重イオン衝突実験と非ガウスゆらぎ

北沢正清

（阪大理）

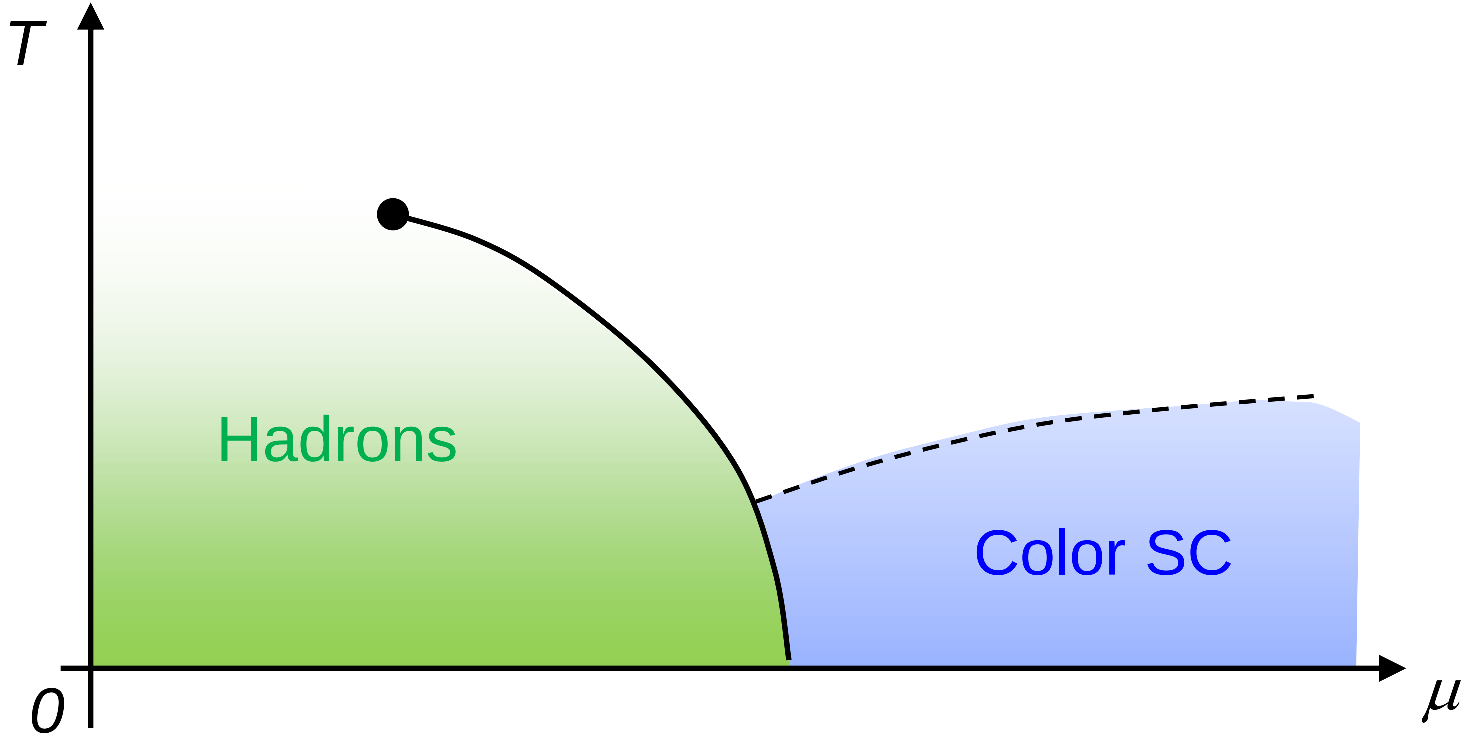
MK, Asakawa, Ono, in preparation

松本研究会, 2013/Jun./22

北沢の生い立ち

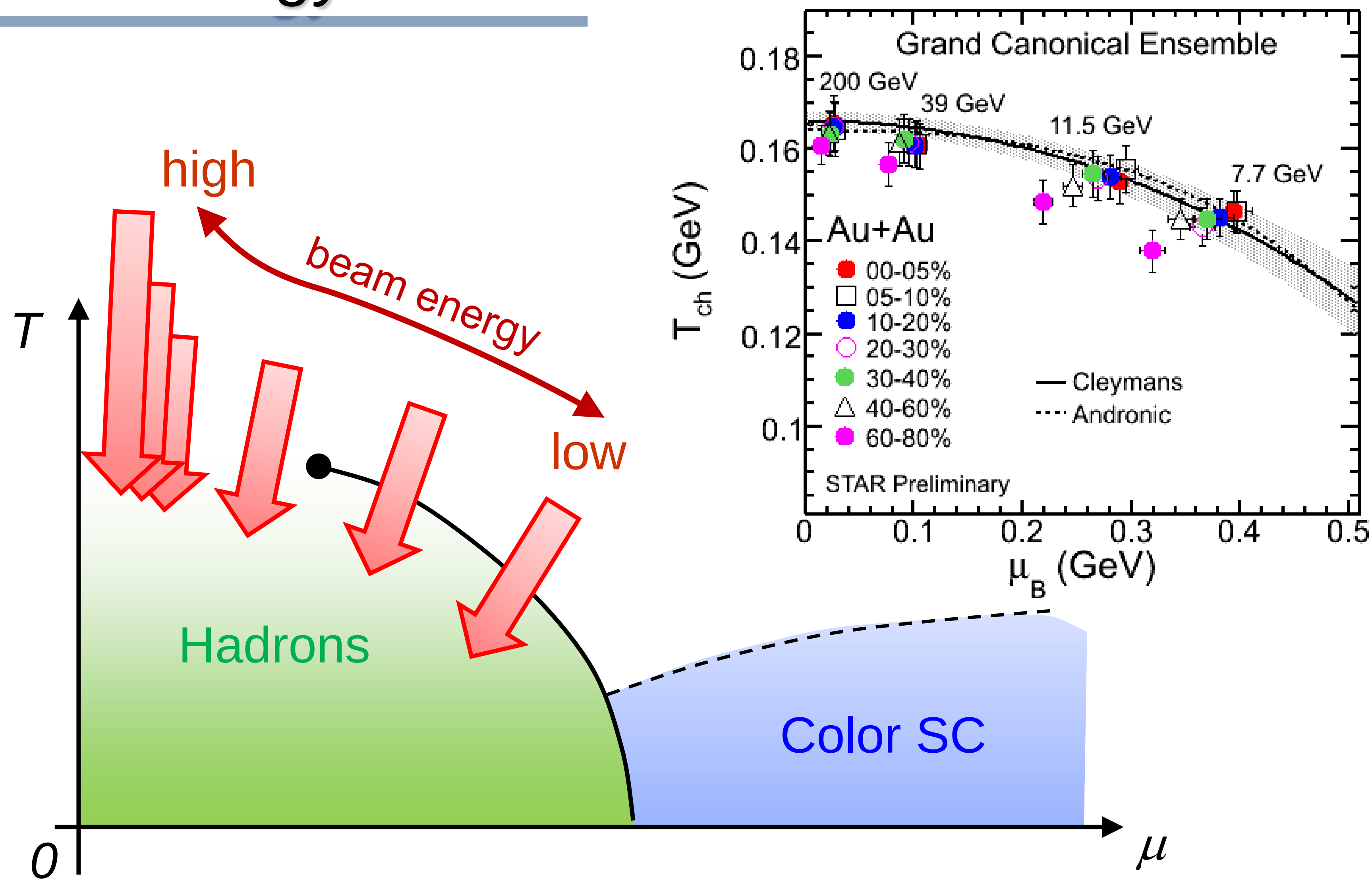
信州新町
小2～小6
長野市
中、高
大町市
誕生～1.5歳
南箕輪村
1.5歳～小1
飯田市
実家
茨城県へ
大学
新潟県
富山県
群馬県
岐阜県
埼玉県
愛知県
静岡県
北部エリア
中部エリア

Beam-Energy Scan



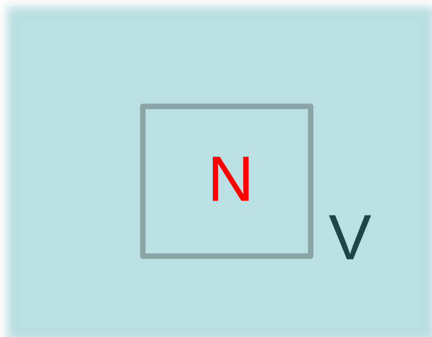
Beam-Energy Scan

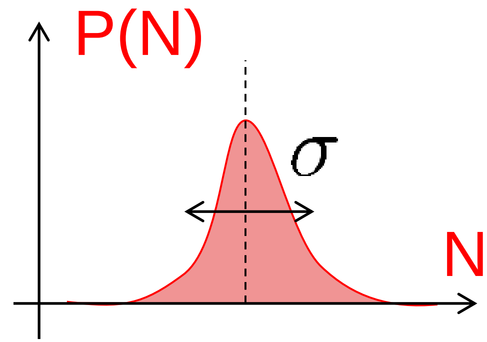
STAR 2012



Fluctuations

Observables in equilibrium are fluctuating.





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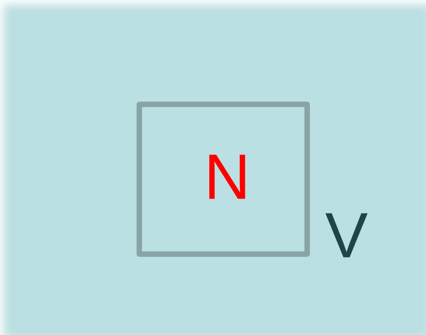
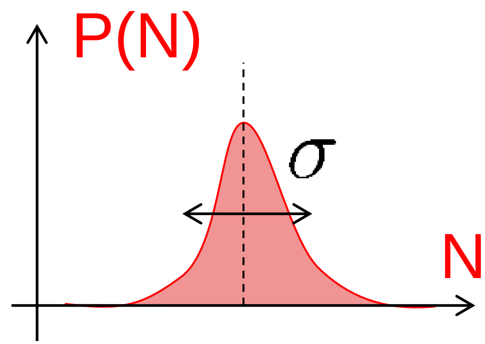


