Geometry / Gauge Theory Duality and the Dijkgraaf–Vafa Conjecture

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Introduction

Gauge theory: hard to study

- Strongly coupled at low E
- Confinement / chiral symmetry breaking
- Even vacua are not known analytically

Supersymmetric gauge theory: more tractable

- Sometimes exact analysis is possible
- Superpotential: determines vacua
- No systematic way to compute superpotential

Introduction

Dijkgraaf–Vafa conjecture

- Based on string theory duality
- One can compute superpotential systematically using matrix model

"Recipe"

- Need to go back to string theory when matrix model is not enough
- Sometimes counter-intuitive from gauge theory viewpoint: e.g., glueball S for $U(1)$, $Sp(0)$
- Inclusion of flavors

Outline

- Introduction $\surd$
- Dijkgraaf–Vafa conjecture
- “Counterexample” [hep-th/0303104, 0304138]
- String theory prescription [0311181]
- Inclusion of flavors [0405101]
- Future problems
- Conclusion

DV conjecture

Dijkgraaf-Vafa conjecture:

For a large class of $\mathcal{N} = 1$ supersymmetric gauge theory,

i) Low E degree of freedom is glueball superfield

$$S \sim \mathrm{tr}[W^\alpha W_\alpha] = \mathrm{tr}\,\lambda^\alpha \lambda_\alpha + \ldots$$

ii) Effective superpotential $W_{\mathrm{eff}}(S)$ which encodes nonperturbative effect can be exactly calculated by matrix model

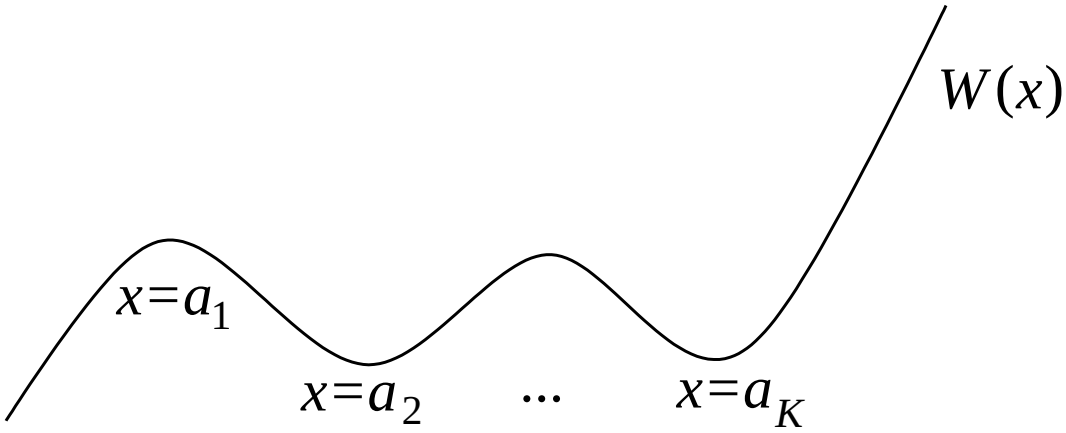
DV conjecture

Example:

$\mathcal{N}=1$ $U(N)$ theory with adjoint chiral superfield Φ_{ij}

$$W_{\text{tree}} = \text{Tr}[W(\Phi)],$$

$$W'(x) = (x-a_1)(x-a_2)\cdots(x-a_K)$$

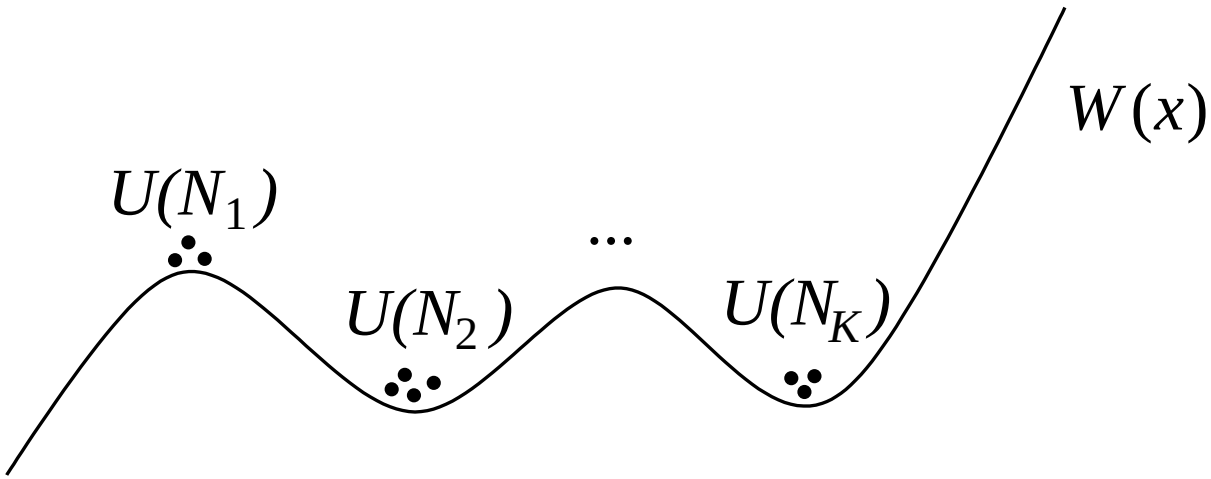


DV conjecture

- Classical vacua:

$$\Phi \cong \mathrm{diag}(\underbrace{a_1, \ldots, a_1}_{N_1}, \underbrace{a_2, \ldots, a_2}_{N_2}, \ \ldots \ , \underbrace{a_K, \ldots, a_K}_{N_K})$$

$$U(N) \to U(N_1) \times U(N_2) \times \cdots \times U(N_K)$$

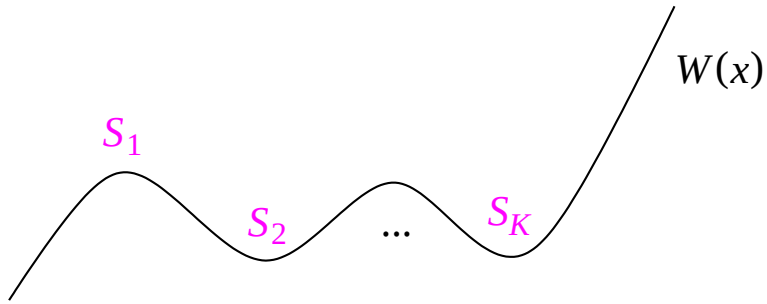


N_i eigenvalues at the i−th critical point

DV conjecture

- Glueballs

$$U(N) \to U(N_1) \times U(N_2) \times \cdots \times U(N_K)$$



- Effective glueball superpotential

$$W_{\text{eff}}(S_1, \ldots, S_K) = \sum_{i=1}^{K} N_i \frac{\partial F_0}{\partial S_i}, \qquad F_0 : \text{ MM free energy}$$

