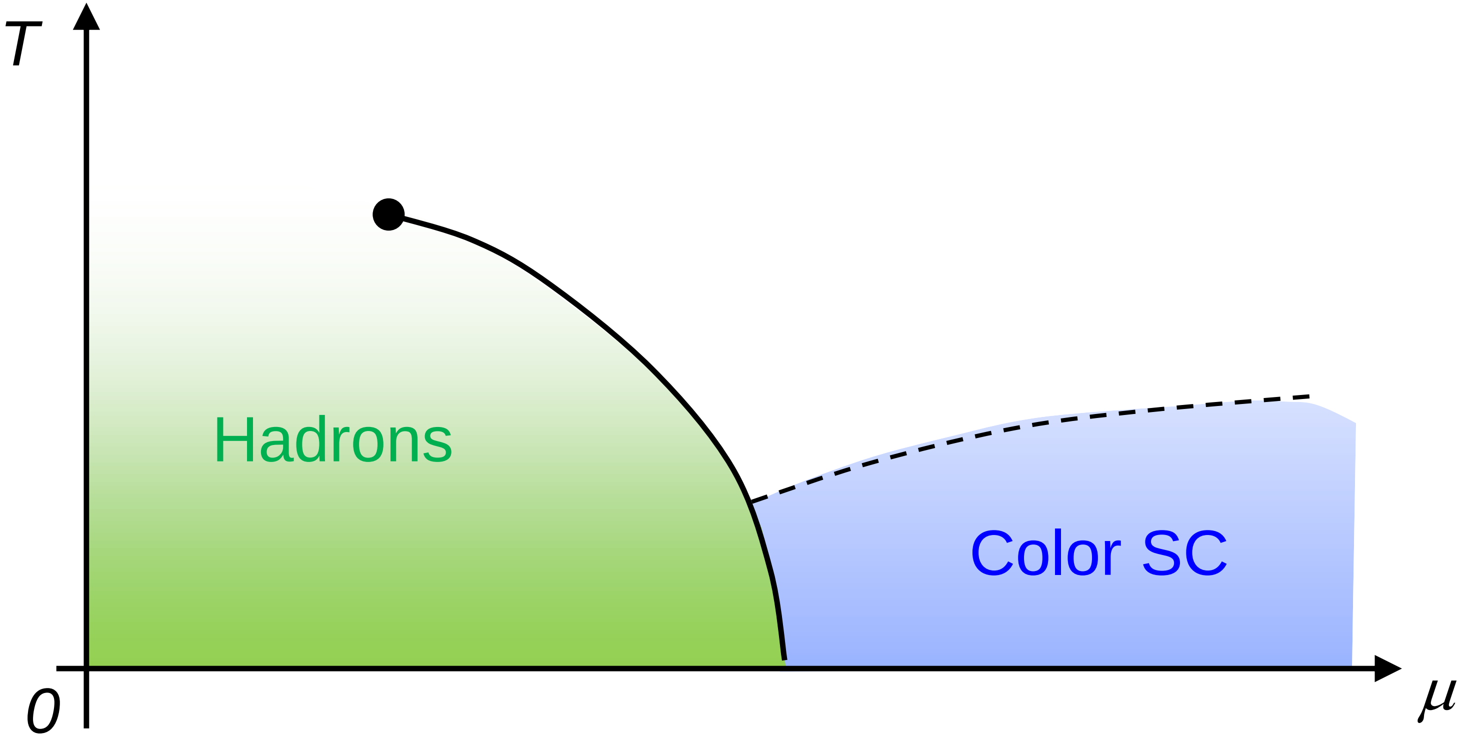
Time Evolution of Non-Gaussianity in Heavy-Ion Collisions

Masakiyo Kitazawa
(Osaka U.)

MK, Asakawa, Ono, in preparation

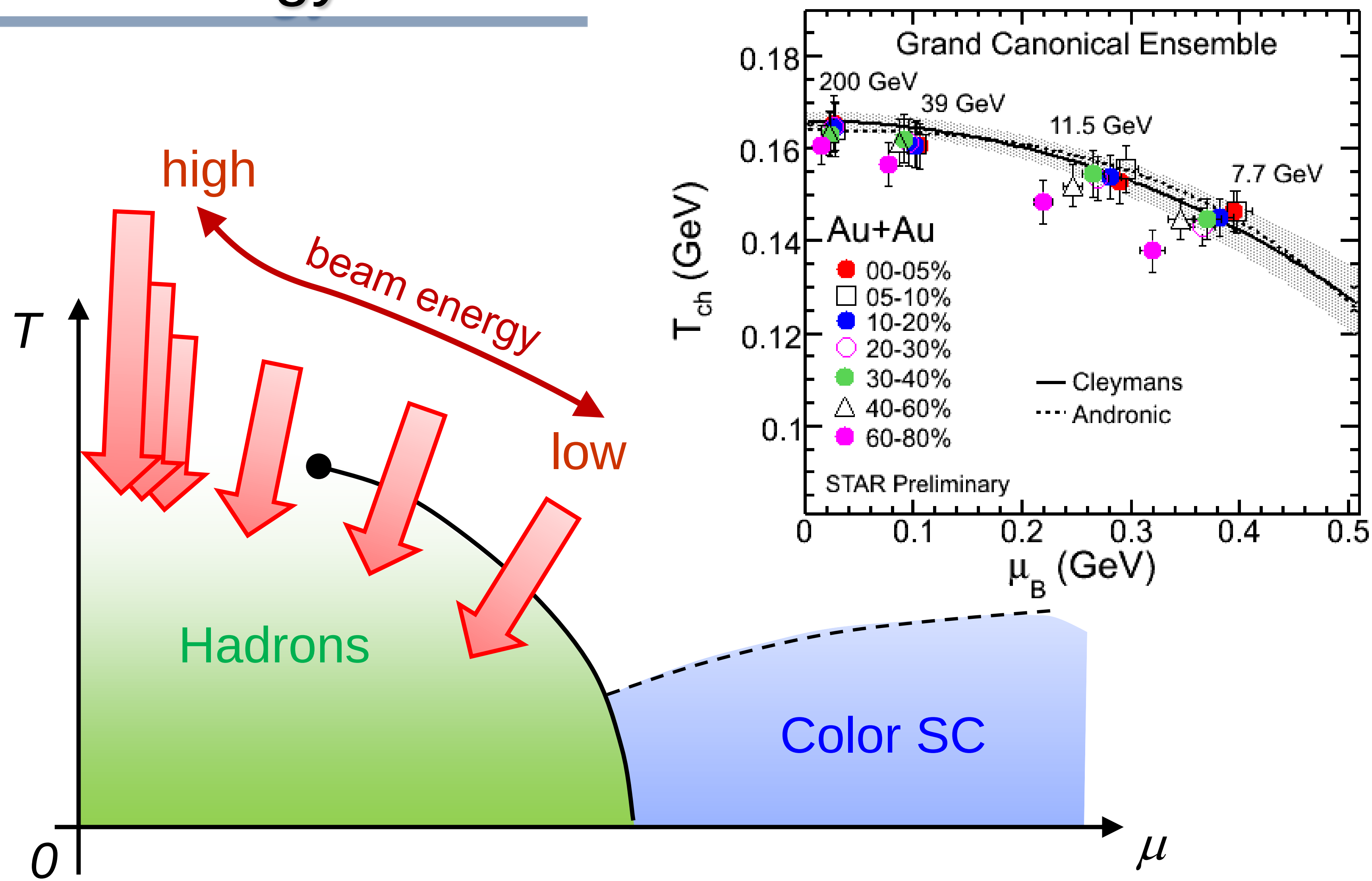
TIFR, 2/May/2013

Beam-Energy Scan



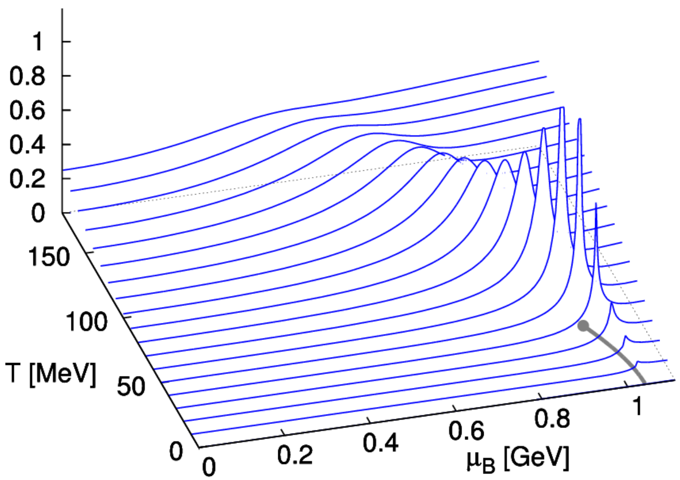
Beam-Energy Scan

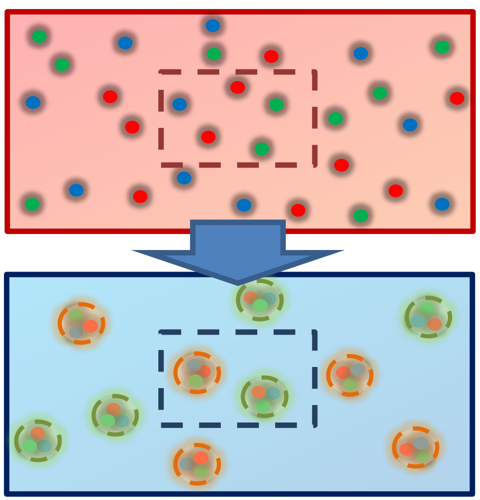
STAR 2012

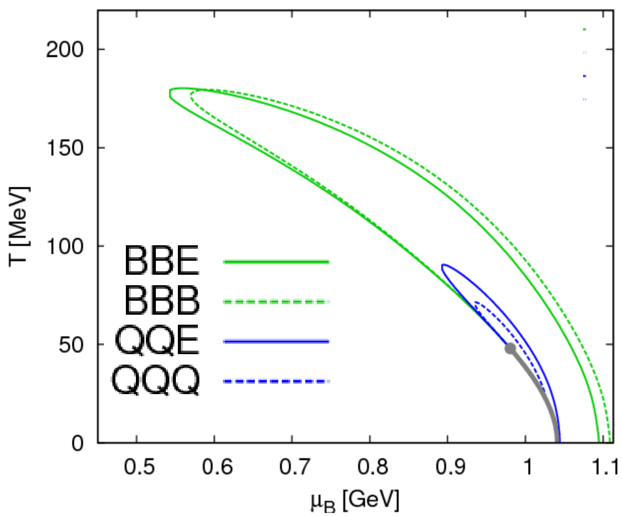


Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point
    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges
    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants
    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



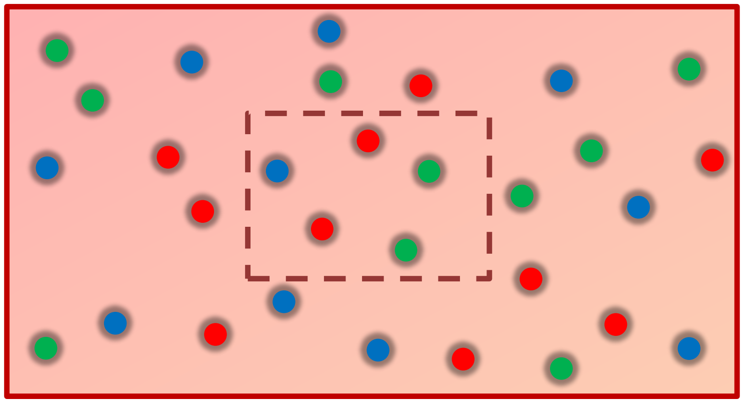




Fluctuations

Free Boltzmann → Poisson

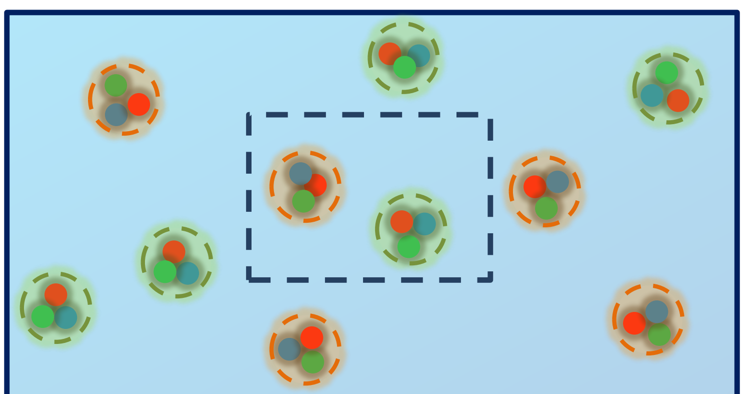
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

Fluctuations

Free Boltzmann → Poisson

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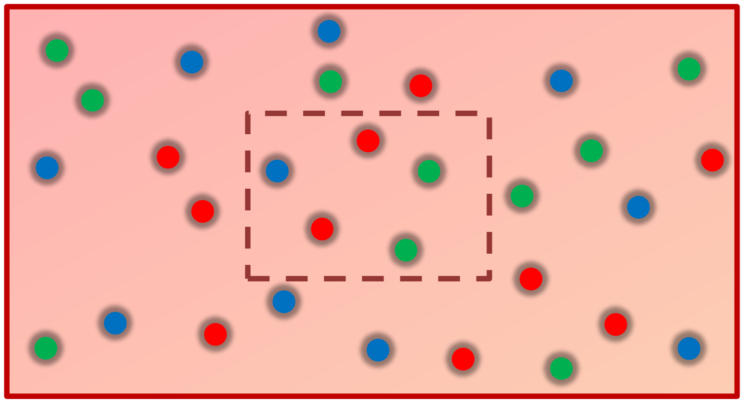

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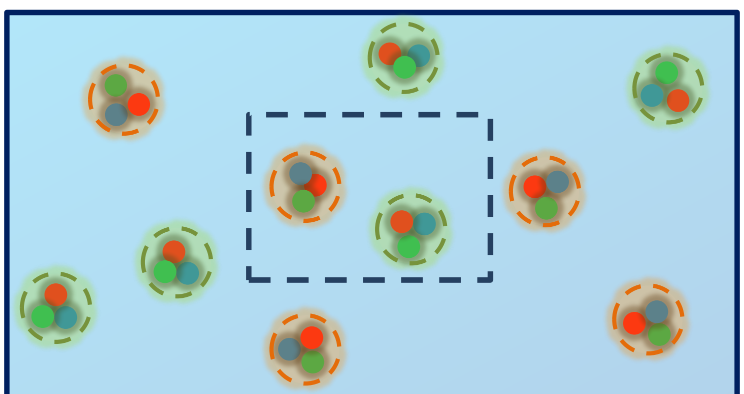
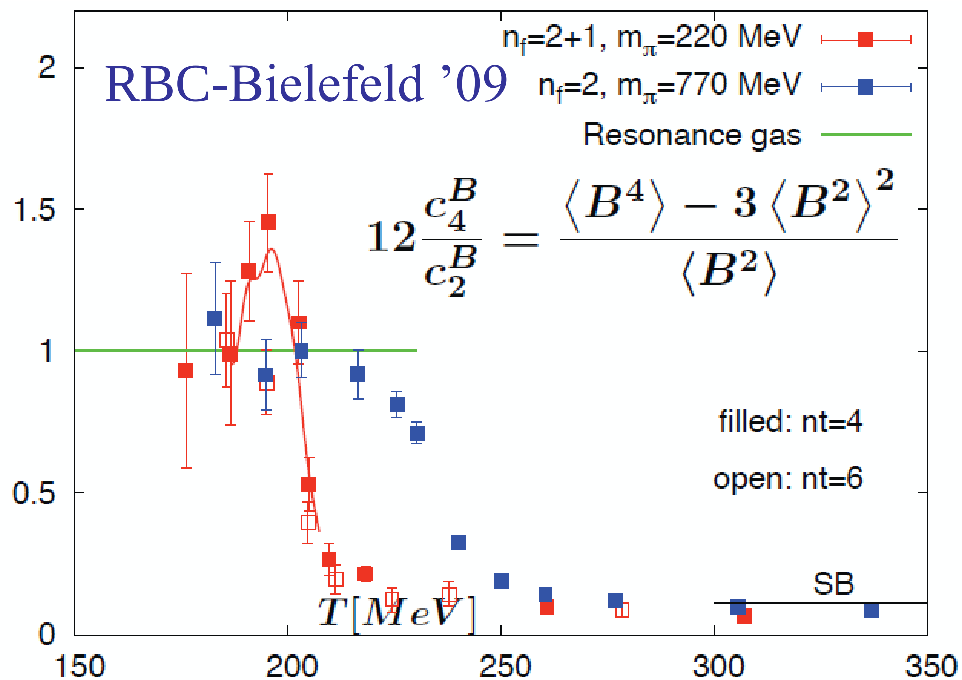
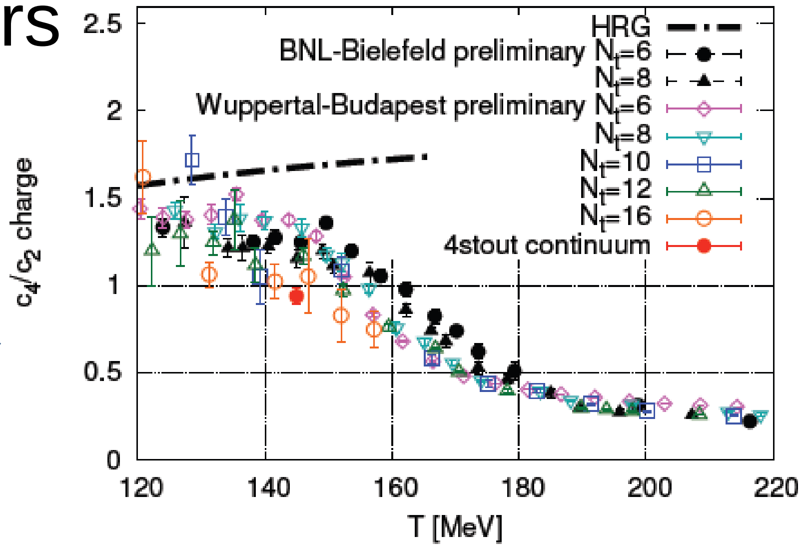




