重イオン衝突における保存電荷高次ゆらぎの時間発展

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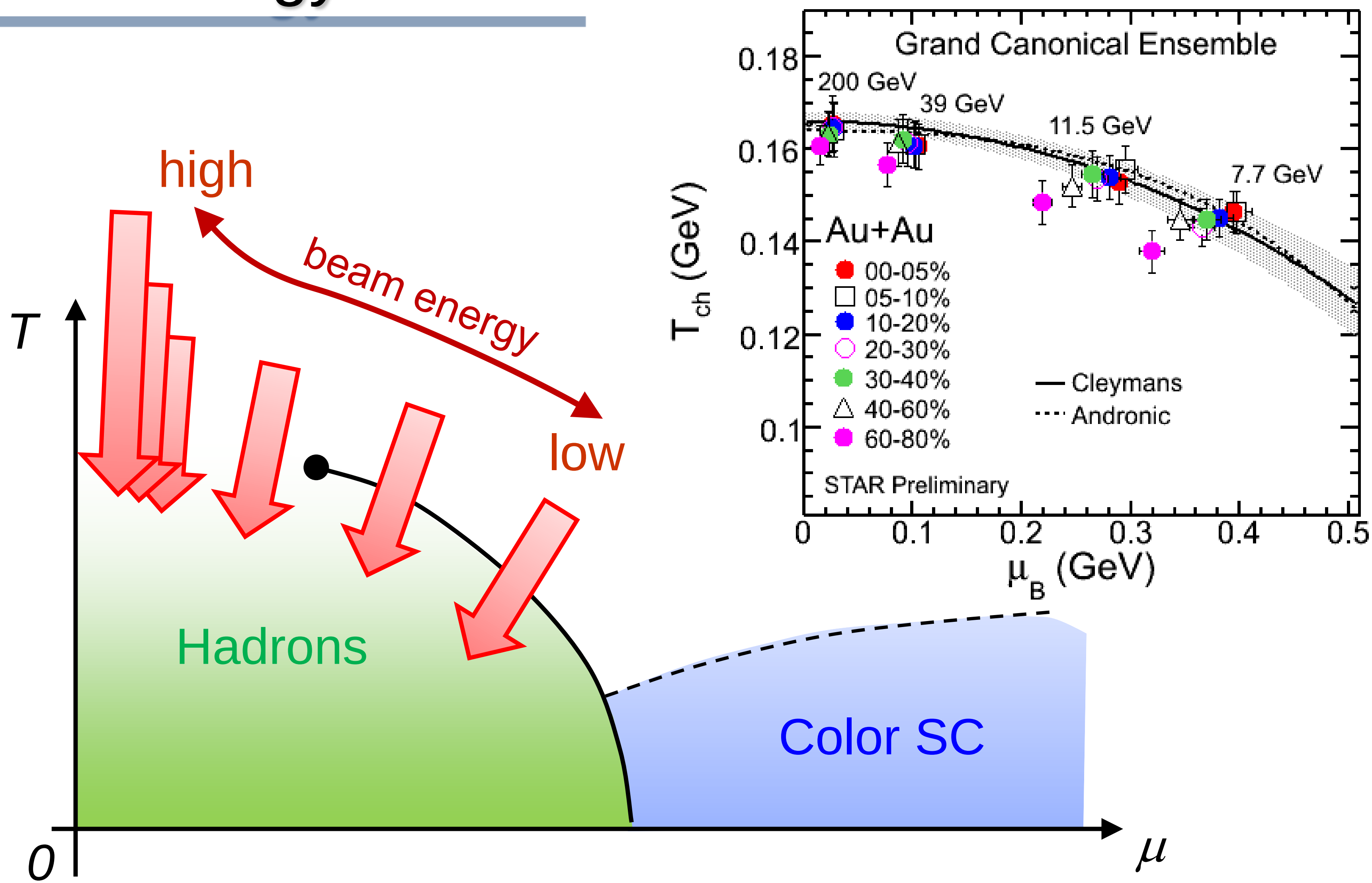
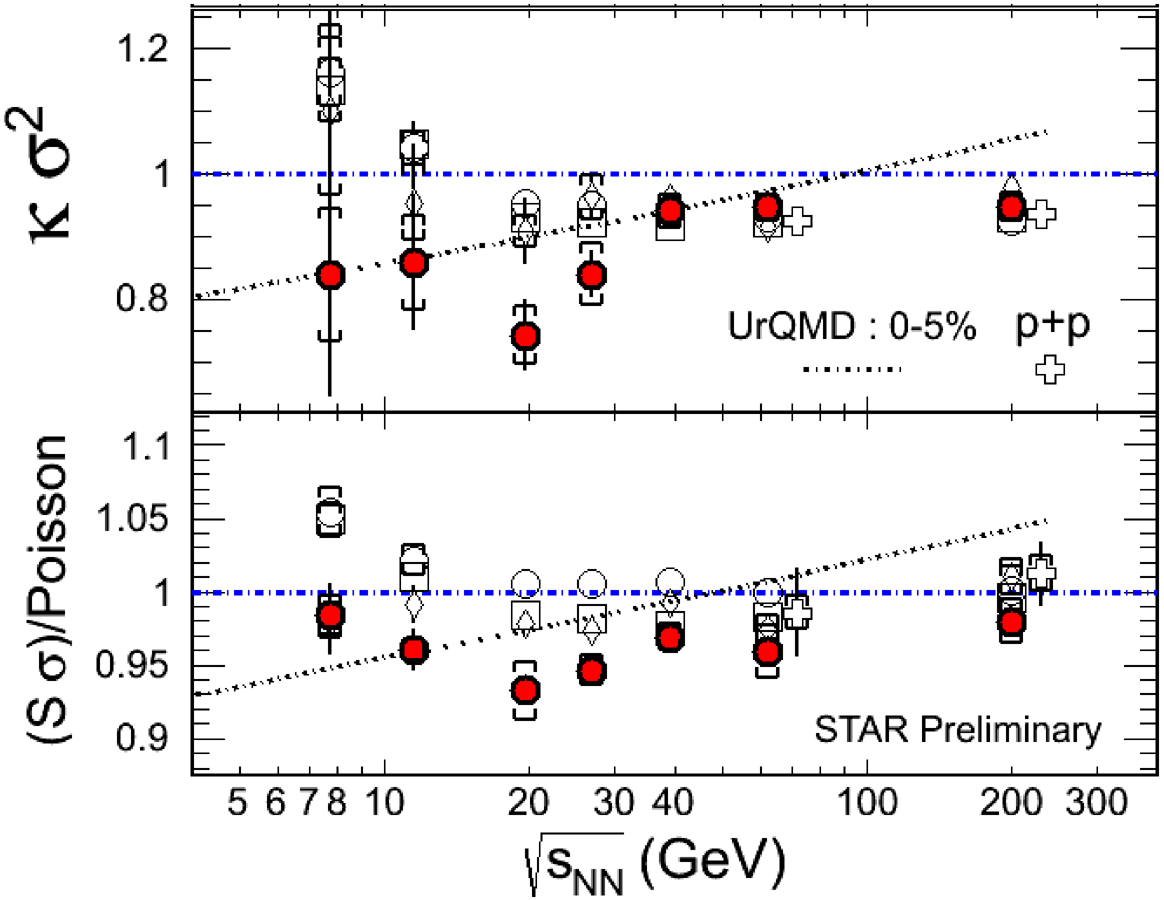
Beam-Energy Scan



