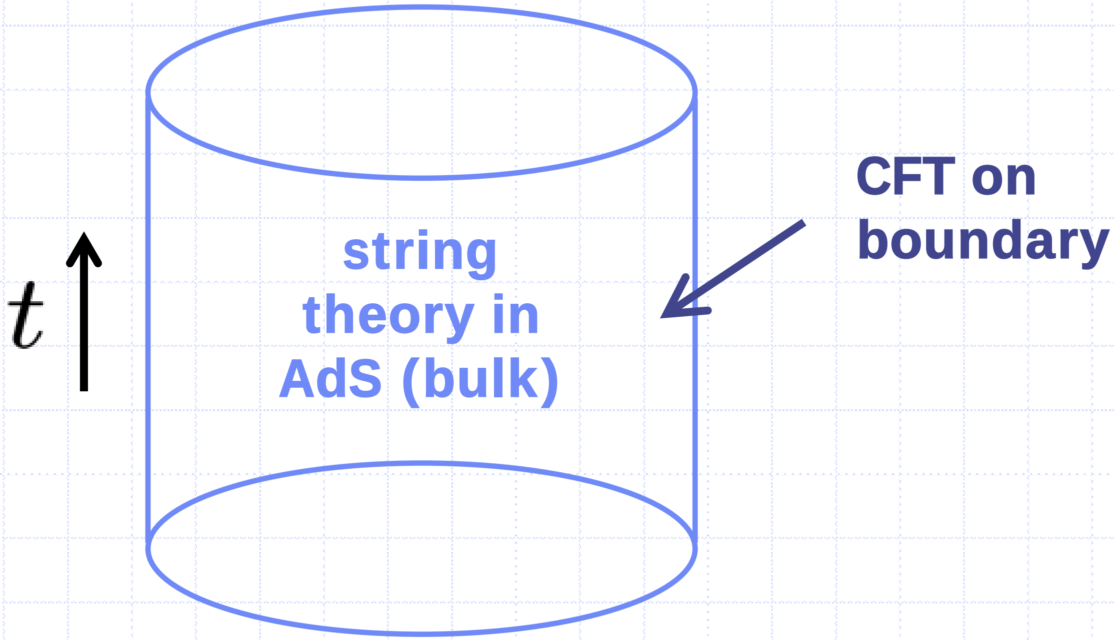
1. Introduction

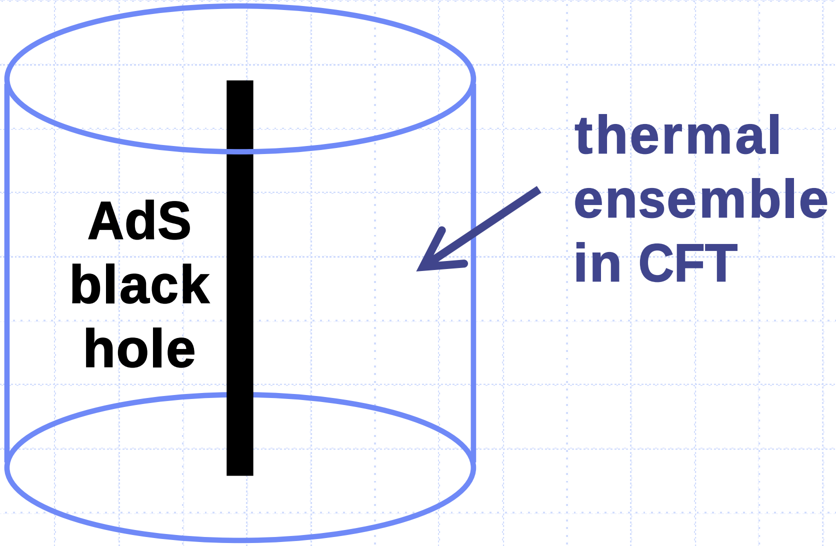
AdS/CFT

- Can study gravity/string theory in AdS using CFT on boundary



AdS/CFT and BH

- Black hole $\longleftrightarrow$ thermal ensemble in CFT
  - Can study BH from CFT:
    entropy, correlation function, …
  - Even valid for "small BH" [Dabholkar 0409148], [DKM], …



Ensemble vs. microstates — CFT side

Thermal ensemble = weighted collection of microstates

→ Nothing stops one from considering individual microstates in CFT

- Individual CFT microstates
  - For large N, most states are very similar to each other —— "typical state"
  - Result of "typical" measurements for "typical state" very well approximated by that for thermal ensemble

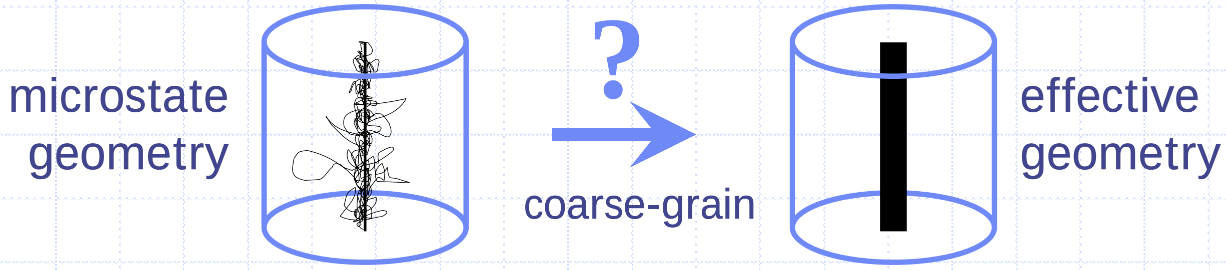
Cf. gas of molecules is well described by thermodynamics, although it's in pure microstate

