Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85,021901C(2012); PRC86 , 024904(2012)

20min

Time Evolution of Higher Order Cumulants

MK, et al., in preparation

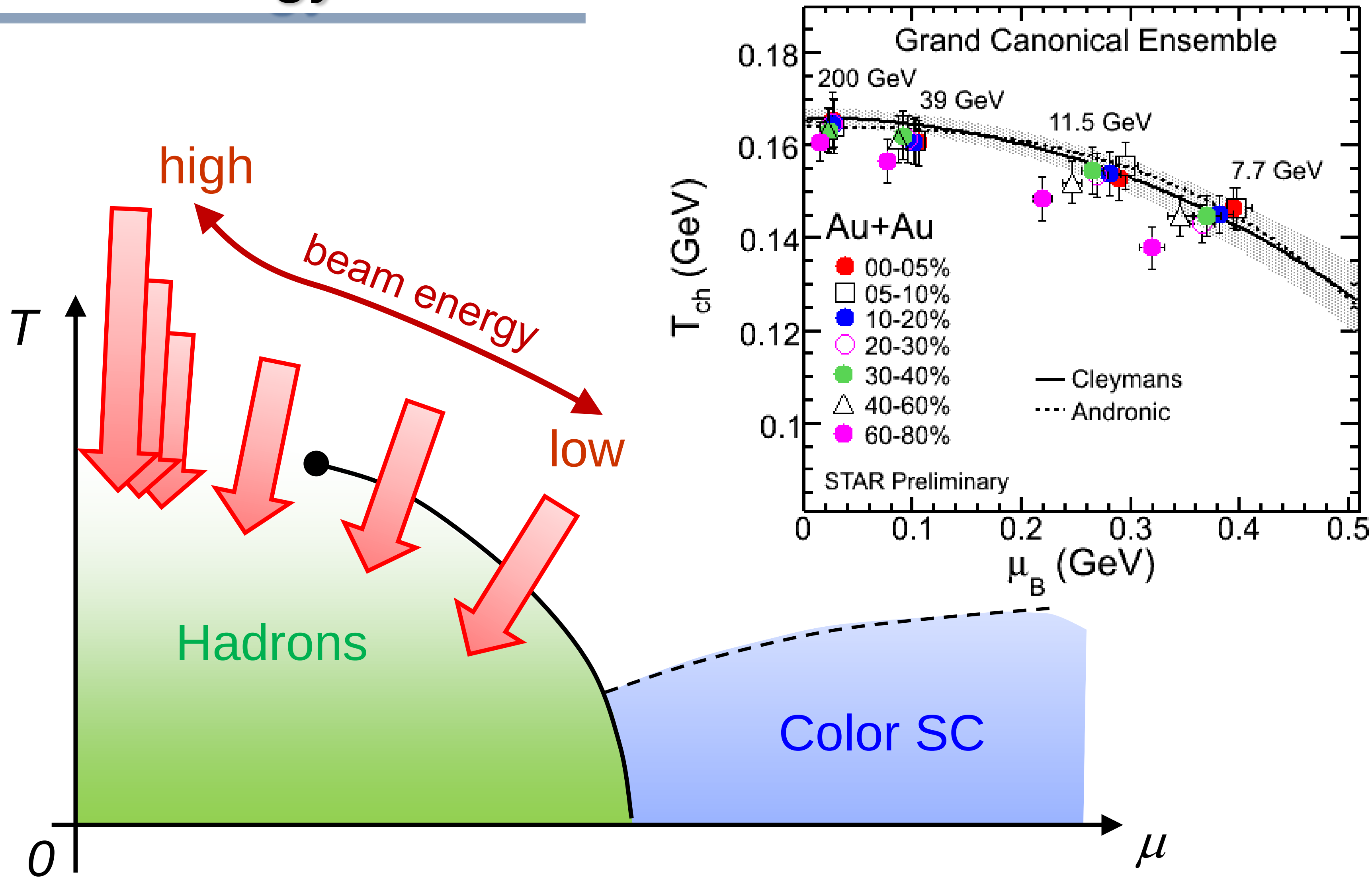
30min

Masakiyo Kitazawa

Osaka U.

Beam-Energy Scan

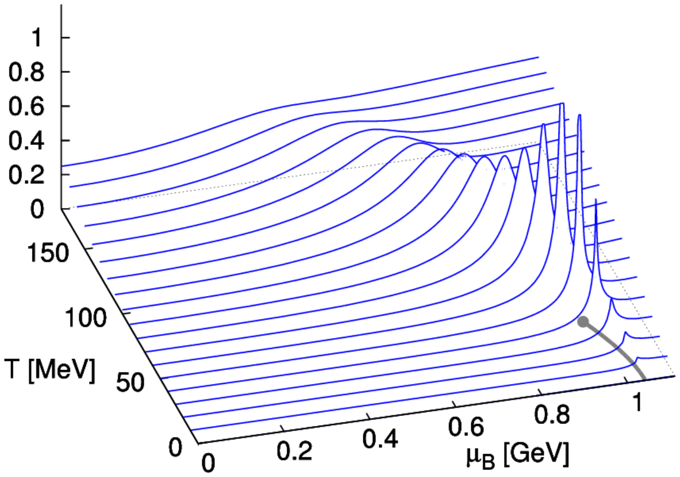
STAR 2012

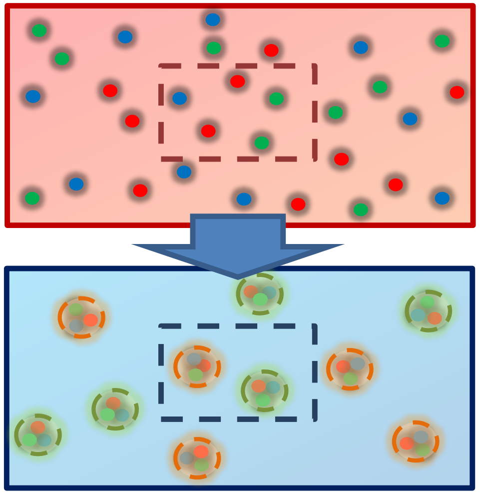


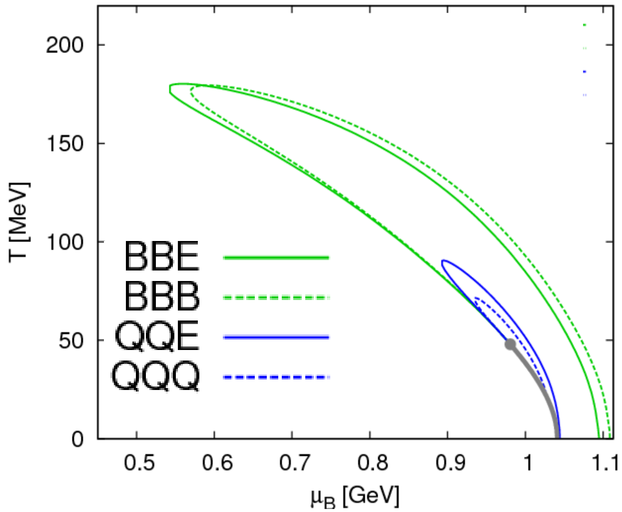
LHC RHIC-BES SPS FAIR,NICA,…

Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point
    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges
    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants
    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



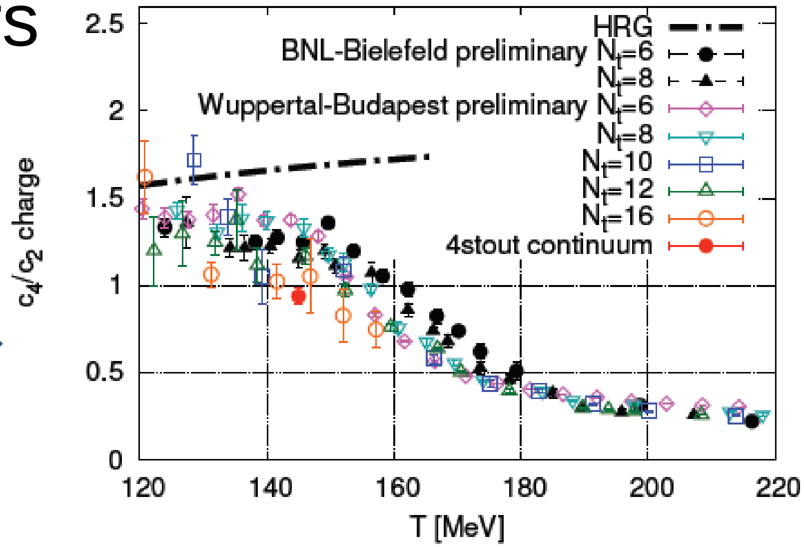




Conserved Charges : Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory

ex: on the lattice



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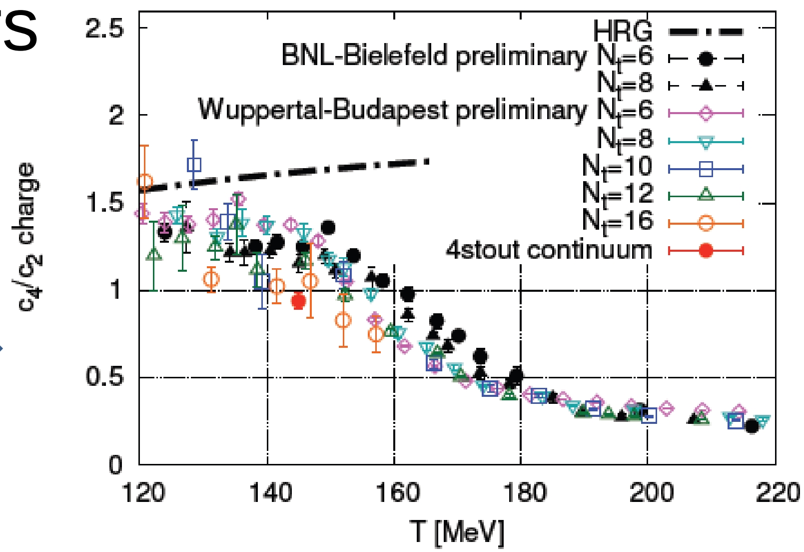


- **Simple thermodynamic relations**

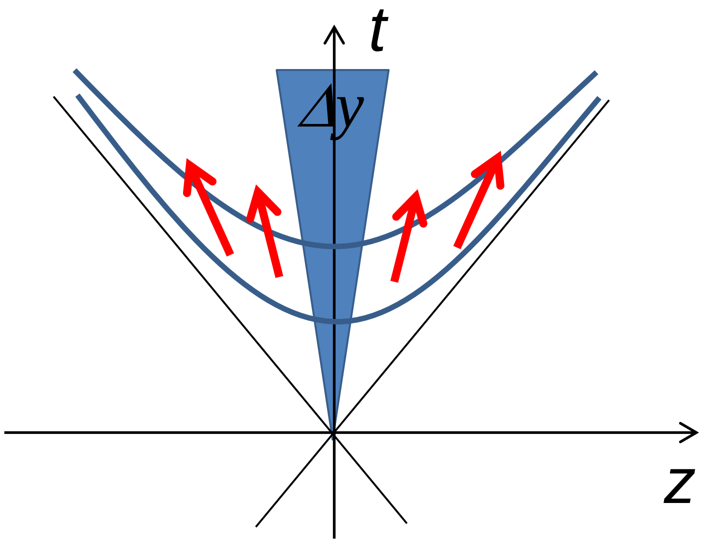
$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

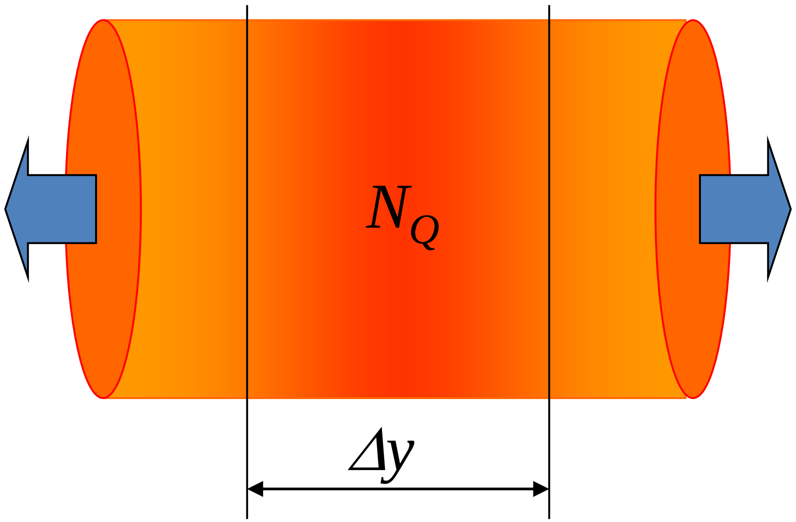
  - Intuitive interpretation for the behaviors of cumulants

ex: $$\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$$

Fluctuations of Conserved Charges

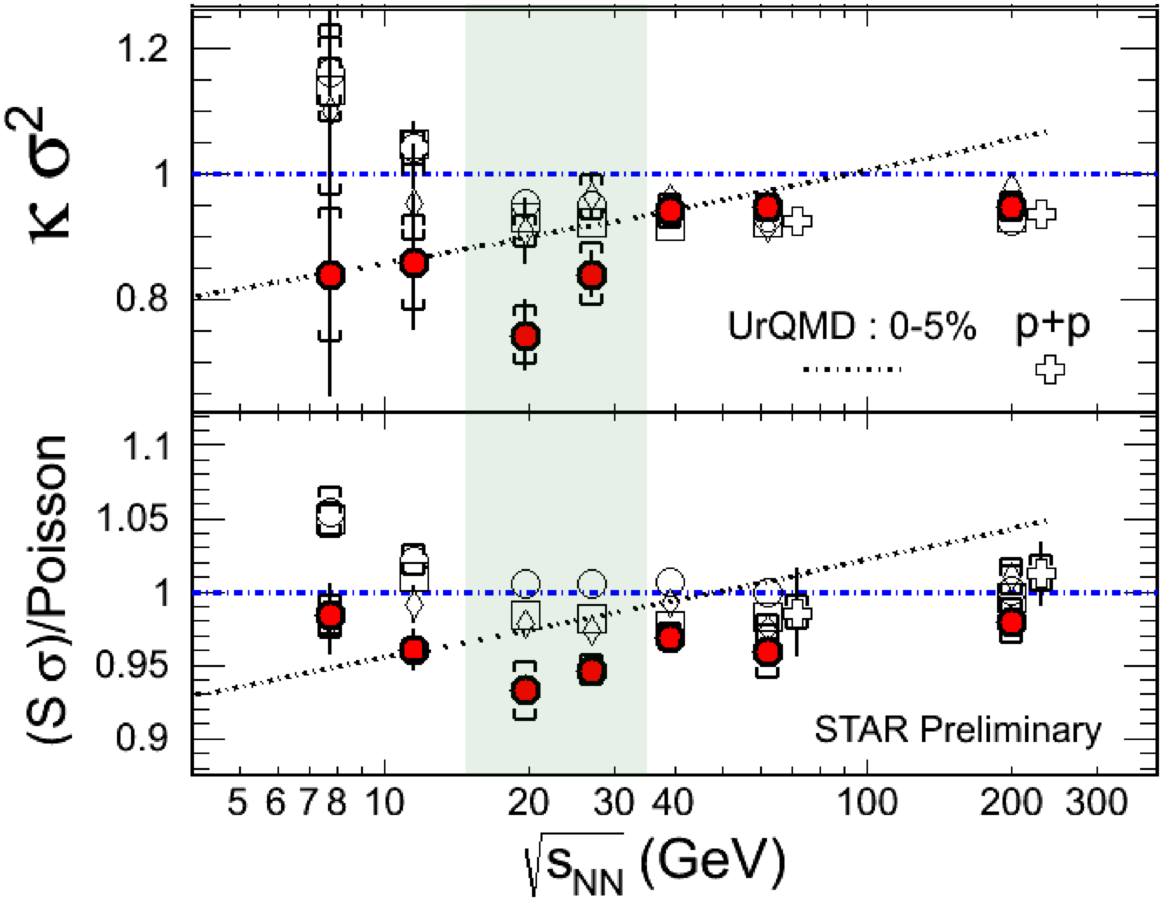
- Under Bjorken expansion





- Count number of particles in Δy in the final state
- Variation of a conserved charge in Δy is **slow**, since it is achieved only through diffusion.
  - The variation is controlled by **transport coefficients**.