DV conjecture

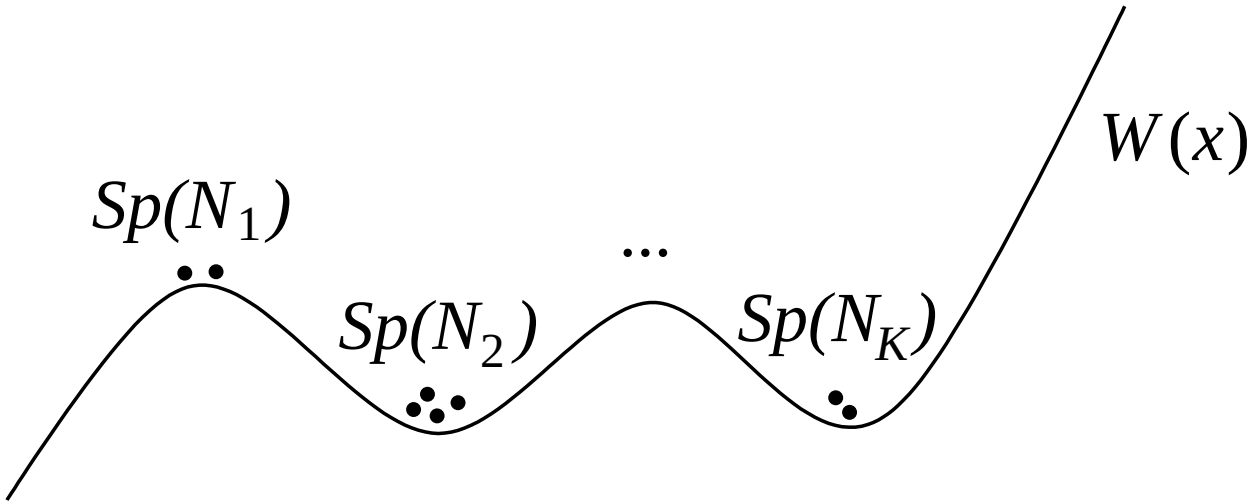
- Systematic way to compute nonpert. superpot.
- Checked for many nontrivial examples
- Second part (reduction to matrix model) can be proven by superspace Feynman diagrams
- Philosophy applicable to any representations [CDSW, AIVW]

“Counterexample”

$Sp(N)$ with antisymmetric tensor

- Breaking pattern:

$$Sp(N) \to Sp(N_1) \times Sp(N_2) \times \cdots \times Sp(N_K)$$



“Counterexample”

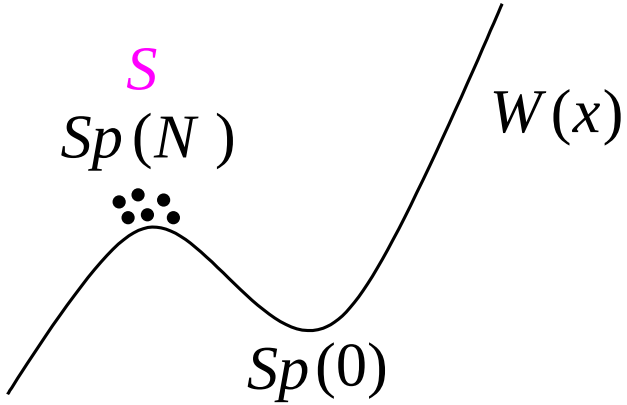
Discrepancy

- Cubic superpotential with

$$Sp(N) \to Sp(N) \times Sp(0)$$

- One glueball S for unbroken $Sp(N)$
- Discrepancy:

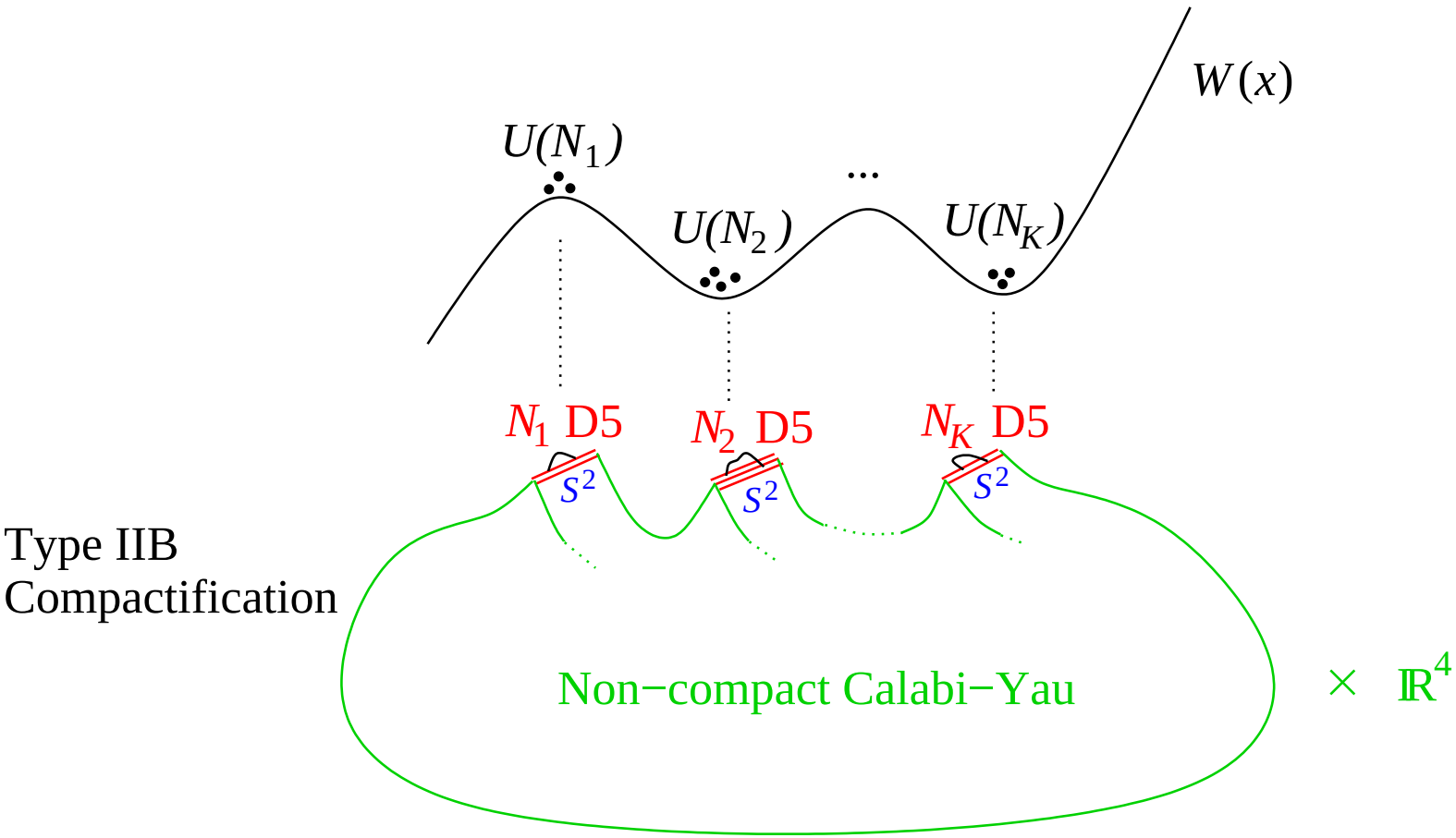
$$W_{\rm DV} = W_{\rm eff}\big(\langle S \rangle\big) \neq W_{\rm GT} \ \ !$$