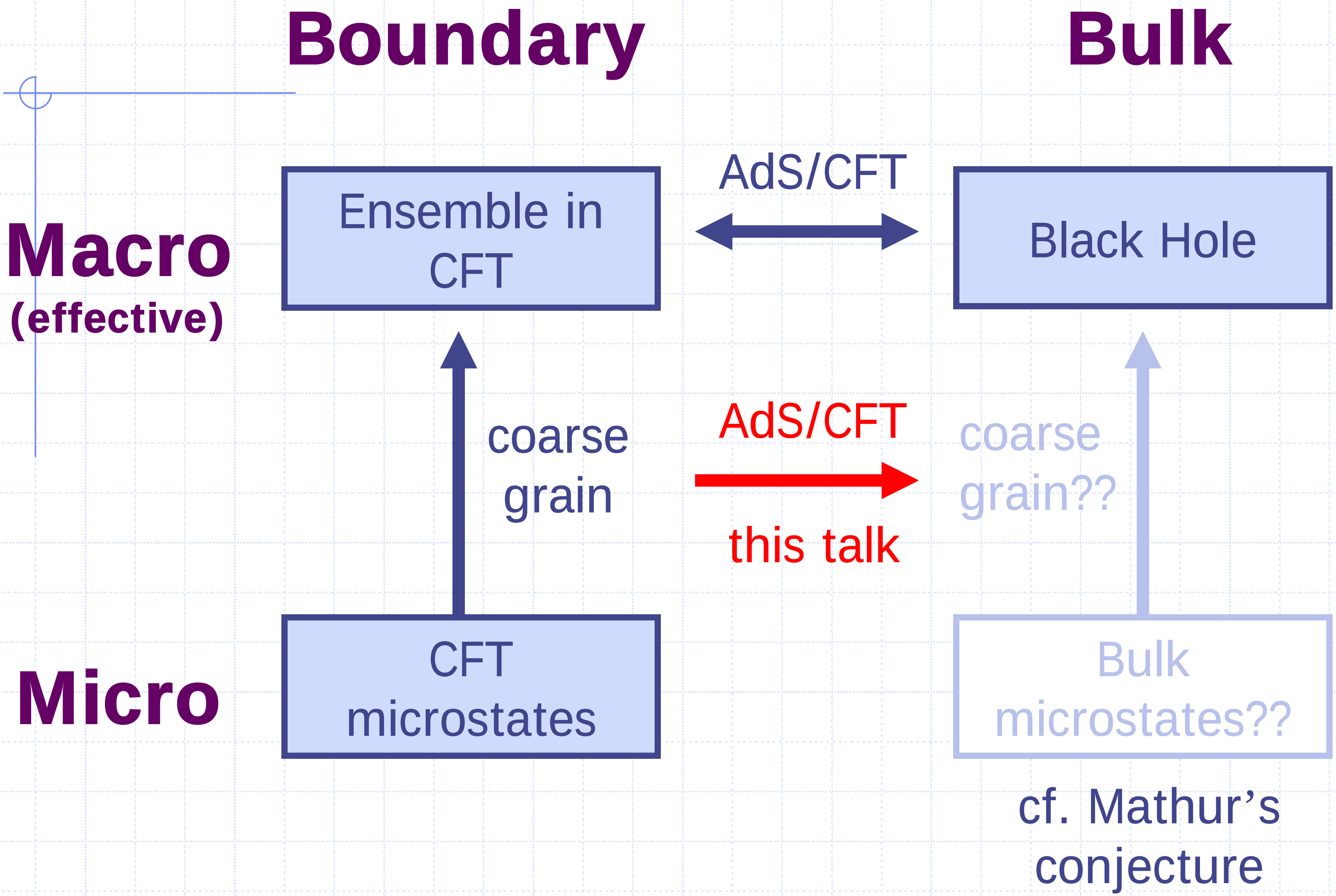
Ensemble vs. microstates — bulk side

AdS/CFT → 1-to-1 correspondence between bulk and CFT microstates

→ There must be bulk microstate geometries (possibly quantum)

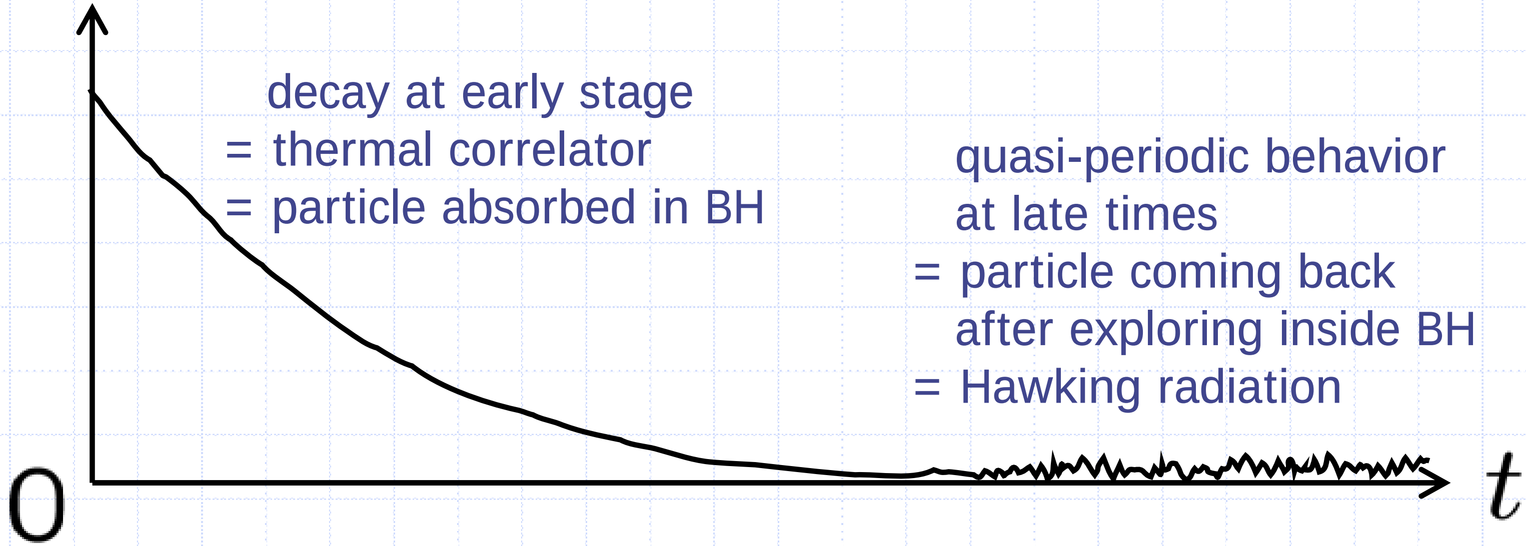
- Expectation for bulk microstates:
  - Result of "typical" measurements for "typical state" is very well approximated by that for thermal ensemble, i.e. classical BH