Conserved Charges : Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory

ex: on the lattice



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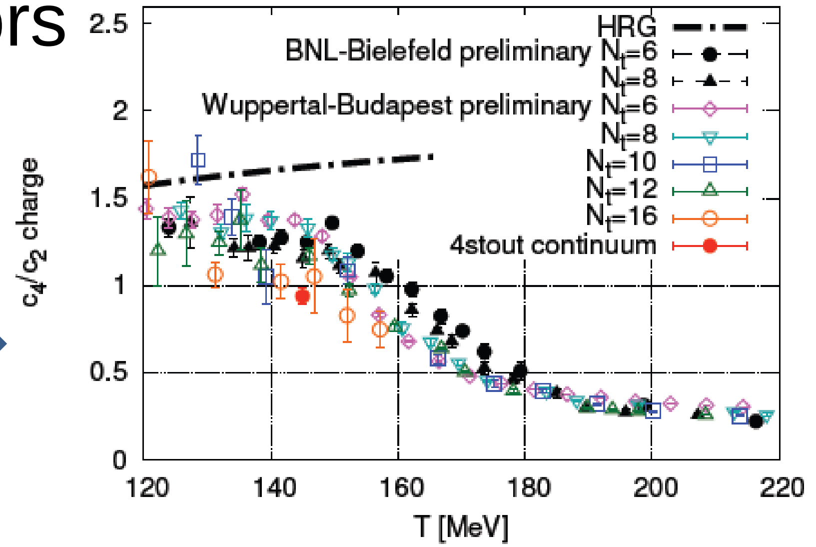


- Simple thermodynamic relations

$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

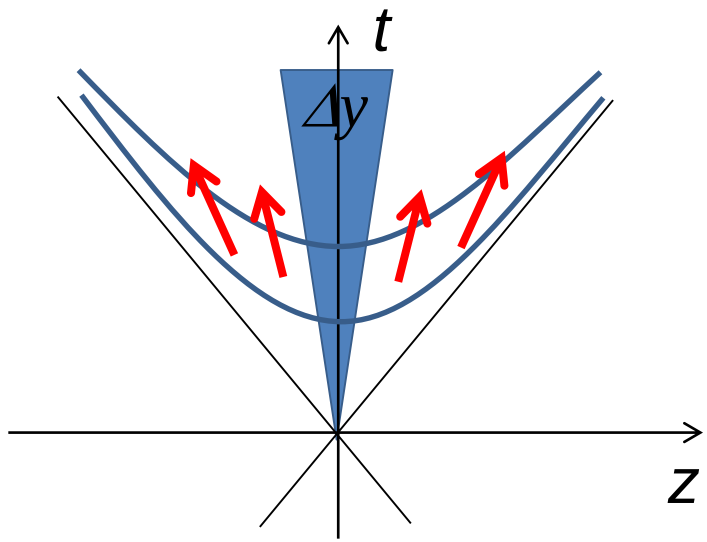
  - Intuitive interpretation for the behaviors of cumulants

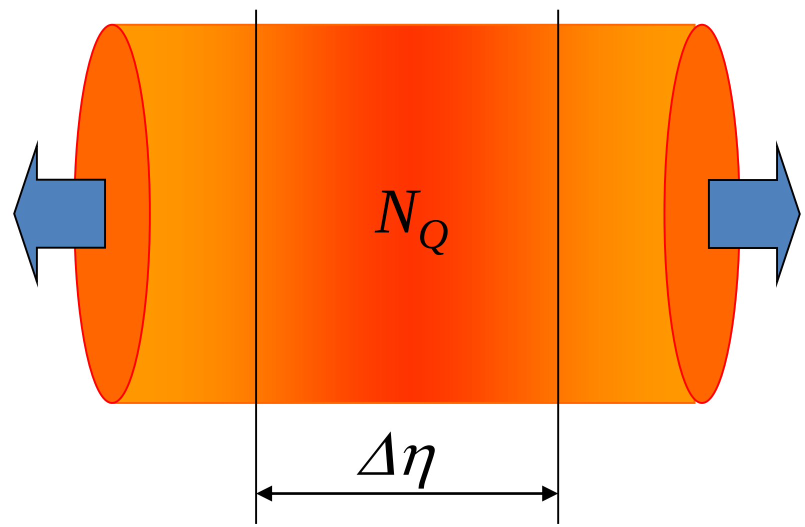
ex: $$\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$$

Fluctuations of Conserved Charges

- Under Bjorken expansion

Asakawa, Heintz, Muller, 2000
Jeon, Koch, 2000
Shuryak, Stephanov, 2001

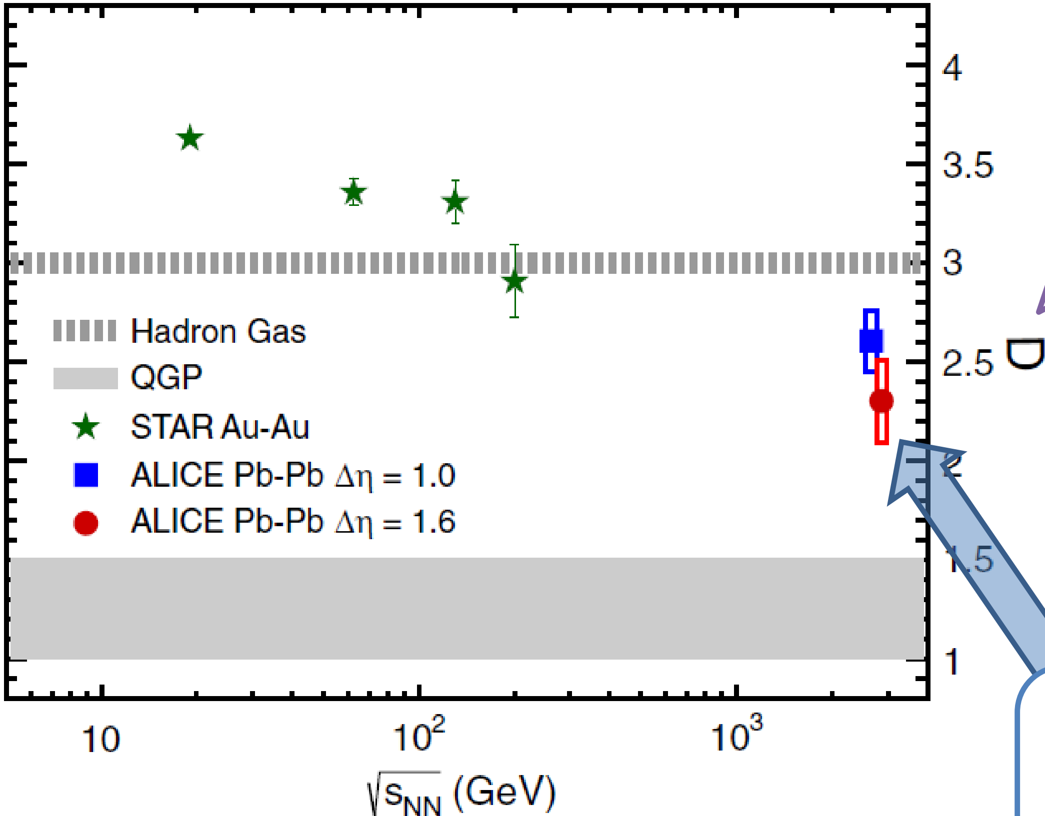




- Variation of a conserved charge in $\Delta\eta$ is **slow**, since it is achieved only through diffusion.

Primordial values can survive until freezeout.
The wider $\Delta\eta$, more earlier fluctuation.

Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

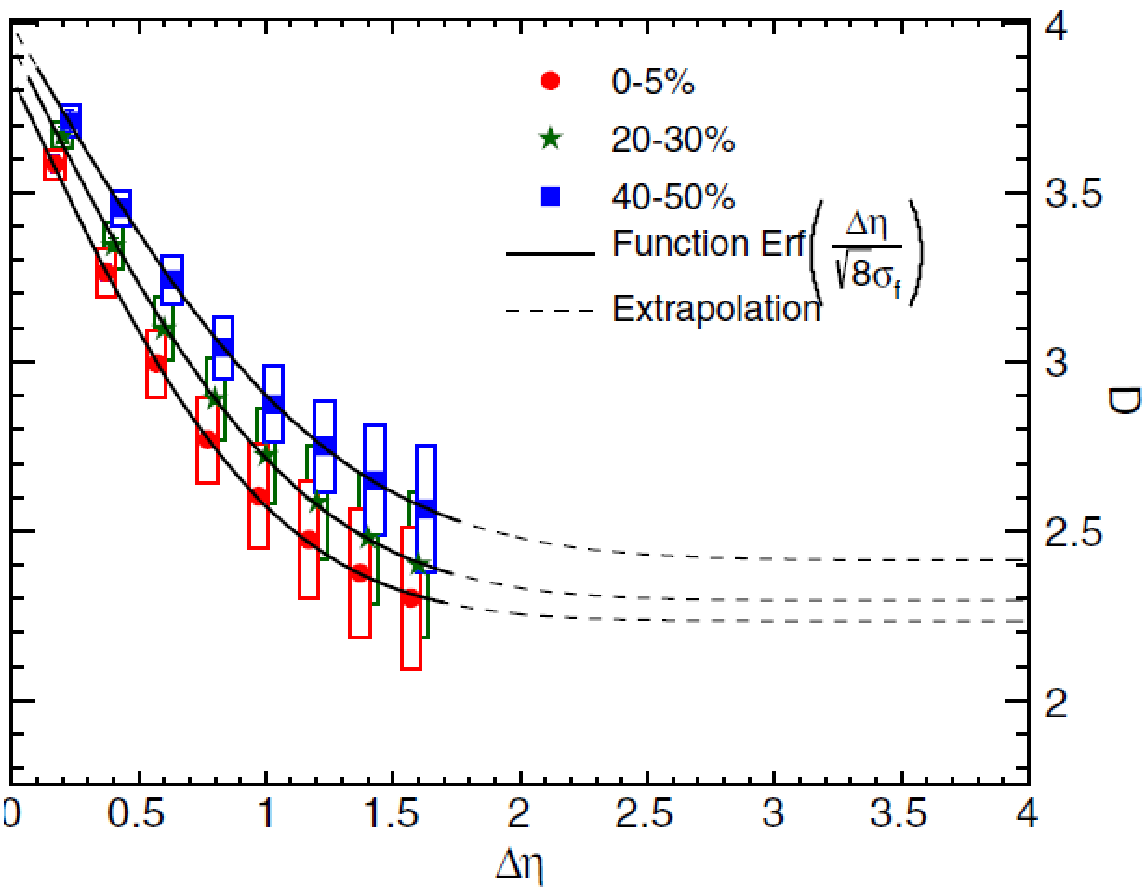
$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- D ~ 3-4 Hadronic
- D ~ 1 Quark

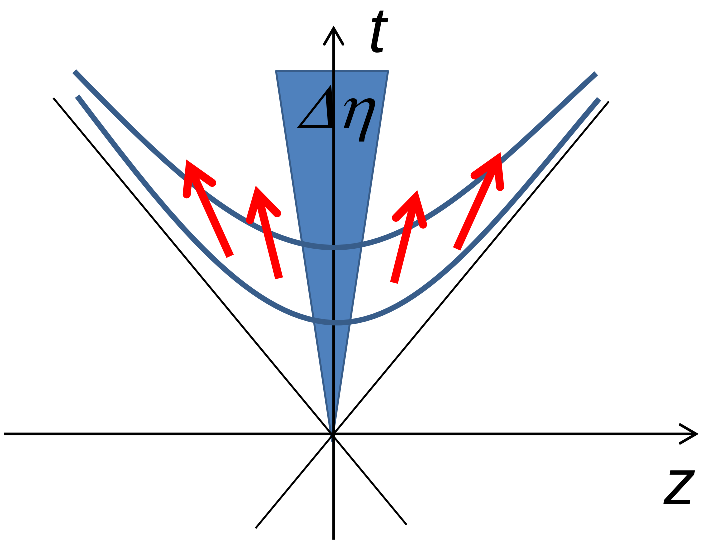
LHC:
significant suppression from hadronic value

$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE



ALICE
PRL 2013



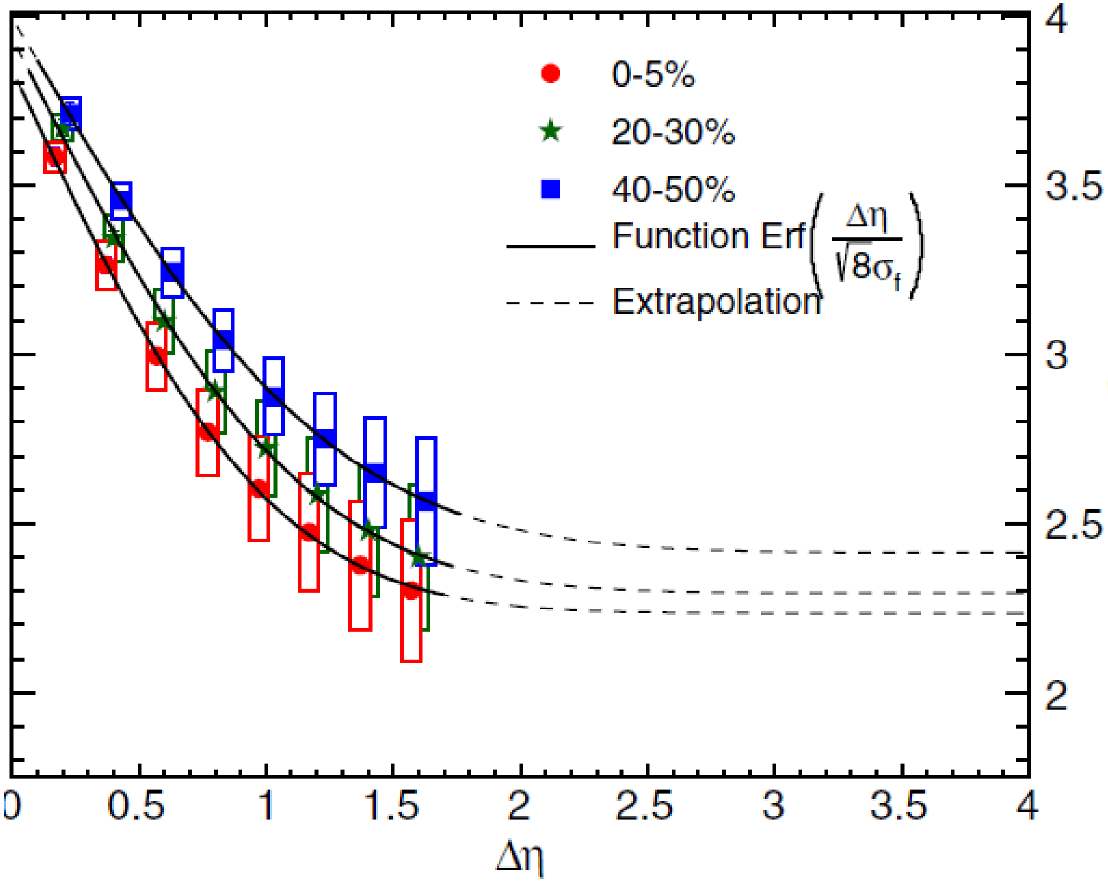
rapidity acceptane

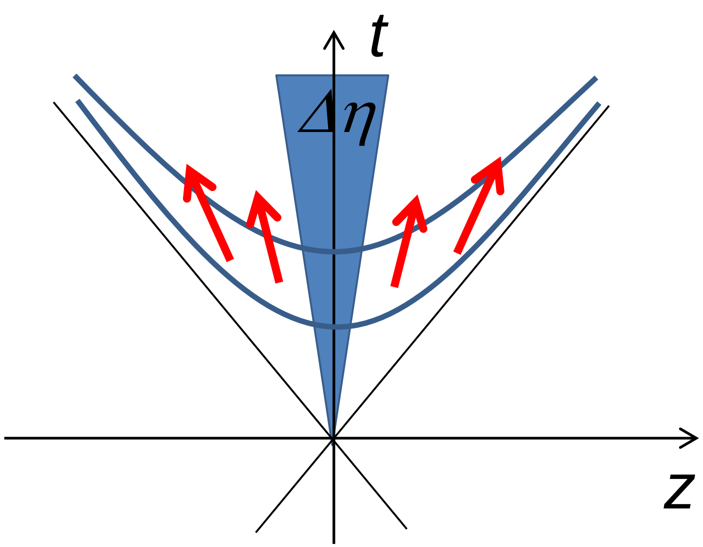
Higher-order cumulants as thermometer?