String theory prescription

Geometric engineering of $U(N)$ theory

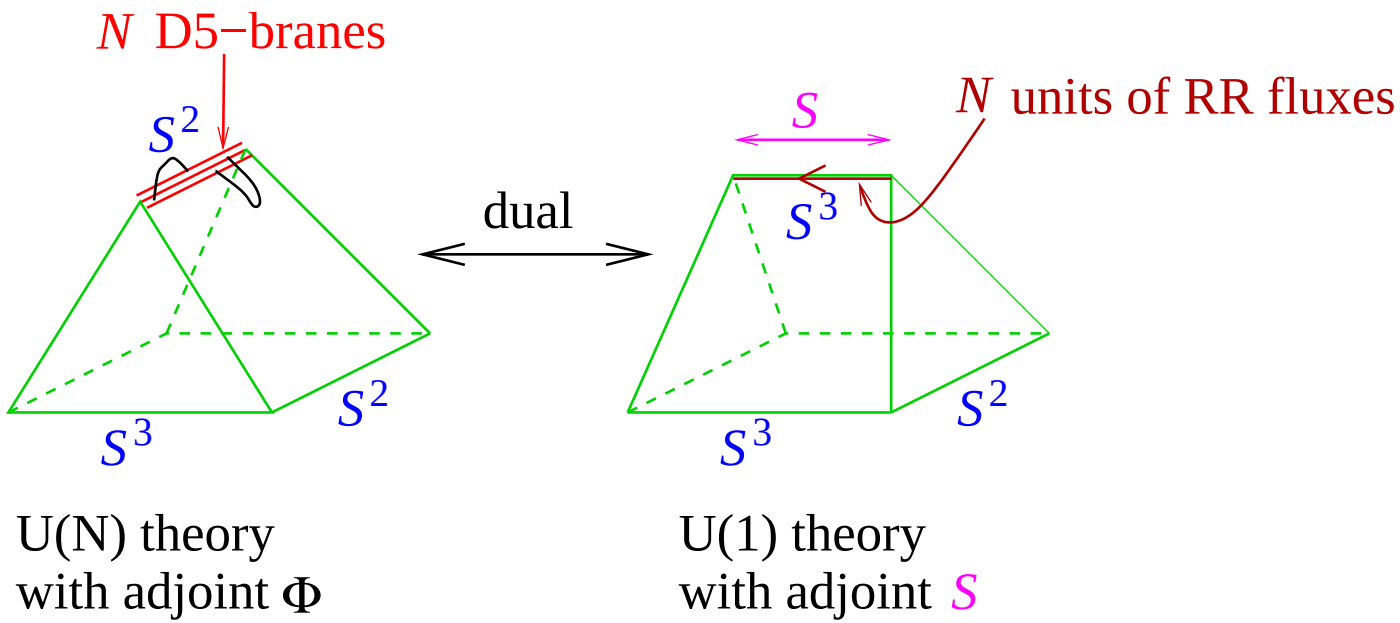
Breaking pattern: $U(N) \to U(N_1) \times \cdots \times U(N_K)$



String theory prescription

Geometric transition — simple case

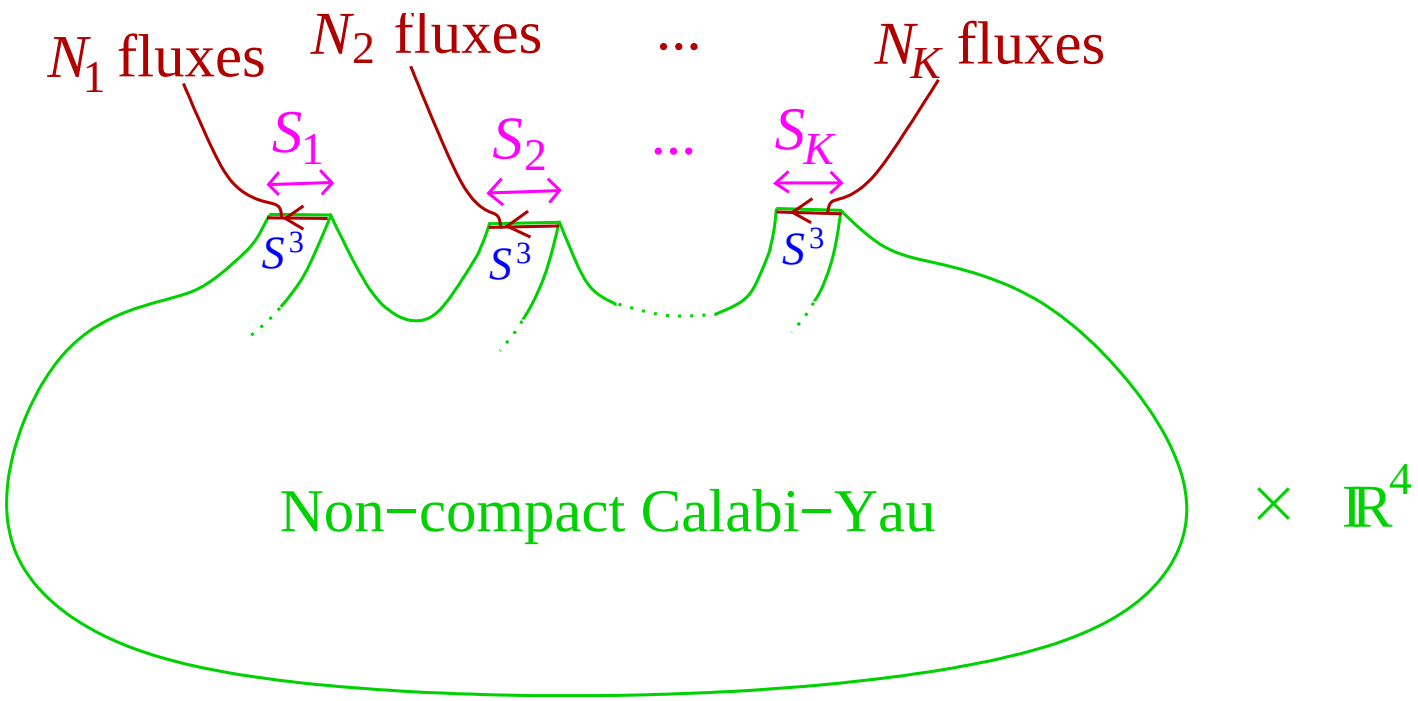
- Open string theory on S^2-blown up conifold
  $\Longleftrightarrow$ closed string theory on S^3-blown up conifold



- A version of AdS/CFT duality

String theory prescription

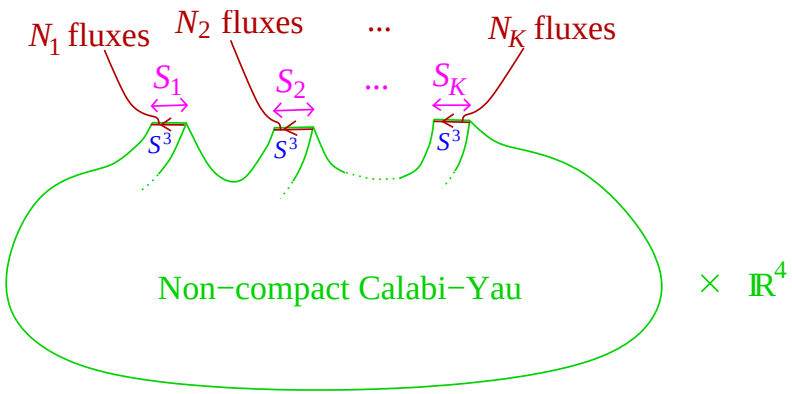
Geometric transition — general case



- 4d theory: $U(1)^K$ theory with $S_1, \ldots, S_K$

String theory prescription

Flux superpotential



- Exact superpotential

$$W_{\rm flux}(S_j) = \sum_{i=1}^{K}\Big[\underbrace{N_i}_{\text{RR flux}}\,\Pi_i(S_j) - \underbrace{2\pi i\tau_0}_{\text{NSNS flux}}\,S_i\Big]$$

- Computation of Π_i reduces to MM $\longrightarrow$ DV conjecture

String theory prescription

Physics near critical points

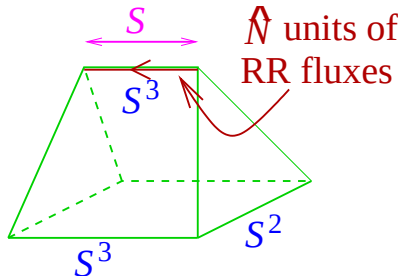
- More generally, for $G(N) \to \prod_i G_i(N_i)$,

$$W_{\text{flux}}(S_j) \sim \sum_{i=1}^{K} \left[\widehat{N}_i S_i \big(1 + \ln(\Lambda_i^3/S_i)\big) - 2\pi i \eta_i \tau_0 S_i \right]$$

$$\widehat{N}_i = \text{(\# of RR fluxes)} = \begin{cases} N_i & U(N_i) \\ N_i/2 \mp 1 & SO/Sp(N_i) \end{cases}$$

$\Downarrow$ focus on one critical point

$$\boxed{W_{\text{flux}}(S) \sim \widehat{N} S \left[1 + \ln\left(\frac{\Lambda^3}{S}\right) \right] - 2\pi i \eta \tau_0 S.}$$



- When is S a good variable?