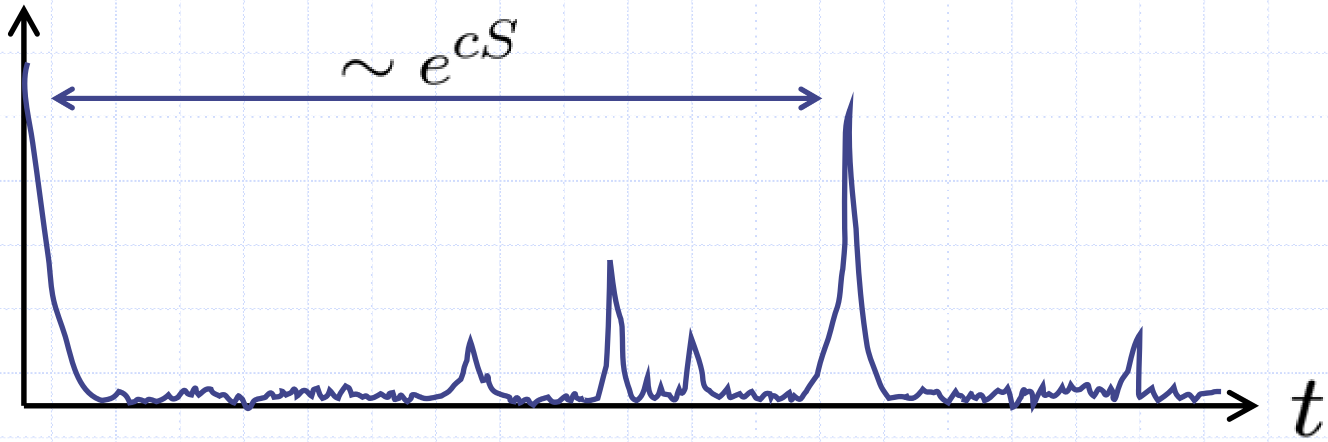

Boundary
Bulk
Macro
(effective)
Ensemble in CFT
AdS/CFT
Black Hole
coarse grain
AdS/CFT
this talk
coarse grain??
Micro
CFT microstates
Bulk microstates??
cf. Mathur's conjecture

Atypical measurements

- Typical measurements see thermal state.
- Atypical measurements can reveal detail of microstates
  - E.g. long-time correlation function



Remark: Poincaré recurrence



- For microstate correlator, Poincaré recurrence is automatic.
- Summing over SL(2,Z) family of BHs can't account for Poincaré recurrence [Maldacena, Kleban-Porrati-Rabadan]
  - BHs are coarse-grained effective description
    Cf. gas of molecules → dissipative continuum

What we use:
D1-D5 sys — ideal arena

- Simplest link between BH & CFT
- P=0: Ramond ground states (T=0)
- $S = 2\sqrt{2}\pi\sqrt{N}$: macroscopic for large $N=N_1N_5$
  - Must have some properties of BH
  - Stringy corrections makes it a small BH [Dabholkar, DKM]
- Large class of microstate geometries are known [Lunin-Mathur]

We'll see...

- Emergence of effective geometry (M=0 BTZ)
- Its breakdown for atypical measurements

Plan

1. Introduction ✓
2. D1-D5 system
3. Typical states
4. The effective geometry
5. Conclusion

2. D1-D5 system

Setup: D1-D5 System

- Configuration:
  - N_1 D1-branes on S^1
  - N_5 D5-branes on S^1 x T^4
  - $SO(4)_E$ x $SO(4)_I$ symmetry

	$\mathbb{R}^4$	S^1	T^4
D1	$\cdot$	$\bigcirc$	$\sim$
D5	$\cdot$	$\bigcirc$	$\bigcirc$

- Boundary CFT:
  - N=(4,4) supersymmetric sigma model
  - Target space: $(T^4)^N/S_N$, $N= N_1N_5$
- We use orbifold point (free) approximation

D1-D5 CFT

- Symmetry:

$$SO(4)_E \times SO(4)_I = [\overbrace{SU(2)_R}^{s} \times \overbrace{SU(2)_R}^{\widetilde{s}}] \times [SU(2)_I \times \overbrace{SU(2)_I}^{\widetilde{\alpha},\widetilde{\beta}}]$$

- R ground states
  - 8+8 single-trace twist ops.:

$$\sigma_n^{s\widetilde{s}},\ \sigma_n^{\widetilde{\alpha}\widetilde{\beta}},\ \tau_n^{s\widetilde{\alpha}},\ \tau_n^{\widetilde{\alpha}\widetilde{s}} \quad \equiv \quad \sigma_n^{\mu}, \tau_n^{\mu}$$

$$1 \le n \le N, \quad s, \widetilde{s}, \widetilde{\alpha}, \widetilde{\beta} = \pm$$

  - General: $\sigma = \prod_{n,\mu} (\sigma_n^{\mu})^{N_{n\mu}} (\tau_n^{\mu})^{N'_{n\mu}}$
  - Specified by distribution $\{N_{n\mu}, N'_{n\mu}\}$ s.t.