HotQCD,2012

Δη Dependence @ ALICE

ALICE
PRL 2013

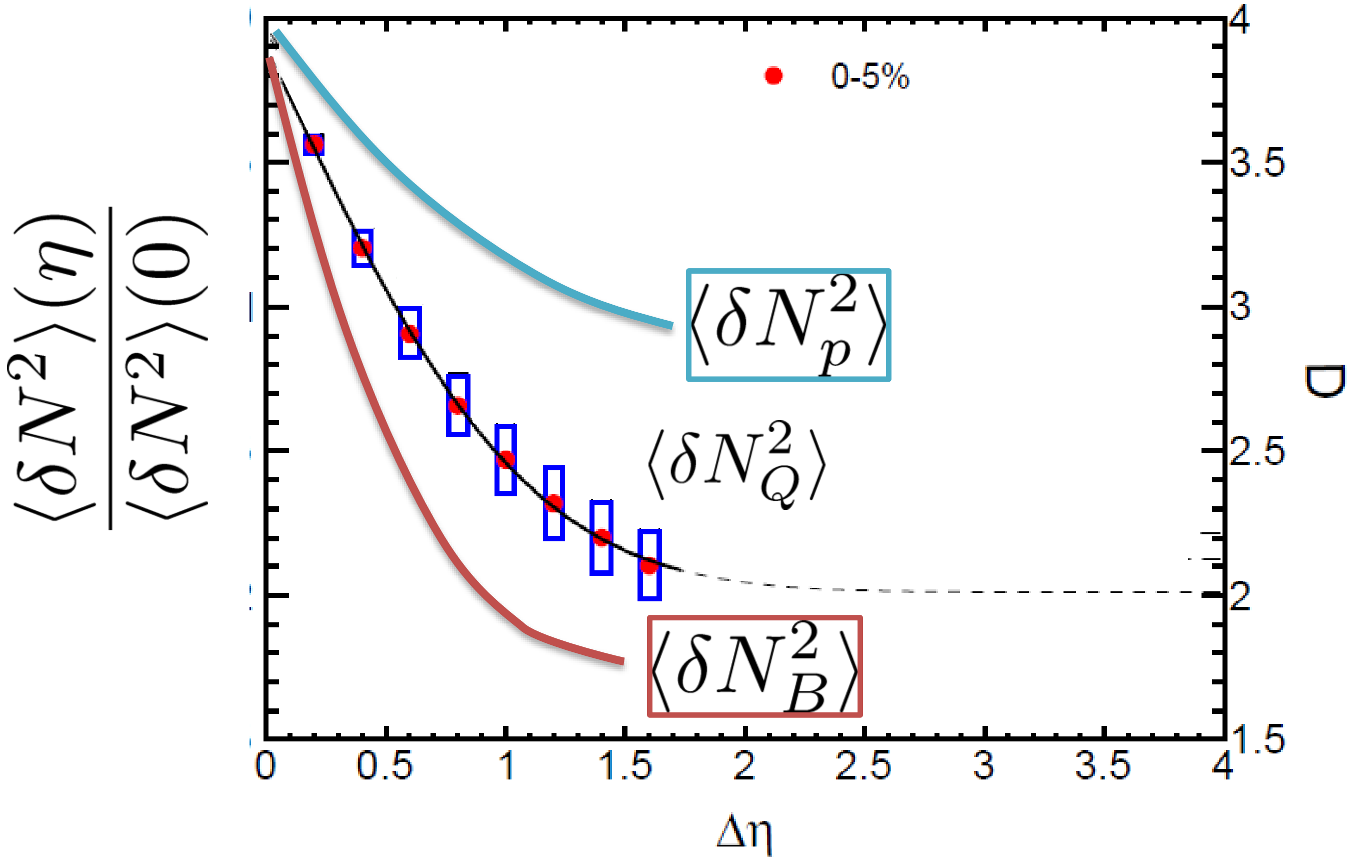




Δη dependences of fluctuation observables encode history of the hot medium!

$<\delta N_B^2>$ and $<\delta N_p^2>$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different Δη dependence.

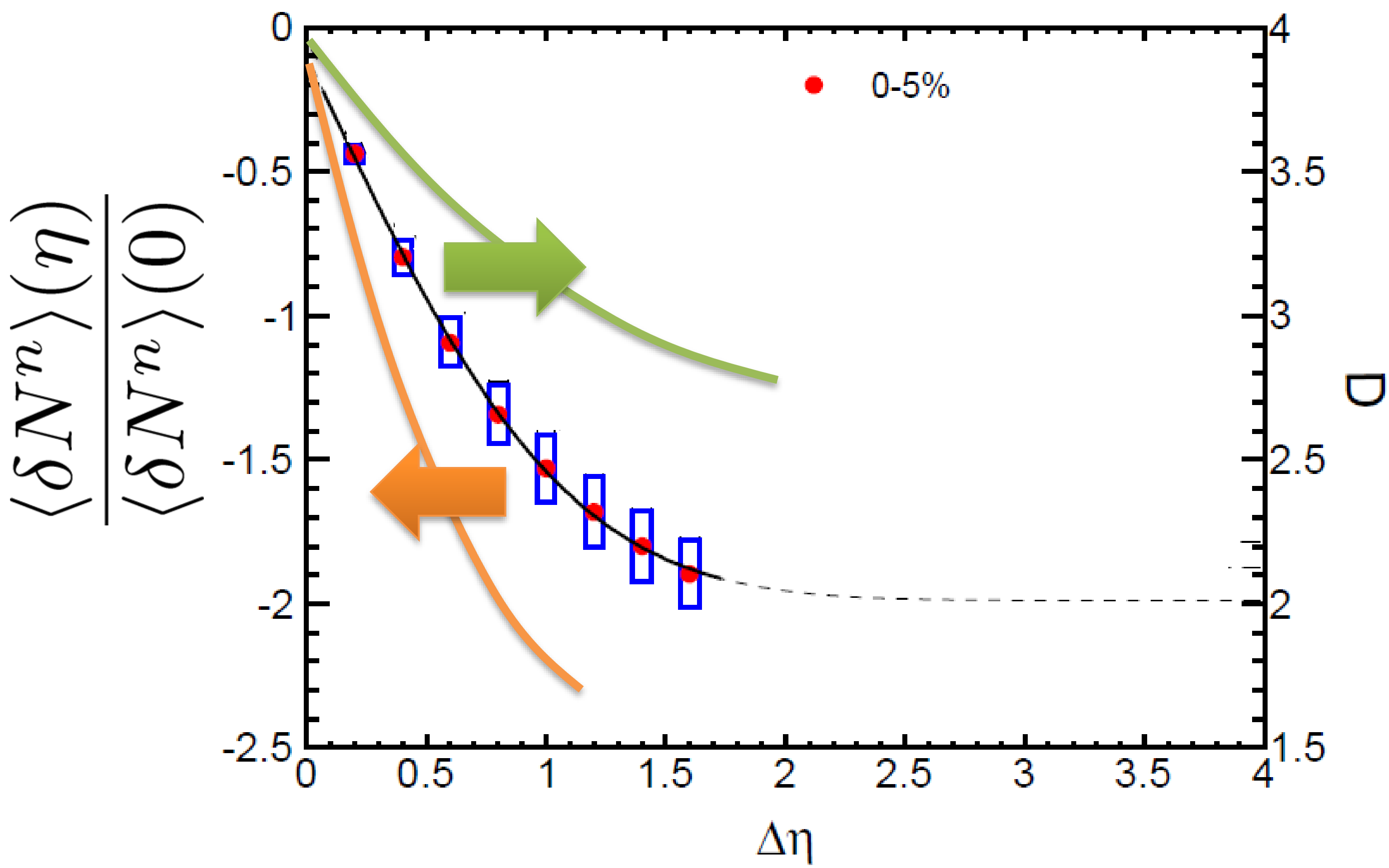


$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_\mathrm{B}^{(\mathrm{net})})^2 \rangle + \frac{1}{4} \langle N_\mathrm{B}^{(\mathrm{tot})} \rangle$$

$<\delta N_Q{}^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

Left (suppression) or Right (hadronic)



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

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$$\partial_\tau n = D\partial_\eta^2 n$$

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Conservation Law

$$\partial_\tau n = -\partial_\eta j$$

Fick's Law

$$j = -D\partial_\eta n + \xi$$

Fluctuation-Dissipation Relation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Stochastic force

- Local correlation (hydrodynamics) $\langle \xi(\eta_1, \tau_1)\xi(\eta_2, \tau_2)\rangle \sim \delta(\eta_1 - \eta_2)\delta(\tau_1 - \tau_2)$
- Equilibrium fluc. $\langle \delta Q(t)^2 \rangle \xrightarrow[t \to \infty]{} \chi_2 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

χ_2 : susceptibility

$$\langle \xi(k_1, \tau_1)\xi(k_2, \tau_2)\rangle = \frac{2\chi_2}{D}\delta(k_1 + k_2)\delta(\tau_1 - \tau_2)$$

$\Delta\eta$ Dependence

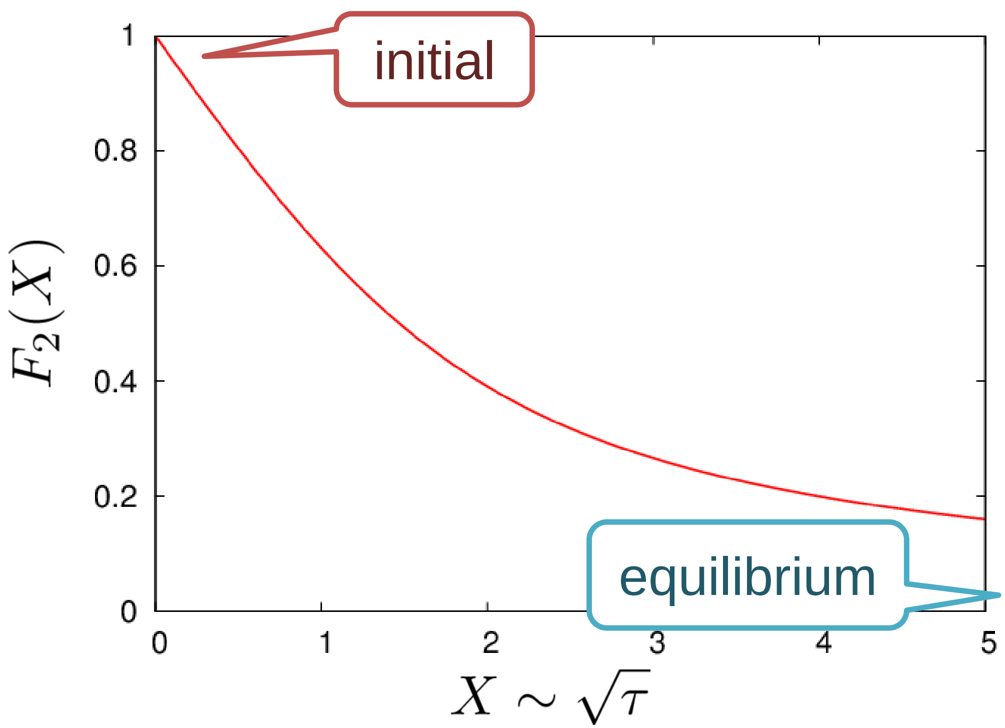
Shuryak, Stephanov, 2001

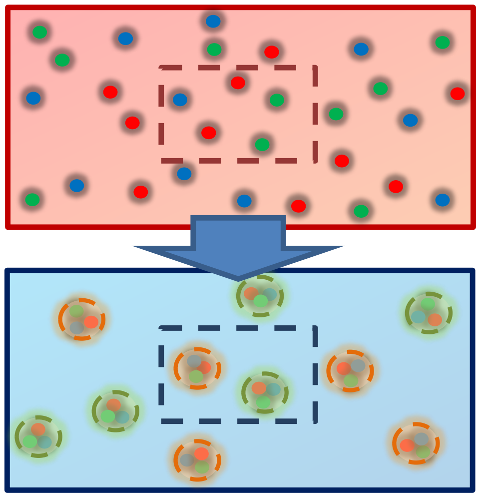
- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2 (1 - F_2(X))}_{\text{equilibrium}}$$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta\, n(\eta, \tau)$$

$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$





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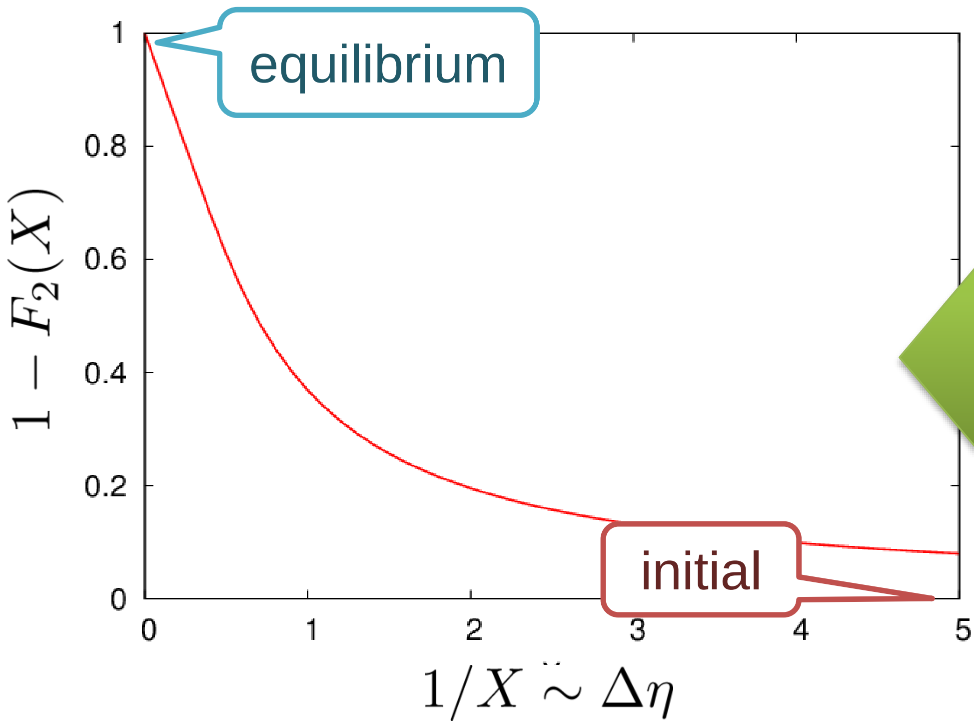
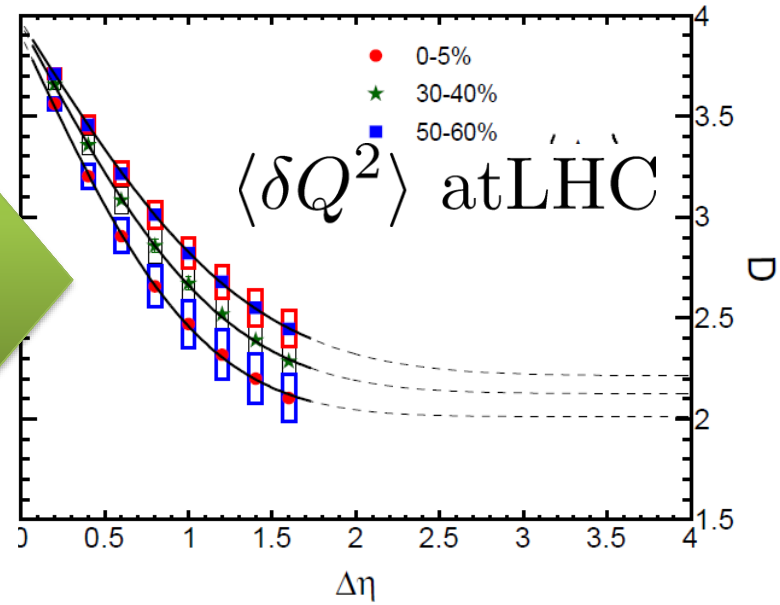




Non-Gaussian Stochastic Force ??