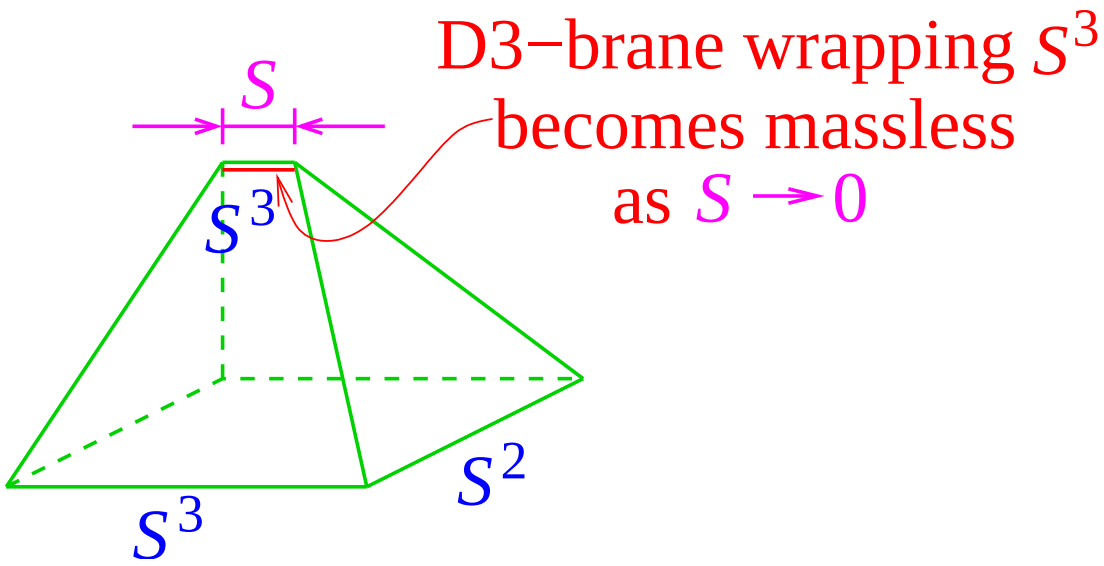
String theory prescription

$\widehat{N} = 0$ **case**

$$W_{\rm flux}(S) = -2\pi i\eta\tau_0 S, \qquad \frac{\partial W_{\rm flux}}{\partial S} = -2\pi i\eta\tau_0 \neq 0$$

$\Longrightarrow$ No susy solutions?

- Extra massless degree of freedom:



String theory prescription

$\widehat{N} = 0$ case

- Taking D3-brane hypermultiplet $h, \widetilde{h}$ into account

$$W_{\text{flux}} = h\widetilde{h}S - 2\pi i\eta\tau_0 S \quad \Longrightarrow \quad S = 0 \ \ \underline{\text{is}} \text{ a solution}$$

- For $U(0)$, $SO(2)$, set $S = 0$ from the beginning

$\widehat{N} > 0$ case

- D3-brane is infinitely massive with nonvanishing RR flux through it, hence there are no $h, \widetilde{h}$.
- There is a glueball for

$$U(N > 0),\ SO(N > 2),\ Sp(N \geq 0).$$

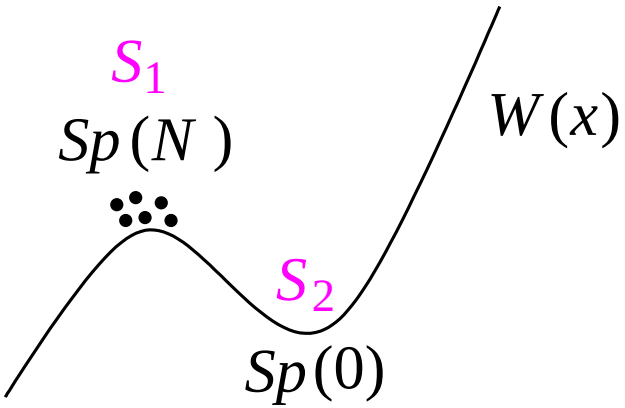
String theory prescription

Resolving discrepancy

- Breaking pattern was

$$Sp(N) \to Sp(N) \times Sp(0)$$

Need glueball S even for $Sp(0)$!



- Taking S_2 into account resolves discrepancy

Inclusion of flavors

$U(N)$ theory with an adjoint Φ and N_f flavors $Q, \widetilde{Q}$

- Tree level superpotential:

$$W_{\rm tree} = {\rm Tr}[W(\Phi)] - \sum_{I=1}^{N_f} \widetilde{Q}^I (\Phi - z_I) Q_I, \quad z_I = -m_I$$

- Classical vacua:

  i) Pseudo-confining vacua: $\langle Q \rangle = \langle \widetilde{Q} \rangle = 0$

$$U(N) \to \prod_{i=1}^{K} U(N_i), \qquad \sum_{i=1}^{K} N_i = N.$$

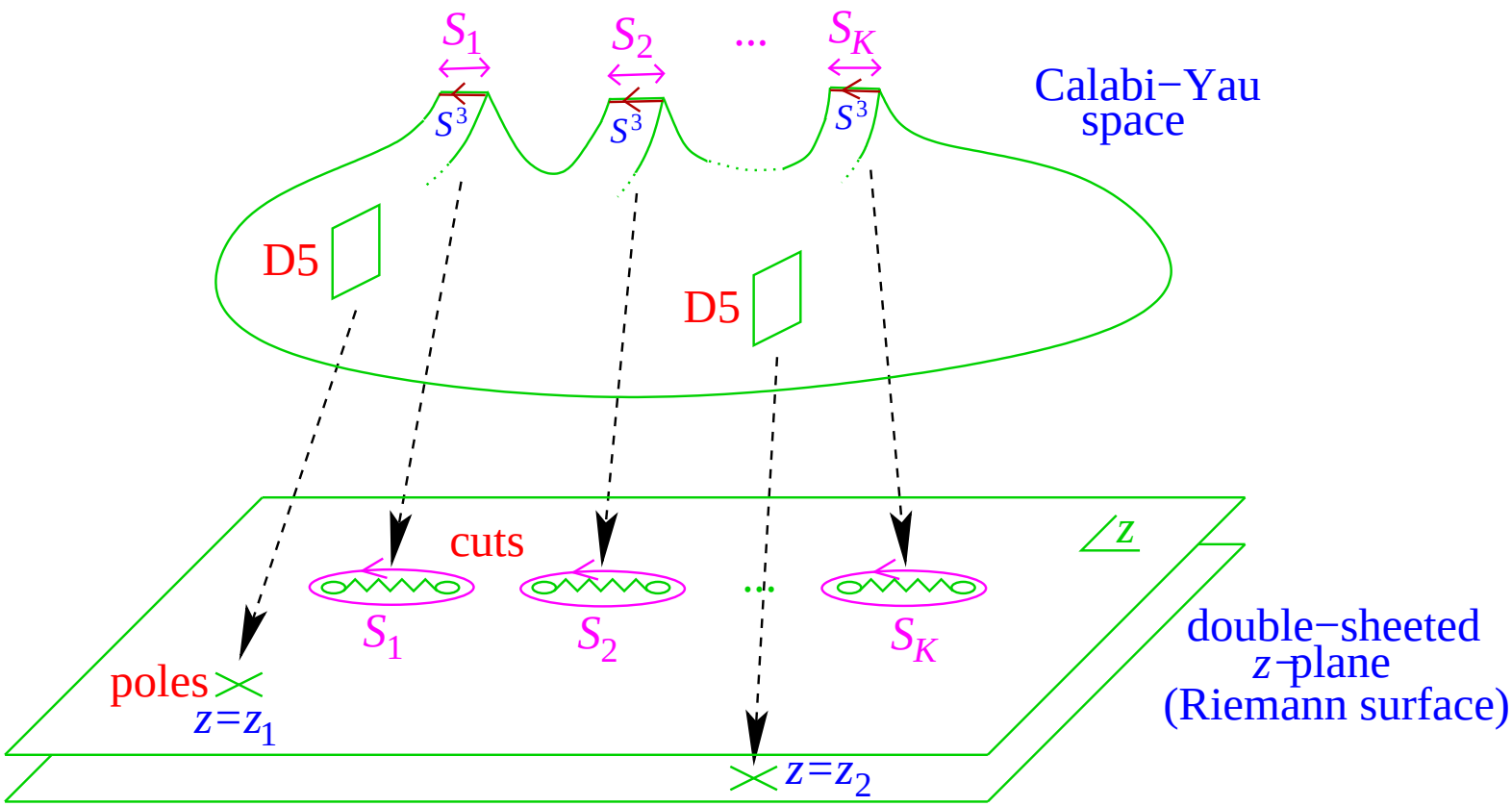
  ii) Higgs vacua: $\langle Q \rangle, \langle \widetilde{Q} \rangle \neq 0$