Proton # Cumulants @ STAR-BES

STAR,QM2012



$$\frac{C_4}{C_2}$$

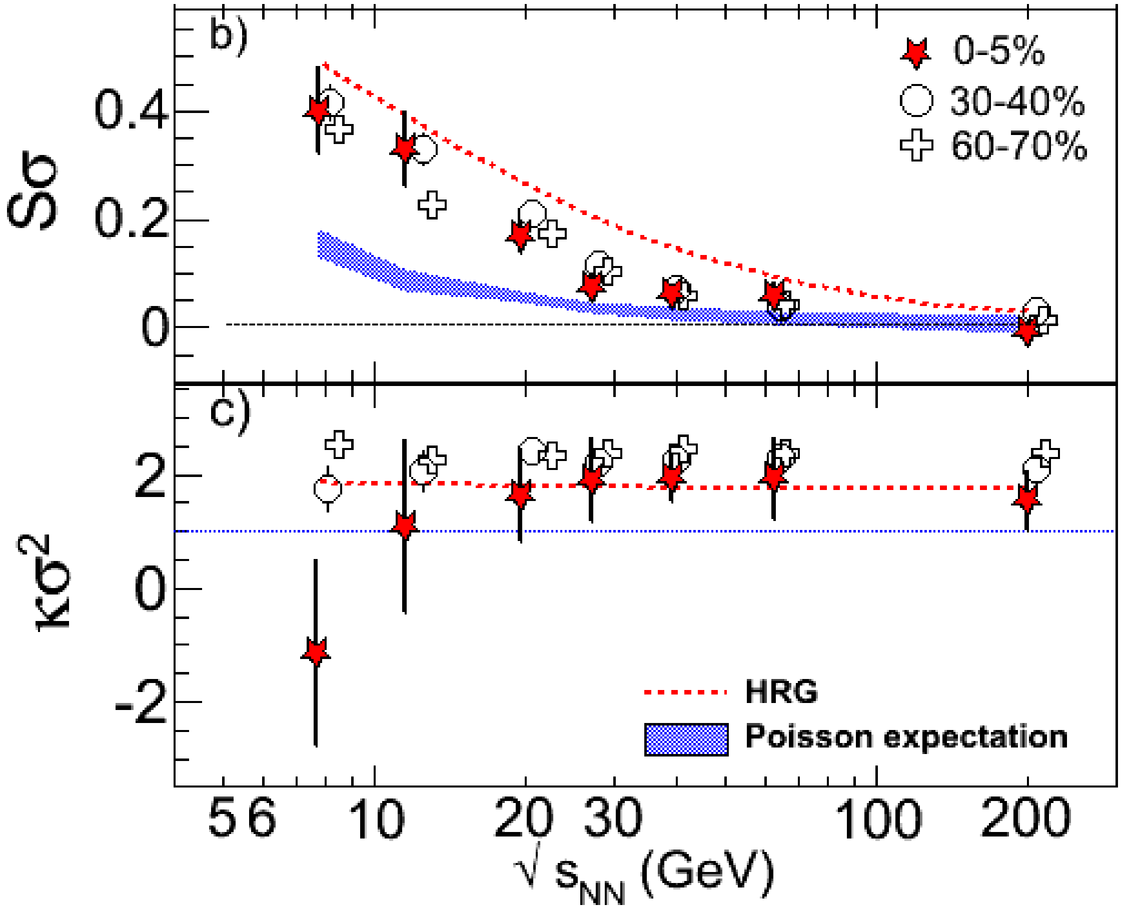
$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

No characteristic signals on
phase transition to QGP nor QCD CP

Charge Fluctuations @ STAR-BES

$$\frac{\langle \delta N_Q^3 \rangle}{\langle \delta N_Q^2 \rangle}$$

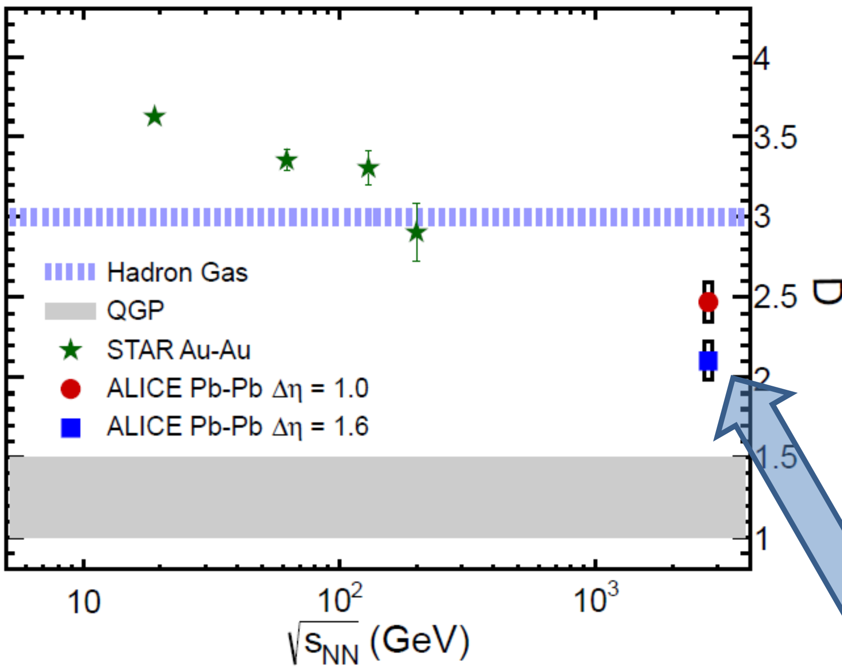
$$\frac{\langle \delta N_Q^4 \rangle_c}{\langle \delta N_Q^2 \rangle}$$



STAR, QM2012

No characteristic signals on
phase transition to QGP nor QCD CP

Charge Fluctuation @ LHC



ALICE, 1207.6068

D-measure

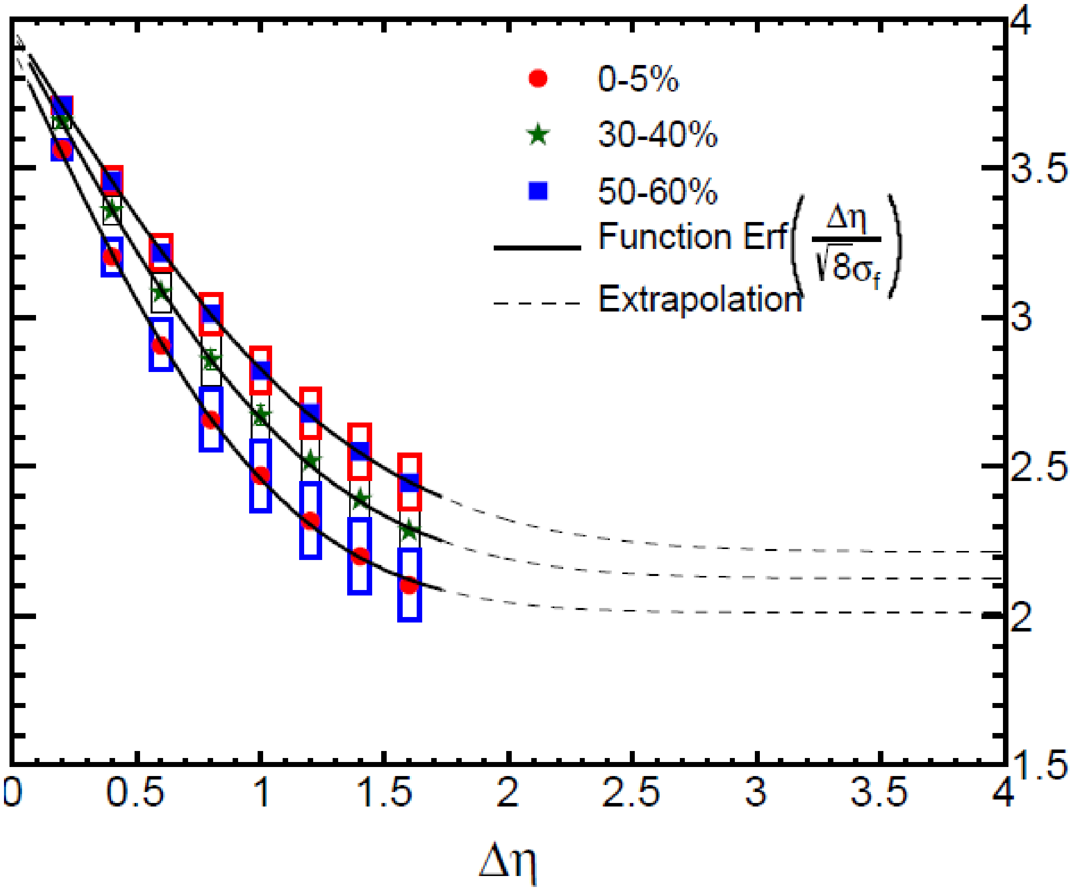
$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- D ~ 3-4 Hadronic
- D ~ 1 Quark