Proton # Fluctuations @ STAR-BES



$$\frac{C_4}{C_2}$$

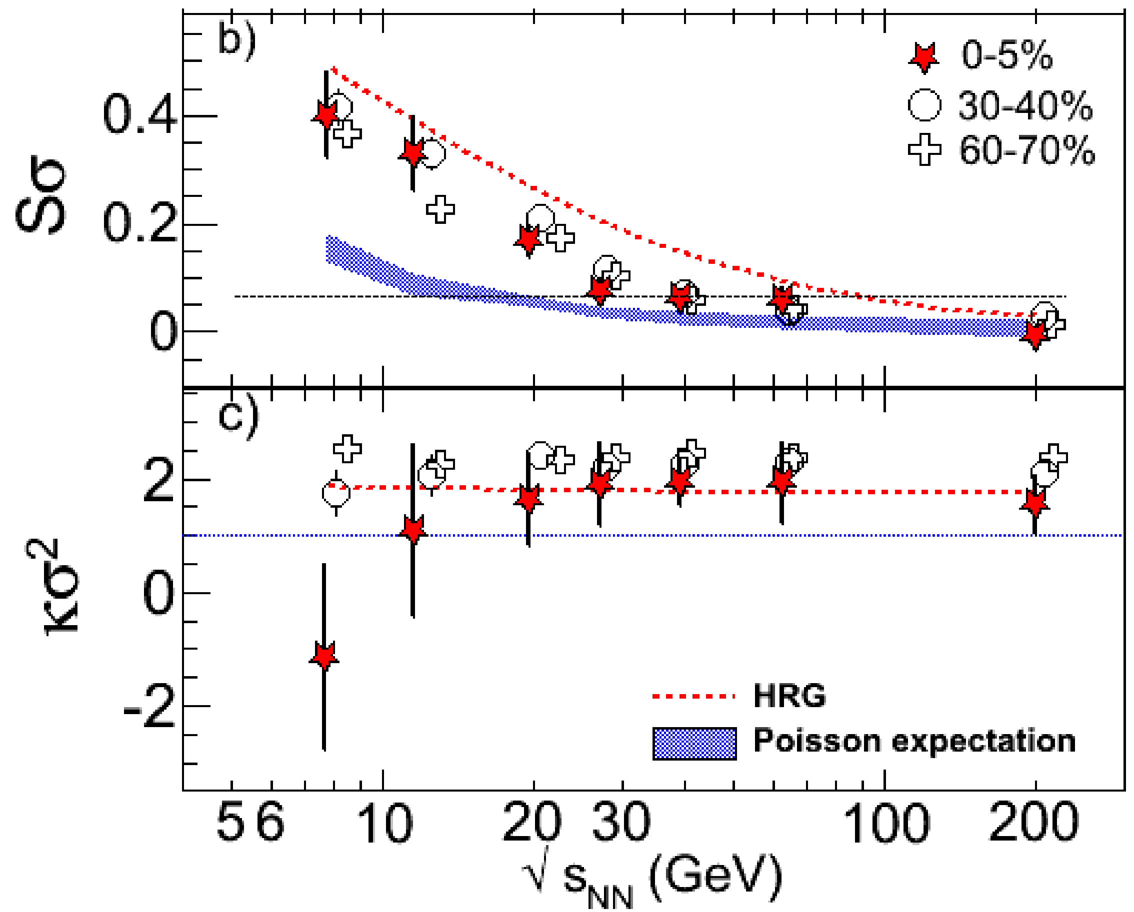
$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

No characteristic signals on
phase transition to QGP nor QCD CP

Charge Fluctuations @ STAR-BES

$$\frac{\langle \delta N_Q^3 \rangle}{\langle \delta N_Q^2 \rangle}$$

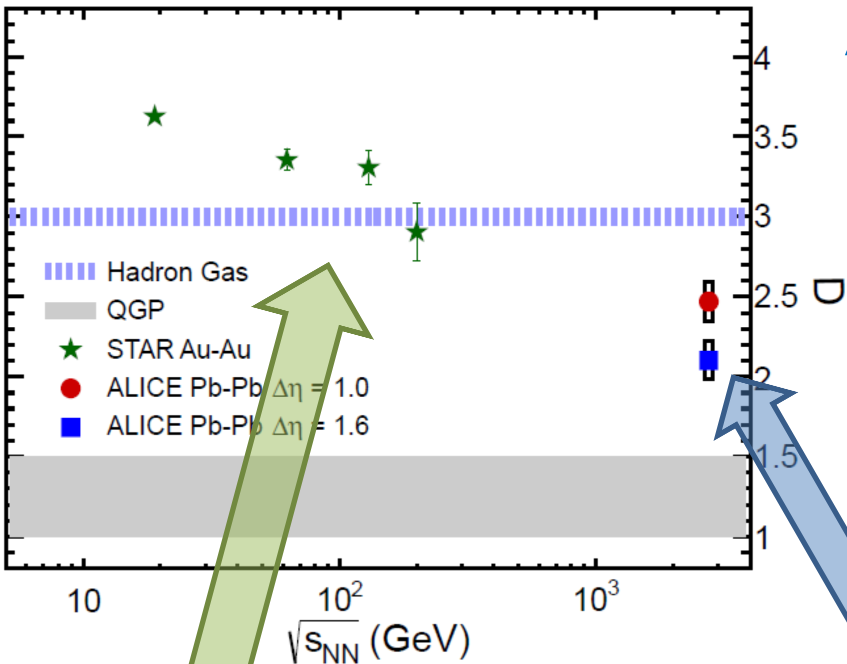
$$\frac{\langle \delta N_Q^4 \rangle_c}{\langle \delta N_Q^2 \rangle}$$



STAR, QM2012

No characteristic signals on
phase transition to QGP nor QCD CP

Charge Fluctuation @ LHC



ALICE, 1207.6068

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- D ~ 3-4 Hadronic
- D ~ 1 Quark

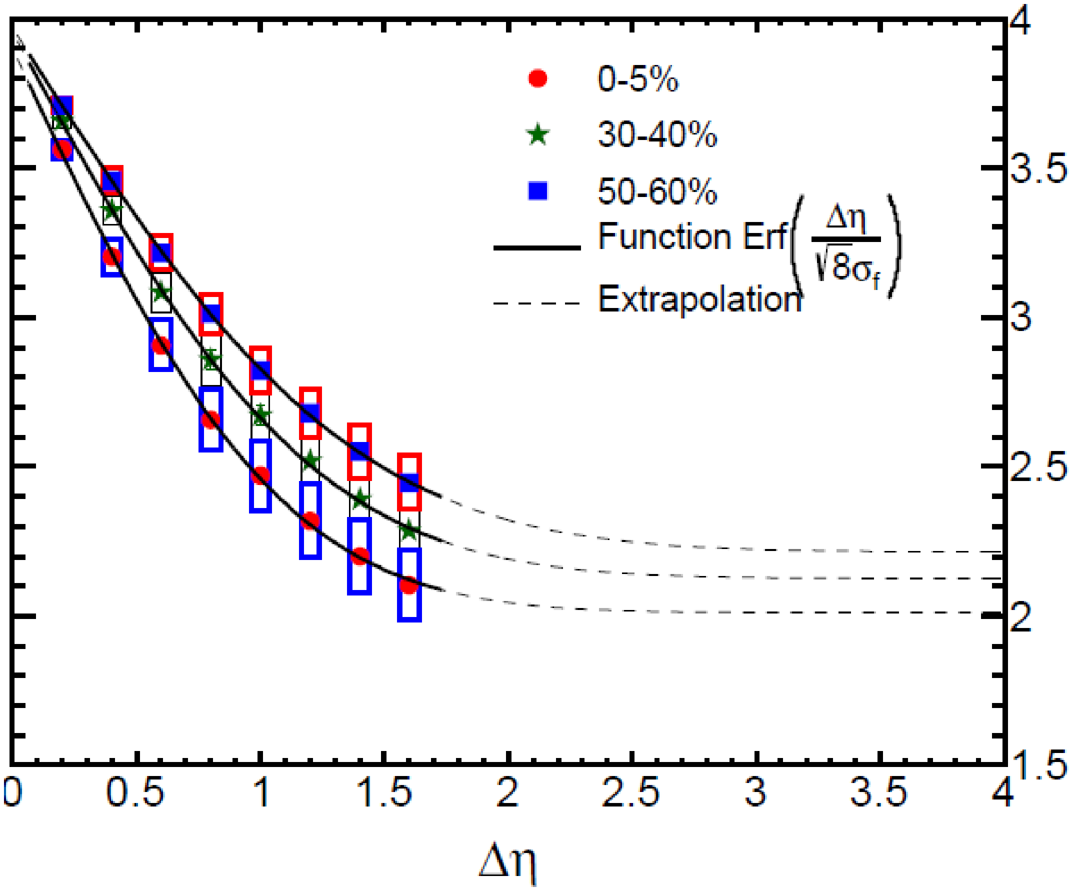
RHIC:
consistent with hadronic value

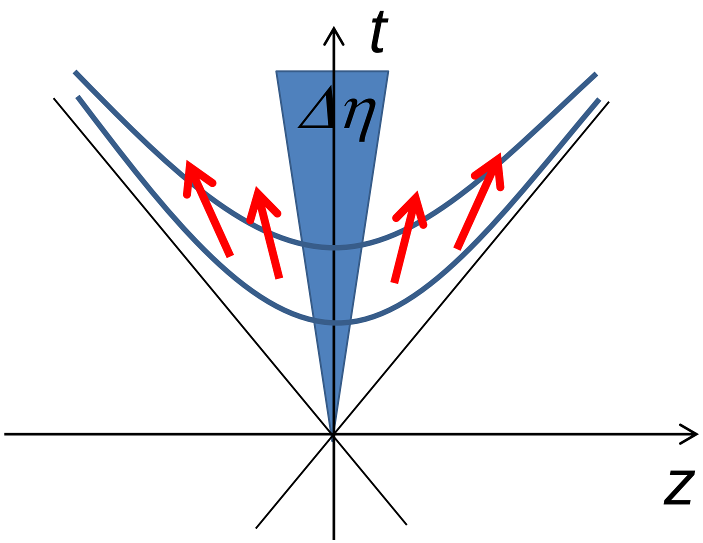
LHC:
significant suppression to quark-gluon value

$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE

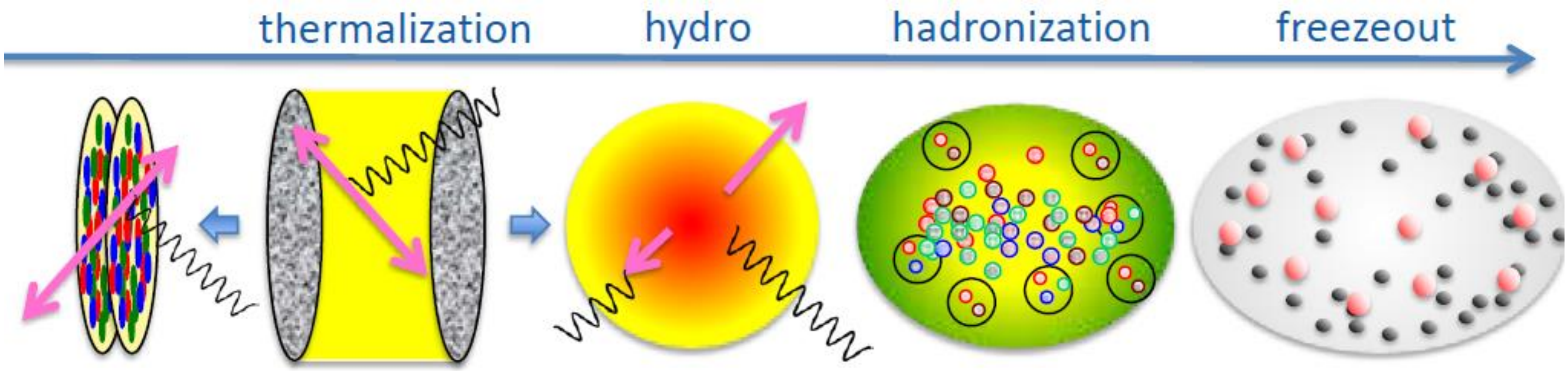
ALICE
1207.6068





- Charge fluctuation is strongly dependent on Δη.
- The larger Δη, the more significant suppression.

Evolution of Fluctuations



Fluctuation in initial state

Time evolution in the QGP

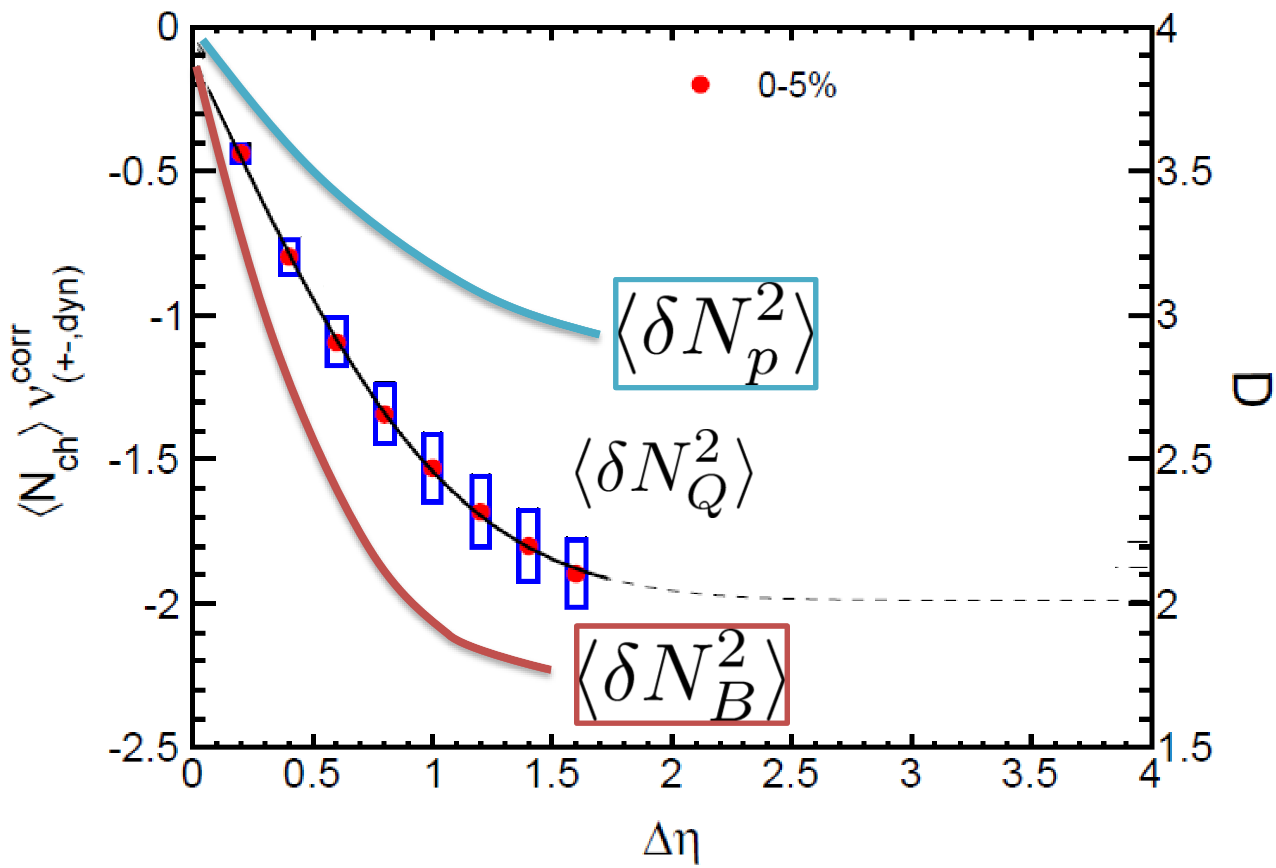
approach to HRG by diffusion

experimental effects
particle missID, etc.

volume fluctuation

$<\delta N_B^2>$ and $<\delta N_p^2>$ @ LHC ?

$\langle\delta N_Q^2\rangle, \langle\delta N_B^2\rangle, \langle\delta N_p^2\rangle$
should have different $\Delta\eta$ dependence.

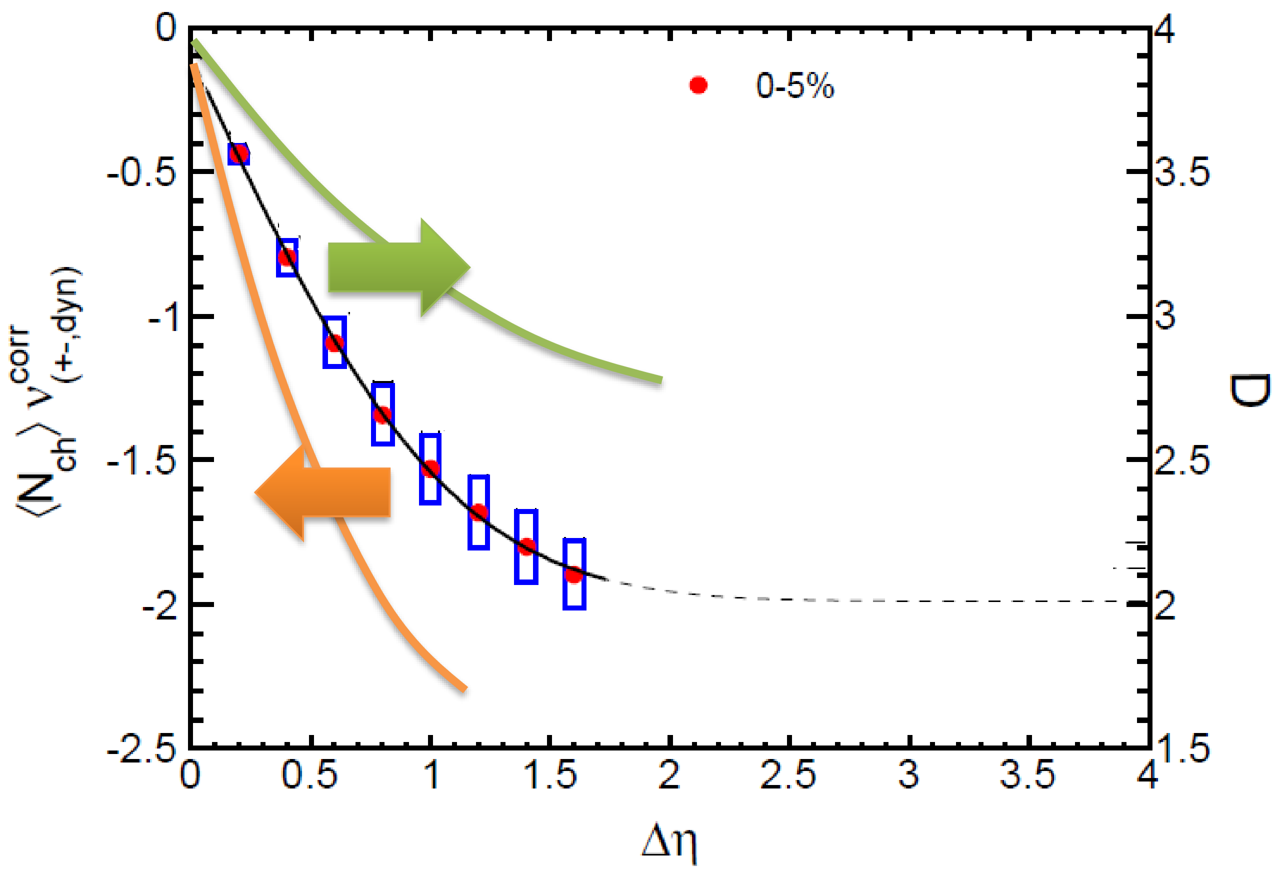


$$\langle(\delta N_p^{(net)})^2\rangle = \frac{1}{4}\langle(\delta N_B^{(net)})^2\rangle + \frac{1}{4}\langle N_B^{(tot)}\rangle$$