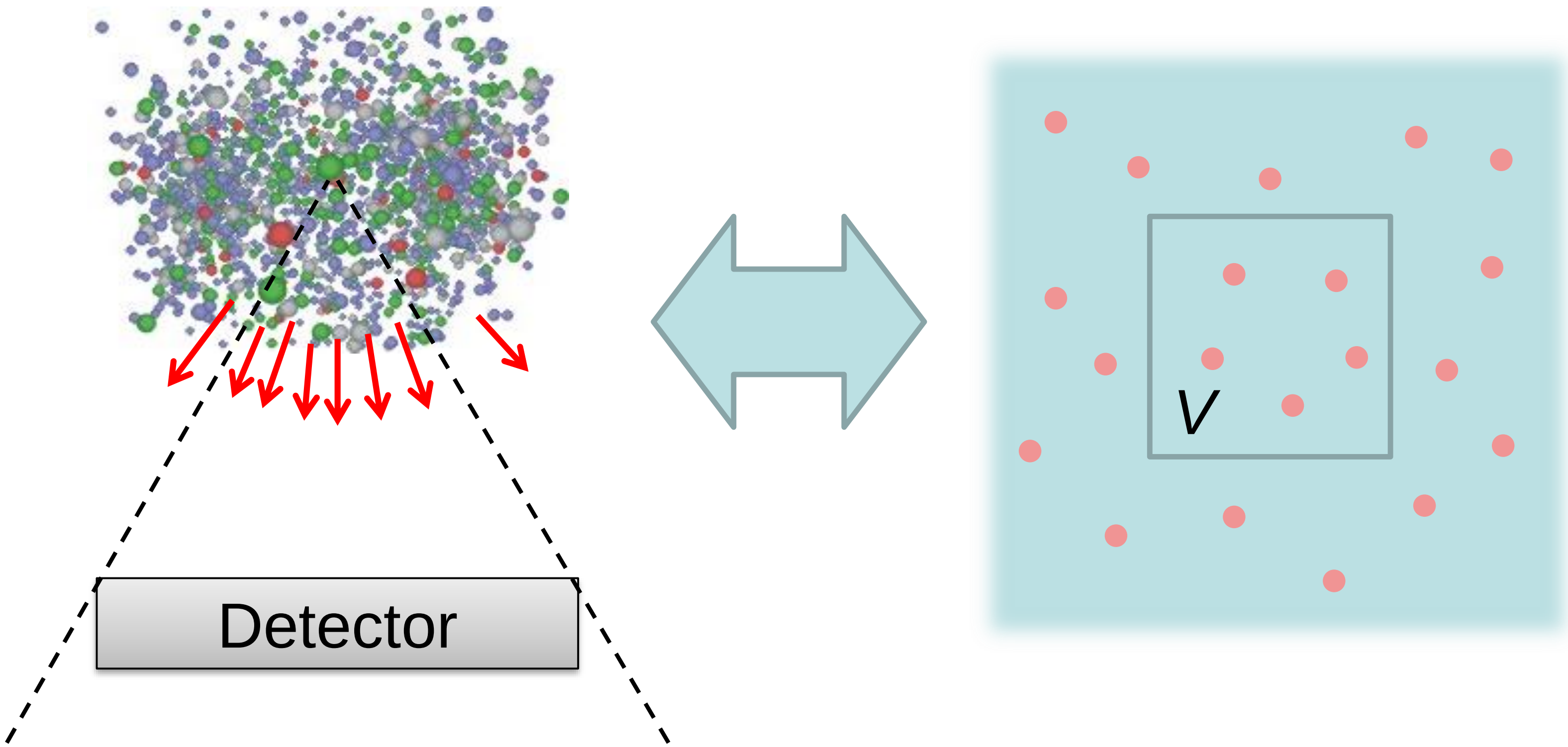


$$\delta N = N - \langle N \rangle$$

- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



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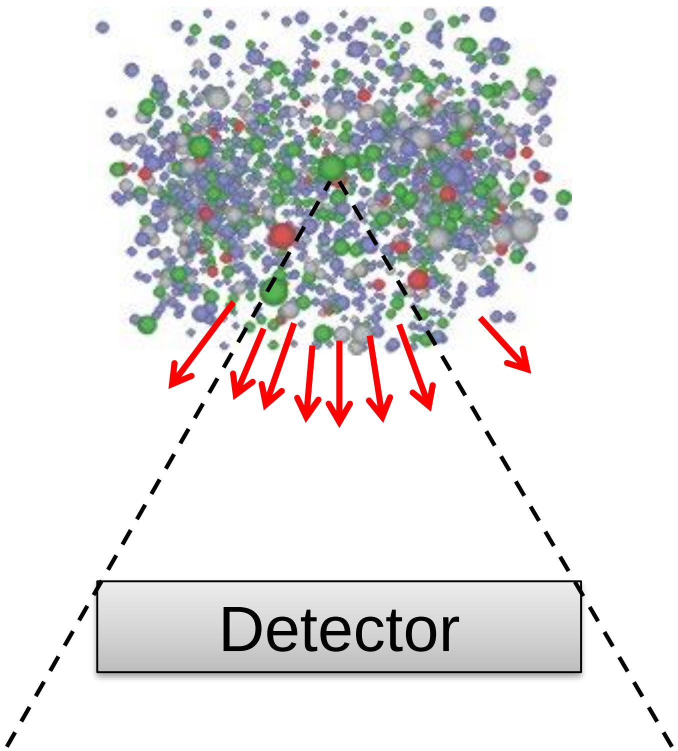
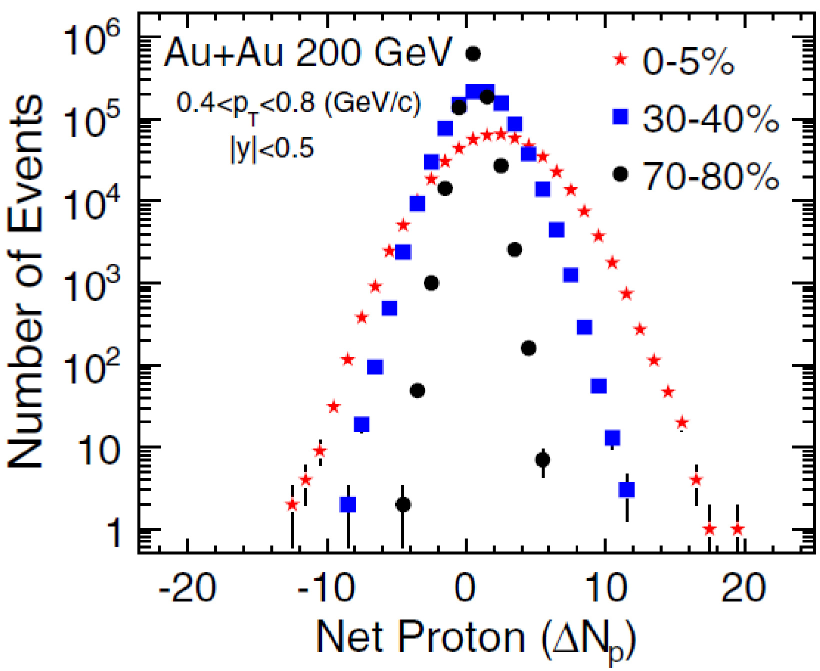


STAR, PRL**105** (2010)



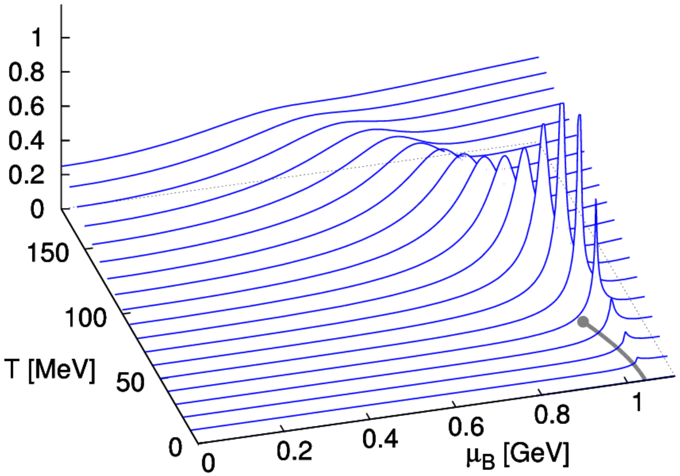
Fluctuations

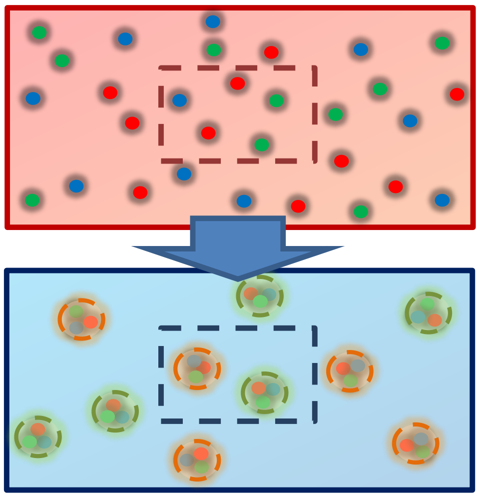
- Fluctuations reflect properties of matter.
  - Enhancement near the critical point

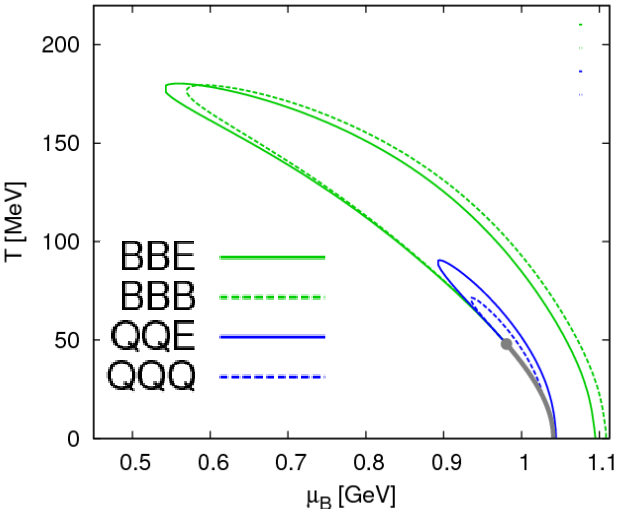
    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges

    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants

    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



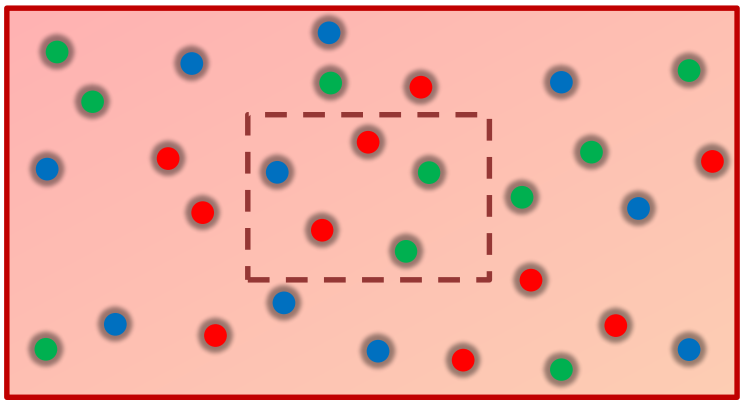




Fluctuations

Free Boltzmann → Poisson

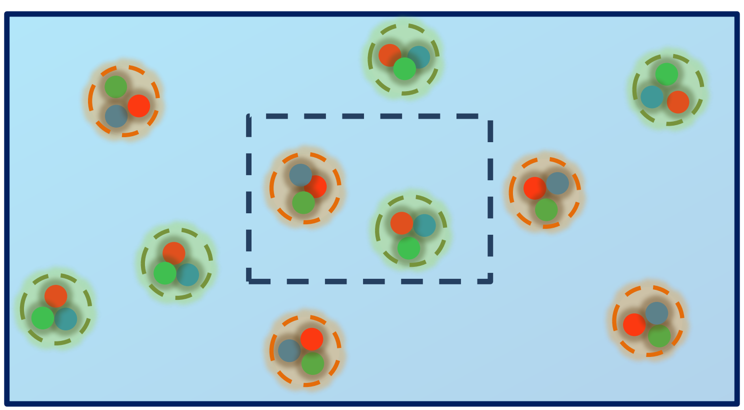
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



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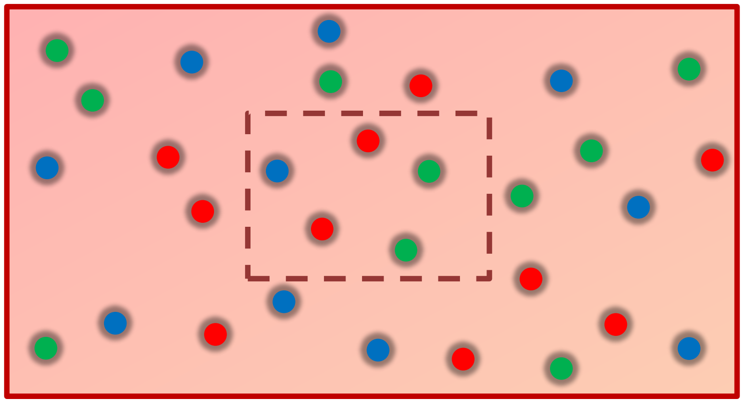
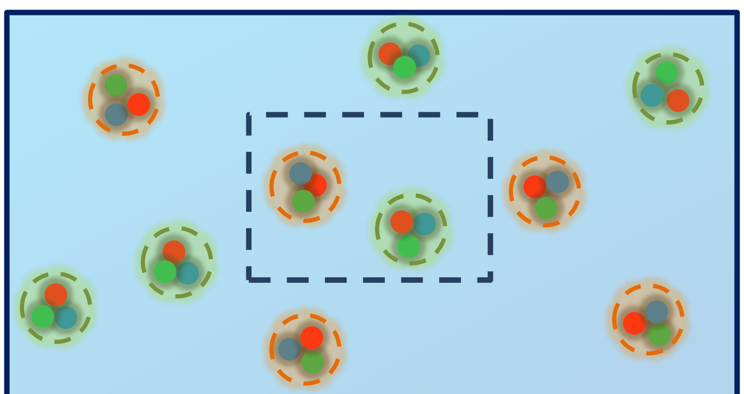
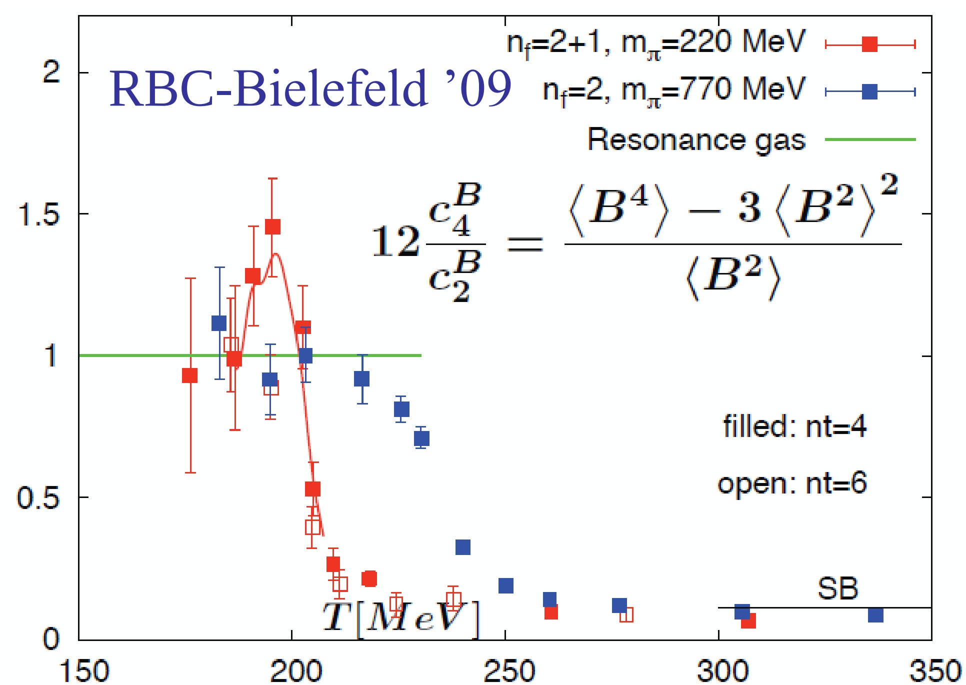


$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$





Central Limit Theorem

CLT

Large system → Gaussian

Higher-order cumulants suppressed
as system volume becomes larger?

Central Limit Theorem

CLT

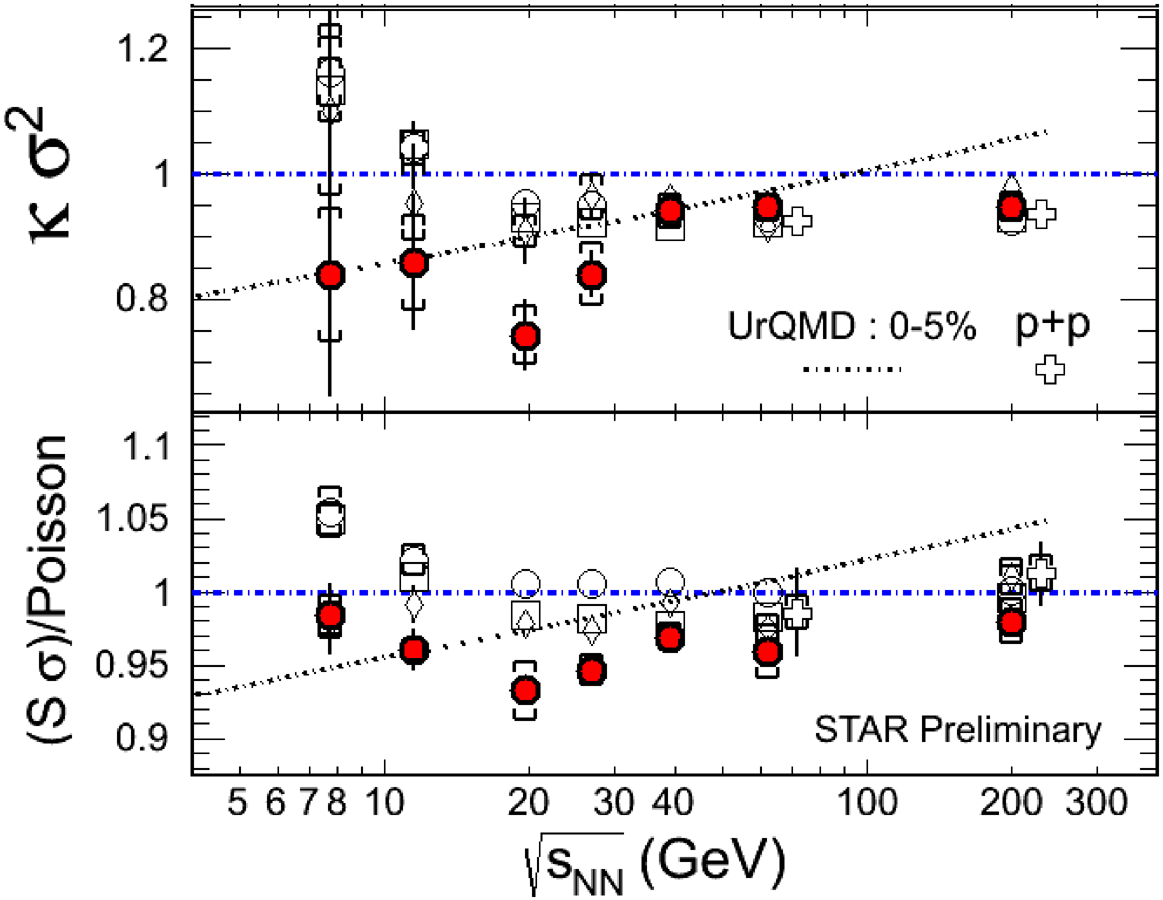
$$\bar{n} = \frac{N}{\sqrt{V}} \quad \text{:Gaussian}$$

$$\frac{\langle \bar{n}^k \rangle_c}{\langle \bar{n}^2 \rangle_c} = \frac{\langle N^k \rangle_c}{\langle N^2 \rangle_c} V^{(2-k)/2} \underset{k \geq 3}{\rightarrow} 0$$

- In a large system,
  - Cumulants $\langle N^k \rangle_c$ are nonzero.
  - Their experimental measurements are difficult.