LHC:
significant suppression from hadronic value

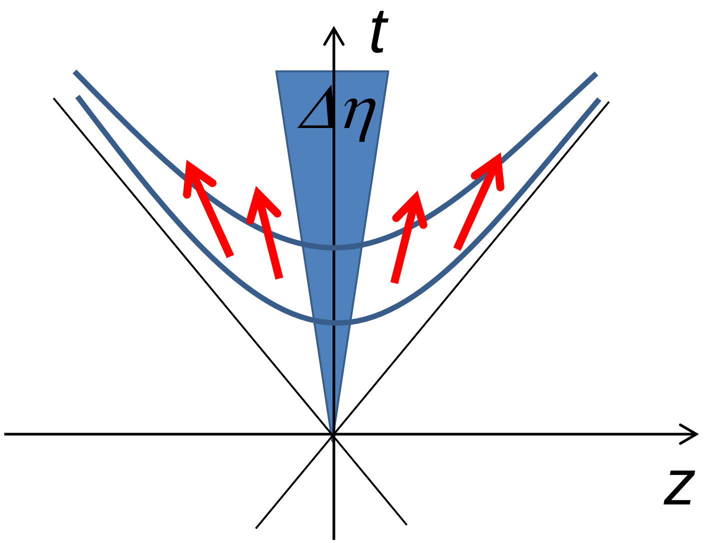
$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE



検出器ラピディティ幅

ALICE
1207.6068



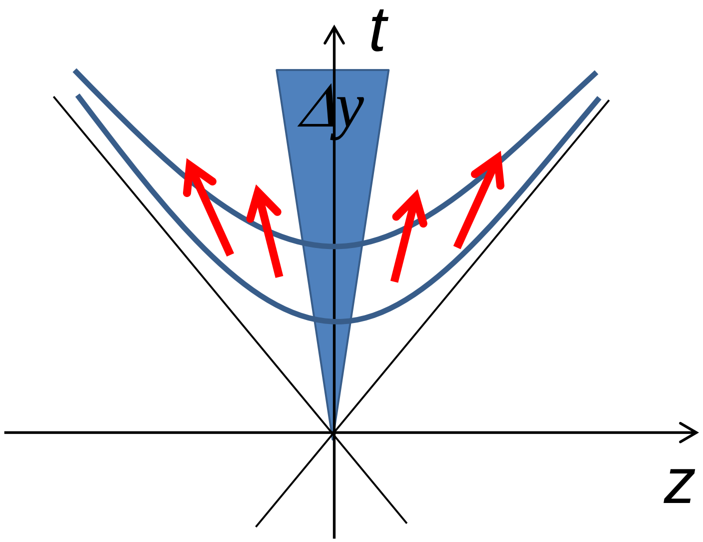
高次ゆらぎは高温物質の温度計？

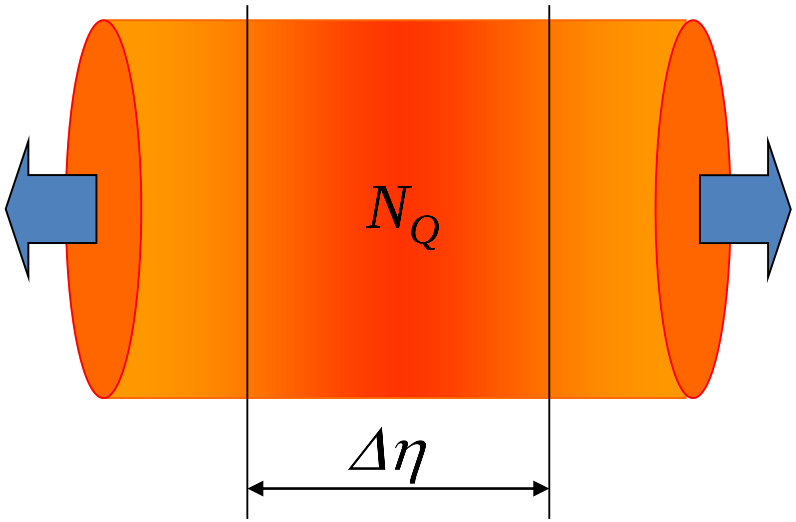
HotQCD,2012

Fluctuations of Conserved Charges

- Under Bjorken expansion

Asakawa, Heintz, Muller, 2000
Jeon, Koch, 2000
Shuryak, Stephanov, 2001



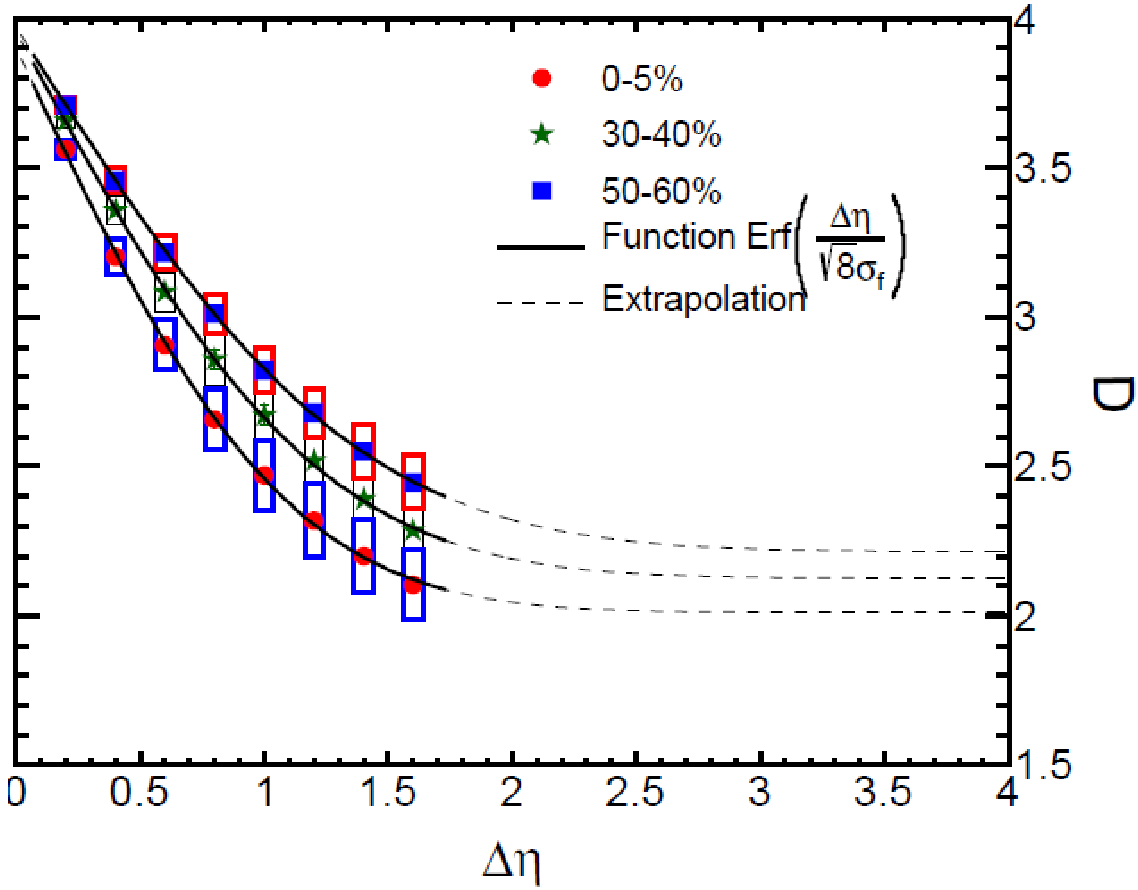


- Variation of a conserved charge in $\Delta\eta$ is **slow**, since it is achieved only through diffusion.

➡ Primordial values can survive until freezeout. The wider $\Delta\eta$, more earlier fluctuation.