$<\delta N_Q^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

Left (suppression) or Right (hadronic)

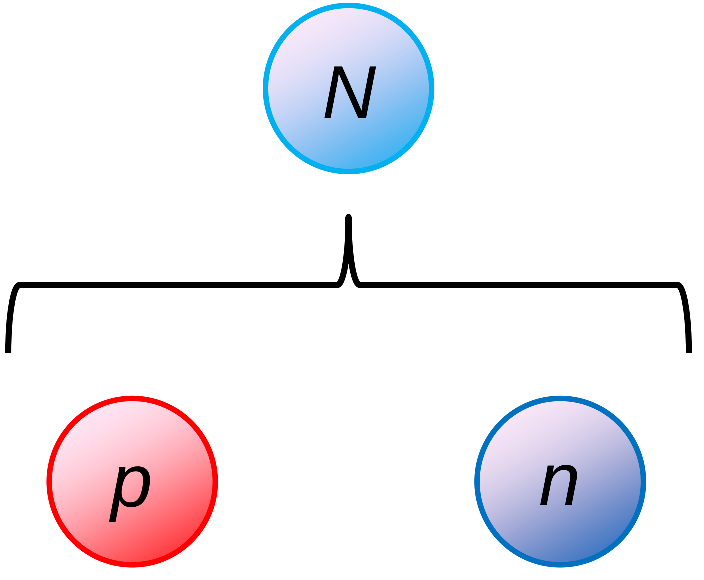


Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85,021901C(2012); PRC86 , 024904(2012)

- $$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} \neq \frac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c}$$
- $\langle \delta N_B^n \rangle_c$ are experimentally observable

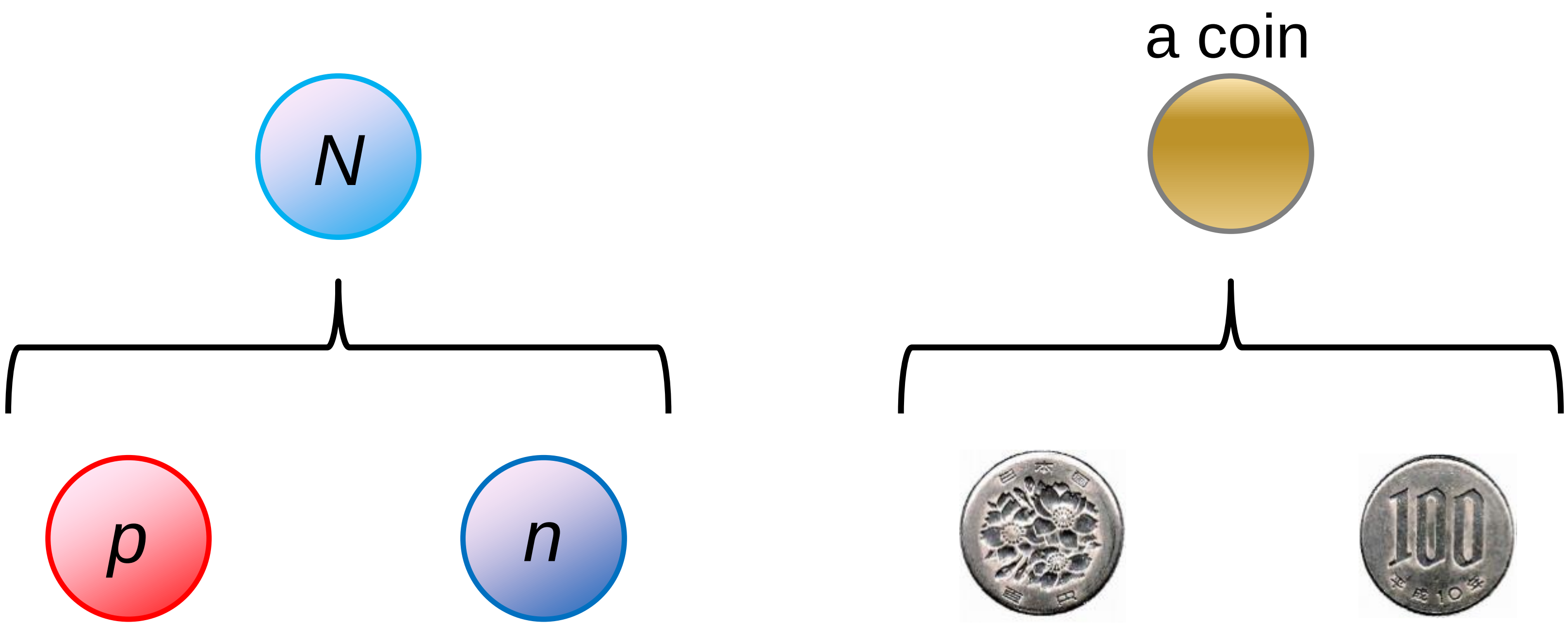
Nucleon Isospin as Two Sides of a Coin



Nucleons have
two isospin states.

MK, Asakawa,2012

Nucleon Isospin as Two Sides of a Coin

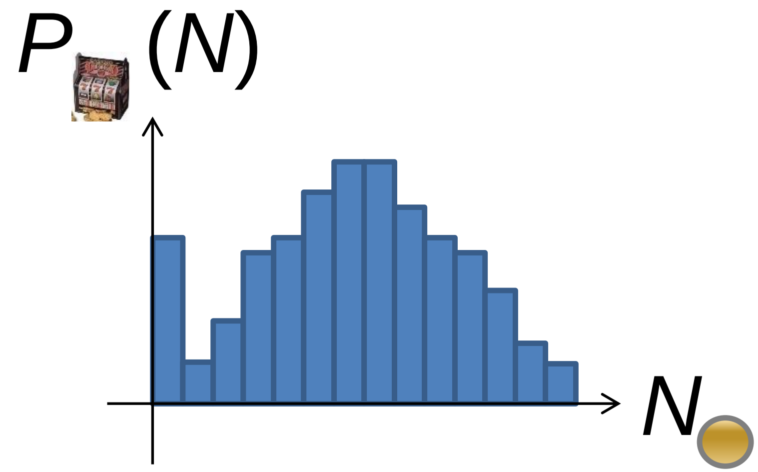


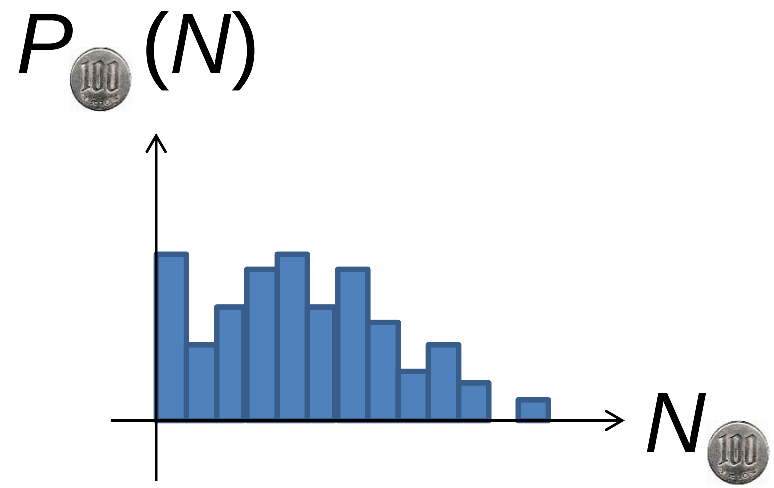
Nucleons have
two isospin states.

Coins have two sides.

MK, Asakawa,2012

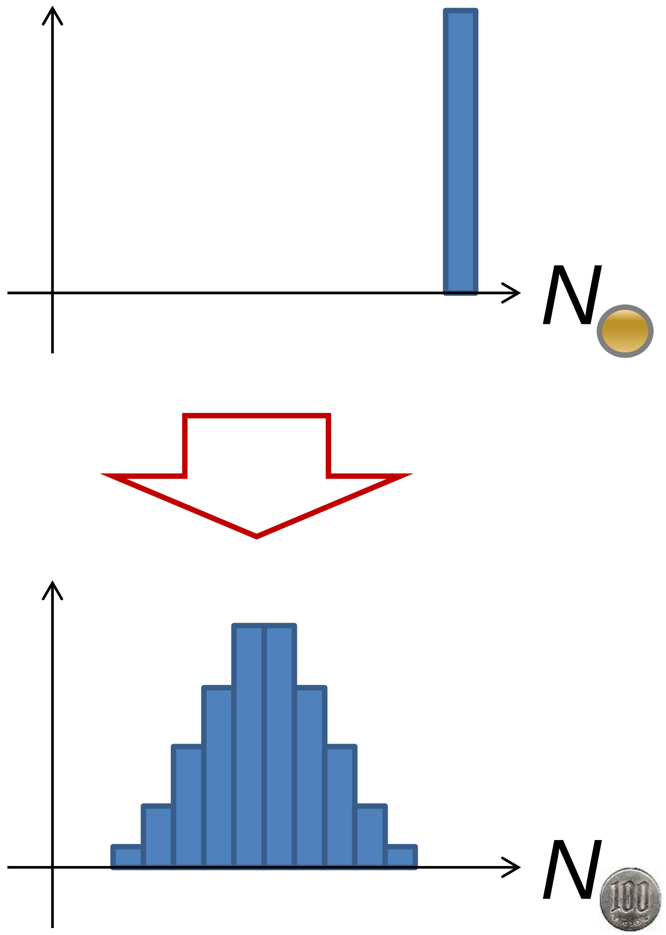
Slot Machine Analogy



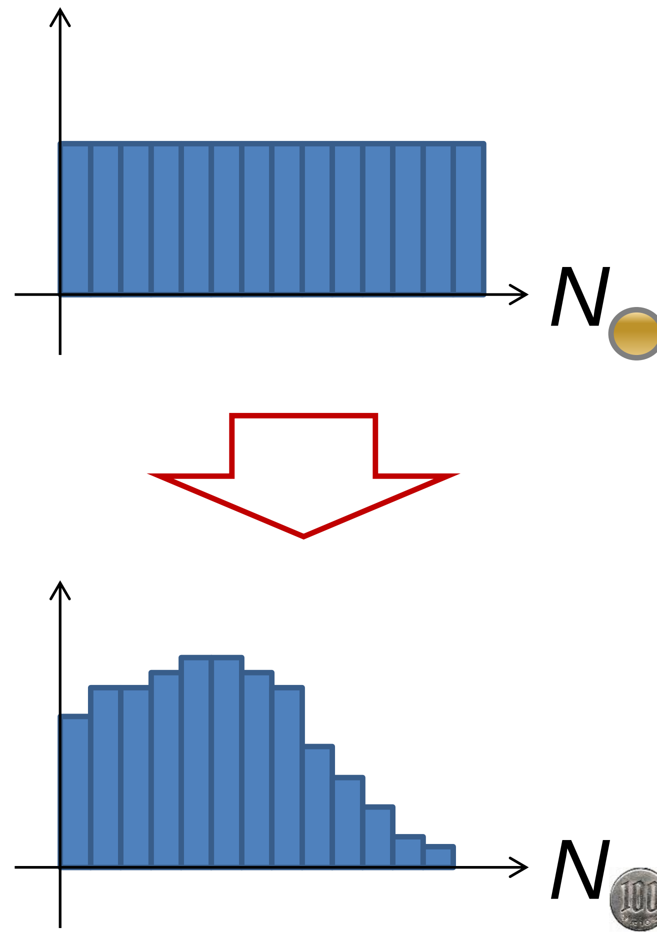


Extreme Examples

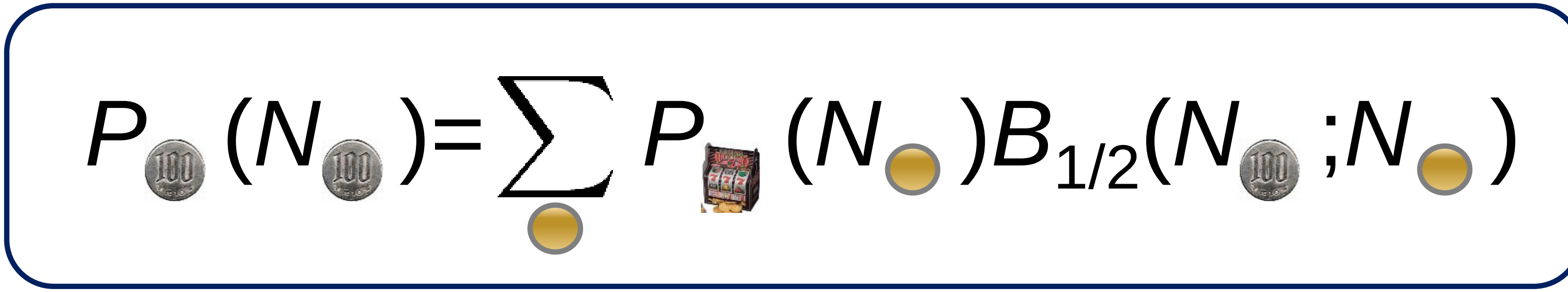
Fixed # of coins

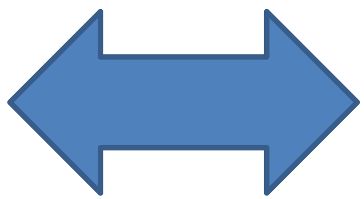


Constant probabilities



Reconstructing Total Coin Number





$$B_p(k;N) = p^k (1-p)^{N-k}\ {}_kC_N$$:binomial distr. func.

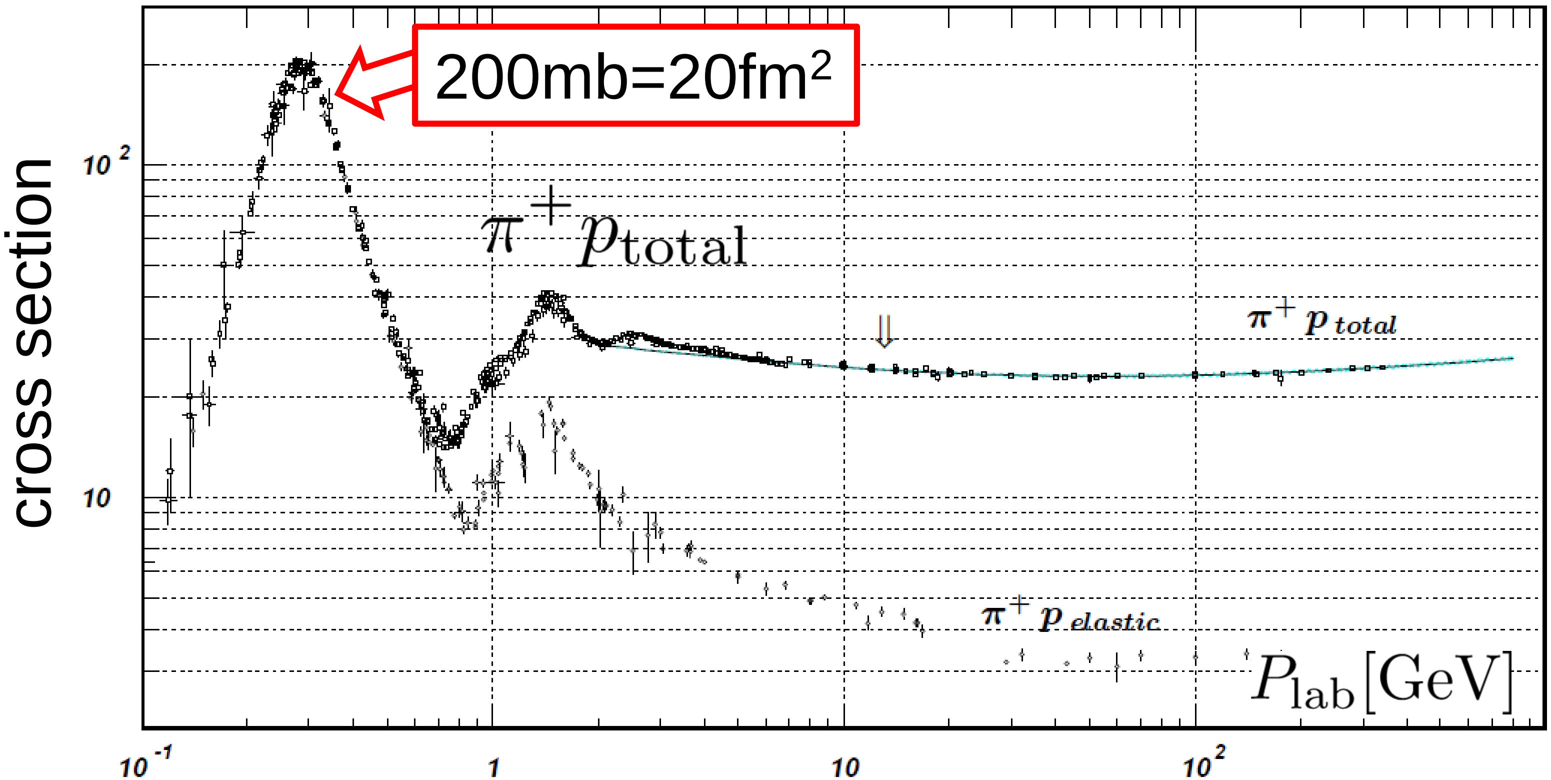
Nucleon Isospin in Hadronic Medium

- Isospin of baryons can vary <u>after chemical freezeout</u> via charge exchange reactions mediated by Δ(1232):