$$U(N) \to \prod_{i=1}^{K} U(N_i), \qquad \sum_{i=1}^{K} N_i < N.$$

Inclusion of flavors

Generalized Konishi anomaly formalism

- Descend from CY to z-plane [CDSW,CSW]

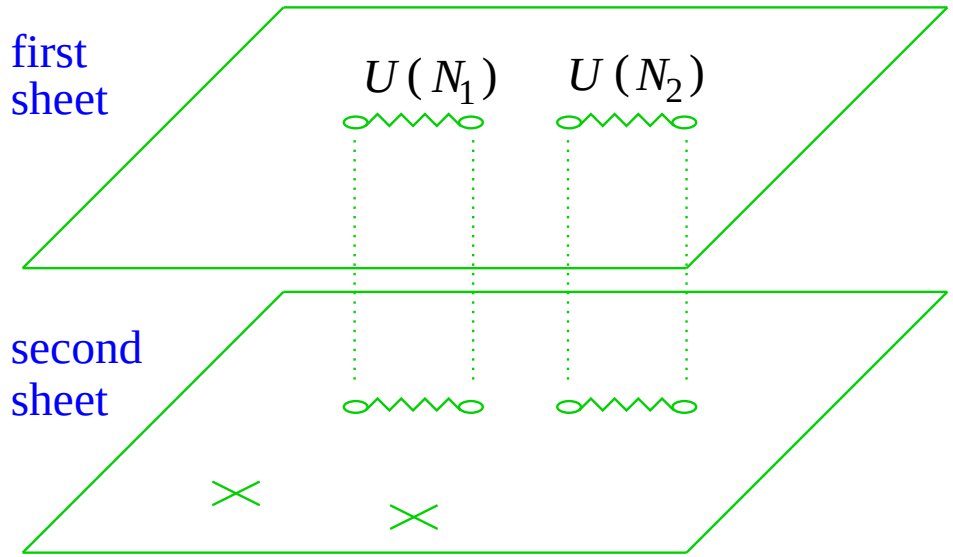


Inclusion of flavors

How are the vacua described?

- Pseudo-confining vacua

$$U(N) \to U(N_1) \times U(N_2), \quad N_1 + N_2 = N.$$



All flavor poles are on the second sheet

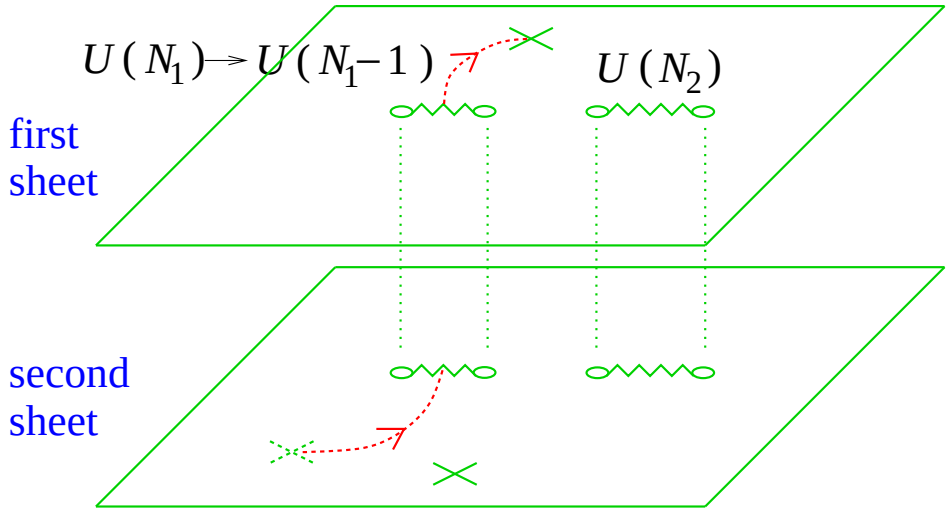
Inclusion of flavors

How are the vacua described?

- Higgs branch

$$U(N) \to U(N_1) \times U(N_2), \quad N_1 + N_2 < N.$$

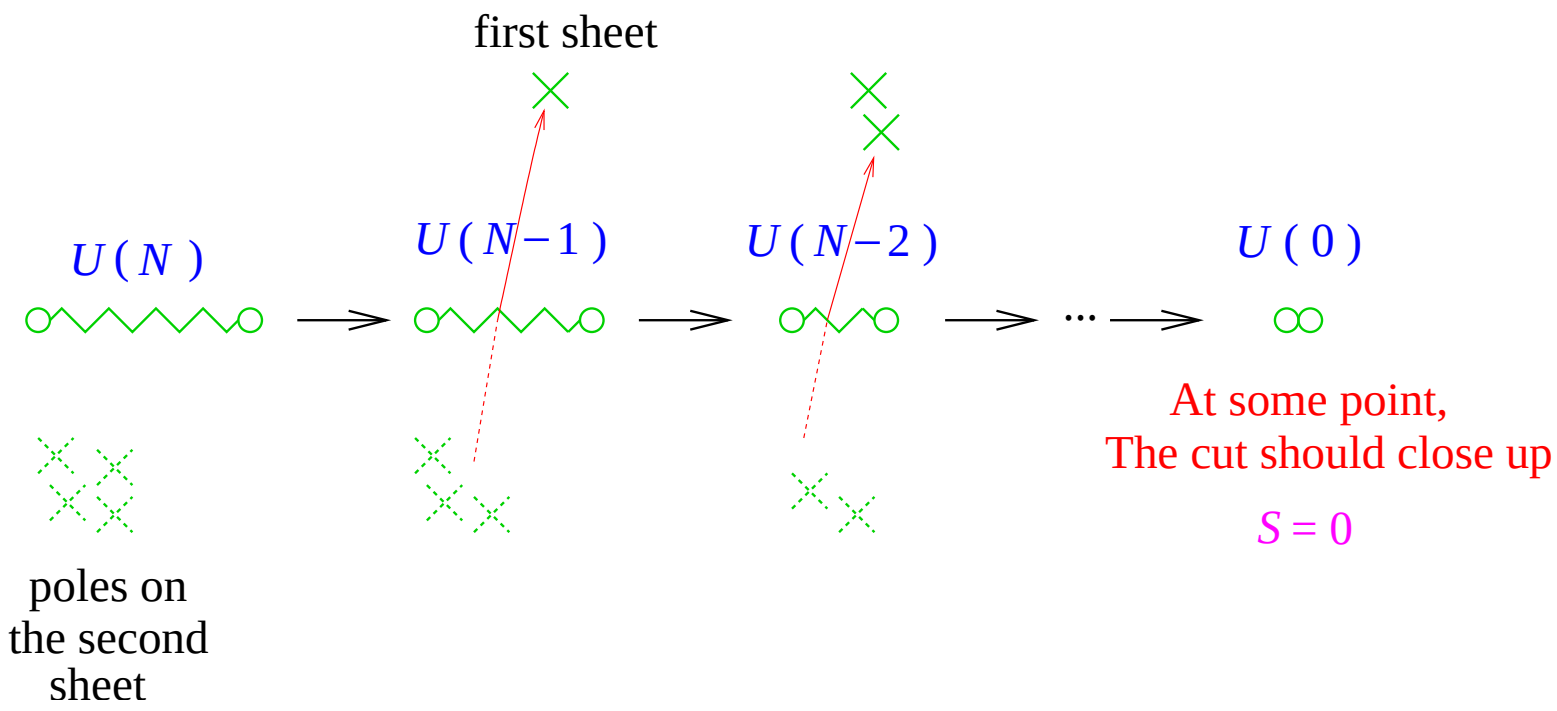
Passing poles through cuts corresponds to Higgsing:



Inclusion of flavors

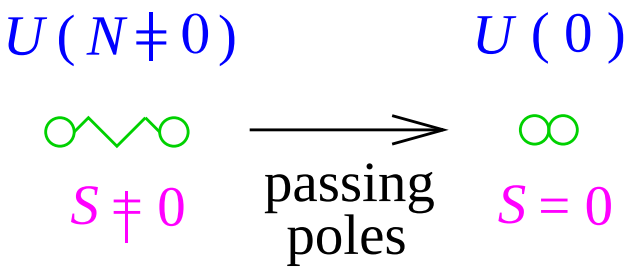
How many poles can pass through a cut?

- There must be a limit to this passing process, *on-shell*:



Inclusion of flavors

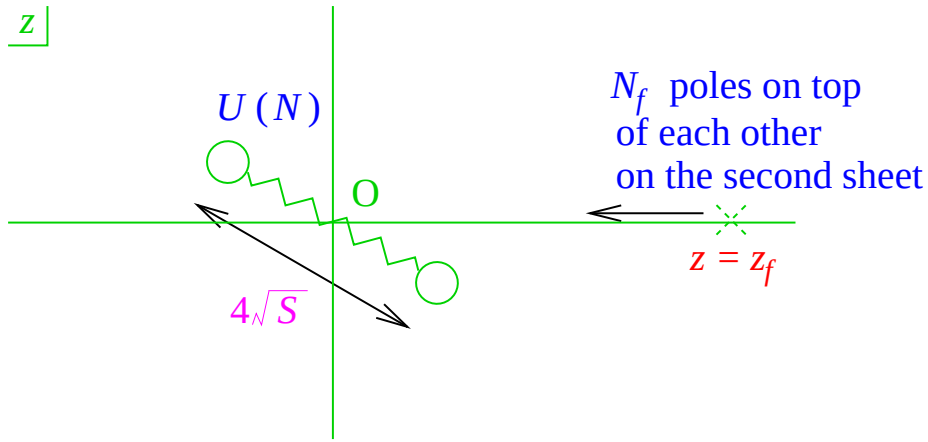
$S = 0$ solutions and matrix model



- $S = 0$ solutions should not be directly describable in matrix model
- $U(N \neq 0) \to U(0)$ is not a smooth process; # of massless photons changes discontinuously
- There must be some extra charged massless DoF condensing

Inclusion of flavors

Try actually passing poles through a cut!



- Take one-cut case, solve the EOM

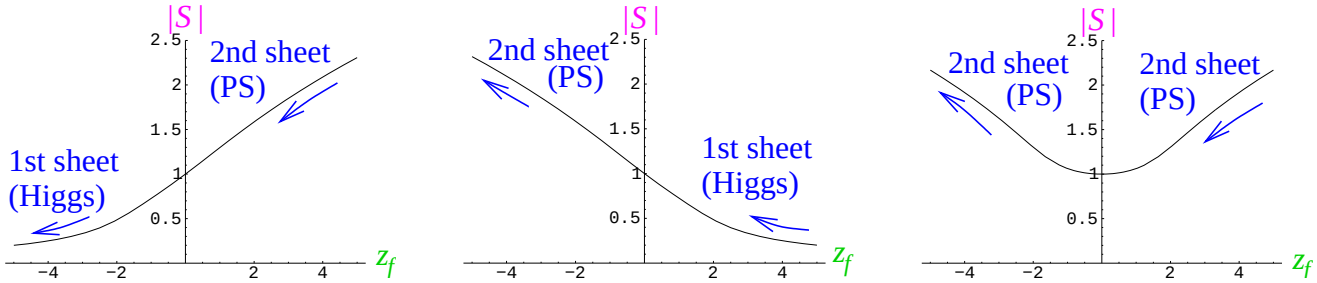
$$\frac{\partial W_{\text{eff}}(S; z_f)}{\partial S} = 0 \quad \Longrightarrow \quad z_f = S^{N/N_f} + S^{1-N/N_f},$$

and study S as a function of z_f for various N, N_f

Inclusion of flavors

$N_f < N$ case

- Typical $|S|$ versus z_f graphs ($N_f = N/2$):

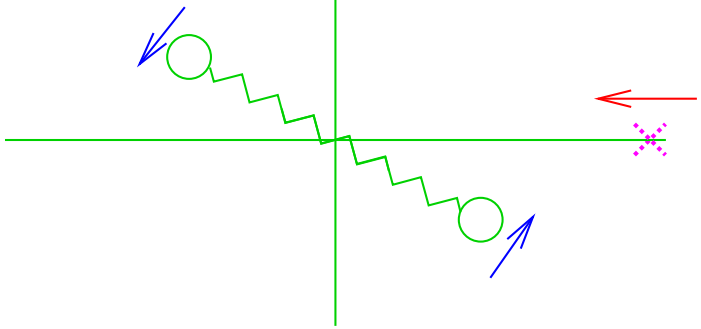


- Three branches:
  - i) Poles pass through and reach 1st sheet, without obstruction
  - ii) Reverse process of i)
  - iii) Poles get reflected back to 2nd sheet
- Cut never closes up: $S \neq 0$
- Corresponds to Higgsing $U(N) \to U(N - N_f)$

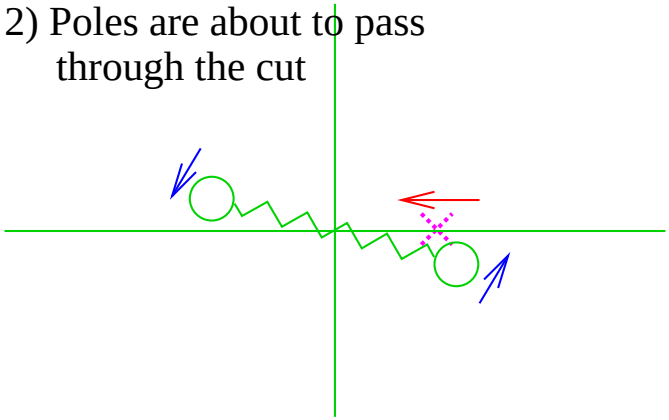
Inclusion of flavors

How can poles be reflected back to 2nd sheet?