Δη Dependence @ ALICE

ALICE
1207.6068

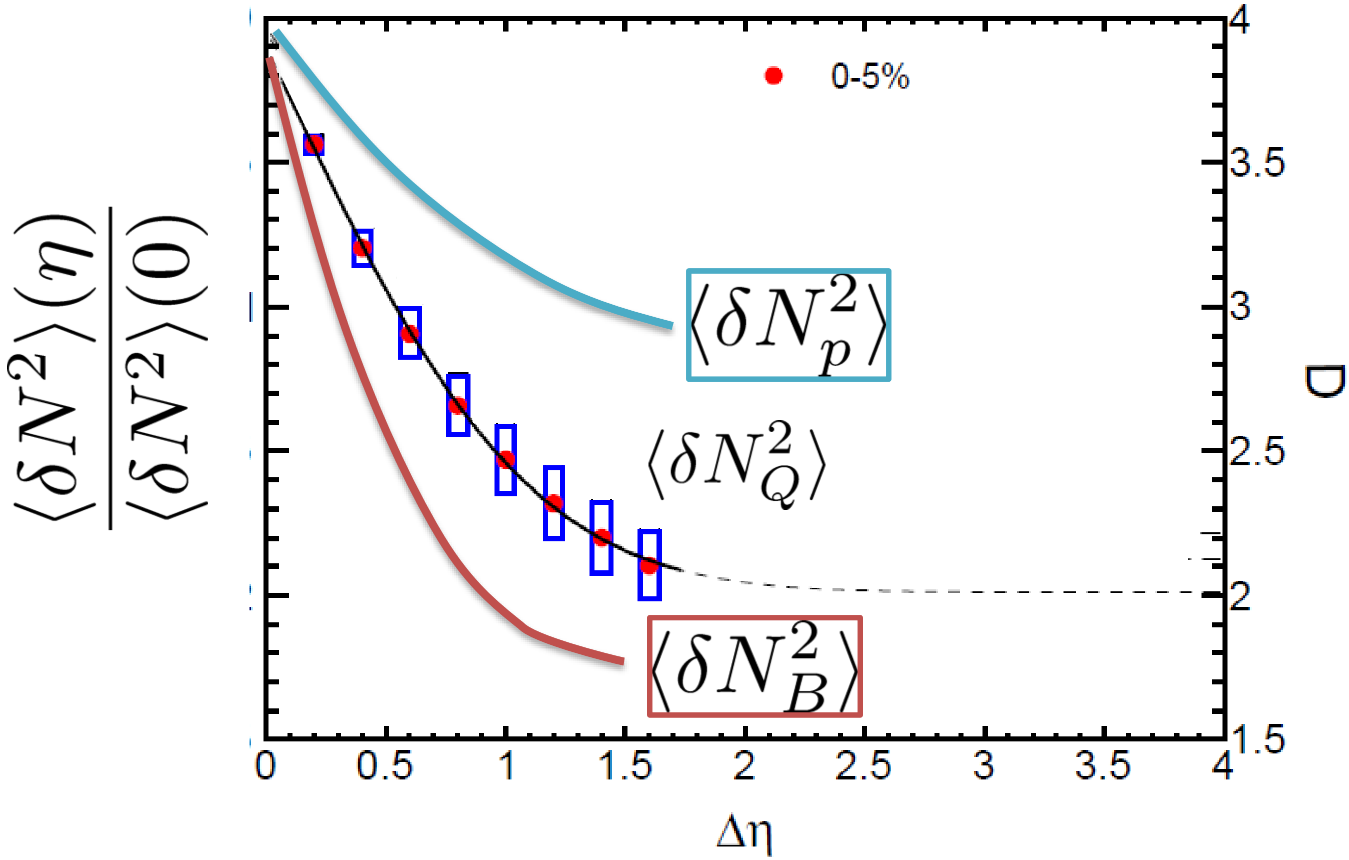


保存電荷ゆらぎのΔη依存性には、
高温物質時間発展の歴史が刻まれている！

$<\delta N_B^2>$ and $<\delta N_p^2>$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$

should have different Δη dependence.

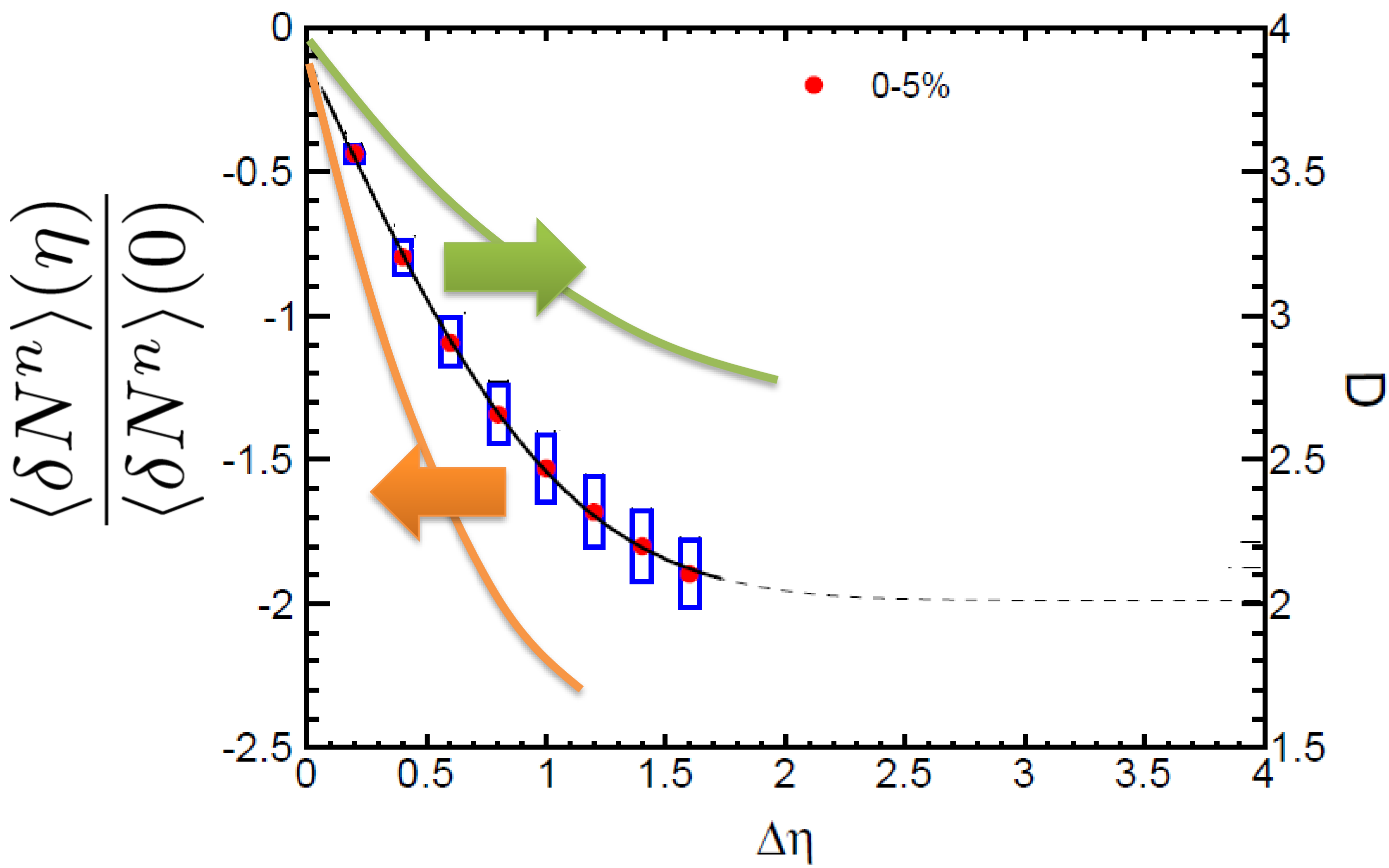


$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \rangle + \frac{1}{4} \langle N_{\mathrm{B}}^{(\mathrm{tot})} \rangle$$

$\langle \delta N_Q^4 \rangle$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

Left (suppression) or Right (hadronic)



Stochasitc Hydrodynamics

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

ミクロな原子相互作用に
由来するノイズ項

Fluctuation-Dissipation Relation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Stochastic force