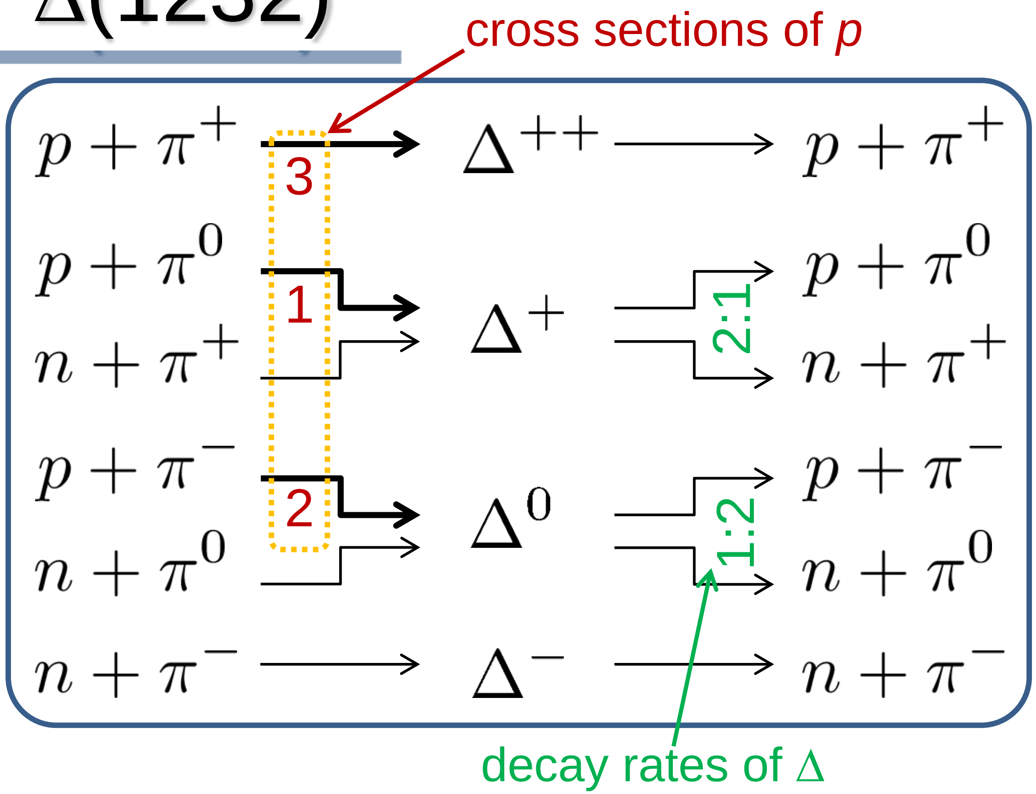
$$p,n \rightarrow \Delta(1232) \rightarrow p,n$$
$$\pi \nearrow \qquad\qquad \searrow \pi$$

$I = 3/2$

$\Gamma \simeq 1.8$ [fm]

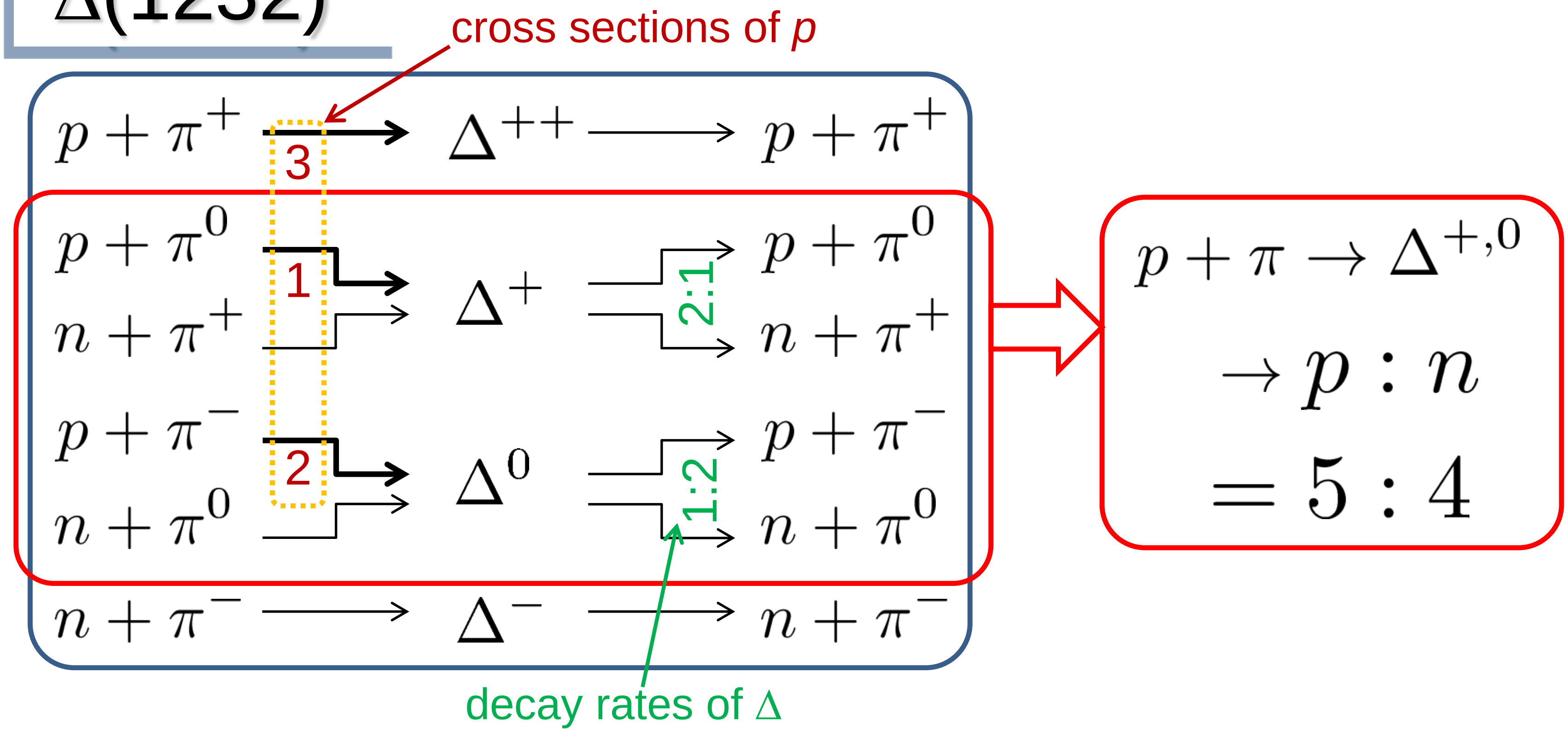


Δ(1232)

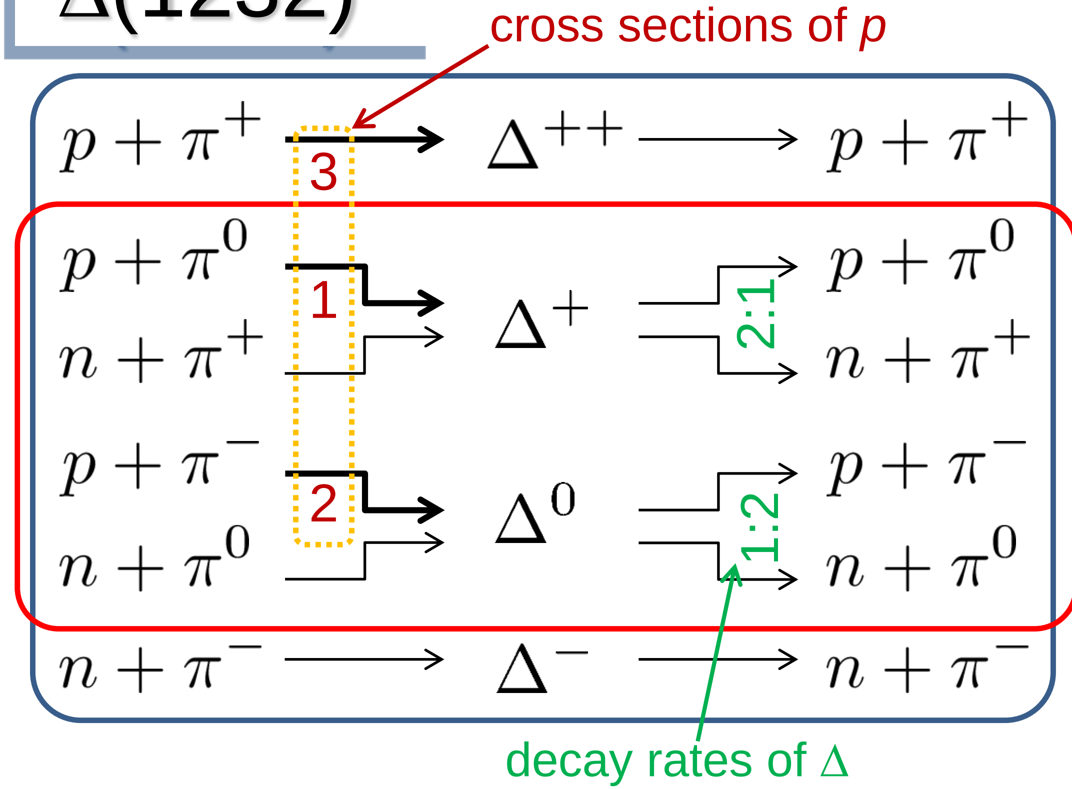


cross sections of p
$p + \pi^+$
$p + \pi^0$
$n + \pi^+$
$p + \pi^-$
$n + \pi^0$
$n + \pi^-$
3
1
2
Δ^{++}
Δ^{+}
Δ^{0}
Δ^{-}
2:1
1:2
$p + \pi^+$
$p + \pi^0$
$n + \pi^+$
$p + \pi^-$
$n + \pi^0$
$n + \pi^-$
decay rates of Δ

Δ(1232)


cross sections of *p*
$p+\pi^+$
3
Δ^{++}
$p+\pi^+$
$p+\pi^0$
1
$n+\pi^+$
Δ^+
2:1
$p+\pi^0$
$n+\pi^+$
$p+\pi^-$
2
$n+\pi^0$
Δ^0
1:2
$p+\pi^-$
$n+\pi^0$
$n+\pi^-$
Δ^-
$n+\pi^-$
decay rates of Δ
$p+\pi \to \Delta^{+,0}$
$\to p:n$
$=5:4$

Δ(1232)

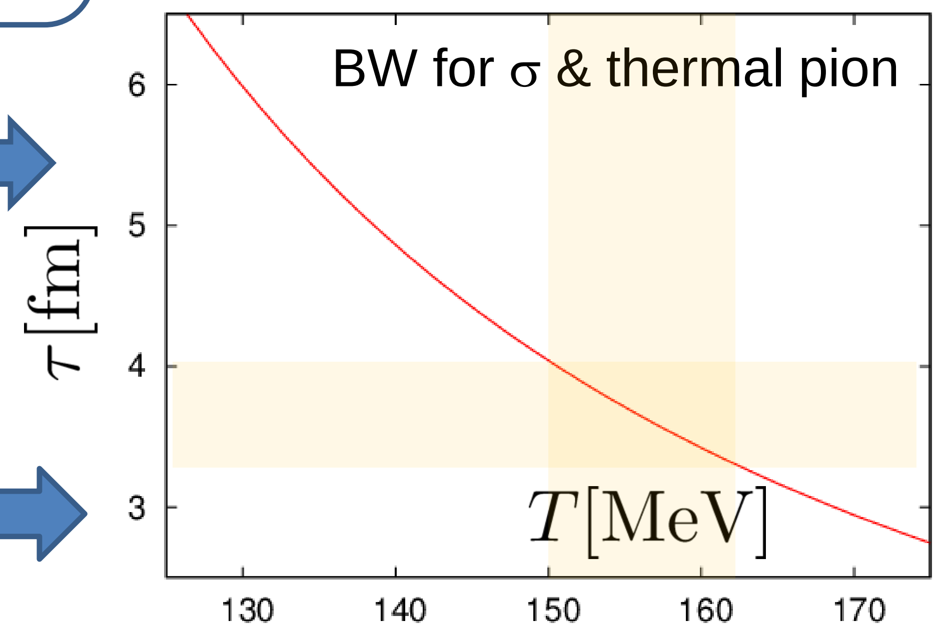


$$p + \pi \to \Delta^{+,0}$$
$$\to p : n$$
$$= 5 : 4$$

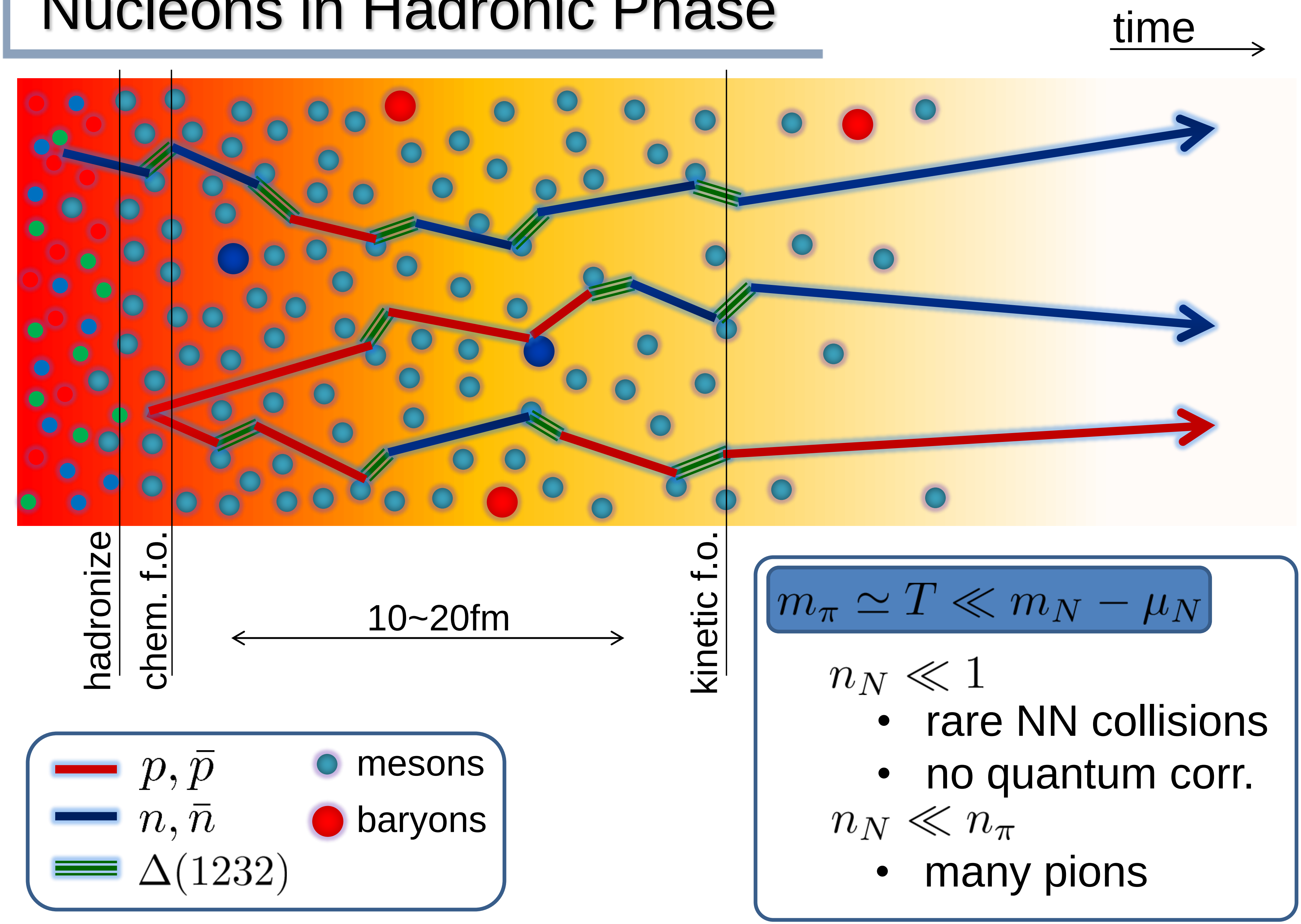
Lifetime to create Δ^+ or Δ^0

$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

$$(\text{freezeout time}) \simeq 20[\text{fm}]$$



Nucleons in Hadronic Phase



$m_\pi \simeq T \ll m_N - \mu_N$

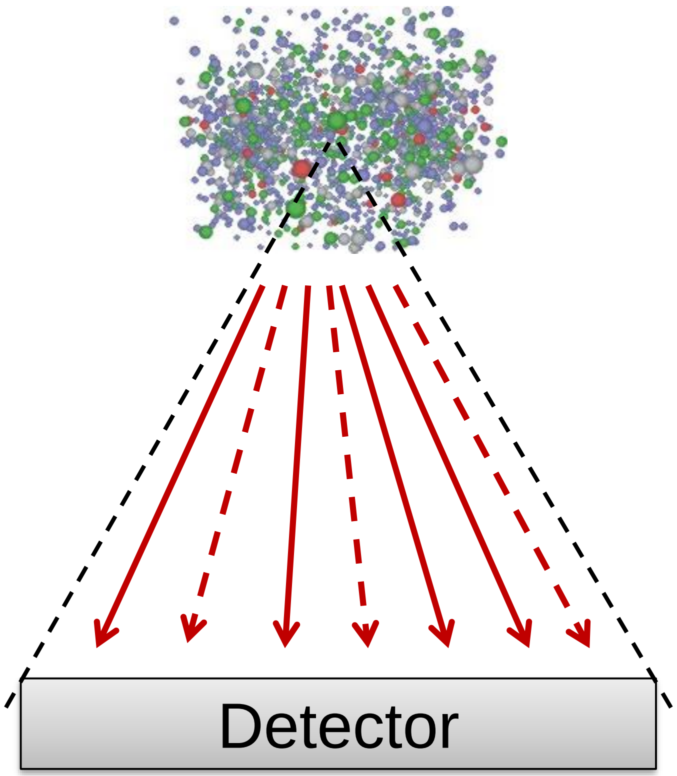
$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- many pions