$$\sum_{n,\mu} n(N_{n\mu} + N'_{n\mu}) = N, \quad N_{n\mu} = 0, 1, 2, \ldots, \quad N'_{n\mu} = 0, 1 \, .$$

Map to FP system

- D1-D5 sys is U-dual to FP sys:
  - F1 winds N_5 times around S^1
  - N_1 units of momentum along S^1

	$\mathbb{R}^4$	S^1	T^4
F1	$\cdot$	$\bigcirc$	$\sim$
P	$\cdot$	$\bigcirc$	$\sim$

- BPS states: any left-moving excitations

$$\prod_{n,\mu} (\alpha^{\mu}_{-n})^{N_{n\mu}} (\psi^{\mu}_{-n})^{N'_{n\mu}} |N_1, N_5\rangle$$

- One-to-one correspondence:

$$\sigma^{\mu}_{n} \leftrightarrow \alpha^{\mu}_{-n} \qquad \tau^{\mu}_{n} \leftrightarrow \psi^{\mu}_{-n}$$

D1-D5 FP D1-D5 FP

D1-D5 microstate geometries

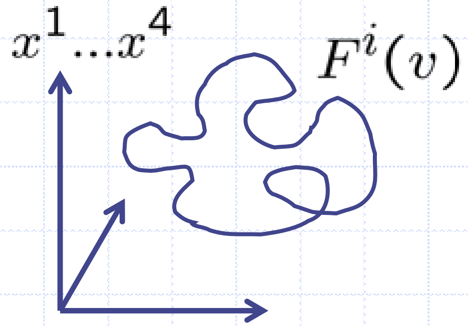
R ground state of D1-D5 sys $\xrightarrow{\text{U-dual}}$ BPS state of FP sys $\longrightarrow$ Classical profile of F1

$$\sigma = \prod_{n,\mu} (\sigma_n^\mu)^{N_{n\mu}} (\tau_n^\mu)^{N'_{n\mu}},$$

$$\prod_{n,\mu} (\alpha_{-n}^\mu)^{N_{n\mu}} (\psi_{-n}^\mu)^{N'_{n\mu}} |N_1, N_5\rangle$$

$$F^i(v), \quad v = t - y$$
$$i = 1, \ldots, 4$$

$\longrightarrow$ FP sys geometry $\xrightarrow{\text{U-dual}}$ D1-D5 geometry



Lunin-Mathur
hep-th/0109154

$$ds^2_{\text{string}} = \frac{1}{\sqrt{f_1 f_5}}[-(dt - A)^2 + (dy + B)^2] + \sqrt{f_1 f_5}\, dx^i dx^i + \sqrt{\frac{f_1}{f_5}} dz^a dz^a,$$

$$e^{2\Phi} = \frac{f_1}{f_5}, \qquad f_5 = \frac{Q_5}{L}\int_0^L \frac{dv}{|\mathbf{x} - \mathbf{F}(\mathbf{v})|^2}, \qquad f_1 = \frac{Q_5}{L}\int_0^L \frac{|\dot{\mathbf{F}}(v)|^2 dv}{|\mathbf{x} - \mathbf{F}(\mathbf{v})|^2},$$

$$A_i = -\frac{Q_5}{L}\int_0^L \frac{\dot{F}_i(v) dv}{|\mathbf{x} - \mathbf{F}(\mathbf{v})|^2}, \qquad dB = - *_4 dA.$$

3. Typical states

Statistics & typical states

- R gnd states: specified by distribution $\{N_{n\mu}, N'_{n\mu}\}$
- Large $N = \sum_{n,\mu} n(N_{n\mu} + N'_{n\mu})$
  → Macroscopic number $(\sim e^{2\sqrt{2}\pi\sqrt{N}})$ of states
  → Almost all microstates have almost identical distribution (typical state)
- Result of "typical" measurements for almost all microstates are very well approximated by that for typical state
- Can also consider ensemble with J≠0

Typical distribution: J=0

- Consider all twists with equal weight

$$\sigma_n^{s\tilde{s}},\ \sigma_n^{\tilde{\alpha}\tilde{\beta}},\ \tau_n^{s\tilde{\alpha}},\ \tau_n^{\tilde{\alpha}\tilde{s}}$$

β is not physical temp

- Microcanonical (N) → canonical (β)

$$Z(\beta)=\mathrm{Tr}[e^{-\beta N}]=\prod_{n=1}^{\infty}\frac{(1+q^n)^8}{(1-q^n)^8}=\left[\frac{\vartheta_2(0|\tau)}{2\eta(\tau)^3}\right]^4,\ q=e^{-\beta}.$$

$$N=\frac{2\pi^2}{\beta^2},\quad N\gg 1\Longleftrightarrow \beta\ll 1;\qquad S=2\sqrt{2}\pi\sqrt{N}$$

- Typical distribution: BE/FD dist.

$$N_{n\mu}=\frac{1}{e^{\beta n}-1},\quad N'_{n\mu}=\frac{1}{e^{\beta n}+1}$$

Typical distribution: J≠0

- Constituent twists with J≠0:

$$\sigma_n^{s\tilde{s}}: \;\; J = -\frac{s+\tilde{s}}{2}, \qquad \tau_n^{s\tilde{\alpha}}: \;\; J = -\frac{s}{2}, \qquad \tau_n^{\tilde{\alpha}\tilde{s}}: \;\; J = -\frac{\tilde{s}}{2}.$$

- Entropy: $S = \log d_{N,J} = 2\sqrt{2}\pi\sqrt{N - |J|}$

→ σ_1^{--} has BE condensed (J>0)

$$\underbrace{(\sigma_1^{--})^J}_{\text{BE condensate}} \times \underbrace{\prod_{n=1}^{\infty}\left[\prod_{\mu}(\sigma_n^{\mu})^{N_{n\mu}}(\tau_n^{\mu})^{N'_{n\mu}}\right]}_{\text{typical states of ensemble with } (N,J)\to(N-J,0)}$$

whole J is carried by σ_1^{--}

4. The Effective Geometry

What we've learned so far:

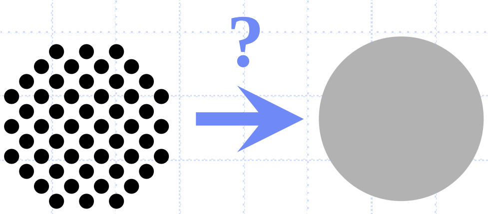
- R ground states of D1-D5 system is specified by $\{N_{n\mu}, N'_{n\mu}\}$
- For large N, there are macroscopic number ($\sim e^S$) of them
- Almost all states have almost identical distribution (typical state).