- Local correlation (hydrodynamics) $\langle \xi(\eta_1,\tau_1)\xi(\eta_2,\tau_2)\rangle \sim \delta(\eta_1-\eta_2)\delta(\tau_1-\tau_2)$
- Equilibrium fluc. $\langle \delta Q(t)^2 \rangle \xrightarrow[t\to\infty]{} \chi_2 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta,\tau)$$

χ_2 : susceptibility

$$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\rangle = \frac{2\chi_2}{D}\delta(k_1+k_2)\delta(\tau_1-\tau_2)$$

$\Delta\eta$ Dependence

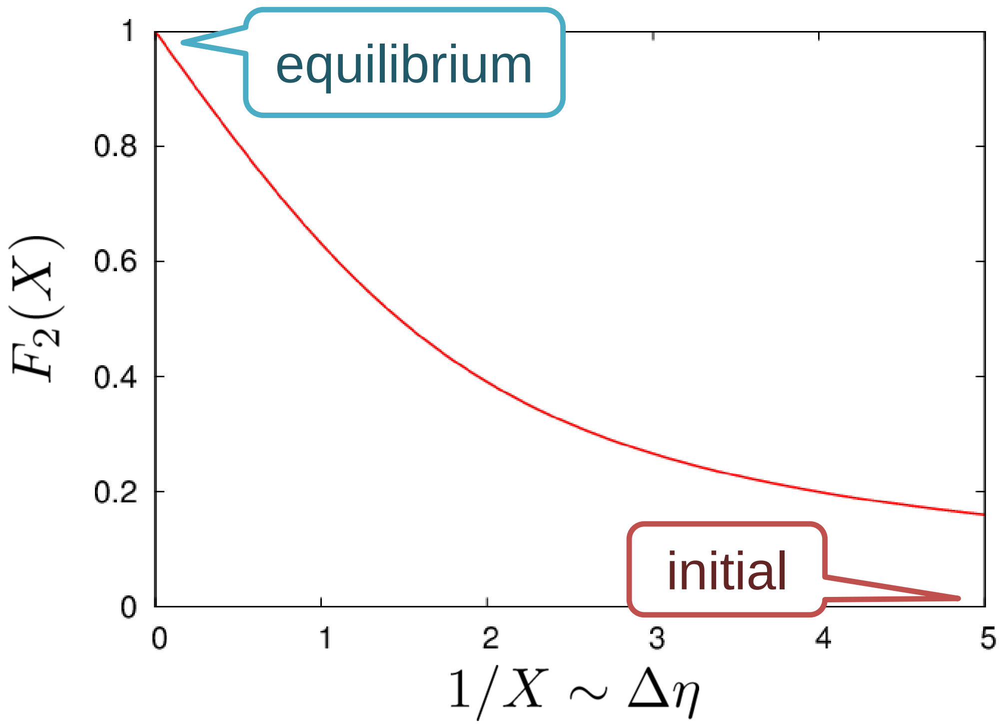
Shuryak, Stephanov, 2001

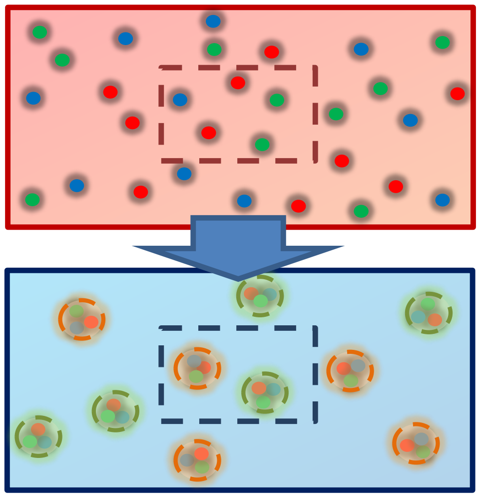
- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2 (1 - F_2(X))}_{\text{equilibrium}}$$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$





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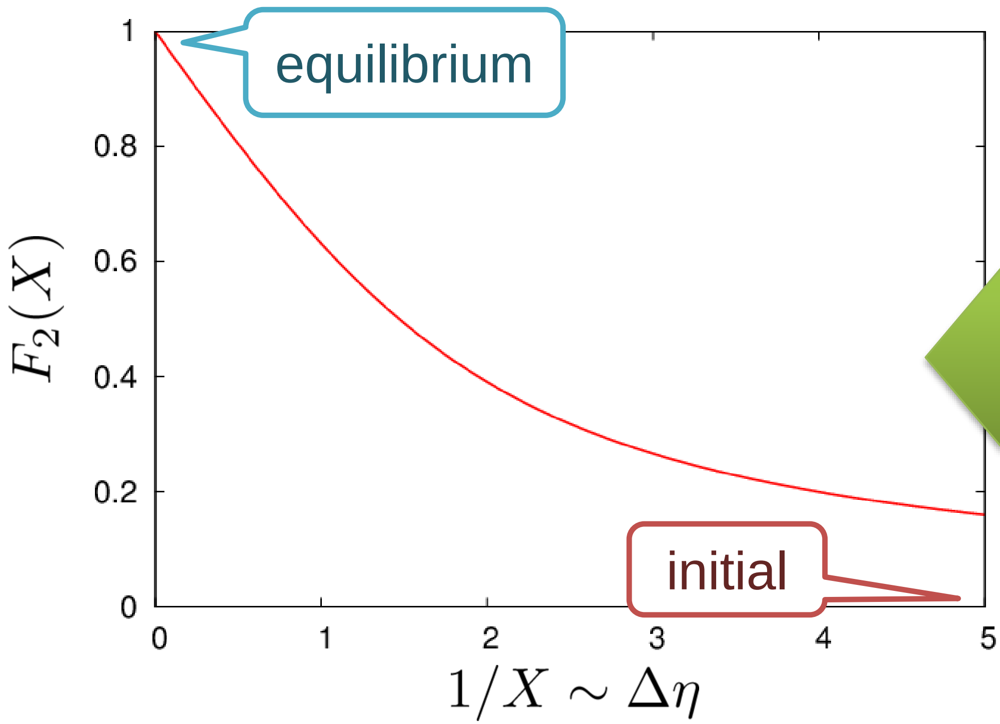

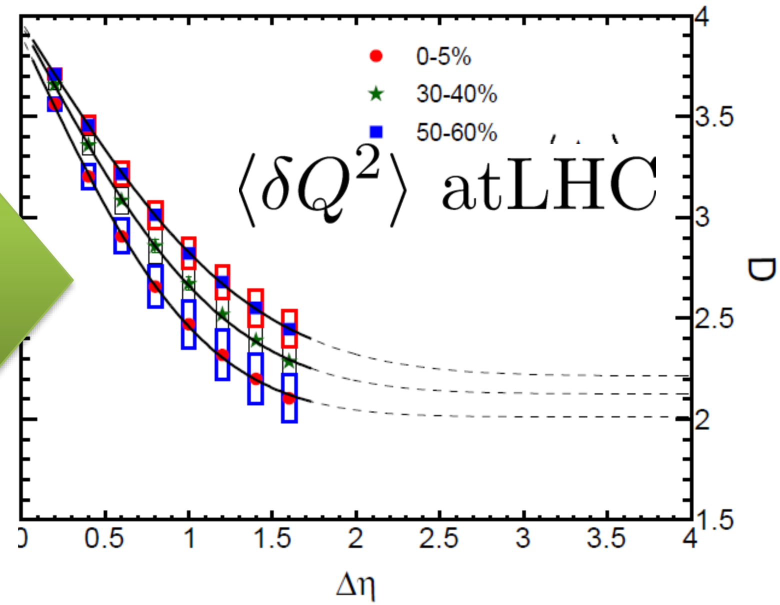

Non-Gaussian Stochastic Force ??

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Stochastif Force : 3rd order

- Local correlation (hydrodynamics) $\langle \xi(\eta_1, \tau_1)\xi(\eta_2, \tau_2)\xi(\eta_3, \tau_3)\rangle$
$$\sim \delta(\eta_1 - \eta_2)\delta(\eta_2 - \eta_3)\delta(\tau_1 - \tau_2)\delta(\tau_2 - \tau_3)$$
- Equilibrium fluc. $\langle \delta Q(t)^3 \rangle \underset{t \to \infty}{\longrightarrow} \chi_3 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

χ_3 : third − moment

Caution!