Proton # Cumulants @ STAR-BES

STAR,QM2012



$$\frac{C_4}{C_2}$$

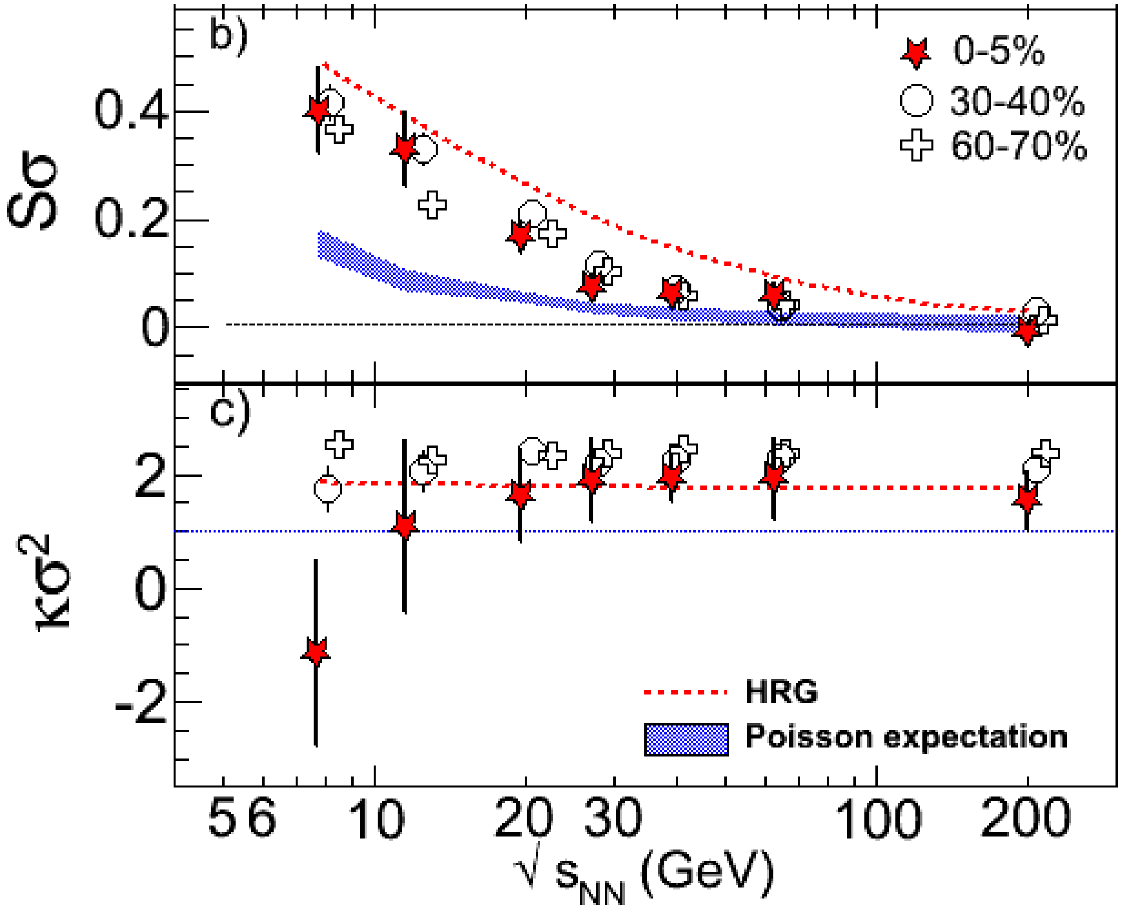
$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

No characteristic signals on
phase transition to QGP nor QCD CP

Charge Fluctuations @ STAR-BES

$$\frac{\langle \delta N_Q^3 \rangle}{\langle \delta N_Q^2 \rangle}$$

$$\frac{\langle \delta N_Q^4 \rangle_c}{\langle \delta N_Q^2 \rangle}$$



STAR, QM2012

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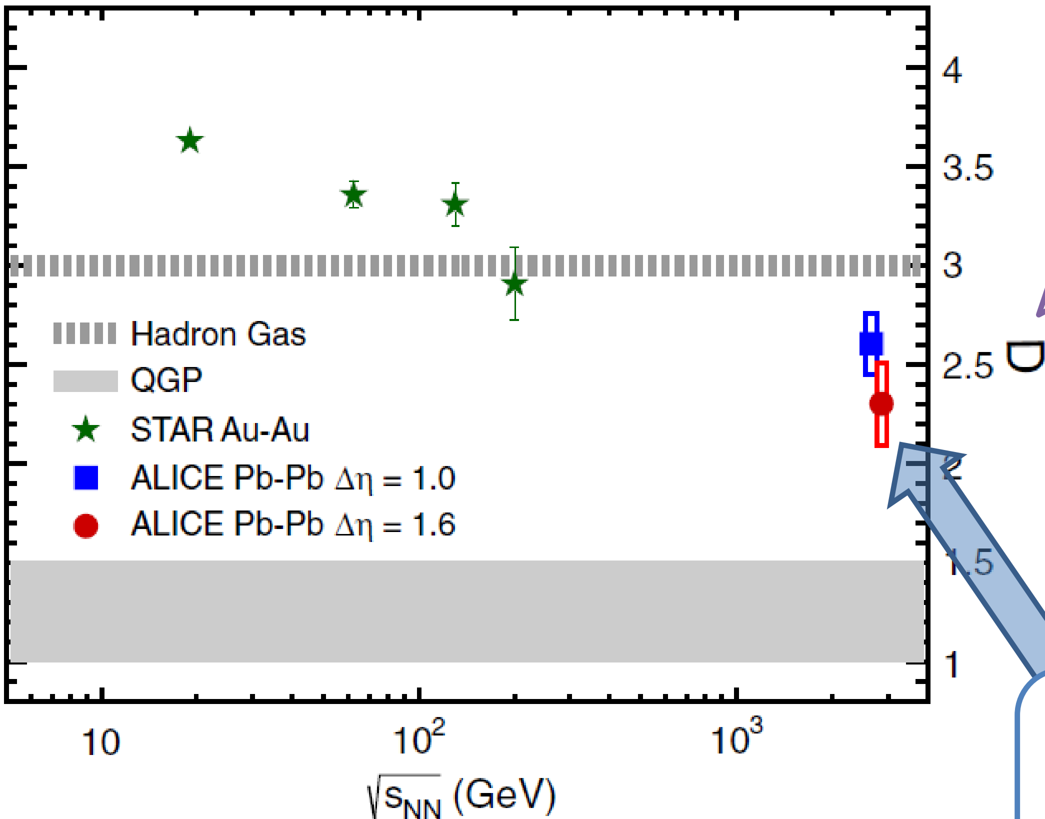
Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

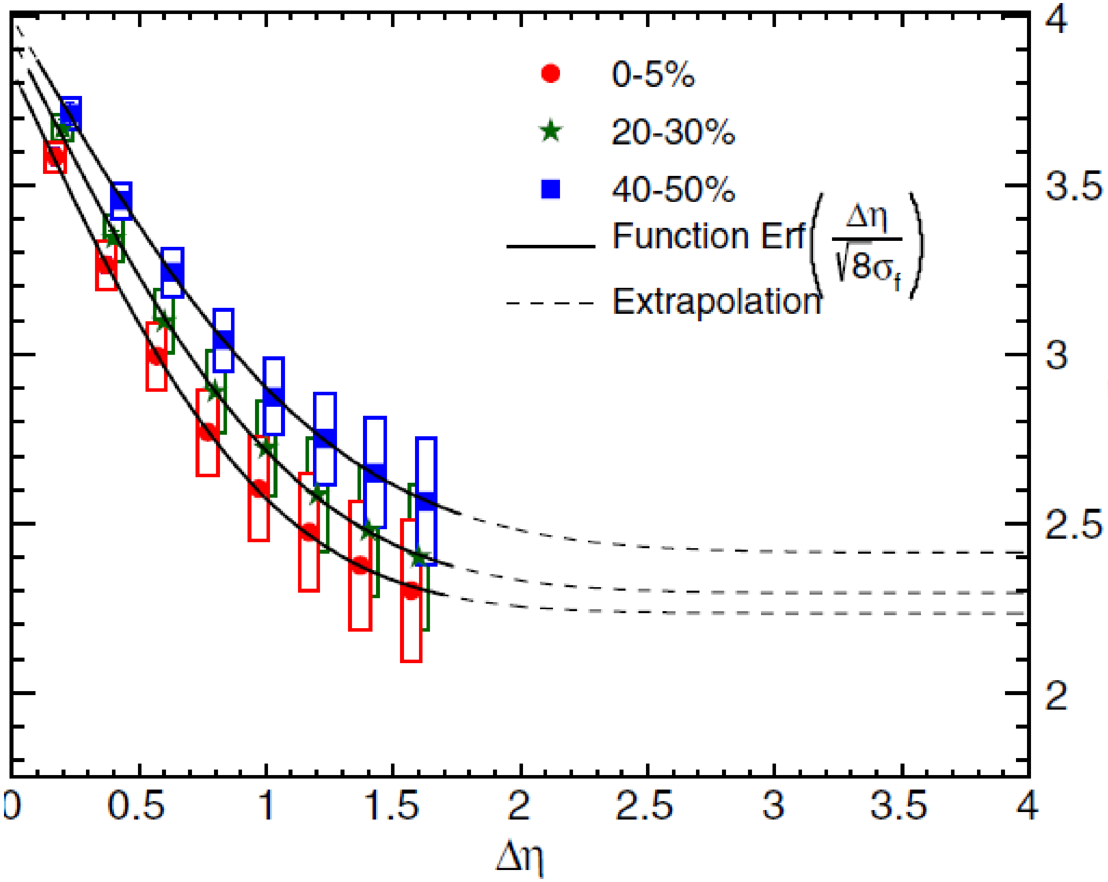
- D ~ 3-4 Hadronic
- D ~ 1 Quark

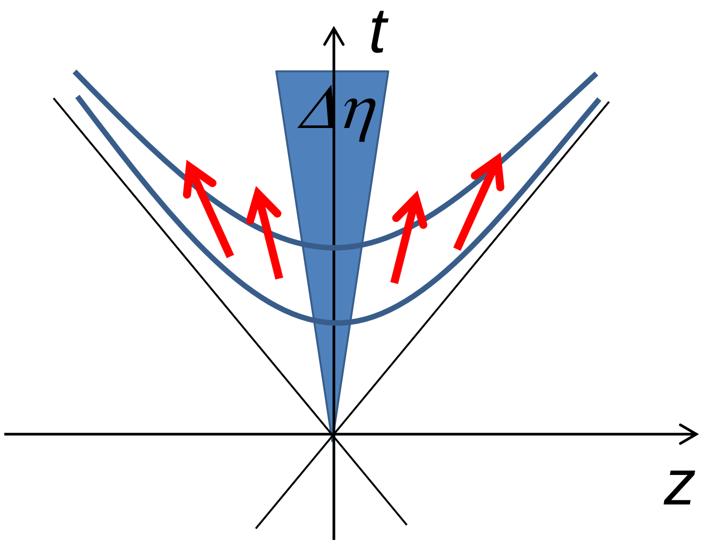
LHC:
significant suppression
from hadronic value