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Stochastif Force : 3rd order

- Local correlation $\langle \xi(\eta_1, \tau_1)\xi(\eta_2, \tau_2)\xi(\eta_3, \tau_3)\rangle$
  (hydrodynamics)
  $$\sim \delta(\eta_1 - \eta_2)\delta(\eta_2 - \eta_3)\delta(\tau_1 - \tau_2)\delta(\tau_2 - \tau_3)$$
- Equilibrium fluc. $\langle \delta Q(t)^3 \rangle \underset{t \to \infty}{\longrightarrow} \chi_3 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

χ_3 : third − moment

Caution!

- $$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\xi(k_3,\tau_3)\rangle = \frac{\chi_3}{\gamma}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\delta(k_1+k_2+k_3)$$
$$\times\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$

diverge in long wavelength

- No a priori extension of FD relation to higher orders

Caution!

- $$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\xi(k_3,\tau_3)\rangle = \frac{\chi_3}{\gamma}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\delta(k_1+k_2+k_3) \times\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$

  diverge in long wavelength

- No a priori extension of FD relation to higher orders

- Theorem

  Markov process＋continuous variable

  ➔Gaussian random force

  cf) Gardiner, "Stochastic Methods"

- Hydrodynamics ➡ Local equilibrium with many particles

  ➡ Gaussian due to central limit theorem

Thee “NON”s

Physics of non-Gaussianity in heavy-ion collisions is a particular problem.

- **Non-Gaussian**

  Non-Gaussianitiy is irrelevant in large systems

- **Non-critical**

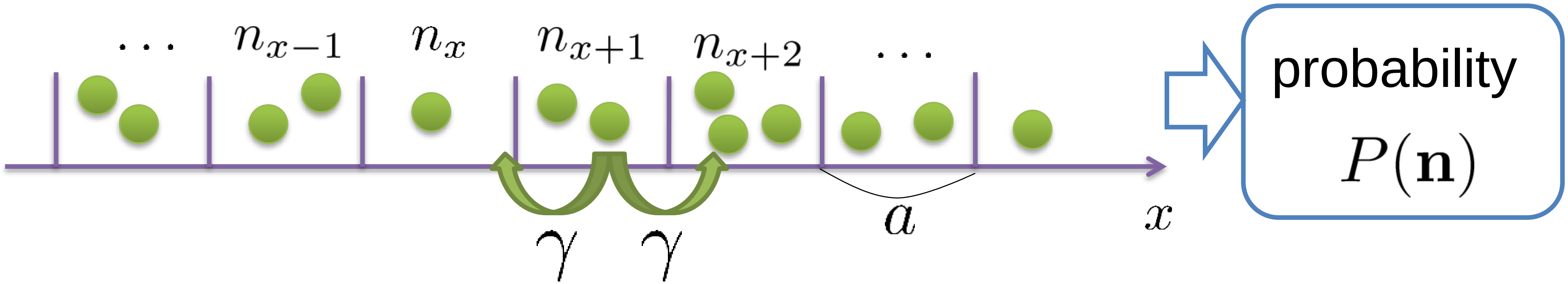
  fluctuations observed so far do not show critical enhancement

- **Non-equilibrium**

  Fluctuations are not equilibrated

Diffusion Master Equation

Divide spatial coordinate into discrete cells



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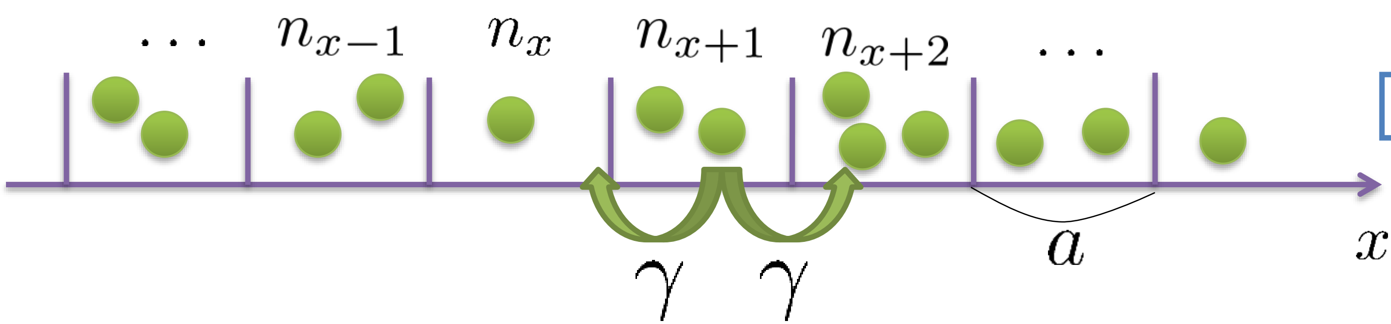


probability

$P(\mathbf{n})$

Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\left\{P(\mathbf{n}+\hat{x}-\widehat{x+1}) + P(\mathbf{n}+\hat{x}-\widehat{x-1})\right\} - 2n_x P(\mathbf{n})]$$

x-hat: lattice-QCD notation

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen's system size expansion

Solution of DME

1st $\langle \tilde{n}_k \rangle (t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$ $\omega_k \simeq \gamma a^2 k^2$

initial

- Deterministic part follows diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

- Appropriate continuum limit with $\gamma a^2 = D$

Solution of DME

1st $\langle \tilde{n}_k \rangle(t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$ $\qquad \omega_k = \gamma a^2 k^2$

(initial)

- Deterministic part follows diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

- Appropriate continuum limit with $\gamma a^2 = D$

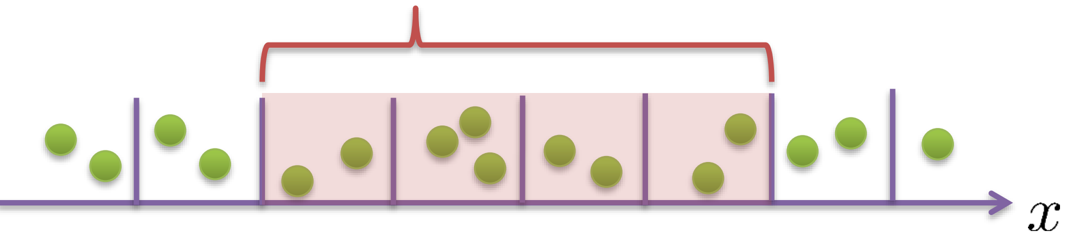
2nd

$$\langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle(t) = \langle \tilde{n}_{k_1+k_2} \rangle_0 \left(e^{-\omega_{k_1+k_2} t} - e^{-(\omega_{k_1}+\omega_{k_2})t}\right) + \langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle_0 e^{-(\omega_{k_1}+\omega_{k_2})t}$$

- Consistent with stochastic diffusion eq. for sufficiently slowly-varying initial condition.

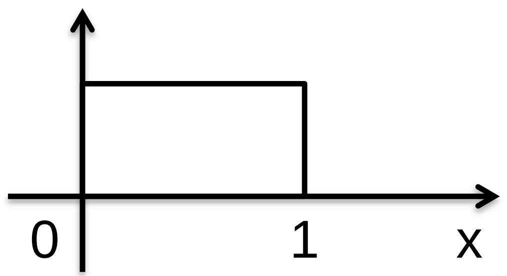
Total Charge in $\Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$



$$I(z, X) = \int \frac{dk}{2\pi} e^{-Xk^2} \int_0^1 dx e^{ik(x+z)}$$

FT of step func.



$$\langle Q\rangle(\tau) = \int dz \langle n(z)\rangle_0 I_{\Delta\eta}(z, X)$$

$$\langle \delta Q^2\rangle(\tau) = \int dz_1 dz_2 \langle \delta n(z_1)\delta n(z_2)\rangle_0 I_{\Delta\eta}(z_1, X) I_{\Delta\eta}(z_2, X)$$

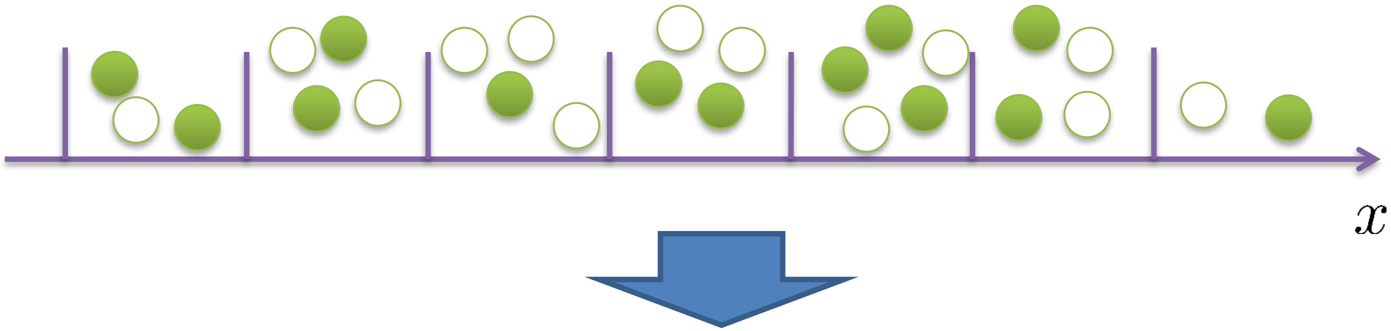
$$+ \int dz \langle n(z)\rangle_0 (I_{\Delta\eta}(z, X) - I_{\Delta\eta}^2(z, X))$$

$$\langle \delta Q^3\rangle(t) = \cdots$$

$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$

Net Charge Number

Prepare 2 species of (non-interacting) particles



$$\bar{Q} = \int_0^{\Delta\eta} d\eta \, (n_1(\eta) - n_2(\eta))$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ at freezeout time t

Initial Condition at Hadronization

- Boost invariance / infinitely long system
- Local equilibration / local correlation
- Initial fluctuations

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ $\quad \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\quad \langle Q^2_{(\mathrm{tot})} \rangle_c$

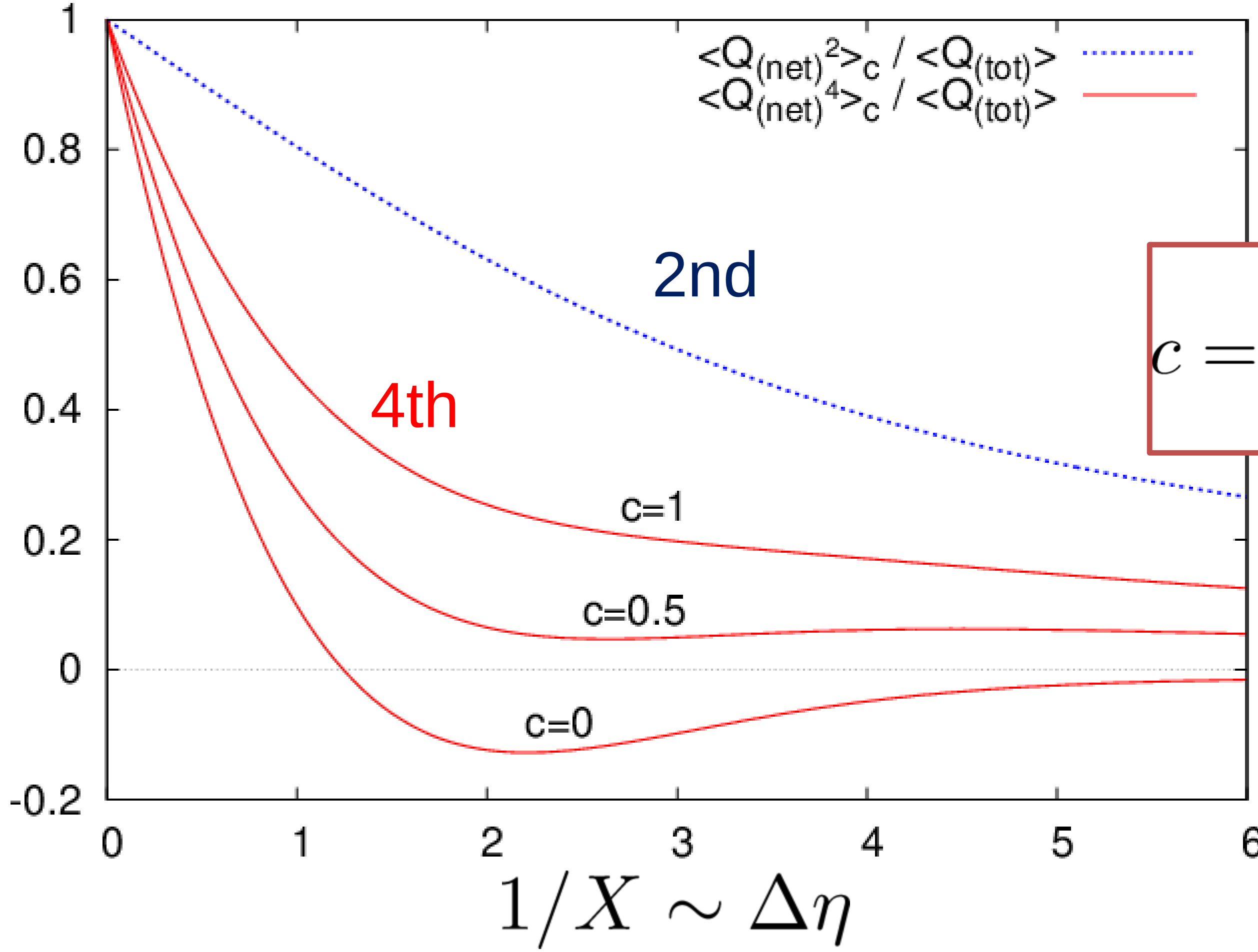
suppression owing to local charge conservation

strongly dependent on hadronization mechanism

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

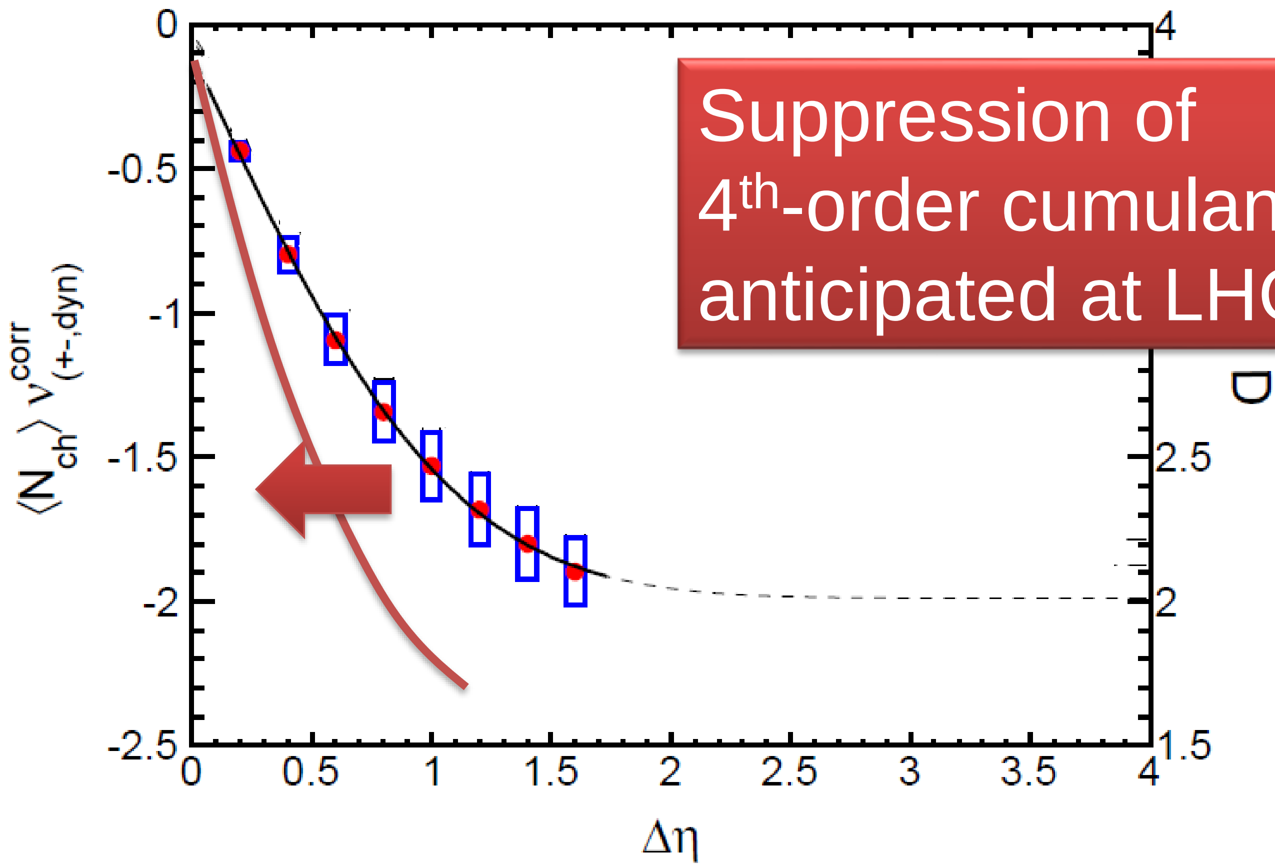
$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$<\delta N_Q{}^4>$ @ LHC