What we'll see:

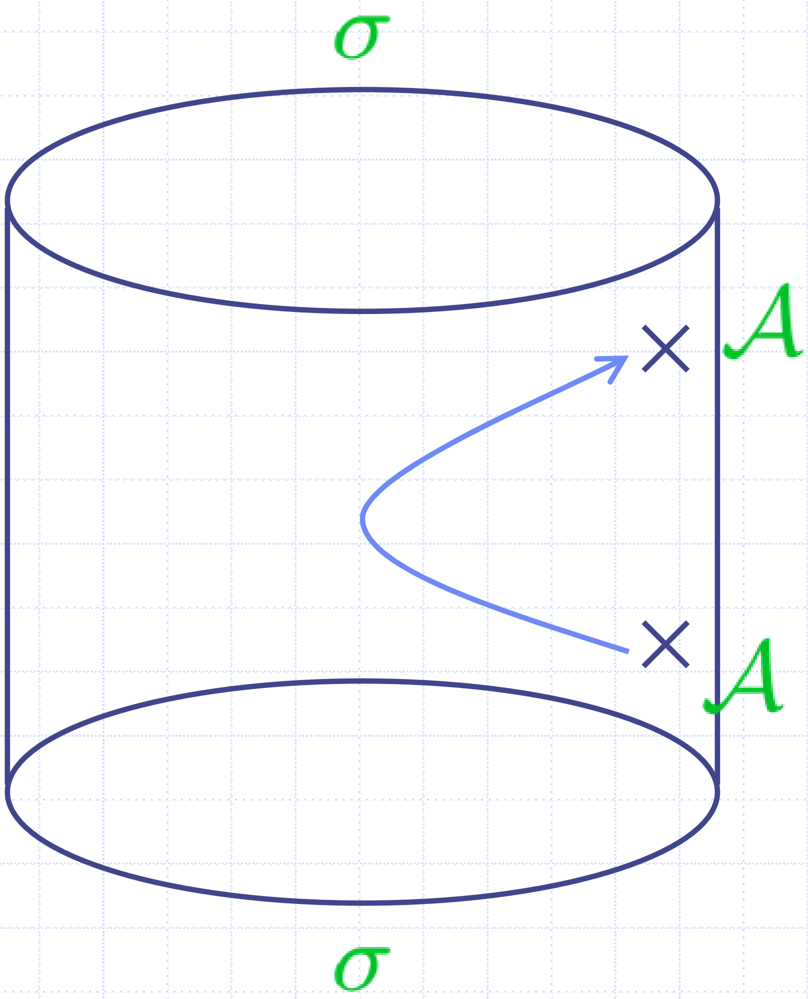
- We will compute CFT correlator $\langle\sigma|\mathcal{A}^\dagger\mathcal{A}|\sigma\rangle$
- For generic probes, almost all states give universal responses
  → effective geometry: M=0 BTZ
- For non-generic probes (e.g. late time correlator), different microstates behave differently

* How about bulk side? Why not "coarse-grain" bulk metric?

- Technically hard
- LLM/Lunin-Mathur is at sugra level



2-point func of D1-D5 CFT



Probe the bulk geometry corresponding to R ground state σ

$$= \langle \sigma^\dagger \mathcal{A}^\dagger \mathcal{A} \sigma \rangle$$

2-point func of D1-D5 CFT

- Background: general RR gnd state

$$\sigma = \prod_{n,\widehat{\mu}} (\sigma_n^{\widehat{\mu}})^{N_{n\widehat{\mu}}}, \qquad (\sigma_n^{s\widetilde{s}}, \sigma_n^{\widetilde{\alpha}\widetilde{\beta}}, \tau_n^{s\widetilde{\alpha}}, \tau_n^{\widehat{\alpha}\widetilde{s}}) \equiv \sigma_n^{\widehat{\mu}}$$

- Probe: non-twist op.

$$\mathcal{A} = \frac{1}{\sqrt{N}} \sum_{A=1}^{N} \mathcal{A}_A \qquad \text{e.g.} \quad \mathcal{A}_A = \partial X_A^a(z) \bar{\partial} X_A^b(\bar{z}),$$

- Correlator decomposes into contributions from constituent twist ops.:

$$\langle \sigma^\dagger \mathcal{A}^\dagger \mathcal{A} \sigma \rangle = \frac{1}{N} \sum_{n,\widehat{\mu}} n N_{n\widehat{\mu}} \sum_{A=1}^{n} \langle [\sigma_n^{\widehat{\mu}}]^\dagger \mathcal{A}_A^\dagger \mathcal{A}_1 \sigma_n^{\widehat{\mu}} \rangle \ .$$

Typical state correlator: example

$$\langle\sigma^\dagger \mathcal{A}^\dagger \mathcal{A}\sigma\rangle = \frac{1}{N}\sum_{n,\hat{\mu}} n N_{n\hat{\mu}} \sum_{A=1}^{n} \langle [\sigma_n^{\hat{\mu}}]^\dagger \mathcal{A}_A^\dagger \mathcal{A}_1 \sigma_n^{\hat{\mu}}\rangle \ .$$