$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE

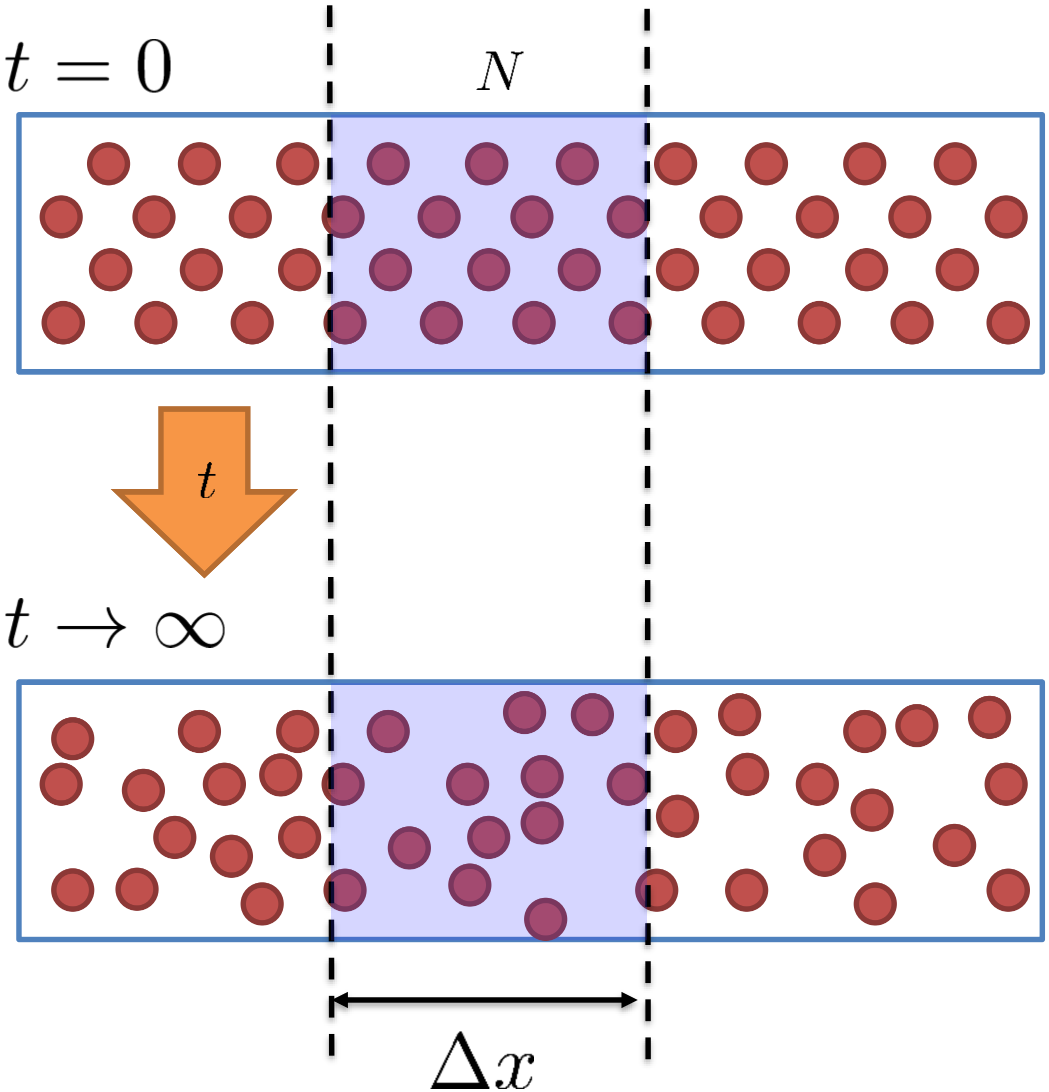
ALICE
PRL 2013

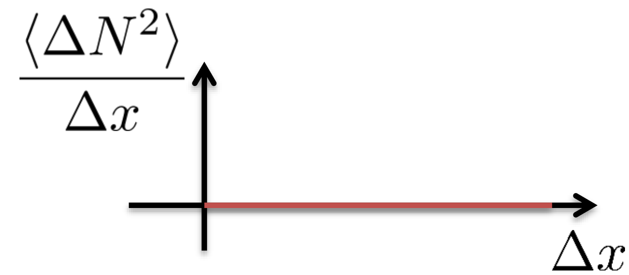




rapidity acceptane

Dissipation of a Conserved Charge





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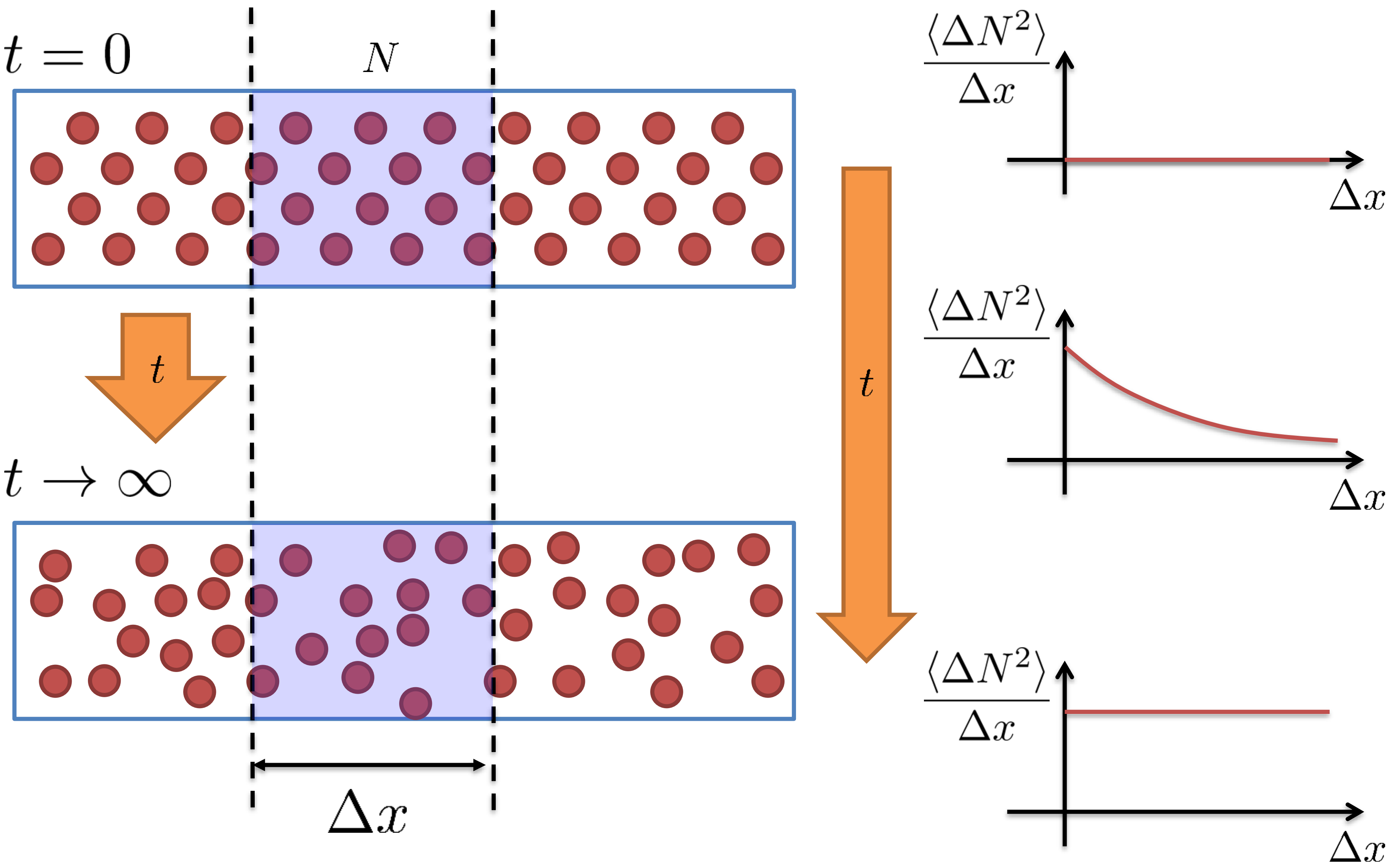
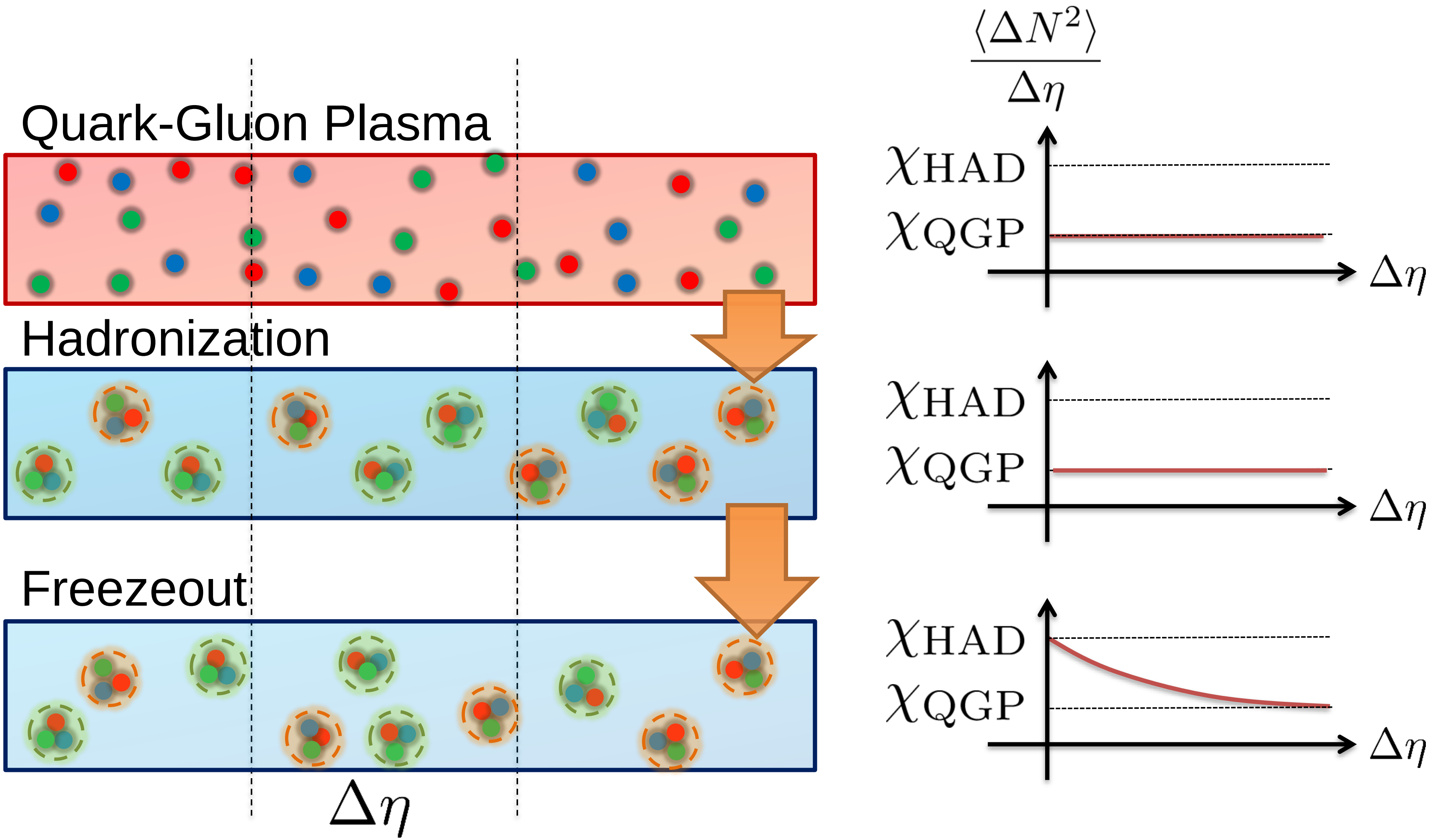


Time Evolution in HIC



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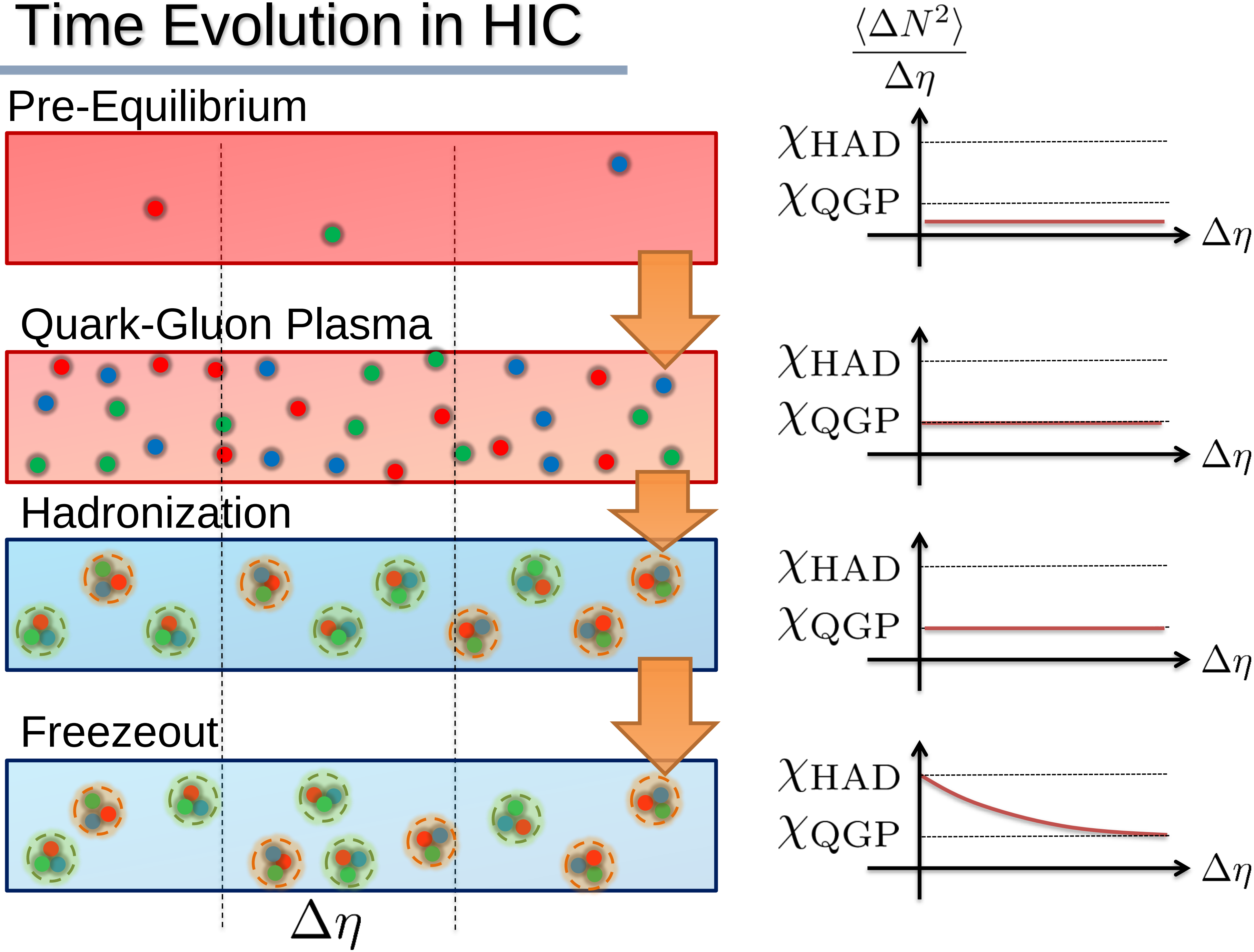