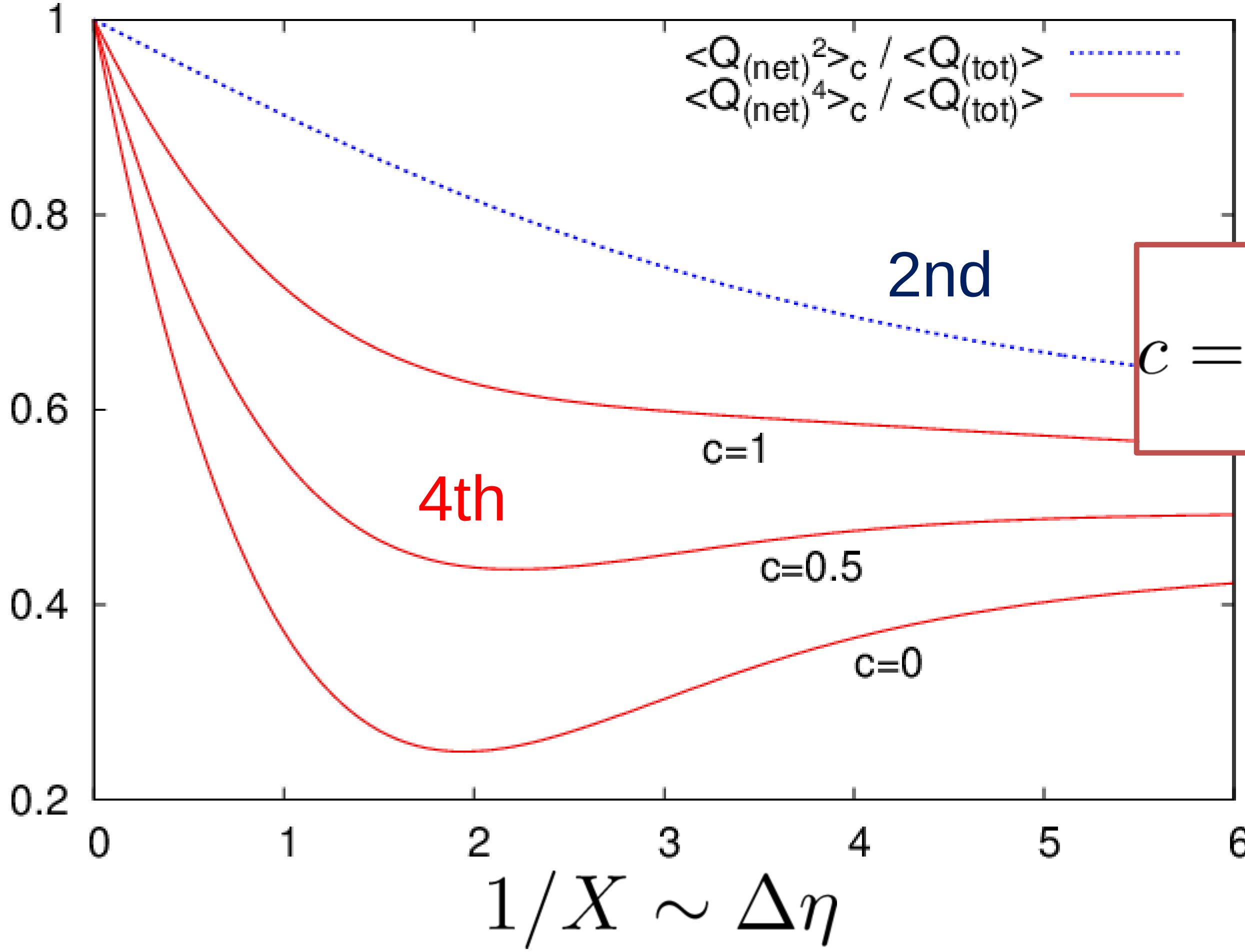
Assumptions
- boost invariant system
- tiny fluctuations of CC at hadronization
- short correlaition in hadronic stage



$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0.5 \langle Q_{(\mathrm{tot})} \rangle$$



$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

Summary

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$\langle N_Q^2 \rangle_c, \ \langle N_B^2 \rangle_c, \ \langle N_Q^4 \rangle_c, \ \langle N_B^4 \rangle_c,$
$\langle N_{ch}^2 \rangle_c, \cdots$

Physical meanings of fluctuation obs. in experiments.

Diagnozing dynamics of HIC

- history of hot medium
- mechanism of hadronization
- diffusion constant

Summary

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$\langle N_Q^2 \rangle_c,\ \langle N_B^2 \rangle_c,\ \langle N_Q^4 \rangle_c,\ \langle N_B^4 \rangle_c,\ \langle N_{ch}^2 \rangle_c, \cdots$

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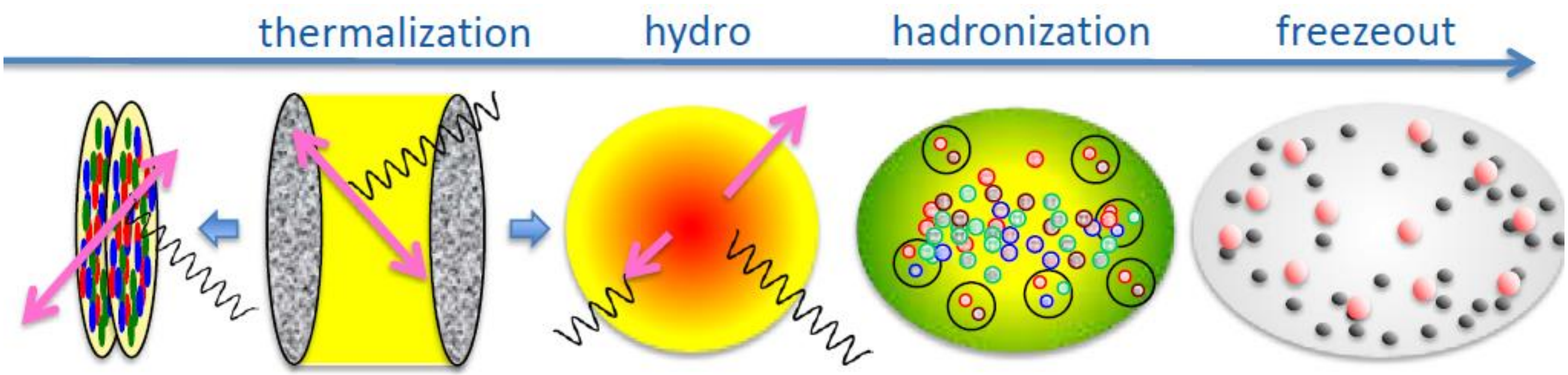
Search of QCD Phase Structure

Open Questions & Future Work

- Why the primordial fluctuations are observed only at the LHC, and not the RHIC ?
- Extract more information on each stage of fireballs using fluctuations

- Model refinement
  - Including the effects of
    nonzero correlation length / relaxation time
    global charge conservation
  - Non Poissonian system ← interaction of particles

Evolution of Fluctuations



Fluctuation in initial state

Time evolution in the QGP

approach to HRG by diffusion

volume fluctuation

experimental effects
particle missID, etc.

Chemical Reaction 1

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

x: # of X
a: # of A (**fixed**)

Master eq.: $$\frac{\partial}{\partial t}P(x,t) = k_2 a P(x-1,t) + k_1(x+1)P(x+1,t) - (k_1 x + k_2 a)P(x,t)$$

Cumulants with fixed initial condition $P(x,0) = \delta_{x,N_0}$

$$\langle x(t)\rangle = N_0 e^{-k_1 t} + N_{eq}(1 - e^{-k_1 t})$$

$$\langle \delta x(t)^2\rangle = N_0(e^{-k_1 t} - e^{-2k_1 t}) + N_{eq}(1 - e^{-k_1 t})$$

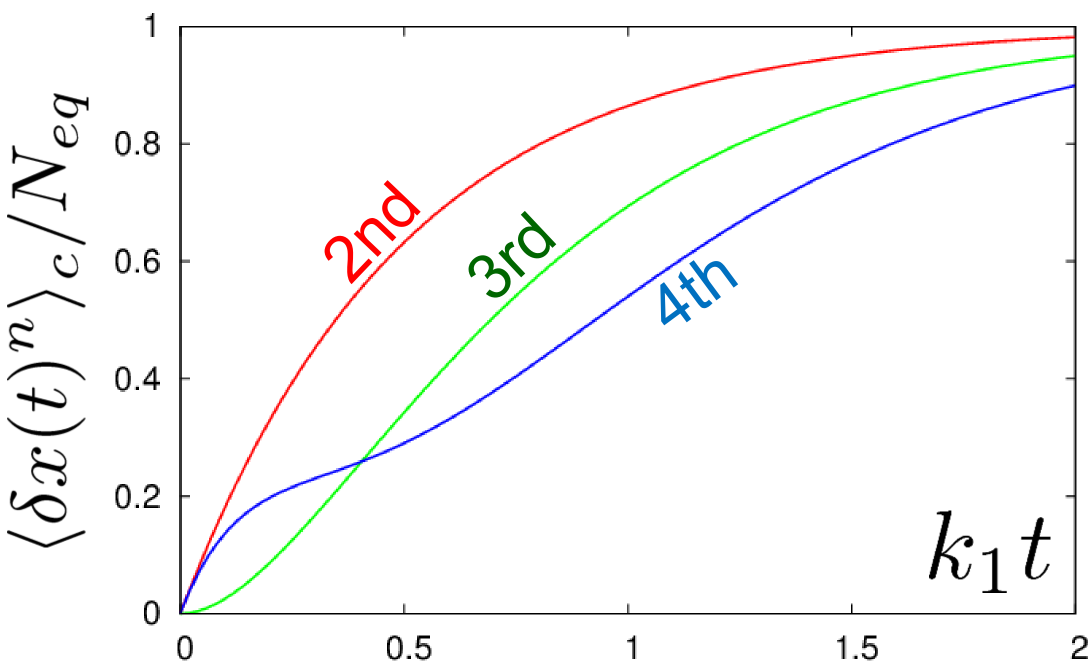
$$\langle \delta x(t)^3\rangle = N_0(e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t}) + N_{eq}(1 - e^{-k_1 t})$$

initial equilibrium

Chemical Reaction 2

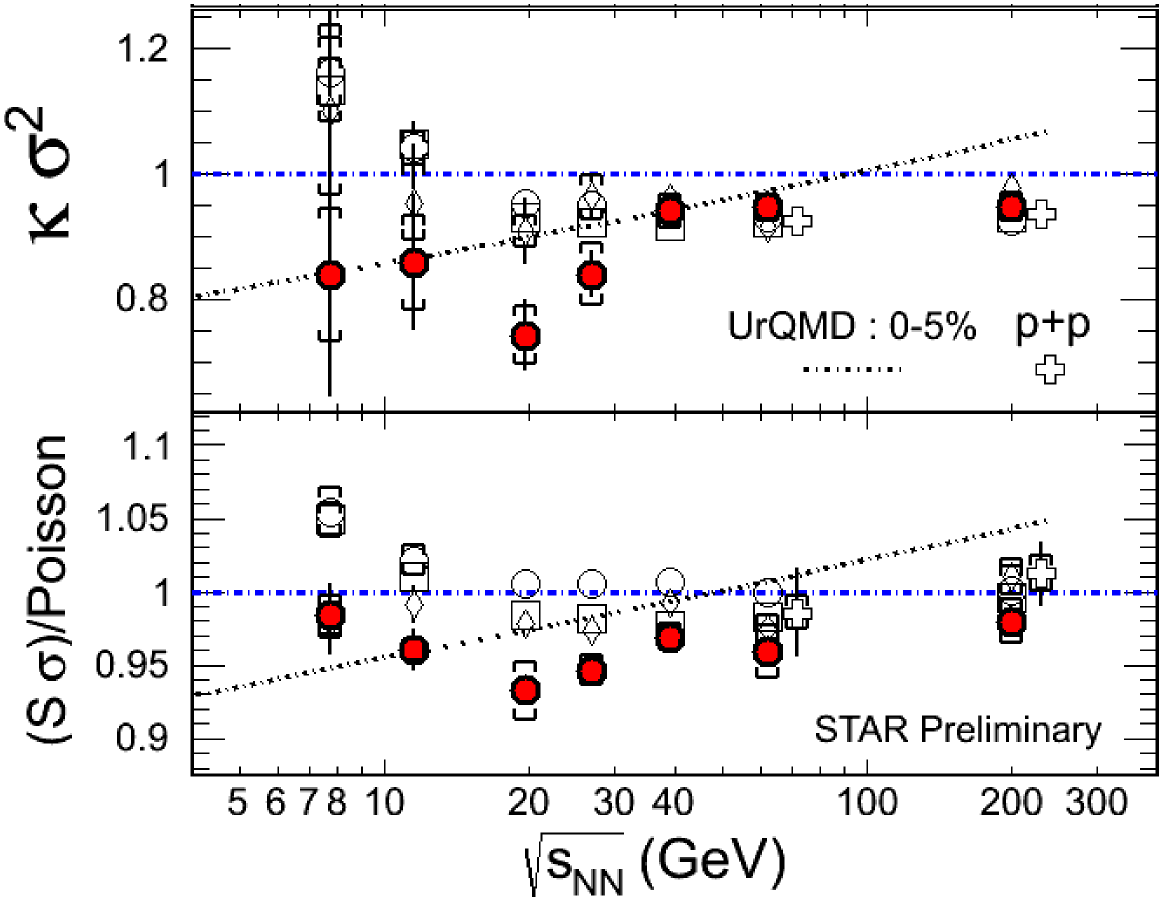
$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

$$N_0 = N_{eq} \Rightarrow \begin{cases} \langle x(t) \rangle = N_{eq} \\ \langle \delta x(t)^2 \rangle = N_{eq}(1 - e^{-2k_1 t}) \\ \langle \delta x(t)^3 \rangle = N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t}) \end{cases}$$



Higher-order cumulants grow slower.

Proton # Cumulants @ STAR-BES



STAR,QM2012

$$\frac{C_4}{C_2}$$

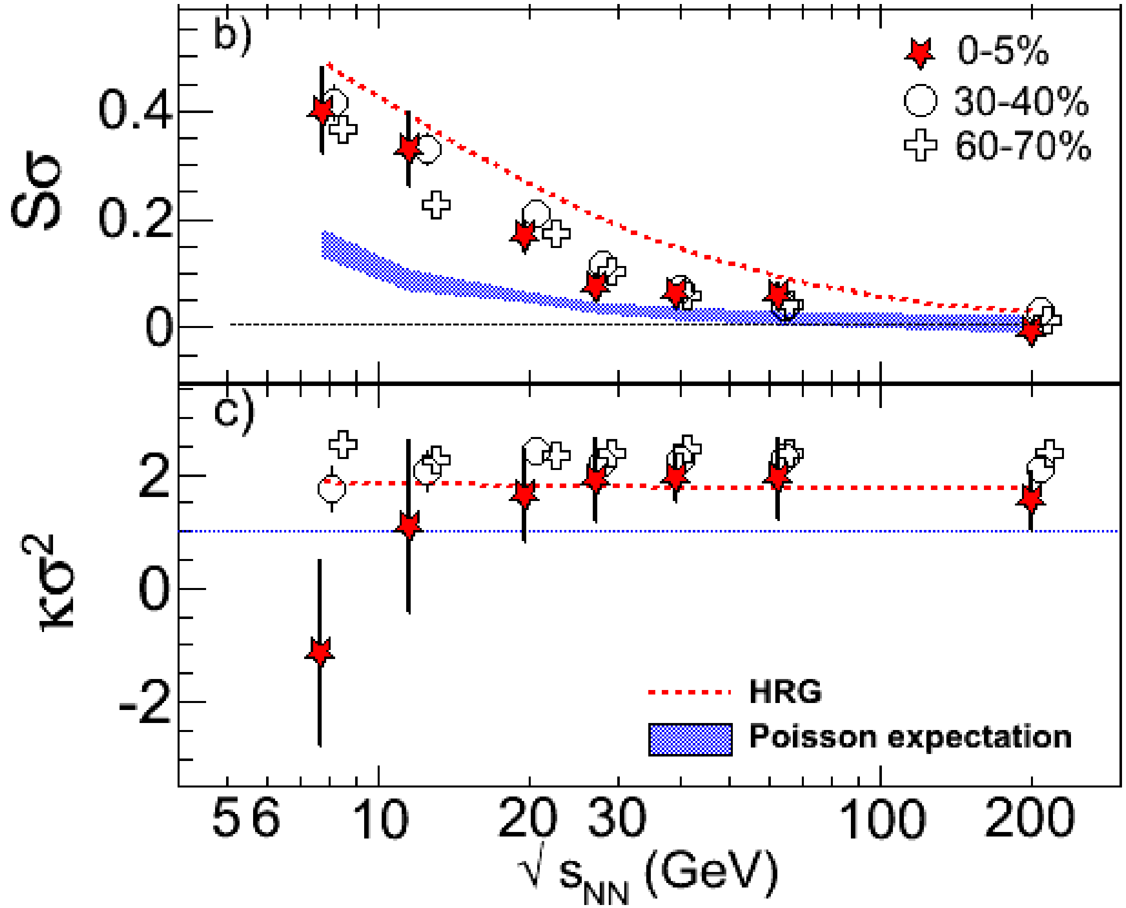
$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