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



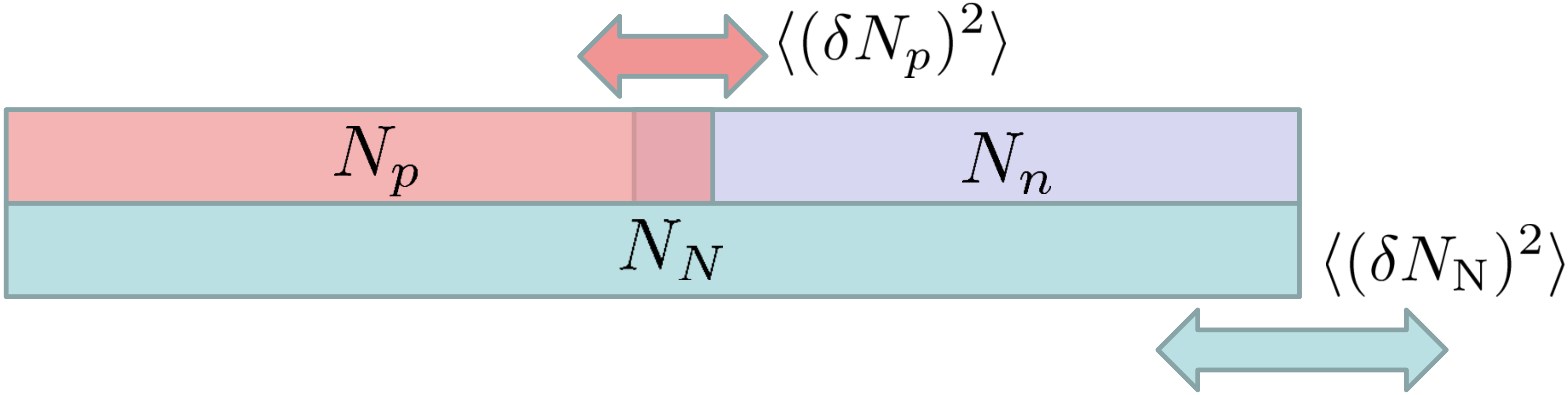
- N_N nucleons; $N_{\bar{N}}$ anti-nucleons ⇒ $F(N_N, N_{\bar{N}})$
- N_N: N_p protons; N_n neutrons ⇒ $B(N_p; N_N)$

binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

- for any phase space in the final state.

Nucleon & Proton Number Fluctuations



$$\begin{cases} \langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{4}\langle(\delta N_\mathrm{N}^{(\mathrm{net})})^2\rangle + \frac{1}{4}\langle N_\mathrm{N}^{(\mathrm{tot})}\rangle \\ \langle(\delta N_\mathrm{N}^{(\mathrm{net})})^2\rangle = 4\langle(\delta N_p^{(\mathrm{net})})^2\rangle - 2\langle N_p^{(\mathrm{tot})}\rangle \end{cases}$$

- for isospin symmetric medium
- effect of isospin density <10%
- Similar formulas up to any order!

For free gas

$$\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_\mathrm{N}^{(\mathrm{net})})^2\rangle$$

3rd & 4th Order Fluctuations

$N_{\rm B} \to N_p$

$$\langle(\delta N_p^{(\rm net)})^3\rangle = \frac{1}{8}\langle(\delta N_{\rm B}^{(\rm net)})^3\rangle + \frac{3}{8}\langle\delta N_{\rm B}^{(\rm net)}\delta N_{\rm B}^{(\rm tot)}\rangle,$$

$$\langle(\delta N_p^{(\rm net)})^4\rangle_c = \frac{1}{16}\langle(\delta N_{\rm B}^{(\rm net)})^4\rangle_c + \frac{3}{8}\langle(\delta N_{\rm B}^{(\rm net)})^2\delta N_{\rm B}^{(\rm tot)}\rangle + \frac{3}{16}\langle(\delta N_{\rm B}^{(\rm tot)})^2\rangle - \frac{1}{8}\langle N_{\rm B}^{(\rm tot)}\rangle,$$

$N_p \to N_{\rm B}$

$$\langle(\delta N_{\rm B}^{(\rm net)})^3\rangle = 8\langle(\delta N_p^{(\rm net)})^3\rangle - 12\langle\delta N_p^{(\rm net)}\delta N_p^{(\rm tot)}\rangle + 6\langle N_p^{(\rm net)}\rangle,$$

$$\langle(\delta N_{\rm B}^{(\rm net)})^4\rangle_c = 16\langle(\delta N_p^{(\rm net)})^4\rangle_c - 48\langle(\delta N_p^{(\rm net)})^2\delta N_p^{(\rm tot)}\rangle + 48\langle(\delta N_p^{(\rm net)})^2\rangle + 12\langle(\delta N_p^{(\rm tot)})^2\rangle - 26\langle N_p^{(\rm tot)}\rangle,$$

Difference btw Baryon and Proton Numbers

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

Difference btw Baryon and Proton Numbers

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

$$
\begin{cases}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{cases}
$$

genuine info.   noise

For free gas

$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_\mathrm{N}^{(\mathrm{net})})^n\rangle_c$$

Strange Baryons

Decay Rates:

Λ $m_\Lambda \simeq 1116[\mathrm{MeV}]$

$$p : n \simeq 1.6 : 1$$

Σ $m_\Sigma \simeq 1190[\mathrm{MeV}]$

$$p : n \simeq 1 : 1.8$$

Decay modes:

$\Lambda \rightarrow p + \pi^-$ 64%

$\Lambda \rightarrow n + \pi^0$ 36%

$\Sigma^+ \rightarrow p + \pi^0$ 52%

$\Sigma^+ \rightarrow n + \pi^+$ 48%

$\Sigma^0 \rightarrow \Lambda \rightarrow p + \pi^-$ 64%

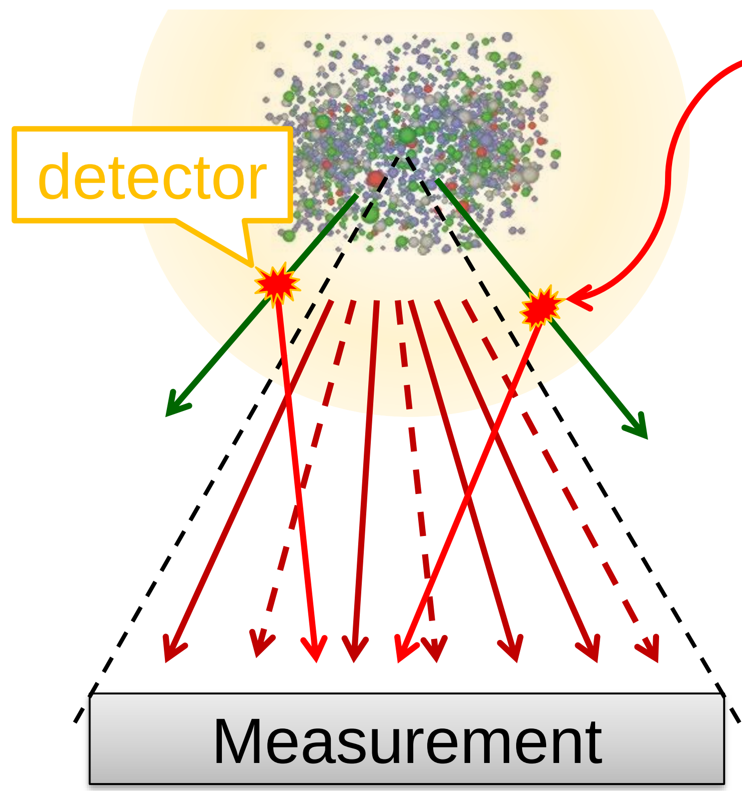
$\Sigma^0 \rightarrow \Lambda \rightarrow n + \pi^0$ 36%

$\Sigma^- \rightarrow n + \pi^-$

Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \rightarrow N_B$

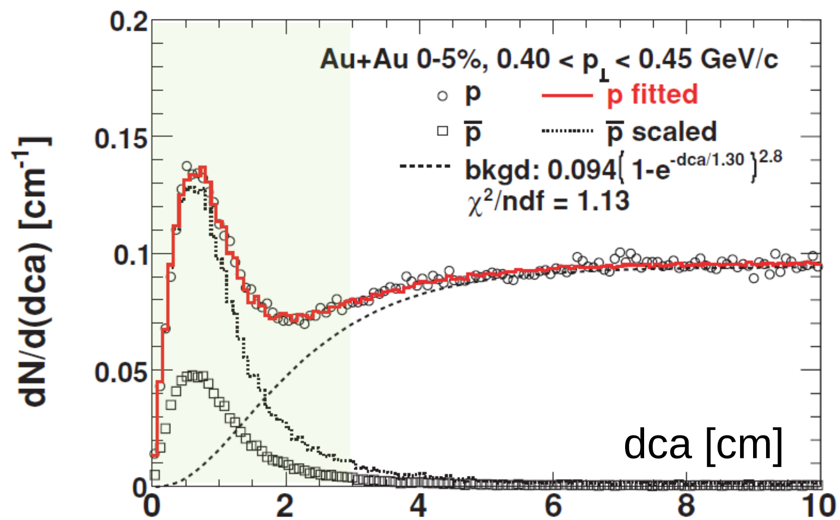
Secondary Protons

MK+, in preparation



Secondary (knockout) protons

20% of observed protons @ STAR



STAR, PRC79,034909(2009)

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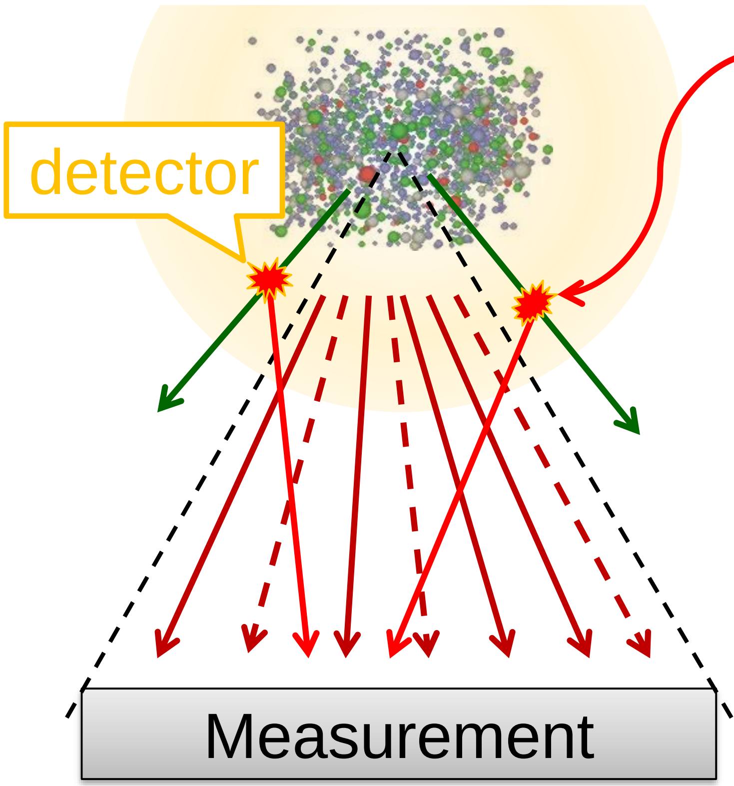

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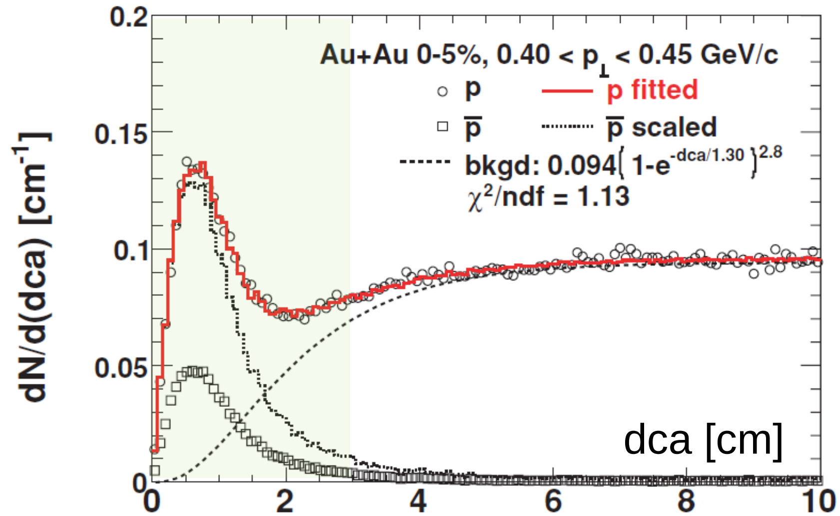
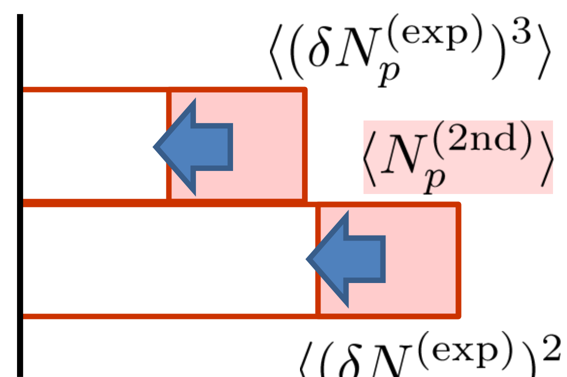


STAR, PRC79,034909(2009)

Their contribution can be eliminated!

$$\langle(\delta N_p^{(\mathrm{QGP})})^n\rangle_c = \langle(\delta N_p^{(\mathrm{exp})})^n\rangle_c - \langle N_p^{(\mathrm{2nd})}\rangle$$



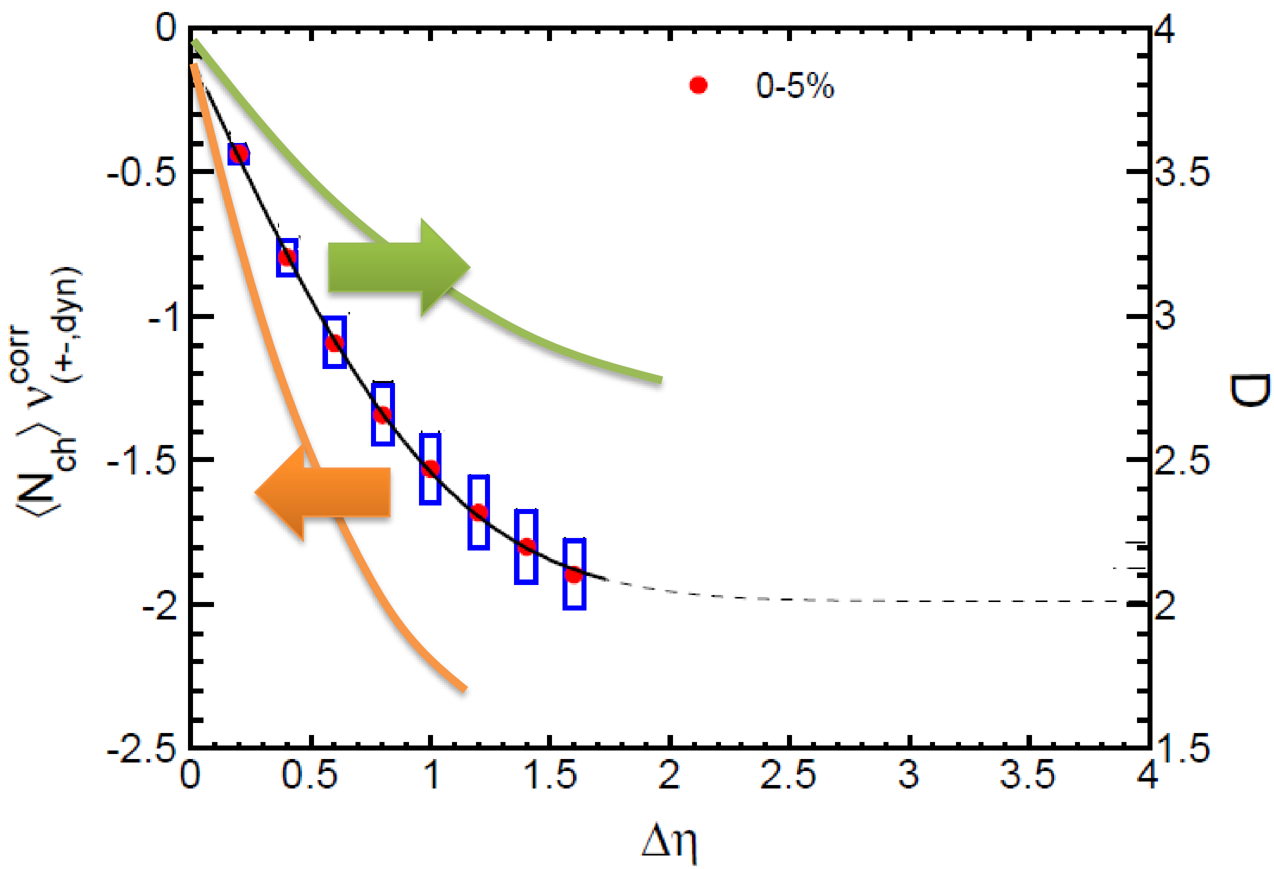
Time Evolution of Higher Order Cumulants