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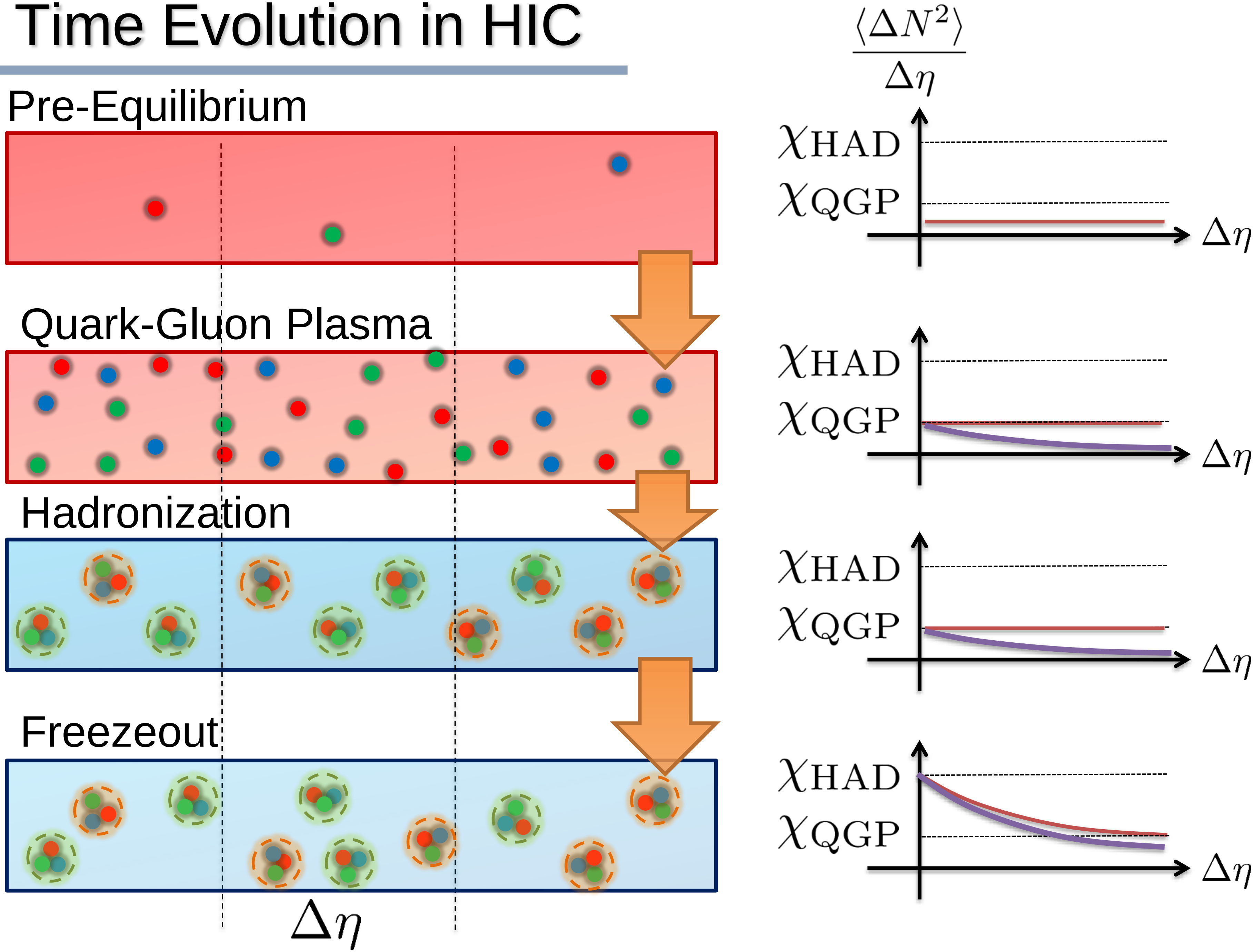
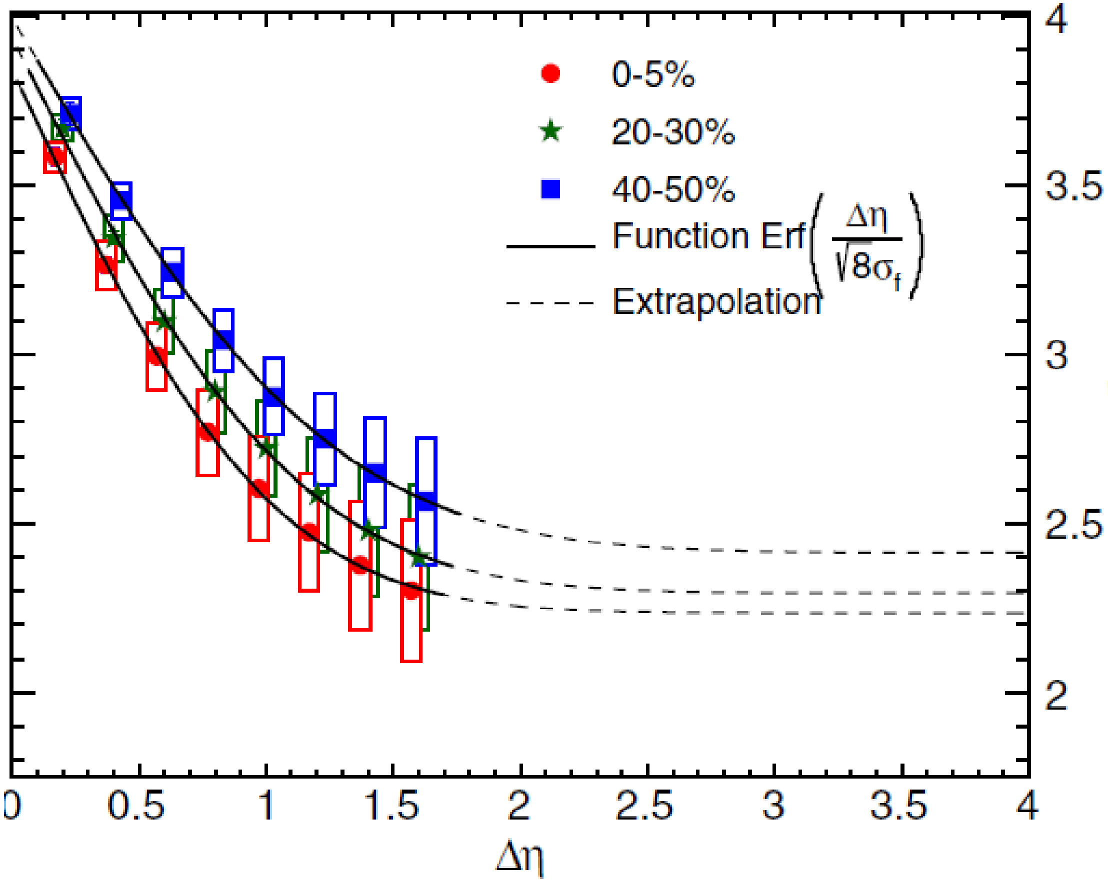
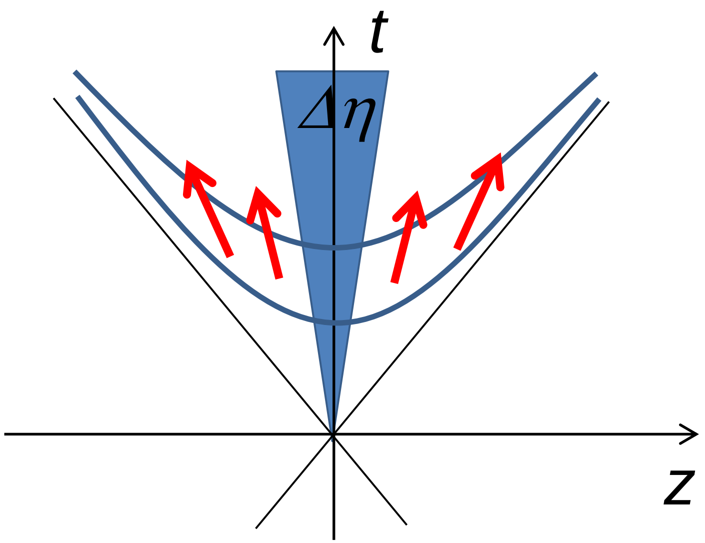


Δη Dependence @ ALICE

ALICE
PRL 2013



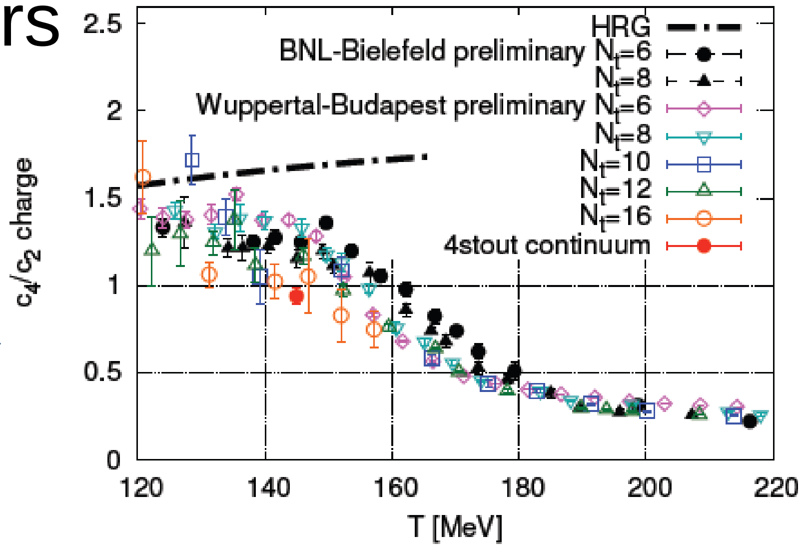


Δη dependences of fluctuation observables encode history of the hot medium!

Conserved Charges : Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory

ex: on the lattice



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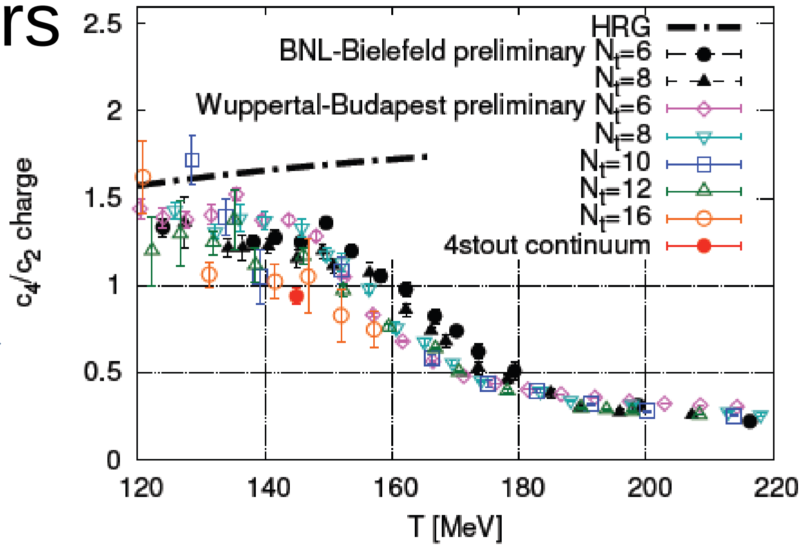


- Simple thermodynamic relations

$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

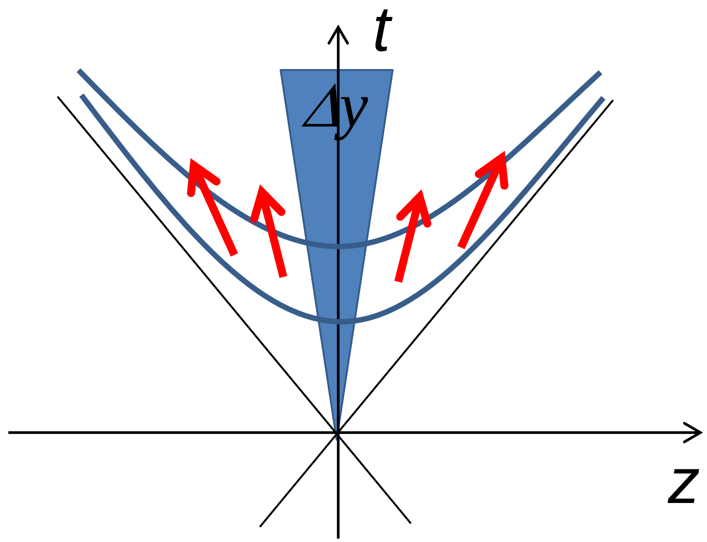
  - Intuitive interpretation for the behaviors of cumulants

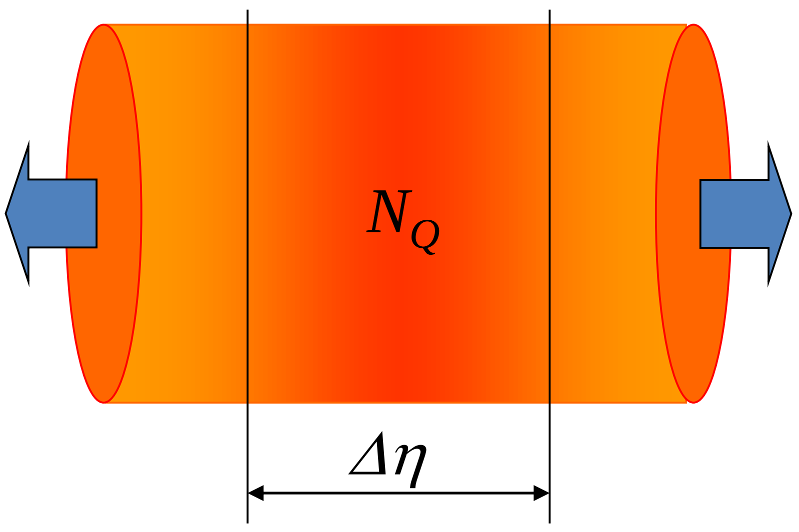
ex: $$\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$$

Fluctuations of Conserved Charges

- Under Bjorken expansion

Asakawa, Heintz, Muller, 2000
Jeon, Koch, 2000
Shuryak, Stephanov, 2001



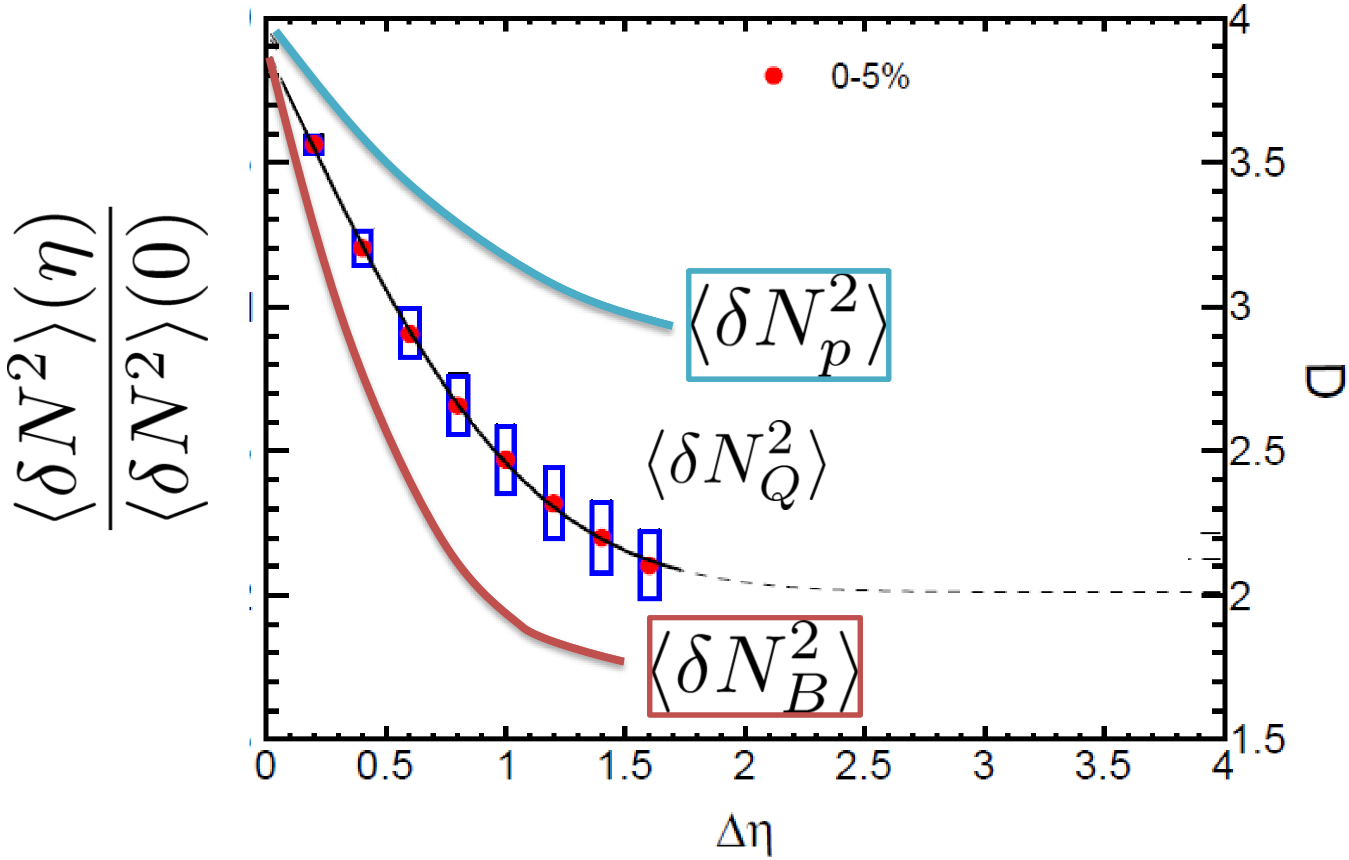


- Variation of a conserved charge in $\Delta\eta$ is **slow**, since it is achieved only through diffusion.