- Operator: $\mathcal{A}_A = \partial X(z)\bar{\partial} X(\bar{z}),$
- Plug in typical distribution:

$$N_{n\mu} = \frac{1}{e^{\beta n}-1}, \quad N'_{n\mu} = \frac{1}{e^{\beta n}+1}$$

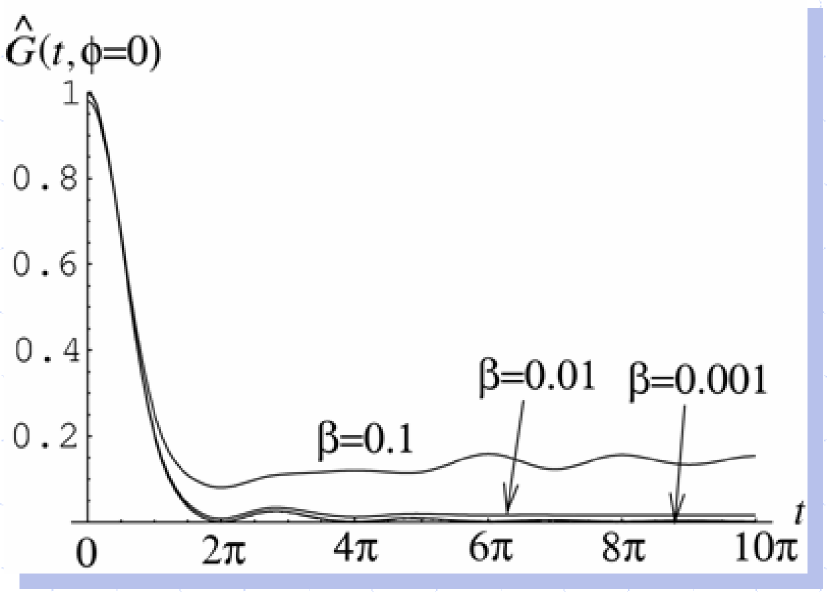
- Regularized 2-pt func:

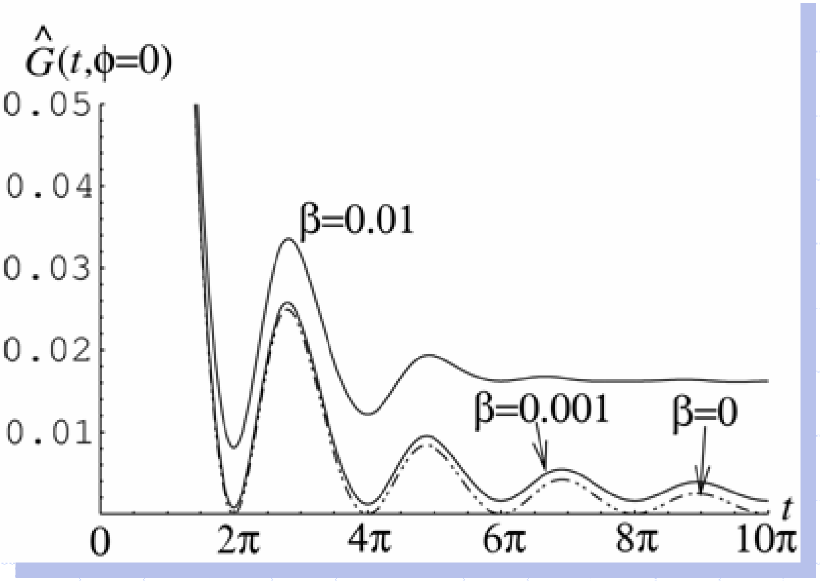
$$\hat{G}(t,\phi) \equiv -16\sin^2\frac{w}{2}\sin^2\frac{\overline{w}}{2}\langle \partial X\bar{\partial}X(w_1)\,\partial X\bar{\partial}X(w_2)\rangle$$
$$= \frac{1}{N}\sum_{n=1}^{\infty}\frac{8n}{\sinh\beta n}\frac{1}{(n\sin\frac{t}{n})^2}\left[\sin^2\frac{w}{2}+\sin^2\frac{\overline{w}}{2}-\frac{2\sin\frac{t}{\ell}\sin\frac{w}{2}\sin\frac{\overline{w}}{2}}{n\tan\frac{t}{n\ell}}\right],$$

$$w = \phi - \frac{t}{\ell}, \quad \overline{w} = \phi + \frac{t}{\ell}$$

Typical state correlator: example

- Short-time behavior:

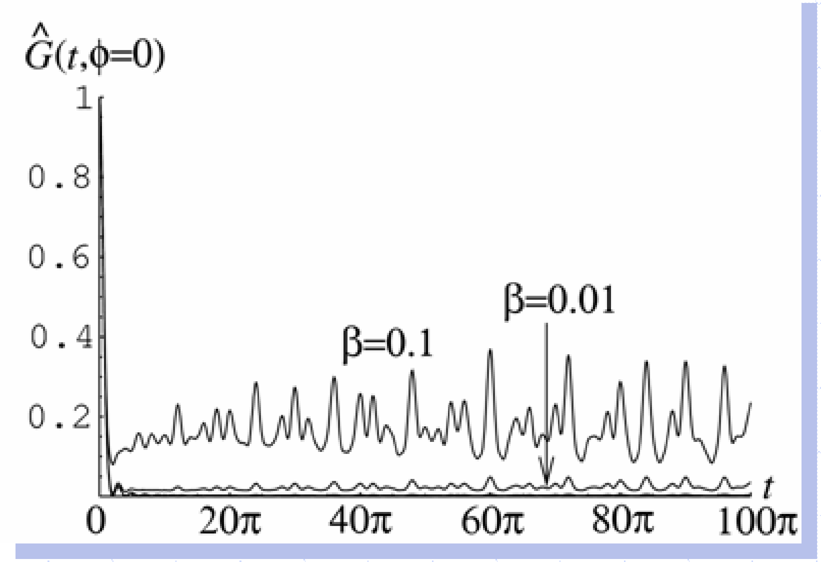


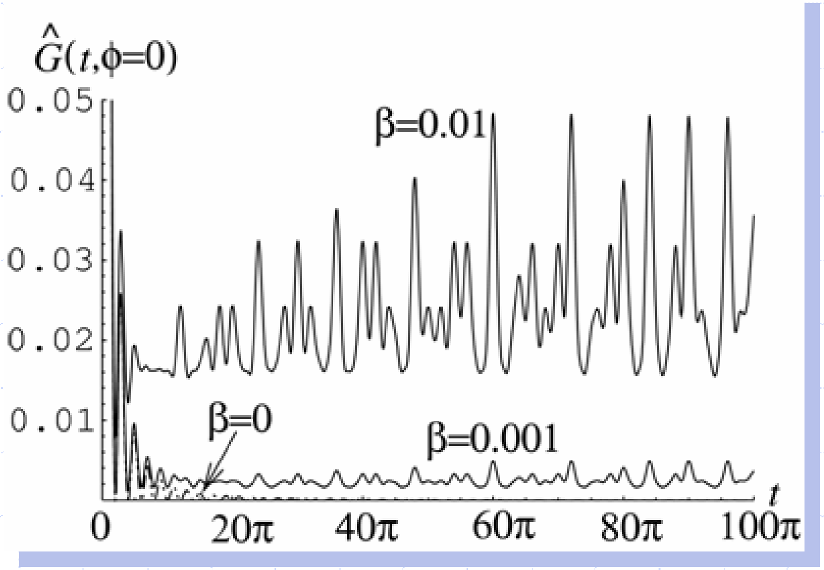


- Decays rapidly at initial times $(t \lesssim \pi \ell_{\mathrm{AdS}})$
- As N→∞ (β→0), approaches a certain limit shape (actually M=0 BTZ correlator!)

Typical state correlator: example

Long-time behavior:





- Becomes random-looking, quasi-periodic
- The larger N is, the longer it takes until the quasi-periodic regime
- Precise functional form depends on detail of microscopic distribution $\{N_{n\mu}, N'_{n\mu}\}$

Effective geometry of microstates with J=0

General non-twist bosonic correlator for $(h,\tilde{h})$

$$\langle \mathcal{A}(w_1)\mathcal{A}(w_2)\rangle = \frac{1}{N}\sum_n nN_n \sum_{k=0}^{n-1} \frac{C}{\left[2n\sin\left(\frac{w-2\pi k}{2n}\right)\right]^{2h}\left[2n\sin\left(\frac{\overline{w}-2\pi k}{2n}\right)\right]^{2\tilde{h}}},$$

$$N_n = \frac{8}{\sinh\beta n}, \quad w = \phi - \frac{t}{\ell}, \quad \overline{w} = \phi + \frac{t}{\ell}$$

- Substantial contribution comes from terms with

$$n \sim 1/\beta \sim \sqrt{N}$$

- For $t \ll n$, can approximate the sum:

$$\sum_{k=0}^{n-1} \frac{1}{\left[2n\sin\left(\frac{w-2\pi k}{2n}\right)\right]^{2h}\left[2n\sin\left(\frac{\overline{w}-2\pi k}{2n}\right)\right]^{2\tilde{h}}} \approx \sum_{k=-\infty}^{\infty} \frac{1}{(w-2\pi k)^{2h}(\overline{w}-2\pi k)^{2\tilde{h}}}$$

For $t \ll t_c = \mathcal{O}(\sqrt{N})$, correlator is indep. of details of microstates:

$$\langle \mathcal{A}(w_1)\mathcal{A}(w_2)\rangle \approx \sum_{k=-\infty}^{\infty} \frac{C}{(w-2\pi k)^{2h}(\overline{w}-2\pi k)^{2\tilde{h}}}.$$

Correlator for M=0 BTZ black hole

- Crucial points:
  - For N≫1, correlator for any state is very well approximated by that for the “typical state”
  - Typical state is determined solely by statistics
  - Correlator decomposed into constituents

Comments:

- M=0 BTZ has no horizon

  → we ignored interaction

- Still, M=0 BTZ has BH properties
  - Well-defined classical geometry
  - Correlation function decays to zero at late times

Notes on correlator

- M=0 BTZ correlator decays like $1/t^2$
- Microstate correlators have quasi-periodic fluctuations with mean $\sim 1/\sqrt{N}$
  Cf. for finite system: mean $\sim e^{-cS}$
  →need effect of interaction?
- Periodicity: $\Delta t \sim \mathrm{LCM}(1, \ldots, \sqrt{N}) \sim e^{c\sqrt{N}} = e^{c'S}$
  as expected of finite system

Fermion correlator also sees M=0 BTZ

Argument using LM metric

$$ds^2_{\text{string}} = \frac{1}{\sqrt{f_1 f_5}}[-(dt-A)^2 + (dy+B)^2] + \sqrt{f_1 f_5}\, dx^i dx^i + \sqrt{\frac{f_1}{f_5}} dz^a dz^a,$$

$$f_5 = \frac{Q_5}{L}\int_0^L \frac{dv}{|\mathbf{x}-\mathbf{F}(\mathbf{v})|^2}, \qquad f_1 = \frac{Q_5}{L}\int_0^L \frac{|\dot{\mathbf{F}}(v)|^2 dv}{|\mathbf{x}-\mathbf{F}(\mathbf{v})|^2}, \qquad A_i = -\frac{Q_5}{L}\int_0^L \frac{\dot{F}_i(v) dv}{|\mathbf{x}-\mathbf{F}(\mathbf{v})|^2},$$