MK, et al., in preparation

$\langle \delta N_Q^4 \rangle$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

Left (suppression) or Right (hadronic)



Stochastic Diffusion Equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

↑

Conservation Law

$$\partial_\tau n = -\partial_\eta j$$

Fick's Law

$$j = -D\partial_\eta n + \xi$$

Stochastic Diffusion Equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Stochastic force

- Local correlation (hydrodynamics) $\langle \xi(\eta_1, \tau_1)\xi(\eta_2, \tau_2)\rangle \sim \delta(\eta_1 - \eta_2)\delta(\tau_1 - \tau_2)$
- Equilibrium fluc. $\langle \delta Q(t)^2 \rangle \xrightarrow[t \to \infty]{} \chi_2 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

χ_2 : susceptibility

$$\langle \xi(k_1, \tau_1)\xi(k_2, \tau_2)\rangle = \frac{2\chi_2}{D}\delta(k_1 + k_2)\delta(\tau_1 - \tau_2)$$

Time Evolution

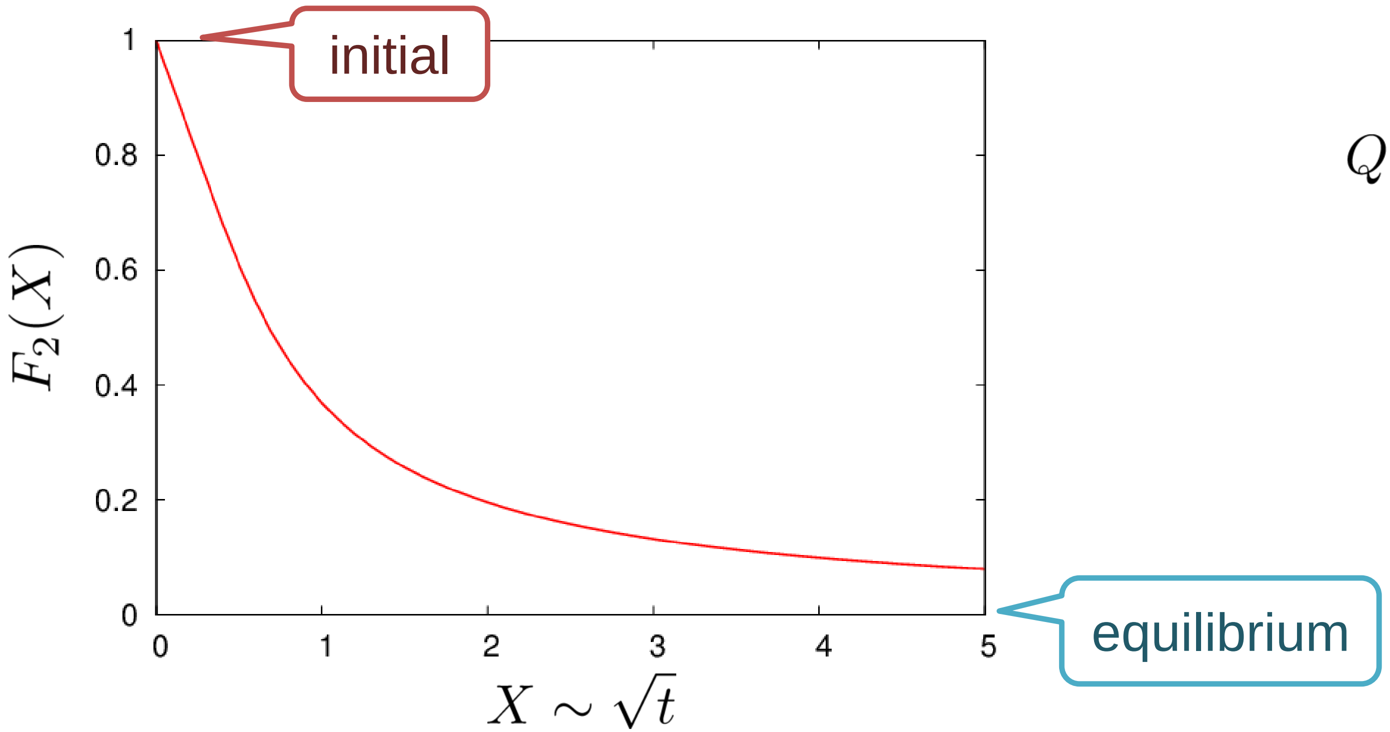
Shuryak, Stephanov, 2001

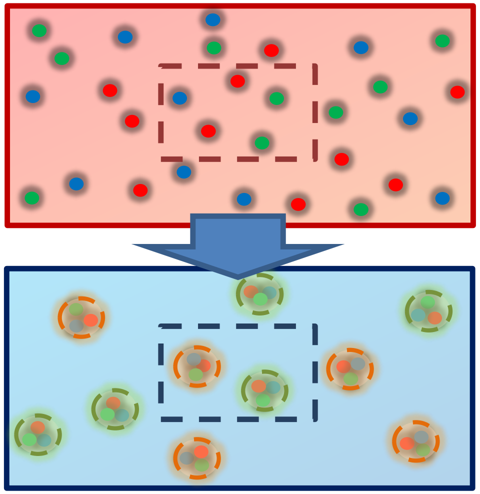
- Initial condition: $\langle \delta n(\eta_1, 0) \delta n(\eta_2, 0) \rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2 (1 - F_2(X))}_{\text{equilibrium}}$$

$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$





$\Delta\eta$ Dependence

Shuryak, Stephanov, 2001

- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2 (1 - F_2(X))}_{\text{equilibrium}}$$

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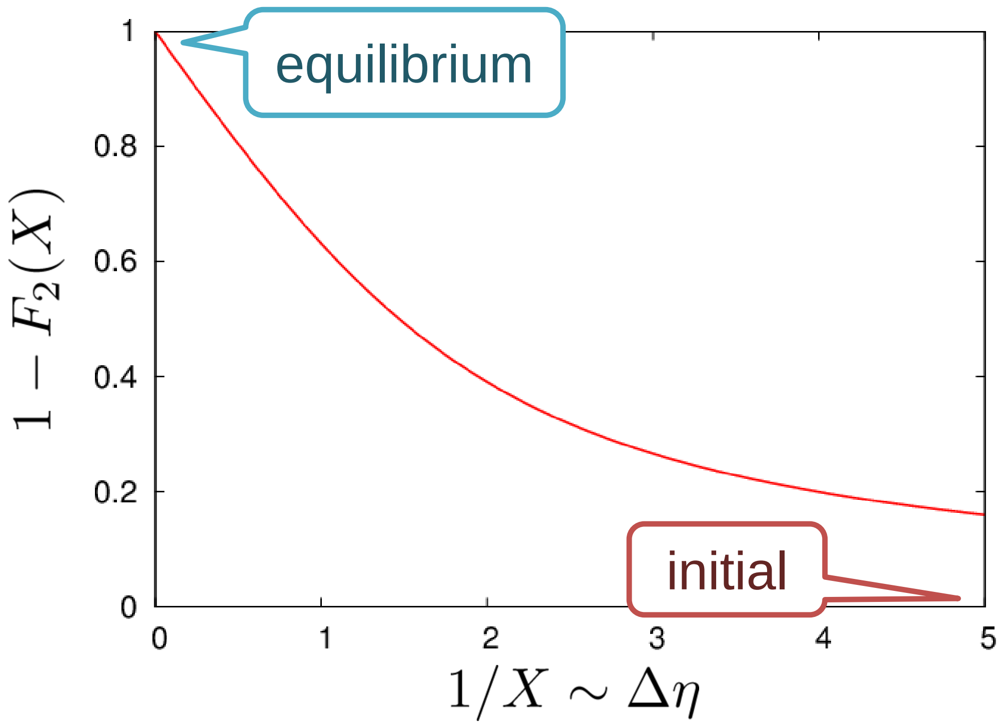

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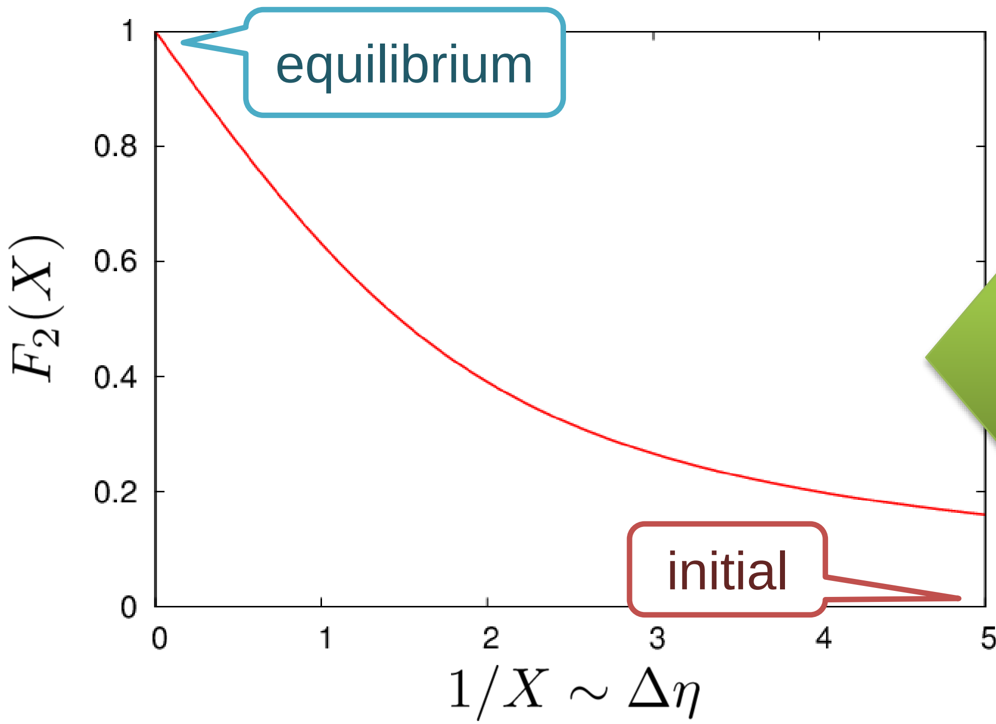
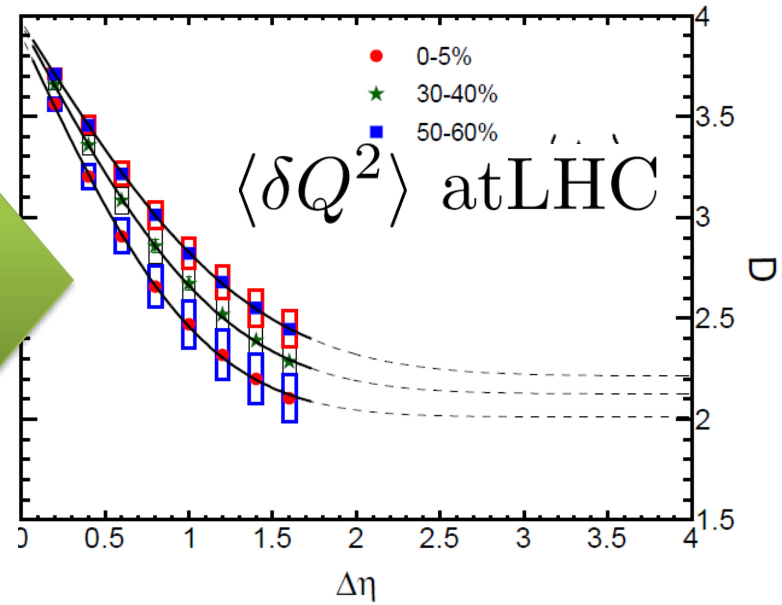




Non-Gaussian Stochastic Force??

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Stochastif Force : 3rd order

- Local correlation (hydrodynamics) $\langle \xi(\eta_1,\tau_1)\xi(\eta_2,\tau_2)\xi(\eta_3,\tau_3)\rangle$
  $$\sim \delta(\eta_1-\eta_2)\delta(\eta_2-\eta_3)\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$
- Equilibrium fluc. $\langle \delta Q(t)^3 \rangle \xrightarrow[t\to\infty]{} \chi_3 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta,\tau)$$

χ_3 : third − moment

Caution!

diverge in long wavelength

- $$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\xi(k_3,\tau_3)\rangle = \frac{\chi_3}{\gamma}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\delta(k_1+k_2+k_3) \times\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$

- Markov process + continuous variable

  ⇒ Gaussian stochastic force

  cf) Gardiner, “Stochastic Methods”

- Hydrodynamics ⇒ Huge particles in a small volume

  ⇒ Gaussian distribution (central limit theor.)

Caution!

diverge in long wavelength

- $$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\xi(k_3,\tau_3)\rangle = \frac{\chi_3}{\gamma}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\delta(k_1+k_2+k_3) \times\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$

- Markov process + continuous variable

  ⇒ Gaussian stochastic force

  cf) Gardiner, "Stochastic Methods"

- Hydrodynamics ⇒ Huge particles in a small volume

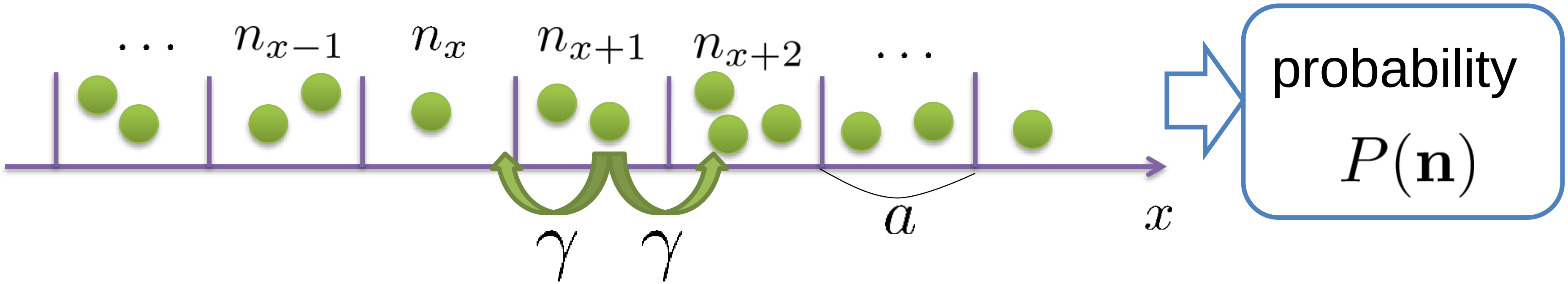
  ⇒ Gaussian distribution (central limit theor.)

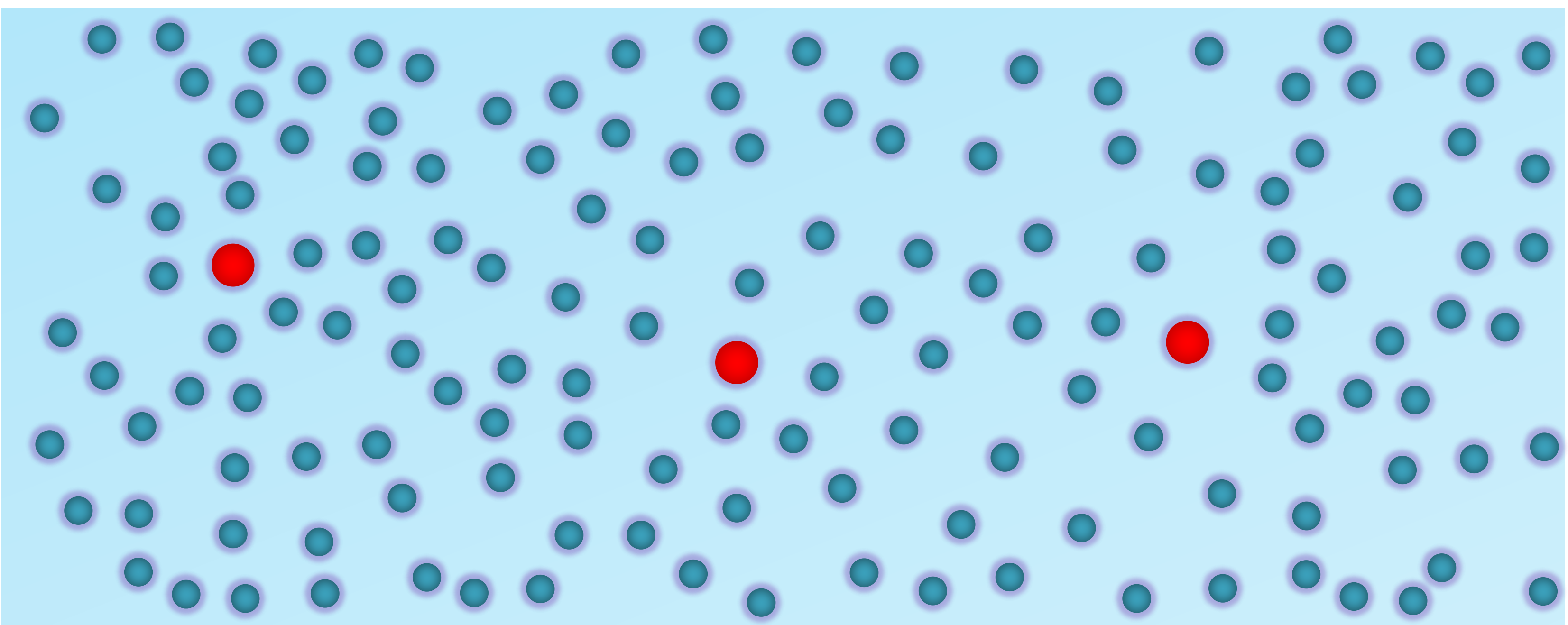
NOTES

- Near the CP, locality is violated.
- Analysis of higher-order cumulants without a critical phenomena should be a **particular** problem in physics!