→ Primordial values can survive until freezeout. The wider $\Delta\eta$, more earlier fluctuation.

$<\delta N_B{}^2>$ and $< \delta N_p{}^2 >$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different $\Delta\eta$ dependence.

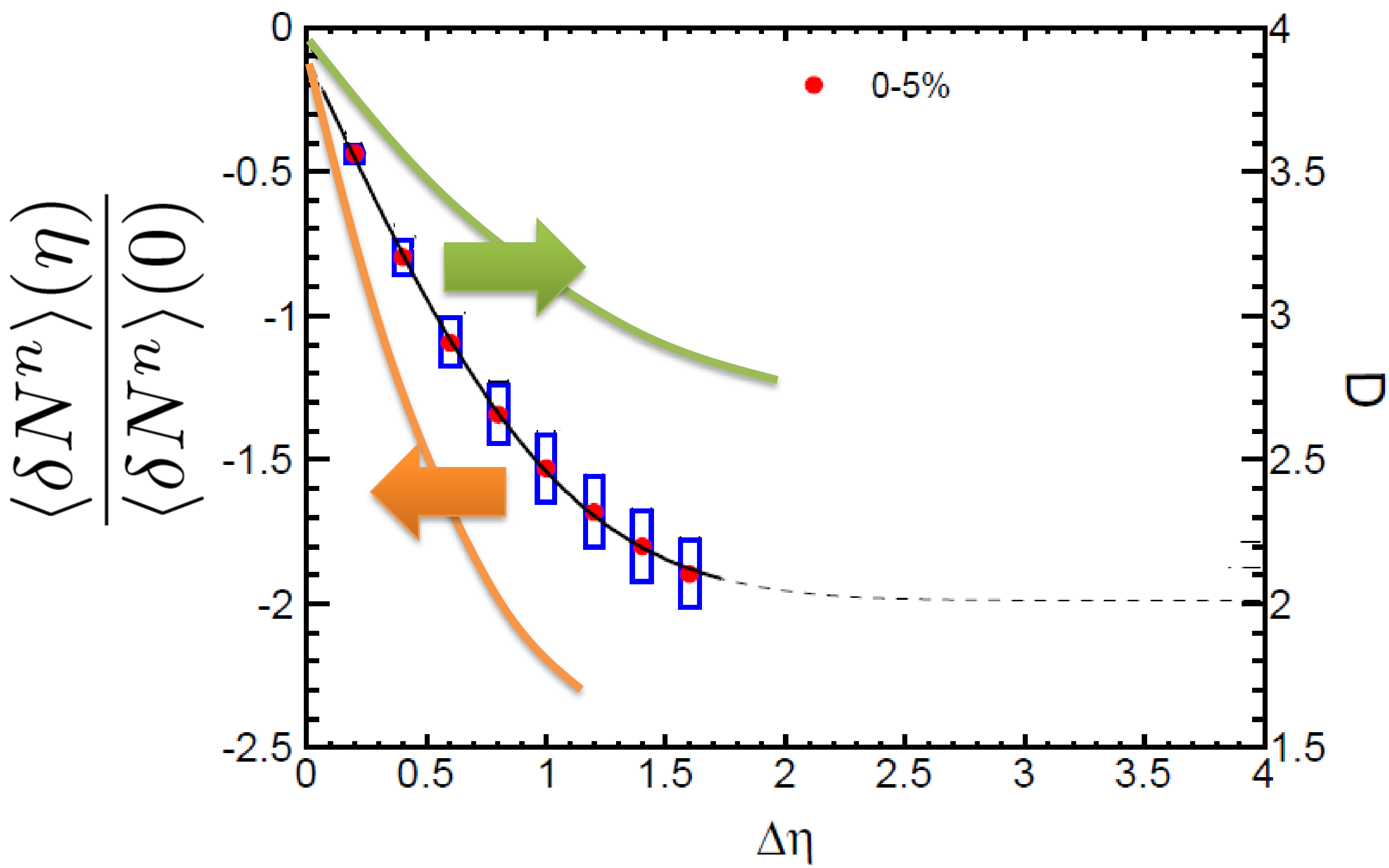


$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \rangle + \frac{1}{4} \langle N_{\mathrm{B}}^{(\mathrm{tot})} \rangle$$

$<\delta N_Q{}^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

Left (suppression) or Right (hadronic)



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\boxed{\partial_\tau n = D\partial_\eta^2 n}$$

Stochastic diffusion equation

$$\boxed{\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)}$$

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$$\partial_\tau n = D\partial_\eta^2 n$$

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$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Conservation Law

$$\partial_\tau n = -\partial_\eta j$$

Fick's Law

$$j = -D\partial_\eta n + \xi$$

Fluctuation-Dissipation Relation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Stochastic force

- Local correlation (hydrodynamics) $\langle \xi(\eta_1,\tau_1)\xi(\eta_2,\tau_2)\rangle \sim \delta(\eta_1-\eta_2)\delta(\tau_1-\tau_2)$
- Equilibrium fluc. $\langle \delta Q(t)^2 \rangle \xrightarrow[t\to\infty]{} \chi_2 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta,\tau)$$

χ_2 : susceptibility

$$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\rangle = \frac{2\chi_2}{D}\delta(k_1+k_2)\delta(\tau_1-\tau_2)$$

$\Delta\eta$ Dependence

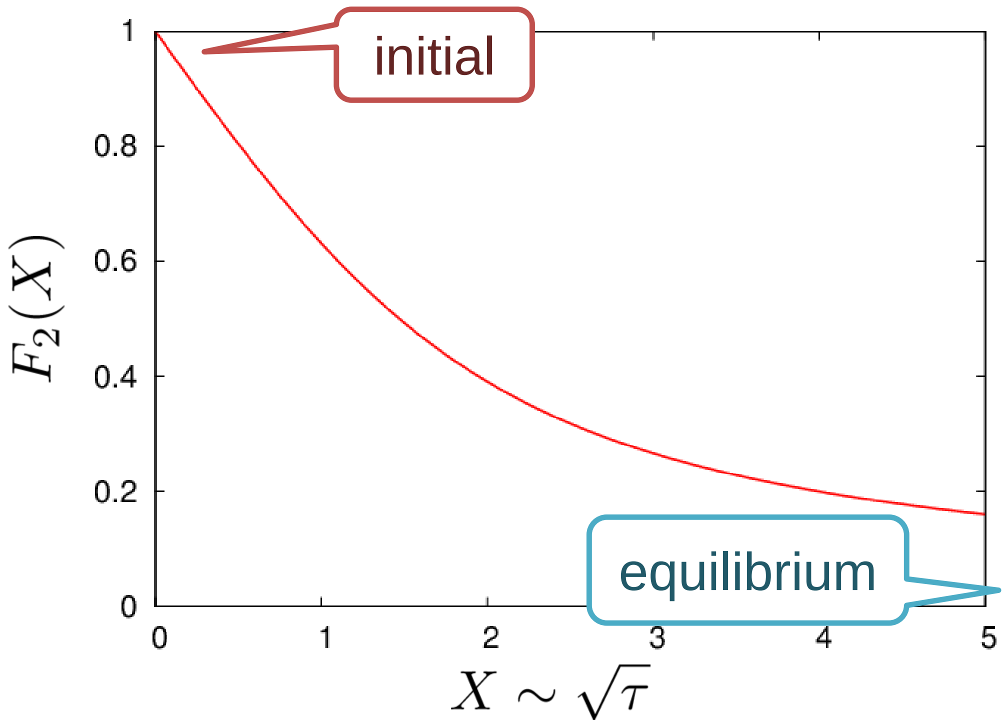
Shuryak, Stephanov, 2001

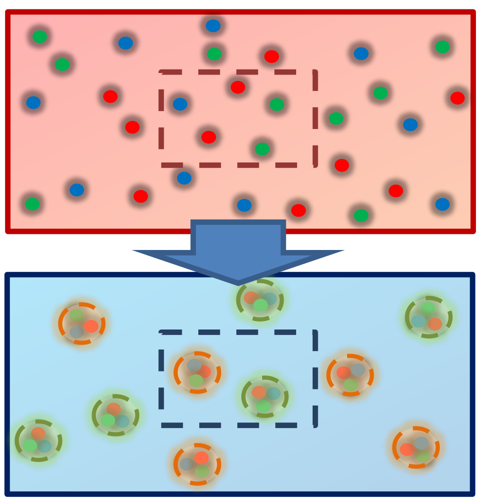
- Initial condition: $\langle \delta n(\eta_1,0)\delta n(\eta_2,0)\rangle = \sigma_2 \delta(\eta_1-\eta_2)$
- Translational invariance

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2(1-F_2(X))}_{\text{equilibrium}}$$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta\, n(\eta,\tau)$$

$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$





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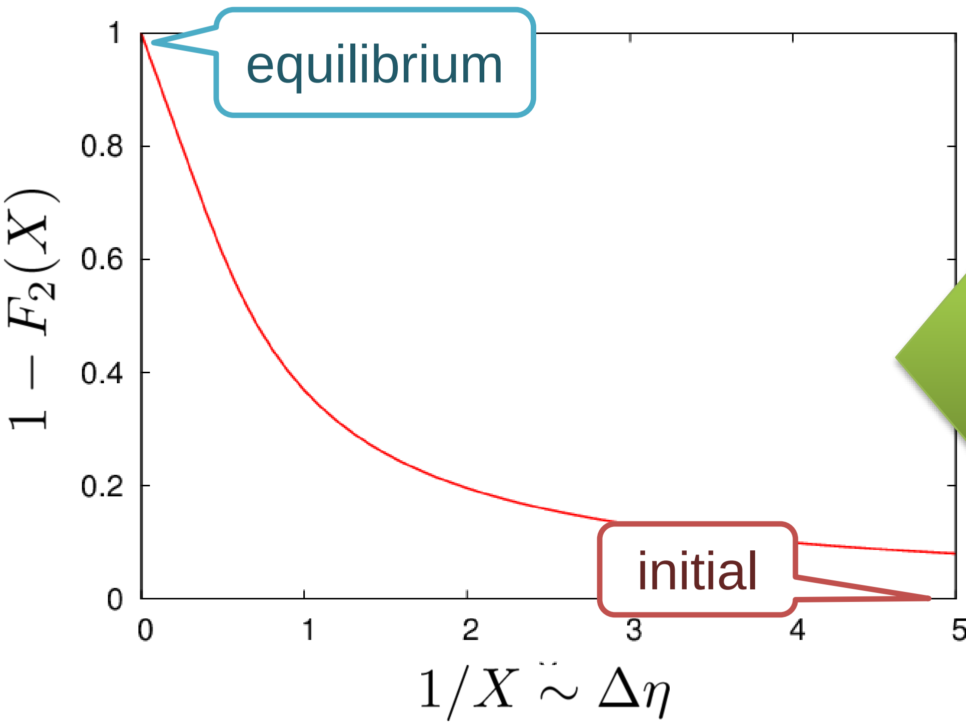
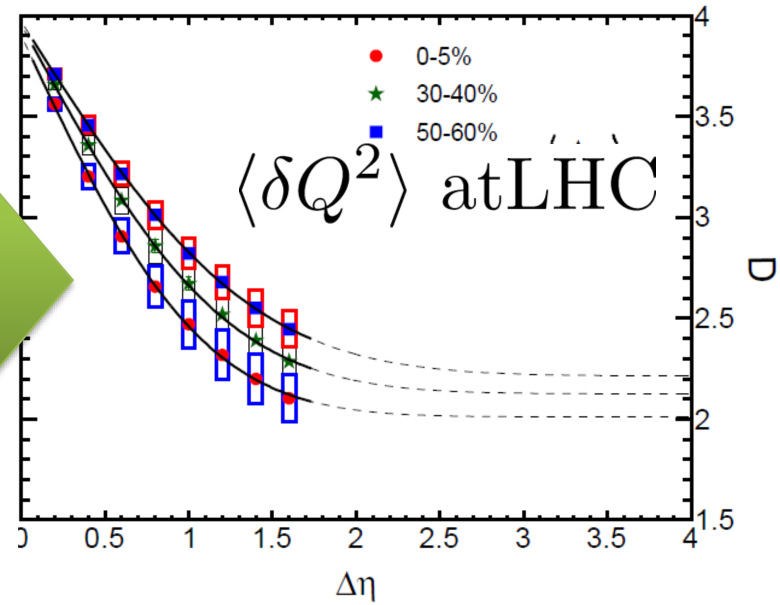




Non-Gaussian Stochastic Force ??

$$\boxed{\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)}$$

Stochastif Force : 3rd order

- Local correlation $\langle \xi(\eta_1, \tau_1)\xi(\eta_2, \tau_2)\xi(\eta_3, \tau_3)\rangle$
  (hydrodynamics)
  $$\sim \delta(\eta_1 - \eta_2)\delta(\eta_2 - \eta_3)\delta(\tau_1 - \tau_2)\delta(\tau_2 - \tau_3)$$
- Equilibrium fluc. $\langle \delta Q(t)^3 \rangle \underset{t \to \infty}{\longrightarrow} \chi_3 \Delta\eta$

$$Q(\tau) = \int_0^{\Delta\eta} d\eta n(\eta, \tau)$$

$$\chi_3 : \mathrm{third-moment}$$

Caution!

- $$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\xi(k_3,\tau_3)\rangle = \frac{\chi_3}{\gamma}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\delta(k_1+k_2+k_3)$$

diverge in long wavelength

$$\times\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$

- No a priori extension of FD relation to higher orders

Caution!

- $$\langle \xi(k_1,\tau_1)\xi(k_2,\tau_2)\xi(k_3,\tau_3)\rangle = \frac{\chi_3}{\gamma}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\delta(k_1+k_2+k_3) \times\delta(\tau_1-\tau_2)\delta(\tau_2-\tau_3)$$

  diverge in long wavelength

- No a priori extension of FD relation to higher orders

- Theorem

  Markov process＋continuous variable

  ➔Gaussian random force

  cf) Gardiner, "Stochastic Methods"

- Hydrodynamics ➡ Local equilibrium with many particles

  ➡ Gaussian due to central limit theorem

Three “NON”s

Physics of non-Gaussianity in heavy-ion collisions is a particular problem.