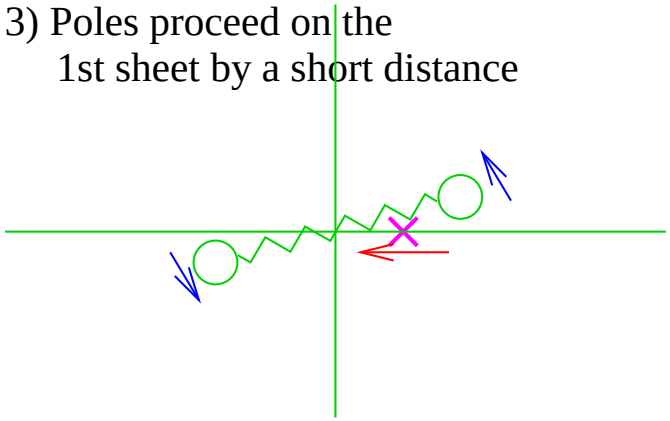
1) Poles approach from infinity
on the 2nd sheet



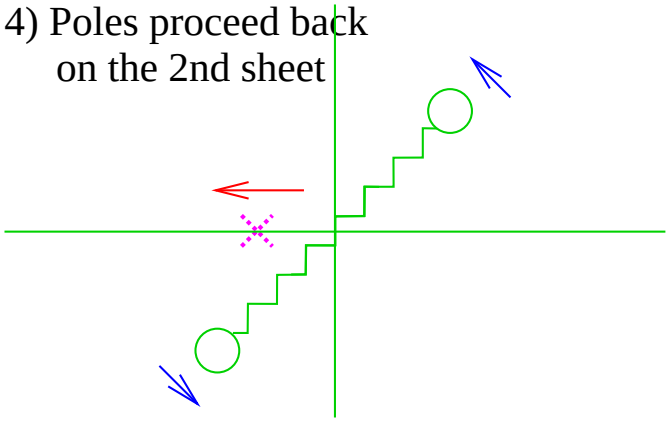
2) Poles are about to pass
through the cut



3) Poles proceed on the
1st sheet by a short distance



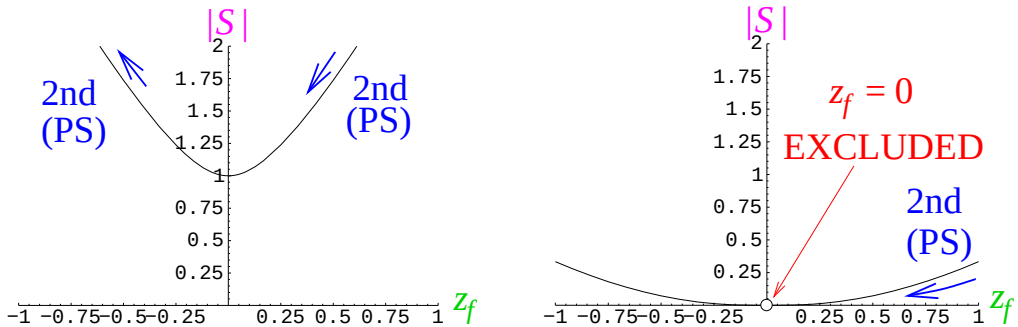
4) Poles proceed back
on the 2nd sheet



Inclusion of flavors

$N < N_f \leq 2N$ case

- Typical $|S|$ versus z_f graphs ($N_f = 3N/2$):



- Two branches:
  - i) Poles get reflected back to 2nd sheet
  - ii) Cut closes up before poles reach it
- Careful study of EOM shows that $z_f = 0$, $S = 0$ is not a solution — exactly what we expected

Inclusion of flavors

Exclusion of $z_f = S = 0$ solution for $N < N_f < 2N$

$$W_{\text{eff}} = S\left[N + \ln\left(\frac{m_A^N \Lambda_0^{2N-N_f}}{S^N}\right)\right] - N_f S\left[-\ln\left(\frac{z_f}{2} + \frac{1}{2}\sqrt{z_f^2 - \frac{4S}{m_A}}\right) + \frac{m_A z_f}{4S}\left(\sqrt{z_f^2 - \frac{4S}{m_A}} - z_f\right) + \frac{1}{2}\right] + 2\pi i \tau_0 S$$

- If $z_f \neq 0$, this leads to $z^f = S^{N/N_f} + S^{1-N/N_f}$.
- If $z_f = 0$

$$W_{\text{eff}} = S\left(N - \frac{N_f}{2}\right)\left[1 + \ln\left(\frac{\Lambda_0'^3}{S}\right)\right] + 2\pi i \tau_0 S$$

$$\frac{\partial W_{\text{eff}}}{\partial S} = \left(N - \frac{N_f}{2}\right)\ln\left(\frac{\Lambda_0'^3}{S}\right) + 2\pi i \tau_0 = 0 \implies S \neq 0$$

Inclusion of flavors

Summary so far:

- For $N < N_f \leq 2N$, there are solutions with $S \to 0$ as poles approach the cut, but $S = 0$ is not a solution to EOM in the MM context.
- But in the GT context (Seiberg–Witten theory), $S = 0$ is a solution.
- This is just as expected — massless DoF is missing in the MM (glueball) framework

- $N_f > 2N$ case cannot be discussed in one-cut case, but IR free so we must set $S = 0$
- $N_f = N$ case is exceptional (discussed later)

Inclusion of flavors

String theory interpretation: $N_f = 2N$

- Cut closes up for $z_f = 0$, for which

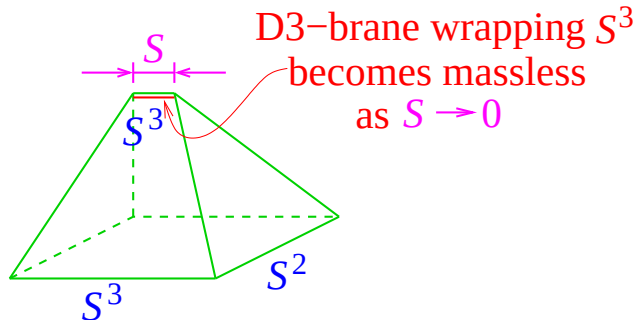
$$W_{\rm eff} = \widehat{N} S \left[1 + \ln \left(\frac{\Lambda_0'^{\,3}}{S} \right) \right] + 2\pi i \tau_0 S$$

$\widehat{N} \equiv N - N_f/2$: effective # of fluxes

This is of the same form as $U(N)$ theory w/o flavors

$\Downarrow$

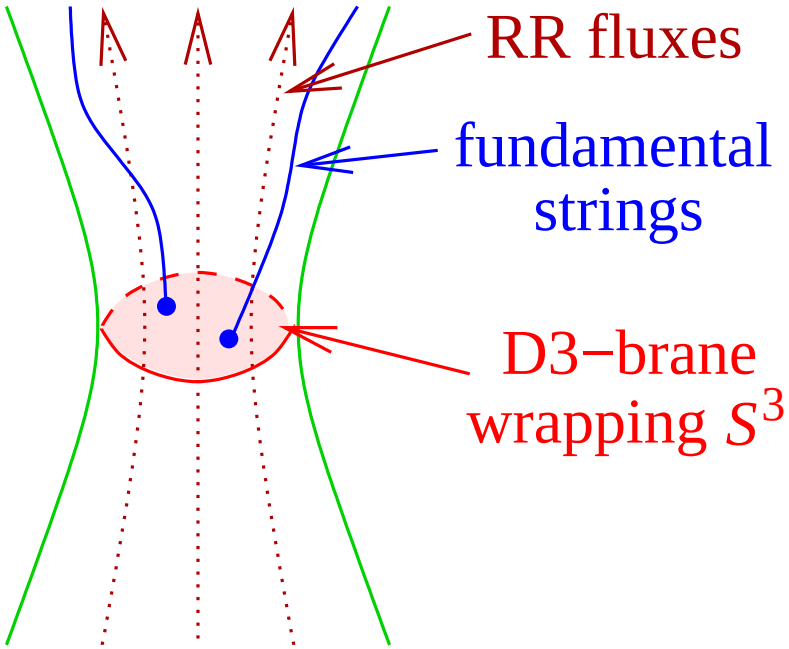
Same mechanism as the case w/o flavors; D3-brane wrapping S^3 makes $S = 0$ a solution.