Diffusion Master Equation

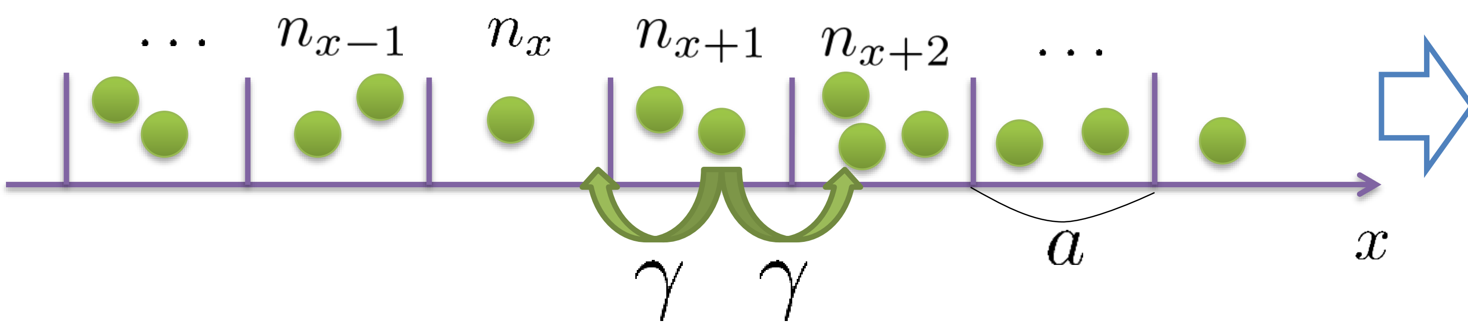
Divide spatial coordinate into discrete cells





Diffusion Master Equation

Divide spatial coordinate into discrete cells



probability

$P(\mathbf{n})$

Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = d\sum_x \Big[(n_x+1)\left\{P(\mathbf{n}+\hat{x}-\widehat{x+1}) + P(\mathbf{n}+\hat{x}-\widehat{x-1})\right\} - 2n_x P(\mathbf{n})\Big]$$

x-hat: lattice-QCD notation

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen's system size expansion

Solution of DME

1st $\langle \tilde{n}_k \rangle (t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$ $\qquad \omega_k \simeq \gamma a^2 k^2$

↑ initial

- Deterministic part is the diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

- Appropriate continuum limit with $\gamma a^2 = D$

Solution of DME

1st $\langle \tilde{n}_k \rangle(t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$ $\qquad \omega_k = \gamma a^2 k^2$

initial

- Deterministic part is the diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

- Appropriate continuum limit with $\gamma a^2 = D$

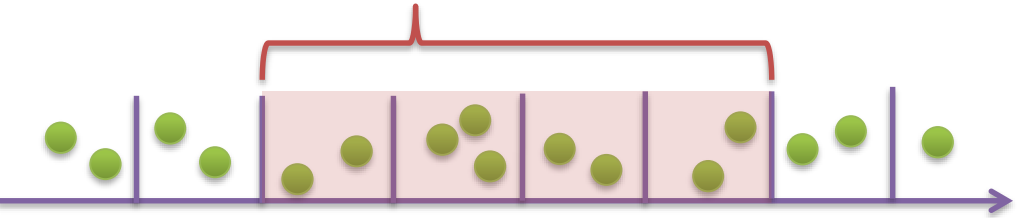
2nd

$$\langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle(t) = \langle \tilde{n}_{k_1+k_2} \rangle_0 \left(e^{-\omega_{k_1+k_2} t} - e^{-(\omega_{k_1}+\omega_{k_2})t}\right) + \langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle_0 e^{-(\omega_{k_1}+\omega_{k_2})t}$$

- Consistent with stochastic diffusion eq. for sufficiently slowly-varying initial condition.

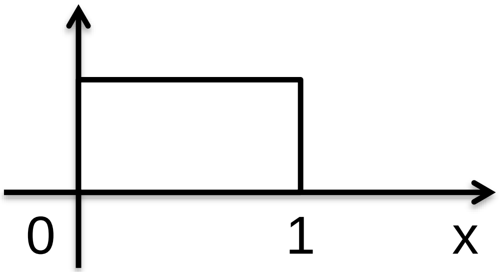
Total Charge in $\Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta,\tau)$$



$$I(z,X) = \int \frac{dk}{2\pi} e^{-Xk^2} \int_0^1 dx e^{ik(x+z)}$$

FT of step func.



$$\langle Q \rangle(\tau) = \int dz \langle n(z) \rangle_0 I_{\Delta\eta}(z,X)$$

$$\langle \delta Q^2 \rangle(\tau) = \int dz_1 dz_2 \langle \delta n(z_1) \delta n(z_2) \rangle_0 I_{\Delta\eta}(z_1,X) I_{\Delta\eta}(z_2,X)$$

$$+ \int dz \langle n(z) \rangle_0 (I_{\Delta\eta}(z,X) - I^2_{\Delta\eta}(z,X))$$

$$\langle \delta Q^3 \rangle(t) = \cdots$$

$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$

Time Evolution of $<\delta Q^n>_c$

Uniform and fixed Initial condition

$$P(\mathbf{n},0) = \prod_x \delta_{n_x,M}$$

Particle numbers in all cells are *M*.

Time Evolution of $\langle \delta Q^n \rangle_c$

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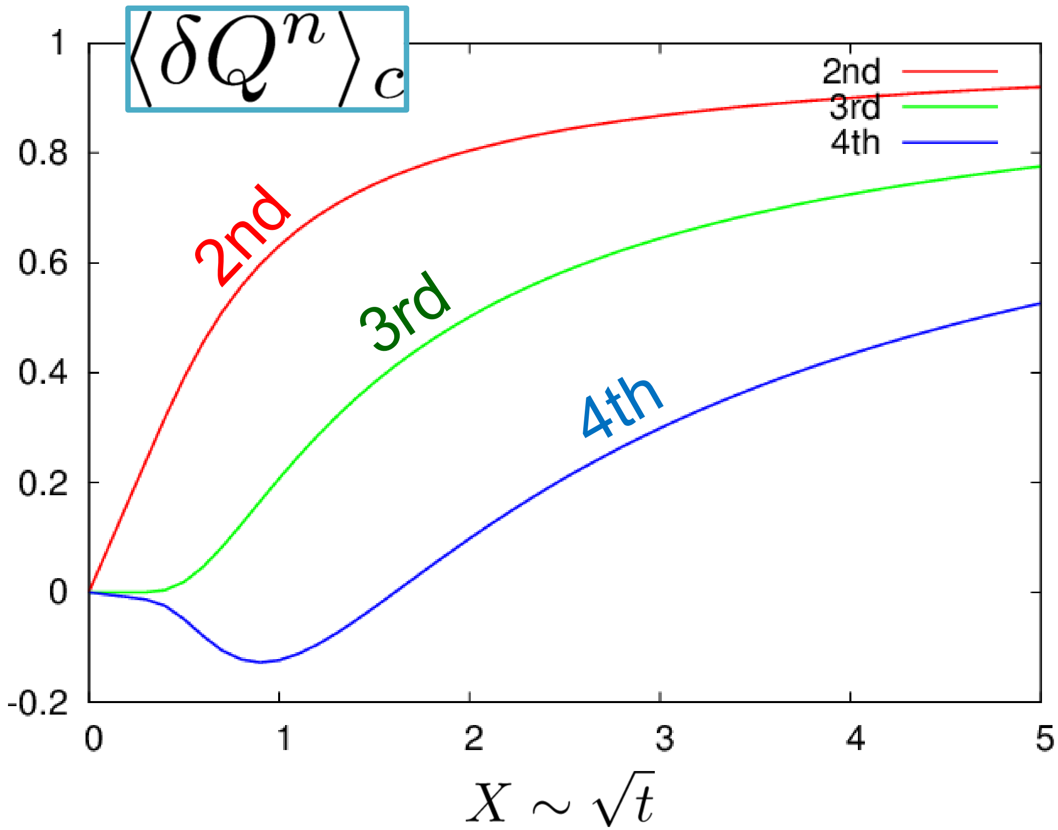


$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta\, n(\eta,\tau)$$

- Slow enhancement for higher order
- Negative 4th cumulant
- Similar behavior for small initial fluc.

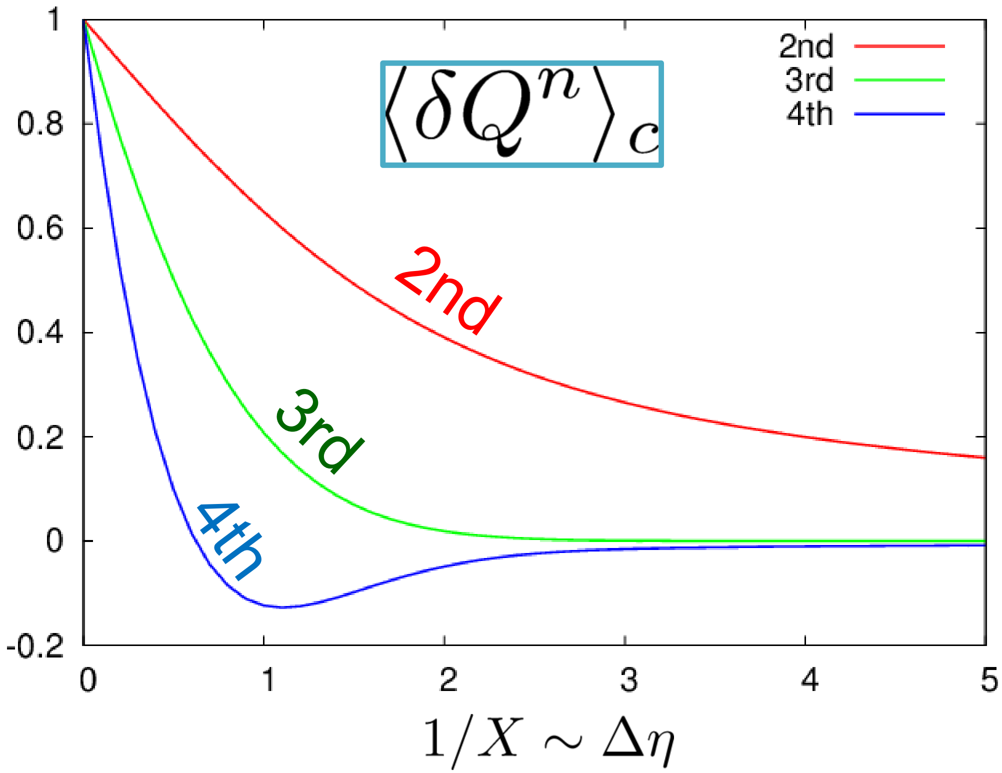
Time Evolution of $\langle \delta Q^n \rangle_c$

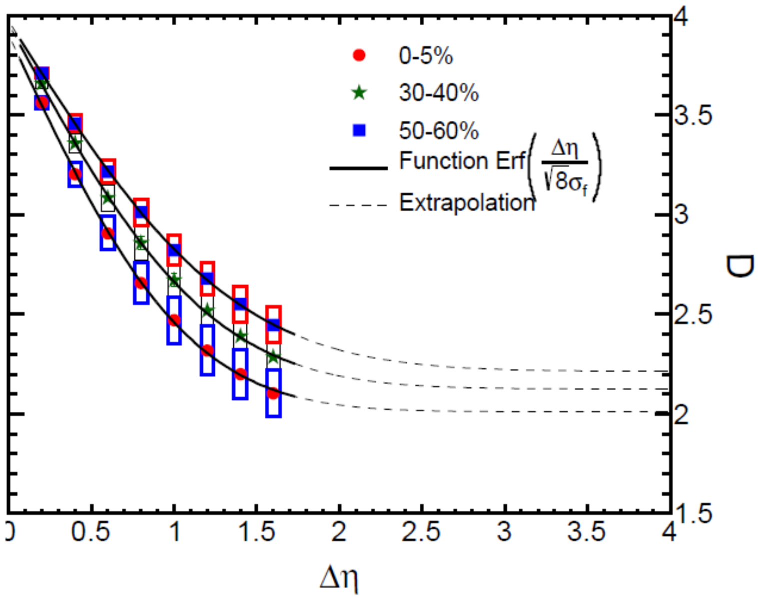
Initial condition

$$P(\mathbf{n},0) = \prod_x \delta_{n_x,M}$$

Particle numbers in all cells are *M*.

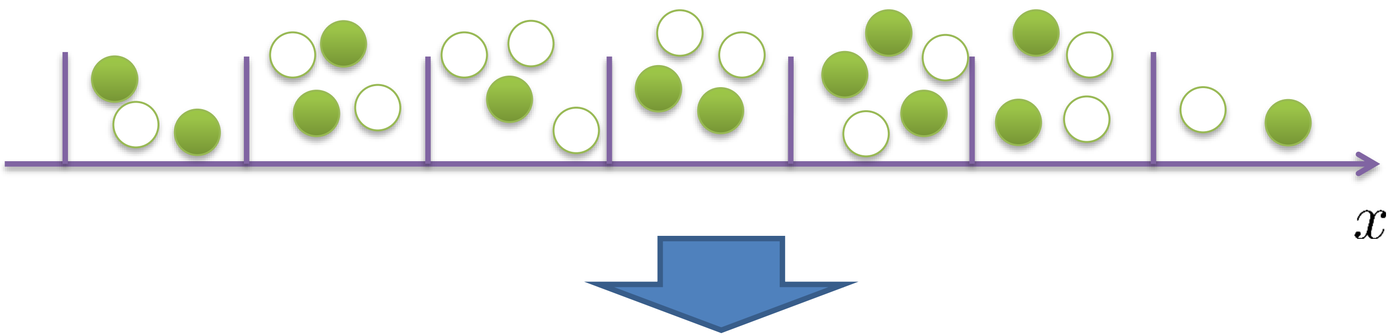
$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$





Net Charge Number

Prepare 2 species of (non-interacting) particles

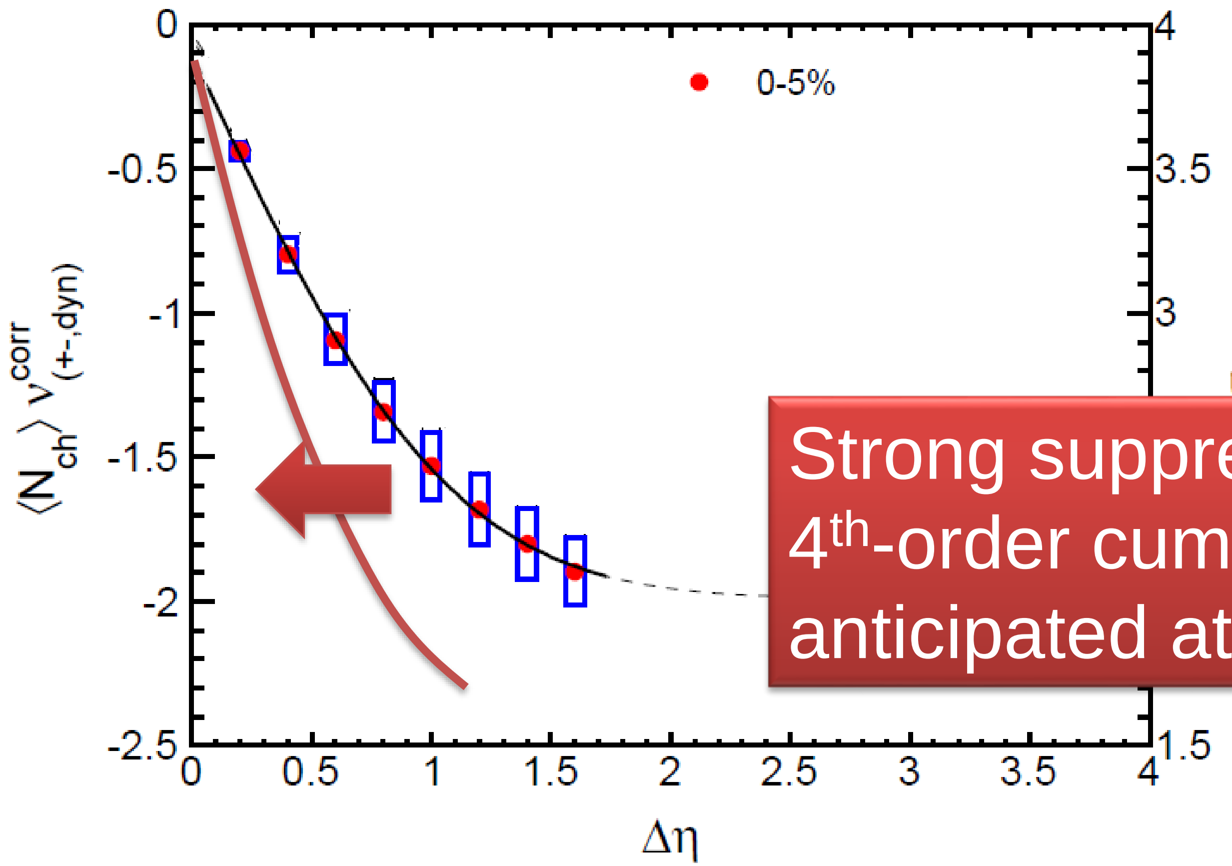


$$\langle \delta Q^n \rangle_c(t)$$

Same time evolution except for
Poisson → Skellam

$<\delta N_Q^4>$ @ LHC

Assumptions
- Bjorken expansion
- small fluctuations at hadronization
- short correlaition in hadronic stage



Strong suppression of 4^{th}-order cumulant is anticipated at LHC energy!