- **Non-Gaussian**

  Non-Gaussianitiy is irrelevant in large systems

- **Non-critical**

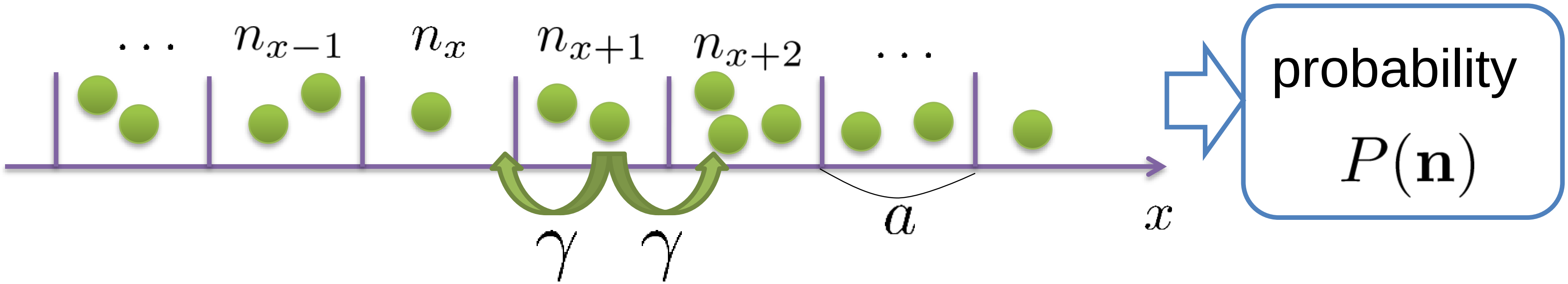
  fluctuations observed so far do not show critical enhancement

- **Non-equilibrium**

  Fluctuations are not equilibrated

Diffusion Master Equation

Divide spatial coordinate into discrete cells



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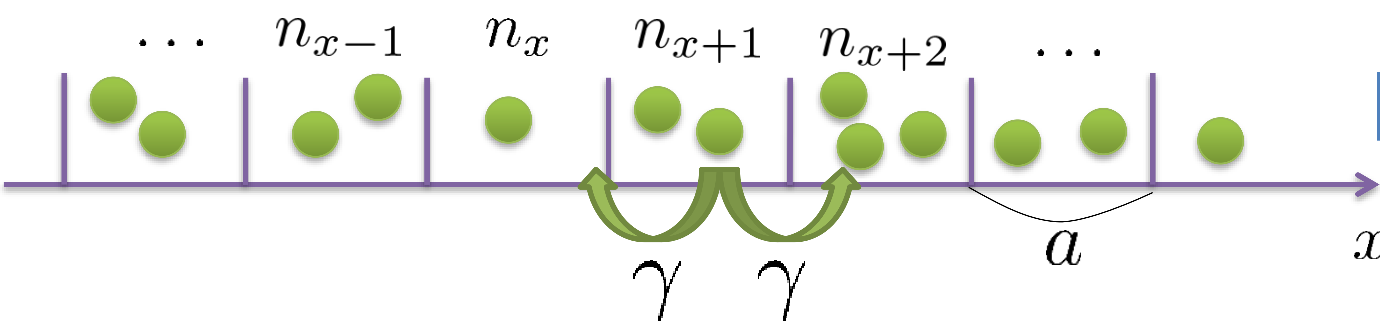


probability

$P(\mathbf{n})$

Master Equation for $P(n)$

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x \Big[(n_x+1)\Big\{P(\mathbf{n}+\hat{x}-\widehat{x+1}) + P(\mathbf{n}+\hat{x}-\widehat{x-1})\Big\}$$

$$-\,2n_x P(\mathbf{n})\Big]$$

x-hat: lattice-QCD notation

Solve the DME **exactly**, and take $a \to 0$ limit

No approx., ex. van Kampen's system size expansion

Solution of DME

1st $$\langle \tilde{n}_k \rangle (t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0 \qquad \omega_k \simeq \gamma a^2 k^2$$

initial

- Deterministic part ←→ diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

- Appropriate continuum limit with $\gamma a^2 = D$

Solution of DME

1st $$\langle \tilde{n}_k \rangle(t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0 \qquad \omega_k = \gamma a^2 k^2$$

initial

Deterministic part ←→ diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

Appropriate continuum limit with $\gamma a^2 = D$

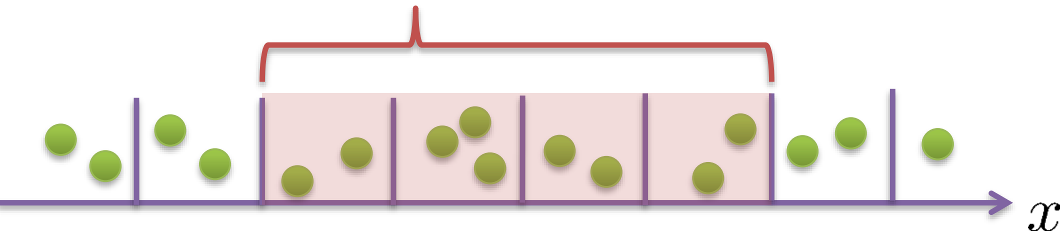
2nd $$\langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle(t) = \langle \tilde{n}_{k_1+k_2} \rangle_0 (e^{-\omega_{k_1+k_2} t} - e^{-(\omega_{k_1}+\omega_{k_2})t}) + \langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle_0 e^{-(\omega_{k_1}+\omega_{k_2})t}$$

Consistent with stochastic diffusion eq.
(for sufficiently smooth initial condition)

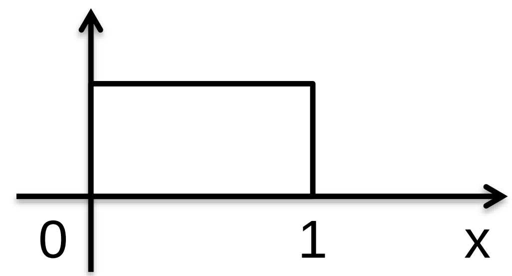
Total Charge in $\Delta\eta$

$$\boxed{Q(\tau) = \int_0^{\Delta\eta} d\eta\, n(\eta,\tau)}$$

$$I(z,X) = \int \frac{dk}{2\pi} e^{-Xk^2} \int_0^1 dx\, e^{ik(x+z)}$$



FT of step func.



$$\langle Q\rangle(\tau) = \int dz \langle n(z)\rangle_0 I_{\Delta\eta}(z,X)$$

$$\langle \delta Q^2\rangle(\tau) = \int dz_1 dz_2 \langle \delta n(z_1)\delta n(z_2)\rangle_0 I_{\Delta\eta}(z_1,X) I_{\Delta\eta}(z_2,X)$$

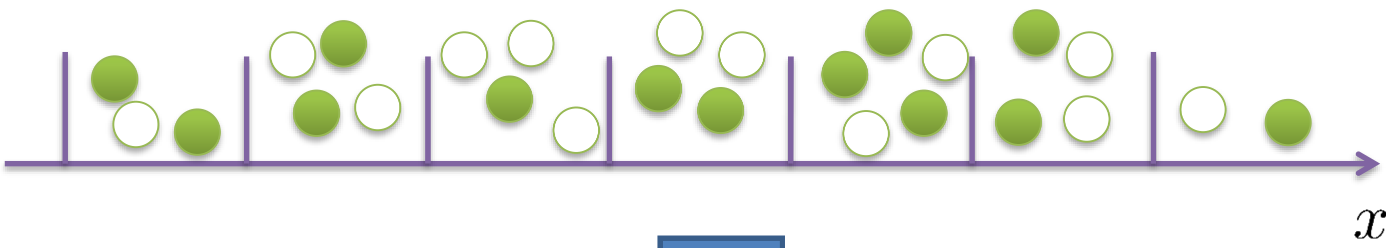
$$+ \int dz \langle n(z)\rangle_0 (I_{\Delta\eta}(z,X) - I_{\Delta\eta}^2(z,X))$$

$$\langle \delta Q^3\rangle(t) = \cdots$$

$$X = \frac{2\sqrt{D\tau}}{\Delta\eta}$$

Net Charge Number

Prepare 2 species of (non-interacting) particles



$$\bar{Q} = \int_0^{\Delta\eta} d\eta \, (n_1(\eta) - n_2(\eta))$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ at freezeout time t

Initial Condition at Hadronization

- Boost invariance / infinitely long system
- Local equilibration / local correlation
- Initial fluctuations

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ $\quad \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\quad \langle Q_{(\mathrm{tot})}^2 \rangle_c$

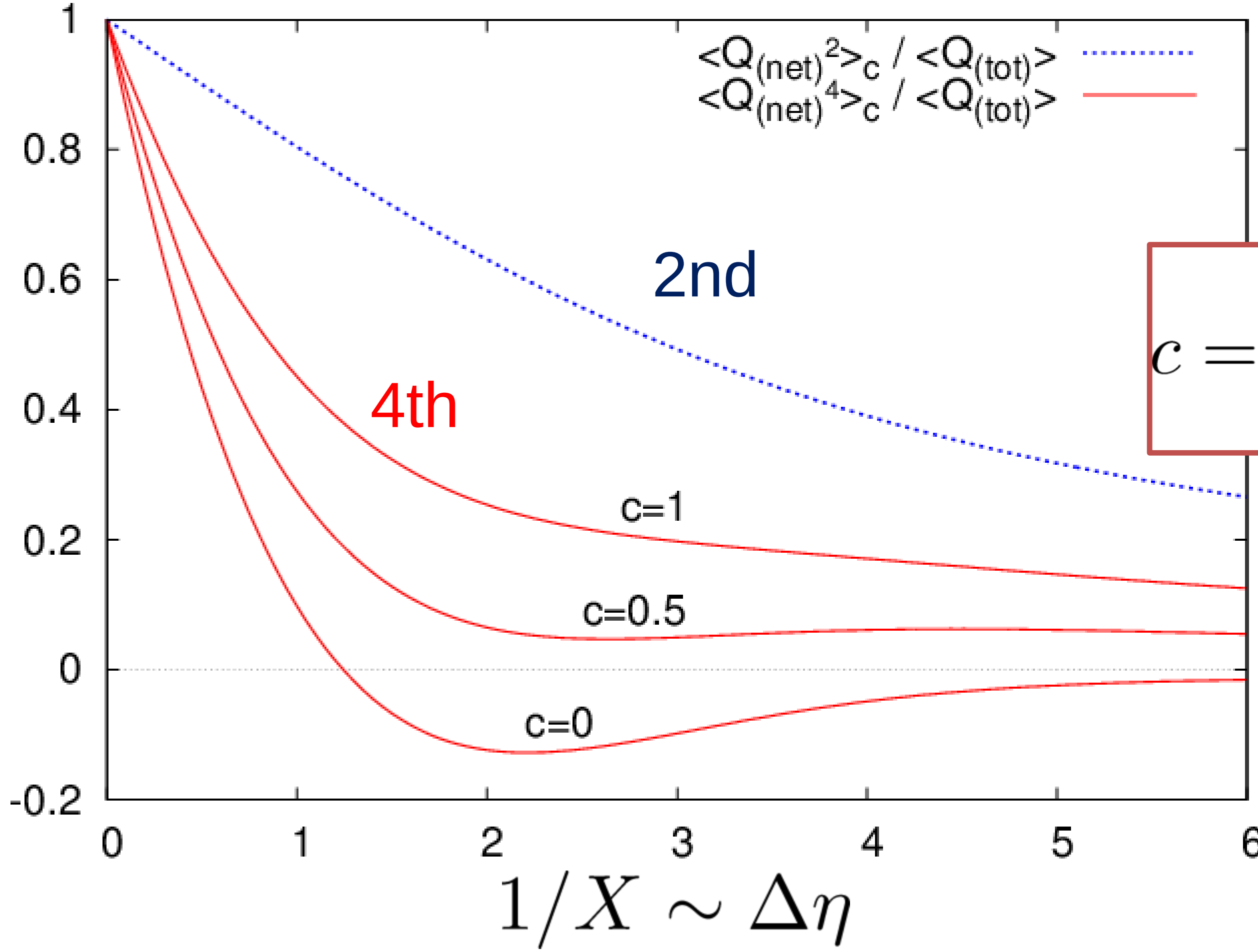
suppression owing to local charge conservation

strongly dependent on hadronization mechanism

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

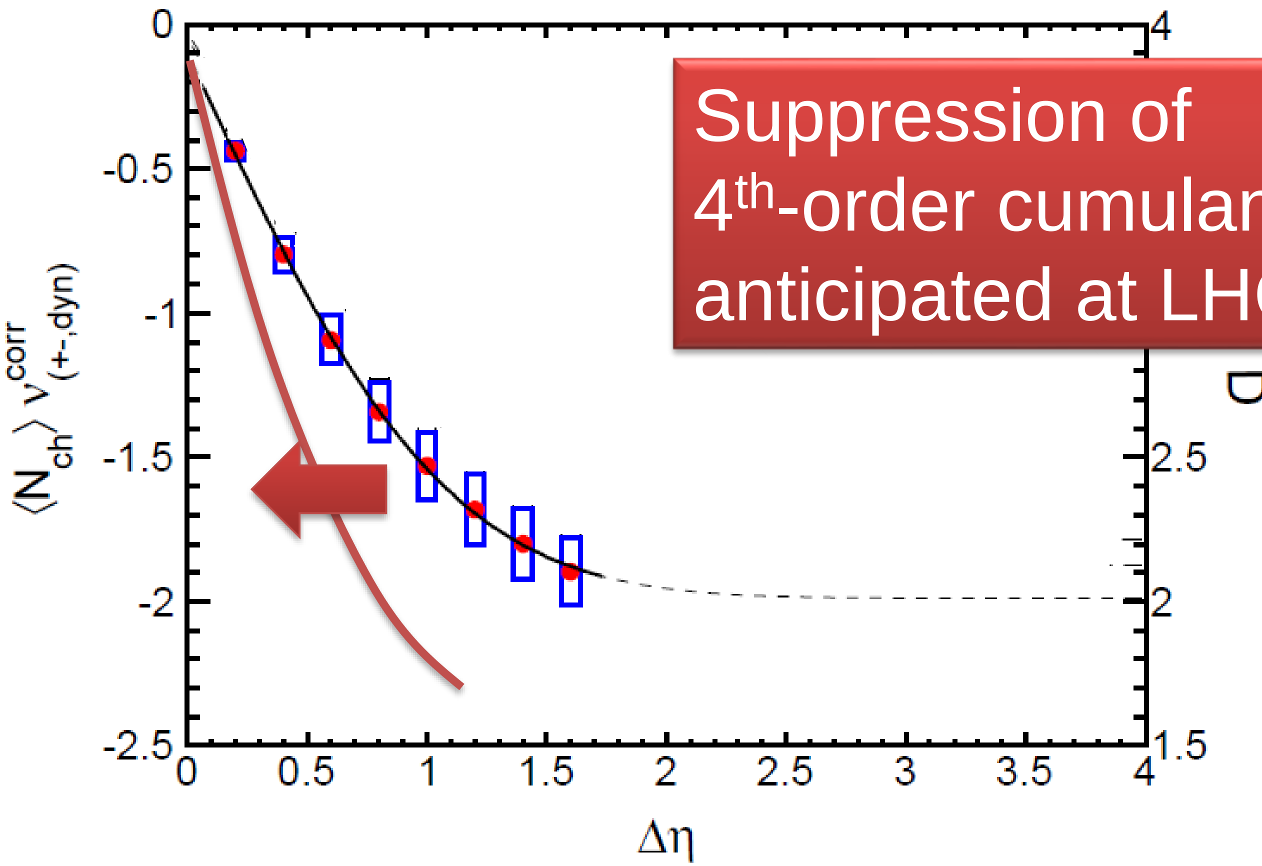
$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$\langle \delta N_Q^4 \rangle$ @ LHC