- 揺動散逸関係の高次への拡張は、非自明。

- 定理

  マルコフ過程＋連続変化する確率変数
  ➔ランダム力はガウスゆらぎ

  cf) Gardiner, “Stochastic Methods”

- 流体力学 ⇨ 微小空間に多数の粒子
  ⇨ 中心極限定理により、ガウス分布

Thee "NON"s

重イオン衝突での高次ゆらぎの観測・解析は、
物理学として**相当に**特殊な問題である。

- **Non-Gaussian**

通常、高次ゆらぎは観測困難。
適度に小さい系

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  観測されたゆらぎの値は、
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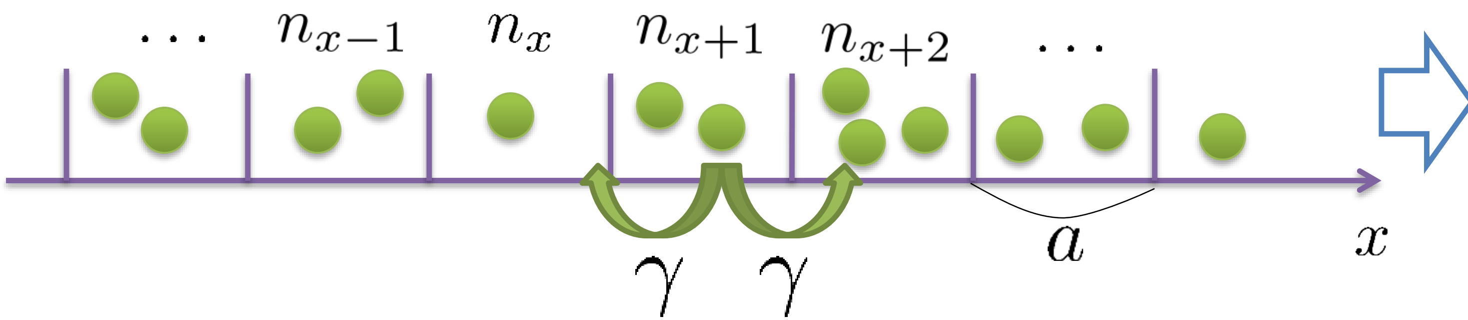
- **Non-equilibrium**
  平衡に至る非定常過程を
  記述する必要性。

Diffusion Master Equation

Divide spatial coordinate into discrete cells



probability

$P(\mathbf{n})$

Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t} P(\mathbf{n}) = d \sum_x [(n_x + 1) \left\{ P(\mathbf{n} + \hat{x} - \widehat{x+1}) + P(\mathbf{n} + \hat{x} - \widehat{x-1}) \right\} - 2 n_x P(\mathbf{n})]$$

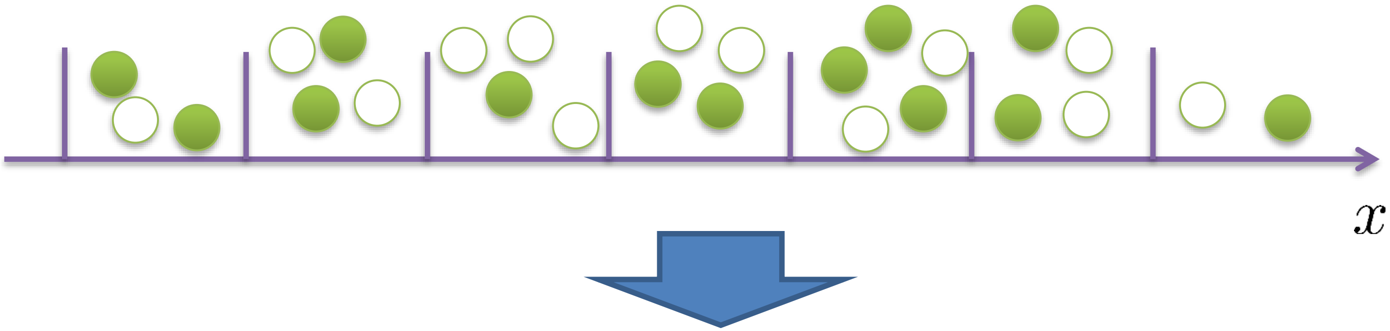
x-hat: lattice-QCD notation

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen’s system size expansion

Net Charge Number

Prepare 2 species of (non-interacting) particles



$$\bar{Q} = \int_0^{\Delta\eta} d\eta \, (n_1(\eta) - n_2(\eta))$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ at freezeout time t

Initial Condition at Hadronization

- Boost invariance / infinitely long system
- Local equilibration / local correlation
- Initial fluctuations

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ $\quad \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\quad \langle Q^2_{(\mathrm{tot})} \rangle_c$

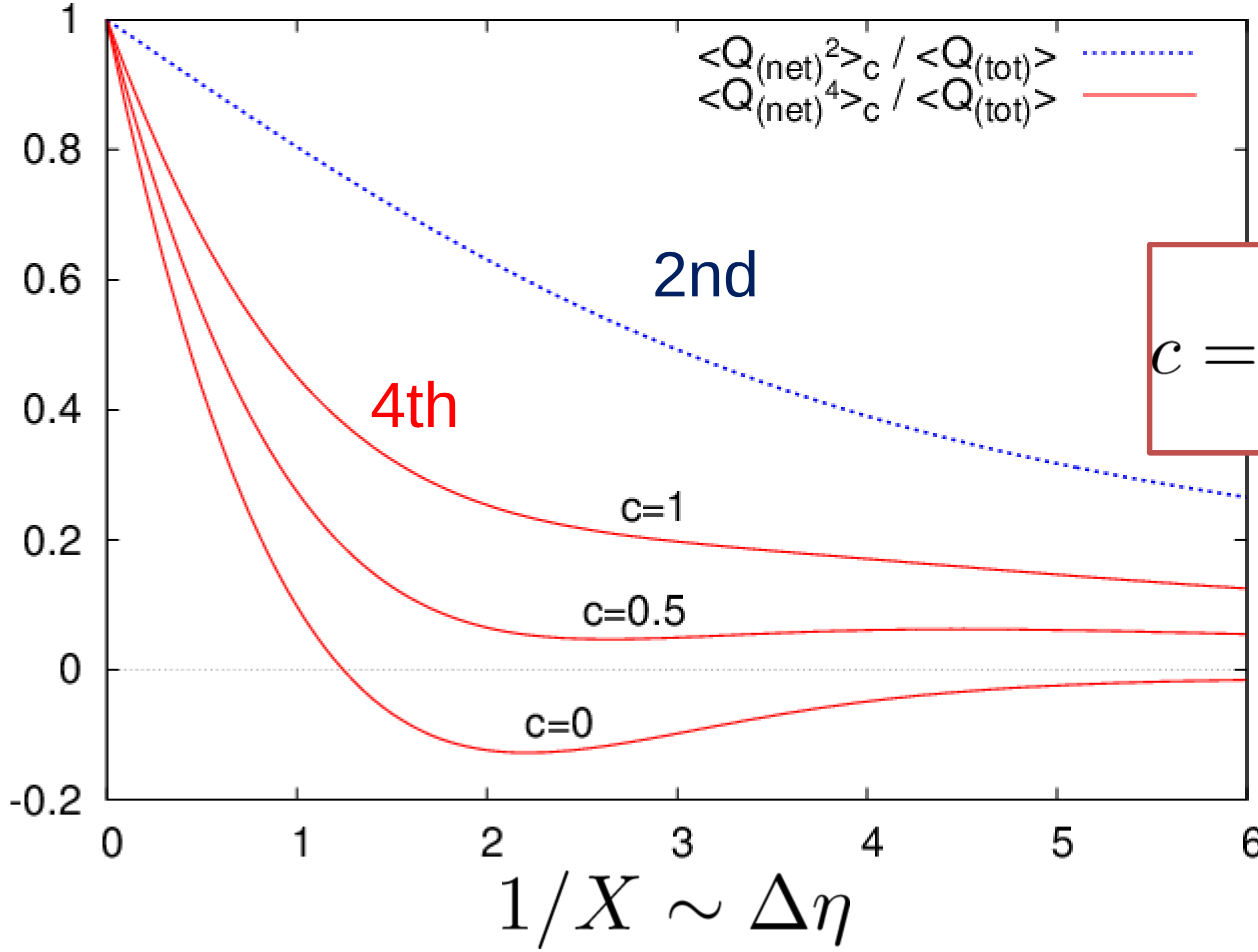
suppression owing to local charge conservation