Chemical Reaction 1

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

x: # of X
a: # of A (**fixed**)

Master eq.: $$\frac{\partial}{\partial t} P(x,t) = k_2 a P(x-1,t) + k_1 (x+1) P(x+1,t) - (k_1 x + k_2 a) P(x,t)$$

Cumulants with fixed initial condition $P(x,0) = \delta_{x,N_0}$

$$\langle x(t) \rangle = N_0 e^{-k_1 t} + N_{eq}(1 - e^{-k_1 t})$$

$$\langle \delta x(t)^2 \rangle = N_0 (e^{-k_1 t} - e^{-2k_1 t}) + N_{eq}(1 - e^{-k_1 t})$$

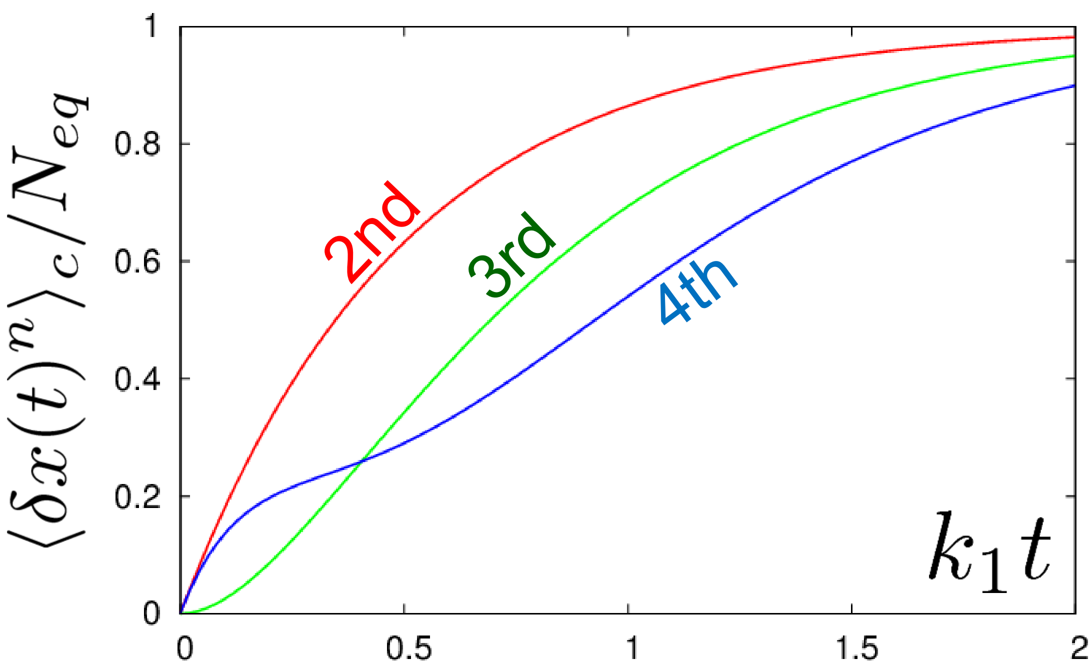
$$\langle \delta x(t)^3 \rangle = N_0 (e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t}) + N_{eq}(1 - e^{-k_1 t})$$

initial — equilibrium

Chemical Reaction 2

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

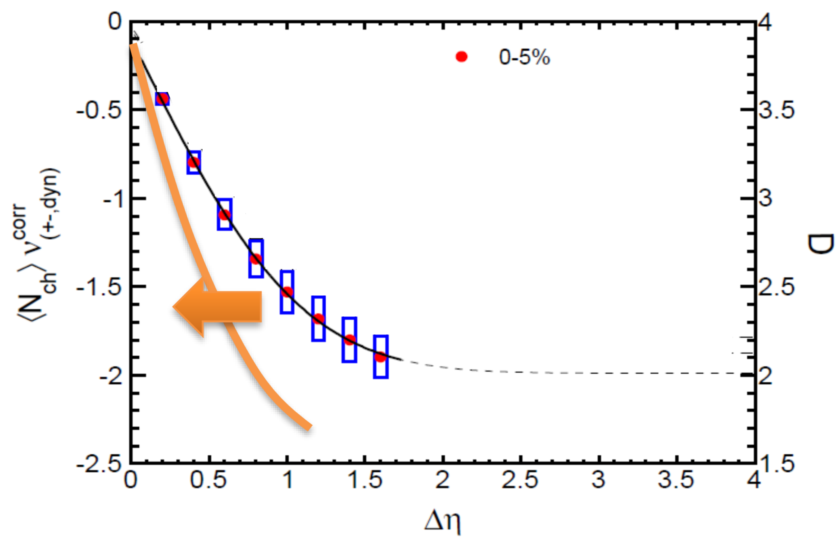
$$N_0 = N_{eq} \Rightarrow \begin{cases} \langle x(t) \rangle = N_{eq} \\ \langle \delta x(t)^2 \rangle = N_{eq}(1 - e^{-2k_1 t}) \\ \langle \delta x(t)^3 \rangle = N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t}) \end{cases}$$



Higher-order cumulants grow slower.

Summary

- $\Delta\eta$ dependence of cumulants encodes plenty of physics.
- Cumulants with different order have different time evolution.
- Our analysis with a diffusion master equation shows that
  - Approach to equilibrium from small fluctuation is slower for higher orders.
  - ALICE will observe a small 4th order cumulant.
  - or, we miss something…



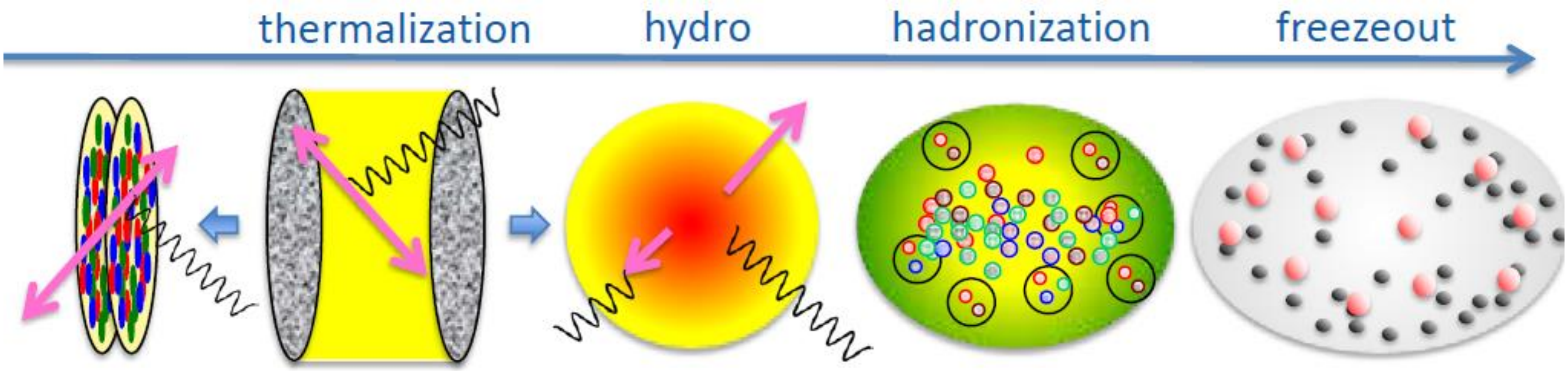
Open Questions & Future Work

- Why the primordial fluctuations are observed only at the LHC, and not the RHIC ?
- Extract more information on each stage of fireballs using fluctuations

- Model refinement
  - Including the effects of
    nonzero correlation length / relaxation time
    global charge conservation

  - Non Poissonian system ← interaction of particles

Evolution of Fluctuations



Fluctuation in initial state

Time evolution in the QGP

approach to HRG by diffusion

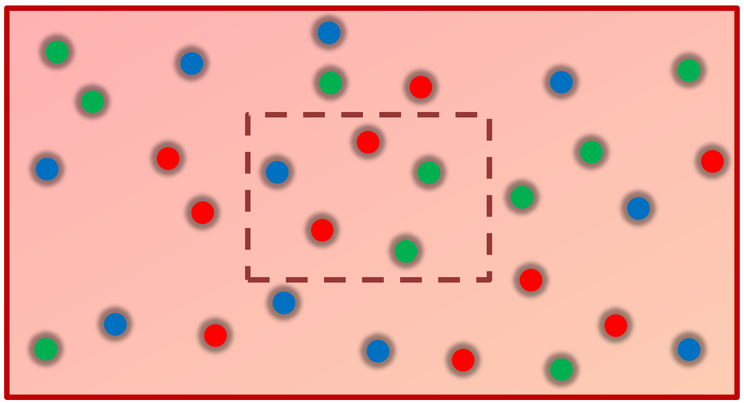
experimental effects
particle missID, etc.

volume fluctuation

Fluctuations

Free Boltzmann → Poisson

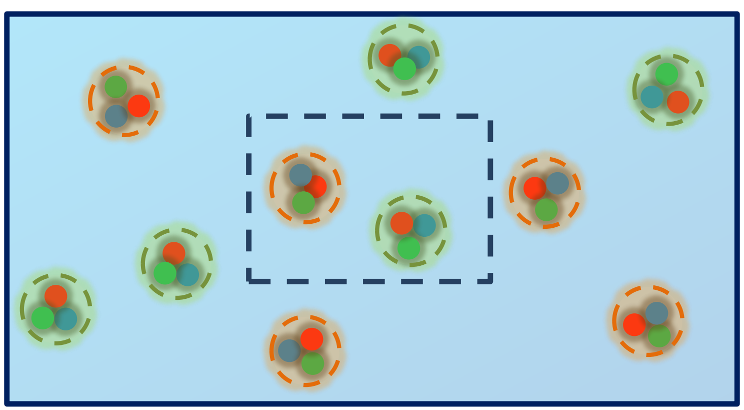
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

Nonzero Isospin Density

$$N_{\mathrm{B}} \to N_p$$

$$\xi_1 = r, \quad \xi_2 = r(1-r), \quad \xi_3 = r(1-r)(1-2r),$$
$$\xi_4 = r(1-r)(1-6r+6r^2), \quad \cdots,$$

$$\langle N_p^{(\mathrm{net})} \rangle = \langle \xi_1 N_{\mathrm{B}} - \bar{\xi}_1 N_{\bar{\mathrm{B}}} \rangle,$$

$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \langle (\xi_1 \delta N_{\mathrm{B}} - \bar{\xi}_1 \delta N_{\bar{\mathrm{B}}})^2 \rangle + \langle \xi_2 N_{\mathrm{B}} + \bar{\xi}_2 N_{\bar{\mathrm{B}}} \rangle,$$

$$\langle (\delta N_p^{(\mathrm{net})})^3 \rangle = \langle (\xi_1 \delta N_{\mathrm{B}} - \bar{\xi}_1 \delta N_{\bar{\mathrm{B}}})^3 \rangle + 3\langle (\xi_2 \delta N_{\mathrm{B}} + \bar{\xi}_2 \delta N_{\bar{\mathrm{B}}})(\xi_1 \delta N_{\mathrm{B}} - \bar{\xi}_1 \delta N_{\bar{\mathrm{B}}}) \rangle + \langle \xi_3 N_{\mathrm{B}} - \bar{\xi}_3 N_{\bar{\mathrm{B}}} \rangle,$$

$$\langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c = \langle (\xi_1 \delta N_{\mathrm{B}} - \bar{\xi}_1 \delta N_{\bar{\mathrm{B}}})^4 \rangle_c + 6\langle (\xi_2 \delta N_{\mathrm{B}} + \bar{\xi}_2 \delta N_{\bar{\mathrm{B}}})(\xi_1 \delta N_{\mathrm{B}} - \bar{\xi}_1 \delta N_{\bar{\mathrm{B}}})^2 \rangle + 3\langle (\xi_2 \delta N_{\mathrm{B}} + \bar{\xi}_2 \delta N_{\bar{\mathrm{B}}})^2 \rangle$$
$$+ 4\langle (\xi_3 \delta N_{\mathrm{B}} - \bar{\xi}_3 \delta N_{\bar{\mathrm{B}}})(\xi_1 \delta N_{\mathrm{B}} - \bar{\xi}_1 \delta N_{\bar{\mathrm{B}}}) \rangle + \langle \xi_4 N_{\mathrm{B}} + \bar{\xi}_4 N_{\bar{\mathrm{B}}} \rangle$$

$$N_p \to N_{\mathrm{B}}$$

Effect of nonzero isospin density is well suppressed down to sqrt{s}~10GeV

$$\langle N_{\mathrm{B}}^{(\mathrm{net})} \rangle = \langle \xi_1^{-1} N_p - \bar{\xi}_1^{-1} N_{\bar{p}} \rangle,$$

$$\langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \rangle = \langle (\xi_1^{-1} \delta N_p - \bar{\xi}_1^{-1} \delta N_{\bar{p}})^2 \rangle - \langle \xi_2 \xi_1^{-3} \delta N_p + \bar{\xi}_2 \bar{\xi}_1^{-3} \delta N_{\bar{p}} \rangle,$$

$$\langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^3 \rangle = \langle (\xi_1^{-1} \delta N_p - \bar{\xi}_1^{-1} \delta N_{\bar{p}})^3 \rangle - 3\langle (\xi_2 \xi_1^{-3} \delta N_p + \bar{\xi}_2 \bar{\xi}_1^{-3} \delta N_{\bar{p}})(\xi_1^{-1} \delta N_p - \bar{\xi}_1^{-1} \delta N_{\bar{p}}) \rangle + \left\langle \frac{3\xi_2^2 - \xi_1 \xi_3}{\xi_1^5} N_p - \frac{3\bar{\xi}_2^2 - \bar{\xi}_1 \bar{\xi}_3}{\bar{\xi}_1^5} N_{\bar{p}} \right\rangle,$$

$$\langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^4 \rangle_c = \langle (\xi_1^{-1} \delta N_p - \bar{\xi}_1^{-1} \delta N_{\bar{p}})^4 \rangle_c - 6\langle (\xi_2 \xi_1^{-3} \delta N_p + \bar{\xi}_2 \bar{\xi}_1^{-3} \delta N_{\bar{p}})(\xi_1^{-1} \delta N_p - \bar{\xi}_1^{-1} \delta N_{\bar{p}}) \rangle + 12\langle (\xi_2^2 \xi_1^{-5} \delta N_p - \bar{\xi}_2^2 \bar{\xi}_1^{-5} \delta N_{\bar{p}})$$
$$\times (\xi_1^{-1} \delta N_p - \bar{\xi}_1^{-1} \delta N_{\bar{p}}) \rangle + 3\langle (\xi_2 \xi_1^{-3} \delta N_p + \bar{\xi}_2 \bar{\xi}_1^{-3} \delta N_{\bar{p}})^2 \rangle - 4\langle (\xi_3 \xi_1^{-4} \delta N_p - \bar{\xi}_3 \bar{\xi}_1^{-4} \delta N_{\bar{p}})(\xi_1^{-1} \delta N_p - \bar{\xi}_1^{-1} \delta N_{\bar{p}}) \rangle$$
$$- \left\langle \frac{15\xi_2^3 - 10\xi_1 \xi_2 \xi_3 + \xi_1^2 \xi_4}{\xi_1^7} N_p - \frac{15\bar{\xi}_2^3 - 10\bar{\xi}_1 \bar{\xi}_2 \bar{\xi}_3 + \bar{\xi}_1^2 \bar{\xi}_4}{\bar{\xi}_1^7} N_{\bar{p}} \right\rangle.$$

Solving DME

(1) Factorial Generating Func.

$$G(\mathbf{s}) = \sum_{\mathbf{n}} \prod_x s_x^{n_x} P(\mathbf{n})$$

$$\frac{\partial}{\partial t} G(\mathbf{s}, t) = d \sum_x (s_{x+1} + s_{x-1} - 2s_x) \frac{\partial}{\partial s} G(\mathbf{s}, t)$$

(2) Solution with Fixed Initial Condition

(3) Time evolution of factorial cumulants

(4) Factorial cumulants → cumulants

(5) Superposition of cumulants

Time Evolution of Cumulants

Fixed initial condition

$$P(\mathbf{n}, 0) = \prod_x \delta_{n_x, M_x}$$

Particle number in each cell is fixed to M_x.

Solution

$$\langle n_k(t) \rangle = e^{-\omega_k t} M_k \qquad \omega_k = \gamma a^2 k^2$$

In coordinate space (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

Deterministic part is controlled by the diffusion eq.
Appropriate continuum limit with $\gamma a^2 = D$

Time Evolution of Cumulants

Fixed initial condition

$$P(\mathbf{n}, 0) = \prod_x \delta_{n_x, M_x}$$

Particle number in each cell is fixed to M_x.

Solution

$$\langle n_k(t) \rangle = e^{-\omega_k t} M_k$$

$$\langle \delta n_{k_1}(t) \delta n_{k_2}(t) \rangle = \cdots$$

Consistent with stochastic diffusion eq.
with sufficiently slow initial condition