No characteristic signals on
phase transition to QGP nor QCD CP

Charge Fluctuations @ STAR-BES

$$\frac{\langle \delta N_Q^3 \rangle}{\langle \delta N_Q^2 \rangle}$$

$$\frac{\langle \delta N_Q^4 \rangle_c}{\langle \delta N_Q^2 \rangle}$$



STAR, QM2012

No characteristic signals on
phase transition to QGP nor QCD CP

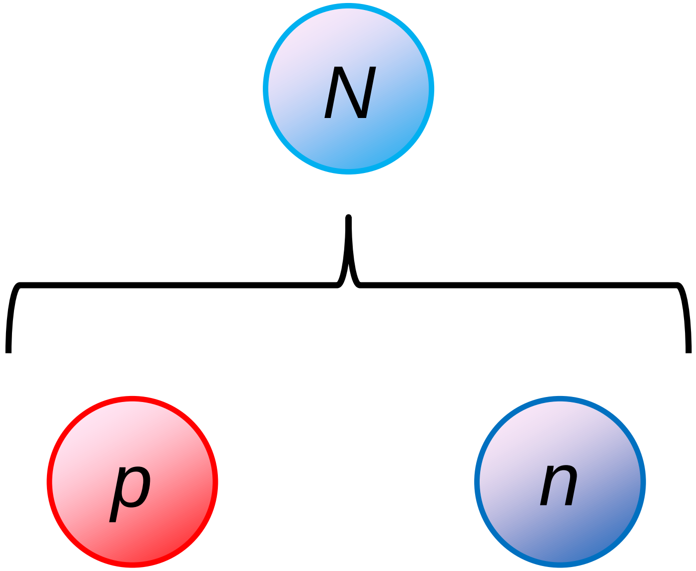
Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85,021901C(2012); PRC86 , 024904(2012)

- $\dfrac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} \neq \dfrac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c}$

- $\langle \delta N_B^n \rangle_c$ are experimentally observable

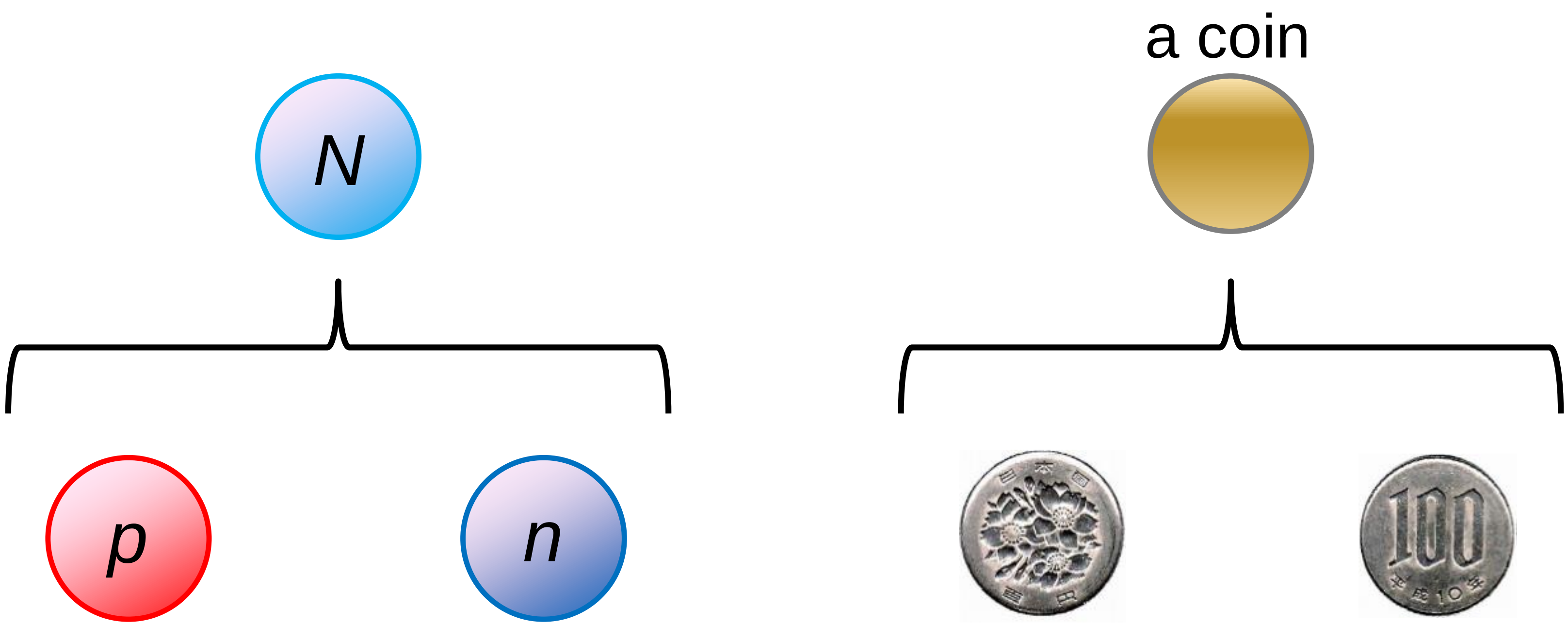
Nucleon Isospin as Two Sides of a Coin



Nucleons have
two isospin states.

MK, Asakawa,2012

Nucleon Isospin as Two Sides of a Coin

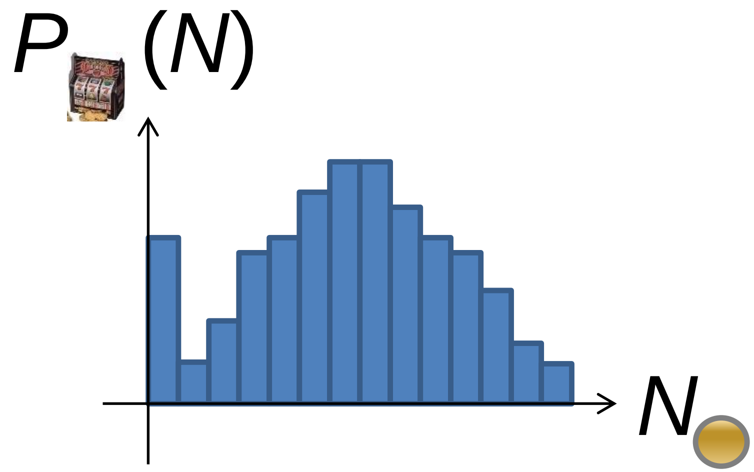


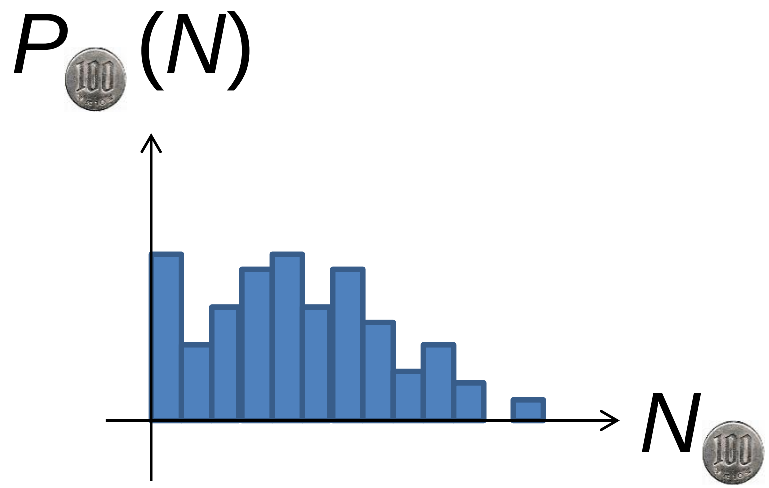
Nucleons have
two isospin states.

Coins have two sides.

MK, Asakawa,2012

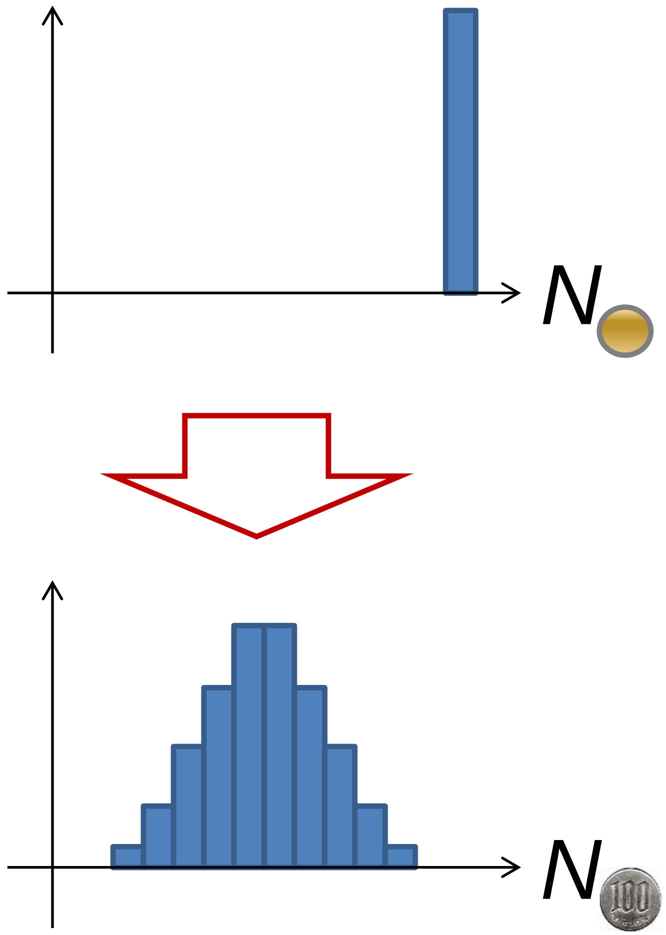
Slot Machine Analogy



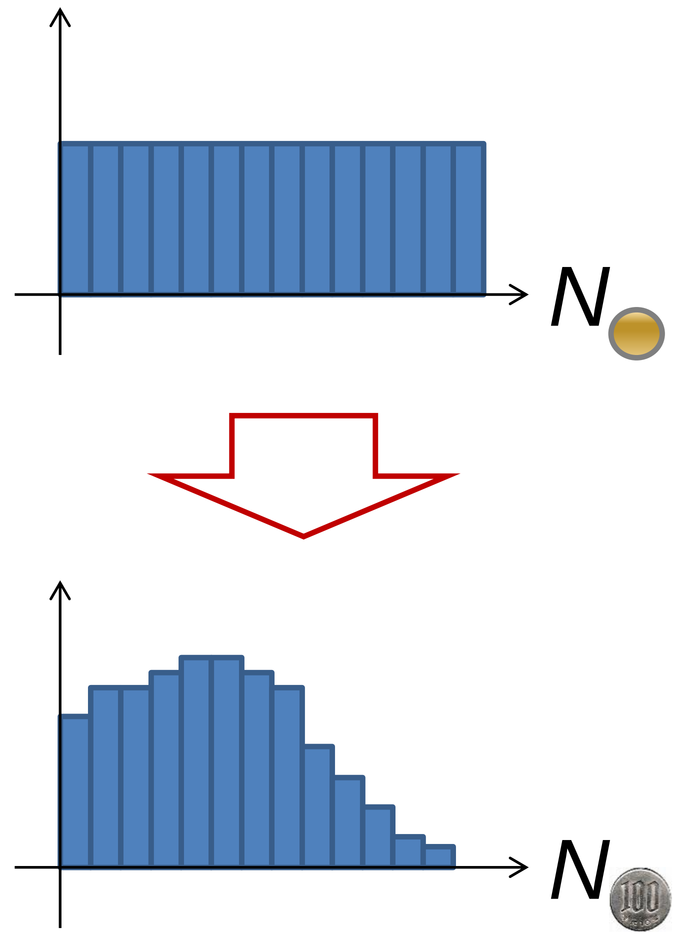


Extreme Examples

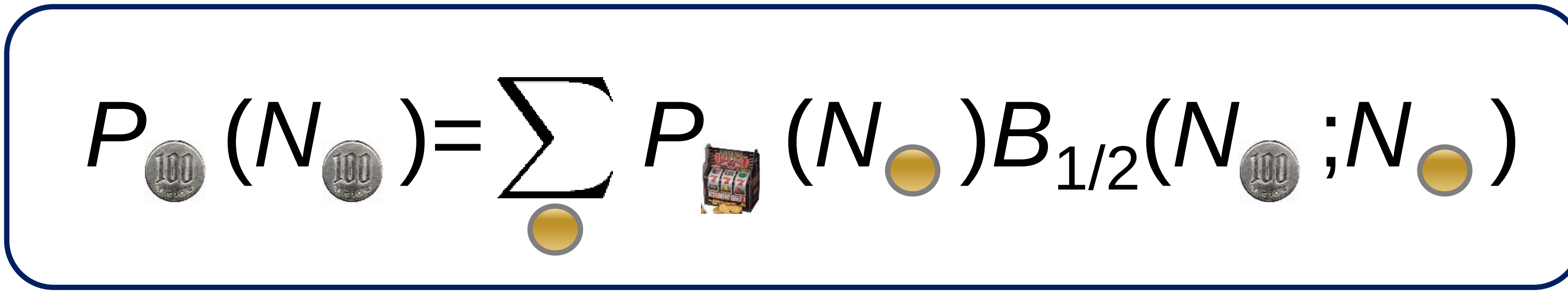
Fixed # of coins



Constant probabilities



Reconstructing Total Coin Number



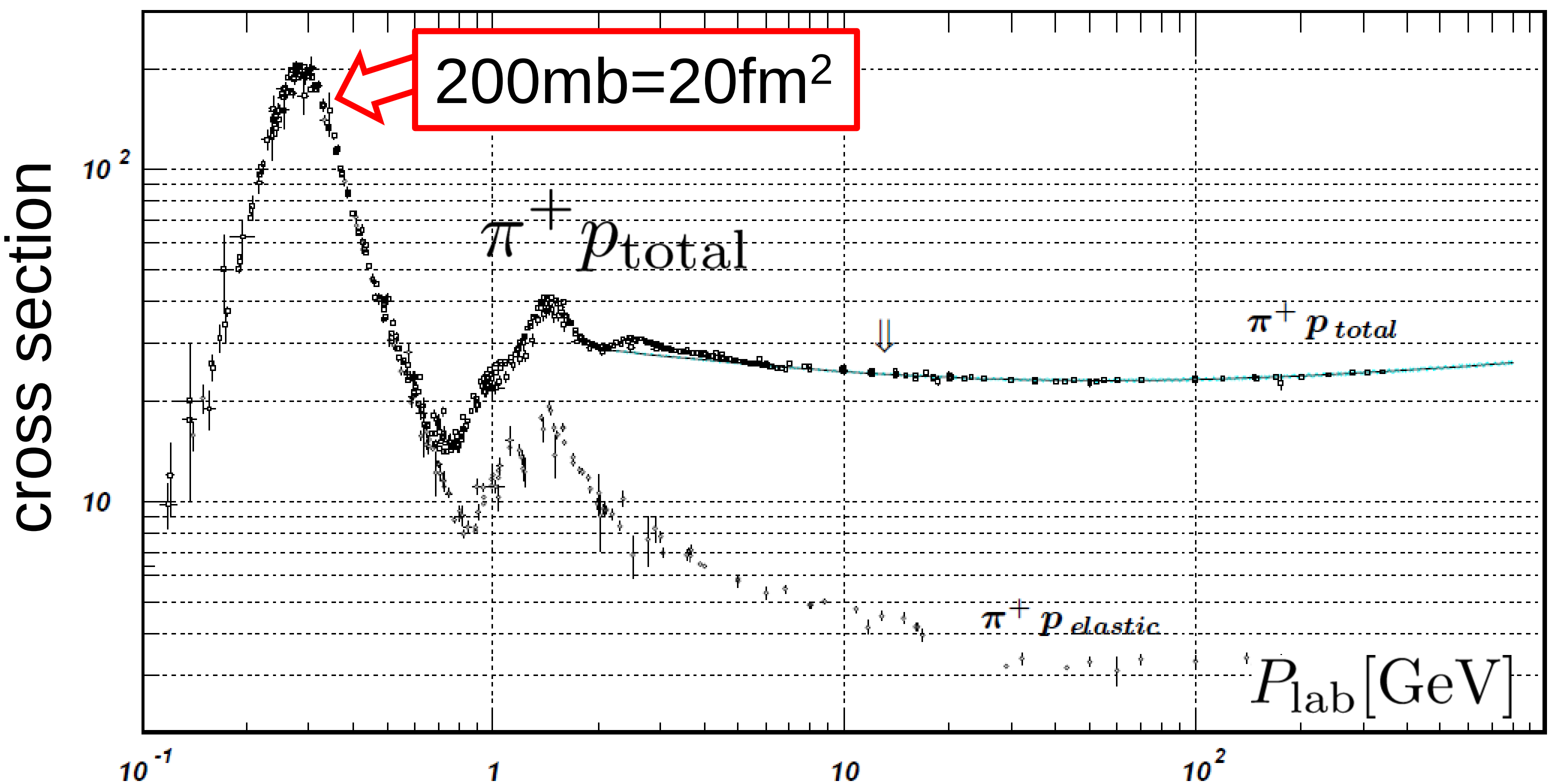
$$B_p(k;N) = p^k (1-p)^{N-k}\ {}_kC_N$$:binomial distr. func.