strongly dependent on hadronization mechanism

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

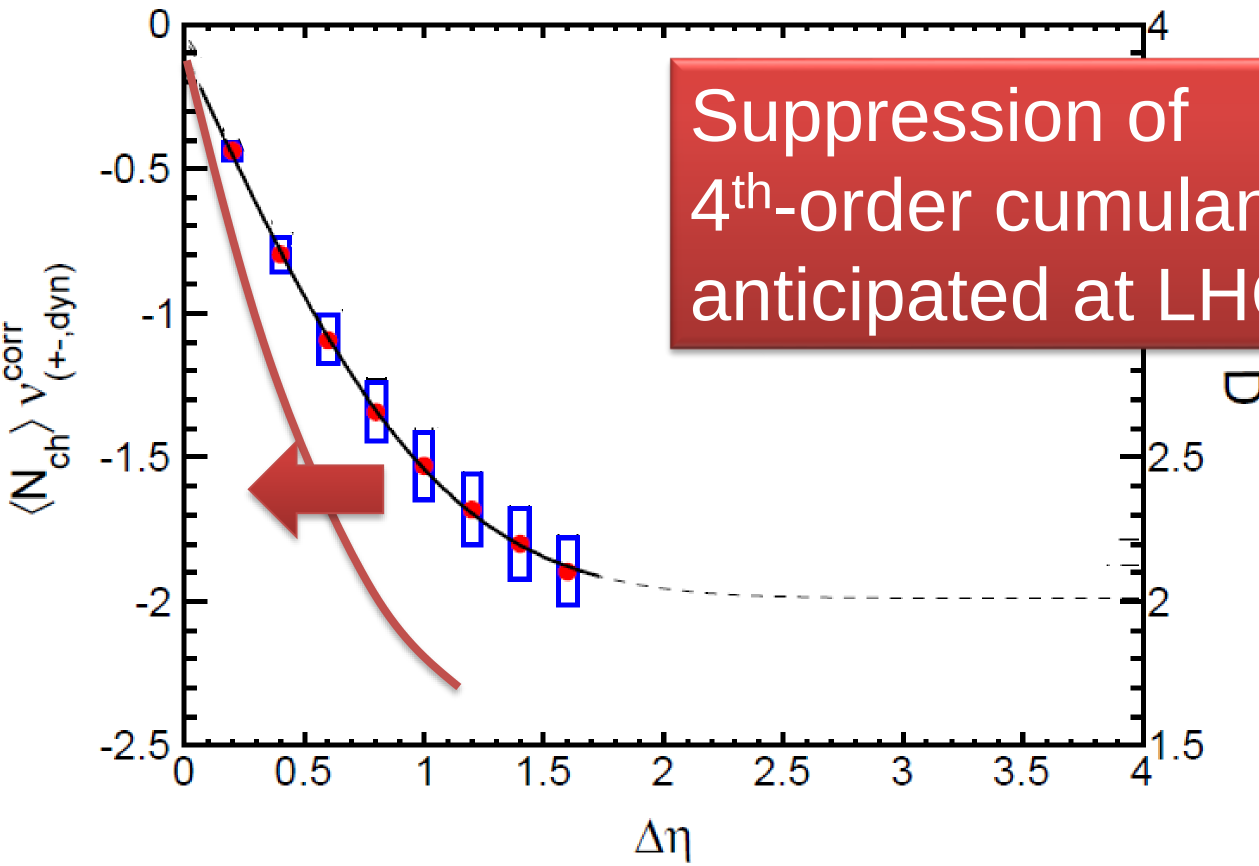
$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$<\delta N_Q^4>$ @ LHC

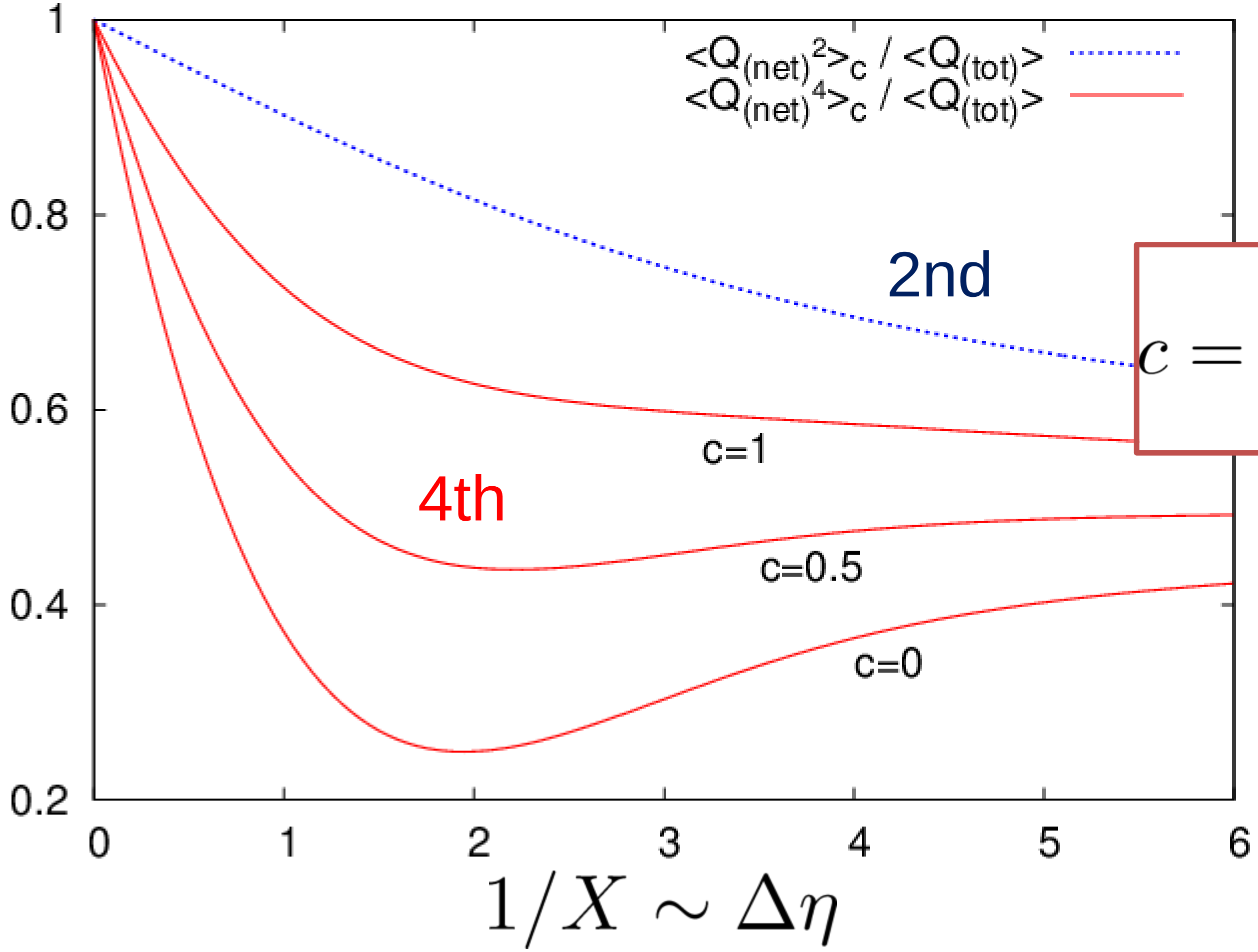
Assumptions
- boost invariant system
- tiny fluctuations of CC at hadronization
- short correlaition in hadronic stage



$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0.5$$



$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

まとめ

キュムラント$\Delta\eta$依存性には豊富な物理が含まれている

$$\langle N_Q^2\rangle_c,\ \langle N_B^2\rangle_c,\ \langle N_Q^4\rangle_c,\ \langle N_B^4\rangle_c,\ \langle N_{ch}^2\rangle_c,\cdots$$