Plug in profile F(v) corresponding to typical distribution

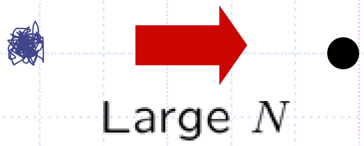
$$n \sim \sqrt{N} \sim 1/\beta, \qquad N_n = \frac{8}{\sinh \beta n} \sim \mathcal{O}(1)$$

$$\Rightarrow |\mathbf{F}| \sim \sqrt{N_n} \sim 1 \ll \ell \sim N^{1/4}, \qquad \ell = N^{1/4} \text{ (AdS}_3\text{ radius)}$$

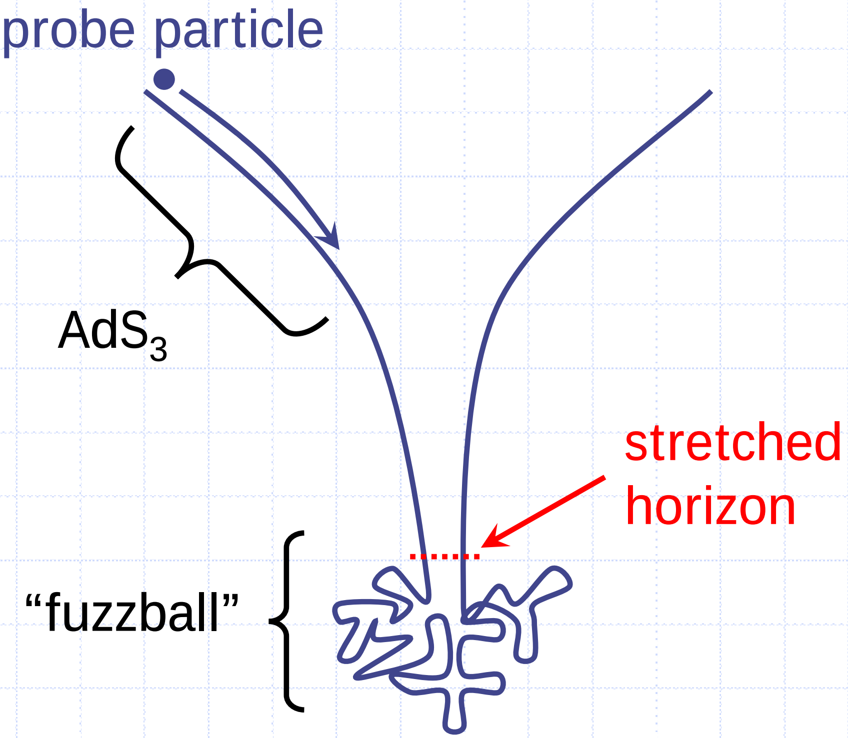
If one assumes that F(v) is randomly fluctuating, for $r \ll l$,

$$ds^2 = -\frac{r^2}{\ell^2}dt^2 + \frac{r^2}{\ell^2}dy^2 + \frac{\ell^2}{r^2}(dr^2 + r^2 d\Omega_3^2) + \sqrt{\frac{Q_1}{Q_5}}\, ds^2_{T^4}$$

This is M=0 BTZ black hole!



Consistency check: where are we probing?



$$t_c \sim \left[\begin{array}{c} \text{time until probe} \\ \text{reaches stretched horizon} \end{array} \right] \sim \sqrt{N}$$

Effective geometry of microstates with J≠0

Typical state:

$$\left[\text{"BE condensate"}\right] \times \left[\begin{array}{l}\text{typical states of ensemble} \\ \text{with } (N,J) \to (N-J,0)\end{array}\right]$$

For bosonic correlator for probe with $(h, \tilde{h})$

$$\langle \mathcal{A}(w_1)\mathcal{A}(w_2)\rangle = \frac{|J|}{N}\langle \mathcal{A}\mathcal{A}\rangle_{\mathrm{AdS}_3} + \left(1 - \frac{|J|}{N}\right)\langle \mathcal{A}\mathcal{A}\rangle_{M=0\ \mathrm{BTZ}}$$

$$\text{for} \quad t \ll t_c = \mathcal{O}(\sqrt{N-|J|})$$

Bulk geometry is "weight sum" of AdS_3 and M=0 BTZ black hole??

Effective geo. for J≠0: Argument using LM metric

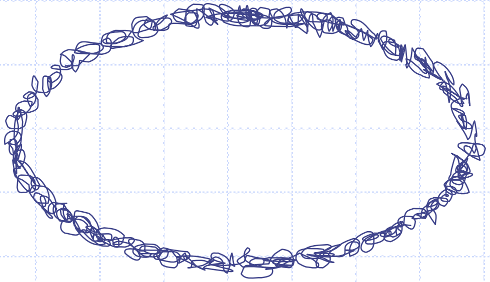
$$\left[\text{"BE condensate"}\right] \times \left[\begin{array}{l}\text{typical states of ensemble} \\ \text{with } (N,J) \to (N-J,0)\end{array}\right]$$

$$\Downarrow$$

Profile: (ring) + (fluctuation)

$$\mathbf{F} = \mathbf{F}^{(0)} + \delta\mathbf{F}, \qquad \mathbf{F}^{(0)} : \text{ring}$$

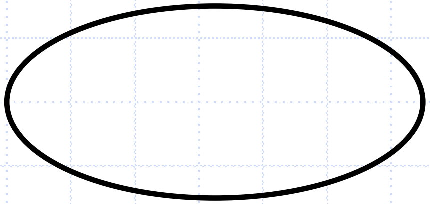
$\delta\mathbf{F}$:small-amplitude, high-frequency fluctuation



$\Rightarrow$

Large N,

large $J = \mathcal{O}(N)$



black ring with
vanishing horizon

Conclusion

- For large N, almost all microstates in D1-D5 ensemble is well approximated by typical state
- Form of typical state is governed solely by statistics
- At sufficiently early times, bulk geometry is effectively described by M=0 BTZ BH
- At later times ($t \gtrsim t_c \sim N^{1/2}$), description by effective geometry breaks down

Message:

A black hole geometry should be understood as an effective coarse-grained description that accurately describes the results of "typical" measurements, but breaks down in general.