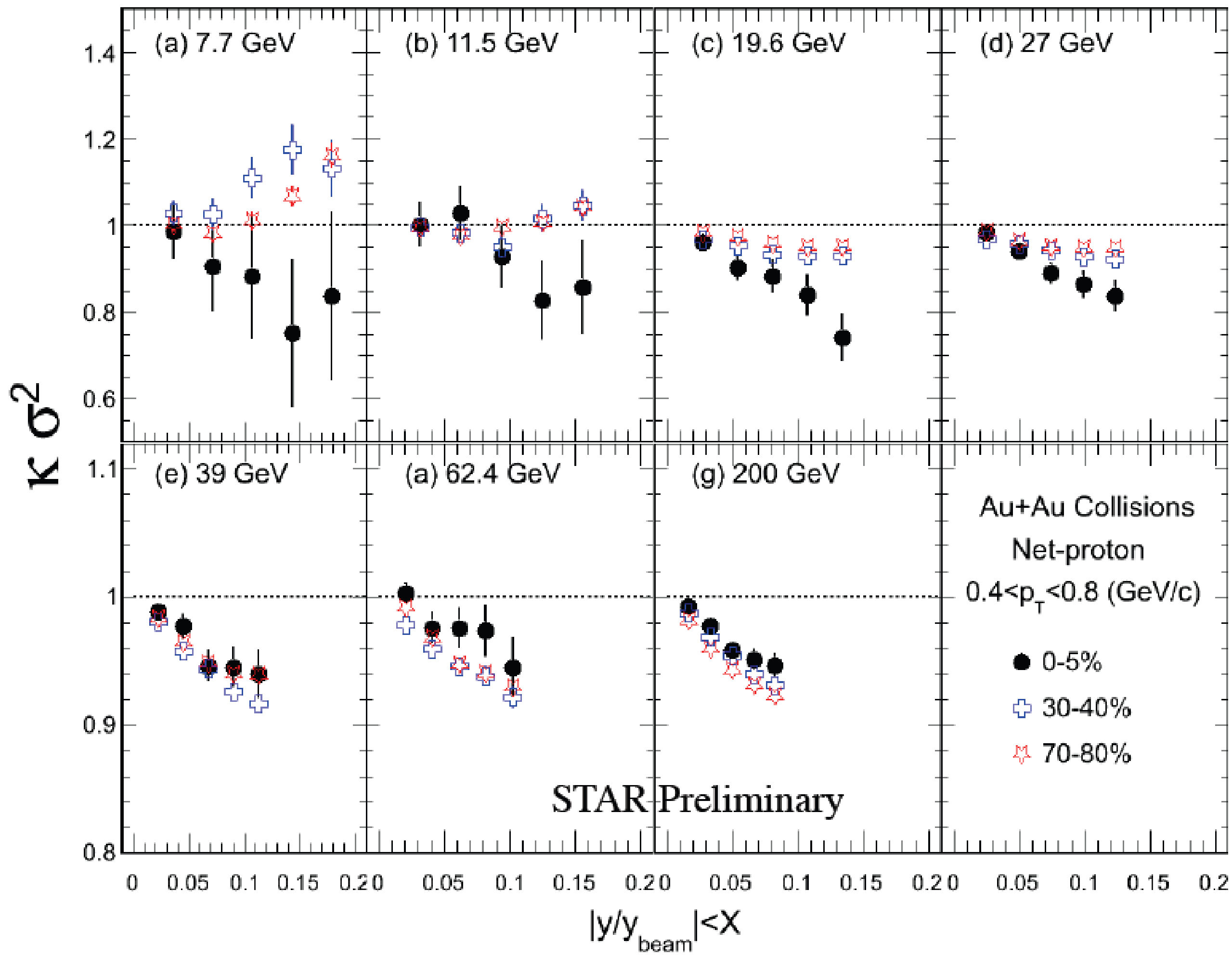
Assumptions
- boost invariant system
- tiny fluctuations of CC at hadronization
- short correlaition in hadronic stage



Suppression of
4^{th}-order cumulant is
anticipated at LHC energy!

$\Delta\eta$ Dependence at STAR

STAR, QM2012



$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$ decreases as $\Delta\eta$ becomes larger at RHIC.

Chemical Reaction 1

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

x: # of X
a: # of A (**fixed**)

Master eq.:
$$\frac{\partial}{\partial t}P(x,t) = k_2 a P(x-1,t) + k_1(x+1)P(x+1,t) - (k_1 x + k_2 a)P(x,t)$$

Cumulants with fixed initial condition $P(x,0) = \delta_{x,N_0}$

$$\langle x(t)\rangle = N_0 e^{-k_1 t} + N_{eq}(1-e^{-k_1 t})$$

$$\langle \delta x(t)^2\rangle = N_0(e^{-k_1 t} - e^{-2k_1 t}) + N_{eq}(1-e^{-k_1 t})$$

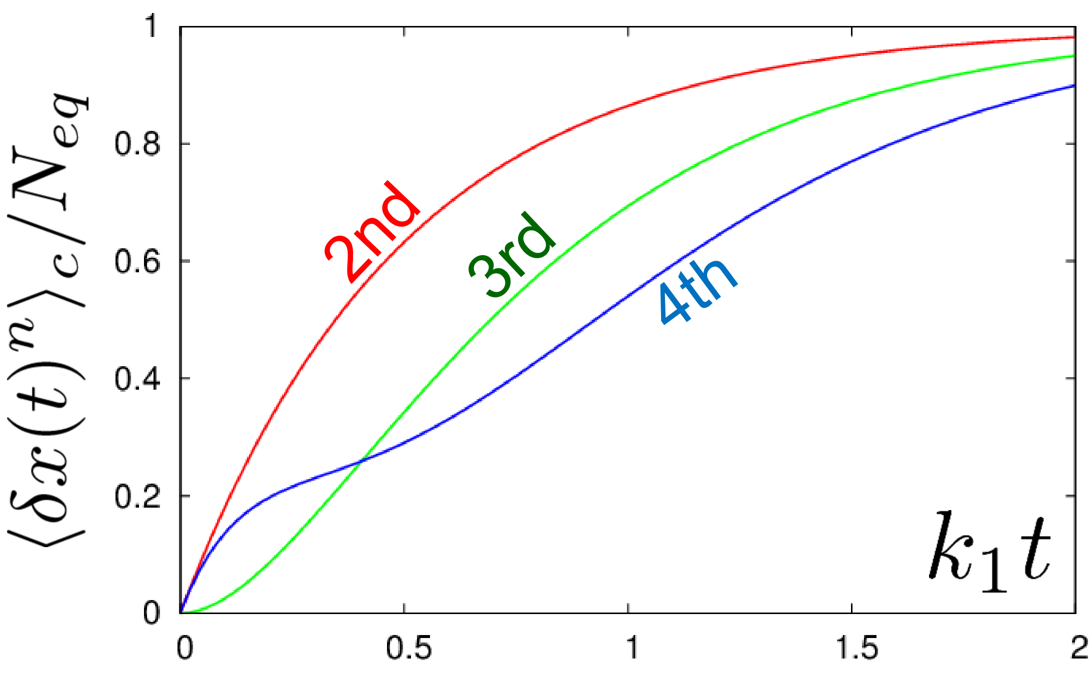
$$\langle \delta x(t)^3\rangle = N_0(e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t}) + N_{eq}(1-e^{-k_1 t})$$

initial equilibrium

Chemical Reaction 2

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

$$N_0 = N_{eq} \Rightarrow \begin{cases} \langle x(t) \rangle = N_{eq} \\ \langle \delta x(t)^2 \rangle = N_{eq}(1 - e^{-2k_1 t}) \\ \langle \delta x(t)^3 \rangle = N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t}) \end{cases}$$

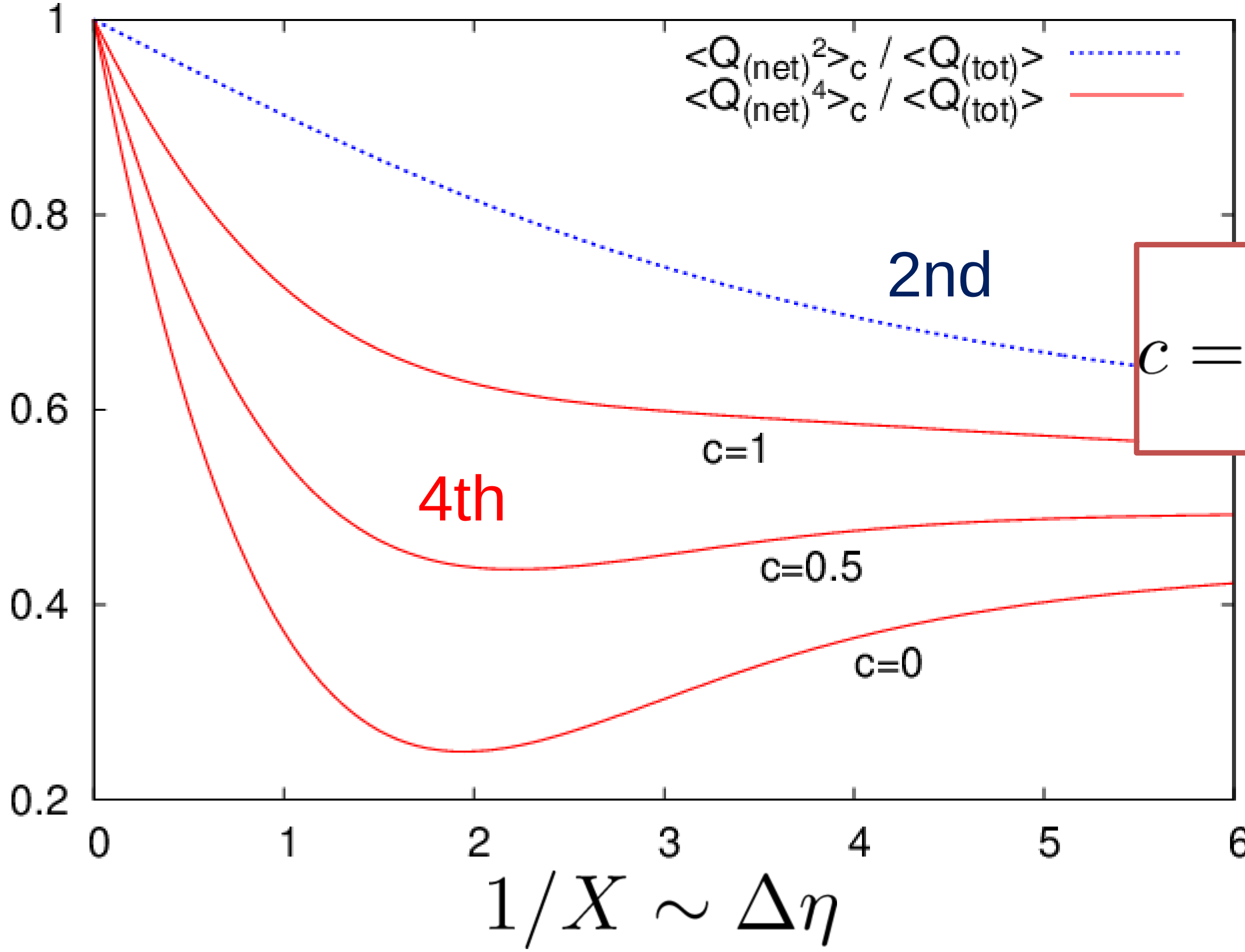


Higher-order cumulants grow slower.

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0.5 \langle Q_{(\mathrm{tot})} \rangle$$



$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

Summary

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2\rangle_c,\ \langle N_B^2\rangle_c,\ \langle N_Q^4\rangle_c,\ \langle N_B^4\rangle_c,\ \langle N_{ch}^2\rangle_c,\cdots$$

Physical meanings of fluctuation obs. in experiments.

Diagnozing dynamics of HIC

- history of hot medium
- mechanism of hadronization
- diffusion constant

Summary

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2\rangle_c,\ \langle N_B^2\rangle_c,\ \langle N_Q^4\rangle_c,\ \langle N_B^4\rangle_c,\ \langle N_{ch}^2\rangle_c, \cdots$$

Physical meanings of fluctuation obs. in experiments.

Diagnozing dynamics of HIC

- ☐ history of hot medium
- ☐ mechanism of hadronization
- ☐ diffusion constant

Search of QCD Phase Structure

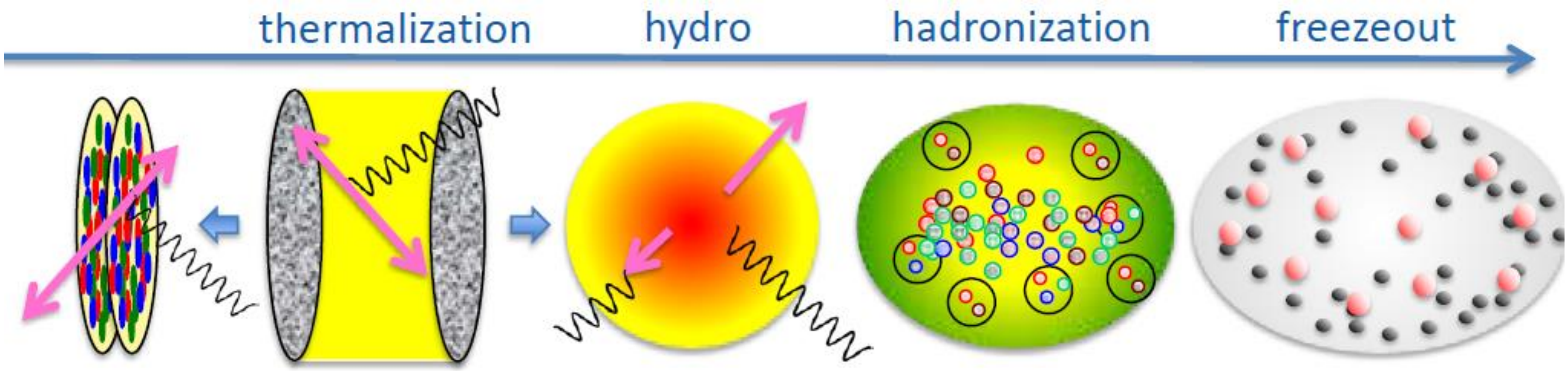
Open Questions & Future Work

- Why the primordial fluctuations are observed only at the LHC, and not the RHIC ?
- Extract more information on each stage of fireballs using fluctuations

- Model refinement
  - Including the effects of
    nonzero correlation length / relaxation time
    global charge conservation

  - Non Poissonian system ← interaction of particles

Evolution of Fluctuations



Fluctuation in initial state

Time evolution in the QGP

approach to HRG by diffusion

experimental effects
particle missID, etc.

volume fluctuation