実験で観測されるゆらぎが持つ物理的意味の理解

高温物質の動的発展の情報

- ビーム軸方向の時空発展の歴史
- ハドロン化の機構
- 拡散係数

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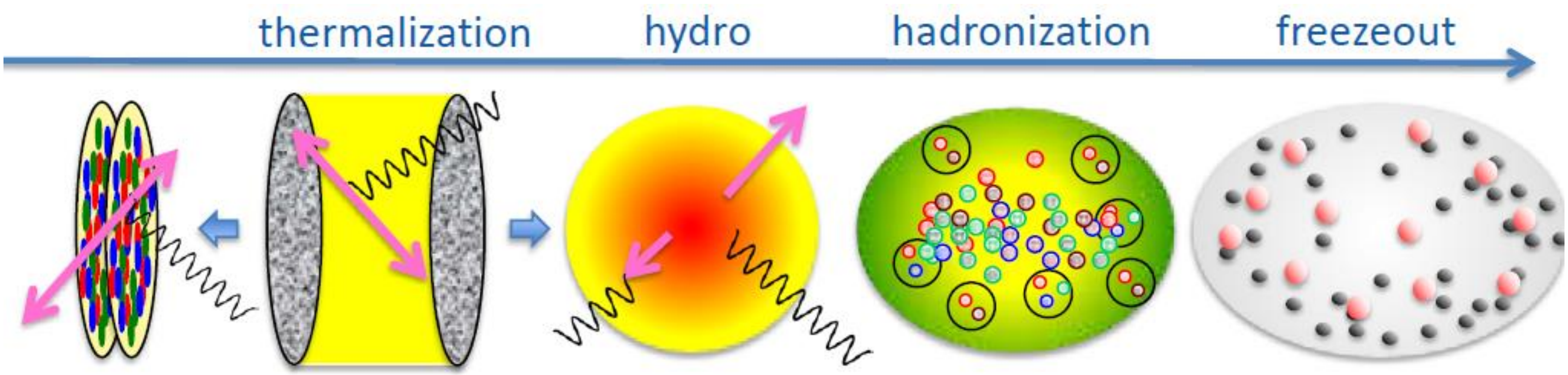
QCD相構造の探索

Evolution of Fluctuations



Fluctuation in initial state

Time evolution in the QGP

approach to HRG by diffusion

volume fluctuation

experimental effects
particle missID, etc.

Open Questions & Future Work

- Why the primordial fluctuations are observed only at the LHC, and not the RHIC ?
- Extract more information on each stage of fireballs using fluctuations

- Model refinement
  - Including the effects of
    nonzero correlation length / relaxation time
    global charge conservation

  - Non Poissonian system ← interaction of particles

Chemical Reaction 1

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

x: # of X
a: # of A (**fixed**)

Master eq.:
$$\frac{\partial}{\partial t}P(x,t) = k_2 a P(x-1,t) + k_1(x+1)P(x+1,t) - (k_1 x + k_2 a)P(x,t)$$

Cumulants with fixed initial condition $P(x,0)=\delta_{x,N_0}$

$$\langle x(t)\rangle = N_0 e^{-k_1 t} + N_{eq}(1-e^{-k_1 t})$$

$$\langle \delta x(t)^2\rangle = N_0(e^{-k_1 t} - e^{-2k_1 t}) + N_{eq}(1-e^{-k_1 t})$$

$$\langle \delta x(t)^3\rangle = N_0(e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t}) + N_{eq}(1-e^{-k_1 t})$$

initial — equilibrium

Chemical Reaction 2

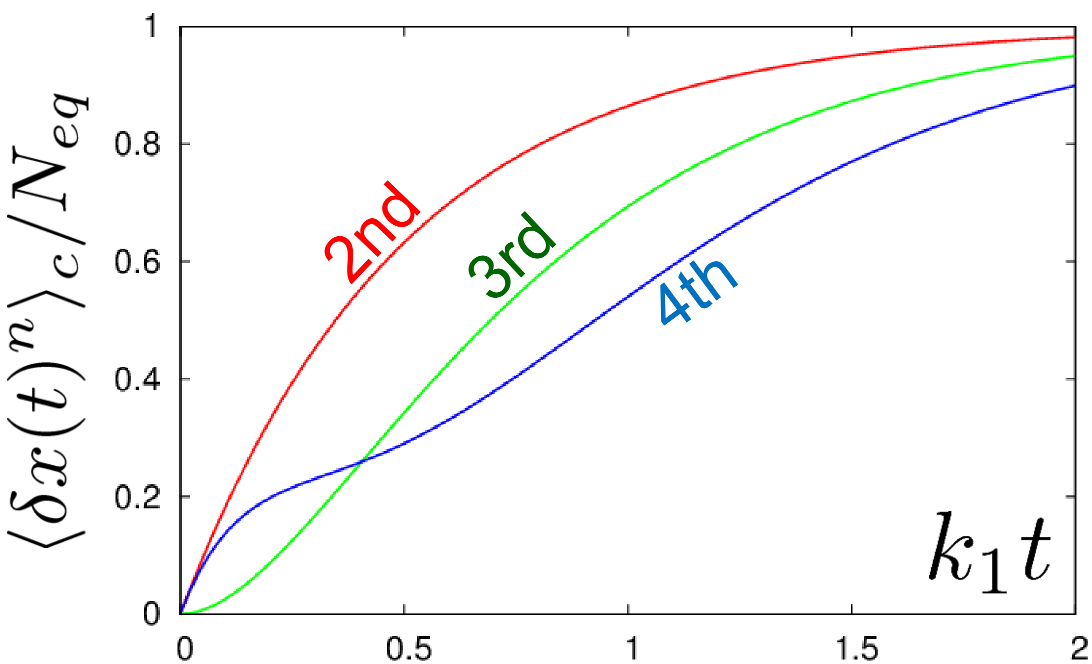
$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

$$N_0 = N_{eq}$$

$$\langle x(t) \rangle = N_{eq}$$

$$\langle \delta x(t)^2 \rangle = N_{eq}(1 - e^{-2k_1 t})$$

$$\langle \delta x(t)^3 \rangle = N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t})$$

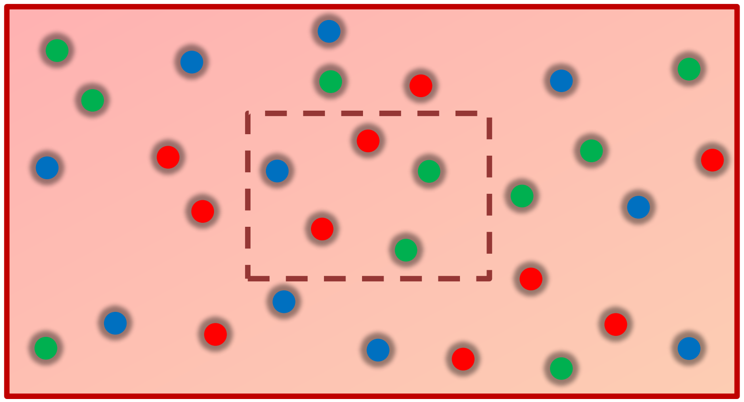


Higher-order cumulants grow slower.

Fluctuations

Free Boltzmann → Poisson

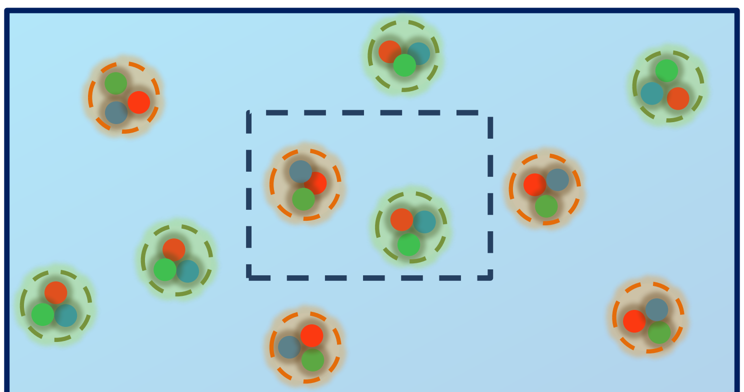
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$