Inclusion of flavors

String theory interpretation: $N < N_f < 2N$

- There is net RR flux through D3 in this case.
- If there weren't flavors, D3 would be infinitely massive:



$$T_{\rm D3} \int d^4\xi \, A_1 \wedge H_3^{\rm RR}$$

$\Rightarrow$ net $F_{\mu\nu}$ induced in D3

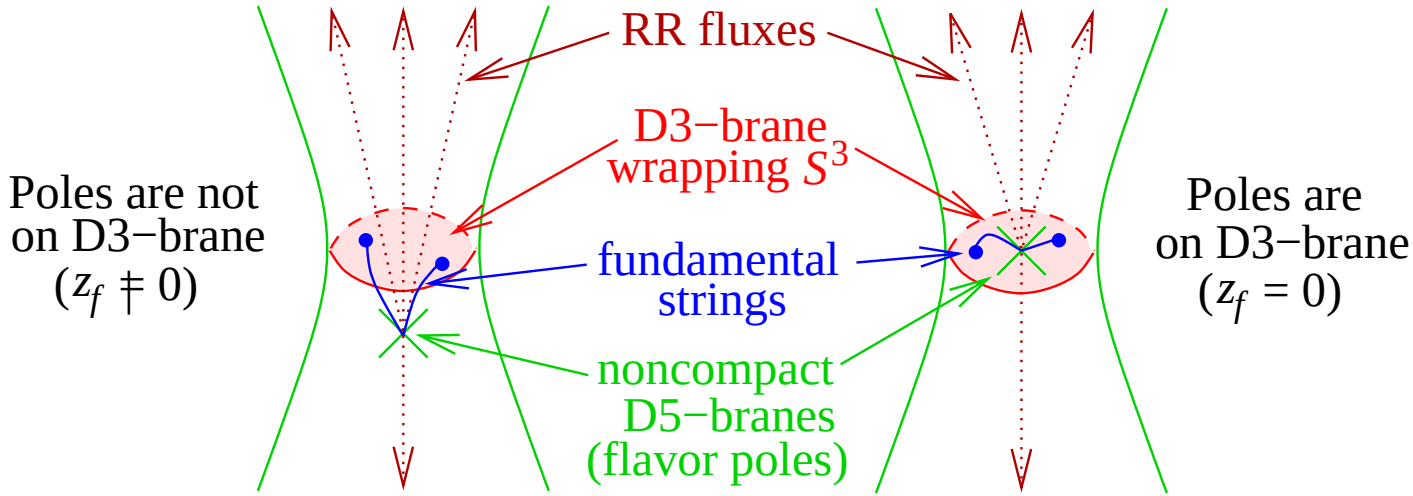
$\Rightarrow$ need to emanate F1

$\Rightarrow$ F1 extends to ∞

Inclusion of flavors

String theory interpretation: $N < N_f < 2N$

- In the presence of flavor poles = noncompact D5-branes, F1 can end!



- If $z_f = 0$, the F1 are massless
  $\Longrightarrow$ D3 with F1 on it ("baryon") is the massless DoF

Inclusion of flavors

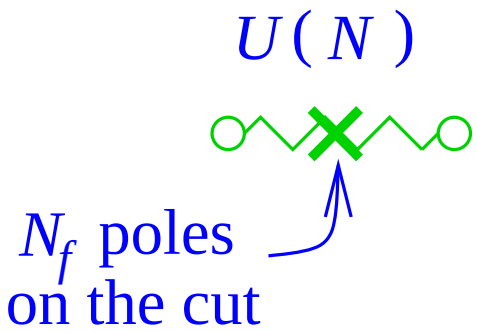
String theory interpretation: $N < N_f < 2N$

- Condensation of massless "baryon" DoF B causes $U(N \neq 0) \to U(0)$.
- We don't know the precise form of $W_{\text{eff}}(S, B)$, but the whole effect should be to make $S = 0$ a solution

Inclusion of flavors

Prescription

- Assume N_f poles are on the cut associated with $U(N)$.

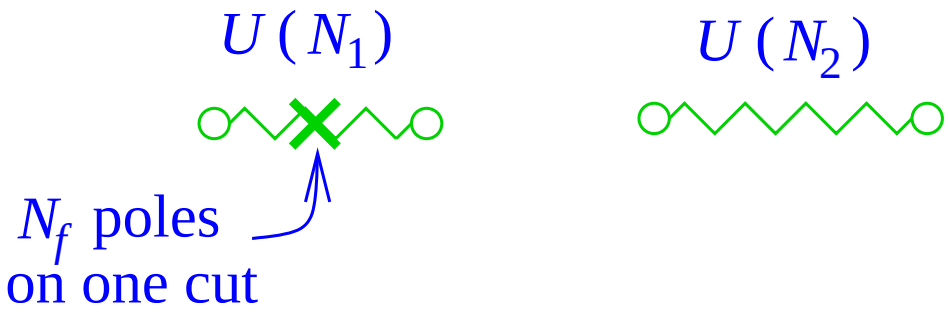


- For $N_f \geq 2N$, one should set $S = 0$ and it's the only solution.

- For $N < N_f < 2N$, $S = 0$ is a physical solution, although there may be $S \neq 0$ solutions too.

Inclusion of flavors

Example

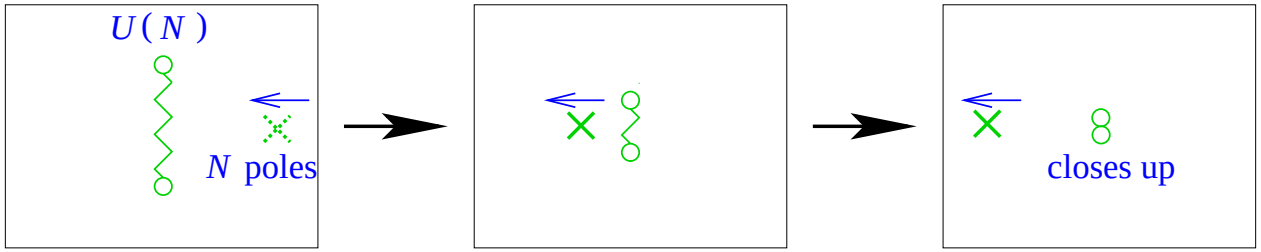


- We considered $U(3)$ theory with cubic $W(x)$, with all possible breaking pattern $U(3) \to U(N_1) \times U(N_2)$.
- We put N_f poles on a cut.
- We checked that W_{GT} can be reproduced by $W_{\mathrm{MM}}(S_1, S_2)$ by setting $S_1 = 0$ following prescription.

Future problems

- Refine string theory interpretation
  - Precise form of $W_{\text{eff}}(S, B)$
  - Baryonic property and $N_f \geq N$
  - Generalize to quiver theories, then descend

- $N = N_f$ case: when cut closes up, poles aren't on the cut



F1 has finite length, so "baryon" isn't massless.
But there must be massless DoF behind the scene

Conclusion

- DV conjecture provides new approach to susy gauge theories.
- Geometry/gauge theory duality clarified how string theory treats glueballs.
- Vanishing glueball $S = 0$ signifies existence of extra massless DoF.
- String theory helps identify the DoF.