Nucleon Isospin in Hadronic Medium

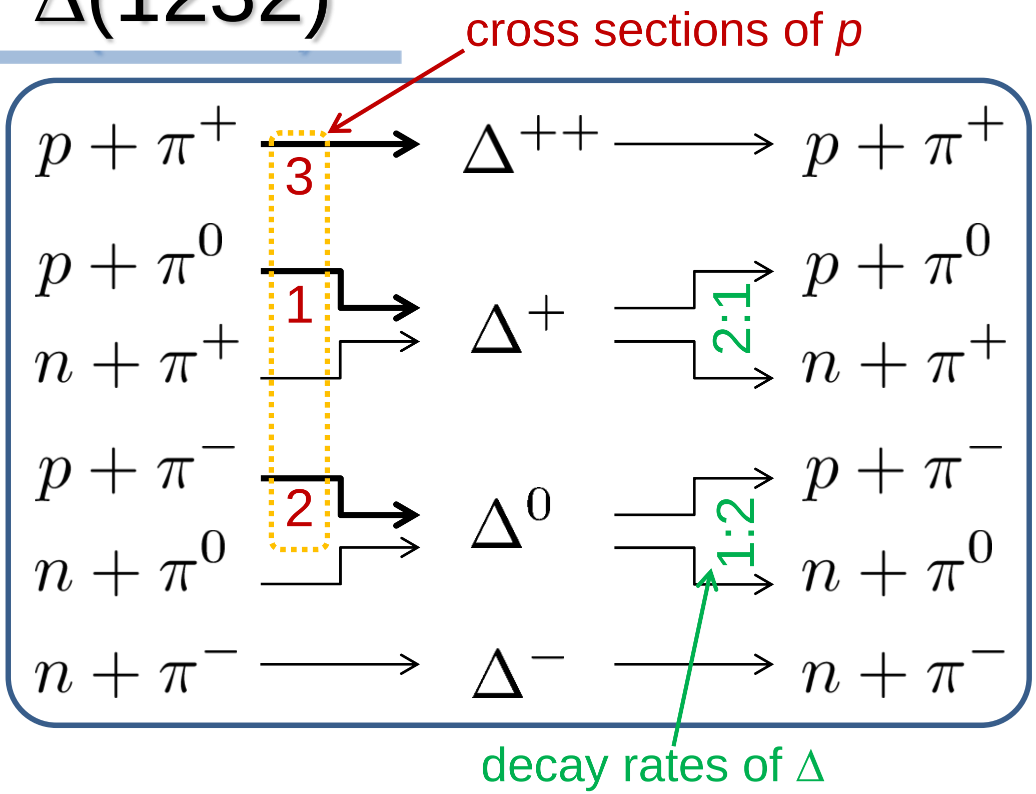
- Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by Δ(1232):

$$p, n \rightarrow \Delta(1232) \rightarrow p, n$$
$$\pi \rightarrow \quad\quad\quad \rightarrow \pi$$

$$I = 3/2$$
$$\Gamma \simeq 1.8 \ [\mathrm{fm}]$$

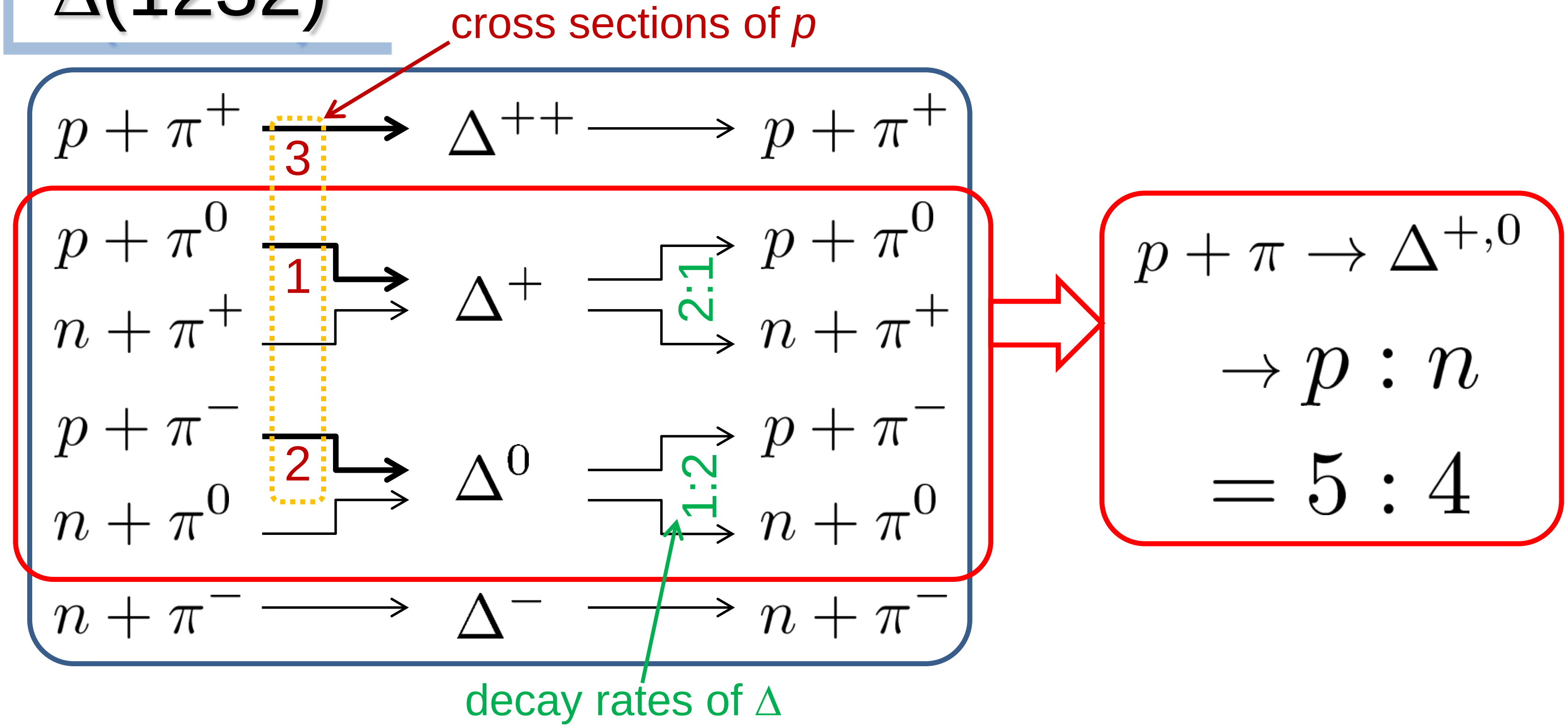


Δ(1232)



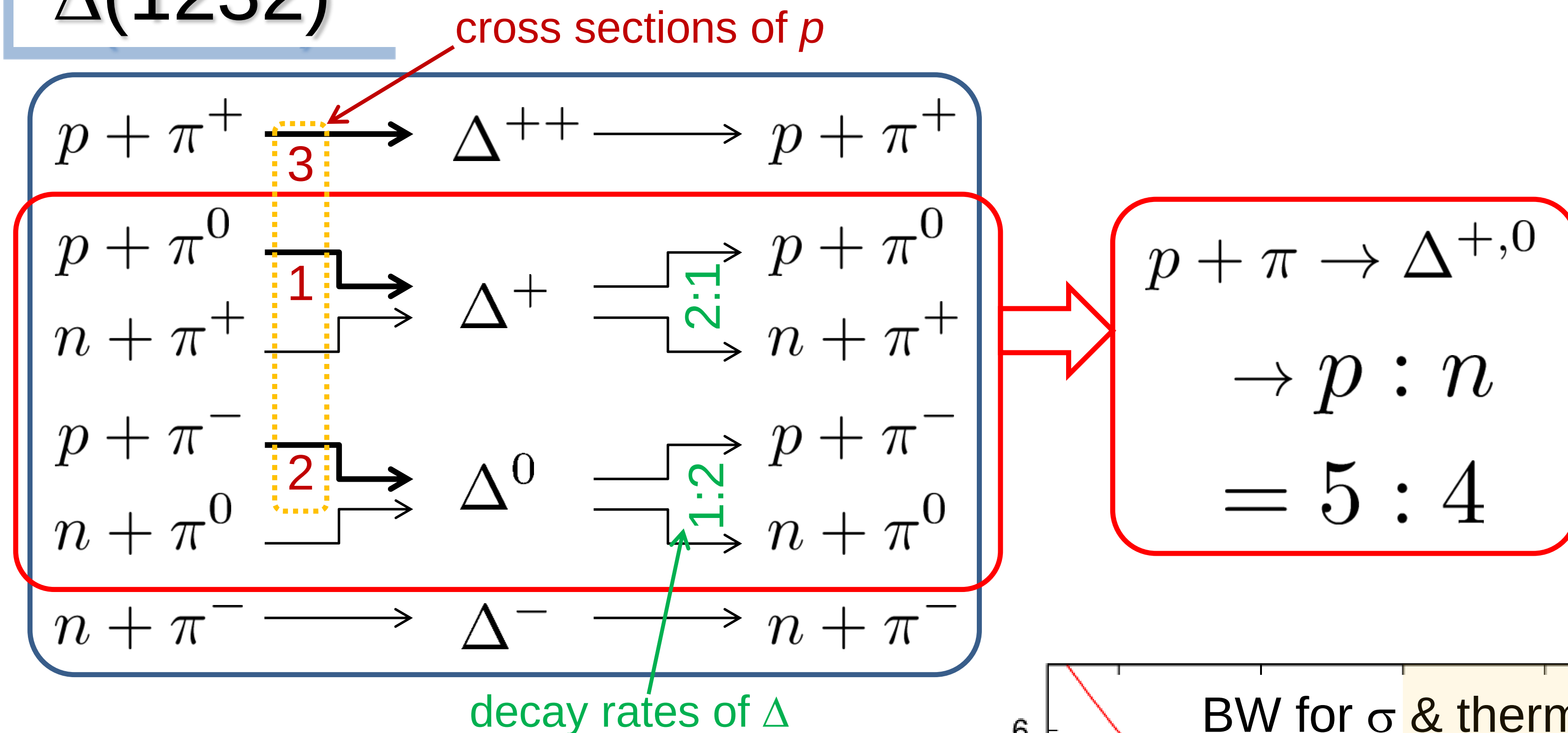
cross sections of p
p + π+
p + π0
n + π+
p + π−
n + π0
n + π−
3
1
2
Δ++
Δ+
Δ0
Δ−
2:1
1:2
p + π+
p + π0
n + π+
p + π−
n + π0
n + π−
decay rates of Δ

Δ(1232)



cross sections of p
3
1
2
2:1
1:2
decay rates of Δ
$p+\pi^+ \to \Delta^{++} \to p+\pi^+$
$p+\pi^0$, $n+\pi^+$ $\to \Delta^+ \to$ $p+\pi^0$, $n+\pi^+$
$p+\pi^-$, $n+\pi^0$ $\to \Delta^0 \to$ $p+\pi^-$, $n+\pi^0$
$n+\pi^- \to \Delta^- \to n+\pi^-$
$p+\pi \to \Delta^{+,0}$
$\to p:n$
$= 5:4$

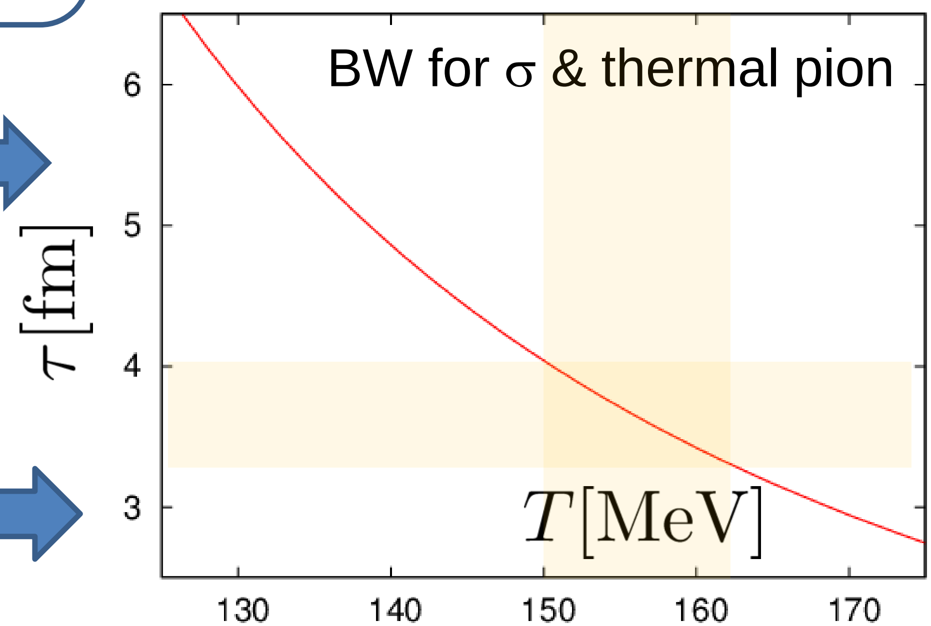
Δ(1232)



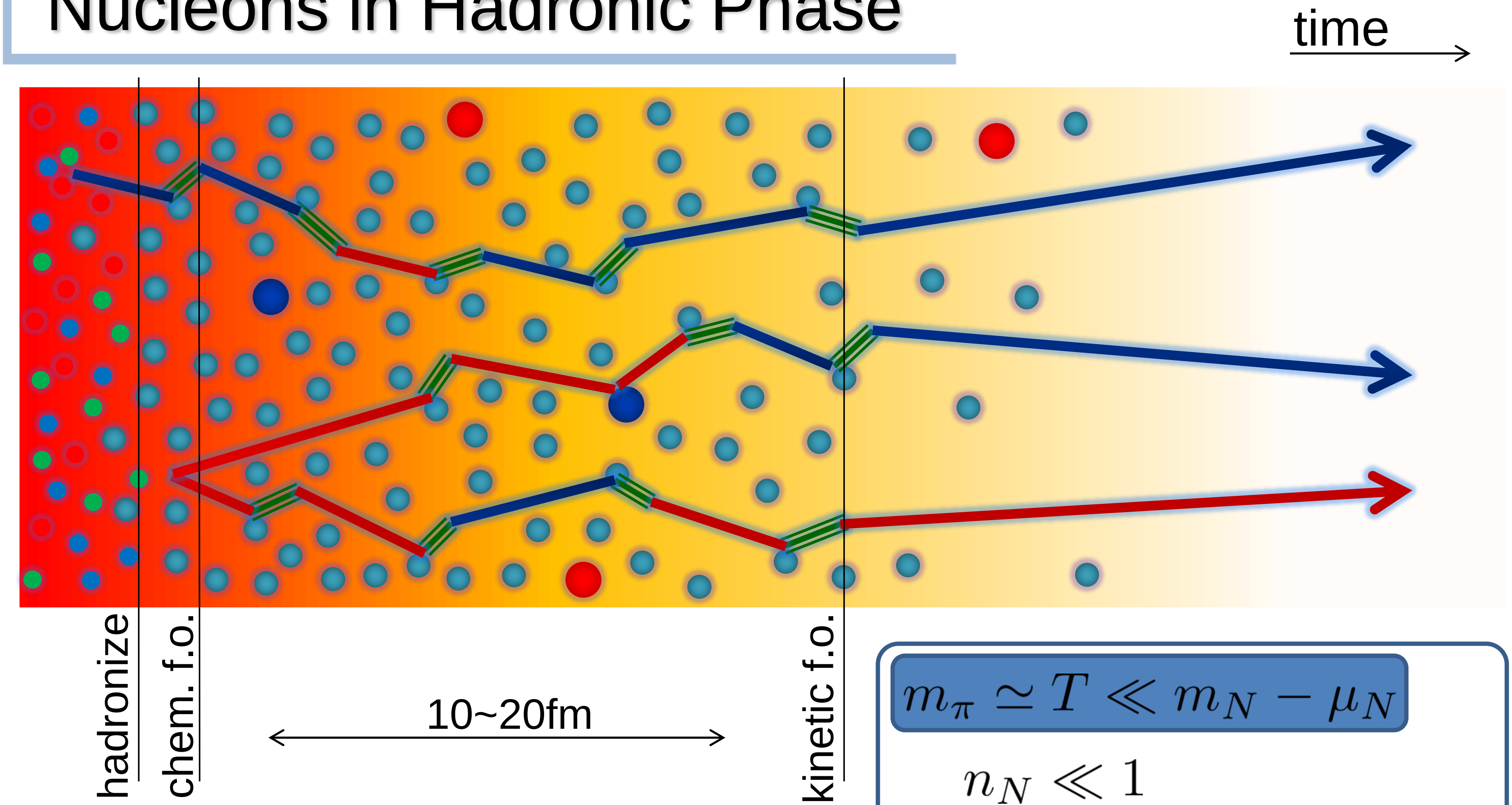
Lifetime to create Δ^+ or Δ^0

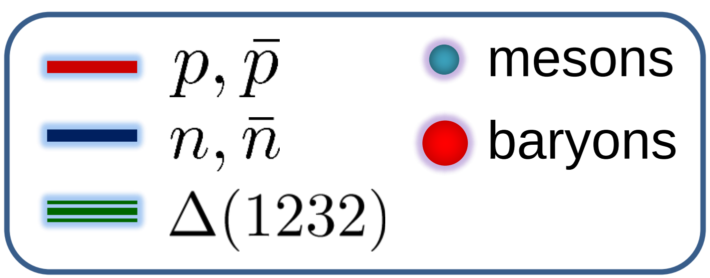
$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

$$(\text{freezeout time}) \simeq 20[\text{fm}]$$



Nucleons in Hadronic Phase





$$m_\pi \simeq T \ll m_N - \mu_N$$

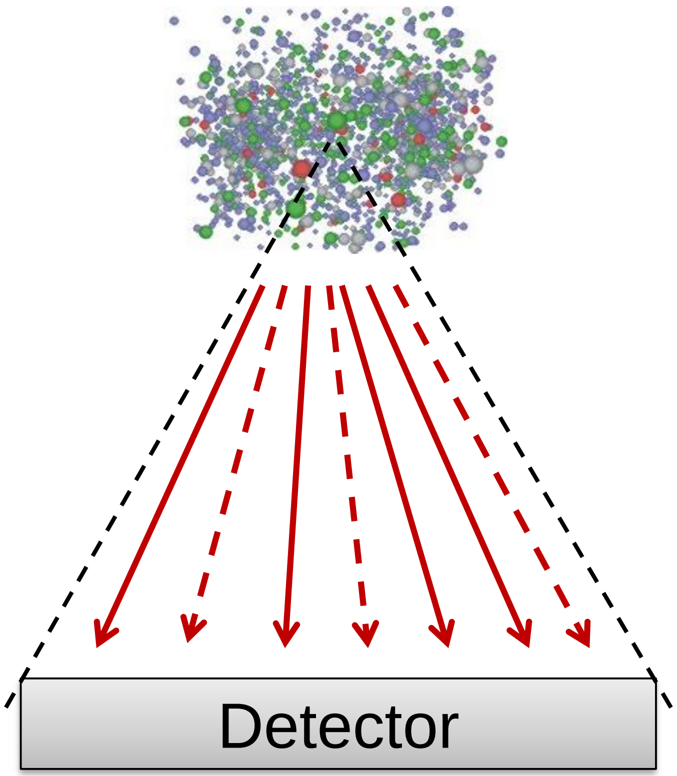
$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- many pions

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



- $\left\{ \begin{array}{l} \rightarrow N_N \text{ nucleons} \\ \dashrightarrow N_{\bar{N}} \text{ anti-nucleons} \end{array} \right.$

  ⇒ $F(N_N, N_{\bar{N}})$

- $N_N \left\{ \begin{array}{l} N_p \text{ protons} \\ N_n \text{ neutrons} \end{array} \right.$

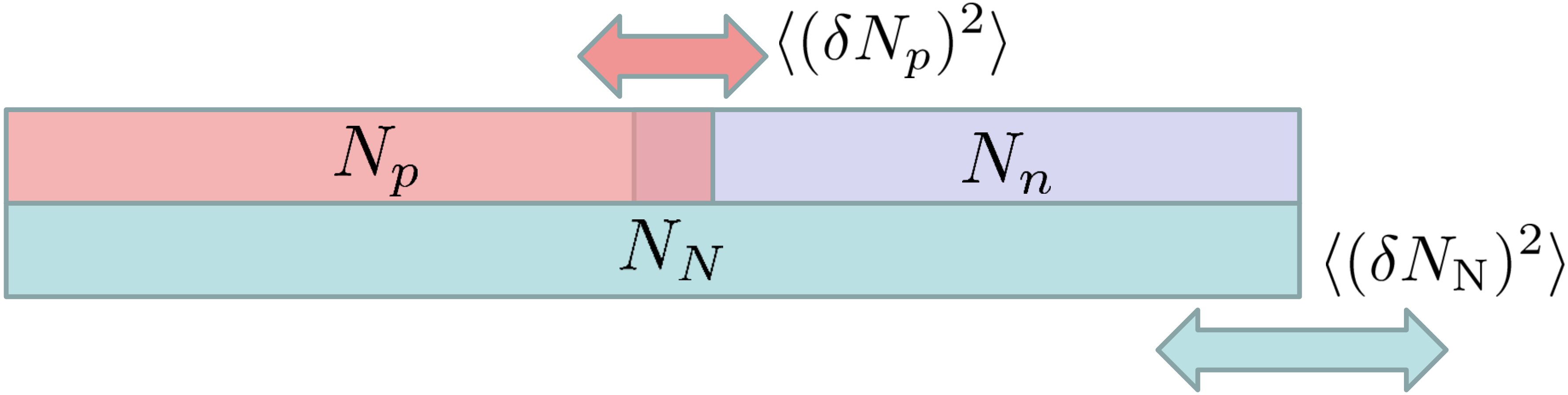
  ⇒ $B(N_p; N_N)$

  binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

- for any phase space in the final state.

Nucleon & Proton Number Fluctuations



$$\begin{cases} \langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{4}\langle(\delta N_\mathrm{N}^{(\mathrm{net})})^2\rangle + \frac{1}{4}\langle N_\mathrm{N}^{(\mathrm{tot})}\rangle \\ \langle(\delta N_\mathrm{N}^{(\mathrm{net})})^2\rangle = 4\langle(\delta N_p^{(\mathrm{net})})^2\rangle - 2\langle N_p^{(\mathrm{tot})}\rangle \end{cases}$$

- for isospin symmetric medium
- effect of isospin density <10%
- Similar formulas up to any order!

For free gas

$$\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_\mathrm{N}^{(\mathrm{net})})^2\rangle$$

3rd & 4th Order Fluctuations

$N_{\rm B} \to N_p$

$$\langle(\delta N_p^{(\rm net)})^3\rangle = \frac{1}{8}\langle(\delta N_{\rm B}^{(\rm net)})^3\rangle + \frac{3}{8}\langle\delta N_{\rm B}^{(\rm net)}\delta N_{\rm B}^{(\rm tot)}\rangle,$$

$$\langle(\delta N_p^{(\rm net)})^4\rangle_c = \frac{1}{16}\langle(\delta N_{\rm B}^{(\rm net)})^4\rangle_c + \frac{3}{8}\langle(\delta N_{\rm B}^{(\rm net)})^2\delta N_{\rm B}^{(\rm tot)}\rangle + \frac{3}{16}\langle(\delta N_{\rm B}^{(\rm tot)})^2\rangle - \frac{1}{8}\langle N_{\rm B}^{(\rm tot)}\rangle,$$

$N_p \to N_{\rm B}$

$$\langle(\delta N_{\rm B}^{(\rm net)})^3\rangle = 8\langle(\delta N_p^{(\rm net)})^3\rangle - 12\langle\delta N_p^{(\rm net)}\delta N_p^{(\rm tot)}\rangle + 6\langle N_p^{(\rm net)}\rangle,$$

$$\langle(\delta N_{\rm B}^{(\rm net)})^4\rangle_c = 16\langle(\delta N_p^{(\rm net)})^4\rangle_c - 48\langle(\delta N_p^{(\rm net)})^2\delta N_p^{(\rm tot)}\rangle + 48\langle(\delta N_p^{(\rm net)})^2\rangle + 12\langle(\delta N_p^{(\rm tot)})^2\rangle - 26\langle N_p^{(\rm tot)}\rangle,$$

Difference btw Baryon and Proton Numbers

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

Difference btw Baryon and Proton Numbers

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

$$
\begin{cases}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{cases}
$$

genuine info. noise

For free gas

$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^n\rangle_c$$

Strange Baryons

Decay Rates:

Λ $\quad m_\Lambda \simeq 1116[\mathrm{MeV}]$

$$\Rightarrow \; p : n \simeq 1.6 : 1$$

Σ $\quad m_\Sigma \simeq 1190[\mathrm{MeV}]$

$$\Rightarrow \; p : n \simeq 1 : 1.8$$

Decay modes:

$\Lambda \rightarrow p + \pi^-$ $\quad 64\%$

$\Lambda \rightarrow n + \pi^0$ $\quad 36\%$

$\Sigma^+ \rightarrow p + \pi^0$ $\quad 52\%$

$\Sigma^+ \rightarrow n + \pi^+$ $\quad 48\%$

$\Sigma^0 \rightarrow \Lambda \rightarrow p + \pi^-$ $\quad 64\%$

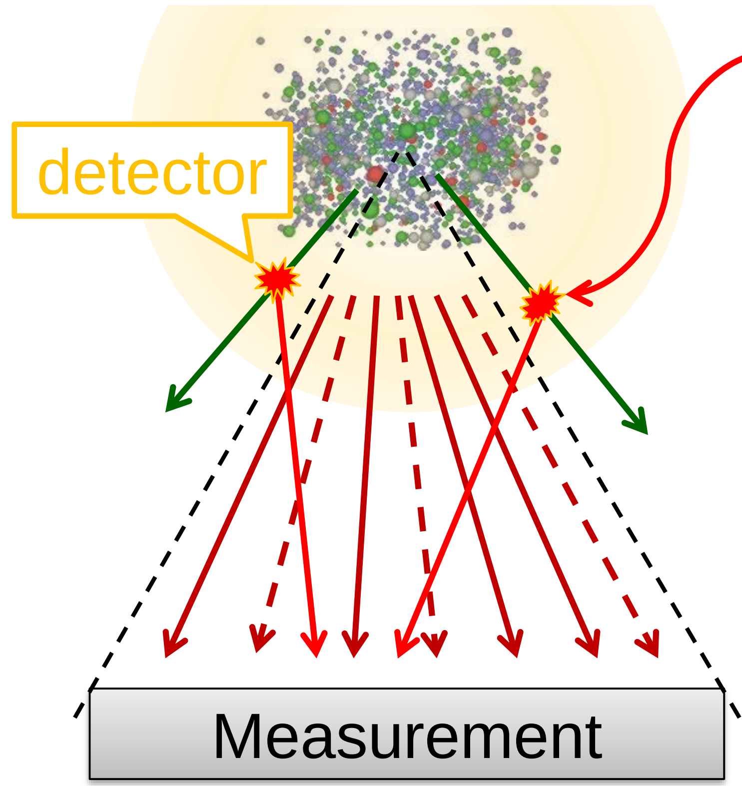
$\Sigma^0 \rightarrow \Lambda \rightarrow n + \pi^0$ $\quad 36\%$

$\Sigma^- \rightarrow n + \pi^-$

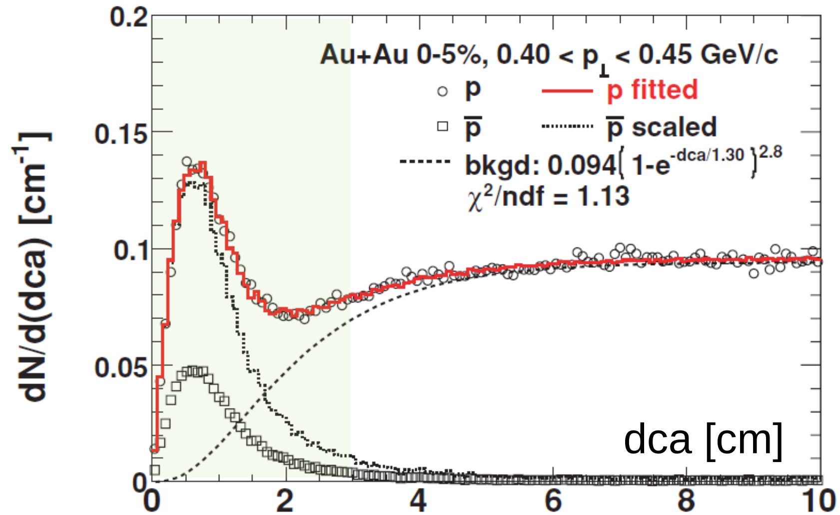
Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \rightarrow N_B$

Secondary Protons

MK+, in preparation



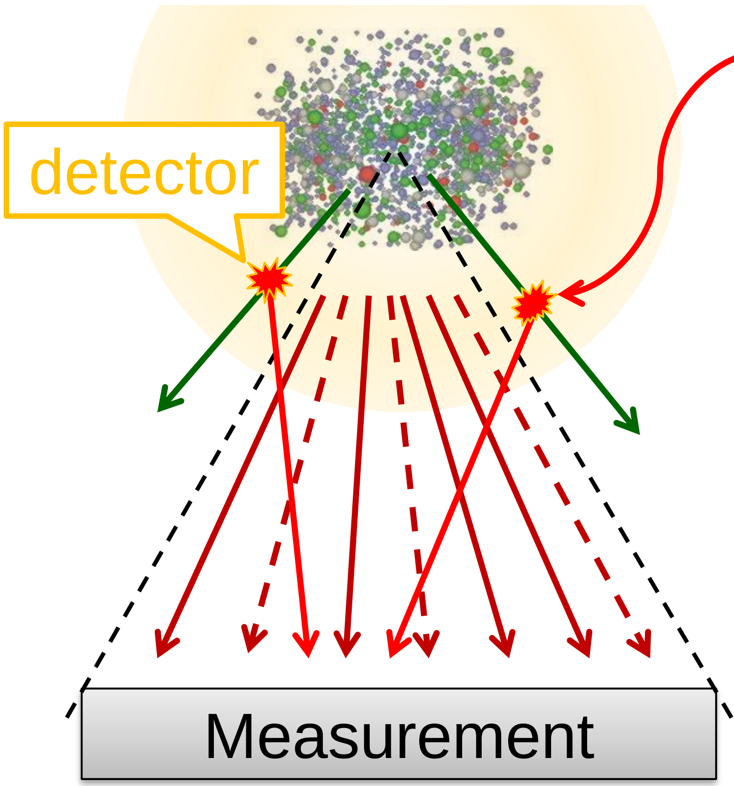
20% of observed protons @ STAR



STAR, PRC79,034909(2009)

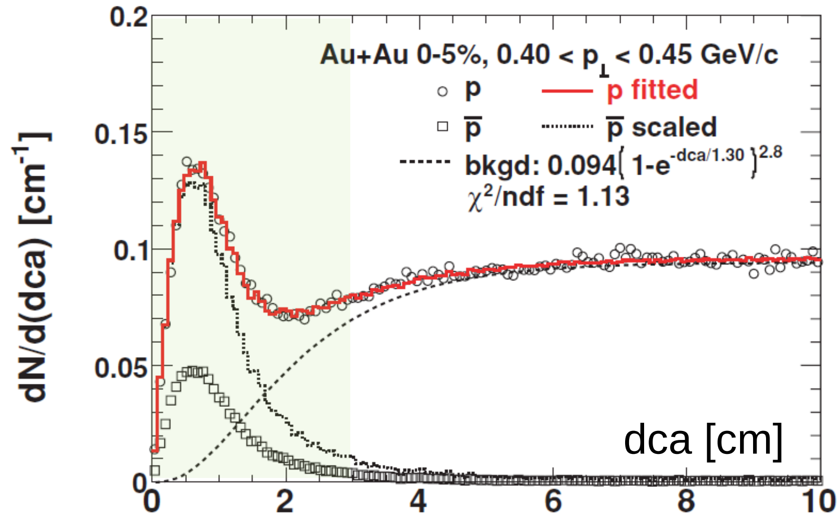
Secondary Protons

MK+, in preparation



Secondary (knockout) protons

20% of observed protons @ STAR



STAR, PRC79,034909(2009)

Their contribution can be eliminated!

$$\langle(\delta N_p^{(\mathrm{QGP})})^n\rangle_c = \langle(\delta N_p^{(\mathrm{exp})})^n\rangle_c - \langle N_p^{(\mathrm{2nd})}\rangle$$